On Bank Disclosure and Subordinated debt*

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Abstract

Following Pillar 3 of the new Basel Capital Adequacy Proposals (Basel II), we analyse the effects of disclosure in the banking sector in a stylised setting of delegated portfolio management. We first consider the interaction between the shareholder and the manager of a bank - the manager has to exert risk monitoring effort in order to decrease the bank’s probability of default. Disclosure is captured through a signal about the manager’s effort and it is shown that the shareholder desires full disclosure (a perfect signal) so as to implement the first best level of effort. We then introduce a third stakeholder that has fixed claims on the bank, the debt-holder. This agent introduces a counteracting effect: the shareholder may not desire full disclosure anymore given that a lower level of disclosure allows the bank to improve its perceived probability of default, which in turn decreases its financing costs. This implies that

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subordinated debt itself may not increase the soundness of the banking sector unless it is accompanied by measures of compulsory disclosure.

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It seems clear that adding more and more layers of arbitrary regulation would be counter productive. We should, rather, look for ways to harness market tools and market-like incentives wherever possible, by using bank’s own policies, behaviors, and technologies in improving the supervisory process

GREENSPAN (1998)

1. **INTRODUCTION**

The purpose of this paper is to analyse using a simple model, the effect of an increase in disclosure on the banking system. And how subordinated debt enhances or counteracts this effect. Following Pillar 3 of the new Basel capital adequacy proposals (Basel II), we want to assess whether there exists a role for market discipline to make prudential regulation more effective. And, in particular, whether effective disclosure of bank’s private information would be conducive to more market discipline.

We have two main issues at hand. First, how market discipline affects the behaviour of the bank’s management; and, second, how disclosure affects market discipline. Three main players appear in our model, the manager of the bank, the shareholders and the debt-holders. From a policy maker’s perspective it seems sensible to focus on the role played by the debt-holders once information is disclosed because their interests are more in line with those of the regulator. However, we also study the strategic interactions that such an increase of information may have on the behaviour of the other agents.

As a first approximation, we expect that more disclosure will enable the market to better assess the risk the bank is taking. It will eliminate noise in the agent’s beliefs about the bank’s fundamentals. Similarly, we expect that subordinated debt will incentivise the bank to better monitor its risks so that its financing costs are also reduced. However, the interaction between both disclosure and subordinated debt remains unclear to us.
In the rest of this section we describe our own view of the underlying arguments embedded in Basel II’s third Pillar. Subsequently we sketch out the model we intend to use to examine the issue of market discipline and disclosure. We finish this section with a review of the existing literature on this topic, and how our model builds on this earlier work.

In sections 2 to 4 we present the formal model setting out the various actors involved and their respective constrained optimisation problems. Section 2 articulates the principal-agent problem between shareholders and bank management. Section 3 introduces a role for information disclosure and examines how the previous agency problem is affected. Section 4 restates the problem where the investor of the bank is characterised by a debt-contract. That is, an investor who receives a fixed payoff in case of non-default. Finally, section five offers some preliminary conclusions.

1.1. **Basel New Adequacy Proposals**

Market discipline performs an essential role in ensuring that the capital of banking institutions is maintained at adequate levels. Effective public disclosure enhances market discipline and allows market participants to assess a bank’s capital adequacy and can provide strong incentives to conduct their business in a safe, sound and efficient manner.

**BASEL COMMITTEE ON BANKING SUPERVISION (2000).**

The third pillar in the Basel New Capital Adequacy Framework addresses the topic of market discipline. Market discipline is the term that describes the monitoring and control of a firm’s management by outside stakeholders to ensure that they act in their best interest. We want to address whether an increase in the information given by the bank about its risk assessment would increase the role of market discipline in making prudential regulation more effective. There are three main issues that follow from the Basel proposals:

1. **An agency problem between the bank and the market participants.** A bank, its shareholders and its debt-holders have different incentives and any change in the regulations may have unexpected consequences due to the strategic interactions between these three stakeholders.
2. **A common agency problem.** Given that the proposal is formulated in an international framework we should realise how the existence of multiple regulators
(central banks) that simultaneously attempt to influence the behaviour of a single privately informed agent (private bank) is affected by a new regulatory environment.

3. Information efficiency and allocation efficiency. It is not clear whether more quantity and/or quality of information is desirable. Examples such as the Hirshleifer effect\(^1\) or Morris-Shin (2001) are concrete cases in which the conjecture that more information is beneficial is shown to be false. Related to this question, it is not clear whether informational efficiency implies an optimal allocation of resources\(^2\).

In this paper we will focus on the first aspect. The common agency problem would be a natural extension of our model if we introduced regulators to our model. The last issue, however, has a much broader scope than the topic analysed here. More importantly, there is a considerable existing literature on this issue although as yet there still is no consensus. In any case, our analysis (in relation to Basel II) is aimed at the impact of disclosure on the viability of the banking system rather than efficiency per se.

1.2. Informal presentation of the model

Given the increased complexity of today’s banking sector there exists a body of opinion that argues that «adding more and more layers of arbitrary regulation would be counter productive» (Greenspan 1998). Instead we should create the incentives for improving the supervisory process through the market. In this sense, more information about the internal behaviour of the bank can complement formal prudential regulation by encouraging banks to more effectively manage the risks they take.

We develop a model studying the interaction between a shareholder and a bank manager and, later, between these two and a debt/bond investors\(^3\). The shareholder

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\(^1\) Too much information may eliminate valuable insurance opportunities.» (1973)

\(^2\) Messner and Vives (2000) characterise the divergence between informational and economic efficiency in a rational expectations competitive market with asymmetric information about the costs of production.

\(^3\) Note that the incentives of the shareholder and manager are totally aligned vis-a-vis the debt-holder: they need him to finance a project and the lower the return they offer him the better because this will reduce the bank’s financing costs.
(principal) will offer a contract to the manager anticipating the reaction from the other agents and acting accordingly - this is the role of market discipline. Disclosure will play a significant role affecting the information on which the contract is written. Note that disclosure, per se, influences the information but will not affect the actions of the players if market discipline is not effective.

We will assume that the manager, in a previous stage, has conducted all relevant activities aside from the risk management tasks (assessments of new projects and monitoring old ones). Hence the modelled activities of the manager are focussed on the ones that increase the soundness of the bank. Two external agents interact with him. First, a shareholder who relies only on disclosed information to evaluate the internal functioning of the bank, and has to write a wage contract with the manager in order to align both of their objectives. Second, an investor that also holds some public information about the bank and is offered a bond at a given interest rate and decides how to allocate his wealth between the bank’s bond and a risk free asset.

In our setting an increase in disclosure means an increase in the precision of information rather than its quantity. Therefore we abstract from the debate about what concrete measures relatives to disclosure should be undertaken. Instead, we study the effect of «meaningful disclosures of [the bank] risk exposures»4.

Note that the degree of disclosure (relates to the degree of misperception about the probability of bank’s default) will be exogenously given. However, we will assess the preferences of the shareholder towards its increase.

1.3. Related literature

Two main branches of the economics literature refer to the issue at hand. First, the directly linked literature on disclosure and market discipline and second the microeconomic and finance literature on moral hazard and delegated portfolio management.

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Disclosure

The incentives of management to disclose some particular inside information has been analysed in a number of previous papers. We will highlight the ones related to the banking industry. Cordella and Yeyati (1998) conclude that whenever the bank has complete control over its risks, the complete disclosure framework (in contrast to the «no disclosure» one) yields a higher stability (i.e. a lower probability of default). A controversial issue is the costs associated with disclosure. On the one hand, there are social earnings in disclosure due to the saving in real resources that would otherwise be invested in acquiring privately the same information. On the other hand, the release of some inside information may hurt the firm’s competitive position and result in higher costs of disclosure (Diamond 1985). Shaffer (1995) points out that the concrete costs and benefits should be analysed in every case.

Nevertheless, the costs of monitoring rather than the real resources costs associated with disclosure drive our study. In this sense Hyytinen and Takalo (2000) paradoxically point out how the monitoring effort, and its implied decrease in profits, reduces the incentives of the shareholders to avoid bankruptcy (since shareholders want to avoid bankruptcy as long as it implies a loss in future profits).

Recent work focuses on the role of market discipline (Blum 2000) and how the disciplining role of depositors is influenced by the timing of when the bank sets its risk level. In any case, to allow depositors to discipline the bank, state guarantees should be diminished —Freixas (1999) and Danielson et al. (2000) suggest that government insurance act as a disincentive for safe banking risk strategies.

5 Shaffer (1995) gives an example (Truth in Savings, FDICIA 1991) where the costs outweighed the benefits.

6 The view that compulsory disclosure may have few real costs might be inferred from the Securities and Exchange Commission Study (1998) report which claims that «disclosures are not terribly costly». Similarly Danielson, Jorgensen and de Vries (2000) suggest that «banks employ dual risk management systems, an elaborate system for internal control, and a scaled-down version for reporting purposes» meaning that enforced disclosure may impose few additional costs on banks.
Moral hazard and delegated portfolio management

At the root of moral hazard lies the idea of asymmetric information: the principal only holds some imperfect signal of the manager’s action so that he can not force any particular action. And, in our setting, the manager’s action is related to the investment of the principal’s wealth.

Usually, the signal of the agent’s action is the amount of realised profits. To avoid any confusion, in the model below we interpret increased disclosure as a better signal of the bank’s risk profile rather than a decrease in the risks inherent in profits. Hence, disclosure does not mean a lower risk exposure of the bank’s activities but instead a better knowledge of the assumed risk. Hermalin and Katz (1996) develop a model where both concepts (information and risk) are clearly differentiated.

We should differentiate two effects of the agent’s actions in a setting of delegated portfolio management: (i) when they affect the expected return (first order stochastic dominance); (ii) when they affect the risk or the variance of the returns (second order stochastic dominance). In our model the risk mitigation effort will only decrease the probability of failure -second effect. Palomino and Prat (1999) mix both types of models but their manager has just the option of spending an exogenous amount of effort to access the set of feasible portfolios (or in an extended version to access a dominated set of portfolios). In our case, the effort is a continuous variable that summarises the manager’s work to move towards a more efficient point under the risk-return frontier.

Finally, related to our work, is the paper by Ackerman et al. (1999) where they analyse empirically how the structure (incentive contracts, compulsory disclosure, etc.) of hedge funds and mutual funds affect their performance.

2. The agency problem

In a context of delegated portfolio management we want to assess how the agency problem encourages greater risk taking by the agent. Our analysis relies on the fact that management and ownership are separate activities with, consequently, different incentives and objectives. The contracts will be constrained -given the asymmetry of information- so that the shareholder will not be able to dictate to the manager which action to implement. Once the incentives for the manager are set through the salary contract (a linear function contingent on the observed profits), the shareholder will just extract the net benefits resulting from the manager’s actions.
The model presented below is based on Holmstrom and Milgrom (1987).

2.1. *The bank*

We analyse an economy with a single bank. The interpretation is that it represents the financial intermediary industry. The bank is summarized by two parameters: an expected return and a risk variable. We will assume that the return of the bank is normally distributed with mean $\mu$ and variance $\sigma^2_x$. A higher monitoring and controlling of the bank’s risk reduces the variance on profits ($\sigma^2_x$). We summarise the invested resources in those activities as risk mitigation effort ($e \geq 0$). Embedded in this idea is that it is costly to invest in an efficient project (a project on the efficient risk-return frontier).

Thus, the available technology is given by the strictly decreasing function $\sigma^2_x(e)$. We will also assume that the marginal efficiency of the risk mitigation effort is decreasing (i.e. the second derivative of $\sigma^2_x(e)$ is positive). Graphically the curve $\sigma^2_x(e)$ could look like:

![Graph showing the relationship between $\sigma^2_x$ and $e$.]
The next list summarises the previous assumptions:

- \( x \sim N(\mu, \sigma_x^2) \)
- \( e \in [0, \infty) \)
- \( \sigma_x^2(e) \in [0, \infty] \)
- \( \frac{\partial \sigma_x^2}{\partial e} < 0 \)
- \( \frac{\partial^2 \sigma_x^2}{\partial e^2} \geq 0 \)

2.2. *The manager*

Moral hazard arises from the fact that the shareholder (principal) hires a manager (agent) with specialised skills to manage the bank because he cannot take the direct control of it given that this task is too complicated or too costly. The manager undertakes a project with a non deterministic return and is the one that can invest effort to reduce the randomness of such a project. The agent faces a quadratic disutility of effort. The principal proposes a salary contract contingent on some observable variable (the realised profits of the bank) that is indirectly linked to the agent’s invested effort with the aim to align the manager’s incentives with his own.

The manager is risk averse with constant absolute risk aversion (CARA) utility and index of risk aversion \( r^2 \). Ex-ante he maximises his expected utility over the wage \( w(x) \) and effort level \( e \):

\[
\max_e \mathbb{E}\left\{ U\left( w(x) - \frac{e^2}{2} \right) \right\}
\]

If the resulting value from such a maximisation is not lower than his reservation utility (i.e. the opportunity cost of labour which is normalised to minus one) he will accept the job. That is, he accepts the job if and only if

\[
\max_e \mathbb{E}\left\{ U\left( w(x) - \frac{e^2}{2} \right) \right\} \geq -1
\]

Note that, as we are working with CARA utility functions and normally distributed profits, we can transform all our expressions to their certainty equivalent: \( \mathbb{E}[U(\cdot)] \rightarrow \mathbb{E}(\cdot) - \frac{1}{2}r^2\text{Var}(\cdot) \). Note that the initial reservation utility of minus one will be now equivalent to zero.
2.3. **The shareholder**

As explained above, the shareholder offers a wage contract to the agent contingent on the realised profits and extracts the net benefits of the business activities \((x-w(x))\). We will also assume that he is risk averse with CARA utility and index of risk aversion \(r_2\). As noted before, we will work with its certainty equivalent: \(E\{V(\cdot)\} \to E(\cdot) - \frac{r_1}{2} \text{Var}(\cdot)\).

We will concentrate our study on linear contracts, i.e. \(w(x) = w^? + w_1(x)\).

When maximising his expected utility, the principal will anticipate the reaction of the manager to the offered contract and will take into account that the manager will accept the job if and only if he is offered at least his reservation utility. Hence his maximisation will be subject to two constraints: (i) the manager will only accept the job if he is offered at least his reservation utility (Participation Constraint, PC); (ii) once the shareholder has offered the contract to the manager, he will not be able to enforce any action so he will internalise the fact that the manager will maximise a different utility function (Incentive Constraint, IC).

Formally, the shareholder’s maximisation program reads as follows:

\[
\max_{w(x), e} E\{V(x-w(x))\}
\begin{aligned}
& \quad \text{s.t.} \\
& \quad E\left[U\left(w(x) - \frac{\epsilon^2}{2}\right)\right] \geq 0 \\
& \quad e \in \arg \max_{e} E\left[U\left(w(x) - \frac{\epsilon^2}{2}\right)\right]
\end{aligned}
\]

Note that in our case the set of values that maximise the utility function of the agent is a singleton. Thus, from now on we will write \(\leftarrow\) instead of \(\in\).

2.4. **First Best**

As a benchmark case, we will first want to know how much effort would be invested if the agent’s action was verifiable. That is, in the absence of the agency problem.

In that case, the shareholder will maximise his ex-ante utility subject to solely the participation constraint of the agent:
\[ \max_{w_0, w_1, e} E\{V(x - w(x))\} \]
\[ s.t. \ E\left[U\left(w(x) - \frac{e^2}{2}\right)\right] \geq 0 \]

One can easily show that the solution of this program (first best solution) is \(^7\): \[
\begin{align*}
w_1^* &= \frac{r_1}{r_1 + r_2} \\
e^* &= -\frac{\partial \sigma}{\partial e} \cdot \frac{r_2}{2} \cdot w_1^* \\
w_0^* \text{ is s.t. PC is binding}
\end{align*}
\]

Note that using the implicit function theorem we have that \(\frac{\partial e^*}{\partial r_1}, \frac{\partial e^*}{\partial r_2} > 0\). The intuition behind this result is straightforward: the first best level of effort will be higher the more risk averse any of the agents is \(^8\).

The first best level of effort is the level of effort that the shareholder would optimally induce.

\subsection*{2.5. Second Best}

Given that the only verifiable variable is the realised profits, the principal will write a contract trying to align the manager’s incentives with his. In this new program the principal will not only have a participation constraint but also an incentive compatibility one (through which he will anticipate the behaviour of the agent given the offered contract).

\(^7\) For a detailed proof see the appendix.

\(^8\) Note that this may raise an adverse selection in the sense that the shareholder will try to hire the manager with an index of risk aversion \(r_2\) that maximises his objective function. Such issue is outside the scope of this paper.
The program of the shareholder reads as follow:

$$\max_{w_0, w_1, e} E\{V(x - w(x))\}$$

s.t. $E\{U(w(x) - e)\} \geq 0$, $e = \arg \max_{\bar{e}} E\{U(w(x) - \bar{e})\}$

Using the First Order Approach and Lagrangian techniques it can be easily proved that the solution of this program (second best solution) is:

$$w_1^{**} = (w_1^*)^{-1} + \gamma \frac{\partial^2 \sigma_x^2}{\partial e} \cdot \frac{r_2}{r_1 \cdot \sigma_x^2}$$

$$w_0'$$ is s.t. PC is binding

where denotes the Lagrange multiplier of the Incentive Compatibility constraint.

It is important to realise that the PC will still be binding. Hence, the manager will always receive his reservation utility because the shareholder will perfectly anticipate the invested effort by the agent and will remunerate him just enough for him to be participating. From the previous maximisation we know that the invested effort will be:

$$e^* = - \frac{\partial \sigma_x^2}{\partial e} \cdot \frac{r_2}{2} \cdot (w_1^{**})^2$$

The salary has been written so that we can easily realise that whenever the Lagrange multiplier $\gamma$ is zero the second best solution will coincide with the first best. As we know, a Lagrange multiplier can also be interpreted as the shadow price of its correspondent constraint. Having the shadow price of a constraint equal to zero is synonymous to having the constraint not restricting the objective function. In our case, this translates in replicating the first best level of effort. Nevertheless, the following proposition tells us that this is never the case.

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9 For a detailed proof see the appendix.
Proposition 1. *The second best optimal effort is always lower than the first best, \( e^{**} < e^* \).*

*Proof* Similarly to the proofs in Holmstrom ‘79, we need to show that the Lagrange multiplier \( \lambda \) is strictly higher than zero. Looking to the derivative of the Lagrangian with respect to \( e \) we realise that \( \lambda \) has to be greater than zero (\( \lambda > 0 \)). Finally, it is immediate to see that \( \lambda \) cannot be zero in order to have a solution to the system of equations.

**Corollary 1.** *The agency problem increases the risk of the bank.*

*Proof* Given that there exists an agency problem the invested level of effort will be lower than the first best. As \( \sigma \) is decreasing in effort, the resulting variance will be higher.

We can observe that, as it is usually highlighted in the standard moral hazard setting, the principal incurs an extra cost when providing incentives to the manager. And this makes the first best situation too costly to enforce so that it is no longer optimal to induce that level of effort.

### 3. Disclosure

The model described so far is one in which the shareholder can only infer the manager’s behaviour through the realised profits. This does not allow him to write very accurate contracts so that only an allocation far from first best can be achieved. On the other hand, the manager cannot be punished very seriously when deviating from the first best situation because bad management cannot be distinguished from bad luck. More explicitly, as the risk of the undertaken projects is not directly observed by the principal, he cannot realise if an event with a very small probability has occurred (bad luck) or if that event was much more probable than he thought given that the manager was not investing sufficient effort to monitor and control the bank’s risks (bad management).

We can argue that in the real world the shareholder would like to hold more information so that he can write better contracts. In this sense, greater disclosure is synonymous with an increase in the precision of the additional signal about the manager’s effort. As noted above, we abstract from any issue related to the effectiveness of disclosure.
We will define $y$ as a signal of the action undertaken by the manager. And we will assume that such a signal takes only values higher than the effort invested by the manager. This assumption plays no important role in this section but it will be crucial later on when we will introduce the debt-holders.

Formally we will assume that $y$ is a positive tailed normal distribution with parameters $e$ and $\gamma$: $y \sim N(e, \gamma^2)$. Note that its density function will be twice the density of a $N(e, \gamma^2)$ and it will be restricted to values above $e$. From the latter fact it can easily be proven that there will not be any difference in the certainty equivalent utilities.

Note that the new wage contract will be contingent on the realised profits and also on the new disclosed signal $y$: $w(x, y) = w_0 + w_1 x + w_2 y$. Finally, also note that an increase in disclosure will be a decrease in the variance of the disclosed signal ($\gamma^2$).

We can now replicate our earlier study and analyse the effects of adding this additional information about the manager’s action.

### 3.1. First best with disclosure

As before, the shareholder will maximise his utility in the absence of the agency problem (i.e. subject only to the participation constraint of the agent and contracting on the invested effort). Hence, the imperfect signal of the effort will not add any useful information to the manager. It is immediate to see that $w_2 = 0$ and that the first best with disclosure coincides with the first best (without disclosure).

### 3.2. Second best with disclosure

Analogous to the previous section, the principal will write a contract contingent on the realised profits and the disclosed signal trying to align the manager incentives with his. In this new program the principal will not only have a participation constraint but also an incentive compatibility one (through which he will anticipate the behaviour of the agent given the offered contract).

Analogously, the program of the shareholder reads as follow:
\[
\max_{w_0, w_1, w_2, e} E\{V(x - w(x))\}
\]
\[
s.t. \left\{\begin{array}{l}
E\{U(w(x), e)\} \geq 0 \\
e = \arg \max_{\hat{e}} E\{U(w(x), \hat{e})\}
\end{array}\right.
\]

Using the First Order Approach and Lagrangian techniques it can be easily proved that the solution of this program (second best solution) is:

\[
\begin{align*}
w_1^D &= (w_1^*)^{-1} + \gamma^D \frac{\partial \sigma_x^2}{\partial e} \cdot \frac{r_2}{r_1 \cdot \sigma_x^2} \\
w_2^D &= \frac{\gamma^D}{\sigma_x^2 (r_1 + r_2)} \\
w_0^D \text{ is s.t. PC is binding}
\end{align*}
\]

where similarly \(w^D\) denotes the Lagrange multiplier of the Incentive Compatibility constraint. It is important to realise that, again, the PC is binding. Hence, the manager will receive his reservation utility.

The second best with disclosure (SBWD) effort is:

\[
e^D = w_2^D - \frac{\partial \sigma_x^2}{\partial e} \cdot \frac{r_2}{2} \cdot (w^D)^2
\]

The same argument about the Lagrange multiplier \(\gamma^D\) being zero could be written and, similarly, the next proposition tells us that this is never the case whenever the signal is not perfectly informative \(\gamma^2 > 0\). But the next proposition tells us more. It tells us that as long as the signal is not perfectly uninformative \(\gamma^2 < \infty\) the second best situation without disclosure can be improved upon. This is because the shareholder now has more tools and can provide better incentives to the manager. In other words, it is less costly to provide incentives to the manager.

Proposition 2. If the two following results hold:

1. The SBWD effort is always lower than the first best level, \(e^D < e^*\).
2. The SBWD effort is always higher than the second best without disclosure, \(e^D < e^{**}\).
Proof. The proof relies on showing that $?^D$ is strictly higher than zero. For the second part we just need to prove that the Lagrange multiplier from this maximisation is strictly lower than the Lagrange multiplier of the second best (without disclosure) case ($?>?^D$).

Corollary 2. First best is achieved when $?^2=0$.

Corollary 3. Second best is replicated when $?^2=\infty$.

To prove the last two propositions we just need to rewrite the programs including the condition on $?^2$ to realise that they are the same as the first best and second best programmes, respectively. The intuition is again the one we expected. If the signal is perfectly informative (i.e. $?^2=0$) we are in a first best world in which the shareholder knows accurately what the manager is doing. In contrast, when the signal is perfectly uninformative ($?^2=\infty$) the shareholder simply disregards it and the situation is analogous to one in which there is no disclosure. It is important to realise that as long as the signal is not perfectly uninformative (even if its variance is close to $\infty$) we will observe an improvement from the first best situation.

The previous propositions can be interpreted as if there was a one to one relationship between the level of effort and the level of disclosure. Graphically:

![Graph](image)

The previous graph tries to capture the fact that given any level of disclosure ($\omega^2 \in [0, \infty]$) only one level of effort between the second best and first best ($e^D \in [e^*, e^*]$) will be observed.

Similarly as before, we can now write a corollary that relates our study with the probability of failure:
Corollary 4. *In the presence of the agency problem between the manager and the shareholder, disclosure decreases the risk of the bank.*

*Proof.* Given that there exists an agency problem, the invested level of effort will be lower than the first best. An increase in disclosure (i.e. a decrease in $?_y$) will increase the information available to the shareholder to use in formulating the contract with the manager. Thus, the invested effort in risk monitoring issues will increase and consequently the variance of the bank’s project will decrease (i.e. $?_x$ will decrease).

It is immediate to realise that the shareholder has clear incentives in favour to disclosure. Full disclosure will allow him to write a first best contract so that he will be able to provide incentives to the manager to implement the first best. It is important to realise that the first best can be approached irrespective of the level of riskiness in the profits. It is only necessary that the additional signal is informative in the sense that it provides a less variable signal of the manager’s effort. In any case, a lower randomness in profits will allow us to achieve first-best easily.

Hence, we can state that the preferred level of disclosure by the shareholder is full disclosure. But, in the real world, why don’t we observe full disclosure in the banking system?\(^{10}\)

To answer this question we need to assess what features are missing in our (highly) stylised model that might explain why the implications of the model are not consistent with reality. As we are going to see in the next section, including a third party -a debt-holder- will go some way to explain why the shareholder might not choose to incentivise the bank management to disclose fully all the internal information of the bank. Later on, we will also discuss some further changes to the model setup which might rationalise why it is that transparency has traditionally not been particularly widespread in banking.

\(^{10}\) «…there are significant gaps in the information disclosed currently» Basel Committee on Banking Supervision (2000).
4. Completing the Model: The Debt-holder

We want to introduce a third party in our model in order to capture an essential characteristic of a bank-agents who have fixed claims on the bank. That is, in a more detailed framework the bank is not only composed of shareholders and managers but also needs to issue debt to finance its investment projects. Of course our setting is still very simple because the most apparent activities that a bank does (lending and depositing) are just implicit in our profit function. Nevertheless, we want to study how disclosure affects the bank’s risk taking behaviour and for this purpose just three stylised agents are needed: managers, shareholders and debt-holders. In this section we will analyse the interaction between the bank and debt-holders and, in particular, the conflicting interests of both parties.

4.1. Subordinated debt

The bank wants to finance a project. We will assume that the return form this project is high enough so that it will be always worth issuing subordinated uninsured debt to finance it\(^{11}\).

We denote the offered return on the bond \( R_b \) and we define \( q_b \) \((q_b \in (0, 1))\) as being its nominal value. Taking into account that the bond is repaid if and only if the bank does not default, the bond is formally defined as

\[
\begin{align*}
  b &= \begin{cases} 
  R_b & \text{with probability } (1-p) \\
  0 & \text{with probability } p 
  \end{cases} 
\end{align*}
\]

where \( p \in [0, 1] \) is the probability of the bank defaulting. We will come back to the parameter \( p \) later. The expected value of the bond is \( E(b) = (1-p) R_b \) and the variance is \( Var(b) = p(1-p) R_b^2 \).

\(^{11}\) It is crucial that the issued debt is uninsured so that the agents have incentives to monitor the bank’s activities.
4.2. The debt-holder

A debt-holder is now introduced\(^\text{12}\). We will assume that he is risk averse and has a quadratic utility\(^\text{13}\) of the form

\[
U_d(\bar{x}) = E(\bar{x}) - \frac{r^3}{2} \text{Var}(\bar{x})
\]

He is endowed with an initial wealth (normalised to one) and has to decide how to invest it between the bond issued by the bank and a risk free asset (the return on which is also normalised to one). Hence the program the debt-holder faces reads as follows:

\[
\max_{z_1,z_2 \geq 0} U_d(z_1 + z_2 \cdot b)
\text{ s.t. } z_1 + z_2 = 1
\]

where \(z_1\) and \(z_2\) are the fractions of his wealth invested in the risk free asset and the bank’s bond, respectively.

We can solve the debt-holder program and obtain the optimal allocation on bank’s bond:

\[
z_2 = \max \left\{ 0, \frac{(1 - p) R_b - 1}{(1 - p) R_b^2 r_3} \right\}
\]

Note that the fraction invested in bank bonds will be higher than zero if and only if the expected return on these bonds is higher than one (the return on the risk free asset). Note also that this quantity will be decreasing in the probability of failure of

\(^{12}\) We are implicitly assuming that the debt-holder does not know the salary contract that the shareholder has offered to the manager so that he cannot rationally anticipate the behaviour of the latter. Therefore, the only information he holds about the probability of failure of the bank comes from the disclosed information (in our model, \(y\)).

\(^{13}\) Given the quadratic utility we should constrain our study to the domain in which the utility is increasing. A sufficient condition for this to be the case is that \(R_b < 2\) (i.e. the return on banks’ bonds is lower than one hundred per cent).
the bank. That is, if the probability of bank failure diminishes (all the rest remaining fix) the investors will buy more bonds.

It is important to note that the debt-holder does not care directly about the relationship between the shareholder and the manager. The interaction between the shareholder and the manager results in a «return on bonds» and a «risk», and these are the only things the bond-holder cares about. Similarly, when the bank needs to finance a project, it must also anticipate the behaviour of the debt-holder and offer an interest rate such that (given the probability of failure of the bank) will cover its financial needs.

The probability of failure of the bank

We need to specify how we can compute the probability of failure of the bank given the specifications of our model. Note that in our model the returns of the bank are normally distributed -hence they can take any real value. It seems clear that the bank would not be able to repay the debt if its profits were below a certain level. Hence we are implicitly assuming that there is a threshold on the level of profits —if profits fall below this level the bank fails. Without loss of generality we can assume that this threshold is zero. Hence the probability of failure of the bank is defined as

\[ p : = \Pr\{x \leq 0\} = \Phi\left( -\frac{\mu}{\sigma_x} \right) \]

where \( \Phi(\cdot) \) is the standard normal distribution function. This definition is consistent with \( p \in (0, 1) \).

4.3. Disclosure

In line with the particular focus of our study, we would like to assess what would happen in the case that the market (in this case the debt-holder) does not hold perfect information about the bank’s activities. As before, we will assume that the manager’s action (or equivalently the variance in the bank’s project) is not verifiable. Hence, the debt-holders have to rely on the disclosed information, \( y \).

Here is where the «positively tailed» nature of the variable \( y \) plays a crucial role. Imposing the restriction that \( y \geq \epsilon \) implies that the disclosed information works to
misperceive the market towards an overstatement of the perceived probability of failure of the bank. And, as we will see later, this will provide a clear incentive for the shareholder not to implement full disclosure: the gains to the shareholder from holding better contractible information may be outweighed by the gains of providing less information to the market (i.e. the debt-holders) so that the financing costs are reduced\(^{14}\).

Hence, depending on the disclosed information we define a perceived probability of failure \( \tilde{p} \). As defined above, the perceived probability of failure is the probability that the profits are negative contingent on the available information in the market (i.e. \( y \)). Formally,

\[
\tilde{p} = \Phi \left( -\frac{\mu}{\sigma_x(y)} \right)
\]

As \( y \geq e \), the perceived probability of failure will be lower than the true one (\( \tilde{p} \leq p \)). Consequently, the debt-holder will allocate a fraction \( z^2 \) of his wealth on the bank’s bonds.

\[
z^2 = \frac{(1 - \tilde{p}) R_b - 1}{(1 - \tilde{p}) \tilde{p} R^2_b r_3}
\]

An increase in disclosure closes the gap (in probability terms) between \( y \) and \( e \), and, consequently, \( \tilde{p} \) approaches to \( p \). That is, more information increases the perceived probability of failure. Therefore, the fraction of wealth invested in the bank’s bond decreases. Equivalently, if the bank wants to keep constant the fraction of wealth invested in its bonds after the increase in disclosure, it has to increase the offered return.

From an ex-ante point of view (i.e. from the perspective of the bank who offers the return on bonds to attract investors), the expected value of the perceived probabi-

\(^{14}\) We are implicitly assuming that the market is myopic in the sense that he believes the signal that he receives - he does not apply any correction to it. Even though this is a strong assumption, we believe that the Basel proposals are induced by a misperception of the true probability of failure of the bank and this misperception is believed to harm the market (i.e. it corresponds to the positive tailed nature of the variable \( y \)).
lity of failure can be computed \( (E_y \{ \overline{p} \}) \). And similarly, the expected value of the fraction invested in bank’s bond \( (E_y \{ \overline{z}_2 \}) \). The importance of the latter expression will be shown below.

4.4. Disclosure vs. subordinated debt

Recall that the bank is issuing debt to finance a project. Therefore it requires all its bonds to be sold. That is, in ex-ante terms, it wants the fraction of wealth invested in his debt to be equal to the issued debt:

\[
E_y \{ \overline{z}_2 \} = q_b
\]

Note that in the case that \( E_y \{ \overline{z}_2 \} < q_b \) there will be an excess supply and the bank will have to increase the offered return to increase the demand for his bonds and finance its project. On the other hand, if \( E_y \{ \overline{z}_2 \} > q_b \) the bank will gain from lowering \( R_b \) until this excess demand vanishes. Hence we have that the offered return will be given by the equation:

\[
E_y \left\{ \frac{(1 - \overline{p}) R_b - 1}{(1 - \overline{p}) \overline{p} R_b^2 r_3} \right\} = q_b
\]

Note that this equation expresses \( R_b \) in terms of the quantity of bonds issued, the index of risk aversion of the debt-holder, the level of disclosure, the expected profits of the bank and the invested effort by the manager (i.e. \( R_b = R_b (q_b, r_3, \mu, \sigma^2, e) \)). It can be proven that the return rate will be higher than one but smaller or equal than the return that the bank would have to offer if the perceived probability coincided with the true default probability. That is, if there was full disclosure (\( ? = 0 \)):

\[
R_b \in [1, R_b (q_b, r_3, \mu, 0, e)]
\]

The following lemma expresses how a change in any of the variables affect the offered return on bonds and, consequently, the financing costs of the bank.

**Lemma 1**

\[
\frac{\partial R_b}{\partial q_b} > 0, \frac{\partial R_b}{\partial r_3} > 0, \frac{\partial R_b}{\partial \mu} > 0, \frac{\partial R_b}{\partial \sigma^2} > 0 \text{ and } \frac{\partial R_b}{\partial e} > 0.
\]
The intuition behind these partial derivatives is quite straightforward. First of all, the higher the quantity of bonds the bank wants to issue the more he has to offer to the investor so that he allocates a higher fraction of his wealth on the bank’s debt. Similarly, the more risk averse the investor is, the more return the bank has to offer him to cover its financial needs.

The third partial derivative expresses the fact that if the bank is in a better position (that is, has a higher mean in profits\textsuperscript{15}), it will be less likely that he incurs in big losses. Thus, the probability of the bank defaulting and not paying back its debt is lower. Consequently he will be able to offer a lower return on its bond than a bank with a lower expected return.

Finally the last two partial derivatives relate directly and indirectly to disclosure. The first one tells us that the lower the level of disclosure, the more biased the perceived probability of failure may be; hence, the lower the financing costs\textsuperscript{16}.

And the second one captures the fact that the higher the risk monitoring effort, the lower the risk will be and consequently, the lower the return. Furthermore, we can also state the relationship between $R_b$ and the expected value of the perceived probability of failure ($E_{\{\tilde{p}\}}$). As we might expect, it will be positive. That is, if in ex-ante terms the perceived probability of failure increases, the bank will have to increase the offered return in order to keep the demand for bonds constant.

We cannot assess what would be the exact effect of an increase in disclosure on $e$. We know that an increase in disclosure will increase the perceived probability of failure and this will imply (ceteris paribus) that the financing costs will increase. Will greater risk monitoring effort be undertaken by the bank manager in order to lower $R_b$. This will depend on whether it is worthwhile for the shareholder to induce higher risk monitoring incentives. It will depend on the relative costs/benefits of an increase in the risk monitoring effort\textsuperscript{17}.

\textsuperscript{15} Note that the mean was a parameter exogenously given that was not affected by the risk monitoring effort.

\textsuperscript{16} Diamond and Verrecchia (1991) argue that an increase in disclosure decreases the cost of capital. Their result comes from the fact that giving more information, the bank increases the liquidity of its debt which attracts large investors and reduces the offered return. We do not assess such an issue here. That is, we do not assume a premium for higher information. Therefore, in our model a contrary effect is observed.

\textsuperscript{17} The costs come from providing more incentives to the manager and the benefits come from reducing $R_b$. 
Following the structure of our reasoning we should conclude with the following proposition:

Proposition 3.  *In the presence of subordinated debt, the effect of disclosure on the risk monitoring effort exerted by the manager is ambiguous.*

The immediate consequence in terms of probability of failure translates in the following corollary:

Corollary 5.  *In the presence of subordinated debt, the effects of disclosure on the probability of failure are ambiguous.*

Nevertheless, these results are not as ambiguous as they seem. We have proved that in this new setting the shareholder may not optimally desire anymore a full disclosure situation. Imagine that he was first in a situation of full disclosure. At this point a decrease in disclosure will imply that the first best will not be achieved anymore but on the other hand it may also imply that the expected perceived probability (and consequently $R_a$) will decrease. If the second effect is important enough we will not be able to state anymore that «the preferred level of disclosure by the shareholder is full disclosure».

Moreover, we should realise that an implication of our model is that subordinated debt introduces some non desirable incentives. We saw in section 3 that the shareholder preferred full disclosure but we noted that this implication was not borne out in practice in the real world. Introducing a third party, we have shown that it might be the case that the debt-holder is providing incentives not to fully disclose.

Hence, notwithstanding the limitations of the present model it seems clear that we can infer that institutions that are issuing subordinated debt have an extra incentive to control their risk but also not to disclose. The latter may allow them to hide their true probability of default and may result in investing many more resources in hiding their real situation than in changing it. This might happen because in the short term the latter is less costly and these institutions might expect that in a longer term conditions will be more favourable to improve their fundamentals.

Summarising, subordinated debt provides incentives to behave in a sound manner but also to reduce the level of disclosure. With compulsory disclosure we eliminate the second negative effect and, even more, we enhance the safe sound and efficient banking sector.
5. Conclusion

In the present paper we have first analysed the effects of disclosure in the delegated portfolio management story that arises between the manager and the shareholder. In this simplified setting the implication was clear: full disclosure should be observed. Later, we have considered the relationship between the bank manager and a debt-holder realising that in this new setting full disclosure may not be optimal any more. This is because subordinated debt introduces incentives for the bank to hide inside information so as to reduce its financial costs. The next graph illustrates the interaction between the three stylised agents:

Hence the shareholder faces a trade-off: (1) he desires full disclosure to reduce the agency problem and (2) he wants to lower disclosure so that he will reduce its financing costs.

However, we should asses how consistent are these results with the real world. In other words, until which extent are our theoretic conclusions relevant? It is important to highlight that the main conclusion in our model is that introducing subordinated debt may induce the bank to disclose less information so that compulsory disclosure will play a very relevant role in enhancing a stable banking sector. Hence the bases of our conclusions rely on comparing the shareholder’s desired level of disclosure with and without subordinated-debt.
It is also important to realise that we have neglected the information premium issue. Under this circumstance, an increase in the information given by the bank is rewarded. That is, the market (the debt-holders) appreciates the fact that is holding more accurate information about the fundamentals of the bank and pays an extra price for it. Note that the implications of this issue are totally opposed with the ones in our model. In our model an increase in disclosure increases the financing costs; instead, the information premium implies that more disclosure reduces the financing costs. In the real world a mixed effect is expected. Hence, if we were to build an extensive model with both effects, no clear conclusion could be drawn about the direct effects of disclosure on the financing costs of the bank.

Still and all, the main point of the present paper still holds. Whenever the banking authority requires banks to issue subordinated debt, compulsory disclosure could just strengthen the soundness of the bank. Precisely because in our setting only the negative effects of disclosure on the safeness of the bank were modelled.

Besides, it is important to realise how credible is the fact that full disclosure will be observed if we only had the agency problem. Note that this claim says that no agency problem could ever exist because the principal would always force the manager to disclose enough information. Indeed this is not the case because there might be many reasons why it is not possible to increase the contractible information. For instance, the information may not be verifiable and consequently non enforceability of the contract could occur or, simply, the costs of such information may outweigh the benefits from writing more accurate contracts.

The friction induced by the costs of disclosure has been studied in a branch of the disclosure literature (Shaffer 95, Blum 2000) and plays a very relevant role in the real world. The problem is that there is no clear evidence on whether disclosure may introduce new real costs in the banking sector and whether it may reduce some competitive advantage of some banks. Our model could easily be extended introducing some real costs for the principal when reducing $\gamma(y)$. Depending on the schedule of these costs the effect could be more acute but the qualitative conclusions of our model will not change. On the other hand, the issue of the competitive advantage needs to be addressed in a setting with a multi bank sector resembling some studies on industrial organisation (Diamond ‘85).

Aside from these questions, it is important to remember that we restricted our contracts to be linear and our disclosure signal to be understating the true probability of default of the bank. The rest of our model has a very standard setup in which all agents are risk averse and profits are normally distributed.

To conclude we should point out that the main policy conclusion that can be
drawn is that forcing banks to issue subordinated debt may worsen its risk situation. This is because less disclosure is observed and the bank’s management may be investing less effort in monitor and mitigate the bank’s risks. Consequently, for such a measure to be effective in reducing the bank’s risks it should be accompanied by measures of compulsory disclosure. Such a conclusion could be contrasted in empirical grounds to reinforce the consistency of our theoretical model.

6. REFERENCES


HERMALIN, B. and KATZ, M. (1996): «Corporate diversification and agency», Incentives, Orga-


6. Appendix (Detailed Proofs)

Section 2.4: derivation of the first best

\[
\max_{w_0, w_1, e} E\{V(x - w(x))\}
\]
\[
s.t. E\left\{U\left(w(x) - \frac{e^2}{2}\right)\right\} \geq 0
\]

Taking into account the certainty equivalent expression for the utilities and the fact that \(w(x) = w^0 + w^1 x\), the correspondent Lagrangian is

\[
L = E\{x - w(x)\} - \frac{r_1}{2} \text{Var}\{x - w(x)\} + \lambda \left[E\{x - w(x)\} - \frac{r_2}{2} \text{Var}\{w(x)\} - \frac{e^2}{2}\right] =
\]
\[
= (1 - w_1)\mu - w_0 - \frac{r_1}{2}(1 - w_1)^2 \sigma_x^2 + \lambda \left[w_0 + w_1\mu - \frac{r_2}{2} w_1^2 \sigma_x^2 - \frac{e^2}{2}\right]
\]

The solution to the previous constrained maximisation is given by equating to zero the derivatives of the Lagrangian with respect to \(w_0, w_1\) and \(e\). The first one,

\[
\frac{\partial L}{\partial w_0} = -1 + \lambda = 0 \Rightarrow \lambda = 1
\]

implies that the constraint is binding. This is, \(w_0^*\) is marginally determined so that the agent’s utility is at its minimum level. The optimal \(w_1^*\) will be given by

\[
\frac{\partial L}{\partial w_1} = -\mu + r_1(1 - w_1)\sigma_x^2 + \lambda [\mu - r_2 w_1\sigma_x^2] = 0
\]

from the fact that \(? = 1\) we have that \(w_1^* = ((r?)/(r?+r?))\).

Finally, the derivative with respect to \(e\) gives us the optimal first best effort

\[
\frac{\partial L}{\partial e} = -\frac{r_1(1 - w_1)^2}{2} \frac{\partial \sigma_x^2}{\partial e} + \lambda \left[-\frac{r_2}{2} w_1^2 \frac{\partial \sigma_x^2}{\partial e} - e\right] = 0 \Rightarrow e^* = -\frac{\partial \sigma_x^2}{\partial e} \cdot \frac{r_2}{2} \cdot w_1^*
Section 2.5: derivation of the second best

\[
\max_{w_0, w_1, e} E\{V(x - w(x))\}
\]
\[
s.t. \quad \begin{cases} 
E\{U(w(x), e)\} & \geq 0 \\
 e = \arg \max_x E\{U(w(x) - \partial)\}
\end{cases}
\]

Using the First Order Approach we have that the second constraint is replaced by its first order condition, hence the correspondent Lagrangian is:

\[
L = (1 - w_1)\mu - w_0 - \frac{r_1}{2}(1 - w_1)^2 \sigma_x^2 + \lambda \left[ w_0 + w_1\mu - \frac{r_2}{2} w_1^2 \sigma_x^2 - \frac{e^2}{2} \right] + 
\gamma \left[ -\frac{r_2}{2} w_1^\prime \frac{\partial \sigma_x^2}{\partial e} - e \right]
\]

Similarly, the participation constraint is binding (\(\gamma = 1\)) and \(w_1^{**}\) is determined marginally. The derivative with respect to \(w_1\) is:

\[
\frac{\partial L}{\partial w_1} = -\mu + r_1(1 - w_1)\sigma_x^2 + \lambda [\mu - r_3w_1\sigma_x^2] + \left[-r_2w_1 \frac{\partial \sigma_x^2}{\partial e}\right] = 0
\]

and consequently, we get \(w_1^{**} = \left((w_1^*)^{-1} + \gamma \cdot \frac{\partial \sigma_x^2}{\partial e} \cdot \frac{r_2}{2}\right)\)

The derivative with respect to \(e\) is

\[
\frac{\partial L}{\partial e} = -\frac{r_1}{2}(1 - w_1)^2 + \lambda \left[ -\frac{r_2}{2} w_1^2 \frac{\partial \sigma_x^2}{\partial e} - e \right] + \gamma \left[ -\frac{r_2}{2} w_1^\prime \frac{\partial \sigma_x^2}{\partial e} \right] + \frac{r_2}{2} = 0 \quad (1)
\]

Note that the second term in expression above is 0 given the Incentive Compatibility constraint. Finally the optimal second best is given by the Incentive Compatibility constraint:

\[
e^{**} = -\frac{\partial \sigma_x^2}{\partial e} \frac{r_2}{2} (w_1^{**})^2
\]
Proof of proposition 1 \((g \geq 0, \ g \neq 0)\)

Note that the LHS on equation (1) has two terms: a first positive term plus lambda times a negative term. Consequently, for the equality to hold we need \(? \geq 0\). To prove that \(? > 0\) we just need to see that \(? = 0\) does not satisfy the system of equations given by the Lagrangian techniques. Observe that for \(? = 0\) we have

\[
\begin{align*}
\dot{w}^* = \dot{w} = \frac{r_1}{r_1 + r_2}
\end{align*}
\]

And substituting \(? = 0\) into equation (1) we have that

\[
\begin{align*}
-\frac{r_1}{2} (1 - w^*) \frac{\partial \sigma^2}{\partial e} = 0
\end{align*}
\]

We can plug the expression for \(w^*\) into the previous expression, and given that \(r_1\) and \(r_2\) are strictly higher than zero, and that \((\frac{\partial \sigma^2}{\partial e})\) is strictly negative, we conclude that there is no \(e\) that solves the previous expression and, consequently, \(? \neq 0\).

**Section 3.2: derivation of the second best with disclosure**

\[
\begin{align*}
\max_{w_0, w_1, w_2, e} E\{V(x - w(x))\}
\end{align*}
\]

s.t. \[
\begin{align*}
E\{U(w(x), e)\} \geq 0
\end{align*}
\]

We just have to repeat the previous analysis taking into account that now the salary is also contingent on the disclosed signal \(y\). Therefore we have an additional condition (the derivative of \(L\) with respect to \(w^*\)).
Again the Participation Constraint is binding ($\zeta = 1$) and $w^D$ is marginally determined.

Note that the derivative with respect to $w^D$ does not change. Thus we have that:

$$w^1 = \left( (w^*_1) + \gamma^D \frac{\partial \sigma^2_x}{\partial e} \cdot \frac{r_2}{r_1 \cdot \partial \sigma^2_x} \right)$$

Where $\gamma^D$ is determined by the condition:

$$\frac{\partial L}{\partial e} = -w_2 - \frac{r_1}{2} (1 - w_1)^2 \frac{\partial \sigma^2_x}{\partial e} + \gamma^D \left[ -\frac{r_2}{2 \cdot w_1} \frac{\partial^2 \sigma^2_x}{\partial e^2} - 1 \right] = 0 \quad (2)$$

The optimal $w^D$ is given by

$$\frac{\partial L}{\partial w_2} = -e - r_1 w_2 \sigma^2_y + [e - r_2 w_2 \sigma^2_y] + \gamma^D = 0 \Rightarrow w^D_2 = \frac{\gamma^D}{\sigma^2_y} \frac{1}{r_1 + r_2}$$

Finally the optimal second best effort with disclosure (SBWD effort) is given by the Incentive Compatibility constraint:

$$e^D = w^D_2 - \frac{\partial \sigma^2_x}{\partial e} \cdot \frac{r_2}{2} (w^1)^2$$
Proof of proposition 2 (Step 1: $e^D \geq 0$, $e^D \neq 0$)

The proof of $\gamma^D > 0$ is totally analogous to the proof of proposition 1. As before we have that the LHS on the equation (2) can be rewritten as having two terms:

\[-w_2 - \frac{r_1}{2} (1 - w_1) \frac{\partial \sigma_x^2}{\partial e} + \gamma^D \left[ - \frac{r_2}{2} w_1^2 \frac{\partial^2 \sigma_x^2}{\partial e^2} - 1 \right] = \]

\[= - \frac{r_1}{2} (1 - w_1)^2 \frac{\partial \sigma_x^2}{\partial e} + \gamma^D \left[ - \frac{r_2}{2} w_1^2 \frac{\partial^2 \sigma_x^2}{\partial e^2} - 1 - \frac{1}{\sigma_y^2} \cdot \frac{1}{r_1 + r_2} \right] = 0\]

The first term is positive and the second is $\gamma^D$ times a negative term. Therefore for this equation to hold we need $\gamma^D \geq 0$. As before, we need to prove that $\gamma^D > 0$. Hence, we just need to see that $\gamma^D = 0$ does not satisfy the system of equations given by the Lagrangian techniques. Observe that for $\gamma^D = 0$ we have

\[w_1^D = w^* = \frac{r_1}{r_1 + r_2} \text{ and } w_2^D = 0\]

And substituting $\gamma^D = 0$ into equation (2) we have that

\[-w^D - \frac{r_1}{2} (1 - w_1^D) \frac{\partial \sigma_x^2}{\partial e} = - \frac{r_1}{2} \left( 1 - \frac{r_1}{r_1 + r_2} \right) \frac{\partial \sigma_x^2}{\partial e^2} = 0\]

Given that $r^1$ and $r^2$ are strictly higher than zero, and that $((\partial _e \gamma^D)/(\partial e))$ is strictly negative, we conclude that there is no $e$ that solves the previous expression and, consequently, $\gamma^D \neq 0$.

Proof of proposition 2 (Step 2: $? > ?^D$)

We need to compare the Lagrangian multipliers from both maximisations. Recall equations (1) and (2):
\[
\frac{-r_1}{2} (1 - \omega_{**})^2 \frac{\partial \sigma_x^2}{\partial e} + \gamma \left[ \frac{r_2}{2} (\omega_{**})^2 \frac{\partial^2 \sigma_x^2}{\partial e^2} - 1 \right] = 0
\]

\[
- \frac{r_1}{2} (1 - w^D) \frac{\partial \sigma_x^2}{\partial e} + \gamma^D \left[ \frac{r_2}{2} w_1^D \frac{\partial^2 \sigma_x^2}{\partial e^2} - 1 - \frac{1}{\sigma_y^2} \frac{1}{r_1 + r_2} \right] = 0
\]

Suppose that \( \omega \leq \omega^D \). Then we have that \( w^D \geq \omega^{**} \) and consequently \( e^D > e^{**} \) (because \( w^D \geq 0 \)). Hence if the first expression holds, the second expression will not hold.

**Proof of corollary 2 (First best is achieved when \( y^? = 0 \))**

Note that the Lagrangian for the program when \( y^? = 0 \) is:

\[
L = (1 - w_1) \mu - w_0 - w_2 e - \frac{r_1}{2} (1 - w_1)^2 \sigma_x^2 + \\
+ \lambda \left[ w_0 + w_1 \mu + w_2 e - \frac{r_2}{2} w_1^D \sigma_x^2 - \frac{e^2}{2} \right] + \\
\gamma^D \left[ w_2 - \frac{r_2}{2} \frac{\partial \sigma_x^2}{\partial e} - e \right]
\]

As before, the Lagrange multiplier of the Participation Constraint is 1,

\[
\frac{\partial L}{\partial w_0} = -1 + \lambda = 0 \Rightarrow \lambda = 1
\]

and the derivative of the Lagrangian with respect to \( w? \)

\[
\frac{\partial L}{\partial w_2} = -e + \lambda e + \gamma^D = 0
\]

tells us that the first best is achieved because \( y^D = 0 \).
Proof of corollary 3 (Second best is replicated when $y^*=\infty$)

It is clear that when $y^*=\infty$, $w^D_D=0$. This is because given that both agents are risk averse, $w^D_D>0$ will introduce an infinite cost in terms of risk bearing. Hence, replacing $y^*=\infty$ and $w^D_D=0$ into equation (2) we realise that we are left with an identical equation to (1). That is, the maximisation problem coincides with the one without disclosure.

$$\max_{w_0, w_1, w_2, e} E\{V(x - w(x))\}$$

s.t. \(E\{U(w(x), e)\} \geq 0\)

$$e = \arg \max_e E\{U(w(x) - \hat{e})\}$$

Proof of the lemma 1 \(\left(\frac{\partial R_b}{\partial q_b} > 0\right)\)

Recall how we defined the perceived probability of failure and which was the condition that offered interest rate should hold:

$$\tilde{p} = \Phi\left(-\frac{\mu}{\sigma_y(y)}\right) \text{ and } E_y \left\{ \frac{(1 - \tilde{p}) R_b - 1}{(1 - \tilde{p}) \tilde{p} R_b^2 r_3} \right\} = q_b$$

We now define

$$F(R_b, q_b) = E_y \left\{ \frac{(1 - \tilde{p}) R_b - 1}{(1 - \tilde{p}) \tilde{p} R_b^2 r_3} \right\} - q_b$$

In equilibrium, $R_b$ and $q_b$ are such that $F(R_b, q_b)=0$. Hence using the implicit function theorem we have that

$$\frac{\partial R_b}{\partial q_b} = -\frac{\partial F}{\partial q_b}$$
Finally we just need to compute the two partial derivatives of $F$: $\frac{\partial F}{\partial q_b} = -1$ and $\frac{\partial q_b}{\partial R_b} > 0$. Note that for the second partial derivative to be true we must assume that the parameters of the model are such that we are restricting our analysis on the increasing part of the debt-holder’s quadratic utility function. As noted in footnote 14, a sufficient condition for this to be true is that $R_b < 2$ (this is, the return on bonds is lower than a 100%). The remaining results on the lemma are proved analogously.