Discussion of:

“The Lost Capital Asset Pricing Model”

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In a Nutshell

- Key idea: econometrician has limited information compared to market participants (Roll (1977), Hansen and Richard (1987), Jagannathan and Wang (1996)) $\Rightarrow$ can lead to spurious rejection of asset pricing models.
- Paper's key ingredient: stochastic supply of assets
  Market return $= M^\top R$, $R \in \mathbb{R}^N$, $M \sim \left(\bar{M} = \frac{1}{N}\mathbf{1}, Var(M)\right)$,
- CARA + everything Gaussian/linear + single aggregate risk source $\Rightarrow$ CAPM holds under the market info-set
$$\mathbb{E}[R] = \beta\mathbb{E}\left[R_{\bar{M}}\right] = \beta\mathbb{E}\left[\bar{M}^\top R\right]$$
- The econometrician does not observe M (but knows $\bar{M}$) $\Rightarrow$ cannot filter out idiosyncratic noise, therefore estimates:
$$\tilde{\beta} = \beta + \delta\left(\beta - \mathbf{1}\right) = (1+\delta)\beta - \delta\mathbf{1}$$

$\Rightarrow$ $\alpha \neq 0$ in her "filtration" $\rightarrow$ wrongly (and naively) rejects CAPM.

$\delta > 0 \Rightarrow$ "flatter" SML

BaB : makes "alpha" on measurement error... (or maybe not)

What about a *less-naive* econometrician?

Note that:

$$\tilde{\beta} \propto \beta$$

$\Rightarrow$ Unconditional expected returns linear in $\tilde{\beta}$, therefore:

- monotone risk premia
- 100% cross-sectional R^2 for unconditional CAPM estimated with a common free intercept.

$\Rightarrow$ let's check in the paper itself...

Monotone risk premia in $\tilde{\beta}$?

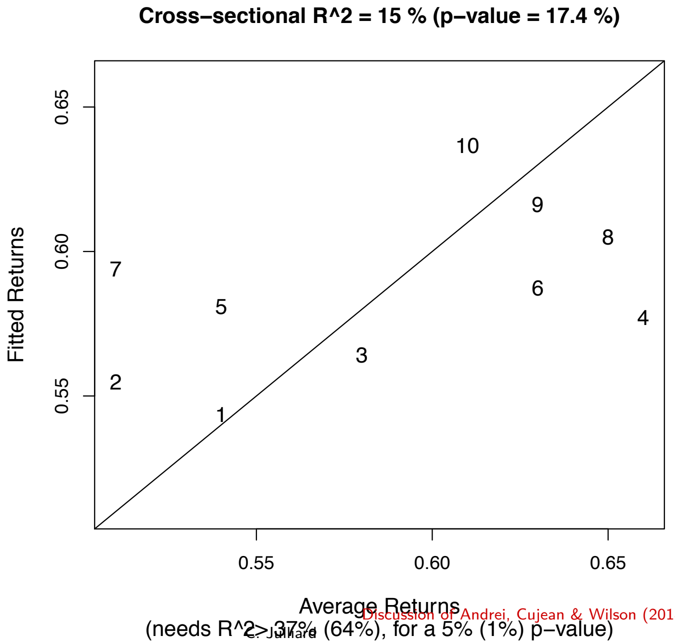
Panel A: Ten beta-sorted portfolios					
	(a)	(b)	(c)	(d)	(e)
Portfolio	Avg. excess returns	Sample betas	Adj. betas ($\delta = 0.5$)	Adj. betas ($\delta = 3$)	Adj. betas ($\delta = 4.5$)
Low	0.54	0.61	0.74	0.90	0.93
2	0.51	0.73	0.82	0.93	0.95
3	0.58	0.83	0.89	0.96	0.97
4	0.66	0.97	0.98	0.99	0.99
5	0.54	1.01	1.01	1.00	1.00
6	0.63	1.08	1.05	1.02	1.01
7	0.51	1.15	1.10	1.04	1.03
8	0.65	1.27	1.18	1.07	1.05
9	0.63	1.39	1.26	1.10	1.07
High	0.61	1.61	1.40	1.15	1.11
Panel B: Securities Market Line					
Intercept		0.49	0.44	0.20	0.06
		(0.09)	(0.09)	(0.23)	(0.32)
Slope		0.09	0.14	0.38	0.52
		(0.06)	(0.09)	(0.23)	(0.32)

Table 2: Resurrecting the CAPM. Columns (a) and (b) of Panel A report average

Violates 42.2% of monotonicity restrictions (19/45,"adjusted" or not)

⇒ worse than "flip of a coin" model: $p\text{-val}_{LR-test}$ 0 vs. 0.22, post $\mathbb{P}r = 0$

100% cross-sectional R^2?



What about a *smart* econometrician?

"*[...] Living with the Roll critique*" — Shanken (1987)*

- The econometrician observes a proxy, $R_{\bar{M}} = \bar{M}^{\top} R$, for the true market portfolio, $R_M = M^{\top} R$.

But: if $\rho \equiv corr(R_{\bar{M}}, R_M) > 0.7$ the data reject the CAPM*

- In this paper:

$$R_M = R_{\bar{M}} + \varepsilon_M^{\top} R, \text{ where } \varepsilon_M \in \mathbb{R}^N \text{ is the independent } M \text{ shock}$$

$\Rightarrow$ $\rho < 0.7$ iff $Var(R_{\bar{M}}) < Var(\varepsilon_M^{\top} R)$

- since $Var(R_M) = Var(R_{\bar{M}} = \bar{M}^{\top} R) + Var(\varepsilon_M^{\top} R)$, the CAPM is *not* rejected only if more than 70% of the market portfolio vol comes from the Stochastic Supply rather than fundamentals $\Rightarrow$ SS-APM rather than CAPM ☺

Other remarks and doubts

I. $\delta \to 0$ as $N \to \infty \Rightarrow \tilde{\beta} \xrightarrow[N\to\infty]{} \beta$?!

$$\delta \equiv \frac{\kappa/N}{\mathbb{V}[R_{\overline{M}}]} = \frac{1}{N\,\mathbb{V}[R_{\overline{M}}]}\left[\frac{\gamma^2}{\tau_M \tau_\epsilon}\left(\frac{1}{\tau_\epsilon} + \frac{\Phi'\Phi}{\tau}\right) + \frac{\tau_v}{\tau\tau_\epsilon}\right] > 0. \qquad (29)$$

$N \to \infty$ 0 ?

So if the econometrician uses a large cross-section the inference problem is solved?

II. alphas and BaB? You say: *"In the eyes of the empiricist [...]:"*

$$\mathbb{E}[R_n] = \underbrace{\delta(1+\delta)^{-1}(1-\widetilde{\beta}_n)\,\mathbb{E}[R_{\overline{M}}]}_{\text{perceived mispricing (alpha)}} + \widetilde{\beta}_n\,\mathbb{E}[R_{\overline{M}}]. \qquad (27)$$

Nope: (less-naive) empiricist runs a cross-sectional regression on $\tilde{\beta}_n$:

$$\alpha_n = \delta(1+\delta)^{-1}\mathbb{E}\left[R_{\bar{M}}\right] \quad \forall n \quad \text{(as in your eq. (30))}$$

⇒ find no excess return from Betting against Beta. This comes from the flatter empirical SML: $\lambda = \mathbb{E}\left[R_{\bar{M}}\right]/(1+\delta)$.

(in eq. (27) you imposed the "right" slope instead)

Other remarks and doubts cont'd

Note: in the model a *very smart* (i.e. knows the model) econometrician would do X-sectional GMM, recover δ, and *not* reject the CAPM.

☺ You can estimate δ directly from the X-sectional $\hat{\alpha}$ & $\hat{\lambda}$ ⇒ do so! You can even do a model specification J-test (over-identified model).

$\hat{\delta}_{MM} \approx$ 1.6 or 4.4 (from α or λ) from your Table 2

III. Inconsistent/unnecessary empirical estimate of δ based on Martin (2017) (and Martin and Wagner (2017)) expected returns. (either log utility or lower bound: the former is inconsistent with the theory, the latter gives inconsistent estimates in eq (34))

IV. Black (1972) CAPM? With stochastic supply (M) the composition of the zero-β portfolio will change a lot ⇒ $Var(R_t^f)$? (eigenvalue problem not invariant to M)

Baseline

"[CAPM] Beta is dead" — Fama and French (1992)

- A clever, elegant and well executed work...

... but probably beats a dead horse: rationalizes $\alpha \neq 0$ for the CAPM (with naive testing), but still implies monotonicity of returns and perfect cross-sectional fit for the model...

$\Rightarrow$ in the data, even if "lost", the CAPM still performs worse than the "flip of a coin" model.

But: actually the paper's argument is much more general, and important, than just CAPM: maybe the *"lost APT"* is the right spin? (with the *un-resurrected* "lost CAPM" as a salient example)

$\Rightarrow$ first order filtering problem for asset pricing.

baseline: recommended reading (but I would market/frame it very differently, and I would make the empiricist a bit smarter).