Discussion of:

"Time Aggregation and Asset Pricing Models"

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Time Aggregation in a Nutshell

Key idea: The use of time aggregation alters the serial correlation of the original series (Working (ECMA1960)).

Example:

$$X_t = X_{t-1} + \varepsilon_t \sim WN(1)$$

$$\Delta_m X_t := X_t - X_{t-m}, \quad m \in \mathbb{Z}^+$$

$$\Rightarrow var(\Delta_m X_t) = m, \qquad cor(\Delta_m X_t, \Delta_m X_{t-m}) = 0$$

If we *average* m elements before taking first differences

$$\Delta_m^* X_t = \frac{1}{m}(X_t + X_{t+1} + \ldots + X_{t+m-1}) - \frac{1}{m}(X_{t-m} + X_{t-m+1} + \ldots + X_{t-1})$$

$$var(\Delta_m^* X_t) = \frac{2m^2+1}{3m} < m, \quad cor(\Delta_m^* X_t, \Delta_m^* X_{t-m}) = \frac{m^2-1}{2(2m^2+1)} > 0$$

$\Rightarrow$ Spurious MA(1) behaviour and correlations.

Baseline: the DGP specification of a model is NOT frequency invariant.

What This Paper Does

Studies time aggregation bias in representative agent models when:

1. Preferences are Epstein-Zin type

$$V_t = \left[(1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1}{\theta}} \right) \right]^{\frac{\theta}{1-\gamma}}$$

$\Rightarrow$ i.e. time separable CRRA is the particular case when $\theta = \frac{1-\gamma}{1-\psi^{-1}} = 1$

2. The vector of state variables, X, is exponentially affine

$$\ln E_t \left[e^{z' X_{t+1}} \right] = b(z) + a'(z) X_t$$

$\Rightarrow$ fits most asset pricing workhorse specifications

3. The model is solved after <u>log-linearization</u> around equilibrium

But: log-linearization is likely to be part of the problem.

Multiplicative Relations and Time Averaging

Example:

$$\frac{X_t}{X_{t-1}} = \varepsilon_t \sim WN$$

$$\Delta^{\%}_m X_t := \frac{X_t}{X_{t-m}} = \prod_{i=0}^{m-1} \varepsilon_{t-i}, \quad m \in \mathbb{Z}^+$$

$$\Rightarrow cor\left(\Delta^{\%}_m X_t, \Delta^{\%}_m X_{t-m}\right) = 0$$

If we *average m* elements before taking the ratio

$$\begin{aligned}
\Delta^{\%*}_m X_t &= \frac{\frac{1}{m}\left(X_t + X_{t+1} + \ldots + X_{t+m-1}\right)}{\frac{1}{m}\left(X_{t-m} + X_{t-m+1} + \ldots + X_{t-1}\right)} \\
&= \left(\prod_{i=0}^{m-1} \varepsilon_{t-i}\right) \frac{1 + \sum_{j=1}^{m-1} \prod_{i=1}^{j} \varepsilon_{t+i}}{1 + \sum_{j=1}^{m-1} \prod_{i=1}^{j} \varepsilon_{t-m+i}}
\end{aligned}$$

$$\Rightarrow cor\left(\Delta^{\%*}_m X_t, \Delta^{\%*}_m X_{t-m}\right) = 0$$

But: if we were to work with the log-linearized version we would be back to altering the serial correlation.

Baseline: I would like to know how much of the time averaging bias is caused by the linearization.

What This Paper Finds with CRRA Utility

Focusing on the time separable CRRA particular case, and *iid* monthly consumption growth, the authors find that

1. In the "most standard" case of misspecification (i.e., ignore time aggregation and sum up consumption growth), there is no bias.

Q: is this due to the fact that there is no linearization at all in this case? (fits the last example)

2. When "trying" to take into account time averaging, the RRA estimate can be upward, or downward biased, depending on the form of misspecification.

⇒ interesting since the common belief was the one of a systematic upward bias.

But: is the bias really due to aggregation, or is it because of the linearization used to link monthly to annual consumption?

⇒ this can be easily checked numerically by: *i*) generate data from the monthly model; *ii*) do exact aggregation to annual data; *iii*) estimate the model using the annual data.

What This Paper Finds with EZ and LRR

Focusing on the recursive utility case, and the LRR specification, and DGP calibration, of Bansal, Kiku and Yaron (2010)

$$
\begin{aligned}
\Delta c_{t+1} &= g_c + x_t + \sigma_t \eta_{t+1} \\
\Delta d_{t+1} &= g_d + \phi x_t + \sigma_t u_{t+1} \\
x_{t+1} &= \rho x_t + \varphi \sigma_t e_{t+1} \\
\sigma_{t+1}^2 &= \sigma^2 + \nu(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1} \\
\eta_t,\ u_t,\ e_t,\ w_t &\sim i.i.d. N(0,1)
\end{aligned}
$$

the authors find that:

1. If the monthly – true – EIS > 1 (< 1), the annual estimate is also > 1 (< 1)
2. EIS can be biased both $\uparrow$ or $\downarrow$.

$\Rightarrow$ relevant and not expected results.

3. If EIS> 1 (< 1) , the RRA estimate biased $\uparrow$ ($\downarrow$).

Q: intuition?

What This Paper Finds with EZ and LRR cont'd

But:

- What drives the results? No intuition/intermediate analytical results presented (e.g. with EIS $><1$ the role of σ changes, is this the key?).

- What happens to the estimates of the DGP parameters?

  E.g. using monthly simulated data from BY, and quarterly aggregation with naive estimation, I find strong upward bias in the estimation of the persistence parameters.

  $$\text{Quartely } \varphi : .956 \text{ vs. } .938; \;\; \text{Quartely } \nu : .973 \text{ vs. } .961$$

  $\Rightarrow$ if EIS >1 (<1) I would expect RRA biased $\downarrow$ ($\uparrow$)

- What does time aggregation imply for estimation approaches based on inversion of state variables from observable P/D and R^f? (Constantinides and Ghosh (2010))

- How much bias comes from linearization? Can check this: *i*) simulate monthly data, *ii*) perform exact aggregation, *iii*) estimate the model non-linearly (with and without observable states)

What I Would Also Like to See

Some Guidance: they show and analyze a problem, but don't really give us a way to address it in our research work. E.g.:

- Bias correction? Bounds for the parameters?
- What can we learn from assets predictability? (e.g. time-aggregation problems increase cash-flow predictability, Campbel and Cochrane (2011))
- Can we extract information from mixed frequency data?
- Since the "true" frequency is unknown, can we do frequency selection via posterior probabilities or moments selection?

Time Dis-aggregation: optimal consumption decision could be taken at <u>lower</u> frequency than collected data (e.g. Parker & Julliard (JPE2005), Jagannathan & Wang (JF2007), ...). What happens then?

Linearization: should be taken seriously since *a*) "nice" errors are no more so and, *b*) big effect on pricing of tail risks.

Habits? the jury is still out, so differentiate your stakes...

Conclusion

- A worthy reading with interesting – and not obvious – findings.
- More of a good starting point for a research with impact, than a finished product.
- So far the paper is only *positive*, but to have a big impact it would require to build the *normative* side.