Human Capital and International Portfolio Choice*

Christian Julliard
Princeton University

October 2004

SECONDARY JOB MARKET PAPER

Abstract

This paper shows that in a non-representative agent model in which households face short selling constraints and labor income risk, in the form of both uninsurable shocks and a common aggregate component, small differences in the correlation between aggregate labor income shocks and domestic and foreign stock market returns lead to a very large home bias in asset holdings. Calibrating this buffer-stock saving model to match both microeconomic and macroeconomic U.S. labor income data, I demonstrate that, consistent with the empirical literature, a) investors that enter the stock market will initially specialize in domestic assets, b) individual portfolios become more internationally diversified, adding foreign stocks one at a time, as the level of asset wealth increases, and c) most importantly, the implied aggregate portfolio of U.S. investors shows a large degree of home bias consistent with observed levels.

Keywords: Home Bias, International Diversification, Human Capital, Labor Income Risk, Buffer-Stock Saving, Optimal Portfolio Choice

JEL Classification: F30, G11, G15

*For helpful comments and discussions, I thank Orley Ashenfelter, Alan Krueger, Per Krusell, Jonathan Parker, Helene Rey, Chris Sims, Noah Williams and seminar participants at Princeton University.
1 Introduction

This paper studies the international diversification puzzle taking seriously the fact that human capital is part of the wealth of nations. I show that, in the presence of short selling constraints, non-tradable labor income risk generates a large degree of home bias in portfolio holdings.

This paper models agents as facing both idiosyncratic - transitory and permanent - labor income risk and a small aggregate labor income risk component. Also, agents are not allowed to short sell financial assets. The model is calibrated to match both the microeconomic (following Gourinchas and Parker (2002)) and macroeconomic (following Julliard (2004)) characteristics of the U.S. labor income processes and the empirical correlations between labor income shocks and stock market returns. Moreover, the model reproduces fairly well the distribution of the asset wealth to labor income ratios observed in the Panel Study of Income Dynamics (PSID).

In this buffer-stock saving setting, extremely small differences in the correlation of aggregate labor income shocks with domestic and foreign stock market returns, lead to very large departures from the optimal portfolio diversification without human capital, and generate a substantial degree of home bias in asset holdings. The main findings of the paper are that, a) investors that enter the stock market with low levels of liquid wealth will initially specialize in domestic assets and, b) only at large ratios of asset wealth to labor income agents significantly diversify their portfolios internationally, c) as a consequence, the aggregate portfolio of U.S. investors shows a large degree of home bias. The paper also shows that a representative agent model without short selling constraints is unlikely to reproduce the degree of home bias observed in the data, unless implausibly high levels of risk aversion are assumed.

What drives the results? Palacios-Huerta (2001) analyzes the differences in human capital of stockholders and non-stockholders and argues that the information contained in the human capital of stockholders can greatly contribute towards explaining the in-
ternational diversification puzzle. Moreover, several studies have found suggestive evidence of a negative correlation of labor income innovation and stock market returns at the country level. Nevertheless, empirical studies find that the conditional correlation of labor income and domestic market returns is not sufficiently smaller than the correlation of labor income and foreign market returns to explain the home bias. But, as stressed in the works of Willen (1999) and Davis and Willen (2000), in a dynamic setting the human capital hedging motive depends crucially on the degree of persistence of the labor income process, and this substantially amplifies the effect of small correlations. Moreover, in the presence of liquidity constraints agents cannot borrow to construct an optimally diversified portfolio. Therefore, when their level of liquid wealth to labor income ratio is sufficiently high and they enter the stock market, agents try to minimize the overall wealth risk, investing first in the assets that have the lower degree of correlation with labor income. Only when the ratio of liquid wealth to labor income is sufficiently high and the labor income risk hedging motive becomes less important relative to the financial risk hedging motive, do agents start investing in foreign assets and diversifying their portfolios internationally. Since the distribution of liquid wealth to labor income is (in the data as in the model) concentrated in the region of low liquid wealth to labor income ratios, the resulting aggregate portfolio is heavily skewed toward domestic assets.

The balance of the paper is organized as follows. Section 2 reviews the related

---

1Palacios-Huerta (2001) finds that if human capital is included in the definition of wealth, gains from international financial diversification for a mean-variance investor appear to be smaller than previously reported.

2Abowd (1989), in a study on the wage bargaining in the U.S., finds a large and negative correlation between unexpected union wage changes and unexpected changes in the stock value of the firm. Gali (1999), Rotemberg (2003) and Francis and Ramsey (2004) find a negative correlation between labor hours and productivity conditioning on productivity shock. See also Davis and Willen (2000).

3Bottazzi, Pesenti, and van Wincoop (1996) find that in the U.S. the correlation of return to human capital ($r_h$) and domestic stock market is $-0.4$ while the correlation of $r_h$ with foreign stocks is about $-0.05$, but this large difference is able to increase the share of domestic assets in the optimal portfolio by a mere 19 percent. See also Pesenti and van Wincoop (2002).
literature on human capital and international diversification. Section 3 discusses a simple incomplete market model of optimal portfolio choice that admits a closed form solution, and stresses the link between labor income risk and international portfolio diversification in a dynamic setting. Moreover, this section shows that, in the absence of short selling constraints and with a low level of risk aversion, the human capital hedging motive is unlikely to generate the degree of home bias observed in the data. Section 4 presents the main model with heterogeneous agents, labor income risk and short selling constraints. Section 5 derives the optimal policy rules and the optimal portfolio choice of individual investors while section 6 presents the model implications for the U.S. aggregate portfolio. Section 7 discusses the main results and possible extensions.

2 Review of related literature

International finance theory emphasizes the effectiveness of global portfolio diversification strategies for cash-flow stabilization and international risk sharing. However, the empirical evidence on portfolio holdings finds a widespread lack of diversification across countries.\(^4\)

Several contributions have argued that when the role of non traded human capital is explicitly taken into account, the discrepancy between theoretical predictions and observed portfolios is wider than commonly assessed.\(^5\) The argument, originally formalized by Brainard and Tobin (1992) with a stylized example, works as follows:

\(^4\)French and Poterba (1991) and Tesar and Werner (1995) estimate that the aggregate stock market wealth share invested in domestic equities in the beginning of the 90s was about 90 percent for U.S. and Japan and about 80 percent for U.K. and Germany. Tesar and Werner (1997) shows that the degree of international diversification has increased in the late 90s, but the level of home bias still remains very high. See also Warnock (2002) for a more recent study.

\(^5\)Cole (1988) asserts that "[...] this result is disturbing, given the apparent lack of international diversification that we observe."
if returns to human capital are more correlated with the domestic stock market than with the foreign ones, the labor income risk can be more effectively hedged with foreign assets than with domestic ones, and equilibrium portfolio holdings should be skewed toward foreign securities. More recently, Michaelides (2003) shows with a calibration exercises that, in the presence of liquidity constraints, if labor income shocks are positively correlated with the domestic stock market returns and orthogonal to foreign asset returns, investors should hold only foreign assets in their portfolios.

But, this line of reasoning does not seem to have found strong support in the data on labor income and asset returns. Bottazzi, Pesenti, and van Wincoop (1996) find a negative correlation between wage and profits rates in many OECD countries. They also argue that shocks that lead to a redistribution of total income between labor and capital are sufficiently important to generate a bias toward home equities, but their model falls short from matching the magnitude of the observed home bias.6

Abowd (1989) finds a large and negative correlation between unexpected union wage changes and unexpected changes in the stock value of the firm. More recently, Davis and Willen (2000), using data from the Panel Study of Income Dynamics (PSID) to construct synthetic cohorts, find that the correlation between labor income shocks and returns on the S&P500 is substantially negative for male workers that are not college graduate.7 Moreover, they find that for six out of the eight sex-education groups considered in their study, a long position on the worker’s own industry represent a good hedge for labor income risk. The empirical works of Gali

---

6Bottazzi, Pesenti, and van Wincoop (1996) derive a continuous-time VAR model of international portfolio choice which allows for intertemporal interactions between wage rates and capital returns, and apply the model to a large set of OECD countries. Their findings account for an average bias of about 30 percentage points toward domestic securities.

7Davis and Willen (2000) generally finds that the degree of correlation between earning shocks and equity returns rises with education, with a lower bound correlation of -.25 for men who did not finish high school. This is in line with empirical studies on the labor demand in modern economies that consistently find that more educated workers are relatively complimentary to physical capital and the use of advanced technologies.
Rotemberg (2003) and Francis and Ramsey (2004) also document a negative correlation between labor hours and productivity conditioning on productivity shocks.

The influential paper of Baxter and Jermann (1997) directly addresses the implications of human capital risk for international portfolio diversification. If the production function were of the Cobb-Douglas type, the portfolio weight on individual’s home country stock market should be negative, that is investors should short sell their domestic capital. Baxter and Jermann (1997) also assess empirically the impact of human capital on optimal portfolio choice. Following the approach of Campbell and Shiller (1988), they express the returns on physical and human capital ($r^k$ and $r^h$ respectively) as

$$r^k_{t+1} = r^k + (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j}$$

$$r^h_{t+1} = r^h + (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta y_{t+1+j}$$

where $\Delta d$ and $\Delta y$ represent, respectively, the dividend and labor income growth rates, $\rho$ is a positive constant strictly smaller than one and $E_t$ is the mathematical expectation operator conditional on the information available at time $t$. Baxter and Jermann (1997) construct the returns series in equation (1) and equation (2) by fitting a vector error correction model (VECM) for dividend and labor income in each of the four country considered in their study (U.S., U.K., Japan and Germany), assuming that each country is block exogenous with respect to other countries. Under these specification, the authors obtain optimal portfolios according to which investors should hold substantial negative portfolio shares of their home country stocks.

Julliard (2002) shows that the empirical result of Baxter and Jermann (1997) is due to the econometric assumption of block exogeneity, and that this assumption is strongly rejected by the data. Moreover, the paper shows that when the misspecification is corrected, the results are reverted: considering the human capital risk does not unequivocally worsen the international diversification puzzle, and to some extent helps explaining it. The difference in results is due to the fact that the assumption
of block exogeneity implies that the countries considered are not economically and technologically integrated. As an outcome, when this restriction is imposed, the correlations between countries are underestimated and the within country correlation is overestimated i.e. the countries appear not to be internationally integrated. Once this restriction is removed, the effective degree of technological and economic integration becomes evident, implying fewer opportunities to hedge human capital risk by investing in foreign marketable assets.

Consistent with this finding, Palacios-Huerta (2001) shows that, considering the returns to human capital, gains from international financial diversification for a mean-variance investor appear to be smaller than previously reported.

The analysis presented in this paper is part of, and complementary to, the literature that has attempted to explain home bias as a hedge against non-tradable risks.\footnote{See, among others, Eldor, Pines, and Schwarz (1988), Stockman and Dellas (1989), Tesar (1993), Baxter, Jermann, and King (1998), Obstfeld and Rogoff (2001), Serrat (2001), Pesenti and van Wincoop (2002), Heathcote and Perri (2004).}

The present paper is also connected to the works of Constantinides and Duffie (1996), Heaton and Lucas (1996), Willen (1999), Davis and Willen (2000), Storesletten, Telmer, and Yaron (2001) and Wei (2003), on the theoretical relation between labor income and market returns at the individual level, and to the empirical literature on asset returns and labor income: Jagannathan and Wang (1996), Jagannathan, Kubota, and Wang (1996) and Palacios-Huerta (2003), on the human capital augmented Capital Asset Pricing Model (CAPM); Santos and Veronesi (2003) that find that the labor income to consumption ratio forecasts asset returns and is a good conditioning variable for the CAPM and has forecasting ability for future asset returns; Julliard (2004) that shows that expected labor income growth rates have high explanatory power for both the time series and the cross-section of asset returns.
3 International portfolio diversification in a representative agent model

This section presents a simple dynamic representative agent model of international portfolio diversification without short selling constraints. In this setting, I find that there are no evidences that human capital hedging for U.S. investors should imply short positions on the domestic stock market. Nevertheless, such a model is unlikely to reproduce the degree of home bias observed in the data, unless implausibly high levels of risk aversion are assumed.

In order to obtain a model that allows for closed form solutions, I follow the works of Willen (1999), Davis and Willen (2000) and Davis, Nalewaik, and Willen (2000). The representative consumer has exponential utility and maximizes the future stream of expected utility given by

$$E_t \left[ \sum_{\tau=0}^{\infty} \beta^{\tau-t} \left( \frac{-1}{A} \right) \exp (-AC_{\tau}) \right]$$

where $\beta$ is the intertemporal discount factor, $C$ is consumption and $A$ is the coefficient of absolute risk aversion. The financial structure of the economy is as follows: there is one risky domestic financial assets with gross return $R^d_t$, and $J$ foreign risky assets with gross returns (in domestic currency) $R^j_t$, $j = 1...J$; there is a risk free asset with gross return $R^f_t$ that is elastically supplied at the exogenous interest rate; unlimited short sales are possible.

The period budget constraint is

$$C_t + B_t + S^d_t + \sum_{j=1}^{J} S^j_t \leq R^d_t B_{t-1} + R^d_t S^d_{t-1} + \sum_{j=1}^{J} R^j_t S^j_{t-1} + Y_t$$

where $B$ is the dollar amount invested in the riskless asset, $S^d$ is the amount invested in the domestic risky asset and $S^j$ is the amount invested in the risky asset of country $j$. 
The representative agent also receives a labor income \((Y_t)\) that follows an ARIMA process with innovation \(\eta_t\). Moreover, let denote with \(\psi_i\) the \(i^{th}\) coefficient of the moving average representation of labor income, that is

\[
E_t [Y_{t+i}] - E_{t-1} [Y_{t+i}] = \psi_i \eta_t
\]

In order to obtain a closed form solution, we need to assume that labor income innovations and risky asset returns are (conditionally) joint normal

\[
\begin{pmatrix}
\eta_t \\
R_t
\end{pmatrix}
\sim
N
\left(
\begin{pmatrix}
0 \\
E_t [R_t]
\end{pmatrix},
\begin{pmatrix}
\sigma_Y & \Omega \\
\Omega' & \Sigma
\end{pmatrix}
\right)
\]

where \(R_t\) is the vector of returns on the risk assets.

Under these assumptions, the risky asset holdings at time \(t\) in the optimal portfolio are given by

\[
S_t = \frac{1}{A_{d,t}} \Sigma^{-1} E_t [\mathbf{R}_t^e] - \Psi_t \Sigma^{-1} \Omega
\]

where \(\mathbf{R}_t^e\) is the vector of excess returns on risky assets, \(S_t = [S_t^d, S_t^1, ..., S_t^J]'\) is the vector of risky asset holdings, \(a_t\) is the marginal propensity to consume out of generalized wealth\(^9\) and \(\Psi_t\) is the net present value multiplier of labor income innovations given by

\[
\Psi_t = \sum_{\tau=t}^{\infty} \frac{1}{\prod_{i=t}^{\tau} R_i^e} \psi_{\tau-t}.
\]

Equation (3) has a familiar structure and interpretation. If labor income innovations and asset returns were uncorrelated \((\Omega = [0, 0, ..., 0]')\) the solution would be the familiar one given a CARA utility function and normal returns, and the portfolio shares of risky assets would be given by the projection of expected excess returns onto the space of asset returns. This is the market portfolio in the Capital Asset Pricing Model (CAPM) two-fund separation theorem. The size of the portfolio would depend inversely on the level of absolute risk aversion. However, when \(\Omega \neq [0, 0, ..., 0]'\), the second component is the hedge portfolio for the labor income risk and has an intuitive form since \(\Sigma^{-1} \Omega\) is the projection of the covariance between labor income innovation

\(^9\)For a formal definition of \(a_t\) see Davis and Willen (2000).
and asset returns onto the space of asset returns. It is scaled by the discounted cumulative effect of a labor income innovation on wealth \( \Psi_t \), which takes into account the intertemporal element of the human capital hedging problem.

Note that in this setting the demand of precautionary savings does not depend on wealth. Moreover, the coefficient of absolute risk aversion does not affect the optimal demand for the hedge portfolio but only the component that reflects the desire to exploit the excess return on risky assets. The size of the first component declines in risk aversion while the size of the hedge component increases with the degree of persistence of the labor income process.

I now use this model to assess the role of human capital risk for international diversification in a representative agents model. That is, I ask whether a model that does not include short-sale constraints can match the degree of home bias observed in the data.

I use the data set of international stock market returns for 14 major industrialized countries constructed by Campbell (1999) and the aggregate U.S. per capita labor income time series\(^{10}\) to estimate the covariance matrix of returns and labor income innovations. In terms of dynamic absolute risk aversion \( A_a_t \), following the Davis and Willen (2000) estimates, I calibrate this as \( 0.03 \times \gamma \times 10^{-4} \) where \( \gamma \) is the desired level of relative risk aversion. The risk free rate is calibrated to the sample mean of the 3 Month Treasury bill rate.

The model requires labor income to follow an ARIMA process in levels (instead than in log levels, as more commonly assumed). Several ARIMA models seem to fit reasonably well the data and in particular, ARIMA(1,1,2) and ARMA(2,2) seem to perform best in terms of residuals’ correlations, fit and Theil’s \( U \)-statistics. Since both specifications deliver similar results (both quantitatively and qualitatively), in what follows I focus only on the former.

The first column of Table 1 reports the optimal portfolio shares without considering labor income risk. Note that these shares are determined from the first component of

\(^{10}\)A detailed description of the data is provided in the Appendix.
equation (3) and are independent of the calibrated level of risk aversion. About one third of the optimal portfolio is invested in domestic assets in the absence of human capital risk. This is entirely the consequence of the observed means and covariance matrix of returns from different countries.\textsuperscript{11} The results is largely due to the fact that the U.S. stock market has the lowest variance (and a high Sharpe ratio) during the sample period.

The second column of Table 1 reports the hedge component of the optimal portfolio in equation (3), in 1992 dollars. The first thing to notice is that hedging human capital risk, for the representative American investor, requires a long position on the domestic asset. This result holds for a wide range of ARIMA specifications for the labor income process. Moreover, the amount that should be invested in domestic assets, $21,408, is extremely large compared to the actual asset holdings data: the median net worth per household in 1993 was $37,587 while the mean per capita net worth was roughly $26,500.\textsuperscript{12} As expected, the optimal hedged portfolio also implies implausible short positions on several markets (Australia, Denmark, Germany, Netherlands, Sweden).\textsuperscript{13}

To compute the equilibrium portfolio in equation (3) we need to calibrate the value of the coefficient of risk aversion. The third and fourth columns of Table 1 focus on a relative risk aversion of 5. Column three reports the dollar amounts of the optimal portfolio. The first thing to notice is that, with such a level of risk aversion, the model implies a far too high a portfolio share in risky assets: almost a quarter million dollars per capita, that is one order of magnitude larger than what is observed in the data. This is entirely driven by the risk aversion calibration and the first term in equation (3). As a consequence, the effect of the hedge portfolio on the optimal portfolio shares in column four is small. In particular, the holding of domestic assets increase by only 6 percent to a 39 percent level. Columns 5 and 6 report optimal portfolio amounts.

\textsuperscript{11}Summary statistics of market returns are reported in the Table A1 in the appendix


\textsuperscript{13}The hedging portfolio is computed under the infinite horizon assumtpion, but assuming a horizon of 25 years leaves the results basically unchanged.
and shares based on a relative risk aversion coefficient of 45. Many view this as an implausibly high level of risk aversion. Nevertheless it is at the lower bound of the estimated risk aversion coefficient required to match the domestic equity premium (Campbell (1999)). Column 5 shows that, with such a level of risk aversion, the size of the portfolio is reduced to a more reasonable $43,723, and column 6 stresses that in this case the human capital hedging component has a large effect, delivering an equilibrium portfolio share in domestic assets of 67.5 percent. Column 7 and 8, instead of calibrating the level of risk aversion, constrains the total value of the portfolio to match the per capita real wealth in 1992 dollars. In this case the effect of the human capital hedging motive is dramatic, bringing the level of domestic assets in the portfolio of domestic investors in the range of the observed home country bias.

The results in Table 1 assume that households can short sell unlimited amounts of assets. This delivers short positions in the equilibrium portfolio that are hard to reconcile with the empirical evidence. Moreover, to generate a home country bias of the magnitude observed in the data, an implausibly high level of risk aversion is required. Nevertheless, this exercises stresses that, at least in this simple setting, there is no evidence that human capital hedging for US investors should imply short positions on the domestic stock market. Moreover, this main finding will carry over in a more general and realistic setup in the next section.

4 International portfolio choice with short-selling constraints and labor income risk

In this section I relax the assumptions needed in section 3 to obtain a close form solution for equilibrium portfolio holdings, and rely on numerical methods to compute the equilibrium outcome of a model that directly takes into account liquidity constraints.

\(^{14}\)Campbell (1999) estimates a level of relative risk aversion of 245, using the end of period timing convention for consumption, and of 47 using the beginning of period timing convention.
The model is a generalization of Heaton and Lucas (1997) to a multiple asset context and of Michaelides (2003), and the solution methods relies on the methodology proposed by Haliassos and Michaelides (2002).

Each household solves the problem

$$\max_{\{C_t, B_t, S^d_t, \{S^i_j\}_{t=1}^\infty\}} E_0 \sum_{t=0}^\infty \beta^t \frac{C^{1-\gamma}_t}{1-\gamma}$$

subject to the short selling constraints

$$B_t, S^d_t, S^i_j \geq 0 \ \forall t \text{ and } j,$$

the period budget constraint

$$C_t + B_t + S^d_t + \sum_{j=1}^J S^i_j \leq R^d_t B_{t-1} + R^d_t S^d_{t-1} + \sum_{j=1}^J R^i_j S^i_j + Y_t, \quad (4)$$

and the standard transversality condition, where $1 > \beta > 0$ is the discount factor (that is calibrated at the value of .99 per quarter), $\gamma$ is the relative risk aversion coefficient (that is calibrated at the benchmark value of 3), $C_t$ is consumption, $B_t$ is the dollar amount invested in domestic bonds, $S^d_t$ is the amount invested in the domestic stock, $S^i_j$ is the amount invested in the stock of country $j$, $Y_t$ is the labor income, $R^d_t$ is the gross risk free rate, $R^d_t$ is the gross return on the domestic stock and $R^i_j$ is the return on the stock of country $j$.

Following Julliard (2004), assume that aggregate per capita labor income growth follows the process

$$g_{t+1} = \log \frac{Y^q_{t+1}}{Y^q_t} = \mu_y + \varepsilon_{t+1} + \vartheta_1 \varepsilon_t + \vartheta_2 \varepsilon_{t-1} \quad (5)$$

where $\varepsilon_t \sim N(0, \sigma^2_{\epsilon})$. Assume also that the individual labor income of agent $i$ follows the process

$$Y^i_t = Y^q_t P^i_t U^i_t \quad (6)$$

$$P^i_t = GP^i_{t-1} N^i_t \quad (7)$$
where $U^i_t$ is independent of $\varepsilon$, $N$ and asset returns, and $\log U^i_t \sim N\left(-\frac{1}{2}\sigma_u^2, \sigma_u^2\right)$ so that $E[U^i_t] = 1$, $P^i_t$ evolves as a random walk with drift, $\log N^i_t \sim N\left(-\frac{1}{2}\sigma_n^2, \sigma_n^2\right)$, so that $E[N^i_t] = 1$, and $N$ is independent of $\varepsilon$ and asset returns. This specification correspond to Gourinchas and Parker (2002) except for the added term $Y^g_t$ that reflects aggregate economic uncertainty.\textsuperscript{15} Following Gourinchas and Parker (2002) estimates, I calibrate $\sigma_u = 0.073$ and $\sigma_n = 0.105$, and following Julliard (2004), I calibrate $\sigma_\varepsilon = 0.005$, $\vartheta_1 = 1.531$, $\vartheta_2 = 0.598$ and $\mu_y = 0.013$,\textsuperscript{16} therefore assuming that the aggregate labor risk component has a standard deviation that is of a unit of magnitude smaller than the ones of the idiosyncratic components. I assume also that log returns on risky assets and shocks to the aggregate labor income process ($\varepsilon$) are jointly normal.

Under equations (5)-(7) the individual labor income growth is given by

$$\Delta \log Y^i_t = g_t + \log G + \log N^i_t + \Delta \log U^i_t$$

and requires the restriction $\log G = \frac{1}{2}(\sigma_u^2 + \sigma_n^2)$ in order to recover the aggregate labor income growth rate from the individual labor income growth rates.

The model implies the following Euler equations

$$C^{-\gamma}_t = \beta R^d E_t \left[C^{-\gamma}_{t+1}\right] + \lambda_B$$

$$C^{-\gamma}_t = \beta E_t \left[C^{-\gamma}_{t+1} R^d_{t+1}\right] + \lambda_d$$

$$C^{-\gamma}_t = \beta E_t \left[C^{-\gamma}_{t+1} R^f_{t+1}\right] + \lambda_j \quad \forall j$$

where $\lambda_B$, $\lambda_d$ and $\lambda_j$ are the Lagrange multipliers on the short selling constraints for domestic bonds, domestic stocks and foreign stocks. Let $X_t$ be the cash on hand at the beginning of period $t$

$$X_t = R^f B_{t-1} + R^d S^d_{t-1} + \sum_{j=1}^{J} R^i_j S^j_{t-1} + Y_t.$$

\textsuperscript{15}Gourinchas and Parker (2002) also add a small positive probability for $U = 0$, therefore allowing the labor income to be zero with positive probability.

\textsuperscript{16}Details of the estimation of the aggregate labor income growth rate process are reported in the Appendix.
Since the utility function implies that there is no satiety in consumption, the budget constraint will hold with equality and

\[ C_t = X_t - 1_{\{B_t > 0\}} B_t - 1_{\{S_t^d > 0\}} S_t^d - \sum_{j=1}^J 1_{\{S_t^i > 0\}} S_t^i \]  (8)

where \( 1_{\{\}} \) is an index function that takes value 1 if the condition in brackets is satisfied and zero otherwise.

To solve the model, I make the problem stationary dividing all the variables at time \( t \) by

\[ Z_t^i := E_t \left[ Y_{t+2}^i \right] = G^2 P_t^i Y_t^g \exp \left[ (\vartheta_1 + \vartheta_2) \varepsilon_t + \vartheta_2 \varepsilon_{t-1} + k \right] \]

where \( k = 2\mu_y + \left[ 1 + (1 + \vartheta_1)^2 \right] \sigma^2 \). Note also that this implies that

\[ \log \frac{Z_{t+1}^i}{Z_t^i} = \mu_y + (1 + \vartheta_1 + \vartheta_2) \varepsilon_{t+1} + \log G + \log N_{t+1} \]

Using equation (8) and the homogeneity of degree \(-\gamma\) of the marginal utility, we can rewrite the Euler equations as

\[
\left( x_t - b_t - s_t^d - \sum_{j=1}^J s_t^j \right)^{-\gamma} = \max \left\{ \left( x_t - s_t^d - \sum_{j=1}^J s_t^j \right)^{-\gamma} ; \beta R_f E_t \left[ \frac{Z_{t+1}^i}{Z_t^i} \right]^{-\gamma} \right\}
\]

\[
\left( x_t - b_t - s_t^d - \sum_{j=1}^J s_t^j \right)^{-\gamma} = \max \left\{ \left( x_t - b_t - \sum_{j=1}^J s_t^j \right)^{-\gamma} ; \beta E_t \left[ R_{t+1}^d c_{t+1}^{\gamma} \left( \frac{Z_{t+1}^i}{Z_t^i} \right)^{-\gamma} \right] \right\}
\]

\[
\left( x_t - b_t - s_t^d - \sum_{j=1}^J s_t^j \right)^{-\gamma} = \max \left\{ \left( x_t - b_t - s_t^d - \sum_{j=1,j\neq j'}^J s_t^j \right)^{-\gamma} ; \beta E_t \left[ R_{t+1}^j c_{t+1}^{\gamma} \left( \frac{Z_{t+1}^i}{Z_t^i} \right)^{-\gamma} \right] \right\}
\]

\( \forall j' = 1, \ldots, J \)

where the small letters represent the ratios of the capital variables to the normalizing variable \( Z \), and the normalized state variable \( x \) (see Deaton (1991)) evolves according to

\[ x_t = \left( R_t^i b_{t-1} + R_t^i s_{t-1}^d + \sum_{j=1}^J R_t^j s_{t-1}^j \right) \frac{Z_{t-1}^i}{Z_t^i} + \frac{Y_t^i}{Z_t^i} \]  (9)
where
\[
\frac{Y^i_t}{Z^i_t} = G^{-2}U^i_t \exp \left[ - (\vartheta_1 + \vartheta_2) \varepsilon_t - \vartheta_2 \varepsilon_{t-1} - k \right]
\]

In order for the individual Euler equations to define a contraction mapping for the normalized asset holdings optimal rules \( \{b(x, \varepsilon), s^d(x, \varepsilon), s^j(x, \varepsilon)\} \), we need (following Theorem 1 of Deaton and Laroque (1992)) that

\[
\beta R^d_{t+1} E_t \left[ \left( \frac{Z_{t+1}}{Z_t} \right)^{-\gamma} \right] < 1
\]

\[
\beta E_t \left[ R^d_{t+1} \left( \frac{Z_{t+1}}{Z_t} \right)^{-\gamma} \right] < 1
\]

\[
\beta E_t \left[ R^i_{t+1} \left( \frac{Z_{t+1}}{Z_t} \right)^{-\gamma} \right] < 1.
\]

Given the assumptions on the primitives, these conditions hold and there will exist a unique set of optimum policies satisfying the Euler equations. Moreover, the Euler equations are contraction mappings.

Due to the curse of dimensionality of numerical problems, I only focus on four countries: the U.S. (as domestic country), U.K., Japan and Germany, since the market capitalization of this four countries represent close to ninety percent of the world market.

Since the U.S. domestic risky asset has enjoyed both the lowest variance and a high Sharpe ratio compared to the other countries, and this pushes the optimal portfolio to be skewed toward the domestic stock as shown in the previous section, I calibrate all the countries as having the same mean return and Sharpe ratio as the U.S..\(^\text{17}\) A summary of the calibrated values is reported in Table 2.

The first five columns of Table 3 reports the estimated correlations of market returns and labor income innovations that are used in the calibration exercise.

The first thing to notice is that the correlations between aggregate U.S. labor income innovations and quarterly market returns is weakly negative for all the countries\(^\text{17}\) As discussed below and shown in the last column of Table 3, this calibration will bias portfolio holdings against the domestic risky asset.
considered. Second, the differences in estimated correlations with labor income innovations are extremely small, but the absolute value of these correlations is slightly larger for the domestic stock market, closely followed by U.K. and Germany. The correlation between U.S. aggregate labor income shocks and Japan’s stock market is very close to zero.

Are these estimated negative correlations surprising? Bottazzi, Pesenti, and van Wincoop (1996) find a negative correlation between the return to human capital and the domestic stock market returns in ten out of ten countries considered in their study. Moreover, they find that the absolute value of the correlation between return to human capital and foreign returns is smaller than the within country correlation for seven countries out of ten. Their point estimate for the correlation of return to human capital with asset returns in the U.S. is $-0.4$ when considering the domestic stock market and $-0.05$ for the foreign stock market. Also, Abowd (1989) finds a large and negative correlation between unexpected union wage changes and unexpected changes in the stock value of the firm. Moreover, the empirical works of Gali (1999), Rotemberg (2003) and Francis and Ramsey (2004) find a negative correlation between labor hours and productivity conditioning on productivity shocks.

The last column of the Table 3 reports the implied optimal portfolio shares in the market portfolio without human capital risk and shows that, according to the estimated market return correlations and calibrated moments of excess returns, the share of U.S. assets in the domestic portfolio should only be of the order of 17 percent in the absence of aggregate labor income risk.\footnote{This also shows that the bias toward U.S. assets in the market portfolio reported in the first column of Table 1, was largely due to the low variance and better Sharpe ratio of the U.S. market in the sample period considered.}
5 Investor’s optimal policy rules and portfolio choice

Having calibrated the model, we can estimate the optimal policy function by standard numerical dynamic programming techniques (see, among others, Carroll (1992) and Haliassos and Michaelides (2002)) to compute the optimal consumption and asset holding rules. Since optimal policy rules will depend both on the normalized cash on hand and on the last labor income shock, I numerically integrate out this last variable to have policy rules as a function of the cash on hand only, obtaining the investment rules \( \{b(x), s^d(x), s^i(x)\} \). Moreover, from equation (8) we can obtain the optimal consumption rule \( c(x) \).

Optimal policy rules are plotted, as a function of normalized cash-on-hand, in Figure 1. The optimal consumption policy rule has the same shape as in the buffer stock saving literature, with consumption being equal to cash on hand (no saving region) until a target level of cash on hand is reached and saving starts taking place. Once the saving region is reached, the consumers specialize in stocks disregarding bonds. This result, well know in the literature, was originally obtained by Heaton and Lucas (1997) in a domestic portfolio choice settings, and it reflects the implication of the large equity premium for the optimal portfolio choice.

More interestingly, when the consumer enters the saving region she initially invests only in the domestic stocks and, only as the level of cash on hand increases, does she gradually diversify her portfolio internationally. This happens for three reasons. First, only a small buffer stock saving is needed for the agent to protect herself from future labor income shocks. Second, when entering the saving region, the agent prefers to invest in the assets that has the lower correlation with labor income shocks, in order not to increase her overall level of risk correlated with income. As a consequence,\(^{19}\)

---

\(^{19}\)Optimal policy functions do not seem to change significantly as a function of past aggregate labor income shocks (manly due to the very small variance of these shocks compared to the idiosyncratic ones). Policy function computed assuming a plus or minus two standard deviation shock in aggregate labor income are almost identical to the ones obtained after integrating out this variable.
the order in which the agents start investing in the different stock markets closely match the inverse rank of the correlations between labor income and asset returns. Third, only for very high levels of liquid wealth to labor income ratio, does the financial portfolio diversification motive become more important than the labor income hedging one, and the agent starts diversifying fully her portfolio.

Comparing this results with the empirical distribution of cash-on-hand in the PSID data set, less than 1 percent of the population should be investing positive amounts in all the four assets considered. Moreover, given the positive correlation between normalized cash-on-hand and asset wealth observed in the data, the results imply that only the richest households will be diversifying their portfolio internationally, coherently with the empirical evidence on households’ portfolio holdings at the micro level.

Using the estimated policy functions we can compute the optimal portfolio shares as a function of cash-on-hand. These optimal shares are reported in Figure 2. The figure shows a large bias toward domestic assets in all the relevant range of standardized cash on hand implying that more than 99 percent of the population should have an asset portfolio strongly biased toward domestic assets. Compared with the optimal share of domestic assets in the market portfolios without aggregate labor income risk (17 percent), this represent a home country bias that ranges from 83 to 28 percent. Even investors in the top one percent of the distribution of cash-on-hand observed in the data, would have on average more than 50 percent of their asset wealth invested in domestic stocks. Interestingly, this large effect is generated by extremely small differences in the correlations between labor income shocks and market returns across countries. Moreover, even calibrating the model using positive correlations but assigning the same ordering as in Table 2 (i.e. with aggregate labor income shocks having a slightly smaller correlation with domestic market returns than with foreign market returns) the outcome would be again a large home country bias.\footnote{This counterfactual exercise (presented in section C of the Appendix) is discussed in section 7.} This means that small shocks that lower the correlation between labor income and market returns at
a country level generate, in the presence of short selling constraints and buffer-stock saving behavior, a very large degree of domestic bias in portfolio holdings.

6 Implications for the aggregate portfolio

This section derives the implications of the optimal investment rules obtained in the previous section for the aggregate portfolio of U.S. investors.

The standardized cash-on-hand in equation (9) follows a renewal process and can be shown to have an associated invariant distribution, and this can be used to compute the implied aggregate portfolio of U.S. investors. Moreover, given the estimated policy functions, the aggregate portfolio can also be computed using the observed empirical distribution of cash-on-hand. The implied model distribution can be computed in two different ways.

First, conditioning on a given past aggregate labor income shock, we can use the policy functions and equation (9) to compute, by repeated simulation over a grid of values, the transition probabilities from one level of cash-on-hand to the other

\[ T_{lm} = \Pr(x = l|x = m). \]

Given the matrix \( T \) of transition probabilities, the probability of each state is updated by

\[ \pi_{l,t+1} = \frac{1}{\sum_m T_{lm}} \pi_{m,t}. \]

Therefore, the invariant distribution \( \pi \) can be found as the normalized eigenvector of \( T \) corresponding to the unit eigenvalue by solving

\[
\begin{pmatrix}
T - I & 1' \\
1 & 0
\end{pmatrix}
\begin{pmatrix}
\pi \\
0
\end{pmatrix}
=
\begin{pmatrix}
0 \\
1
\end{pmatrix}
\]

where \( I \) is the identity matrix and \( 1 \) is a vector of ones of appropriate dimension. Since this procedure produces invariant distributions conditioned on the past labor income

\footnote{See Deaton and Laroque (1992), Carroll (1997), Szeidl (2002) and Carroll (2004).}
shock, we can integrate out the conditioning variable to obtain the unconditional *model distribution* of $x$.

Second, we can instead draw random initial levels of $x$ to reproduce the initial heterogeneity in wealth among agents, and then simulate dynamically the evolution of normalized cash-on-hand over time, generating what I refer to as the *dynamic distribution* of the model. I perform both procedures since the first one requires fixing ex-ante the relevant range of $x$ while the second one instead determines the relevant range autonomously, therefore providing a robustness check of the construction of the model ergodic distribution.

Figure 3 reports the distributions of normalized cash-on-hand implied by the model and the observed distribution of normalized cash-on-hand in the Panel Study of Income Dynamics (*PSID*) data.\footnote{The *PSID* data contains accurate information on wealth holding of households in 1984, 1989 and 1994. Moreover, the *PSID* provides weights to map the data to a nationally representative sample. A description of the data is provided in the Appendix.} The model distribution and the dynamic distribution are hardly distinguishable, suggesting that the former is an accurate representation on the ergodic distribution of the model. Moreover, the model seems to reproduce fairly well the location of the mode and the shape of the right tail of the empirical distribution, but the model distribution is much less concentrated than the data around the boundary between the saving and the no saving zone implying a higher participation rate in the market than what is observed in the *PSID* data, probably due to the absence of stock market entry costs in the set up of the model.

With these distributions in hand, we can compute the aggregate market portfolio shares of U.S. investors. The first column of Table 4 reports, as benchmark comparison, the CAPM market portfolio implied by the calibrated covariance structure of returns in the absence of labor income risk. The implied aggregate portfolio shares of the model are reported in the second column. There is a dramatic effect of labor income risk on the aggregate portfolio with three quarter of the market portfolio invested in domestic assets. Moreover, the relative investments in foreign stocks are strongly affected, with
a reduction of the portfolio share of the Japanese stock from 30 percent to about zero, and a reduction of the share of German stock from 29 percent to 5 percent. The least affected asset is the U.K. market, since its correlation with aggregate labor income growth is a mere 0.03 higher than the one between domestic assets and labor income: its share in the optimal portfolio declines from 24 percent to 21 percent.

Since agents with different levels of normalized cash-on-hand are likely to have very different amounts of wealth invested in the stock market, the simple computation of the aggregate portfolio reported in column two of Table 4 could be a poor approximation of the aggregate portfolio. To address this issue, the third column of Table 4 weights the model distribution by the contribution to the aggregate portfolio of agents having different levels of cash-on-hand. This weighting of the distribution also corrects for the fact that the model implies a higher degree of market participation than what is observed in the data. The weights are constructed from the PSID data, and are proportional to the total stock market holdings of households belonging to each category of normalized cash-on-hand.

This weighting marginally reduces the degree of home bias reported in column two, but still delivers a portfolio share of domestic stocks of 64 percent, implying that hedging human capital increases the portfolio share of domestic stocks by as much as 47 percent and decreases the portfolio shares of Japanese and German stocks, respectively, by 30 and 18 percent, while the share of U.K. is almost unaffected.

The last two columns of the table show that the main result also holds if I aggregate using the distribution of cash-on-hand observed in the PSID data, with (column five) and without (column four) weighting, instead of the model distribution. The aggregate portfolio shares implied by the empirical distribution are, in both case, extremely similar to the ones obtained weighting the model distribution (column three) and carry the same message: the human capital hedging motive generates a very large home country bias with an increase of the portfolio shares of domestic assets between 45 and 49 percent.
7 Discussion and robustness of results

This section discusses extensions and comparative statics of the buffer-stock saving model presented in the previous sections.

First, what’s the key mechanism delivering a large home bias in the model presented? Does the results depend on the estimated correlations of aggregate labor income innovations and market returns being negative?

The key mechanism is that small differences in the correlation of aggregate labor income innovations and market returns, in the presence of short-selling constraints, lead to a gradual international diversification of investors’ portfolio as their level of normalized cash-on-hand increases. With liquidity constraints, agents cannot borrow to construct an optimally diversified portfolio. Therefore, when their level of liquid wealth to labor income ratio is sufficiently high and they enter the stock market, agents try to minimize the overall wealth risk, investing first in the assets that have the lower degree of correlation with labor income. Only when the ratio of liquid wealth to labor income is sufficiently high and the labor income risk hedging motive becomes less important relative to the financial risk hedging motive, do agents start diversifying their portfolios. Since the distribution of liquid wealth to labor income is - in the data as in the model - concentrated in the region of low liquid wealth to labor income ratios, the resulting aggregate portfolio is heavily skewed toward the asset with the lowest correlation with aggregate labor income shocks. A counterfactual calibration of the model, that assumes that the correlation of labor income shocks and market returns are positive but that, as in Table 3, the correlation with the domestic asset is smaller, delivers results qualitatively similar to the one presented here, therefore generating a large home country bias.\textsuperscript{23} This means that domestic shocks that lead to a redistribution of total income between capital and labor, therefore lowering the correlation between return on physical and human capital, are likely to skew portfolio holdings toward domestic assets. Bottazzi, Pesenti, and van Wincoop (1996) finds that

\textsuperscript{23}This result is reported in section C of the Appendix and in Figure A1.
the correlations of returns to human capital with domestic market returns is smaller than the one with foreign market returns in seven out of ten countries in their study (with an average difference of .19). Therefore, the human capital hedging motive is likely to explain a large fraction of the home country bias in several countries.

Second, the results derived in sections 4-6 have been obtained without considering the exchange rate risk connected with the investment in foreign assets. In the sample period considered, the lower bound on the estimated standard deviation of exchange rates in the three countries considered is about one third of the standard deviation of market returns. Moreover, the exchange rates show a weakly positive correlation with the stock market of the foreign country and seem to be uncorrelated with the U.S. stock market and with labor income innovations. Therefore, adding exchange rate risk to the model would reduce the Sharpe ratio of foreign assets, making foreign investment less attractive and increasing the degree of home bias.

Third, the results do not seem to depend crucially either on the level of relative risk aversion nor on the intertemporal discount factor, since small changes in the calibration of these parameters delivers results that are in line with the ones presented in sections 5 and 6.

8 Conclusion

This paper studies the international diversification puzzle when human capital is considered part of the wealth of nations and heterogeneous agents face short-selling constraints.

This paper models agents as facing both idiosyncratic labor income risk and a small aggregate labor income risk component. Calibrating the model to match the characteristics of the U.S. labor income process and the observed covariance structure

\footnote{Hau and Rey (2003) finds that, at higher frequencies, higher returns in the home equity market relative to the foreign equity market are associated with a home currency depreciation.}
of labor income, domestic returns, and foreign returns, I find that, consistent with the empirical literature, a) investors that enter the stock market with low levels of liquid wealth will initially specialize in domestic assets and, b) only as the level of asset wealth increases, individual portfolios become more internationally diversified adding one foreign stock at a time, c) as a consequence, the aggregate portfolio of U.S. investors shows a large degree of home bias.

This happens for three reasons. First, only a small buffer stock saving is needed for the agent to protect herself from future labor income shocks. Second, when entering the saving region, the agent prefers to invest in the assets that has the lower correlation with labor income shocks, in order not to increase her overall level of risk correlated with income. Third, only for very high levels of liquid wealth to labor income ratio, does the financial portfolio diversification motive become more important than the labor income hedging one, and the agent starts diversifying fully her portfolio. Since the distribution of liquid wealth to labor income is concentrated in the region of low liquid wealth to labor income ratios, the resulting aggregate portfolio is heavily skewed toward domestic assets.

The origin of the lower correlation between labor income innovations and domestic assets returns, with respect to the correlation between labor income innovations and foreign asset returns, documented in this paper and in the previous literature, are not explored here and should be object of future research.
References


FRANCIS, N., AND V. A. RAMSEY (2004): “A New Measure of Hours Per Capita with Implications for the Technology-Hours Debate,” UCSD manuscript.


HEATHCOTE, J., AND F. PERRI (2004): “The International Diversification Puzzle is Not as Bad as You Think,” mimeo.


Macroeconomic Dynamics, 1, 76–101.


Labor-Income Risk and Average Return: Empirical Evidence from the Japanese


Cross-Section of Expected Returns,” The Journal of Finance, 51, 3–53.

JAPPPELLI, T., C. JULLIARD, AND M. PAGANO (2001): “Italian Households’

Portfolio Diversification,” in XIX Rapporto Sul Risparmio e Sui Risparmiatori in

Italia, pp. 91–121. BNL/Centro Einaudi.

JONES, C. I. (2004): “The Shape of Production Functions and the Direction of

Technical Change,” U.C. Berkeley manuscript.

JULLIARD, C. (2002): “The International Diversification Puzzle is not Worse Than

You Think,” Princeton University Manuscript.


Manuscript.


Structure of Earnings in a Longitudinal Data Analysis,” Journal of Econometrics,

pp. 83–114.


and the Home Equity Bias Puzzle,” Journal of Economic Dynamics and Control,

28(3), 555–594.


Macroeconomics: Is There A Common Cause?,” in 2000 NBER Macroeconomics

Annual, ed. by B. S. Bernanke, and K. Rogoff. MIT Press.


International Diversification Puzzle,” Journal of International Economics, 54(2),

309–331.


Appendix

A Data description

The proxy chosen for the market return is the value weighted CRSP (CRSP-VW) market return index. The CRSP index includes NYSE, AMEX and NASDAQ, and should provide a better proxy for market returns than the Standard & Poor (S&P) index since it is a much broader measure. The proxy for the risk free rate is the return on the 3 months Treasury bill.

International quarterly market returns data are taken from the dataset used in Campbell (1999, 2003) and are available over the sample 1972-1999. Summary statistics of international asset returns are reported in Table A1.

Aggregate labor income data are taken from the BEA National Income and Product table 1.14 available through DRI. Population data are three-month averages of monthly data from the U.S. Census data available through DRI.

Asset wealth data are taken from the Panel Study of Income Dynamics (PSID) as in Parker (1999). The PSID data contains accurate information on wealth holding of household in 1984, 1989 and 1994 and do an excellent job at reproducing the wealth distribution of the bottom 99 percent of the wealth distribution. Moreover, the PSID provides weights to map the data to a nationally representative sample.

B Estimation of the aggregate labor income risk component

In order to model the labor income process, we experimented with several specification in the ARIMA class, and performed the standard set of Box-Jenkins selection procedures. In particular, among the model considered, MA(2) and ARMA(1,1) processes fit well to first differences of log labor income. These specifications deliver similar
results, we henceforth restrict attention to the ARIMA(0,1,2) specification for log income since it simplifies the exposition and it has previously used in the literature in similar contexts.\textsuperscript{25} Thus, the fitted earning specification is

$$\Delta y_t = \mu y + \varepsilon_t + \vartheta_1 \varepsilon_{t-1} + \vartheta_2 \varepsilon_{t-2}$$  \hspace{1cm} (B.1)$$

where $\varepsilon_t$ is the earning innovation at time $t$ and $\vartheta$'s are moving-average coefficients. Estimated coefficients are reported in Table A2.

<table>
<thead>
<tr>
<th>Table A2: Estimated Labor Income Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_y$</td>
</tr>
<tr>
<td>0.013</td>
</tr>
<tr>
<td>(0.0008)</td>
</tr>
</tbody>
</table>

Note: Newey-West standard errors reported in brackets

The estimated correlations between log market returns and aggregate labor income growth shocks are reported in Table 3.

C A counterfactual calibration

This section show that the results presented in sections 4-6 are not the outcome of the estimated correlations of aggregate labor income innovations and market returns being negative, but is instead due to the fact that the absolute value of the correlation between labor income innovations and domestic market returns is marginally smaller than the correlation of labor income innovations and foreign market returns.

To stress this point, I perform a counterfactual calibration of the model. First, to focus only on the effect of the correlations of labor income and asset returns, I calibrate the between assets correlation to be all equal to .5. Second, I consider four assets that have equally spaced positive correlations with labor income innovations that ranges from 0.0 to 0.15, therefore reproducing the range of difference in correlations reported in Table 3.

\textsuperscript{25}See Julliard (2004), Davis and Willen (2000) and Macurdy (1982).
Optimal policy functions of this counterfactual exercise are reported in Panel A of Figure A1. As in the baseline model of sections 4-6, when the agent enters the saving region she starts investing only in the asset with the lowest correlation with aggregate labor income shocks. Only at higher levels of liquid wealth to labor income ratios, the agent starts diversifying her portfolio, adding stocks one at a time. Moreover, the order in which the agents start investing in the different stocks matches the inverse rank of the correlations between labor income and asset returns. As a consequence, the optimal portfolio shares reported in Panel B of Figure A1 are heavily skewed toward the asset with the lowest correlation.
### Table A1: Summary Statistics of Quarterly Market Returns

<table>
<thead>
<tr>
<th>Country</th>
<th>Australia</th>
<th>Belgium</th>
<th>Canada</th>
<th>Denmark</th>
<th>France</th>
<th>Germany</th>
<th>Italy</th>
<th>Japan</th>
<th>Netherlands</th>
<th>Spain</th>
<th>Sweden</th>
<th>Switzerland</th>
<th>UK</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.016</td>
<td>0.449</td>
<td>0.582</td>
<td>0.323</td>
<td>0.425</td>
<td>0.395</td>
<td>0.363</td>
<td>0.366</td>
<td>0.517</td>
<td>0.396</td>
<td>0.449</td>
<td>0.479</td>
<td>0.471</td>
<td>0.556</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.006</td>
<td>0.011</td>
<td>0.462</td>
<td>0.406</td>
<td>0.705</td>
<td>0.692</td>
<td>0.473</td>
<td>0.544</td>
<td>0.799</td>
<td>0.480</td>
<td>0.585</td>
<td>0.730</td>
<td>0.541</td>
<td>0.506</td>
</tr>
<tr>
<td>Canada</td>
<td>0.007</td>
<td>0.004</td>
<td>0.008</td>
<td>0.359</td>
<td>0.354</td>
<td>0.332</td>
<td>0.281</td>
<td>0.379</td>
<td>0.543</td>
<td>0.270</td>
<td>0.461</td>
<td>0.459</td>
<td>0.477</td>
<td>0.795</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.004</td>
<td>0.004</td>
<td>0.003</td>
<td>0.011</td>
<td>0.320</td>
<td>0.442</td>
<td>0.369</td>
<td>0.359</td>
<td>0.468</td>
<td>0.214</td>
<td>0.421</td>
<td>0.430</td>
<td>0.245</td>
<td>0.353</td>
</tr>
<tr>
<td>France</td>
<td>0.007</td>
<td>0.010</td>
<td>0.004</td>
<td>0.004</td>
<td>0.017</td>
<td>0.681</td>
<td>0.562</td>
<td>0.422</td>
<td>0.687</td>
<td>0.411</td>
<td>0.450</td>
<td>0.681</td>
<td>0.511</td>
<td>0.472</td>
</tr>
<tr>
<td>Germany</td>
<td>0.005</td>
<td>0.008</td>
<td>0.003</td>
<td>0.005</td>
<td>0.009</td>
<td>0.012</td>
<td>0.463</td>
<td>0.372</td>
<td>0.730</td>
<td>0.355</td>
<td>0.519</td>
<td>0.793</td>
<td>0.425</td>
<td>0.454</td>
</tr>
<tr>
<td>Italy</td>
<td>0.007</td>
<td>0.007</td>
<td>0.004</td>
<td>0.006</td>
<td>0.011</td>
<td>0.007</td>
<td>0.022</td>
<td>0.477</td>
<td>0.429</td>
<td>0.507</td>
<td>0.553</td>
<td>0.452</td>
<td>0.339</td>
<td>0.327</td>
</tr>
<tr>
<td>Japan</td>
<td>0.006</td>
<td>0.007</td>
<td>0.004</td>
<td>0.005</td>
<td>0.007</td>
<td>0.005</td>
<td>0.009</td>
<td>0.016</td>
<td>0.535</td>
<td>0.480</td>
<td>0.487</td>
<td>0.497</td>
<td>0.391</td>
<td>0.453</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.006</td>
<td>0.008</td>
<td>0.005</td>
<td>0.008</td>
<td>0.007</td>
<td>0.006</td>
<td>0.006</td>
<td>0.009</td>
<td>0.391</td>
<td>0.391</td>
<td>0.561</td>
<td>0.768</td>
<td>0.667</td>
<td>0.666</td>
</tr>
<tr>
<td>Spain</td>
<td>0.006</td>
<td>0.007</td>
<td>0.003</td>
<td>0.003</td>
<td>0.007</td>
<td>0.005</td>
<td>0.010</td>
<td>0.008</td>
<td>0.005</td>
<td>0.017</td>
<td>0.515</td>
<td>0.344</td>
<td>0.269</td>
<td>0.340</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.006</td>
<td>0.007</td>
<td>0.005</td>
<td>0.005</td>
<td>0.007</td>
<td>0.006</td>
<td>0.009</td>
<td>0.007</td>
<td>0.006</td>
<td>0.008</td>
<td>0.013</td>
<td>0.572</td>
<td>0.400</td>
<td>0.548</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.006</td>
<td>0.008</td>
<td>0.004</td>
<td>0.005</td>
<td>0.009</td>
<td>0.009</td>
<td>0.007</td>
<td>0.007</td>
<td>0.008</td>
<td>0.005</td>
<td>0.007</td>
<td>0.011</td>
<td>0.541</td>
<td>0.595</td>
</tr>
<tr>
<td>UK</td>
<td>0.008</td>
<td>0.007</td>
<td>0.006</td>
<td>0.003</td>
<td>0.008</td>
<td>0.006</td>
<td>0.006</td>
<td>0.008</td>
<td>0.005</td>
<td>0.005</td>
<td>0.006</td>
<td>0.007</td>
<td>0.017</td>
<td>0.586</td>
</tr>
<tr>
<td>USA</td>
<td>0.006</td>
<td>0.005</td>
<td>0.006</td>
<td>0.003</td>
<td>0.005</td>
<td>0.004</td>
<td>0.004</td>
<td>0.005</td>
<td>0.006</td>
<td>0.004</td>
<td>0.005</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
</tr>
</tbody>
</table>

| Mean Return  | 0.019     | 0.031   | 0.017  | 0.028   | 0.032  | 0.028   | 0.014 | 0.031 | 0.032       | 0.020 | 0.034  | 0.027       | 0.032 | 0.022  |
| Sharpe Ratio | 0.152     | 0.290   | 0.187  | 0.269   | 0.245  | 0.257   | 0.092 | 0.241 | 0.341       | 0.149 | 0.301  | 0.256       | 0.246 | 0.247  |

Note: all returns are in US dollars
Figure A1: Policy Functions and Optimal Portfolio Shares with Positive Correlations

**Panel A: Policy Functions**

**Panel B: Portfolio Shares**
<table>
<thead>
<tr>
<th>Country</th>
<th>Optimal portfolio shares without labor income risk</th>
<th>Hedge portfolio</th>
<th>Relative Risk Aversion = 5</th>
<th>Relative Risk Aversion = 45</th>
<th>Matching per capita net wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimal portfolio</td>
<td>Optimal portfolio shares</td>
<td>Optimal portfolio</td>
<td>Optimal portfolio shares</td>
<td>Optimal portfolio</td>
</tr>
<tr>
<td>USA</td>
<td>33.4%</td>
<td>$21,408</td>
<td>$94,354</td>
<td>39.7%</td>
<td>$29,513</td>
</tr>
<tr>
<td>Australia</td>
<td>-13.4%</td>
<td>-$22,956</td>
<td>-$52,125</td>
<td>-21.9%</td>
<td>-$26,197</td>
</tr>
<tr>
<td>Belgium</td>
<td>3.3%</td>
<td>$31,910</td>
<td>$39,109</td>
<td>16.4%</td>
<td>$32,710</td>
</tr>
<tr>
<td>Canada</td>
<td>-47.4%</td>
<td>$19,358</td>
<td>-$84,097</td>
<td>-35.4%</td>
<td>$7,863</td>
</tr>
<tr>
<td>Denmark</td>
<td>38.7%</td>
<td>-$21,026</td>
<td>$63,553</td>
<td>26.7%</td>
<td>-$11,629</td>
</tr>
<tr>
<td>France</td>
<td>30.5%</td>
<td>-$23,154</td>
<td>$43,348</td>
<td>18.2%</td>
<td>-$15,765</td>
</tr>
<tr>
<td>Germany</td>
<td>3.4%</td>
<td>-$3,737</td>
<td>$3,778</td>
<td>1.6%</td>
<td>-$2,902</td>
</tr>
<tr>
<td>Italy</td>
<td>-46.9%</td>
<td>$596</td>
<td>-$101,695</td>
<td>-42.8%</td>
<td>-$10,769</td>
</tr>
<tr>
<td>Japan</td>
<td>21.1%</td>
<td>$3,384</td>
<td>$49,466</td>
<td>20.8%</td>
<td>$8,504</td>
</tr>
<tr>
<td>Netherlands</td>
<td>49.3%</td>
<td>-$15,884</td>
<td>$91,619</td>
<td>38.5%</td>
<td>-$3,939</td>
</tr>
<tr>
<td>Spain</td>
<td>-7.6%</td>
<td>$12,768</td>
<td>-3,793</td>
<td>-1.6%</td>
<td>$10,928</td>
</tr>
<tr>
<td>Sweden</td>
<td>61.5%</td>
<td>-$10,486</td>
<td>$123,744</td>
<td>52.0%</td>
<td>$4,428</td>
</tr>
<tr>
<td>Switzerland</td>
<td>-40.0%</td>
<td>$18,273</td>
<td>-$69,144</td>
<td>-29.1%</td>
<td>$8,560</td>
</tr>
<tr>
<td>UK</td>
<td>14.0%</td>
<td>$9,016</td>
<td>$39,631</td>
<td>16.7%</td>
<td>$12,418</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>$19,469</td>
<td>$237,749</td>
<td>100%</td>
<td>$43,723</td>
</tr>
</tbody>
</table>

Note: Values expressed in 1992 dollars
Table 2: Calibrated Parameters

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>3</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma_U$</td>
<td>0.105</td>
</tr>
<tr>
<td>$\sigma_N$</td>
<td>0.073</td>
</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>0.005</td>
</tr>
<tr>
<td>$\mu_Y$</td>
<td>0.013</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>1.531</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.598</td>
</tr>
<tr>
<td>Mean Market Return</td>
<td>0.022</td>
</tr>
<tr>
<td>Market Return s.d.</td>
<td>0.084</td>
</tr>
<tr>
<td>Risk Free Rate</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>Germany</td>
</tr>
<tr>
<td>---------------</td>
<td>---------</td>
</tr>
<tr>
<td>U.S.</td>
<td>0.44</td>
</tr>
<tr>
<td>Germany</td>
<td>0.36</td>
</tr>
<tr>
<td>Japan</td>
<td>0.38</td>
</tr>
<tr>
<td>U.K.</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Market Returns and Aggregate Labor Income Shocks
<table>
<thead>
<tr>
<th>Country</th>
<th>Market Portfolio without Labor Income Risk</th>
<th>Model Distribution</th>
<th>Weighted Model Distribution</th>
<th>Empirical Distribution</th>
<th>Weighted Empirical Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>17%</td>
<td>74%</td>
<td>64%</td>
<td>66%</td>
<td>62%</td>
</tr>
<tr>
<td>U.K</td>
<td>24%</td>
<td>21%</td>
<td>25%</td>
<td>23%</td>
<td>26%</td>
</tr>
<tr>
<td>Japan</td>
<td>30%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Germany</td>
<td>29%</td>
<td>5%</td>
<td>11%</td>
<td>10%</td>
<td>12%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>
Figure 1: Optimal Policy Functions
Figure 2: Optimal Portfolio Shares
Figure 3: Distributions of Normalized Cash-on-Hand

Normalized Cash on Hand

Fraction of the sample

Model Distribution
Empirical Distribution
Dynamic Distribution