Network Risk and Key Players: A Structural Analysis of Interbank Liquidity*

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Abstract

We estimate the liquidity multiplier in an interbank market and study systemic risk using a structural network model. In the model, banks hold liquidity to buffer shocks. They borrow liquidity from neighbours and update their valuation based on neighbours actions. When the former (latter) motive dominates, the equilibrium exhibits strategic substitution (complementarity) of holdings, and a reduced (increased) liquidity multiplier dampening (amplifying) shocks. Empirically, we find substantial and procyclical network-generated risks driven mostly by changes of equilibrium type rather than network topology. We identify the banks generating most aggregate risk and solve the planner’s problem, providing guidance to macro-prudential policies.

Keywords: financial networks; liquidity; interbank market; key players; systemic risk.

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I Introduction

The 2007-09 financial crisis has stimulated strong interest in understanding financial intermediation and its role in creating liquidity. As emphasized by Bianchi and Bigio (2014) and Piazzesi and Schneider (2017) among others, intermediaries face their own liquidity management problem, and in particular, banks hold central bank reverses to buffer liquidity shocks. Their choices are crucial for liquidity production, payment activities, and asset prices in the macroeconomy. Another area that has drawn increasing attention is financial networks. The interbank network, where banks borrow and lend reverses, has been at the heart of studies of systemic risk. These two themes merge in our paper. We study both theoretically and empirically how the interbank network affects banks’ liquidity holding decisions, and its implications for systemic risk.

This paper structurally estimates a liquidity-holding game where banks obtain credit from an interbank network and their liquidity management objective incorporates different economic sources of network externality. Applying our framework to U.K. banks, we find that the dominant type of network externality varies over the business cycle. In the boom before 2008, banks’ liquidity holdings exhibit strategic complementarity, and thereby, the network amplifies liquidity shocks. As the financial crisis unfolds, the degree of strategic complementarity declines, and after the introduction of Quantitative Easing (QE) in the U.K., banks’ liquidity holdings exhibit strategic substitution, and the interbank network turns from a shock amplifier to a shock buffer. To the best of our knowledge, we provide the first evidence of a procyclical interbank network externality.

Our framework also offers several novel metrics to guide the monitoring of banks and the design of policy intervention during a crisis. Specifically, we identify the banks that contribute the most to the volatility of aggregate liquidity holdings. Banks’ contribution to systemic risk varies significantly over time. We show that such variation is mostly driven by changes of the type of equilibrium on the network (i.e., strategic complementarity or substitution) rather than changes of the network topology. Furthermore, we compare the decentralized equilibrium with the planner’s solution that achieves constrained efficiency.

We model banks’ decision of holding reserves to manage their exposure to liquidity shocks in a linear-quadratic framework (à la Ballester, Calvo-Armengol, and Zenou (2006)), assuming a predetermined interbank network where banks borrow and lend reserves. Bank char-

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1 In the recent macro-finance literature, intermediaries play the role of marginal investor in asset markets (Brummermeier and Sannikov (2014); He and Krishnamurthy (2013)), credit supplier (Gertler and Kiyotaki (2010); Klenkno, Pfeil, Rochet, and Nicolo (2016)), and money issuer (Brummermeier and Sannikov (2016); Hart and Zingales (2014); Li (2017); Quadroni (2017)).

2 Recent theories on interbank network and systemic risk include Freixas, Parigi, and Rochet (2000), Allen, Carletti, and Gale (2008), and Freixas, Martin, and Skeie (2011).
acteristics and the macroeconomic environment affect banks’ decision, but reserve holdings also depend on the network topology and the nature of network externality. The interbank network generates two counteracting effects for the banks’ liquidity management problem. First, since banks can borrow reserves from their neighbours, the marginal benefit of holding reserves on their own decreases when neighbours hold more reserves. This free-riding incentive gives rise to strategic substitution (Bhattacharya and Gale (1987)). Consequently, the network acts as a risk buffer for liquidity shocks since neighbouring banks’ liquidity holdings are negatively correlated. Second, when banks see neighbouring banks holding more reserves, they positively update their belief on the value of liquidity (e.g., DeMarzo, Vayanos, and Zwiebel (2003)). Due to such informational spillover, the marginal benefit of holding liquidity increases in neighbours’ liquidity, which leads to strategic complementarity. In this case, the network amplifies the liquidity shocks originating from individual banks due to the positive correlation among neighbouring banks’ liquidity holdings. Another channel through which banks’ liquidity holdings may increase in their neighbours’ is the leverage stack mechanism in Moore (2012). At the (unique interior) Nash equilibrium of our model, the overall impact of network on banks’ liquidity holdings is summarized by a parameter $\phi$, the network attenuation factor. If strategic substitution dominates, $\phi$ is negative. If strategic complementarity dominates, $\phi$ is positive.

In equilibrium, the individual banks’ reserve holding decision is affected by the magnitude of the liquidity shocks of all banks in the network. However, not all shocks are equally important. In particular, network features such as the dominant type of network externality, $\phi$, and bank $i$’s indegree Katz-Bonacich centrality (network topology) determine how a bank weights the importance of all liquidity shocks in the network to optimize its own liquidity management problem. The network indegree centrality gives a direct metric of such a weight since it counts the direct and indirect links from other banks towards bank $i$, weighting connections by $\phi^k$, where $k$ is the number of steps needed to reach bank $i$. In other words, the liquidity holding decision of a bank is related to its own shocks, the shocks of its neighbours, of the neighbours of its neighbours, etc., with distant shocks becoming increasingly less important as they are weighted by $\phi^k$. It is important to emphasize that the network topology (e.g., Katz–Bonacich centrality measures) is not the only determinant of banks’ liquidity holding decision. The magnitude of idiosyncratic shocks of all banks in the network and the network attenuation factor $\phi$ are also crucial in the equilibrium liquidity holding

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3In Moore (2012), by signalling its credit worthiness through a higher liquidity buffer, banks can borrow more from other banks, and thereby, finance more positive NPV projects.

4This centrality measure takes into account the number of both immediate and distant connections in a network. For more on the Bonacich centrality measure, see Bonacich (1987) and Jackson (2003). For other economic applications, see Ballester, Calvo-Armengol, and Zenou (2006) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012). For an excellent review of the literature, see Jackson and Zenou (2012).
decisions of the banks. For example, banks that receive extremely large liquidity shocks, regardless of their network location, may have a large impact on all banks’ liquidity holding decisions in equilibrium. Similarly, whether the network effect is dominated by substitution or complementarity determines the nature of shock transmission in the network.

Based on the network equilibrium results, we conduct further welfare analysis for potential policy interventions to remedy the negative impact of network externalities. We characterise the volatility of the aggregate liquidity and identify key banks that contributes the most to the systemic risk – i.e. the risk key players. We find that the contribution by each bank to the network risk is related to 1) the network attenuation factor $\phi$, 2) the bank specific liquidity risks (captured by bank-specific standard deviations), and 3) its outdegree Katz–Bonacich centrality measure. The outdegree centrality is similarly defined as the indegree centrality but the connections are outbound from bank $i$ to measure the impact of bank $i$ on its neighbours, neighbours of its neighbours, etc. Moreover, we introduce the concept of network impulse response function (NIRF), and show that the contribution of each bank to aggregate risk is measured by the NIRF to that bank’s individual shock. The NIRFs are true impulse response function in that the total volatility of aggregate liquidity can be decomposed into NIRFs to bank specific shocks, and the risk key player is precisely the bank with the largest NIRF. Furthermore, we solve for the planner’s solution and contrast it with the decentralised equilibrium level of systemic liquidity level and risk. This analysis allows us to characterise the sources of the externalities and pinpoint possible interventions to achieve the social optimum.

Using daily data from the Bank of England, we apply our model to study the reserve holding decisions of the member banks of the sterling large payment system, CHAPS, in the period of January 2006 to September 2010. Member banks conduct transaction for their own purpose and on behalf of their clients and hundreds of other non-member banks. The stability of this system is crucial for supporting the real economic activities. On average in 2009, £272 billions of transactions in the U.K. were settled every day in CHAPS (U.K. nominal GDP every 5.5 days). CHAPS banks regularly have intraday liquidity exposures in excess of £1 billion to individual counterparties, and they hold reserves to buffer payment imbalances.\textsuperscript{5} Variation in payment imbalances is as close as we can get to a pure liquidity shocks, because CHAPS transactions are settled in real time and on \textit{gross} terms (“RTGS”) to eliminate counterparty credit risks.\textsuperscript{6} In addition to banks’ own reserve holdings, banks

\textsuperscript{5}The U.K. monetary framework leaves reserves management largely at individual banks’ discretion (even during the QE period). In Appendix A.1, we provide background information on the monetary framework (i.e. reserve regimes) including QE, the payment system, and the overnight interbank markets.

\textsuperscript{6}CHAPS uses RTGS instead of DNS (deferred net settlement). The DNS model is more liquidity efficient but creates credit risk exposure for recipient banks until the end of a clearing cycle. Such risks do not exist under RTGS since all payments are settled individually and on a gross basis. A detailed description of the
can borrow reserves from each other on an unsecured basis in the overnight market. These interbank connections form a network – a link between two banks is quantified by the fraction of borrowing by one bank from another in the recent past, so the network is directional and its adjacency matrix is weighted (i.e., right stochastic). Note that the links between two banks can be interpreted as the (frequentist) probabilities of their borrowing-lending relationships per unit of currency. We study the impact of this interbank network on banks’ reserve holdings.\(^7\)

For our empirical analysis we exploit the fact that the equilibrium of our model maps exactly into the spatial error framework, which naturally separates the hypothetical liquidity holdings of a standalone bank and the network induced component. We allow the non-network component to load on bank characteristics and macro variables, and confine the network component of liquidity holdings only in the spatial error term. This conservative approach leaves a minimal amount of variation in liquidity holdings to be driven by the interbank network.\(^8\) Yet, we are able to uncover rich, pro-cyclical, dynamics for the network externality: the network amplifies shocks in good times, and dampens them during the crisis and its aftermath. To show the importance of modelling the role of the network to analyze aggregate risks, we compute the ratio of the volatility of the aggregate liquidity implied by our estimate of network multiplier to the counterfactual volatility where the network and its externalities play no role (i.e. the network attenuation factor is zero). We find that during the boom, this ratio reaches 559%, during the crisis, it is 125% and during the QE, it is reduced to 89%.

Our finding of time-varying network externality sheds light on the relative importance of different economic forces over the business cycle. Because it is costly to hold liquidity at the expense of forgoing other investments, banks free ride their peers by borrowing reserves in interbank network in response to liquidity shocks (Bhattacharya and Gale (1987)). This common wisdom is only part of the story, because strategic complementarity arises from informational spillovers and/or a “leverage stack” type mechanism. Our framework accommodates all these facets of network externality, and allows us to identify from the data the dominant forces at different points in time.

In addition we empirically characterize the shock propagation mechanism, quantify the

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\(^7\)In addition to central bank reserves, member banks may pledge government bonds as collateral to borrow reserves from the Bank of England. Therefore, in our definition of reserve holdings, we include both actual reserves and the holdings of collateral eligible for repo with the Bank of England. Our results are robust if we use actual reserves instead.

\(^8\)We also estimated a spatial Durbin model, in which the network not only propagates shocks in the error terms but also from bank characteristics. This more general model also serves as a specification test of our benchmark framework.
individual banks’ contribution to aggregate liquidity risk, and identify the risk key players. We find that most of the volatility of aggregate liquidity in the banking system is driven by a small group of banks, and that each bank’s contribution varies substantially over time. Moreover, we find that the risk key player is typically not the largest net borrower – even net lenders can generate substantial risk in the system. These findings are particularly relevant for monitoring and regulating the banking system, and policy interventions during crisis.

Since in our sample the network topology changes over time, we decompose the time variation of banks’ risk contributions into two components: the changes ascribable to time variation in $\phi$ and in the network topology. We find that the former is clearly the main driver. This suggests that endogenous network formation plays a limited role in the variation of network effect in our context. It is the type of equilibrium on the network (i.e., strategic complementarity or substitution) that matters most, not the network itself.\footnote{Our finding is related to the empirical literature that critically examines the systemic consequence of network linkages. While there is an extensive literature on network contagion, simulation studies based on reasonably realistic network shows small impact of contagion on systemic bank failure (summarized in Upper (2011)). Related, using a unique dataset of all Austrian banks, Elsinger, Lehar, and Summer (2006) find that contagion happens rarely and that the necessary funds to prevent contagion are surprisingly small. By applying an Eisenberg and Noe (2001) style model to German banks, Chen, Wang, and Yao (2016) find that the lack of bank capital is the key contributor to bank failure rather than the network contagion.}

Finally, we compute the planner’s solution based on our estimates of network multiplier, the sizes of bank-level shocks and other parameters. We find that during the boom period, the amount of aggregate liquidity held by banks is not too far from the planner’s equilibrium but the network generates too much systemic liquidity risk through shock amplification. During the crisis period, the decentralized equilibrium generates smaller aggregate liquidity than the planner’s solution, and the systemic liquidity risk is still too high. After the introduction of QE, banks hold too much liquidity and the volatility of aggregate liquidity is below the level obtained in planner’s solution. These findings may guide policy makers in monitoring banks, designing crisis interventions, and assessing the impact of QE.

This paper is related to the literature on bank liquidity management and monetary policy, and more broadly, the interaction between financial intermediation and the macroeconomy. Bernanke and Blinder (1988) embed bank reserve management in an IS-LM model. Kashyap and Stein (2000) find that the impact of monetary policy on bank lending is stronger for banks with less liquid assets. Bianchi and Bigio (2014) study monetary policy transmission in a dynamic model of banks’ liquidity management. In a model where risk tolerant bankers are the marginal investor in asset markets, Drechsler, Savov, and Schnabl (2014) link risk premia to monetary policy by highlightings bankers’ need to hold reserves as a buffer against liquidity shocks. Piazzesi and Schneider (2017) provide a macroeconomic model of the interaction between interbank transactions and payment activities of the rest of the economy.
The linkages between payment system, reserves, and interbank credit are crucial elements in liquidity production that affects asset prices, money and credit supply, and gives roles to monetary policy intervention. Our paper complements this line of inquiry by providing evidence on these linkages, especially the state-dependent impact of interbank credit network on banks’ liquidity management.

We contribute to the literature on bank liquidity regulation by providing an empirical framework to attribute systemic risk to individual banks, and by characterizing the wedge between decentralised outcome and the planner’s solution. Liquidity regulation has attracted a lot of attention after the financial crisis. Stein (2012) argues that reserves requirement may serve as a tool for financial stability regulation. Diamond and Kashyap (2016) study bank liquidity regulation in the setting of Diamond and Dybvig (1983). Allen and Gale (2017) review earlier theories that may provide foundations (i.e., sources of market failures) for bank liquidity regulations, such as liquidity coverage ratio and net stable funding ratio in Basel III. Our findings of pro-cyclical network externality and banks’ time-varying contribution to systemic risk lend support to a macro-prudential perspective on liquidity regulation.

Our work also advances the literature on interbank market dynamics and banks’ liquidity demand. Fecht, Nyborg, and Rocholl (2010) find that the prices of liquidity depend on counterparties’ liquidity levels. Acharya and Merrouche (2010) document evidence of precautionary liquidity demands of U.K. banks during the subprime crisis. Ashcraft, McAndrews, and Skeie (2010) find that in response to heightened payment uncertainty, banks hold excess reserves in the Fed fund market. There is also a related theoretical literature pioneered by Bhattacharya and Gale (1987). Recent theoretical works in this area highlight the externalities in interbank markets and the associated inefficiencies (e.g. Freixas, Parigi, and Rochet (2000); Allen, Carletti, and Gale (2008); Freixas, Martin, and Skeie (2011); Moore (2012); Castiglionesi, Feriozzi, and Lorenzoni (2017) among others). Our paper differs by modeling banks’ liquidity holdings as outcome of a network game, and estimate the time-varying network externality.\(^{10}\)

Our structural estimation contributes to the broad literature of network and systemic demand. Fecht, Nyborg, and Rocholl (2010) find that the prices of liquidity depend on counterparties’ liquidity levels. Acharya and Merrouche (2010) document evidence of precautionary liquidity demands of U.K. banks during the subprime crisis. Ashcraft, McAndrews, and Skeie (2010) find that in response to heightened payment uncertainty, banks hold excess reserves in the Fed fund market. There is also a related theoretical literature pioneered by Bhattacharya and Gale (1987). Recent theoretical works in this area highlight the externalities in interbank markets and the associated inefficiencies (e.g. Freixas, Parigi, and Rochet (2000); Allen, Carletti, and Gale (2008); Freixas, Martin, and Skeie (2011); Moore (2012); Castiglionesi, Feriozzi, and Lorenzoni (2017) among others). Our paper differs by modeling banks’ liquidity holdings as outcome of a network game, and estimate the time-varying network externality.\(^{10}\)

Our structural estimation contributes to the broad literature of network and systemic

\(^{10}\)There is another line of research that focuses on the topology and formation of linkages. Afonso and Lagos (2015) use a search theoretical framework to study the interbank market and banks’ trading behavior. The empirical literature on the topology of interbank networks starts with Furfine (2000, 2003). Other earlier empirical studies of the interbank network topology include Upper and Worms (2004); Boss, Elsinger, Summer, and Thurner (2004); Soramäki, Bech, Arnold, Glass, and Beyeler (2007); Becher, Millard, and Soramäki (2008); and Bech and Atalay (2008). Recent works study the impact of the crisis on the structure of these networks, which include (but are not limited to): Gai and Kapadia (2010); Wetherill, Zimmerman, and Soramäki (2010); Benos, Garett, and Zimmerman (2010); Ball, Denbee, Manning, and Wetherill (2011); and Afonso, Kovner, and Schoar (2011). We differ from this literature by studying, rather than network formation, the types of equilibria on a predetermined network. We empirically show that the variation of network externality is driven by the type of equilibrium on network instead of the changes in network topology.
risks. Networks have proved to be a useful analytical tool for studying financial contagion and systemic risk from both theoretical and empirical perspectives. Starting from Allen and Gale (2000), recent theories feature increasingly sophisticated networks and shock transmission mechanisms.\textsuperscript{11} Recent empirical works also cover a wide range of economic networks.\textsuperscript{12} We differ from these papers by building upon the linear-quadratic approach of Ballester, Calvo-Armengol, and Zenou (2006) to analyze the impact of economic agents’ optimal liquidity holding decision on a network game for systemic liquidity risk and estimate its time-varying properties.

The remainder of the paper is organised as follows. In Section II we present and solve a liquidity holding decision game in a network, and define key players in terms of level and risk. Section III casts the equilibrium of the liquidity network game in the spatial econometric framework, and outlines the estimation methodology. In Section IV, we describe the data, the construction of the network, and the basic network characteristics throughout the sample period. In Section V, we present and discuss the estimation results. Section VI concludes.

\section{The Network Model}

In this section, in order to study how aggregate liquidity risk is generated within the interbank system, we present a network model of interbank liquidity holding decisions, where the network reflects bilateral borrowing and lending relationships.

The \textit{network}: there is a finite set of \( n \) banks. The time \( t \) network, denoted by \( g_t \), is endowed with an \( n \)-square adjacency matrix \( G_t \) where \( g_{ii,t} = 0 \) and \( g_{ij} \neq i,t \) is the fraction of borrowing by bank \( i \) from bank \( j \). The network \( g_t \) is therefore weighted and directed.\textsuperscript{13} Banks \( i \) and \( j \) are directly connected (in other words, they have a direct lending or borrowing

\textsuperscript{11}This line research includes but is not limited to Leitner (2005); Babus (2009); Babus and Allen (2009) (for a review); Afonso and Shin (2011); Zawadowski (2012); Acemoglu, Carvalho, Ozdaglar, and Tabbaz-Salehi (2012); Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015); Elliott, Golub, and Jackson (2015); Atkeson, Eisfeldt, and Well (2015); Ozdagli and Weber (2015); Glasserman and Young (2015); Cabrales, Gale, and Gottardi (2015); Cabrales, Gottardi, and Vega-Redondo (2016).

\textsuperscript{12}The recent empirical network literature include but is not limited to Diebold and Yilmaz (2009, 2014); Billioa, Getmanskyb, Lo, and Pelizzona (2012); Hautsch, Schaumburg, and Schienle (2012); Aldasoro and Angeloni (2013); Kelly, Lustig, and Nieuwerburgh (2013); Duarte and Eisenbach (2013); Greenwood, Landier, and Thesmar (2015); and Gofman (2017).

\textsuperscript{13}We also explore other definitions of the adjacency matrix, where \( g_{ij} \) is either the sterling amount of borrowing by bank \( i \) from bank \( j \), or 1 (0) if there is (no) borrowing or lending between Bank \( i \) and \( j \). Note that, in this latter case, the adjacency matrix is unweighted and undirected. In the theoretical part of the paper, we provide results and intuitions for the case when \( G_t \) is a right stochastic matrix. However, the results are easily extendable to other forms of adjacency matrices with some restrictions on the parameter values which we will highlight when needed.
relationship) if $g_{ij,t}$ or $g_{ji,t} \neq 0$. The coefficient $g_{ij,t}$ can be interpreted as the frequentist estimate of the probability of bank $i$’s receiving one pound from bank $j$ via direct borrowing.

The matrix $G_t$ is a (right) stochastic (hollow) matrix by construction, is not symmetric, and keeps track of all direct connections – links of order one – between network players. That is, it summarises all the paths of length one between any pair of banks in the network. Similarly, the matrix $G^k_t$, for any positive integer $k$, encodes all links of order $k$ between banks, that is, the paths of length $k$ between any pair of banks in the network. For example, the coefficient in the $(i, j)$th cell of $G^k_t$ – i.e. $\{G^k_t\}_{ij}$ – gives the amount of exposure of bank $i$ to bank $j$ in $k$ steps. Since, in our baseline construction, $G_t$ is a right stochastic matrix, $G_t$ can also be interpreted as a Markov chain transition kernel, implying that $G^k_t$ can be thought of as the $k$-step transition probability matrix, i.e. the matrix with elements given by the probabilities of reaching bank $j$ from bank $i$ in $k$ steps. Simply put, the matrix $G_t$ measures how liquidity travels in the interbank network.

**Banks and their liquidity preferences in a network:** we study the amount of liquidity buffer stocks (reserves) banks choose to hold at the beginning of day $t$ when they have access to the interbank borrowing and lending network $g_t$. We define the total liquidity holding by bank $i$, denoted by $l_{i,t}$. In the main text, we model $l_{i,t}$ as the sum of two components: bank $i$’s liquidity holdings absent of any bilateral effects (i.e. the level of liquidity that a bank would hold if it were not part of a network), and bank $i$’s level of liquidity holdings made available to the network, which depends (potentially) on its neighbouring banks’ liquidity contributions to the network. We use $q_{i,t}$ and $z_{i,t}$ to denote these two components respectively, and $l_{i,t} = q_{i,t} + z_{i,t}$. Before modelling the network effect on banks’ liquidity choices, we assume $q_{i,t}$ load on all bank-specific and macro variables as follows:

$$q_{i,t} = \alpha_i + \sum_{m=1}^{M} \beta_m x^m_{i,t} + \sum_{p=1}^{P} \beta_p x^p_{i,t},$$  \hspace{1cm} (1)

where $\alpha_i$ is a bank fixed-effect, $x^m_{i,t}$ belongs to a set of $M$ variables accounting for observable bank characteristics, and $x^p_{i,t}$ belongs to a set of $P$ variables controlling for time-series variation in systematic factors. Thus, $q_{i,t}$ captures the liquidity need specific to an individual bank from its balance sheet and fundamental conditions (e.g., leverage ratio, lending and borrowing rate), and its exposure to macro shocks (e.g., aggregate activities, monetary policy, etc.).

The network component $z_{i,t}$ is thus modelled as a residual term. This specification can be interpreted as a conservative approach to model the network effects since, empirically, we allow the residual variation of $l_{i,t}$ to be driven by the network component. As a robustness check, we also model and estimate the network effect when $l_{i,t}$ is not decomposed into a
standalone and a bilateral component.\textsuperscript{14}

To study a bank’s choice of $z_{i,t}$, that is, its liquidity holdings in response to network access, we need to model the various sources of bilateral effects. We assume that banks are situated in different locations of the interbank relationship network $g_t$. The network allows banks to borrow and lend reserves, and may also transmit information relevant for liquidity management (more on this later). Each bank decides simultaneously how much liquid stock $z_{i,t}$ to hold given a predetermined $g_t$.\textsuperscript{15} The vector $z_t$ records all banks’ choices.

We assume that banks derive utility from having an accessible buffer stock of liquidity, but at the same time they dislike the variability of this quantity. The accessible network liquidity for bank $i$ has two components: direct holdings, $z_{i,t}$, and what can be borrowed from other banks connected through the network. This second component is proportional to the neighbouring banks direct holdings, $z_{j,t}$, weighted by the network linkage, $g_{ij,t}$, and a technological parameter $\psi$, that is, $\psi \sum_j g_{ij,t} z_{j,t}$. This component can be thought as unsecured borrowing through the interbank network. The marginal benefit of accessible liquidity for bank $i$ is $\tilde{\mu}_{i,t}$ per unit. In sum, banks’ liquidity management objective is represented by the following quadratic utility function:

$$u_i(z_t | g_t) = \underbrace{\tilde{\mu}_{i,t}}_{\text{Per Unit Value}} \left( z_{i,t} + \psi \sum_{j \neq i} g_{ij,t} z_{j,t} \right) - \frac{1}{2} \underbrace{\gamma}_{\text{Risk Aversion}} \left( z_{i,t} + \psi \sum_{j \neq i} g_{ij,t} z_{j,t} \right)^2,$$

where $\gamma$ is the banks’ risk aversion parameter. By establishing bilateral relationships in the banking network $g_t$, a bank also exposes itself to the shocks from its neighbouring banks. We assume that banks dislike the volatility of their own liquidity and of the liquidity they can access given their links, hence the last term in the objective function (2) arises.\textsuperscript{16}

We further decompose $\tilde{\mu}_{i,t}$ into a bank-specific stochastic component $\hat{\mu}_{i,t}$, and a network component, so

$$\tilde{\mu}_{i,t} := \hat{\mu}_{i,t} + \delta \sum_{j \neq i} g_{ij,t} z_{j,t},$$  \hfill (3)

\textsuperscript{14}In the Appendix A.2, we report the results of this alternative formulation where the network effect on banks’ liquidity holding decisions is not modelled a residual. The empirical counterpart of this model is a spatial Durbin model, which serves as a specification test for our baseline model.

\textsuperscript{15}Since we model and estimate banks’ liquidity holdings at the daily level, it is intuitive to take $g_t$ as predetermined as the beginning of the day, as it is unlikely that the interbank relationships change significantly within a day.

\textsuperscript{16}This objective function has the same spirit as a mean-variance utility in portfolio theory.
where
\[ \hat{\mu}_{i,t} / \gamma =: \bar{\mu}_i + \nu_{i,t}, \]
the parameter \( \bar{\mu}_i \) denotes the bank specific average valuation of liquidity (absent any valuation spillovers and scaled by \( \gamma \)), and \( \nu_{i,t} \) denotes the bank-specific shock to the valuation. The randomness of \( \hat{\mu}_{i,t} \) is the ultimate source of uncertainty in this system. As it is part of banks’ valuation of liquidity, we interpret this randomness as capturing banks’ revision of belief on the forthcoming intraday payment imbalance (i.e., the liquidity shock). The network component of \( \hat{\mu}_{i,t} \) is motivated by potential informational spillover. Even though banks may value liquidity differently (due to private value), neighbours’ liquidity holdings can be informative about the common value of reserves. We assume bank \( i \) follows a simple updating rule that adds \( \delta \sum_j g_{i,j,t} z_{j,t} \) to the standalone valuation \( \hat{\mu}_{i,t} \). This updating rule is in the spirit of the boundedly-rational model of opinion formation in DeMarzo, Vayanos and Zwiebel (2003) (see also DeGroot (1974)).\(^{17}\) Therefore, a smaller coefficient \( \delta \) reflects a larger informational discount on neighbouring banks’ holdings, and the network linkages direct information flows via the interbank network.

The bilateral network influences are captured by the following cross derivatives for \( i \neq j \):
\[ \frac{\partial^2 u_i (z_t | g_t)}{\partial z_{i,t} \partial z_{j,t}} = (\delta - \gamma \psi) g_{i,j,t}. \]
When the cross derivative is negative, i.e. when \( \delta < \gamma \psi \), banks’ liquidity holdings exhibit strategic substitution. That is, an individual bank sets aside a smaller amount of liquid assets when its neighbouring banks hold more liquidity, which it can draw upon. This reflects the typical free-riding incentive as in Bhattacharya and Gale (1987).\(^{18}\) In our model, strategic substitutability arises from the fact that banks dislike volatility in their accessible liquidity, and therefore prefer to hold buffer stocks of liquidity that are less correlated with the ones of the neighbouring banks. Since the degree of accessibility of neighbours’ liquidity increase in \( \psi \), and the dislike of uncertainty is captured by \( \gamma \), the degree of strategic substitutability in increasing in these two parameters.

Strategic complementarity arises when \( \delta > \gamma \psi \). Through our interbank network not only flows liquidity (via borrowing and lending) but also the information on the common value

\(^{17}\)Note that this updating rule is not Bayesian. We choose this updating rule for expositional clarity in capturing two opposing network bilateral effects, as shown later. There is a separate but growing literature that studies the role of information aggregation in network settings (DeMarzo, Vayanos, and Zwiebel (2003); Babus and Kondor (2013)).

\(^{18}\)Bhattacharya and Gale (1987) show that banks’ liquidity holdings are strategic substitutes, because liquidity holdings come at a cost of forgoing higher interest revenue from long-term investments. Banks would like to free-ride their neighbouring banks for liquidity rather than conducting precautionary liquidity savings themselves.
of reserves. Strategic complementarity arises precisely from the informational spillover. We would expect a higher \( \delta \), and stronger strategic complementarity, when the common value of reserves is more prominent than the private value among banks.

Even if we restrict the interbank network to be only relevant for fund flows rather than information flow, the strategic complementary may still arise as a result of leverage stack as in Moore (2012). Moore (2012) models a chain of borrowing/lending relationships that starts from the bank who borrows from households and ends at the bank with investment project. Interbank loans can be pledged to upstream lenders as collateral, so the collateral haircut is higher. Under this alternative formulation, we may posit bank \( i \)'s objective function as follows:

\[
\begin{align*}
   u_i(z_i|g_i) &= \hat{\mu}_{i,t} \left( \frac{z_{i,t} + \psi \sum_{j \neq i} g_{ij,t} z_{j,t}}{\gamma} \right)^{-1} \left( \frac{z_{i,t} + \psi \sum_{j \neq i} g_{ij,t} z_{j,t}}{\gamma} \right)^2 + \frac{z_{i,t} \delta \sum_{j \neq i} g_{ij,t} z_{j,t}}{\gamma}, \tag{4}
\end{align*}
\]

The “collateralised” liquidity term, \( z_{i,t} \delta \sum_{j} g_{ij,t} z_{j,t} \) has two parts: the available reserves that could be borrowed from neighbours, \( \sum_{j} g_{ij,t} z_{j,t} \), and the multiplication factor \( z_{i,t} \delta \), which can be thought of as collateral posted by bank \( i \) with the parameter \( \delta \) reflecting a haircut. Since in the empirical context banks borrow and lend reserves on an unsecured basis, we may also interpret the multiplication factor as “information collateral,” i.e., by holding liquidity \( z_{i,t} \), bank \( i \) signals its creditworthiness to neighbouring banks in the interbank network. Note that whether banks’ liquidity management objective is from Equation (2) or Equation (4) does not change the equilibrium outcome (i.e., their first-order conditions, best response functions, and equilibrium, stay the same).

**Equilibrium behaviour**: now, we solve banks’ optimal reserve holdings in the Nash equilibrium. Banks choose their liquidity level \( z_{i,t} \) simultaneously. A representative bank \( i \) maximises (2), and we obtain the following best response function for each bank:

\[
\begin{align*}
   z_{i,t}^* &= \hat{\mu}_{i,t} + \left( \frac{\delta}{\gamma} - \psi \right) \sum_{j \neq i} g_{ij,t} z_{j,t} + \mu_{i,t} + \phi \sum_{j} g_{ij,t} z_{j,t} 
\end{align*}
\]

where \( \phi := \delta/\gamma - \psi \), \( \mu_{i,t} \) is defined earlier in equation (3) and \( \nu_{i,t} \) denotes the bank-specific shock of valuation with variance denoted by \( \sigma_i^2 \).

---

19The only difference between these two objective functions is that Equation (2) has an additional second-order term \( \left( \delta \sum_{j \neq i} g_{ij,t} z_{j,t} \right) \left( \psi \sum_{j \neq i} g_{ij,t} z_{j,t} \right) \), but it only contains other banks' choice of liquidity holdings, not bank \( i \)'s, so this additional term does not affect bank \( i \)'s first-order condition. In the planner’s problem that we discuss later, since the planner internalizes any spillover effect, the solutions differ slightly depending on whether we take Equation (2) or Equation (4) as banks’ objective function.
The “network attenuation factor” $\phi$ is the key parameter that determines the type of equilibrium on network: i.e., strategic substitution if $\phi < 0$ or complementarity if $\phi > 0$. In our paper, we are agnostic about the sign of $\phi$ and we instead estimate it empirically.

**Proposition 1** Suppose that $|\phi| < 1$. Then, there is a unique interior solution for the individual equilibrium outcome given by

$$z_{i,t}^* (\phi, g) = \{M (\phi, G_t)\}_i \mu_t,$$

where $\{\}_i$ is the operator that returns the $i$-th row of its argument, $\mu_t := [\mu_{1,t}, ..., \mu_{n,t}]'$, $z_{i,t}$ denotes the bilateral liquidity holding by bank $i$, and

$$M (\phi, G_t) := I + \phi G_t + \phi^2 G_t^2 + \phi^3 G_t^3 + ... \equiv \sum_{k=0}^{\infty} \phi^k G_t^k = (I - \phi G_t)^{-1}.$$

where $I$ is the $n \times n$ identity matrix.

**Proof.** Since $\gamma > 0$, the first order condition identifies the individual optimal response. Applying Theorem 1, part b, in Calvo-Armengol, Patacchini, and Zenou (2009) to our problem, the necessary equilibrium condition becomes $|\phi \lambda^{\max} (G_t)| < 1$ where the function $\lambda^{\max} (\cdot)$ returns the largest eigenvalue. Since $G_t$ is a stochastic matrix, its largest eigenvalue is 1. Hence, the equilibrium condition requires $|\phi| < 1$, and in this case the infinite sum in equation (7) is finite and equal to the stated result (Debreu and Herstein (1953)).

To gain intuition about the above result, note that a Nash equilibrium in pure strategies $z_t^* \in \mathbb{R}^n$, where $z_t := [z_{1,t}, ..., z_{n,t}]'$, is such that equation (5) holds for all $i = 1, 2, ..., n$. Hence, if such an equilibrium exists, it solves $(I - \phi G_t) z_t = \mu_t$. If the matrix is invertible, we obtain $z_t^* = (I - \phi G_t)^{-1} \mu \equiv M (\phi, G_t) \mu_t$. The rest follows by simple algebra. The condition $|\phi| < 1$ states that network externalities must be small enough in order to prevent the feedback triggered by such externalities to escalate without bounds.

The matrix $M (\phi, G_t)$ has an important economic interpretation: it aggregates all direct and indirect links among banks using an attenuation factor, $\phi$, that penalises (as in Katz (1953)) the contribution of links between distant nodes at the rate $\phi^k$, where $k$ is the length of the path between nodes. In the infinite sum in equation (7), the identity matrix captures the (implicit) link of each bank with itself, the second term in the sum captures all the direct links between banks, the third term in the sum captures all the indirect links corresponding to paths of length two, and so on. The elements of the matrix $M (\phi, G_t)$, given by $m_{ij} (\phi, G_t) := \sum_{k=0}^{\infty} \phi^k \{G_t^k\}_{ij}$, aggregates all the exposures in the network of $i$ to $j$, where the contribution of the $k$th step is weighted by $\phi^k$. 

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In equilibrium, the matrix $M(\phi, G_t)$ contains information about the centrality of network players. Multiplying the rows (columns) of $M(\phi, G_t)$ by a unit vector of conformable dimensions, we recover the indegree (outdegree) Katz–Bonacich centrality measure.\(^{20}\) The indegree centrality measure provides the weighted count of the number of ties directed to each node, while the outdegree centrality measure provides the weighted count of ties that each node directs to the other nodes. That is, the $i$-th row of $M(\phi, G_t)$ captures how bank $i$ loads on the network as a whole, while the $i$-th column of $M(\phi, G_t)$ captures how the network as a whole loads on bank $i$.

However, as equation (6) shows, the matrix $M(\phi, G_t)$ (which includes the network topology and the network attenuation factor $\phi$) is not enough to determine the importance of a bank from a systemic perspective. Banks' equilibrium reserve holdings depend on $M$ and $\mu$, suggesting that bank-specific shocks are equally important. Intuitively, when deciding its optimal liquidity holding level, bank $i$, weights its own shock, neighbouring and more centrally located banks' shocks relatively more heavily. Banks that receive large liquidity shocks, regardless of their network location, may have a larger influence on the other banks' liquidity holding in the network.

**Equilibrium properties:** we can decompose the network contribution to the total bilateral liquidity into a level effect and a risk effect. To see this, note that the total network generated liquidity, $Z_t := \sum z_{i,t}$, can be written at equilibrium as

$$Z_t^* = \underbrace{1' M(\phi, G_t) \bar{\mu}}_{\text{level effect}} + \underbrace{1' M(\phi, G_t) \nu_t}_{\text{risk effect}}$$  \hfill (8)

where $\bar{\mu} := [\bar{\mu}_1, \ldots, \bar{\mu}_n]'$, $\nu_t := [\nu_{1,t}, \ldots, \nu_{n,t}]'$. The first component captures the network level effect, and the second component (that aggregates bank-specific shocks) captures the network risk effect. It is clear that if $\bar{\mu}$ has only positive entries, both the network liquidity level and liquidity risk will be increasing in $\phi$. That is, a higher network multiplier leads the interbank network to produce more liquidity and also generate more risk.

The equilibrium solution in equation (8) implies that bank $i$'s marginal contribution to the volatility of aggregate liquidity can be summarised as

$$\frac{\partial Z_t^*}{\partial \nu_{i,t}} \sigma_i = \underbrace{1' \{M(\phi, G_t)\}_{i,i}}_{\text{outdegree centrality}} \sigma_i =: b_{i,\text{out}}^*(\phi, G_t).$$  \hfill (9)

The above expression is the outdegree centrality for bank $i$ weighted by the standard deviation.

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\(^{20}\)Newman (2004) shows that weighted networks can in many cases be analysed using a simple mapping from a weighted network to an unweighted multigraph. Therefore, the centrality measures developed for unweighted networks apply also to the weighted cases.
of its own shocks. Moreover, the conditional volatility of the aggregate liquidity level in our model is

\[
Var_t(Z^*_t (\phi, G_t)) = \text{vec} \left( \{ b_i^{\text{out}} (\phi, G_t) \}_{i=1}^n \right) \text{vec} \left( \{ b_i^{\text{out}} (\phi, G_t) \}_{i=1}^n \right)' = 1'M(\phi, G_t) \text{diag} \left( \{ \sigma^2_i \}_{i=1}^n \right) M(\phi, G_t)' 1.
\]

Therefore, equation (9) provides a clear ranking of the riskiness of each bank from a systemic perspective. This allows us to define the concept of “systemic risk key player”.

**Definition 1 (Risk key player)** The risk key player $i^*_t$, given by the solution of

\[
i^*_t = \arg \max_{i=1,...,n} b_i^{\text{out}} (\phi, G_t),
\]

is the one that contributes the most to the volatility of the overall network liquidity.

Similarly, we can identify the bank that can cause the maximum expected level of reduction in the network liquidity when removed from the system.\(^{21}\)

**Definition 2 (Level key player)** The level key player is the player that, when removed, causes the maximum expected reduction in the overall level of bilateral liquidity. We use $G_{\tau,t}$ to denote the new adjacency matrix obtained by setting to zero all of $G_t$’s $\tau$-th row and column coefficients. The resulting network is $g_{\tau,t}$. The level key player $\tau^*_t$ is found by solving

\[
\tau^*_t = \arg \max_{\tau=1,...,n} \mathbb{E} \left[ \sum_i z^*_i(\phi, g_t) - \sum_{i \neq \tau} z^*_i(\phi, g_{\tau,t}) \right | g_t, \tau
\]

where $\mathbb{E}$ is the rational expectation operator.

In this definition, the level key player is the one with the largest impact on the total expected bilateral liquidity, under the assumption that when the player $\tau$ is removed, the remaining other banks do not form new links – i.e., we consider the short-run effect of removing a player from the network.

Using Proposition 1, we have the following corollary.

\(^{21}\)This definition is in the same spirit as the concept of the key player in the crime network literature as defined in Ballester, Calvo-Armengol, and Zenou (2006). There, it is important to target the key player for maximum crime reduction. Here, it is useful to consider the ripple effect on the network liquidity when a bank fails and exits from the system. Bailouts for key level players might be necessary to avoid major disruptions to whole interbank network.
Corollary 1: A player \( \tau^*_t \) is the level key player that solves (13) if and only if

\[
\tau^*_t = \arg \max_{\tau = 1, \ldots, n} \left\{ \mathbf{M}(\phi, \mathbf{G}_t) \right\}_{\tau} \bar{\mu} + \sum_{i \neq \tau} m_{i\tau}(\phi, \mathbf{G}_t) \bar{\mu}_\tau
\]  

(14)

This follows from the fact that when bank \( \tau \) is removed, the expected reduction in the total bilateral liquidity can be written as

\[
\mathbb{E} \left[ \sum_i z^*_i(\phi, g_t) - \sum_{i \neq \tau} z^*(\phi, g_{\setminus \tau}) \bigg| g_t, \tau \right] = \left\{ \mathbf{M}(\phi, \mathbf{G}_t) \right\}_{\tau} \bar{\mu} + \sum_{i \neq \tau} \mathbf{M}(\phi, \mathbf{G}_t) \bar{\mu}_\tau - m_{\tau \tau}(\phi, \mathbf{G}_t) \bar{\mu}_\tau
\]  

(15)

That is, the removal of the level key player results in a direct (indegree) effect on its own liquidity generation and an indirect (outdegree) bilateral effect on other banks’ liquidity generation. Instead of being the bank with the largest amount of liquidity buffer stock (captured by the first term on the right-hand side of equation (15)), the level key bank is the one with the largest expected contribution to its own and as well as its neighbouring banks’ liquidity. This discrepancy exists because, in the decentralised equilibrium, no bank internalises the effect of its own liquidity holding level on the utilities of the other banks in the network. That is, no bank internalises the spillover of its choice of liquidity on other banks’ liquidity valuation. Therefore, a relevant metric for a planner to use when deciding whether to bail out a failing bank should not be merely based on the size of the bank’s own liquidity, but should also include its indirect network impact on other banks’ liquidity.

In summary, the two measures (defined in equations (12) and (14) ) help to identify the key players in the determination of aggregate liquidity levels and systemic liquidity risk in the network. However, the network topology alone is not enough. Both network multiplier as well banks’ idiosyncratic shocks and their variabilities are important inputs in computing the key players.

The Planner’s Solution: this discussion leads us to analyse formally a planner’s problem in this interconnected system. A planner that equally weights the utility of each bank (in equation (2)) chooses the network liquidity holdings by solving the following problem:

\[
\max_{\{z_i t\}_{i=1}^n} \sum_{i=1}^n u_i(z_t | g_t)
\]  

(16)

where \( u_i(z_t | g_t) \) is bank’s \( i \) utility from holding liquidity in the network defined in equation
The first order condition for the liquidity holding of the $i$-th bank ($z_{i,t}$) yields

$$z_{i,t} = \mu_{i,t} + \phi \sum_{j \neq i} g_{ij,t} z_{j,t} + \psi \sum_{j \neq i} g_{ji,t} \mu_{j,t} + \phi \sum_{j \neq i} g_{ji,t} z_{j,t}$$  \hspace{2cm} (17)$$

where

- $\phi$ denotes the value of liquidity available to neighbours' valuation of liquidity
- $\psi$ captures the neighbours' idiosyncratic valuation of the liquidity provided by agent $i$
- $\frac{2\delta}{\gamma}$ reflects bank $i$'s impact on bank $j$'s network-dependent valuation of liquidity, so the outbound linkage $g_{ji,t}$ is weighted by bank $j$'s liquidity holdings, $z_{j,t}$, to arrive at the overall impact on bank $j$'s utility level.\textsuperscript{22}
- The fifth term measures bank $i$'s contribution to the volatility of network liquidity accessible by neighbouring banks.

Rewriting equation (17) in matrix form, we obtain $z_t = (I + \psi G_t') \mu_t + P(\phi, \psi, \delta, G_t) z_t$ where $P(\phi, \psi, \delta, G_t) := \phi (G_t + G_t') - \psi (\psi - 2\delta/\gamma) G_t' G_t$. This allows us to formally state the planner’s solution.

**Proposition 2** Suppose $|\lambda_{\text{max}}(P(\phi, \psi, \delta, G_t))| < 1$. Then, the planner’s optimal solution is uniquely defined and given by

$$z^*_t(\phi, \psi, \delta, g_t) = \{M^p(\phi, \psi, \delta, G_t)\}_{i} \mu_t,$$  \hspace{2cm} (18)$$

where $M^p(\phi, \psi, \delta, G_t) := [I - P(\phi, \psi, \delta, G_t)]^{-1} (I + \psi G_t')$.

**Proof.** The proof follows the same argument as in the proof of Proposition 1. ■

As in the decentralised solution, one can solve for the aggregate network liquidity level

\textsuperscript{22}The total impact on bank $j$’s valuation of liquidity also depends on $\phi$, which is equal to $\frac{2\delta}{\gamma} - \psi$ - the positive impact on valuation through informational spillover is captured by $\delta$, and the availability of liquidity through interbank borrowing decreases the benefit of holding liquidity on its own (captured by $\psi$).
and risk in the planner’s problem. We can write the level and volatility as follows.

\[ Z^p_t = 1'M^p (\phi, \psi, \delta, G_t) \bar{\mu} + 1'M^p (\phi, \psi, \delta, G_t) \nu_t \]  
(19)

\[ \text{Var}_t (Z^p (\phi, \psi, \delta, G_t)) = 1'M^p (\phi, \psi, \delta, G_t) \text{diag}(\{\sigma^2_i\}) M^p (\phi, \psi, \delta, G_t)' 1. \]  
(20)

To see what drives the difference between the network liquidity in the decentralised equilibrium \((z^*)\) and in the planner’s solution \((z^p)\), one can rewrite the planner’s first order condition at the equilibrium as:

\[ z^p_t = z^*_t + M (\phi, G_t) \left[ \psi G'_{it} \mu_t + \left( \phi G'_{it} - \psi \left( \psi - \frac{2\delta}{\gamma} \right) G'_{it} G_t \right) z^p_t \right]. \]  
(21)

The extra terms (in the square brackets) of the planner’s solution arise from the banks’ failure, in the decentralized equilibrium, to internalise the network externalities they generate. Intuitively, among these terms: the first one reflects the contribution to the neighbours’ valuations of liquidity holdings; the second one measures the contribution to the neighbouring nodes’ indegree centrality and hence their network liquidity production level; and the last one is the contribution to their neighbouring nodes’ volatility.\(^{23}\)

Therefore, the discrepancy between the planner’s optimum and the decentralised equilibrium rests on the planner’s tradeoff between the liquidity level and the liquidity risk in the network. When the planner cares more about the level of liquidity production than the liquidity risk in the network, the first two terms are more pronounced relative to the last term. In this case, banks that have higher outdegree centralities tend to hold less than the socially optimal amount of liquidity. The planner might subsidise or inject liquidity to these banks to increase the liquidity generated by the network. Conversely, when the planner cares more about the liquidity risk in the network (which happens when \(\psi >> 2\delta/\gamma\), e.g. very large \(\psi\) or \(\gamma\) and small \(\delta\)), banks that have higher second-degree centralities tend to hold more than the socially optimal amount of liquidity. The planner might impose a tax on these banks to reduce the risk in the banking network.

The following corollary offers a closed-form characterisation of the wedge between the planner’s solution and the decentralised outcome.

**Corollary 2** Let \(H_t := \phi G'_{it} - \psi (\psi - 2\delta/\gamma) G'_{it} G_t\). Then, the aggregate network liquidity in the planner’s solution can be expressed as

\[ Z^p_t = Z^*_t + 1' \left[ \psi M_t G'_{it} + M_t H_t M_t (I - H_t M_t)^{-1} (I + \psi G'_{it}) \right] \mu_t \]  
(22)

\(^{23}\)Note that the term \(\phi G'_{it} - \psi \left( \psi - \frac{2\delta}{\gamma} \right) G'_{it} G_t\) vanishes only in the unlikely case of \(\frac{\phi}{\psi(\psi - \frac{2\delta}{\gamma})}\) being an eigenvalue of \(G_t\).
where $Z^*_t$ denotes the aggregate bilateral liquidity in the decentralised equilibrium in equation (8) and $M_t := M(\phi, G_t)$. Moreover, if $H_t$ is invertible, we have

$$Z^p_t = Z^*_t + 1' \left[ \psi M_t G'_t + M_t (H_t^{-1} - M_t)^{-1} M_t (I + \psi G'_t) \right] \mu_t. \quad (23)$$

**Proof.** If $H_t$ is invertible, observing that

$$M^p(\phi, \psi, \delta, G_t) \equiv [M(\phi, G_t)^{-1} - \phi G'_t + \psi (\psi - 2\delta / \gamma) G'_t G_t]^{-1} (I + \psi G'_t)$$

and using the Woodbury matrix identity (see, e.g. Henderson and Searle (1981)) gives

$$M^p(\phi, \psi, \delta, G_t) = M_t + M_t (H_t^{-1} - M_t)^{-1} M_t,$$

hence the result is immediate. If $H_t$ is not invertible, using equation (26) in Henderson and Searle (1981), we obtain

$$M^p(\phi, \psi, \delta, G_t) = M_t + M_t H_t M_t (I - H_t M_t)^{-1}$$

and the result follows. \[\Box\]

The above implies that the discrepancy in the planner and market solutions for both expected total liquidity in the system ($E[Z^p_t - Z^*_t | g_t]$), as well as for the individual expected liquidity holdings ($\{E[z^p_t - z^*_t | g_t]\}_i$), might be positive or negative depending on the parameters and the topology of the network. In particular, one can show that the sign of the discrepancy between the solution of the planner and the decentralised solution depends on the parameters and the eigenvalues of the canonical operator of $G_t$ (see, e.g. Gorodentsev (1994) for a definition of the canonical operator).\[24\]

### III Empirical Methodology

In order to estimate the network model presented in Section II, we need to map the observed total liquidity holding of a bank at time $t$, $l_{i,t}$, into its two components: the liquidity holding absent of any bilateral effect (defined in equation (1)) and the network-dependent component (defined in equation (6)). This can be done by reformulating the theoretical model in the fashion of a spatial error model (SEM). That is, we decompose the total bank liquidity holdings into a function of the observables and a latent term that captures the spatial

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\[24\]The proof of this result is very involved, hence we present it in an appendix available upon request.
dependence generated by the network:

\[ l_{i,t} = \alpha_{t}^{time} + \alpha_{i}^{bank} + \sum_{m=1}^{M} \beta_{m}^{bank} x_{i,t}^{m} + \sum_{p=1}^{P} \beta_{p}^{time} x_{t}^{p} + z_{i,t} \]  

(24)

\[ z_{i,t} = \bar{\mu}_{i} + \phi \sum_{j=1}^{n} g_{ij,t} z_{j,t} + \nu_{i,t} \sim iid \left(0, \sigma_{i}^{2}\right), \ i = 1, \ldots, n, \ t = 1, \ldots, T. \]  

(25)

The only differences between the theoretical model and the econometric reformulation above are that: \( i) \) we have made explicit that one of the aggregate factors is a set of common time dummies, \( \alpha_{t}^{time} \), meant to capture potential trends in the size of the overall interbank market; \( ii) \) well when there are two links, \( g_{ij} \), to potentially vary over time (but we construct them, as explained in the data description section below, in a fashion that makes them predetermined with respect to the information set for time \( t \).\footnote{To allow for potential time variation in \( \phi \) instead we also perform estimations in subsamples and over a rolling window.} The coefficients \( \beta_{m}^{bank} \) capture the effect of observable bank characteristics while the coefficients \( \beta_{p}^{time} \) capture the effects of systematic risk factors on the choice of liquidity.

Equation (25) describes the process of \( z_{i} \), which is the residual of the individual bank \( i \)'s level of liquidity in the network that is not due to bank specific characteristics or systematic factors. Moreover, defining \( \epsilon_{i} \) as the demeaned version of \( z_{i} \), we have that \( \sum_{j=1}^{n} g_{ij,t} \epsilon_{j,t} \) is a standard spatial lag term and \( \phi \) is the canonical spatial autoregressive parameter. That is, the model in equations (24)–(25) is a variation of the Anselin (1988) spatial error model (see also Elhorst (2010a, 2010b)). This specification makes clear the nature of the network as a shock propagation mechanism: the shock to the liquidity of any bank, \( \epsilon_{i,t} \), is a function of all the shocks to the other banks’ liquidity; the intensity of the shock spillover is a function of the intensity of the network links between banks captured by the network weights \( g_{ij} \); and whether the network amplifies or damps the effect of the individual liquidity shocks on aggregate liquidity depends, respectively, on whether the banks in the network act as strategic complements (\( \phi > 0 \)) or strategic substitutes (\( \phi < 0 \)). To illustrate this point, note that the vector of shocks to all banks at time \( t \) can be rewritten as

\[ \epsilon_{t} = (I - \phi G_{t})^{-1} \nu_{t} \equiv M(\phi, G_{t}) \nu_{t} \]  

(26)

where \( \epsilon_{t} = [\epsilon_{1,t}, \ldots, \epsilon_{n,t}]' \) and \( \nu_{t} = [\nu_{1,t}, \ldots, \nu_{n,t}] \). This implies that if \( G_{t} \) is a right stochastic matrix\footnote{If \( G_{t} \) is a right stochastic matrix, then \( G_{t}1 = 1 \), and therefore} (and this is the case when we model the network weights \( g_{ij,t} \) as the fraction of

\[ 1 = (I - \phi G_{t})^{-1} (I - \phi G_{t}) 1 = (I - \phi G_{t})^{-1} 1 (1 - \phi) \implies M(\phi, G_{t}) 1 = (1 - \phi)^{-1} 1. \]  

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borrowing by bank $i$ from bank $j$), then a unit shock to the system equally spread across banks (i.e. $\nu_t = (1/n) \mathbf{1}$) would imply a total change in aggregate liquidity equal to $(1 - \phi)^{-1}$—that is, $\phi$ captures the ‘average’ network multiplier effect of liquidity shocks.

Moreover, equation (26) implies that any time variation in the network structure, $G_t$, or in the network multiplier, $1/(1 - \phi)$, would result in a time variation in the volatility of total liquidity since the variance of the shocks to the total network liquidity ($\mathbf{1}' \epsilon_t$) is

$$Var_t (\mathbf{1}' \epsilon_t) = \mathbf{1}' \mathbf{M} (\phi, G_t) \Sigma_v \mathbf{M} (\phi, G_t)' \mathbf{1}.$$ 

Here we have used the fact that $G_t$ is pre-determined with respect to the time $t$ information, $\Sigma_v := \mathbb{E} [\nu_t \nu_t']$ is a diagonal matrix with the variances of the idiosyncratic shocks $\{\sigma^2_i\}_{i=1}^n$ on the main diagonal.

As outlined in Section A.3.1 of the Appendix, we can estimate the parameters of the spatial error model jointly using a quasi-maximum likelihood approach. In order to elicit the time variation in the network coefficient $\phi$, we perform subsample and rolling window estimates. The estimation frequency is daily, with a $G_t$ network matrix based on a rolling monthly average lagged by one day.

An estimation issue for network models is the well-known reflection problem (Manski (1993)): the neighbouring banks’ decisions about their liquidity holdings affect each other, so that we cannot distinguish between whether a given bank’s action is the cause or the effect of its neighbouring banks’ actions. To address this problem, Bramoullé, Djebbari and Fortin (2009) have shown that the network effect $\phi$ can be identified if there are two nodes in the network with different average connectivities of their direct connected nodes. This condition is satisfied in our data.\(^{27}\)

As a test of the model specification of our theory-driven formulation, we also consider a more general specification that allows for a richer set of network interactions. That is, we model liquidity holding as a spatial Durbin model (SDM—see, e.g. LeSage and Pace (2009)) where bank specific liquidity is allowed to depend directly on other banks’ liquidity

\(^{27}\)The separate identification of the fixed effects $\mu_i$ and $\alpha_{i, \text{bank}}$ is more complex, and is discussed in detail in Appendix A.3.1. In particular, when $G_t$ is a right stochastic matrix, the identification of $\mu_i$ and $\alpha_{i, \text{bank}}$ requires at least one bank to not borrow from any other bank at some point in the sample (in our data, this happens 13.5% of the time spread over all subsamples and rolling windows we consider). Alternatively, one can normalise one of the $\mu_i$ to zero and identify the remaining ones in deviation from it. But note that the separate identification of the fixed effects does not affect the identification of $\phi$. 

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and characteristics, and pairwise control variables

\[
l_{i,t} = \alpha^\text{time}_t + \alpha^\text{bank}_i + \sum_{m=1}^M \beta^\text{bank}_m x^m_{i,t} + \sum_{p=1}^P \gamma^\text{time}_p x^p_t + \\
+ \rho \sum_{j=1}^n g_{i,j,t} l_{j,t} + \sum_{j=1}^n g_{i,j,t} x_{i,j,t} \theta + \nu_{i,t} \sim iid \left(0, \sigma^2_i \right),
\]

(27)

where \(x_{i,j,t}\) denotes match specific control variables and the characteristics of other banks. The above formulation\textsuperscript{28} allows a specification test of our structural model since, setting \(x_{i,j,t} := vec(x^m_{j \neq i,t})^t\), and restricting \(\theta = -\phi vec(\beta^\text{bank}_m)\), \(\gamma^\text{time}_p = (1 - \phi)\beta^\text{time}_p \forall p\), and most importantly \(\rho = \phi\), we are back to the SEM specification implied by our structural model. These restrictions are tested formally in Section V. Such test is not only for validation of the empirical specification: given the close relation between our empirical and theoretical models, we effectively test our theoretical framework.

With the SEM estimated parameter at hand, we can also identify the risk key players of the interbank liquidity market. To do so, we define the network impulse response function as follows.

**Definition 3 (Network Impulse-Response Functions)** Let \(L_t \equiv [l_{1t},...,l_{Nt}]\) denote the total liquidity in the interbank network. The network impulse response function of total liquidity, \(L_t\), to a one standard deviation shock to a given bank \(i\), is given by

\[
\text{NIRF}_i(\phi, \sigma_i, G_t) \equiv \frac{\partial L_t}{\partial \nu_{i,t} \sigma_i} = 1' \{M(\phi, G_t)\} _i \sigma_i
\]

(28)

where the operator \(\{\}_i\) returns the \(i\)-th column of its argument.

The network impulse response is identical to the shock size weighted outdegree centrality of bank \(i\) defined in equation (9). Note that \(\text{NIRF}_i(\phi, 1, G_t)\) is less than or greater than 1 depending on whether \(\phi\) is positive or negative – that is, if \(\phi > 1 (< 1)\) individual bank shocks are amplified (reduced) through the system.

The network impulse response provides a metric to identify which bank’s shocks have the largest impact on the overall liquidity. Moreover, it does so taking into account both the size of the bank (via \(\sigma_i\)), the network multiplier, \(\phi\), and all the direct and indirect links.
between banks, since \(1' \{M(\phi, G_t)\}_i\) is the solution, for \(|\phi| < 1\), of

\[
1' \{M(\phi, G_t)\}_i = 1' \{I + \phi G_t + \phi^2 G_t^2 + \ldots\}_i = 1' \left\{ \sum_{k=0}^{\infty} \phi^k G_t^k \right\}_i
\]

where the first element in the series captures the direct effect of a unit idiosyncratic shock to bank \(i\), the second element captures the effects through the first order network links, the third element captures the effect through the second order links, and so on. This also implies that \(\{M(\phi, G_t)\}_ji\) measures the total (direct and indirect) effect of a shock to bank \(i\) on the liquidity of bank \(j\).

Furthermore, the network impulse response functions provide a natural decomposition of the variance of the total liquidity in the network system, since

\[
Var_t (1' \epsilon_t) \equiv \text{vec} \left\{ \{NIRF_i(\phi, \sigma_i, G_t)\}_{i=1}^n \right\}' \text{vec} \left\{ \{NIRF_i(\phi, \sigma_i, G_t)\}_{i=1}^n \right\}.
\]

where \(Var_t\) denotes the time \(t\) variance conditional on time \(t-1\) information.\(^{29}\)

We can also isolate the purely network part of the impulse response function, that is, the liquidity effect in excess of the direct effect of a shock to a bank (which we call “excess NIRF”):

\[
NIRF^e_i(\phi, \sigma_i, G_t) \equiv NIRF_i(\phi, \sigma_i, G_t) - \sigma_i = 1' \left\{ (I - \phi G_t)^{-1} \phi G_t \right\}_i \sigma_i, \quad (29)
\]

and the above, setting \(\sigma_i = 1\), i.e. considering a unit shock, is exactly the centrality measure of Katz (1953). Note that \(NIRF^e_i(\phi, \sigma_i, G_t)\) has by construction the same sign as \(\phi\).

Note also that it is straightforward to compute confidence bands for the estimated network impulse response functions (using the delta method), since they are simply a function of the distribution of \(\hat{\phi}\), and \(\hat{\phi} - \phi_0\) has the canonical Quasi-MLE asymptotic Gaussian distribution (see Section A.3.2 in the Appendix).

### IV Description of the Network and Other Data

We study the sterling interbank network over the sample period January 2006 to September 2010. The estimation frequency is daily, but we also use higher frequency data to construct several of the control variables defined below. The network we consider comprises of all banks in the CHAPS system during the sample – a set of 11 banks. These banks play a key role in the sterling large value payment system since they make payments both on

\(^{29}\)Note that, by construction, \(G_t\) is in the time \(t-1\) information set.
their own behalf and on behalf of banks that are not direct members of CHAPS. The banks in the network are: Halifax Bank of Scotland (owned by Lloyds Banking Group); Barclays; Citibank (the consumer banking arm of Citigroup); Clydesdale (owned by National Australia Bank); Co-operative Bank (owned by The Co-operative Group); Deutsche Bank; HSBC (that incorporated Midland Bank in 1999 – one of the historical “big four” sterling clearing banks\(^{30}\)); Lloyds TSB; Royal Bank of Scotland (including Natwest); Santander (formerly Abbey, Alliance & Leicester and Bradford & Bingley, owned by Banco Santander of Spain); and Standard Chartered.

We split our sample into three sub-samples of similar length: the Pre-crisis period (1 January 2006 to 9 August 2007); the Post Northern Rock/ Hedge Fund Crisis period (10 August 2007 to 19 September 2009); the Post Asset Purchase Programme period (20 September 2009 to 30 September 2010). This is explained in more detail below.

Our proxies for the intensity of network links are the interbank overnight borrowing relations. This data is identified from overnight payment data between banks by applying an algorithm developed by Furfine (2000). This is an approach which is common to most papers on the interbank money market. The algorithm identifies pairs of payments between two payment system counterparties where the outgoing payment (the loan) is a multiple of 100,000 and the incoming payment (the repayment) happens the following day and is equivalent to the outgoing payment plus a plausible interest rate. This algorithm has been tested thoroughly, and tracks accurately the LIBOR rate on the whole.\(^{31}\) Furfine (2000) showed that the algorithm accurately identifies the Fed Funds rate when applied to Fedwire data.\(^{32}\)

\(^{30}\)For most of the 20th Century, the phrase “the Big Four” referred to the four largest sterling banks, which acted as clearing houses for bankers’ cheques. These were: Barclays Bank; Midland Bank (now part of HSBC); Lloyds Bank (now Lloyds TSB Bank and part of Lloyds Banking Group); and National Westminster Bank (“NatWest”, now part of The Royal Bank of Scotland Group). Currently, the largest four U.K. banks are Barclays, HSBC, Lloyds Banking Group and The Royal Bank of Scotland Group (with a combined market capitalization of more than £254bn) closely followed by Standard Chartered (with a market cap of over £37bn) – and all of these banks are part of our network.

\(^{31}\)The data are only for banks which are participants in the payment systems. This creates two problems. First, some loans may be attributed to the settlement bank involved when in fact the payments are made on behalf of one of their customers. Second, where a loan is made between one customer of a settlement bank and another, this transaction will not be settled through the payment system but rather across the books of the settlement bank. This is a process known as internalisation. Internalised payments are invisible to the central bank, so they are a part of the overnight money market that will not be captured.

\(^{32}\)As documented in Armantier and Copeland (2012), the Furfine’s algorithm is affected by Type I and, to a lesser extent, Type II, errors. Nevertheless, this is less of a concerns in our application since: first, as documented in Kovner and Skeie (2013), at the overnight frequency we focus on, interbank exposures measured by the algorithm are highly correlated with the Fed funds borrowing and lending reported in bank quarterly regulatory filings; second, and most importantly, instead of using the daily identified borrowing and lending relationships, we smooth these exposures by computing rolling monthly averages, therefore greatly reducing the relevance of false positives and negatives in the identification of the interbank relationships. Furthermore, in the empirical application, we apply several robustness checks to our measure of interbank
The loan data are compiled to form an interbank lending and borrowing network. In particular, the elements $g_{ij,t}$ of the adjacency matrix $G_t$ are given by the fraction of bank’s $i$ overnight loans that come from bank $j$. In the baseline specification, the weights at time $t$ are computed as monthly averages for the previous month ending on day $t - 1$.

By construction, $G_t$ is a square right stochastic matrix. Its largest eigenvalue is therefore equal to one. This implies that the potential propagation of shocks within the system will be dominated by the second largest eigenvalue of the adjacency matrix. The time series of the second largest eigenvalue of $G_t$ is presented in Figure 1. As can be seen in the figure, there was a substantial increase of the eigenvalue in the aftermath of the Northern Rock/Hedge Fund Crisis period (September 2007), but what is striking is the substantial increase in the volatility of the network links in the post QE period.

One way to characterise time variation in the cohesiveness of the network is to examine the behaviour of the Average Clustering Coefficient (ACC – see Watts and Strogatz (1998)) defined as

$$ACC_t = \frac{1}{n} \sum_{i=1}^{n} CL_i(G_t), \quad CL_{i,t} = \frac{\# \{jk \in G_t \mid k \neq j, j \in n_i(G_t), k \in n_i(G_t) \}}{\# \{jk \mid k \neq j, j \in n_i(G_t), k \in n_i(G_t) \}}$$

33 This is because $G^k$ can be rewritten in Jordan normal form as $PJ^kP^{-1}$ where $J$ is the (almost) diagonal matrix with eigenvalues (or Jordan blocks in case of repeated eigenvalues) on the main diagonal.
where \( n_i(G_t) \) is the set of players that have a direct link with player \( i \) and \( \#\{\} \) is the count operator. The numerator of \( CL_{i,t} \) is the number of pairs of banks linked to \( i \) that are also linked to each other, while its denominator is simply the number of pairs of banks linked to \( i \). Therefore, the average clustering coefficient measures the average proportion of banks that are connected to bank \( i \) who are also connected to each other. By construction this value ranges from 0 to 1.

The time series of the ACC is reported in Figure 2. The figure shows that at the beginning of the sample the network is highly cohesive since, on average, around 80% of the pairs of banks connected to any given bank are also connected to each other. The degree of connectedness seems to have a decreasing trend during 2007–2008, and a substantial and sudden decrease following the Asset Purchase Programme, when the average clustering coefficient decreased by about one-quarter of its pre-crises average. This might be the outcome of reduced interbank borrowing needs during the QE period thanks to the availability of additional reserves from the Bank of England (combined with a move towards increased collateralisation of borrowing and an overall deleveraging of banks balance sheets, see, e.g. Westwood (2011)). This interpretation is consistent with Figure 3, which depicts the (rolling monthly average of) daily gross overnight borrowing in the interbank network. The data record a substantial increase in overnight borrowing as the initial response to the turmoil in the financial market, possibly caused by a shift to very short borrowing due to increased difficulties in obtaining long term financing (Wetherilt, Zimmerman, and Soramaki (2010)),

Figure 2: Average clustering coefficient of the interbank network.
Figure 3: Daily gross overnight borrowing in the interbank network (rolling monthly average).

and a substantial decrease in overnight borrowing after the beginning of the QE period.

To measure the dependent variable $l_{i,t}$, that is, the liquidity holdings of each bank, we use central bank reserve holdings. We supplement this with the collateral that is repo’ed with the Bank of England in return for intraday liquidity (these repos are unwound at the end of each working day). For robustness, we also analyse separately the behaviour of each of these two liquidity components.\footnote{These results are reported in an appendix available upon request.}

The weekly average of the total liquidity in the system (computed as the sum of the bank specific liquidity holdings) is reported in Figure 4. The figure depicts a substantial upward trend in the available liquidity in the post subprime default subsample and during the various financial shocks registered in 2008–2009, consistently with the evidence of banks’ hoarding liquidity in response to the financial crisis (Acharya and Merrouche (2010)), but this upward trend is dwarfed by the steep run-up registered in response to the Asset Purchase Programme (aka Quantitative Easing) that almost tripled the average liquidity in the system. Interestingly, as shown in Figure (11) in the Appendix, the sharp increase in liquidity in the last part of the sample is associated with a dramatic reduction in the velocity of money.

As covariates, in addition to common monthly time dummies meant to capture time effects, and bank fixed effects, meant to capture unobserved heterogeneity, we use a large set of aggregate ($x_p^T$) and bank specific ($x_{m_i,t}$) control variables. Note that since in the econometric specification in equations (24) and (25) the network effects are elicited through their contri-
Figure 4: Weekly average of aggregate liquidity available at the beginning of the day.

bution to the residuals, any overfitting from those control variables will reduce the variation in residuals, and thus, lead to a conservative estimate of network effects.

**Aggregate Control Variables ($x^p_t$):** All the common control variables, meant to capture aggregate market conditions, are lagged by one day so that they are predetermined with respect to time $t$ innovations. To control for aggregate market liquidity condition we use the total liquidity in the previous day. To proxy for the overall cost of funding liquidity we use the lagged LIBOR rate and the interbank rate premium in the sample network (computed as the difference between the overnight interest rate, averaged over banks in the sample sterling network, and the LIBOR rate).

Since banks’ decisions to hold liquidity are likely to be influenced by the volatility of their daily payment outflows, we construct a measure of the intraday payments volatility, defined as

$$VolPay_t = \sqrt{\frac{1}{88} \sum_{\tau=1}^{88} (P^{out}_{t,\tau})^2}$$

where $P^{out}$ denotes payment outflows and 88 is the number of 10-minute time intervals (the unit of time for payment recording) within a day. The time series of this variable is reported in Figure 5. It is characterised by a strong upward trend before the subprime default crisis, and a distinctively negative trend during the period of financial turmoil preceding the beginning of QE. During the QE period, this variable has no clear trend but is characterised
Intraday volatility of aggregate outflows.

Instead by a substantial increase in volatility.

We also control for the turnover rate in the payment system (see Benos, Garratt, and Zimmerman (2010)). This variable is

\[ TOR_t = \frac{\sum_{i=1}^{N} \sum_{\tau=1}^{88} Pout_{i,t,\tau}}{\max_i \max_{\tau \in [1,88]} [CNP(\tau; i, t)], 0} \]

where the cumulative net debit position (CNP) is defined as the difference between payment outflows and inflows. The numerator captures the total payments in the system in a day, while the denominator is the sum of the maximum net debt positions of all banks in a given day. This variable is meant to capture the velocity of transactions within the interbank system and its time series is reported in Figure 12 of Appendix A.4, and indicates an increased turnover during the financial turmoil, followed by a reduction to levels below the historical average during the QE period.

Since banks have some degree of freedom in deciding on the timing of their intraday outflows, they could use this strategically. Therefore, we control for the right kurtosis\(^{35}\) \((rK_t)\) of intraday payment times. The time series of this variable is reported in Figure 13 of Appendix A.4 and shows a substantial increase during the QE period.

\(^{35}\)We define as right and left kurtosis (denoted, respectively, by \(rK_t\) and \(lK_t\)) the fractions of kurtosis of payment times generated by payment times that are, respectively, above and below the average payment time of the day:
Bank Characteristics \((x_{it}^{m})\): As for the aggregate control variables, all bank characteristic variables are lagged by one day\(^{36}\) so that they are predetermined with respect to innovations at time \(t\). Despite the fact that we control for average interest rates (LIBOR and average overnight borrowing rate), we also control for the bank specific overnight borrowing rate (computed as the average weighted by the number of transactions). We include these variables (reported in Figure 14 of Appendix A.4) because in response to each of the collapses, of Northern Rock and of Lehman Brothers, there was a substantial increase in the cross-sectional dispersion of the overnight borrowing rates, and this increase in dispersion persisted during the QE period (see Figure 15 of Appendix A.4). We also control for: bank specific right kurtosis of the time of intraday payments in \((rK_{i,t}^{in})\), to capture a potential incentive to increase bank liquidity) and out \((rK_{i,t}^{out})\), since banks in need of liquidity might have an incentive to delay their outflows); the intraday volatility of the used liquidity \((VolPay_{i,t})\), defined as in equation (30) but using bank specific flows); the total amount of intraday payments \((LevPay_{i,t} = \sum_{\tau=1}^{\tau_{88}} P_{OUT_{i,t}^{\tau}})\); the liquidity used \((LU_{i,t})\) as defined in Benos, Garratt, and Zimmerman (2010)\(^{37}\); the ratio of repo liabilities to total assets; the cumulative change in the ratio of retail deposits to total assets; the total lending and borrowing in the interbank market; the cumulative change in the 5-year senior unsecured credit default swap (CDS) premia; and a dummy variable for the top four banks in terms of payment activity.

V Estimation Results

As first exercise, we estimate our empirical network model specified in equations (24) and (25) using three sub-periods of roughly equal size. These are the sub-period before the Northern Rock/Hedge Fund Crisis (Period 1), the sub-period immediately after the Northern Rock/Hedge Fund Crisis but before the announcement of the Assets Purchase Programme (Period 2), and the sub-period running from the announcement of the Assets Purchase Programme (Period 3).

\[ rK_t = \frac{\sum_{\tau \geq m_t} (\frac{\tau - m_t}{\sigma_t})^4}{\sum_{\tau=1}^{88} (\frac{\tau - m_t}{\sigma_t})^4}; \quad lK_t = \frac{\sum_{\tau < m_t} (\frac{\tau - m_t}{\sigma_t})^4}{\sum_{\tau=1}^{88} (\frac{\tau - m_t}{\sigma_t})^4}; \]

where \(m_t\) and \(\sigma_t\) are defined as

\[ m_t = \frac{1}{88} \sum_{\tau=1}^{88} \left( \frac{P_{OUT_{t,\tau}}}{\sum_{\tau=1}^{T} P_{OUT_{t,\tau}}} \right), \quad \sigma_t^2 = \frac{1}{88} - \frac{\sum_{\tau=1}^{88} \left( \frac{P_{OUT_{t,\tau}}}{\sum_{\tau=1}^{T} P_{OUT_{t,\tau}}} - m_t \right)^2}{88}. \]

\(^{36}\) For controls variables available at lower than daily frequency. i.e. monthly, we use the latest lagged observation. These variables are: the repo liabilities to asset ratio, total assets, and the cumulative change in the ratio of retail deposits to total assets.

\(^{37}\) Liquidity used on day \(t\) is defined as \(LU_{i,t} = \max\{\max_{\tau \in [1,88]}[CNP(\tau; i,t)], 0\}\).
Programme to the end of the period of study (Period 3). We split our sample into these three parts since (a) they correspond to very different overall market conditions, and (b), as documented in Section IV, the network structure and behaviour of these sub-periods seem to differ substantially. Period 1 is a relatively tranquil period for the banking sector. Period 2 is characterised by several significant events in world financial markets, such as: the run on Northern Rock (the first U.K. bank run in 150 years), the subprime mortgage hedge fund crisis, the Federal Reserve intervention in Bear Stearns and its subsequent sale to JP Morgan Chase, and the bankruptcy of Lehman Brothers. Period 3 is characterised by a real regime shift – the beginning of Quantitative Easing – in U.K. monetary policy.\textsuperscript{38}

The estimation results for these three subsamples are reported in Panel A of Table 1, where we report only the estimates of the spatial dependency parameter $\phi$ (first row), the $R^2$ of the regression (second row), the implied average network multiplier (third row) $1/(1-\phi)$,\textsuperscript{39} as well as the ratio of the volatility of network liquidity to the counterfactual volatility that would have been generated if $\phi = 0$. Omitted from the table are the coefficient estimates associated with the control variables, which are reported in Table A1 of the Appendix.

The first row of the panel reports the estimates of the network coefficient $\phi$. Recall that $\phi > 0$ ($< 0$) implies that banks’ liquidity holding decisions are strategic complements (substitutes) and that this tends to amplify (reduce) the effect of bank specific liquidity shocks. In the first period, the point estimate of this coefficient is about 0.8137 (and highly significant) indicating the presence of a substantial network multiplier effect for liquidity shocks: a £1 idiosyncratic shock equally spread across banks would result in a $1/\left(1 - \hat{\phi}\right) = £5.3677$ shock to aggregate liquidity.

In the second period, the coefficient $\phi$ is still statistically significant but it is substantially reduced in magnitude, to 0.3031, implying weak complementarity, with an (average) shock multiplier of about 1.4349. This finding suggests that in response to the turbulence in the financial market that have characterised Period 2, banks’ liquidity management objective increasingly tilted to free riding neighbours, and away from responding to informational spillover.

In Period 3, the coefficient $\phi$ becomes negative, $-0.1794$, but is still highly significant, implying an average network shock multiplier of about 0.8479. This is particularly interesting


\textsuperscript{39}Note that from equation (26) we can compute the average network multiplier as the total impact on aggregated liquidity resulting from a unit shock equally spread across the $n$ banks. This is given by

$$1'\mathbf{M}\left(\phi, \mathbf{G}_t\right) \frac{1}{n} = \frac{1}{1-\phi}.$$
Table 1: Spatial Error Model Estimation

<table>
<thead>
<tr>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: ( G_t ) based on borrowing</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.8137</td>
<td>0.3031</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>66.01%</td>
<td>92.09%</td>
</tr>
<tr>
<td>( 1/ \left( 1 - \phi \right) )</td>
<td>5.3677</td>
<td>1.4349</td>
</tr>
<tr>
<td>( \sqrt{\frac{\text{Var}(Z_t</td>
<td>\phi)}{\text{Var}(Z_t</td>
<td>\phi=0)}} )</td>
</tr>
<tr>
<td><strong>Panel B: ( G_t ) based on lending</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.8209</td>
<td>0.2573</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>66.02%</td>
<td>91.63%</td>
</tr>
<tr>
<td>( 1/ \left( 1 - \phi \right) )</td>
<td>5.5835</td>
<td>1.3464</td>
</tr>
<tr>
<td>( \sqrt{\frac{\text{Var}(Z_t</td>
<td>\phi)}{\text{Var}(Z_t</td>
<td>\phi=0)}} )</td>
</tr>
<tr>
<td><strong>Panel C: ( G_t ) based on borrowing and lending</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.8204</td>
<td>0.3258</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>63.98%</td>
<td>92.22%</td>
</tr>
<tr>
<td>( 1/ \left( 1 - \phi \right) )</td>
<td>5.5679</td>
<td>1.3464</td>
</tr>
<tr>
<td>( \sqrt{\frac{\text{Var}(Z_t</td>
<td>\phi)}{\text{Var}(Z_t</td>
<td>\phi=0)}} )</td>
</tr>
</tbody>
</table>

Estimation results for equations (24) and (25). Periods 1, 2 and 3, correspond, respectively, to before the Northern Rock/Hedge Fund Crisis, after the Hedge Fund Crisis but before the Asset Purchase Programme, and after the Asset Purchase Programme announcement. The \( t \)-statistics are reported in parentheses under the estimated coefficients. Standard errors are QMLE robust ones. For the average network multiplier, \( 1/ (1 - \phi) \), the delta method is employed. In Panel A, the adjacency matrix is computed using the borrowing relationships, while in Panels B and C we use, respectively, total lending and total borrowing plus lending (row normalized)

since a negative \( \phi \) implies strategic substitution in liquidity holdings, as in Bhattacharya and Gale (1987), that is, a situation in which individual banks decide to hold less liquidity when neighbouring banks hold more liquidity. As a result, the network has a damping effect on shock transmission. The fact that negative network effect is observed during the Quantitative Easing sub-period suggests that the liquidity multiplier effect was not working at the time of the large inflow of liquidity from the central bank through Asset Purchase Programme. The total liquidity injection from the program was, up to October 2011, of about £275 billion.\(^{40}\) Overall, the fit of the model is quite good in all sub-periods, with an \( R^2 \) in the range 66\% – 92\%.

\(^{40}\)See [http://www.bankofengland.co.uk/monetarypolicy/Pages/qe/qe_faqs.aspx](http://www.bankofengland.co.uk/monetarypolicy/Pages/qe/qe_faqs.aspx).
The last row of Panel A reports \( \sqrt{\text{Var}(Z_t|\hat{\phi})/\text{Var}(Z_t|\phi = 0)} \) i.e. the ratio of the volatility of the network liquidity implied by our estimate of \( \phi \) to the (counterfactual) volatility of network liquidity that would have arisen if there were no network externalities. \(^{41}\) This last statistic makes clear that the large positive network multiplier in the first period generates a 459\% increase in volatility. In the second period instead the reduced network multiplier generates an excess volatility of only about 25\%, while in the third period the negative network multiplier generates a reduction in the volatility of network liquidity of about 11\%.

Finally, for robustness, in Panels B and C, we reestimate our network model with two alternative constructions of the adjacency matrix \( G_t \). In particular, in Panel B we use the lending flows, while in Panel C we use the combined borrowing and lending flows (in both cases the adjacency matrix is row normalized). \(^{42}\) Overall, the estimates in Panels B and C are extremely similar, both qualitatively and quantitatively, to the ones reported in Panel A. \(^{43}\)

With the subperiod estimates at hand, we can compute the network impulse response functions to identify the risk key players in the interbank market. The results for Period 1 are reported in the upper panel of Figure 6. In particular, in the upper panel we report the excess network impulse response functions to a unit shock \( \text{NIRF}^{e}(\hat{\phi}, 1, \bar{G}_1) \) defined in equation (29) (where \( \bar{G}_j \) denotes the average \( G_t \) in the \( j \)-th subsample), as well as the two standard deviation error bands. Also, as a point of reference, we report in the same panel the average network multiplier in excess of the unit shock (i.e. \( (1 - \hat{\phi})^{-1} - 1 \)). As mentioned earlier, the point estimate in Period 1 implies a large average network multiplier of shocks to individual banks, and the picture shows that in response to a £1 idiosyncratic shock equally spread across banks, the final compounded shock to the overall liquidity would be increased by another £4.3677. Nevertheless, what the upper panel of Figure 6 stresses is that this large network amplification of shocks is due to a small subset of banks. In particular: a £1 idiosyncratic shock to the liquidity of either Bank 5 or Bank 9 would generate an excess reaction of aggregate liquidity of about £13.9 and £13.8; the same shock to Bank 6 would result in an excess reaction of aggregate liquidity of about £8.9; instead, a shock

\(^{41}\)For completeness, in Table A2 in the Appendix we also report \( \sqrt{\text{Var}(z_{i,t}|\hat{\phi})/\text{Var}(z_{i,t}|\phi = 0)} \) for each bank.

\(^{42}\)Note that when constructing the theoretical model, we emphasize that the network linkages reflect the interbank relationship, which may transmit information and/or liquidity (i.e., interbank credit). Thus, the network linkage is not necessarily just about borrowing. If a bank lends to another, the relationship formed through such transaction may facilitates future borrowing or information transmission.

\(^{43}\)Note that in the theoretical model, the network linkages reflect the interbank relationship, which may transmit information and/or liquidity. Thus, the network linkages are not necessarily only about borrowing. If a bank lends to another, the relationship formed through such transaction may facilitates future borrowing or information transmission.
Figure 6: The period before the Northern Rock/Hedge Fund Crisis. Network excess impulse response functions to a unit shock (upper panel); net borrowing (central panel); borrowing and lending flows (lower panel) where the ellipses identifying individual banks are (log) proportional to their total gross borrowing in the system, incoming arrows to a node indicate borrowing flows to that node, while outgoing arrows indicate lending flows from that node, and the thickness of the arrows is (log) proportional to the sterling value of the flows.
to Bank 4 would have an effect that is roughly of the same size as the average network multiplier while a shock to any of the remaining seven banks would be amplified much less by the network system. That is, the network impulse response functions stress that there is a small subset of key players in the interbank liquidity market that generate most of the network risk.

The central panel of Figure 6 shows the average net borrowing during Period 1. Comparing the upper and central panels of the figure, it is interesting to notice that simply looking at the individual net borrowing behaviour one cannot identify the riskiest players for the network. In particular, the two riskiest players identified through our structural estimation are not the largest net borrowers in the network – the largest net borrower, Bank 4, is instead an average bank in network risk terms. Moreover, Bank 5, one of the two largest network risk contributors, is not a net borrower – it is instead the second largest net lender.

The comparison between the top two panels also makes clear that the risk key players are not necessarily the net borrowing banks – net borrowers and net lenders are roughly as likely to be the network risk key players. This result is intuitive: negative liquidity shocks to a bank that lends liquidity to a large share of the network can be, for the aggregate liquidity level, as bad as a negative shock to a bank that is borrowing liquidity from other banks. But the comparison between the two panels also makes it clear that simply looking at the largest players in terms of net borrowing or lending would not identify the key risk players for the system.

The reasons behind this finding can be understood by looking at the lower panel of the figure, where we present the average network structure during Period 1. In particular, the size of the ellipses identifying individual banks are (log) proportional to their total gross borrowing in the system, incoming arrows to a node indicate borrowing flows to that node while outgoing arrows indicate lending flows from that node, and the thickness of the arrows is (log) proportional to the sterling value of the flows. The lower panel shows that key risk contributors tend to be banks with the most connections and largest flows (and with most links to other well connected banks), i.e., banks with relatively high centrality, but are not necessarily the players that borrow or lend more in either gross nor net terms.

Figure 7 reports excess impulse response functions (upper panel), average net borrowing positions (central panel), and network flows (lower panel) for Period 2 – the period characterised by a high degree of stress in the financial market. The first thing to notice is that despite the overall increase in activity in the interbank borrowing and lending market (outlined by both the central and lower panels and by Figure 3), there is a drastic reduction in the average network multiplier reported in the top panel: the average excess network reaction to a unit shock is only about 0.43. That is, in a period of financial stress, banks
Figure 7: After the hedge fund crisis but before QE. Network excess impulse response functions to a unit shock (upper panel); net borrowing (central panel); borrowing and lending flows (lower panel), where the ellipses identifying individual banks are (log) proportional to their total gross borrowing in the system, incoming arrows to a node indicate borrowing flows to that node, while outgoing arrows indicate lending flows from that node, and the thickness of the arrows is (log) proportional to the sterling value of the flows.
seem, on average, to have radically reduced their network risk exposure, and they have done so despite having increased the amount of overnight borrowing and lending used to fund their liquidity needs. Nevertheless, as stressed by the first panel, the network risk profile, even though substantially reduced overall, is still quite high for a small subset of banks. In particular, a unit shock to Bank 5, Bank 9 and Bank 6, would result, respectively, in an excess network liquidity change of 1.77, 1.36 and 0.85, while the same shock to Bank 4 would have an effect very similar to the average one, and a shock to the remaining banks would receive minimal amplification from the network system.

The results for Period 3 – the one starting at the onset of QE – are reported in Figure 8, and are radically different from the ones of the previous two periods. First, banks tend to behave as strategic substitutes in their liquidity holdings in this period, therefore the network has a buffering effect to individual bank shocks, implying a negative average excess multiplier of $-0.15$, that is, a unit liquidity shock equally spread across banks would result in a $1 - 0.15 = 0.85$ shock to aggregate liquidity. But, once again, there is substantial heterogeneity among the banks, in the sense that for most banks (Bank 1, 3, 7, 8, 10 and 11) the network has basically no effect on how their own shocks propagate to the system, while for a few other banks (4, 5, 6, and 9), the network structure helps reduce the impact of their own idiosyncratic shocks on aggregate liquidity.

This behaviour arises in a period in which the degree of connectedness of the network was substantially reduced (see Figure 2 and the lower panel of Figure 8), the gross borrowing in the system had been substantially reduced (see Figure 3), most banks held net borrowing positions close to zero (central panel of Figure 8), but at the same time the overall liquidity in the system had been substantially increased (Figure 4).

What is also interesting to notice is that the same banks that were the riskiest players in the previous two periods (Banks 5, 6 and 9) are now the least risky ones for the system. Thanks to their centrality and more importantly the overall strategic substitution behaviour on the network, these banks become the biggest shock absorbers.

A natural question is whether we can explain the large heterogeneity of individual banks’ contribution to system risk using banks’ characteristics. To answer this question Table 2 reports the rank correlations of individual bank characteristics with the bank-specific network impulse-response functions in the three periods considered. Only a few bank characteristics seems to correlate significantly with the magnitude of the bank specific $NIRF_i$ and several observations are in order. Considering the total level of payments channeled by the bank, in periods 1 and 2 the rank correlations for this variable are, respectively, 82.73% and 95.45%, while in period 3 we have -85.45%. This implies that banks that channel a larger amount of payments are more likely to be central for the network, but the implications of this centrality

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Figure 8: The QE period: Network excess impulse response functions to a unit shock (upper panel); net borrowing (central panel); borrowing and lending flows (lower panel) where the ellipses identifying individual banks are (log) proportional to their total gross borrowing in the system, incoming arrows to a node indicate borrowing flows to that node, while outgoing arrows indicate lending flows from that node, and the thickness of the arrows is (log) proportional to the sterling value of the flows.
Table 2: Rank Correlation of Bank Characteristics and $NIRF^e_i$

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interbank Rate</td>
<td>20.91%</td>
<td>37.27%</td>
<td>-64.55%</td>
</tr>
<tr>
<td>ln $LevPay_{i,t-1}$</td>
<td>82.73%***</td>
<td>95.45%***</td>
<td>-85.45%***</td>
</tr>
<tr>
<td>$rK^i_{i,t-1}$</td>
<td>20.00%</td>
<td>-34.55%</td>
<td>10.91%</td>
</tr>
<tr>
<td>$rK^o_{i,t-1}$</td>
<td>-45.45%</td>
<td>-89.09%***</td>
<td>73.64%**</td>
</tr>
<tr>
<td>ln $VolPay_{i,t-1}$</td>
<td>48.18%</td>
<td>56.36%*</td>
<td>-54.55%*</td>
</tr>
<tr>
<td>ln $LU_{i,t-1}$</td>
<td>21.82%</td>
<td>35.45%</td>
<td>-23.64%</td>
</tr>
<tr>
<td>Repo Liability/Assets</td>
<td>39.45%</td>
<td>48.18%</td>
<td>-37.27%</td>
</tr>
<tr>
<td>Total Assets (log)</td>
<td>12.73%</td>
<td>25.45%</td>
<td>4.55%</td>
</tr>
<tr>
<td>$\frac{\Delta Deposit}{Assets}$</td>
<td>12.73%</td>
<td>-50%</td>
<td>68.18%**</td>
</tr>
<tr>
<td>CDS (log)</td>
<td>38.18%</td>
<td>18.18%</td>
<td>-40.00%</td>
</tr>
<tr>
<td>Stock Return (Inc. Dividend)</td>
<td>13.64%</td>
<td>-17.27%</td>
<td>-56.36%*</td>
</tr>
<tr>
<td>Total Lending and Borrowing (log)</td>
<td>86.36%***</td>
<td>95.45%***</td>
<td>-89.09%***</td>
</tr>
<tr>
<td>Total Lending (log)</td>
<td>97.27%***</td>
<td>99.09%***</td>
<td>-89.09%***</td>
</tr>
<tr>
<td>Total Borrowing (log)</td>
<td>66.36%**</td>
<td>91.82%***</td>
<td>-76.36%***</td>
</tr>
<tr>
<td>Net Borrowing (log)</td>
<td>-17.27%</td>
<td>10.91%</td>
<td>54.55%*</td>
</tr>
</tbody>
</table>

* represents 10% significance, ** 5% significance, and *** 1% significance.

depend on the type of equilibrium in the interbank market: when strategy complementarity is the dominant force (i.e. when $\phi > 0$ as in the first two periods), banks that channel more payments amplify more the shocks in the system; instead, when the equilibrium is characterized by strategic substitution ($\phi < 0$, as in the last period), such banks dampen the effect of shocks on the system. Second, the last row of Table 2 shows that net borrowing has no significant rank correlation with banks’ $NIRF^e_i$, consistent with Figures (6)-(8). That is, whether a bank is a large net lender or borrower is irrelevant for the shock propagation in the system. Nevertheless, gross lending and gross borrowing, and their sum, are all highly correlated with banks’ $NIRF^e_i$. That is, banks that borrow and/or lend a lot (in gross terms) are key players in our network. Nevertheless, this centrality has a different impact on the system depending on the sign of $\phi$: central banks amplify the shocks when $\phi$ is positive and dampen them when $\phi$ is negative. Interestingly, the rank correlations are, in absolute terms, marginally larger for total lending than for total borrowing. Finally, bank size measured by total assets is very weakly (and not significantly) correlated with the $NIRF^e$. 

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V.1 Central Planner vs Market Equilibrium

With the estimates of the structural parameters at hand, we can quantitatively assess the discrepancy, if any, between the banks’ liquidity holdings generated in the decentralized equilibrium, and the level of liquidity buffer that a benevolent central planner would have wanted the banks to hold. That is, from equations (8) and (19), we can compute the (average) difference between the aggregate liquidity of planner’s choice and the aggregate decentralized liquidity as $1' \left[ \mathbf{M}^p (\phi, \psi, \delta, \mathbf{G}) - \mathbf{M} (\phi, \mathbf{G}) \right] \bar{\mu}$. Similarly, from equations (10) and (20) we can compute the difference in the level of volatility of the planner’s choice and of the aggregate decentralized outcome: $\sqrt{\text{Var}(Z^p (\phi, \psi, \delta, \mathbf{G}))} - \sqrt{\text{Var}(Z^* (\phi, \mathbf{G}))}$.

The challenge in computing the above quantities is that we have consistent estimates of $\phi$ and $\bar{\mu}$, but we cannot directly estimate $\psi$, $\delta$ and $\gamma$. Nevertheless, we can calibrate $\psi$ to a natural benchmark: $\psi = 1$. This corresponds to the case in which each bank values in an identical manner the liquidity it holds directly in the network, and the liquidity available via its borrowing links to other banks. Moreover, with $\psi = 1$ we have that $\delta / \gamma = \phi + 1$. Hence we do not need to choose values of $\gamma$ and $\delta$ if $\psi = 1$ and $\phi$ is set to the estimated value.

Table 3 reports the discrepancies between the central planner’s solutions and the market equilibria, based on the point estimates of the structural parameters in Table 1, and the average value of the adjacency matrix $\mathbf{G}_t$, in the three sub-periods.

In Period 1 – when the (average) network multiplier was extremely large – the market equilibrium features excessive risk from the perspective of a central planner: the central planner would prefer the volatility of liquidity to be reduced by almost 91%. Moreover, albeit marginally, the liquidity level in the system is also excessive. Given the high network multiplier in this period, the network has the capacity to greatly amplify the individual liquidity shocks. Hence, a small reduction in the equilibrium buffer stock holdings (in response to liquidity shocks), from central planner’s perspective, will come with a greater reduction in network volatility, therefore delivering a better level–risk trade-off.

In Period 2, given the reduction in $\phi$, the market equilibrium produces ceteris paribus less volatility than in the Period 1. Nevertheless, the market volatility is still too large (by about 65%) from the central planner’s perspective. Moreover, the level of liquidity buffer in this sub-period is much smaller than what is considered optimal by the central planner. That is, Period 2 is characterised by too much risk and too little buffer stock of liquidity. The latter phenomenon is partially due to the fact that the individual average valuations ($\bar{\mu}$) of accessible liquidity are substantially reduced in this period, implying a general decline in the ‘belief’ of the interbank market’s capacity to generate liquidity, causing a significant reduction in buffer stock holdings in the decentralised equilibrium.

In the last sub-period, the (average) network multiplier in the market equilibrium is
Table 3: Central Planner vs. Market Equilibria

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ% Volatility of Total Liquidity</td>
<td>-90.8%</td>
<td>-64.8%</td>
<td>30.7%</td>
</tr>
<tr>
<td>Δ Network Liquidity</td>
<td>-3.47</td>
<td>15.5</td>
<td>-27.5</td>
</tr>
</tbody>
</table>

The three sub-periods are indicated by $j = 1, 2, 3$, and $\bar{G}_j$ is the average $G_t$ in sub-period $j$. The table reports: first row, $100 \times \left[ \left( \frac{\text{Var}(Z^*(\phi_j, G_j))}{\text{Var}(Z^*(\phi_j, \bar{G}_j))} \right)^{\frac{1}{2}} - 1 \right]$; second row, $\text{I} \left[ \text{M}^p (\phi_j, G_j) - \text{M} (\phi_j, \bar{G}_j) \right] \hat{\mu}_j$ (unit: £10bn).

smaller than 1, hence overall the system dampens the volatility of shocks. From the central planner’s perspective, not enough volatility is generated (by about 31%) while at the same time the aggregate network liquidity buffer is too high. This implies that banks hold idle reserves, and thus, the transmission of monetary policy (i.e., QE in this context) to the broad economy tends to be less effective than envisioned.

V.2 Time Varying Network Effects

The results presented so far indicate a substantial change over time in the role played by the network interactions in determining aggregate liquidity level and risk. In this section, we analyse the drivers of this time variation.

V.2.1 The Drivers of the Time Variation in the Network Amplification

The network impulse response functions depicted in Figures (6)-(8) show substantial time variation in the amplification of shocks between sub-periods. This could be caused by either the time variation in the network topology $G$ or in the network multiplier $\phi$.

To examine these drivers, we compute the changes in the network impulse response functions across the three subperiods. In particular, Panel A of Figure 9 reports the total change in NIRF between Periods 1 and 2 ($\text{NIRF}_i(\hat{\phi}_2, 1, \bar{G}_2) - \text{NIRF}_i(\hat{\phi}_1, 1, \bar{G}_1)$, dashed line with circles), the change due to the variation of $G$ ($\text{NIRF}_i(\hat{\phi}_1, 1, \bar{G}_2) - \text{NIRF}_i(\hat{\phi}_1, 1, \bar{G}_1)$, dotted line with triangles), and the change due to the variation of $\phi$ ($\text{NIRF}_i(\hat{\phi}_2, 1, \bar{G}_1) - \text{NIRF}_i(\hat{\phi}_1, 1, \bar{G}_1)$, dash-dotted line with +).

A striking feature of the graph is that most of the total change comes from the reduction in the network multiplier $\phi$ for all banks. In fact, ceteris paribus, the outdegree centrality (hence the NIRF) of Bank 5 would have increased from Period 1 to 2 due to its increased borrowing and lending activity (captured by $\bar{G}_2$). However, this effect is dwarfed by the reduction of its NIRF caused by the change in $\phi$. 
Panel B reports the same decomposition of the change in NIRFs between Periods 2 and 3. Once again the changes are mostly driven by the change in the network multiplier rather than the change in network topology.

Overall, Figure 9 shows that the time variation of the network multiplier has the first order effect on the network amplification mechanism.

V.2.2 Time Varying Network Multiplier

The results in the previous sections indicate the importance of the time variation of \( \phi \). Therefore, to capture this time variation, we now estimate the structural model in equations (24) and (25) using a 6-month rolling window. These rolling estimates of the network coefficient \( \phi \) are reported (blue line), together with 95% confidence bands (red lines), in Figure 10.

The figure also reports the rolling point estimates of the coefficient \( \phi \) implied by the spatial Durbin model (green line) in equation (27) which, as a more general model, serves as a specification test of our benchmark spatial error model. If the two estimated \( \phi \) are close to each other, this indicates that our theory-driven spatial error specification of the interbank network cannot be rejected for a more general specification.

At the beginning of the sample, the figure shows an extremely large network coefficient, \( \phi \), implying a substantial network amplification of shocks to banks in the system. The estimated coefficient has its first sharp reduction around the 18th of May 2006 when the Bank of England introduced the reserve averaging system described in Section A.1. The network multiplier is relatively stable after May 2006, except for a temporary decrease during the 2007 subprime default, until the Northern Rock bank run when the network multiplier is drastically reduced for several months. After this reduction, the coefficient goes back to roughly the previous period average but shows a trend decline that culminates in a sharp drop following the Bear Stearns collapse. From this period onward, and until long after the Lehman Brothers bankruptcy, the coefficient is statistically indistinguishable from zero, implying no network amplification of bank specific shocks. That is, the estimation suggests that in this period there was basically no added risk coming from the network structure of

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\[^{44}\text{Recall that when } G_i \text{ is a right stochastic matrix, separate identifications of the bank } (\alpha^{\text{bank}}) \text{ and network } (\mu) \text{ fixed effects require that there is a subset of banks that does not borrow at least at one point in time} \text{ in each subsample. This condition is not satisfied in all the rolling sub-samples. But since the separate identification of these fixed effect does not affect the identification of } \phi, \text{ we normalise the unidentified fixed effects. Moreover, given the very short length of the rolling window, we drop time fixed effects from the specification and the heteroskedastic specification of the shocks. Estimates with the full sets of fixed effects and heteroskedasticity show a very similar behaviour, but with somewhat larger confidence intervals, hence making it easier not to reject the SEM specification. As a consequence, we focus on the more parsimonious specification.}\]

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Figure 9: Decomposition of total change in the NIRFs between periods.
the interbank market, and that individual bank shocks would not be amplified by some sort of domino effect in the U.K. interbank market. This figure suggests that the banks’ reaction to the financial market turmoil was to reduce the amplification of risk generated through the interbank network. This reduction could come from any of these three sources: a) a reduction in the availability of collateralization and/or information spillovers, i.e. $\delta$, b) an increase in risk aversion, $\gamma$, and c) an increased availability of accessible liquidity due to $\psi$.

Interestingly, the coefficient $\hat{\phi}$ becomes negative, and statistically significant, right before the announcement of the Asset Purchase Programme, and remains stably so throughout the QE period. This indicates that during the liquidity inflow coming from the Bank of England’s QE policy (and also in expectation of it), banks started behaving as strategic substitutes in their liquidity holding decision (as implied by Bhattacharya and Gale (1987)). Note that this is a period in which the aggregate supply of central bank reserves was almost completely price inelastic since QE set a target level for asset purchases and let market forces determine their price. This overall change of the BoE supply of reserves is unlikely to be the driver of our estimates of the network multiplier coefficient during this period since: a) the change in $\phi$ actually occurred before the announcement of QE; b) we estimate the identified optimal response of the banks to market conditions (effectively, the banks’ equilibrium demand function), and we control for variation in aggregate price and quantities.

Figure 10: Spatial Error (blue line) and Durbin (green line) rolling estimates of $\phi$. 
of liquidity, as well as bank deposits held by the private sector.

Lastly, this figure outlines that the point estimates of $\hat{\phi}$ coming from our theory-driven spatial error specification and the ones coming from the more general spatial Durbin model are always very similar, both numerically and in terms of their overall evolution during the sample. Moreover, testing formally for a discrepancy between the two types of estimates, we find that they are statistically different at the 5% confidence level less than 95% of the time, providing support for the spatial error formulation of our network model.

VI Conclusion

In this paper, we develop and estimate a network model of interbank liquidity that is flexible enough to incorporate both strategic complementarity and substitution as potential network equilibria. Based on network topology, the estimated network effects and bank-specific structural shocks, we construct measures of systemic risks and identify the network players that are most important in contributing to the aggregate liquidity and its risk in the banking system.

We find that the network effect varies significantly through the sample period, January 2006 to September 2010. Prior to the Northern Rock/Hedge Fund crisis, liquidity provision in the network was driven by strategic complementarity of the holding decisions. That is, liquidity shocks were amplified by the network and each bank had a large exposure to network shocks. In contrast, during the crisis, the network itself also became less cohesive, and the network amplification was greatly reduced. Finally, during the QE period, in response to the large injection of liquidity in the system, the network became characterised by strategic substitution: that is, individual players free ride each other and the liquidity inflow from the central bank.

To the best of our knowledge, we are the first to estimate substantial time variation in the nature of the equilibrium in a financial network. Moreover, we show that, for risk generation, the change in the type of equilibrium is the dominant force (rather than the change in the network topology itself). This could rationalise the empirical puzzle of network changes having little impact on aggregate quantities in calibration/simulation exercises on interbank networks (Elsinger, Lehar, and Summer (2006)).

Moreover, we estimate the individual bank contributions to aggregate liquidity risk and document that most of the systemic risk is generated by a small subset of key players. Last, but not least, we also solve for the benevolent central planner equilibrium. This allows us to estimate the gap between central planner and decentralised optima for both liquidity level and risk. In particular, we find that during both the pre-crises and crises periods the system
was characterised by an excessive amount of risk and (during the crises) too little liquidity relative to the social optimum.

References


A Appendix: For Online Publication

A.1 Reserves Schemes, Payment Systems, and Interbank Overnight Borrowing

Banks in the UK choose the amount of central bank reserves that they hold to support a range of short term liquidity needs. Reserves are the ultimate settlement asset for interbank payments. Whenever payments are made between the accounts of customers at different commercial banks, they are ultimately settled by transferring central bank money (reserves) between the reserves accounts of those banks. Reserve balances are used to buffer against intraday payment imbalances (i.e., cumulative outflows larger than inflows). Additionally, central bank reserves are the most liquid asset that banks can draw upon in the presence of unexpected outflows of funds. Since 2006, the starting year of our sample, banks choose their reserve holdings on a discretionary basis, i.e., reserve holdings are not mandatory. However, their reserve holding decisions depend on the policy framework in which they operate.

A.1.1 Monetary Policy Framework

Since the 1998 Banking Act, the Bank of England (BoE) has had independent responsibility for setting interest rates to ensure that inflation, as measured by the Consumer Price Index (CPI), meets the inflation target of 2%. Each month the Monetary Policy Committee (MPC) meets to decide the appropriate level of the Bank rate (the policy interest rate) to meet the inflation target in the medium term. The Sterling Monetary Framework changed over time. During our sample period, the Bank of England had three distinct monetary frameworks: prior to 18 May 2006, the Bank of England operated an unremunerated reserve scheme; this was then replaced by a reserves average scheme; since March 2009 and the initiation of Quantitative Easing, the reserves average scheme has been suspended.

Pre-2006 Reform: Prior to the 2006 reforms, the Sterling Monetary Framework (SMF) was based upon a voluntary unremunerated reserves. There were no reserve requirements and no reserve averaging over a maintenance period. The only requirement was that banks were obliged to maintain a minimum zero balance at the end of each day. In practice, due to uncertainties from end of day cash positions, banks opted for small positive reserve balances.

Reserve Averaging: In May 2006, the Bank of England undertook a major reform of the Sterling Monetary Framework. The new scheme was voluntary remunerated reserves with a period-average maintenance requirement. Each maintenance period – the period between two meetings of the Monetary Policy Committee – banks were required to decide upon a reserves target. This voluntary choice of reserves target is a feature unique to the UK system. Over the course of each maintenance period, the banks would manage their balance sheets so
that, on average, their reserve balances hit the target. Where banks were unable to hit the target, standing borrowing and deposit facilities were available. Within a range of ±1% of the target, reserves are remunerated at the Bank Rate. Holding an average level of reserves outside the target range attracts a penalty charge. But an SMF participant can ensure it hits its target by making use of the Bank’s Operational Standing Facilities (OSFs). These bilateral facilities allow SMF participants to borrow overnight from the Bank (against high-quality collateral) at a rate above Bank Rate or to deposit reserves overnight with the Bank at a rate below Bank Rate. The possibility of arbitrage between interbank market rates and reserves remunerated at Bank Rate is the main mechanism through which market rates are kept in line with Bank Rate. In both schemes before Quantitative Easing (QE), the BoE would ensure sufficient reserves supply for banks to meet their reserves target. Banks then use the interbank market to reallocate reserves from banks in surplus to banks in deficit.

Post Quantitative Easing: Quantitative Easing in UK started in March 2009 when the MPC decided that in order to meet the inflation target in the medium term, it would need to supplement the use of interest rate (which had hit the practical lower bound of 0.5%) with the purchase of assets using central bank reserves. This consisted of the BoE’s boosting the money supply by creating central bank reserves and using them to purchase assets, predominantly UK gilts. Furthermore, the BoE suspended the average reserve targeting regime, and now remunerates all reserves at the Bank rate.

A.1.2 Payment and Settlement Systems

Banks use central bank reserves to, inter alia, meet their demand for intraday liquidity in the payment and settlement systems. Reserves act as a buffer to cover regular timing mismatches between incoming and outgoing payments, for example, due to exceptionally large payments, operational difficulties, or stresses that impact upon a counterparty’s ability, or willingness to send payments. There are two major payment systems in the UK: CHAPS and CREST. These two systems play a vital role in the UK financial system. On average, in 2011, £700 billion of transactions was settled every day within the two systems. This equates to the UK 2011 nominal GDP being settled every two days.

CHAPS is the UK’s large-value payment system. It is used for real time settlement of payments between its member banks. These banks settle payments on behalf of hundreds of other banks through correspondent relationships. Typical payments are business-to-business

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45At various points during the crisis, this ±1% range was increased to give banks more flexibility to manage their liquidity.

46Settlement banks also pay a penalty if their reserves account is overdrawn at the end of any day.

47There are also four retail payment systems (Bacs, the Faster Payments Service (FPS), Cheque and Credit Clearing (CCC) and LINK) that are operated through the BoE.
payments, home purchases, and interbank transfers. Payments relating to unsecured inter-
bank money markets are settled in CHAPS. CHAPS opens for settlement at 8 am and closes
at 4:20 pm. Payments made on behalf of customers cannot be made after 4 pm. The system
has throughput guidelines which require members to submit 50% of their payments by noon
and 75% by 14:30. This helps ensure that payments are settled throughout the day and do
not bunch towards the end of the day.

In 2011, CHAPS settled an average of 135,550 payments each day valuing £254bn. CHAPS is a real-time gross settlement (RTGS) system. This means that payments are
settled finally and irrevocably in real time. To fund these payments, banks have to access
liquidity intraday. If a bank has, at any point during the day, cumulatively sent more pay-
ments than it has received, then it needs liquidity to cover this difference. This comes either
from central bank reserves or intraday borrowing from the BoE. Furthermore, when a bank
sends funds to another bank in the system, it exposes itself to liquidity risk. That is, the
risk that the bank may not get those fund inflows back during the day, and so will run
down their own liquidity holdings or borrow from the BoE. Therefore, it is important to
choose an appropriate level of liquidity buffer. Besides maintaining a liquidity buffer, banks
manage liquidity by borrowing from and lending to each other in the unsecured overnight
markets. According to Bank of England estimates, payments relating to overnight market
activity (advances and repayments) account for about 20% of CHAPS values (Wetherilt,
Zimmerman, and Soramaki (2010)).

CREST is a securities settlement system. Its Delivery-vs-Payment (DVP) mechanism en-
sures simultaneous transfer of funds and securities. The CREST system?s intraday liquidity
mechanism with the BoE is automatic through the ”Self Collateralising Repos” (SCRs), once
a liquidity need is identified. If a CREST settlement bank would otherwise have insufficient
funds to settle a CREST transaction, a secured intraday loan is automatically generated
using as eligible collateral either the purchased security (if eligible) or other securities.

A.1.3 The Sterling Unsecured Overnight Interbank Market

Interbank markets are the markets where banks and other financial institutions borrow
and lend assets, typically with maturities of less than one year. At the shortest maturity,
overnight, banks borrow and lend central bank reserves. Monetary policy aims at influencing
the rate at which these markets transact, so as to control inflation in the wider economy.
There is limited information available about the size and the structure of the sterling money
markets. The Bank of England estimates suggest that the overnight unsecured market is approximately £20–30 billion per day during our sample period. Wetherilt, Zimmerman,
and Soramaki (2010) describe the network of the sterling unsecured overnight money market.
They find that the network has a small core of highly connected participants, surrounded by a wider periphery of banks loosely connected with each other, but with connections to the core. It is believed that prior to the recent financial crisis, the unsecured market was much larger than the secured one. We identify interbank borrowing and lending transactions in CHAPS settlement data.

A.2 An Alternative Model

In this section, we present an alternative model where the network effect on banks’ liquidity holding decisions is not modelled as a residual. Specifically, we let the total liquidity holding by bank \( i \), i.e., \( l_{i,t} \), to be accessible to the network. Hence, the valuation of liquidity for bank \( i \) in network \( g_t \) becomes:

\[
\tilde{\mu}_{i,t} := \tilde{\mu}_{i,t} + \delta \sum_j g_{ij,t,l_{j,t}} + \sum_{m=1}^{M} \tilde{\beta}_m x_{i,t}^{m} + \sum_j g_{ij,t} x_{i,j,t} \tilde{\theta} + \sum_{p=1}^{P} \tilde{\gamma}_p x_t^p
\]

where \( x_{i,j,t} \) denotes match specific control variables and the characteristics of other banks, and \( \tilde{\theta} \) is a vector of suitable dimension. That is, in addition to the aggregate information embedded in the neighbouring banks’ holdings, also macro variables and the neighbouring banks’ characteristics affect the per unit valuation of the liquidity.

In this setup, bank \( i \)’s utility from holding liquidity is specified as:

\[
u_i(l_t | g_t) = \left( \tilde{\mu}_{i,t} + \psi \sum_{j \neq i} g_{ij,t,l_{j,t}} \right) \left( l_{i,t} + \psi \sum_{j \neq i} g_{ij,t,l_{j,t}} \right) - \frac{1}{2} \gamma \left( l_{i,t} + \psi \sum_{j \neq i} g_{ij,t,l_{j,t}} \right)^2.
\]

The optimal response function for each bank is then:

\[
l_{i,t}^* = \frac{\tilde{\mu}_{i,t} + \sum_{m=1}^{M} \tilde{\beta}_m x_{i,t}^{m} + \sum_j g_{ij,t} x_{i,j,t} \tilde{\theta} + \sum_{p=1}^{P} \tilde{\gamma}_p x_t^p}{\gamma} + \left( \frac{\delta}{\gamma} - \psi \right) \sum_{j \neq i} g_{ij,t,l_{j,t}}
\]

\[
= \mu_{i,t} + \sum_{m=1}^{M} \beta_m x_{i,t}^{m} + \sum_j g_{ij,t} x_{i,j,t} \theta + \sum_{p=1}^{P} \gamma_p x_t^p + \phi \sum_{j \neq i} g_{ij,t,l_{j,i,t}}
\]
where $\phi := \delta / \gamma - \psi$, $\mu_{i,t} := \bar{\mu}_{i,t} / \gamma =: \bar{\mu} + \nu_{i,t}$, $\beta_m = \tilde{\beta}_m / \gamma$, $\gamma_p = \tilde{\gamma}_p / \gamma$, and $\theta = \tilde{\theta} / \gamma$. Note that the empirical counterpart of the above best response is the spatial Durbin model in equation (27).

Let us denote $\mu_{i,t} + \sum_{m=1}^{M} \beta_m x_{i,j,t}^m + \sum_{j} g_{i,j,t} x_{i,t} + \sum_{p=1}^{P} \gamma_p x_{i,t}^p$ by $\bar{\mu}_t$. The following result is immediate following similar steps of the proof in the main text.

**Proposition 3** Suppose that $|\phi| < 1$. Then, there is a unique interior solution for the individual equilibrium outcome given by

$$l^*_t (\phi, g) = \{M (\phi, G_t) \} \bar{\mu}_t,$$

where $\{ \}$ is the operator that returns the $i$-th row of its argument, $\bar{\mu}_t := [\bar{\mu}_1, \ldots, \bar{\mu}_N]^t$, $l_{i,t}$ denotes the total liquidity holding by bank $i$.

The above result implies that, even in this more general model, the definitions of conditional volatility of liquidity (equation (10)), risk key player (definition 1), level key player (definition 2), and network impulse response functions (definition 3), as well as their dependency on the network topology and equilibrium parameter $\phi$, stay unchanged.

### A.3 Details of the Empirical Methodology

#### A.3.1 Quasi-Maximum Likelihood Formulation and Identification Issues

Writing the variables and coefficients of the spatial error model in equations (24) and (25) in matrix form as

$$B := [\alpha_1^{time}, \ldots, \alpha_T^{time}, \alpha_1^{bank}, \ldots, \alpha_i^{bank}, \ldots, \alpha_N^{bank}, \beta_1^{bank}, \ldots, \beta_m^{bank}, \beta_M^{bank}, \beta_1^{time}, \ldots, \beta_P^{time}]^t,$$

$$L := [l_{1,1}, \ldots, l_{1,T}, \ldots, l_{N,1}, \ldots, l_{N,T}], \quad z := [z_{1,1}, \ldots, z_{N,1}, \ldots, z_{i,t}, \ldots, z_{1,T}, \ldots, z_{N,T}]^t,$$

$$\nu := [\nu_1, \ldots, \nu_N, \ldots, \nu_{i,t}, \ldots, \nu_{1,T}, \ldots, \nu_{N,T}]^t, \quad \bar{\mu} := 1_T \otimes [\bar{\mu}_1, \ldots, \bar{\mu}_N]^t,$$

$$G := \text{diag} \left( G_t \right)_{t=1}^T$$

where $G_1 = 0 \ldots 0 \ G_2 \ldots \ G_T$, $X := [D, F, X^{bank}, X^{time}]$, $X := [D, F, X^{bank}, X^{time}]$.

---

48In this case, $l^*$ should replace $z^*$ in equation (13).

49This is similar to the spatial formulation in Lee and Yu (2010).
where $\mathbf{D} := I_T \otimes \mathbf{1}_N$, $\mathbf{F} := \mathbf{1}_T \otimes I_N$, and

\[
\mathbf{X}^{\text{time}} = \begin{bmatrix}
x_1^1 & \ldots & x_1^P \\
\vdots & \ddots & \vdots \\
x_T^1 & \ldots & x_T^P 
\end{bmatrix} \otimes \mathbf{1}_N, \quad \mathbf{X}^{\text{bank}} = \begin{bmatrix}
x_{1,1}^1 & \ldots & x_{1,1}^P \\
\vdots & \ddots & \vdots \\
x_{N,1}^1 & \ldots & x_{N,1}^P 
\end{bmatrix},
\]

we can then rewrite the empirical model as

\[
L = \mathbf{X} \mathbf{B} + \mathbf{z}, \quad \mathbf{z} = \mu + \phi \mathbf{G} \mathbf{z} + \nu, \quad \nu_{i,t} \sim \text{iid} \left(0, \sigma_i^2\right).
\]

This, in turn, implies that

\[
\nu \left(\mu, \phi\right) = \left(I_{N \times T} - \phi \mathbf{G}\right) \left(L - \mathbf{X} \mathbf{B}\right) - \mu. \tag{36}
\]

Finally, using the Gaussian distribution to model the exogenous error terms $\nu$ yields the log likelihood

\[
\ln L \left(B, \phi, \mu, \sigma_i^2 \right) = -\frac{T N}{2} \ln (2\pi) - \frac{T}{2} \sum_{i=1}^{N} \ln \sigma_i^2 - \sum_{i=1}^{N} \frac{1}{2\sigma_i^2} \sum_{t=1}^{T} \nu_{i,t} \left(B, \mu, \phi\right)^2, \tag{37}
\]

and the above can be estimated using standard optimization methods.

In the above formulation, the identification of $\phi$ is ensured by the usual conditions on $\mathbf{G}$ (see, e.g. Bramoullé, Djebbari, and Fortin (2009)). Instead, the separate identification of the bank fixed effects, $\alpha_{i,\text{bank}} := \left[\alpha_{i,\text{bank}}^1, \ldots, \alpha_{i,\text{bank}}^N\right]'$, and the network-bank fixed effects, $\bar{\mu} := \left[\bar{\mu}_1, \ldots, \bar{\mu}_N\right]'$, deserve some further remarks. Isolating the role of these fixed effects, equation (36) can be rewritten as

\[
\nu \left(B, \mu, \phi\right) = \left(I_{N \times T} - \phi \mathbf{G}\right) \left(L - \tilde{\mathbf{X}} \tilde{\mathbf{B}} - \mathbf{F} \alpha_{\text{bank}}\right) - \mu
\]

\[
= \left(I_{N \times T} - \phi \mathbf{G}\right) \left(L - \tilde{\mathbf{X}} \tilde{\mathbf{B}}\right) \otimes \mathbf{1}_T \otimes \left(\bar{\mu} + \alpha_{\text{bank}}\right) + \phi \mathbf{G} \mathbf{F} \alpha_{\text{bank}}
\]

where $\tilde{\mathbf{X}} := [\mathbf{D}, \mathbf{X}^{\text{bank}}, \mathbf{X}^{\text{time}}]$ and $\tilde{\mathbf{B}}$ is simply the vector $\mathbf{B}$ without the $\alpha_{i,\text{bank}}$ elements.

Several observations are in order. First, the above implies that if $\phi = 0$, then $\bar{\mu}$ and $\alpha_{i,\text{bank}}$ cannot be separately identified (nevertheless the parameters $\tilde{\mathbf{B}}$ are still identified). Second, if there is no time variation in the network structure, i.e. if $\mathbf{G}_t = \mathbf{G}$ $\forall t$, $\bar{\mu}$ and $\alpha_{i,\text{bank}}$ cannot be separately identified even if $\phi \neq 0$. Third, if a bank never lends to any other bank in the sample, its fixed effects $\bar{\mu}_i$ and $\alpha_{i,\text{bank}}$ cannot be separately identified. Fourth, if $\mathbf{G}_t$ is a
right stochastic matrix, separate identification of $\bar{\mu}$ and $\alpha^{\text{bank}}$ can be achieved only up to a parameter normalization, since for any scalar $\kappa$ and vector $\bar{\kappa} := 1_N \otimes \kappa$, we have

$$\nu (B, \mu, \phi) = (I_{N \times T} - \phi G) \left( L - \bar{X}\bar{B} \right) - 1_T \otimes (\bar{\mu} + \alpha^{\text{bank}} + \phi \bar{\kappa}) + \phi G F (\alpha^{\text{bank}} + \bar{\kappa})$$

The above also makes clear that a handy normalisation is to set one of the network-bank fixed effect (say the $i$-th one) to zero since it would imply the restriction $\{\alpha^{\text{bank}} + \phi \bar{\kappa}\}_i = \{\alpha^{\text{bank}} + \bar{\kappa}\}_i$ that, for any $\phi \neq 0$ and 1, can only be satisfied with $\kappa = 0$. Under this normalisation, the remaining estimated bank-network fixed effects are then in deviation from the normalised one. Fifth, note that the lack of separate identification for $\bar{\mu}$ and $\alpha^{\text{bank}}$ is due to the fact that when $G_t$ is a right stochastic matrix, and if all banks borrow from at least one bank at each point in time (i.e. $G_t$ has no rows of zeros), then $G_t 1_N = 1_N$ and $G 1_N = 1_N$. Fortunately, in our dataset, the condition $G_t 1_N = 1_N$ does not hold every day in the sample because there are periods in which certain banks do not borrow (in this case, the corresponding rows of $G_t$ contain all zeros and sum to zero, instead of one). In our sample, except for bank 7 and bank 11, all the other banks borrow every period from at least one of their counterparties. There are fourteen days when bank 7 does not borrow at all, and 145 days in which bank 11 does not borrow at all. Moreover, the no borrowing days of bank 7 and bank 11 do not overlap, so we have a total of 159 days in which either the sum of the 7th row of $G_t$ or the sum of the 11th row of $G_t$ is equal to zero, not one (13.5% of the days).

### A.3.2 Confidence Bands for the Network Impulse Response Functions

The $\phi$ estimator outlined in the previous section has an asymptotic Gaussian distribution with variance $s_\phi^2$ (that can be readily estimated from the QMLE covariance matrix based, as usual, on the Hessian and gradient of the log likelihood in equation (37)). That is,

$$\sqrt{T} \left( \hat{\phi} - \phi_0 \right) \overset{d}{\rightarrow} N \left( 0, s_\phi^2 \right),$$

where $\phi_0$ denotes the true value of $\phi$. Writing

$$a_1 (\phi) := \frac{\partial 1'}{\partial \phi} \left( I - \phi G \right)^{-1}, \quad a_2 (\phi) = \frac{\partial 1'}{\partial \phi} \left( (I - \phi G)^{-1} \phi G \right),$$

we have from Lemma 2.5 of Hayashi (2000) that

$$\sqrt{T} \left[ NIRF_i (\hat{\phi}, 1) - NIRF_i (\phi_0, 1) \right] \overset{d}{\rightarrow} N \left( 0, a_1 (\phi_0)^2 s_\phi^2 \right),$$

$$\sqrt{T} \left[ NIRF^e_i (\hat{\phi}, 1) - NIRF^e_i (\phi_0, 1) \right] \overset{d}{\rightarrow} N \left( 0, a_2 (\phi_0)^2 s_\phi^2 \right).$$

56
Therefore, since $a_j \left( \hat{\phi} \right) \xrightarrow{p} a_j (\phi_0)$, $j = 1, 2$, by the continuous mapping theorem, and by Slutsky’s theorem, $a_j \left( \hat{\phi} \right) s^2_{\hat{\phi}} \xrightarrow{p} a_j (\phi_0)^2 s^2_{\phi}$, where $s^2_{\hat{\phi}}$ is a consistent variance estimator, we can construct confidence bands for the network impulse response functions using the sample estimates of $\phi$ and $s^2_{\phi}$.

### A.3.3 Details of the Construction of the Variables

#### Macro control variables

- **$rK_{t-1}$**: lagged right kurtosis of the intraday time of aggregate payment outflow:

  $$rK_t = \frac{\sum_{\tau > m_t} (\frac{\tau - m_t}{\sigma_t})^4}{\sum_{\tau = 1}^{88} (\frac{\tau - m_t}{\sigma_t})^4}$$

  where

  $$m_t = \frac{1}{88} \sum_{\tau = 1}^{88} \tau \left( \frac{P_{OUT}^{T, t, \tau}}{\sum_{\tau = 1}^{88} P_{OUT}^{T, t, \tau}} \right), \quad \sigma_t^2 = \frac{1}{88 - 1} \sum_{\tau = 1}^{88} (\tau - m_t)^2 \left( \frac{P_{OUT}^{T, t, \tau}}{\sum_{\tau = 1}^{88} P_{OUT}^{T, t, \tau}} \right)$$

  and $P_{OUT}^{T, t, \tau}$ is the aggregate payment outflow at time interval $\tau$. Note that transactions are recorded for 88 10-minute time intervals within each day (from 5:00 to 19:30). The variable $m_t$ is the average of payment time weighted by the payment outflow.

- **$\ln VolPay_{t-1}$**: intraday volatility of aggregate liquidity available (lagged and in logarithms). “Liquidity available” is defined in the construction of bank specific control variables below.

- **$TOR_{t-1}$**: lagged turnover rate in the payment system. To define the turnover rate, we need first to define the Cumulative Net (Debit) Position (CNP):

  $$CNP(T; i, s) = \sum_{t=1}^{T} (P_{OUT}^{i, s, t} - P_{IN}^{i, s, t}),$$

  where $P_{OUT}^{i, s, t}$ is bank $i$’s total payment outflow at time $t$ in day $s$, $P_{IN}^{i, s, t}$ is the payment inflow. The turnover rate (in day $s$) is defined as

  $$TOR_s = \frac{\sum_{i=1}^{N} \sum_{t=1}^{88} P_{OUT}^{i, s, t}}{\sum_{i=1}^{N} \max\{\max_{T}[CNP(T; i, s)], 0\}}$$

  The numerator is the total payment made in the system at day $s$. The denominator sums the maximum cumulative net debt position of each bank at day $s$. Note that
in the denominator, if the cumulative net position of a certain bank is always below zero (that is, this bank’s cumulative inflow always exceeds its cumulative outflow), this bank actually absorbs liquidity from the system. If there are banks absorbing liquidity from the system, there must be banks injecting liquidity into the system. When we calculate the turnover rate (the ratio between the total amount circulating and the base), we should only consider one of the two. That’s why we take the first (outside) maximum operator. The reason for the inside operator goes as follows: Any increase in the cumulative net debit position (wherever positive) incurs an injection of liquidity into the system, so the maximum of the cumulative net position is the total injection from the outside to the payment system. And, the sum over the different banks gives the total injection through all the membership banks. A higher turnover rate means a more frequent reuse of the money injected from outside into the payment system.

- **LIBOR**: lagged LIBOR rate.
- **Interbank Rate Premium**: lagged average interbank market rate minus lagged LIBOR.

**Bank-specific variables**

- Liquidity Available (LA) is the amount of liquidity to meet payment requirements and is measured as the sum of reserves (SDAB, Start of Day Account Balance) plus the value of intraday repo available with the BoE (PC, Posting of Collateral). As time passes, the liquidity available in CHAPS is calculated by subtracting the money moved to CREST from the liquidity available in the previous time interval. In this way, we can trace for bank i the liquidity available at any time t in day s:

  \[ LA(t, i, s) = SDAB_{i,s} + PC_{i,s} - \sum_{\tau=1}^{t} CREST_{i,s,\tau} \]

- Liquidity holdings at the beginning at the day (\( l_{i,t} \), i.e., the dependent variable): the logarithm of reserve balances plus posting of collateral (the value of intraday repo available with the BoE) at the start of the day.

- **Interbank Rate**: lagged interbank rate.
- \( \ln LevPay_{i,t-1} \): total intraday payment level (lagged, in logarithms).
- \( rK_{i,t-1}^{in} \): lagged right kurtosis of incoming payment time.
- \( rK_{i,t-1}^{out} \): lagged right kurtosis of outgoing payment time.
- \( \ln VolPay_{i,t-1} \): intraday volatility of liquidity available (lagged, in logarithms).
• \( \ln LU_{i,t-1} \): liquidity used (lagged, in logarithms) defined as follows

\[
LU(i, s) = \max\{\max_{T}[CNP(T; i, s)], 0\}.
\]

A positive cumulative net debit position means that in this time interval the bank is consuming its own liquidity. If a positive cumulative net position never happens for a bank, this bank only absorbs liquidity from the system. That is the reason for the first (outside) maximum operator. The second (inside) maximum operator helps us to trace the highest amount of liquidity a bank uses.

• \( \frac{\text{repo Liability}}{\text{Assets}} \): repo liability to total asset ratio (lagged, monthly).

• \( \text{Total Assets (log)} \): total asset (lagged, monthly, in logarithms).

• \( \frac{\Delta\text{Deposit}}{\text{Assets}} \): cumulative change in ratio of retail deposits to total assets \( \times 100 \) (lagged, monthly).

• \( \text{Total Lending and Borrowing (log)} \): total lending and borrowing in the interbank market (lagged, in logarithms).

• \( \text{CDS (log)} \): CDS price (lagged, in logarithms).

• \( \text{Stock Return (Inc. Dividend)} \): stock return including dividends (lagged).

### A.4 Additional Figures and Tables
Figure 11: Velocity of money in the payment system.

Figure 12: Turnover rate in the payment system.
Figure 13: Weekly average of the right kurtosis of aggregate payment times.

Figure 14: Interest rates in the interbank market.
Figure 15: Cross-sectional dispersion of interbank rates.
Table A1: Full Spatial Error Model Estimation

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\phi} )</td>
<td>0.8137*</td>
<td>0.3031*</td>
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<tr>
<td></td>
<td>(21.47)</td>
<td>(1.90)</td>
<td>(-4.96)</td>
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<tr>
<td>( 1/(1 - \hat{\phi}) )</td>
<td>5.3677*</td>
<td>1.4349*</td>
<td>0.8479*</td>
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<tr>
<td></td>
<td>(4.92)</td>
<td>(4.37)</td>
<td>(32.61)</td>
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**Macro Control Variables**

<table>
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<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( rK_{t-1} )</td>
<td>0.1845</td>
<td>0.0084</td>
<td>-0.0032*</td>
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<td></td>
<td>(1.30)</td>
<td>(0.55)</td>
<td>(-3.88)</td>
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<tr>
<td>( \ln VolPay_{t-1} )</td>
<td>-0.4451</td>
<td>0.0308</td>
<td>0.0291</td>
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<tr>
<td></td>
<td>(-1.00)</td>
<td>(1.17)</td>
<td>(1.72)</td>
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<tr>
<td>( TOR_{t-1} )</td>
<td>0.0166</td>
<td>0.0007</td>
<td>0.0018</td>
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<tr>
<td></td>
<td>(1.80)</td>
<td>(0.69)</td>
<td>(1.75)</td>
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<td>LIBOR</td>
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<td>0.0928</td>
<td>0.5800*</td>
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<td>(0.27)</td>
<td>(1.28)</td>
<td>(2.52)</td>
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<td>Interbank Rate Premium</td>
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<td>-0.0405</td>
<td>0.6973*</td>
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<tr>
<td></td>
<td>(1.61)</td>
<td>(0.33)</td>
<td>(3.00)</td>
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**Bank Characteristics/Micro Control Variables**

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<th>Period 3</th>
</tr>
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<tbody>
<tr>
<td>( \ln LevPay_{i,t-1} )</td>
<td>-0.0235</td>
<td>0.0802*</td>
<td>0.0808*</td>
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<td></td>
<td>(-0.62)</td>
<td>(3.29)</td>
<td>(5.09)</td>
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<tr>
<td>( rK^{in}_{i,t-1} )</td>
<td>0.0010</td>
<td>-0.0086</td>
<td>0.0045</td>
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<tr>
<td></td>
<td>(0.14)</td>
<td>(0.63)</td>
<td>(1.03)</td>
</tr>
<tr>
<td>( rK^{out}_{i,t-1} )</td>
<td>0.0090</td>
<td>0.0320*</td>
<td>-0.0061</td>
</tr>
<tr>
<td></td>
<td>(0.92)</td>
<td>(3.62)</td>
<td>(-1.32)</td>
</tr>
<tr>
<td>( \ln VolPay_{i,t-1} )</td>
<td>0.0129*</td>
<td>0.0039</td>
<td>0.0196*</td>
</tr>
<tr>
<td></td>
<td>(4.59)</td>
<td>(1.92)</td>
<td>(5.96)</td>
</tr>
<tr>
<td>( \ln LU_{i,t-1} )</td>
<td>-0.0038*</td>
<td>-0.0039*</td>
<td>-0.0027*</td>
</tr>
<tr>
<td>Asset Repo Liability</td>
<td>(-2.86)</td>
<td>(-3.41)</td>
<td>(-3.79)</td>
</tr>
<tr>
<td>Total Assets (log)</td>
<td>1.2590*</td>
<td>0.6328*</td>
<td>1.0170*</td>
</tr>
<tr>
<td></td>
<td>(5.39)</td>
<td>(10.31)</td>
<td>(18.92)</td>
</tr>
<tr>
<td>( \Delta Deposit )</td>
<td>-0.0014</td>
<td>0.0149*</td>
<td>0.0481*</td>
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<tr>
<td>Assets</td>
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<td>(11.76)</td>
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<td>Total Lending and Borrowing (log)</td>
<td>-0.1882*</td>
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<td>-0.0025</td>
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<tr>
<td></td>
<td>(-5.57)</td>
<td>(2.95)</td>
<td>(-1.27)</td>
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<tr>
<td>CDS (log)</td>
<td>0.0051</td>
<td>-0.1212*</td>
<td>-0.0383*</td>
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<tr>
<td></td>
<td>(0.13)</td>
<td>(-6.61)</td>
<td>(-4.00)</td>
</tr>
<tr>
<td>Stock Return (Inc. Dividend)</td>
<td>-0.5667</td>
<td>0.1927</td>
<td>0.2574</td>
</tr>
<tr>
<td></td>
<td>(-0.88)</td>
<td>(1.49)</td>
<td>(1.88)</td>
</tr>
</tbody>
</table>

\( R^2 \) 66.01% 92.09% 91.53%

Estimation results of equations (24) and (25). Periods 1, 2 and 3, correspond, respectively, to before the Northern Rock/Hedge Fund Crisis, after Hedge Fund Crisis but before the Asset Purchase Programme, and after the Asset Purchase Programme announcement. The \( t \)-statistics are reported in parentheses under the estimated coefficients, where * denotes statistically significant estimates at a 10% or higher confidence level. Standard errors are QMLE robust ones. For the average network multiplier, \( 1/(1 - \hat{\phi}) \), the delta method is employed.
Table A2: ratio of network to idiosyncratic volatility.

<table>
<thead>
<tr>
<th>Bank</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
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<tbody>
<tr>
<td>Bank 1</td>
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<td>2.10</td>
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<td>1.02</td>
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<tr>
<td>Bank 3</td>
<td>1.83</td>
<td>1.06</td>
<td>0.87</td>
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<td>Bank 4</td>
<td>2.62</td>
<td>1.06</td>
<td>1.08</td>
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<tr>
<td>Bank 5</td>
<td>2.41</td>
<td>1.10</td>
<td>1.02</td>
</tr>
<tr>
<td>Bank 6</td>
<td>1.65</td>
<td>1.09</td>
<td>1.03</td>
</tr>
<tr>
<td>Bank 7</td>
<td>1.47</td>
<td>0.97</td>
<td>1.13</td>
</tr>
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<td>Bank 8</td>
<td>1.69</td>
<td>1.09</td>
<td>1.03</td>
</tr>
<tr>
<td>Bank 9</td>
<td>2.12</td>
<td>1.09</td>
<td>1.03</td>
</tr>
<tr>
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<td>1.62</td>
<td>0.99</td>
<td>1.09</td>
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<tr>
<td>Bank 11</td>
<td>2.04</td>
<td>1.14</td>
<td>1.31</td>
</tr>
<tr>
<td>Mean</td>
<td>2.01</td>
<td>1.06</td>
<td>1.05</td>
</tr>
</tbody>
</table>

The table reports $\sqrt{\frac{\text{Var}(z_{i,t})}{\text{Var}(v_{i,t})}}$ for each bank and each period considered.