Information Asymmetries, Volatility, Liquidity, and the Tobin Tax

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Outline

1. The Bigger Picture
   - Introduction
   - (Other) Related Literature
2. The Model
   - Set Up
   - Equilibrium
   - Time Change(s) and Lower Frequencies
3. Conclusion

Appendix

In market data:

- Volatility is time varying and clusters at high/medium frequency $\Rightarrow$ ARCH/GARCH/SV models
- Highest degree of clustering at high frequency.
- Market vol $\neq$ fundamental vol (e.g. Campbell-Kyle (1993)).
- A relation between volatility and number/volume of trade (e.g. Gallant-Rossi-Tauchen (1992), Jones-Kaul-Lipson (1994)) & frequency matters (e.g. Engle-Sun (2007))

$\Rightarrow$ Gaussian log returns under a number of trades (stochastic) time change (Ané-Geman (2000))

- A link between information asymmetries, volatility and liquidity, and return dynamics (e.g. Kelly-Ljungqvist (2013), Easly-Hvidkjaer-O'Hara (2002)).

Our paper: a (non trivial) theory that can explain all the above facts and, more broadly, the equilibrium determinants of volatility (at different frequencies), and liquidity (tightness, depth, resilience).

(Other) Related Literature

- Sequential trade models e.g. Glosten-Milgrom (1985), Easley-O'Hara (1987), Glosten (1989), Brunnermeier-Pedersen (2009) etc.

But: *a*) complete order book; *b*) dynamic info; *c*) *weakly* exogenous arrival process; *d*) arrival intensity to infinity → approximate continuous market → make arrival process irrelevant;

- Time Deformation and Volatility e.g. Clark (1973), Ghysels-Gouriroux-Jasiak (1995), Yor-Madan-Geman (2002), Andersen-Bollerslev-Dobrev (2007), Kalogeropoulos-Roberts-Dellaportas (2007), etc.

⇒ a distributional characterisation (via stochastic time change) of equilibria on different time scales (trade, calendar, business).

- M.M. Invariance Kyle-Obizhaeva (2011, 2013) → same M.M. characteristics for different stocks in "business time."

But: in our case it is an equilibrium property.

- Information aggregation in markets e.g. Grossman-Stiglitz (1980), Hellwig (1980), Admati (1985), Kyle (1985), Wang (1993), Easley-O'Hara (2004), Vayanos-Wang (2012) etc.

Market Structure

Assets: a riskless bond ($r = 0$) and a stock with final value e^{D_T}

$$dD_t = \mu dt + \sigma dW_t^d, \quad D_0 = const, \quad W_t^d \text{ is B.M. w.r.t. to } \mathcal{F}_t.$$

Utilities: risk neutral traders and (competitive) market maker (M).

A1: Traders arrive to the market and meet M according to a stochastic counting process, N_t, with stopping times $\theta_i = \inf\{t \geq 0 : N_t = i\}$ and $\sigma\left\{N_{\theta_i + t} - N_{\theta_i}, t \geq 0\right\} \perp \mathcal{F}_{\theta_i} \forall i$, $N_T < \infty$ a.s.

- When the trader arrives at time θ_i, she observes bid, $B_{\theta_i}(v^-)$, and ask, $A_{\theta_i}(v^+)$, prices per-share posted by M, and decides if and how much to trade ($v \in \mathbb{R}$).

Friction: proportional transaction cost δ (like Tobin tax), aka M receives $v^+ A_t(v^+)(1-\delta)$ (spends $v^- B_t\left(v^-\right)(1+\delta)$)

if $v = 0$ M does not observe the arrival.

Notation: L_t= cumulated # of trades, V_t= cumulated volume, P_t = price.

Information Structure

- Common knowledge: preferences, parameters, and $\mathcal{G}_t^M := \mathcal{F}_t^P \vee \mathcal{F}_t^V$

I type: i-th (more) informed trader, in share $1-q$, knows $\mathcal{G}_t^{I,i} = \mathcal{G}_t^M \vee \mathcal{F}_t^D \vee \sigma\left\{\theta_i^I \wedge s, s \leq t\right\}$

U type: i-th uniformed/liquidity/noisy, in share q, with $\delta \in (0,q)$, trader knows $\mathcal{G}_t^{U,i} = \mathcal{G}_t^M \vee \sigma\left\{S_{\theta_i^U}\right\} \vee \sigma\left\{\theta_i^U \wedge s, s \leq t\right\}$

A2: $\mathcal{F}_T^W$, $\mathcal{F}_T^N$ and S_{θ_i} are conditionally independent given $\mathcal{H}_{i-1} \forall i$, where $\mathcal{H}_i = \mathcal{G}_{\theta_i}$, $\mathcal{G}_t = \mathcal{F}_t^V \vee \mathcal{F}_t^N$.

A3: I_i is independent of $\mathcal{F}_T^{N,S,D} \vee \sigma\left(U_k\right)_{k \neq i}$

A4: $\mathbb{P}\left(v_i \in C | \mathcal{H}_{i-1}, I_i, \theta_i\right) = \mathbb{P}\left(v_i \in C | \mathcal{H}_{i-1}, U_i, \theta_i\right)$ for $C \in \mathcal{B}\left(\mathbb{R}\right)$

Traders' optimisation problem

Notation: z_i (shadow price) is the expected value of holding one share of the asset for the agent that arrives at time θ_i

$$z_i = \mathbf{1}_{\{I_i\}} z_i^I + \mathbf{1}_{\{U_i\}} z_i^U.$$

- The expected utility from holding v^+ shares until time T for an agent of type $k \in \{I, U\}$ that arrived at θ_i^k is

$$\mathbb{E}\left[v^+ e^{D_T} \middle| \mathcal{H}_i^k \right] =: v^+ z_i^k.$$

- The expected utility from investing in the risk free asset the amount needed to buy v^+ shares at time θ_i^k is $v^+ A_{\theta_i^k}(v^+)$.

$\Rightarrow$ the **expected utility** can be expressed as:

$$\max_{v^+, v^-} \quad v^+ \left[z_i^k - A_{\theta_i^k}(v^+) \right] + v^- \left[B_{\theta_i^k}(v^-) - z_i^k \right]. \qquad (1)$$

Trader's Demand

Lemma (Trader's optimal demand)

Suppose $A_t(v^+)$, $B_t(v^-)$ satisfy regularity conditions C1-C6. Consider a trader who arrives on the market at time θ_i and observes the posted prices $A_{\theta_i}(v^+)$ and $B_{\theta_i}(v^-)$. Then

- *if $z_i > A_{\theta_i}(0)$, $v^* > 0$ is the unique solution of*

$$z_i = A_{\theta_i}(v) + vA'_{\theta_i}(v) \tag{2}$$

- *if $z_i < B_{\theta_i}(0)$, $v^* < 0$ is the unique solution of*

$$z_i = B_{\theta_i}(-v) - vB'_{\theta_i}(-v) \tag{3}$$

- *if $B_{\theta_i}(0) \leq z_i \leq A_{\theta_i}(0)$, then the optimal order size is $v^* = 0$.*

where z_i is the stock's valuation of the trader.

Market Maker's optimisation problem

- Need a (non-falsifiable) belief of M about N_t. We assume that $N_t = L_t \Rightarrow$ M doesn't update her beliefs if no trade occurs.

Notation: M's utility from owning one share of the stock until T is

$$Z_t^M := \mathbb{E}\left[e^{D_T} \middle| \mathcal{G}_t^M, N_t = L_t \right].$$

- M sets time t bid and ask prices as a functions of the order size v:

$$A_t\left(v^+\right)(1-\delta) = \underbrace{\sum_{i=1}^{\infty} \mathbf{1}_{\{i=1+L_{t-}\}} \mathbb{E}\left[e^{D_T} | \tilde{\mathcal{H}}_i^M, N_{\tau_i} = L_{\tau_i}\right]\Big|_{\tilde{v}_i = v^+, \tau_i = t}}_{\text{M's valuation}}, \tag{4}$$

$$B_t\left(v^-\right)(1+\delta) = \sum_{i=1}^{\infty} \mathbf{1}_{\{i=1+L_{t-}\}} \mathbb{E}\left[e^{D_T} | \tilde{\mathcal{H}}_i^M, N_{\tau_i} = L_{\tau_i}\right]\Big|_{\tilde{v}_i = -v^-, \tau_i = t}. \tag{5}$$

Equilibrium: Price Setting

Proposition (Optimal ask and bid functions)

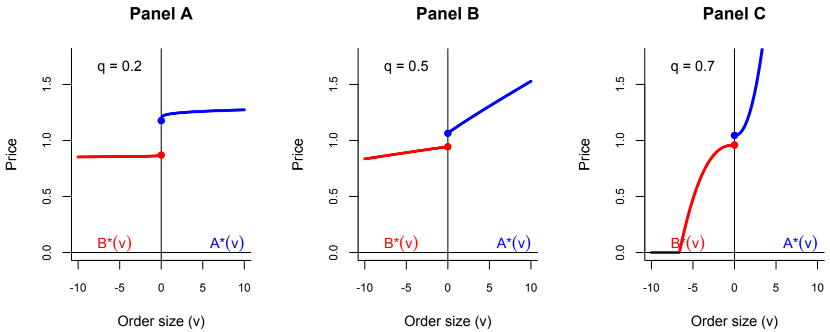
Suppose assumptions A1-A5 are satisfied. Then there exist optimal ask, $A_t(v^+)$, and bid, $B_t(v^-)$, prices that satisfy conditions C1-C5, and the market maker's optimality conditions. Moreover, optimal $A_t(v)$ and $B_t(v)$ have the following forms:

$$A_t^*(v) = \frac{q}{q-\delta}\left(1 + \alpha v^{\frac{q-\delta}{1-q}}\right) \underbrace{\sum_{i=0}^{\infty} \mathbf{1}_{\{i=L_{t-}+1\}} Z^M_{\tau_i - 1}}_{M\text{'s valuation}} \tag{6}$$

$$B_t^*(v) = \begin{cases} \frac{q}{q+\delta}\left(1 - \beta v^{\frac{q+\delta}{1-q}}\right) \sum_{i=0}^{\infty} \mathbf{1}_{\{i=L_{t-}+1\}} Z^M_{\tau_i - 1} & \text{if } \beta v^{\frac{q+\delta}{1-q}} \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

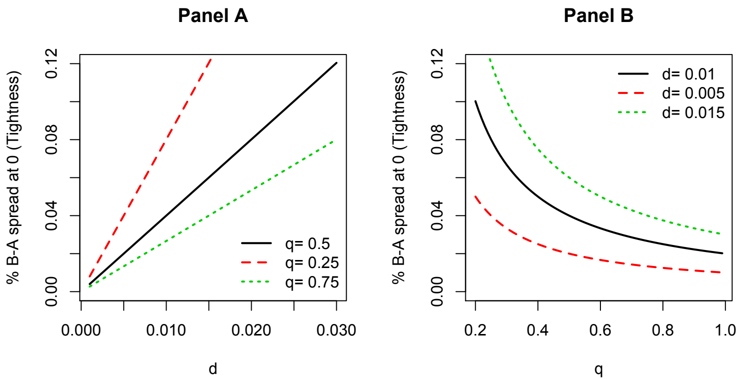
where α and β are strictly positive arbitrary constants, and Z^M denotes the market maker valuation.

Equilibrium Bid and Ask functions



- order book interpretation (M analogy).
- flexible parametrisation and empirically promising.

Liquidity: Tightness

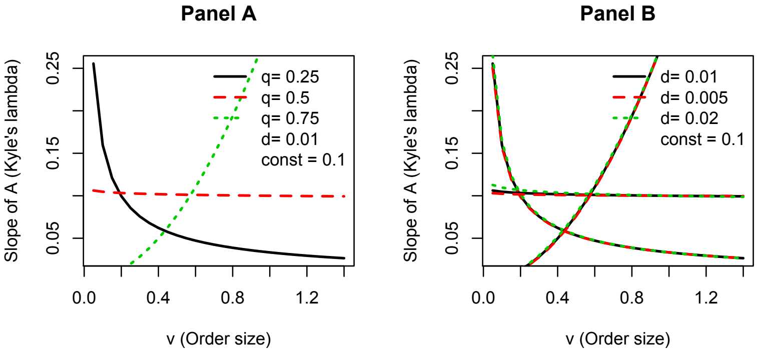


$$\% \text{ Bid-Ask spread at } 0 = \frac{2q\delta}{q^2 - \delta^2}$$

B-A $\uparrow$ in adverse selection $(1-q)$ and trading cost (δ)

- mutually reinforcing effects

Liquidity: Depth



Slope of the Ask schedule, normalised by M's valuation: $\frac{q}{1-q}\alpha(v^{+})^{\frac{2q-\delta-1}{1-q}}$.

Note: Loeb (1983) and Keim and Madhavan (1996) find that the price impact per unit trade is smaller for large orders.

High Frequency Price Process

- Since trades happen either at ask or bid, we can characterise the price process for any volume process:

$$\log \frac{P_{t+s}}{P_t} = \sum_{i=L_t}^{L_{t+s}} \left\{ \log \left(1 + \xi_i \left| V_{\tau_i} - V_{\tau_{i-1}} \right|^{\gamma_i} \right) + \log c_{1,i} + \log c_{2,i-1} \right\}. \tag{8}$$

where ξ_i, γ_i, $c_{1,i}$ and $c_{2,i}$ are known functions of δ, q, and whether trades are at ask or bid (the latter is a binomial r. v.).

$\Rightarrow$ consistent with non-lin model of Gallant, Rossi, and Tauchen (1992) (Tauchen-Pitts (1983), Epps-Epps (1976), Clark (1973), etc.)

if $|\xi_i| \left| V_{\tau_i} - V_{\tau_{i-1}} \right|^{\gamma_i}$ is

Small $\approx$ power law relationship (e.g. Farmer and Lillo (2004) and Farmer, Lillo, and Mantegna (2003))

Large $\approx$ log-log relationship (e.g. Potters-Bouchaud (2003)).

Equilibrium: Volume

Theorem

Suppose Assumptions A1-A5 are satisfied. For strictly positive constants α and β, there is a unique market equilibrium, $A_t^*(v)$, $B_t^*(v)$, v_i^*, where $A_t^*(v)$ and $B_t^*(v)$ are given, respectively, by equations (6) and (7), and

$$v_i^* = \begin{cases} \left[\frac{1-q}{\alpha(1-\delta)}\left(\frac{q-\delta}{q}\frac{z_i}{z_i^M} - 1\right)\right]^{\frac{1-q}{q-\delta}} & \text{if } \frac{q}{q-\delta} z_i^M < z_i, \\ -\left[\frac{1-q}{\beta(1+\delta)}\left(1 - \frac{q+\delta}{q}\frac{z_i}{z_i^M}\right)\right]^{\frac{1-q}{q+\delta}} & \text{if } z_i < \frac{q}{q+\delta} z_i^M, \\ 0 & \text{if } \frac{q}{q+\delta} z_i^M \le z_i \le \frac{q}{q-\delta} z_i^M \end{cases}$$

where $z_i^M := Z_{\theta_i}^M$

From fundamentals to price process

Lemma (price process as map of fundamentals)

Suppose that Assumptions A1-A5 are satisfied and the market is at the equilibrium. Then the trading times are defined recursively $(\tau_0 = 0)$

$$\tau_i = \inf\left\{\theta_j > \tau_{i-1} : \log z_j - \log \tilde{p}_{i-1} \notin \left(b\left(c_{2,i-1}\right), a\left(c_{2,i-1}\right)\right)\right\},$$

where $a(x) = \log\left(\frac{qx}{q-\delta}\right)$ *and* $b(x) = \log\left(\frac{qx}{q+\delta}\right)$*, and prices are*

$$\tilde{p}_0 = e^{D_0 + \left(\mu + \frac{1}{2}\sigma^2\right)T}, \quad \tilde{p}_i = \frac{1}{c_{2,i}}\left[\left(1-q\right)z_i + q\tilde{p}_{i-1}c_{2,i-1}\right], \tag{9}$$

$$c_{2,i} = \begin{cases} 1-\delta & \textit{if } \log \tilde{z}_i - \log \tilde{p}_{i-1} > a\left(c_{2,i-1}\right) \textit{ and } i > 0 \\ 1+\delta & \textit{if } \log \tilde{z}_i - \log \tilde{p}_{i-1} < b\left(c_{2,i-1}\right) \textit{ and } i > 0 \\ 1, & \textit{if } i = 0 \end{cases}$$

The process Z (Shadow Price)

We will work with the value of the log profit of the last agent that arrived before t, D_t^{tr}, given by

$$d_i^{tr} = \begin{cases} \log z_i - \left(\mu + \frac{\sigma^2}{2}\right)(T - \theta_i) & \forall i \geq 1 \\ D_0 & i = 0 \end{cases}, \quad D_t^{tr} = \sum_{i=0}^{\infty} \mathbf{1}_{\{i=N_t\}} d_i^{tr}.$$

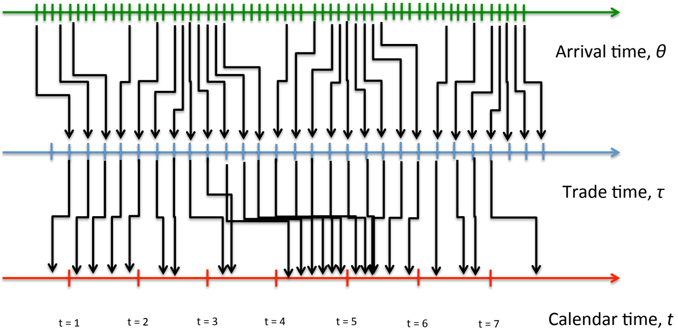
The distribution of the process D^{tr} is (Lemma 3):

$$\begin{aligned} \mathbb{P}\left[d_i^{tr} \leq x | \mathcal{H}_{i-1}, \theta_i\right] &= (1-q)\sum_{j=1}^{i-1} q^{i-1-j} \mathbb{P}\left[d_j^{tr} + \varepsilon_{i,j} \leq x | d_j^{tr}, \Delta_{i,j}\right] \\ &+ q^{i-1}\mathbb{P}\left[d_0^{tr} + \varepsilon_{i,0} \leq x | d_0^{tr}, \Delta_{i,0}\right] \end{aligned}$$

where $\Delta_{i,j} := \theta_i - \theta_j$, $\varepsilon_{i,j} := \mu\Delta_{i,j} + \sigma\sqrt{\Delta_{i,j}}\eta_{i,j}$, and $\eta_{i,j} \sim N(0,1)$ is independent of d_j^{tr} and $\Delta_{i,j}$ for all $j < i$.

Sequence of Markets in a Nutshell

Arrival intensity $\to \infty$ ("business time") $\Rightarrow$ valuation of i-th arrival $(D^{tr}) \xrightarrow{\mathcal{L}}$ to B.M. (Prop. 9) $\Rightarrow$ Trade occurs when B.M. touches B-A bounds $+$ The map from D^{tr} to P is continuous (Lem. 5) $\Rightarrow$ Price $\xrightarrow{\mathcal{L}}$ on trade time (Thm. 7) $\Rightarrow$ Calendar time dist. $=$ time change wrt trades per time (Prop. 9) $\Leftrightarrow$ S.V. driven by # of trades

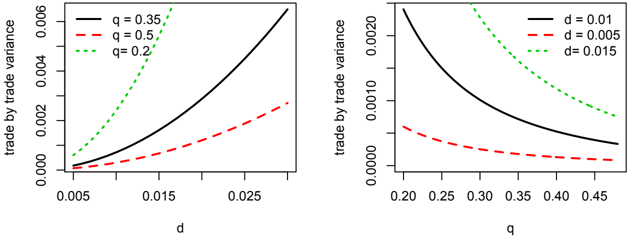


Trade-by-trade Volatility (Med. Frequency)

Corollary (Volatility of the Limiting Price Process)

The conditional variance on the trade time scale, for $i > 1$, is:

$$Var\left(\frac{\tilde{p}_i}{\tilde{p}_{i-1}}|\mathcal{F}^W_{\tau_{i-1}}\right) = \frac{\delta^2(1-q^2)}{q^2-\delta^2}.$$



- $\uparrow$ in adverse selection and δ (cf. Hau (2006), Jones-Seguin (1997), Umlauf (1993))
- $\Rightarrow$ Tobin Tax reduces (increases) Vol. in calm (hectic) times.
- mutually reinforcing effects

Low/Ultra-Low Frequency Price Distribution

Proposition (Low/Ultra-Low Frequency Price Distribution)

$$\frac{\log \frac{P_t}{P_s} - \frac{\sigma^2}{2}(s-t)}{\sqrt{L_t^p - L_s^p}} \xrightarrow[t-s\to\infty]{d} \mathcal{N}\left(0, \sigma^2 \mu_\tau\right), \tag{10}$$

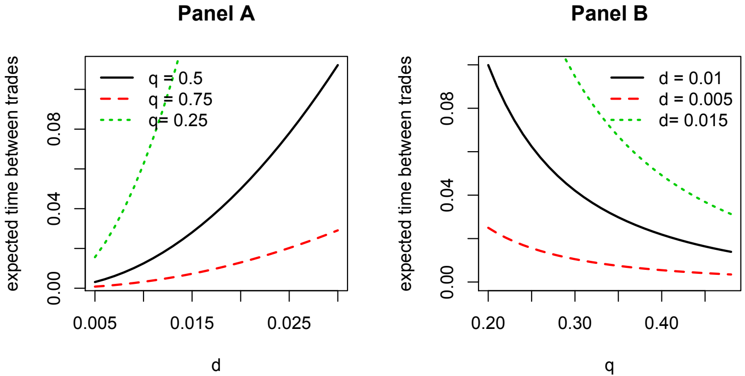
where μ_τ is the expected time between trades. And at Ultra-low frequency

$$\frac{\log \frac{P_t}{P_s} - \frac{\sigma^2}{2}(s-t)}{\sqrt{t-s}} \xrightarrow[t-s\to\infty]{d} \mathcal{N}\left(0, \sigma^2\right). \tag{11}$$

Eq. (10) consistent with Jones, Kaul, and Lipson (1994), Ané and Geman (2000), Engle and Sun (2007) etc.

Note: second result due to $L_t^p/t \xrightarrow[t\to\infty]{a.s} 1/\mu_\tau$

Expected time between trades (μ_τ)



$$\mu_\tau := \frac{2}{\sigma^2}\left[\log\frac{q-\delta}{q(1-\delta)} + \frac{(q+\delta)(1+\delta)}{2\,(q+\delta^2)}\log\frac{(1-\delta)(q+\delta)}{(1+\delta)(q-\delta)}\right]$$

- $\uparrow$ in adverse selection $(1-q)$ and trade cost (δ)
- mutually reinforcing effects

Liquidity: Resilience

Note: on the trade time scale, the market maker valuation evolves as

$$\tilde{z}_i^M = (1-q)\,\tilde{z}_i + q\tilde{z}_{i-1}^M. \tag{12}$$

(i.e. an AR(1)) where $\tilde{z}_i$ is the valuation of the i-th trader.

Hence: half-life (reciprocal of resilience) on the calendar time scale:

$$\underbrace{\frac{\ln 1/2}{\ln q}}_{\text{trade-by-trade half-life}} \times \underbrace{\mu_\tau}_{\text{expected time between trades}}$$

w.r.t. δ: same properties as $\mu_\tau \Rightarrow \uparrow$ in δ (consistent with Umlauf (1993)):
resilience $\downarrow$ in δ (Tobin Tax)

w.r.t. q: two opposing effects:

1. $\mu_\tau \downarrow$ in q
2. trade-by-trade half-life $\uparrow$ in q

overall: calendar time half-life $\downarrow$ in q: resilience $\downarrow$ in adverse selection

mutually reinforcing negative effects

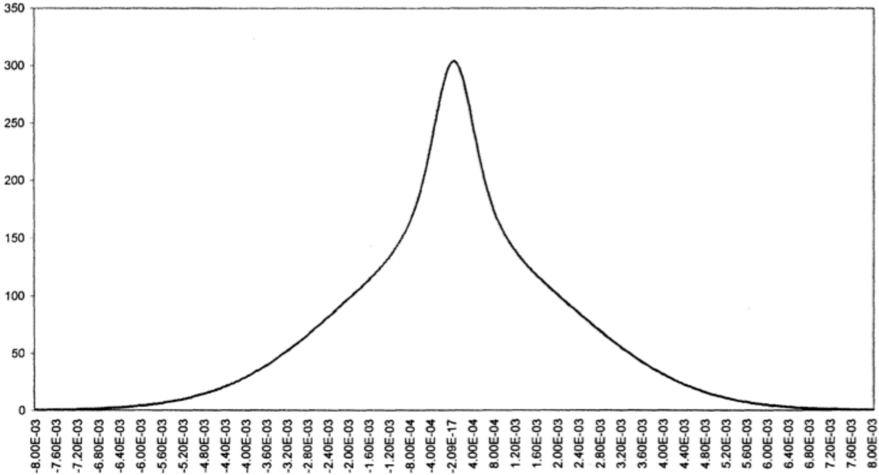
Conclusion

A simple and tractable equilibrium framework that:

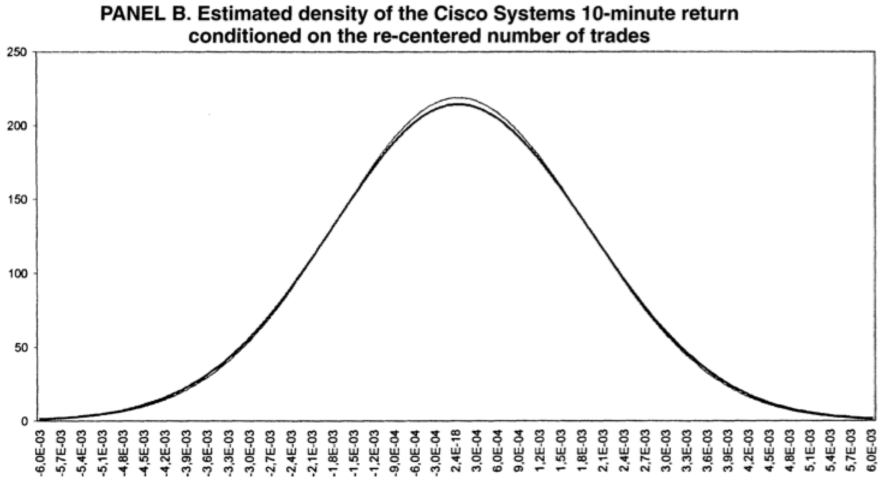
1. can rationalise a very large set of empirical findings about financial market volatility and returns at different frequencies.
2. identifies the equilibrium determinants of the 3 main liquidity dimensions, and can rationalise related empirical findings.
3. delivers policy relevant (and empirically consistent) predictions about the Tobin Tax.
4. can be structurally estimated to pin down asset specific measures of: asy. info., frictions to trade, liquidity, fundamental vol etc. $\Rightarrow$ empirical follow up.
5. provides a novel approach for the study of equilibrium dynamics (at multiple frequencies) for very different economic problems (e.g. sticky prices/wages/information, endogenous consumption optimisation etc.)

Appendix

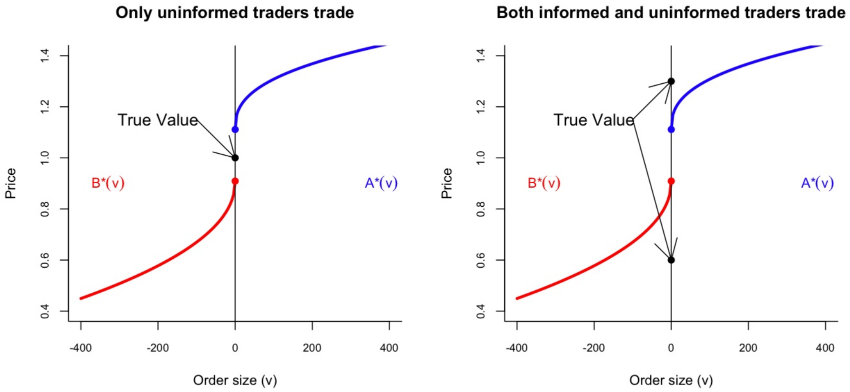
PANEL B. Estimated density of the 10-minute Cisco Systems returns



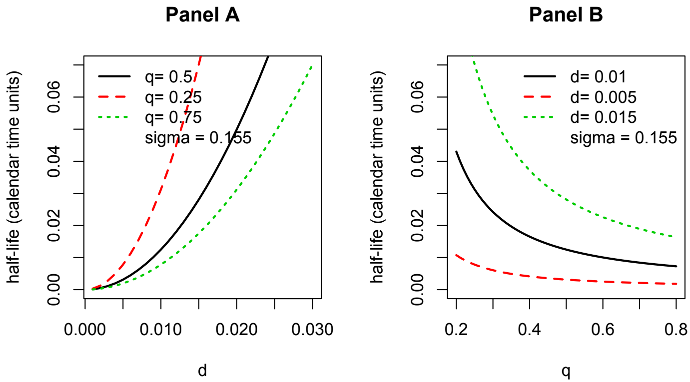
Source: Ané and Geman (2000)



Source: Ané and Geman (2000)


Only uninformed traders trade
True Value
B*(v)
A*(v)
Price
0.4
0.6
0.8
1.0
1.2
1.4
-400
-200
0
200
400
Order size (v)
Both informed and uninformed traders trade
True Value
B*(v)
A*(v)
Price
Order size (v)

Resilience: calendar time half-life of M's update



negative effects of trade and adverse selection costs on resilience mutually reinforce

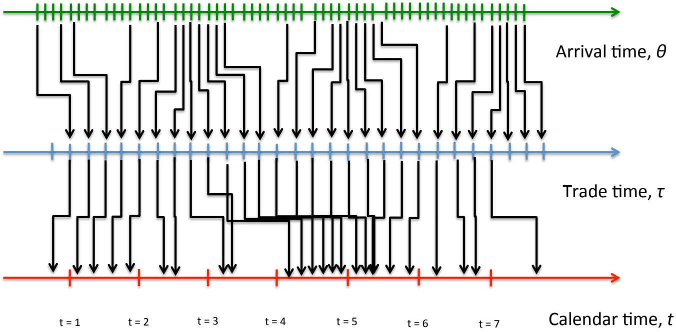
In a Nutshell

- for (non trivial) heteroscedasticity, need the conditional and unconditional information reflected into prices to be different.

$\Rightarrow$ Asym. information + trade friction (δ) (for B-A spread)

High freq. : trade-by-trade price process adapted to info process: no SV in the latter $\Leftrightarrow$ no SV in the former.

But : trade $t \neq$ calendar t (in equilibrium)



In a Nutshell cont'd

Med freq. := arrival rate $\to \infty$ i.e. "business time" $\Rightarrow$ Number of trades becomes the relevant information (no residual info in volume).

- Trade-by-trade Price Vol $\uparrow$ in δ and adverse selection

$\Rightarrow$ Tobin tax: $\downarrow$ Vol in calm times, and $\uparrow$ Vol in hectic ones.

- price SV on calendar time driven by number of trades.

Low freq. := number of trades per time is "large"

- Trades per time $\downarrow$ in δ and adverse selection.

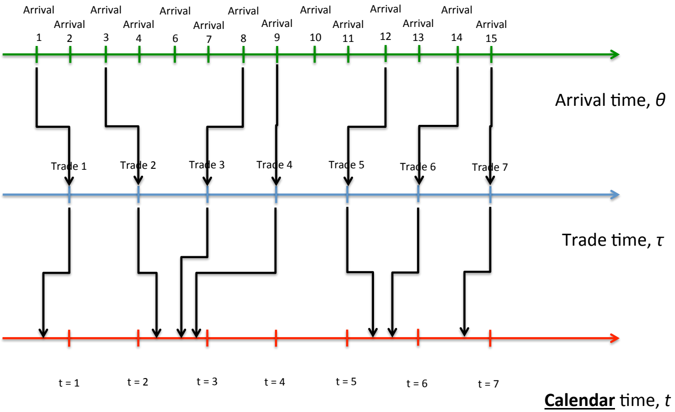
tradeoff: calendar Vol $\approx$ Trade-by-trade Vol $\times$ Trades per time.

$\Rightarrow$ Vol $\uparrow$ in δ and adverse selection.

Tobin Tax :

- Vol $\uparrow$
- $\downarrow$ "Tightness" & "Resilience" (small impact on depth)
- stronger effect in less liquid markets

Review of Time Scales



Regularity Conditions

C1 For a fixed v, the processes $B_t\left(v^-\right)$, $A_t\left(v^+\right)$, are *cáglád*

aka: M can change prices at any time but the time of trade.

C2 For a fixed t, $A_t\left(v^+\right) : \mathbb{R}_+ \to \bar{\mathbb{R}}_+ \backslash \{0\}$ is continuous, nondecreasing and $\lim_{v^+ \to \infty} A_t(v^+) = +\infty$.

C3 For a fixed t, $B_t\left(v^-\right) : \mathbb{R}_+ \to \bar{\mathbb{R}}_+$ is continuous, nonincreasing and $\lim_{v^- \to \infty} B_t(v^-) = 0$.

aka: no: free disposal, infinite trade size, decreasing price per-share.

C4 For a fixed t, $A_t\left(0\right) \geq B_t\left(0\right)$ for all $\omega \in \Omega$.

C5 For any fixed t, $A_t\left(\cdot\right)$ is continously differentiable, and $B_t\left(\cdot\right)$ is continously differentiable on the set $\{v : B_t\left(v\right) > 0\}$

C6 For a fixed t, $vA_t\left(v\right)$ is strictly convex, and $vB_t\left(v\right)$ is striclty concave on the set $\{v : B_t\left(v\right) > 0\}$

aka: C5-C6 ensures strict concavity of traders' problem.

Equilibrium: Definition

Definition (Equilibrium)

A market equilibrium is a set of policy fuctions $A_t\left(v^+\right)$, $B_t\left(v^-\right)$ satisfying regularity conditions and $v_i\left(A_{\theta_i}\left(v^+\right), B_{\theta_i}\left(v^-\right)\right)$ such that:

1. $A_t\left(v^+\right)$ and $B_t\left(v^-\right)$ solve the market maker optimisation problem $\forall v, t$;
2. $v_i\left(A_{\theta_i}\left(v^+\right), B_{\theta_i}\left(v^-\right)\right)$ solves the trader's problem.