Technical note on 'Strategic Firms and Endogenous Consumer Emulation'

Philipp Kircher
University of Pennsylvania

Andrew Postlewaite
University of Pennsylvania

April, 2007

This note provides the technical details on the omitted proof in "Strategic Firms and Endogenous Consumer Emulation". For notation, please see that paper.

Lemma 6: *There exists $M > 0$ such that in any stationary equilibrium with $\Pi^{\theta h} > M$, a high quality firm will promise service $\bar{s}$ in any period to type θ consumers.*

Proof: Let s be the equilibrium strategy of a high quality firm to a type θ customer that generates the benefit of $\Pi^{\theta h}$ for the firm. Let $\check{s} \neq s$ be a one-shot deviation in the service promise.

If promising $\check{s}$ instead of $s = \bar{s}$ results in the customer searching for another firm and never returning, then for $\Pi^{\theta h} > M = c$ offering the service is optimal, since by offering service the firm retains the business of this consumer and gains $\Pi^{\theta h} - c$ when he returns.

For the case in which the consumer returns even in the absence of service, the proof is divided into two parts. The first establishes that increasing (decreasing) the service promise increases (decreases) the probability with which the consumer returns by a finite amount. The second provides the lower bound for the profitability of the consumer such that the threat of a potential time delay warrants service promises.

For the first part we discuss the consumer's reaction in terms of the timing of consumption in response to a deviation. In equilibrium the customer returns whenever $\rho \geq \hat{\rho}_h^{\theta} = u_0^{\theta} - q_h - s$, otherwise he does not consume. Let $V^C(q_h, \rho) = V^C(\rho)$ and V^F be the flow payoff of this strategy for the customer and the firm respectively. Consider the customer's response to a one-shot deviation by the firm. The value function $\check{V}^C(\rho)$ of the customer for the period directly after the deviation is

$$\check{V}^C(\rho) = \max\{(1-\delta)(q_h + \check{s} + \rho) + \delta E V^C(\rho), (1-\delta)u_0^{\theta} + \delta E \check{V}^C(\rho)\}. \tag{1}$$

Let $\check{\rho}$ be the value for which the first term in the max operator is equal to the second term, i.e.,

$$(1-\delta)(q_h+\check{s}+\check{\rho})+\delta EV^C(\rho)=(1-\delta)u_0^\theta+\delta E\check{V}^C(\rho). \tag{2}$$

This implies that in the periods between the deviation and the next time of consumption the customer will return to the firm when $\rho \geq \check{\rho}$, and will not consume otherwise. Then

$$\begin{aligned} E\check{V}^C(\rho) &= \int_{\check{\rho}}^{\bar{\rho}} \left[(1-\delta)(q_h+\check{s}+\rho)+\delta EV^C(\rho)\right] f(\rho)d\rho \\ &\quad + \int_{\underline{\rho}}^{\check{\rho}} \left[(1-\delta)u_0^\theta+\delta E\check{V}^C(\rho)\right] f(\rho)d\rho. \end{aligned}$$

Therefore

$$\begin{aligned} (1-\delta F(\check{\rho}))E\check{V}^C(\rho) &= (1-F(\check{\rho}))\left[(1-\delta)(q_h+\check{s})+\delta EV^C(\rho)\right] \\ &\quad + \int_{\check{\rho}}^{\bar{\rho}} (1-\delta)\rho f(\rho)d\rho + F(\check{\rho})(1-\delta)u_0^\theta. \end{aligned} \tag{3}$$

Substituting (3) into the equation (2), integration by parts and rearranging yields:

$$(1-\delta)(\check{\rho}+q_h+\check{s})-\delta\int_{\check{\rho}}^{\bar{\rho}}(1-F(\rho))d\rho-u_0^\theta+\delta EV^C(\rho)=0.$$

The value of $EV^C(\rho)$ is given by lemma 1. Substitution leads to

$$(1-\delta)(\check{\rho}+q_h+\check{s}-u_0^\theta)+\delta\int_{u_0^\theta-q_h-s}^{\check{\rho}}(1-F(\rho))d\rho=0. \tag{4}$$

Equation (4) has a unique solution. It also reveals that for $s=0$ and $\check{s}=\bar{s}$ we have $\check{\rho}<u_0^\theta-q_h$, which implies that the frequency of consumption is increased by the deviation. Let ζ_s be the probability of returning each period under the equilibrium strategy s, and let $\check{\zeta}_s$ be the probability of returning next period after a one-shot deviation in the service promise. Then $\check{\zeta}_0-\zeta_0=(1-F(\check{\rho}))-(1-F(u_0^\theta-q_h))>0$. On the other hand for $s=\bar{s}$ and $\check{s}=0$ equation (4) reveals that $\check{\rho}<u_0^\theta-q_h-\bar{s}$, which implies that the deviation decreases the frequency of consumption. That is, $\check{\zeta}_{\bar{s}}-\zeta_{\bar{s}}\equiv(1-F(\check{\rho}))-(1-F(u_0^\theta-q_h-\bar{s}))<0$. Hence, a change in service provision changes the frequency of consumption by a finite amount, i.e. $\triangle\zeta\equiv\min\{\check{\zeta}_0-\zeta_0,|\check{\zeta}_{\bar{s}}-\zeta_{\bar{s}}|\}>0$.

For the second part we discuss the firm's incentive to deviate. We show that for Π large enough $s=0$ cannot be an equilibrium strategy since a one-shot deviation would be profitable. We also show that $s=\bar{s}$ is an equilibrium strategy.

Consider first the case where the candidate equilibrium strategy is $s = 0$, the one-shot deviation is $\check{s} = \bar{s}$. In this case $(\check{\zeta}_0 - \zeta_0) > 0$. Note that the effective discount factor for the firm in this case is $\delta_F = \delta\beta$ because the firm discounts with β and the survival probability of the customer is δ. Normalizing profits by $(1 - \delta_F)$, the equilibrium value to the firm is $V^F = \zeta_0 \Pi^{\theta h}$. The value to the firm from period $t + 1$ onward after a one-shot deviation in period t is

$$\check{V}^F = \check{\zeta}_0((1 - \delta_F)(\Pi^{\theta h} - c) + \delta_F V^F) + (1 - \check{\zeta}_0)\delta_F \check{V}^F.$$

A one-shot deviation is *profitable* if $\check{V}^F > V^F$, or equivalently $\Pi^{\theta h} > \frac{\check{\zeta}_0}{\check{\zeta}_0 - \zeta_0} c$. This is fulfilled if $M \geq \frac{1}{\triangle\zeta} c$.

Consider now the case of $s = \bar{s}$ and $\check{s} = 0$. In this case $(\check{\zeta}_{\bar{s}} - \zeta_{\bar{s}}) < 0$. The equilibrium (flow) value to the firm is $V^F = \zeta_{\bar{s}} \Pi^{\theta h}$. The flow value to the firm from period $t + 1$ onward after a one-shot deviation in period t is

$$\check{V}^F = \check{\zeta}_{\bar{s}}((1 - \delta_F)(\Pi^{\theta h} + c) + \delta_F V^F) + (1 - \check{\zeta}_{\bar{s}})\delta_F \check{V}^F.$$

A one-shot deviation is *not profitable* if $\check{V}^F \leq V^F$, or equivalently $\Pi^{\theta h} \geq \frac{\check{\zeta}_{\bar{s}}}{\zeta_{\bar{s}} - \check{\zeta}_{\bar{s}}} c$. This is fulfilled if $M \geq \frac{1}{\triangle\zeta} c$. Therefore for $\Pi^{\theta h} > M \geq \frac{1}{\triangle\zeta} c$ the only equilibrium strategies for high quality firms is $s = \bar{s}$. *Q.E.D.*