Fund Managers, Career Concerns, and Asset Price Volatility

By VERONICA GUERRIERI AND PETER KONDOR

WEB APPENDIX

Proof of Proposition 2

Taking logs of expression (A1), one obtains

$$\log p(s) = \begin{cases} -\log R & \text{if } z(\chi, b) \in [\underline{b} - N^I, \underline{b}) \\ \log P(q) & \text{if } z(\chi, b) \in [\underline{b}, \overline{b} - N^I] \\ \log 0 & \text{if } z(\chi, b) \in (\overline{b} - N^I, \overline{b}] \end{cases},$$

where

$$P(q) = \frac{1-q}{R}\left[\frac{1-\left[\alpha + (1-\alpha) E(q)\right]\delta\omega\beta}{1-\left[\alpha + (1-\alpha) E(q)\right]\delta\omega\beta - (1-2q)\,\delta\omega\beta}\right],$$

with $\alpha = N^I/(\overline{b} - \underline{b})$. Recall that Assumption 3 ensures that $P(q) \in (0, 1/R)$ for any $q \in [\underline{q}, \overline{q}]$. Taking logs of this expression and of equation (11) we can define

$$\begin{aligned} h(q) &\equiv \log P^B(q) = \log(1-q) - \log R, \\ g(q) &\equiv \log P(q) = \log(1-q) - \log R - \log\left[\frac{1-\left[\alpha + (1-\alpha) E(q)\right]\delta\omega\beta}{1-\left[\alpha + (1-\alpha) E(q)\right]\delta\omega\beta - (1-2q)\,\delta\omega\beta}\right]. \end{aligned}$$

Differentiating these two expressions with respect to q gives $h'(q) = -1/(1-q)$ and $g'(q) = -1/(1-q) + 2\beta\delta\omega/\left[1 - \beta\delta\omega(1-2q+\alpha+(1-\alpha)E(q))\right]$. Assumptions 1 and 3 guarantee that $\left|g'(q)\right| > \left|h'(q)\right|$. Define $\bar{h} \equiv \int h(q)\,dF(q)$, $\bar{g} \equiv \int g(q)\,dF(q)$, and q_0 such that $g(q_0) = \int g(q)\,dF(q)$. Then

$$Var(g(q)) = \int (g(q) - \bar{g})^2\,dF(q) = \int (g(q) - g(q_0))^2\,dF(q) > \int (h(q) - h(q_0))^2\,dF(q),$$

where the last inequality follows from $\left|g'(q)\right| > \left|h'(q)\right|$ and the monotonicity of both h and g. Moreover, from a standard property of the second moment, we can write

$$\int (h(q) - h(q_0))^2\,dF(q) = \int \left(h(q) - \bar{h}\right)^2 dF(q) + \int \left(\bar{h} - h(q_0)\right)^2 dF(q).$$

Combining the last two expressions we then obtain $Var(g(q)) > \int \left(h(q) - \bar{h}\right)^2 dF(q) + \int \left(\bar{h} - h(q_0)\right)^2 dF(q) \geq Var(h(q))$. This implies $Var(\log P(q, \chi, b)) > Var\left(\log P^B(q)\right)$ whenever $z(\chi, b) \in [\underline{b}, \overline{b} - N^I]$. For any other $z(\chi, b) \notin [\underline{b}, \overline{b} - N^I]$, $\log P(q, \chi, b) = \log P^B(q)$, completing the proof that $Var(\log P(q, \chi, b)) \geq Var\left(\log P^B(q)\right)$.

Proof of Proposition 3

From the proof of Proposition 2, it is straightforward that the higher the absolute value of $d \log P(q)/dq$, or $|g'(q)|$ in the notation of the proof, the stronger the amplification effect. Such an object is larger, the larger is $y(q;x)$ with

$$y(q;x) \equiv 2\beta\delta\omega\left[1-\beta\delta\omega\left(1-2q+\frac{N^I(x)}{\overline{b}-\underline{b}}(1-E(q))+E(q)\right)\right]^{-1},$$

where, with some slight abuse of notation, $N^I(x)$ denotes the equilibrium measure of informed employed managers as a function of the parameter x, which can be equal to κ, γ, or M^I. Recall that Assumption 3 is sufficient to ensure that $y(q;x) > 0$ for all $q \in [\underline{q},\overline{q}]$. We can then differentiate this expression and obtain that for any given $q \in [\underline{q},\overline{q}]$ we have $dy(q;x)/d\kappa = \left(dy/dN^I\right)\left(dN^I/dx\right)$, where

$$\frac{dy}{dN^I} = 2(\beta\delta\omega)^2\left(\frac{1-E(q)}{\overline{b}-\underline{b}}\right)\left[1-\beta\delta\omega\left(1-2q+\frac{N^I(\kappa,\gamma)}{\overline{b}-\underline{b}}(1-E(q))+E(q)\right)\right]^{-2} > 0.$$

Next, note from (A4) that N^I depends on γ and κ only through μ, so $dN^I/d\kappa = \left(dN^I/d\mu\right)(d\mu/d\kappa)$ and $dN^I/d\gamma = \left(dN^I/d\mu\right)(d\mu/d\gamma)$, where

$$dN^I/d\mu = (1-\delta)M^I/(1-\delta(1-\mu))^2 > 0.$$

Instead M^I affects N^I both directly and through μ, so that

$$dN^I/dM^I = \mu/[1-\delta(1-\mu)] + \left(dN^I/d\mu\right)\left(d\mu/dM^I\right).$$

First, we can rewrite the implicit function for μ as $\upsilon(\mu;x) = 0$ with

$$\upsilon(\mu;x) \equiv \frac{x}{\mu} - R\left\{1-\delta\omega\beta\left[E(q)+\frac{M^I(1-E(q))}{(\overline{b}-\underline{b})\left(\delta+\frac{1-\delta}{\mu}\right)}\right]\right\}^{-1}, \tag{A11}$$

where $x \equiv \kappa/\gamma$. Applying the implicit function theorem, we obtain $d\mu/d\kappa = -\upsilon_x/\left(\gamma\upsilon_\mu\right)$ and $d\mu/d\gamma = \kappa\upsilon_x/\left(\gamma^2\upsilon_\mu\right)$. We can derive $\upsilon_x = 1/\mu$ and

$$\upsilon_\mu = -\frac{1}{\mu^2}\left[x + R\delta\omega\beta(1-\delta)\frac{M^I(1-E(q))}{(\overline{b}-\underline{b})\left(\delta+\frac{1-\delta}{\mu}\right)^2}\left\{1-\delta\omega\beta\left[E(q)+\frac{M^I(1-E(q))}{(\overline{b}-\underline{b})\left(\delta+\frac{1-\delta}{\mu}\right)}\right]\right\}^{-2}\right].$$

It is immediate that $\upsilon_x > 0$ and $\upsilon_\mu < 0$, so that $d\mu/d\kappa > 0$ and $d\mu/d\gamma < 0$. Combining these results with $dy/dN^I > 0$ and $dN^I/d\mu > 0$, we obtain

$dy\left(q;\kappa\right)/d\kappa > 0$ and $dy\left(q;\gamma\right)/d\gamma < 0$. Next, we can rewrite the implicit function for μ as a function $\upsilon\left(\mu; M^I\right) = 0$, where $\upsilon\left(\mu; M^I\right)$ is equal to the right-hand-side of expression (A11). Applying the implicit function theorem we now obtain $d\mu/dM^I = -\upsilon_{M^I}/\upsilon_\mu$, where

$$\upsilon_{M^I} = -\delta\omega\beta R \frac{\left(1-E\left(q\right)\right)}{\left(\overline{b}-\underline{b}\right)\left(\delta+\frac{1-\delta}{\mu}\right)}\left\{1-\delta\omega\beta\left[E\left(q\right)+\frac{M^I\left(1-E\left(q\right)\right)}{\left(\overline{b}-\underline{b}\right)\left(\delta+\frac{1-\delta}{\mu}\right)}\right]\right\}^{-2}.$$

After some algebra we can show that $dN^I/dM^I > 0$, so $dy\left(q;M^I\right)/dM^I > 0$, completing the proof.

Proof of Proposition 5

Given that we assumed there exist three functions $W\left(q,N^I\right)$, $\mu\left(q,N^I\right)$, and $G\left(q,N_I\right)$ satisfying equations (14), (15), and (17), the proof follows closely the proof of proposition 1. The only slightly different step is to prove that the investors' firing strategy is optimal, which we analyze next.

Here we show that Assumption 5 is sufficient to ensure that the belief that an employed manager is informed if he did not reveal to be uninformed is always higher than the probability that a newly hired manager is informed. That is, the posterior probability, $\eta_{i,t+1}$, that manager i is informed if $\sigma_{i,t} = 0$ and either $p_t = 1/R$ or $\theta_{i,t} = 1-\chi_t$ is larger than the probability that an unemployed manager at time t is informed, ε_t. The proof follows closely the one for the iid case, except that now the job flows are not constant over time. First, consider an investor who has just hired manager i so that his prior belief $\eta_{i,t} = \varepsilon_t$. In this case, if $\sigma_{i,t} = 0$ and either $p_t = 1/R$ or $\theta_{i,t} = 1-\chi_t$, then $\eta_{i,t+1} = \varepsilon_{t-1}/\left[\varepsilon_{t-1}+\left(1-\xi_t^U\right)\left(1-\varepsilon_{t-1}\right)\right]$. Next, we want to show that $\eta_{i,t+1} \geq \varepsilon_t$. This condition can be rewritten as (A10) and, substituting for ε_t using expression (A9), we obtain

$$1-\xi_t^U \leq \frac{Z_t^U/\left(M^I-\delta N_t^I\right)}{Z_{t-1}^U/\left(M^I-\delta N_{t-1}^I\right)}, \tag{A12}$$

where $N_{t+1}^U = \delta\left(1-\xi_t\right)N_t^U + \mu_t Z_t^U$ and $N_t^U = 1-N_t^I$. Hence, we can rewrite condition (A12) as follows:

$$1-\xi_t^U \leq \frac{1-N_{t+1}^I-\delta\left(1-\xi_t\right)\left(1-N_t^I\right)}{1-N_t^I-\delta\left(1-\xi_{t-1}\right)\left(1-N_{t-1}^I\right)}\frac{\mu_t\left(M^I-\delta N_{t-1}^I\right)}{\mu_{t+1}\left(M^I-\delta N_t^I\right)}.$$

Given that $\xi_t^U \in [1-\omega, 1]$ and $\mu_t = \kappa/W_t$, a sufficient condition is then

$$\omega \leq \left[\frac{1 - N_{t+1}^{I}}{1 - N_{t}^{I}} - \delta\omega \right] \frac{W_{t+1}}{W_{t}} \frac{M^{I} - \delta N_{t-1}^{I}}{M^{I} - \delta N_{t}^{I}}.$$

where $N_t^I \in \left[0, M^I\right]$. From expression (17), it is straightforward that $W_t \in \left[\gamma R, \gamma R/\left(1 - \delta\omega\beta\right)\right]$ and hence a stricter condition is $\omega \leq \left[1 - M^I - \delta\omega\right](1 - \delta)(1 - \delta\omega\beta)$, which ensures that Assumption 5 is sufficient for condition (A12) to be satisfied. An argument similar to the iid case applies when managers have been employed for more than 1 period, completing the proof.