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The More We Know about the Fundamental, the Less We Agree on the Price

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I allow trading horizon heterogeneity across groups in a standard differential information model of a financial market. This approach can explain the well-established phenomenon that, after a public announcement, trading volume increases, more private information is incorporated into prices and volatility increases. In such environments, public information has the important secondary role of helping agents learn about the information of other agents. Therefore, whenever the correlation between the private information of different groups is sufficiently low, a public announcement increases disagreement among short-horizon traders regarding the expected selling price even as it decreases disagreement about the fundamental value of the asset. Additional testable implications are also suggested.

Key words: Higher-order expectations, Noisy rational expectations model, Trading volume, Public information

JEL Codes: D82, D84, G12, G14

1. INTRODUCTION

Why do announcements of public information set off a frenzy of trading? Intuition suggests that public information brings beliefs closer to each other. With less disagreement, there should be less reason to trade.

For a fresh look at this puzzle, my starting point in this paper is that the trading horizon differs across market participants. That is, some groups of traders buy assets knowing that later they will have to resell them to others. Public information, in such an environment, aside from reducing uncertainty about fundamentals, has an important secondary role of helping each agent guess the private information of other agents. I show that this observation provides a novel explanation for the well-established stylized facts that, after a public announcement, trading volume increases, more private information is incorporated into prices and volatility increases. In particular, I show that these facts arise naturally in a generalized Grossman–Stiglitz type model where agents' trading horizons vary and there is sufficient heterogeneity in their information sets. I also suggest additional testable implications.

The main result is based on a simple observation. Agents' opinions about the opinions of others (higher-order expectations) respond differently to public information than agents' opinion about the fundamentals of an economic object (fundamental expectations). In particular, a public announcement might increase disagreement among agents in higher-order expectations even if it decreases disagreement in their fundamental expectations. A typical case of this is

when agents collect private information on different dimensions of the fundamental. For an extreme example[1], consider two groups: I-agents and J-agents. Suppose that whereas the J-agents form their expectation about the fundamental, each I-agent has to guess the average fundamental expectation of the J-agents. That is, the I-agents form second-order expectations. This might be the case in financial markets if I-agents trade first and then expect to resell their assets to J-agents. Suppose that the fundamental is the sum of two independent factors, $\theta = \theta_I + \theta_J$. While each I-agent observes a different noisy signal on θ_I, each J-agent observes a different noisy signal on θ_J. The public announcement, observed by all, is a noisy version of the fundamental, $y = \theta + \eta$. Without a public announcement, the I-agents agree in their guess because their private signals do not reveal any information about the J-agents' signal. However, a public announcement generates disagreement. For example, an I-agent with a high private signal on the first factor relative to the announcement concludes that the other factor is most likely low. Therefore, the average signal of the J-agents and their average fundamental expectation must also be low. An I-agent with a low private signal relative to the announcement reaches the opposite conclusion. Thus, the announcement *polarizes* second-order expectations. Interestingly, first-order expectations are not polarized, as disagreement regarding fundamental expectations decreases among members of all the groups after the public announcement.

I incorporate this intuition into an economic context by analysing a generalized differential information model of financial markets in the tradition of Grossman and Stiglitz (1980) and Hellwig (1980). Importantly, I allow agents to have heterogeneous trading horizons and to observe private signals with weak unconditional correlations. In particular, I assume two groups of traders; I-traders and J-traders. There are three periods. In the baseline case, only I-traders trade in the first period. In the second period, I-traders have to liquidate their assets and consume the proceeds. J-traders trade in the second period and consume their financial wealth in the third period when the fundamental value of the asset is realized. I interpret the setup as a model of an asset being traded in geographically distinct locations. Examples include currencies and cross-listed stocks. For example, consider the U.K. pound/U.S. dollar market. A large share of the trading goes through dealers located in either London or New York. Thus, in terms of the model, I-traders are dealers located in London and J-traders are dealers located in New York. Then, Period 1 represents trading hours in London and Period 2 represents trading hours in New York. The main consequence of this structure is that I-traders in the first period know that their consumption depends on the second period equilibrium price they receive from J-traders for their assets as opposed to the fundamental value of the asset.

The information structure is general in the sense that the unconditional correlation of private signals across groups can range from 0 to 1 depending on the parameters. Consistent with the example above, whenever this correlation is sufficiently low, a public announcement in the first period increases the dispersion in the I-traders' forecast of the second period price. I refer to an information structure that satisfies this condition on the correlation structure as a weakly correlated information structure. As a main result, under a weakly correlated information structure, the announcement induces an upward shift in trading and in the amount of private information incorporated into prices in both periods. Moreover, the volatility of the first period price can also increase. As I discuss below, these implications are consistent with a vast body of empirical work. Polarization creates trading volume in the first period because the increased disagreement among the I-traders translates into active speculative trading after the announcement. Interestingly, it also induces more trade among the J-traders in the second period because the I-traders' more active trading makes first period prices more informative. This reduces uncertainty for the J-traders, making them more aggressive at trading on their private information.

1. I expand on this example in Section 2.

I also consider a second variant where both I-traders and J-traders are present in the first period. I analyse the effect of an increase in the share of I-traders in the population. I interpret the increase of this share as a proxy for increased market segmentation and/or an increased fraction of short-horizon traders in the economy. This variant of the model helps to find new ways to test my theory. First, my results imply that the trading volume of assets with a larger share of short-horizon investors in their investor base should respond more strongly to public announcements. This is testable, as recent work has constructed empirical proxies for the investment horizon of the investor base of financial assets.[2] Second, I connect my results to an observation in Bailey, Karolyi, and Salva (2006) that the volume and information content of prices of recently cross-listed stocks respond more strongly to a release of public information after the event than before.

This paper is the first to highlight that public announcements can polarize market participants' valuation of an asset without polarizing their fundamental expectations. It is also the first to point out the potential for this observation to explain empirical patterns around public announcements in financial markets. Thus, my model provides a common-prior alternative to its most successful rivals based on heterogeneous priors. I argue below that these two approaches have a natural complementary role in explaining stylized facts related to public announcements on financial markets.

There is a large previous literature focusing on trading and price patterns associated with public announcements. The stylized fact that the trading volume of stocks increases around earnings announcements has been known for decades.[3] Recent studies based on high-frequency data sets give a more detailed picture.[4] First, this stylized fact is true across various markets and various types of public information releases. Second, within the day, trading volume drops for a period before the expected announcement and increases only afterwards. Third, the private information incorporated into prices through trading increases significantly after announcements.[5] Finally, public announcements also increase return volatility.

Neither in representative agent models nor in standard differential information models are the price adjustments caused by new public information accompanied by abnormal trading volume or volatility; thus, even the basic stylized facts are puzzling from the viewpoint of the most standard models.[6] Therefore, the majority of the literature is settled on the conclusion that a viable explanation for these patterns requires models wherein public announcements increase the disagreement among agents regarding the valuation of the asset. Observing that public signals in common prior environments generally decrease disagreement about fundamentals, this literature developed in two directions. The first group (Kim and Verrecchia, 1997; Evans and Lyons, 2001) relaxes the assumption that public announcements are modelled by public signals. Instead, public announcements are modelled as combinations of public and private signals.[7]

2. See Wahal and McConnell (2000) and Gaspar, Massa, and Matos (2005).

3. See Beaver (1968), Bamber (1987), and Kandel and Pearson (1995).

4. Fleming and Remolona (1999) and Green (2004) focus on the market for U.S. treasuries, while Evans and Lyons (2008) and Love and Payne (2008) focus on currency markets and Chae (2005) and Krinsky and Lee (1996) focus on equity markets.

5. Krinsky and Lee (1996) and Fleming and Remolona (1999) decompose the bid-ask spread around announcements, whereas Evans and Lyons (2008) analyse the joint distribution of the order-flow and prices to arrive at this conclusion.

6. Motivated by this fact, the early literature (Kim and Verrecchia, 1991; He and Wang, 1995) made small modifications to the standard framework resulting in some trading volume around announcements. However, in these models, trading volume increases because agents build up speculative positions before the announcement that they liquidate after observing the announcement. Informative trading does not increase after the announcement because disagreement decreases. This is hard to reconcile with the stylized facts above.

7. Rabin and Schrag (1999) uses the same modelling strategy to explain confirmation bias.

Thus, the announcement can increase disagreement. The disadvantage of this line of work is that its generality is limited. As the new assumption is on the nature of the information content of announcements, its potential to explain empirical patterns unrelated to announcements is naturally small. This is in contrast to the second group starting with Varian (1989), Harris and Raviv (1993), and Kandel and Pearson (1995) assuming heterogeneous priors. That is, agents with differing priors process the same public signal but reach a different posterior valuation. The assumption of heterogeneous priors proved to be useful for addressing various other empirical puzzles[8].

Like the aforementioned two groups of papers, this paper also argues that trading volume increases around public announcements because traders' disagreement increases regarding the valuation of the asset. Importantly, I point out that this is consistent with the combination of common priors and public signals as long as a certain fraction of agents has a short trading horizon. As short-horizon agents focus on the future price instead of the fundamental, in this case, a public announcement can polarize market participants' valuation of the asset without polarizing their fundamental expectations. This model shares with heterogeneous prior models the advantage that its main assumption, the presence of short-horizon investors, has proven to be a fruitful approach in a wide range of economic contexts.[9]

In the given context, there is a fundamental trade-off between heterogeneous prior models and common priors-differential information models (including the one in this paper). On the one hand, unlike models with heterogeneous priors, common prior models are constrained by the no trade theorems (Milgrom and Stokey, 1982). That is, differential information does not generate trade in itself without some type of noise in price determination. Thus, these models analyse changes in trading volume for a given amount of noise. Because of this constraint, the assumption of heterogeneous priors appears to be a natural candidate to explain patterns related to the enormous trading volume of financial markets. On the other hand, heterogeneous priors imply a lack of learning from other agents' actions. Thus, these models tend to be inconsistent with the evidence that, after a public announcement, a large flow of private information is incorporated into the price. Because of this trade-off, these two classes of assumptions have a complementary role in explaining patterns of trade and prices around announcements.[10]

More broadly, this paper fits into the recent flow of papers analysing the effects of higher-order expectations in various contexts. The most closely related part of this literature[11] analyses environments where various groups of agents act sequentially and the pay-off of early actors depends on the actions of groups acting later (Allen, Morris, and Shin, 2006; Banerjee, Kaniel, and Kremer, 2009; Makarov and Rytchkov, 2009; Angeletos, Lorenzoni, and Pavan, 2010; Cespa and Vives, 2011; Goldstein, Ozdenoren, and Yuan, 2011). Thus, early actors have to guess the

8. For example, heterogeneous prior models were shown to explain puzzles related to bubbles (Harrison and Kreps, 1978; Biais and Bossaerts, 1998), overpricing of initial public offerings (Morris, 1996), and momentum and post-announcement drift (Banerjee, Kaniel, and Kremer, 2009). See also Dixit and Weibull (2007) and Acemoglu, Chernozhukov, and Yildiz (2009) for a discussion of polarization due to the relaxation of the common prior assumption in other contexts.

9. For example, Tirole (1985) and Santos and Woodford (1997) analyse the role of short horizons (OLG models) in rational bubbles, Froot, Scharftstein, and Stein (1992) connect short horizon of traders to herding, and Vives (1995) relates short-horizon traders to the informational efficiency of prices. See Chapter 8.3 in Vives (2008) for more related literature on short-horizon traders in rational financial models. Also, starting with Holmstrom and Costa (1986) and Shleifer and Vishny (1990), there is a series of models providing micro-foundations for the existence of investors with a short horizon.

10. See Banerjee and Kremer (2010) as a notable example of mixing these two sets of assumptions.

11. A related group of papers focuses on "Beauty contest" environments where the pay-off of agents is a weighted sum of the deviation of their actions from an optimal level plus the deviation of their actions from the average actions of others (see Woodford, 2001; Hellwig, 2002; Morris and Shin, 2002; Angeletos and Pavan, 2007).

information of agents acting later. Applications include financial markets, currency attacks, and the interaction between stock prices and real investment. None of these papers considers information structures with the possibility of polarized higher-order expectations.

The structure of this paper is as follows. In the next section, I illustrate with an example how public signals can polarize higher-order expectations in Gaussian information structures. In section 3, I present the financial application, characterize the equilibrium, and discuss additional empirical implications. Finally, I conclude.

2. POLARIZED SECOND-ORDER EXPECTATIONS: AN EXAMPLE

Before introducing a model of a financial market, I illustrate the driving force behind the results using a simple example. In this example, a public announcement increases disagreement in second-order expectations without increasing disagreement in first-order expectations.

Consider groups I and J with a unit mass of agents in each group indexed by i and j. J-agents form expectations about a fundamental, θ. I-agents form expectations about the average expectation of J-agents.[12] These are second-order expectations on θ. The fundamental value is the sum of two independent factors, $\theta = \theta_I + \theta_J$. Each I-agent i observes a private signal about the first factor, $x^i = \theta_I + \varepsilon^i$, and each J-agent j observes a private signal about the second factor, $z^j = \theta_J + \varepsilon^j$. The difference across the two groups' information sets represents an unmodelled specialization in information acquisition. I am interested in the change of dispersion in first- and second-order expectations after the release of a public signal, $y = \theta + \eta$. I assume that each factor and noise term are drawn from an independent distribution

$$\theta_I, \theta_J \sim N\left(0, \frac{1}{\nu}\right), \varepsilon^i, \varepsilon^j \sim N\left(0, \frac{1}{\alpha}\right), \eta \sim N\left(0, \frac{1}{\beta}\right).$$

First, consider the case before the public announcement. The fundamental expectation of each J-agent is a linear function of the private signal

$$E(\theta|z^j) = b^n z^j,$$

where $b^n > 0$ and the n superscript stands for no announcement. The average expectation in group J is

$$\bar{E}(\theta|z^j) \equiv \int_0^1 E(\theta|z^j)dj = b^n \theta_J.$$

Then, a measure of the dispersion in fundamental expectations is

$$\int_0^1 |E(\theta|z^j) - \bar{E}(\theta|z^j)|dj = \frac{|b^n|}{\sqrt{\alpha}}\sqrt{\frac{2}{\pi}}.$$

Each agent i in group I forms a second-order expectation

$$E(\bar{E}(\theta|z^j)|x^i) = b^n E(\theta_J|x^i) = 0.$$

The second-order expectation is 0 independently of the private signal of agent i because I-agents have private information about the I factor only, whereas J-agents have private information about the J factor only, and the two factors are independent. As the fundamental expectations of J-agents depend only on their private signal, I-agents' private information is useless for

12. In the main model, each agent in group I eventually wants to sell her asset to someone in group J, this is why she is interested in the expectations of group J. However, to keep our example simple, in this section, we do not model the motivations of agents.

forming expectations about the fundamental expectation of the average J-agent. Consequently, the dispersion in the second-order expectations of I-agents is also 0.

Consider now the case with a public announcement. Any J-agent's fundamental expectation is linear and can be written as follows:

$$E(\theta|z^j, y) = bz^j + cy,$$

where $b, c > 0$ are constants. As above, the dispersion of fundamental expectations is

$$\int_0^1 |E(\theta|z^j, y) - \bar{E}(\theta|z^j, y)|dj = \frac{|b|}{\sqrt{\alpha}}\sqrt{\frac{2}{\pi}}.$$

By calculating the coefficients[13], it is easy to show that the dispersion decreases after the announcement as

$$b = \frac{\alpha\nu}{\nu^2 + \alpha\nu + \beta(\alpha + 2\nu)} < \frac{\alpha\nu}{\nu^2 + \alpha\nu} = b^n.$$

This is intuitive. Each J-agent has more precise knowledge about the fundamental after observing the public signal; thus, the disagreement among J-agents decreases.

Note that I-agents second-order expectations are no longer independent of private signals

$$E(\bar{E}(\theta|z^j, y)|x^i, y) = bE(\theta_J|x^i, y) + cy = b(ax^i + c'y) + cy,$$

where $a, c' \neq 0$ are the coefficients in $E(\theta_J|x^i, y)$. Consequently, the dispersion in second-order expectations increases from zero to

$$\int_0^1 |E(\bar{E}(\theta|z^j, y)|x^i, y) - \bar{E}(\bar{E}(\theta|z^j, y)|x^i, y)|di = \frac{|ba|}{\sqrt{\alpha}}\sqrt{\frac{2}{\pi}} > 0.$$

Thus, second-order expectations are *polarized* by the public announcement. The idea is that the information that a public signal gives about the sum of the two factors can be combined with a private signal to reveal the likely value of the other factor. For example, an I-agent with a high private signal regarding θ_I relative to the announcement concludes that the other factor is most likely low and, therefore, that the average signal of the J-agents and their fundamental expectation must also be low. In contrast, an I-agent with a low private signal regarding θ_I reaches the opposite conclusion. Thus, there will be dispersion among the I-agents about the expectation of the average J-agent.

Due to the critical role of the strength of connections across private information sets, polarization in higher-order expectations differs from polarization in first-order expectations. When an I-agent forms expectations regarding the fundamental expectations of the average J-agent, she has to forecast the private signal of that agent. When the I-agents' private signal x^i is informative

13. Whenever I calculate the coefficients of conditional expectations of normal variables throughout the paper, I use the projection theorem. This states that if $\mathbf{v}_\theta$ and $\mathbf{v}_s$ are vectors of variables that are jointly normally distributed with the vector of expected values μ_θ and μ_s, respectively, and the covariance matrix of the vector $[\mathbf{v}_\theta, \mathbf{v}_s]$ is

$$\begin{bmatrix} \Sigma_\theta & \Sigma_{\theta,s} \\ \Sigma_{s,\theta} & \Sigma_s \end{bmatrix},$$

where $\Sigma_\theta, \Sigma_{\theta,s}, \Sigma_{s,\theta}, \Sigma_{s,\theta}$ are the appropriate submatrices, then

$$(\mathbf{v}_\theta|\mathbf{v}_s) \sim N(\mu_\theta + \Sigma_{\theta,s}\Sigma_s^{-1}(\mathbf{v}_s - \mu_s), \Sigma_\theta - \Sigma_{\theta,s}\Sigma_s^{-1}\Sigma_{s,\theta}).$$

about the private signal z^j, then the dispersion of the I-agents' second-order expectations is high. Polarization occurs when conditional on the public signal this informativeness increases.

One might wonder why this property has not received any attention in the previous literature. There are two likely reasons. The first is that interest in models where higher-order expectations play an important role is relatively recent. The second is that, even in such models, the information structure is virtually always assumed to be of the form where both private and public signals are noisy observations of the fundamental: $x^i = \theta + \varepsilon^i, z^j = \theta + \varepsilon^j, y = \theta + \eta$. Virtually, all models of financial markets with constant absolute risk aversion (CARA) and normal distribution impose this information structure. This is why I refer to this structure as the *standard information structure*. As I show in this section, polarization is inconsistent with the standard information structure.

To highlight the effect of moving from the standard information structure towards the extreme specification in the above example, in the rest of the paper, I use a more general information structure than the one in the example. In particular, I assume that the fundamental is the sum of three factors

$$\theta = \theta_I + \theta_J + \theta_K \tag{1}$$

and the private signals of I and J-agents and the public signal are

$$x^i = \theta_I + \theta_K + \varepsilon^i, \tag{2}$$

$$z^j = \theta_J + \theta_K + \varepsilon^j \tag{3}$$

and the public signal is

$$y = \theta + \eta. \tag{4}$$

All factors and noise terms are drawn from independent normal distributions

$$\theta_I, \theta_J \sim \boldsymbol{N}\left(0, \frac{1}{\nu}\right),\ \theta_K \sim \boldsymbol{N}\left(0, \frac{1}{\omega}\right),\ \varepsilon^i, \varepsilon^j \sim \boldsymbol{N}\left(0, \frac{1}{\alpha}\right), \eta \sim \boldsymbol{N}\left(0, \frac{1}{\beta}\right). \tag{5}$$

Note that, apart from the group-specific factors θ_I and θ_J, there is also a common factor θ_K that all agents learn about. The advantage of this structure is that, by choosing $\nu \to \infty$, it nests the standard single-factor information structure, whereas choosing $\omega \to \infty$ yields the structure of the current example wherein the private information sets are independent. Nevertheless, this structure is simple enough to give tractable expressions. I refer to the information structure given by equations (1)–(5) as the *general information structure*.

Throughout the paper, instead of comparing equilibrium objects with and without announcement, I consider only the situation when the public signal is observed, and the announcement is considered an increase in the precision of the public information, β. I also refer to β as the amount of public information. The following proposition gives a necessary and sufficient condition for polarization in the informational environment given by equations (1)–(5).

Proposition 1. *Given the information structure, equations (1)–(5), a public announcement always decreases disagreement among agents' fundamental expectations in each group. That is,*

$$\frac{\partial \int_0^1 |E(\theta|z^j, y) - \bar{E}(\theta|z^j, y)|dj}{\partial \beta}, \frac{\partial \int_0^1 |E(\theta|x^i, y) - \bar{E}(\theta|x^i, y)|di}{\partial \beta} < 0.$$

Furthermore, a public announcement increases disagreement among I-agents about the average fundamental expectation of J-agents, i.e.

$$\frac{\partial \int_0^1 |E[\bar{E}(\theta|z^j, y)|x^i, y] - \bar{E}[\bar{E}(\theta|z^j, y)|x^i, y]|di}{\partial \beta} > 0,$$

if and only if

$$\beta > \frac{\nu^2}{\omega} \tag{6}$$

holds.

To see the intuition behind condition (6), note that in our information structure, it is equivalent to the condition

$$\text{corr}(x^i, z^j) < \text{corr}(x^i, y)\text{corr}(z^j, y), \tag{7}$$

where $\text{corr}(\cdot,\cdot)$ is the correlation between the variables. Thus, the proposition states that more public information polarizes I-agents' second-order expectations if and only if the correlation in private information across groups is small relatively to the product of the correlation between private and public information in the two groups. This condition trivially holds in our example wherein the correlation between the private signals of the agents in the different groups is zero. In contrast, the standard information structure imposes a rigid structure on the correlation structure of signals and θ. In particular,

$$\text{cov}(x^i, z^j) = \text{cov}(z^j, z^n) = \text{cov}(x^i, x^m) = \text{cov}(x^i, y) = \text{cov}(z^j, y) = \text{var}(\theta) \tag{8}$$

for any agents i, j, n, m. It is easy to confirm that this structure violates equations (7) and (6).

Throughout the paper, I refer to the combination of equations (1)–(5) and assumption (6) as a *weakly correlated information structure*.

In the next section, I argue that the statistical property highlighted in this section has important economic consequences by modifying a standard workhorse model of financial markets with differential information.

3. TRADING WITH HETEROGENEOUS HORIZONS AND DISPERSED INFORMATION

In this section, I consider the effects of public information on strategies and prices in modified versions of a standard three-period rational expectation equilibrium (REE), Grossman–Stiglitz-type model.[14] I deviate from the basic model along two main dimensions. First, I consider the general information structure instead of the standard information structure and second, I allow for the interaction of two groups who consume at different time points.

The effect of public announcement on trading positions can differ from its effect on expectations derived in the example of the previous section. First, in the model, endogenously determined prices serve as public signals and pay-off-relevant variables. Second, as traders are risk averse, their position depends not only on their expectation of the pay-off from their portfolio but also on the uncertainty about that pay-off. The main purpose of this section is to analyse how these channels affect the mapping between the example and observables in a financial market.

As in the example, I consider two groups of traders, I and J, trading the same risky asset and a riskless bond in a three-period model. I consider two main versions of the model. In both versions, I-traders sell all their holdings to $J-$traders at the beginning of Period 2 and consume the proceeds, whereas J-traders consume only in Period 3. However, in Case 1, only I-traders are present in Period 1, and J-traders arrive only in Period 2. In Case 2, both groups are present in Period 1. I will show that these variants nest several classic models, including Hellwig (1980) and Brown and Jennings (1989), and the two-period versions of He and Wang (1995) and Allen, Morris, and Shin (2006). In this sense, my structure is general.

14. See Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), Brown and Jennings (1989), He and Wang (1995), and Allen, Morris, and Shin (2006).

The two cases differ in their interpretations and in their analytical complexity, but they share the same main results. The main finding of this section is that the combination of heterogeneous trading horizons and a weakly correlated information structure implies polarization in higher-order expectations. This leads to increasing trading volume, increased information content of prices and potentially increased volatility of prices around announcements in otherwise standard Grossman–Stiglitz-type models. This pattern is consistent with the vast body of empirical work cited in the introduction.

3.1. *Case 1: Local traders in a global market*

There are two groups of traders each with unit mass I and J trading a risky asset and a riskless bond. The return on the bond is normalized to 1. There are three periods, $t = 1, 2, 3$. I-traders trade in Period 1 and sell all their holdings to J-traders at the beginning of Period 2 and consume. J-traders trade only in Period 2 and consume in Period 3 when the uncertain fundamental value θ is realized. Each agent has CARA utility over final wealth with the identical risk-aversion parameter γ. The total supply of the risky asset in each period, u_1 and $u_2 \equiv u_1 + \Delta u_2$, is normally distributed and independent.[15] Each trader forms her demand d_1^i or d_2^j conditional on her information set $\mathcal{I}_1^i$ or $\mathcal{I}_2^j$, which includes current and past prices. In equilibrium, price p_t has to clear the market in period $t = 1, 2$.

The demand of each I-trader solves

$$\max_{d_1^i} E[-e^{-\gamma W_I^i} | \mathcal{I}_1^i] \tag{9}$$

$$W_I^i = d_1^i (p_2 - p_1), \tag{10}$$

and the demand of each J-trader solves

$$\max_{d_2^j} E[-e^{-\gamma W_J^j} | \mathcal{I}_2^j] \tag{11}$$

$$W_J^j = d_2^j (\theta - p_2). \tag{12}$$

Random supply shocks u_1 and u_2 are drawn independently from the distributions

$$u_1 \sim N\left(0, \frac{1}{\delta_1^2}\right) \text{ and } u_2 \sim N\left(0, \frac{1}{\delta_2^2}\right).$$

There are various potential interpretations of this case. For example, one can see this case as a (part of the) 24-hour day in the market of global currencies. In reality, the main direct participants of these markets are dealers. Dealers receive orders from their customers[16] but also trade on their own account. They trade with each other either directly or through interdealer electronic brokerage services. The structure of Case 1 emphasizes two stylized facts about global currency markets. First, a large proportion of the trading volume is generated by dealers operating in

15. The independence of u_1 and u_2 implies that $\frac{\text{cov}(u_1, \Delta u_2)}{\text{var}(u_1)} = -1$. This is clearly a stark assumption, but it leads to the simplest analysis. The model can be generalized to include any correlation structure across the noise terms. The main results are robust to this treatment.

16. Starting from Diamond and Verrecchia (1981), it is common to interpret the random supply u_1, u_2 as the sum of the initial endowments of traders. In our context, this random endowment could be interpreted as the customer orders of dealers. There is a setup that is formally equivalent to our version but uses this alternative interpretation of random supply.

distinct geographical locations and during their local daylight hours. Second, dealers tend not to hold positions overnight.[17] That is, they do not pass on positions at the end of the day, even to other dealers of the same financial firm located in an other geographical location. For example, consider the U.K. pound/U.S. dollar market. A large share of the trading goes through dealers located in either in London or in New York. Thus, in terms of the model, I-traders are dealers located in London and J-traders are dealers located in New York. Then, Period 1 represents trading hours in London and Period 2 represents trading hours in New York. Because dealers do not want to hold positions overnight, they maximize their end-of-day utility. Because the trading hours in London end shortly after trading hours start in New York, Londoners sell their excess positions to New Yorkers at the end of their trading day.[18] Cross-listing of stocks is another example where geographical segmentation seems to play an important role.[19] In general, I will refer to this setup as a model of local traders in global markets. This structure is especially useful for my purposes because of its analytical tractability.

The information structure is described in equations (1)–(5). The information sets of agents are

$$\mathcal{I}_1^i = \{x^i, y, p_1\},$$
$$\mathcal{I}_2^j = \{z^j, y, p_1, p_2\}.$$

I look for a linear rational expectation equilibrium defined as follows.

Definition 1. A linear REE is given by the linear price functions p_1, p_2, mapping the aggregate random variables to prices and individual demands, d_1^i, d_2^j, such that d_1^i and d_2^j solve problems (9)–(11), respectively, and p_t clears the market in period $t = 1, 2$.

Before proceeding to the analysis of the model, it is useful to sum up how our structure nests the usual assumptions made in the literature.

1. If $\nu \to \infty$, the information structure becomes the standard informational structure.
2. If $\delta_1 \to 0$, the second period environment is the same as a static environment, as first period prices become uninformative.
3. If $\beta \to 0$, the environment converges to an environment with no public announcement.

Thus, combining different subsets of these limits, this model nests many models in the literature. For example, with $\delta_1 \to 0, \nu \to \infty$, the second period is close to Hellwig (1980). The limit $\nu \to \infty$ is the two-period version of Allen, Morris, and Shin (2006). In this sense, the presented framework is general.

In this section, I characterize the equilibrium. First, I show that an equilibrium always exists in this model. Second, I analyse trading volume. I show that both I-traders in the first period and

17. See (Lyons, 2001, p. 46).

18. As understanding this particular market is not the main purpose of this model, I keep the framework close to the standard models of trading with differential information. Thus, I abstract from many other institutional features of this market such as the interdealer trade, the structure of price quotations and market orders and intraday dynamics.

19. Although in theory, local markets could work as one global market, several studies find significant segmentation in trading activity in these markets. For example, Pulatkonak and Sofianos (1999) finds that 40% of the variation in the U.S. market share of trading volume of cross-listed stocks can be explained by the hours of overlap in trading between the New York Stock Exchange and the home market for the stock. See also Rosenthal and Young (1990) and Froot and Dabora (1999).

J-traders in the second period trade more after a public announcement whenever $\frac{\nu^2}{\omega} < \beta$, *i.e.* whenever the expectations of I-traders about the second period price are polarized by the public announcement. Finally, I analyse the volatility of prices.

3.1.1. Equilibrium. The derivation of the equilibrium is standard. First, I conjecture the price functions

$$p_2 = \frac{\mathbf{b}_2(\theta_J + \theta_K) + \mathbf{c}_2 y + \mathbf{g}_2 q_1 - u_2}{\mathbf{e}_2}, \tag{13}$$

$$p_1 = \frac{\mathbf{a}_1(\theta_I + \theta_K) + \mathbf{c}_1 y - u_1}{\mathbf{e}_1}, \tag{14}$$

where $\mathbf{a}_1, \mathbf{b}_2, \mathbf{c}_1, \mathbf{c}_2, \mathbf{e}_1, \mathbf{e}_2$, and $\mathbf{g}_2$ are undetermined coefficients. Second, I derive the optimal demand given these price functions. For this, observe that p_1 and y are informationally equivalent to y and the "price signal" q_1 of the first period defined as follows:

$$q_1 \equiv \frac{\mathbf{e}_1 p_1 - \mathbf{c}_1 y}{\mathbf{a}_1} = (\theta_I + \theta_K) - \frac{u_1}{\mathbf{a}_1}. \tag{15}$$

The conditional precision of q_1 is as follows:

$$\tau_1^2 \equiv \frac{1}{\mathrm{var}(q_1|\theta_I + \theta_K)} = \delta_1^2 \mathbf{a}_1^2.$$

Similarly, p_2, y, and q_1 are informationally equivalent to y, q_1 and the price signal q_2 of the second period defined as follows:

$$q_2 \equiv \frac{\mathbf{e}_2 p_2 - \mathbf{c}_2 y - \mathbf{g}_2 q_1}{\mathbf{b}_2} = (\theta_J + \theta_K) - \frac{u_2}{\mathbf{b}_2} \tag{16}$$

with a conditional precision of

$$\tau_2^2 \equiv \frac{1}{\upsilon ar\,(q_2|\theta_J + \theta_K)} = \mathbf{b}_2^2 \delta_2^2. \tag{17}$$

I also define b_2, c_2, e_2, g_2, and a_1, c_1, e_1 as the linear coefficients of the conditional expectations

$$E(\theta|z^j, y, q_1, q_2) = b_2 z^j + c_2 y + e_2 q_2 + g_2 q_1 \tag{18}$$

$$E(q_2|x^i, y, q_1) = a_1 x^i + c_1 y + e_1 q_1 \tag{19}$$

and

$$\tau_\theta^2 \equiv \frac{1}{\mathrm{var}(\theta|z^j, y, q_1, q_2)}$$

$$\tau_q^2 \equiv \frac{1}{\mathrm{var}(q_2|x^i, y, q_1)}$$

as the corresponding precision. Note that each boldface letter refers to a coefficient in the price function, whereas its nonboldface version refers to its closest equivalent in the expressions for conditional expectations. Note also that all the expectational coefficients and precisions are functions of the primitive parameters and the equilibrium values of τ_1, τ_2. Then, the first-order condition of the problem of J-traders, equation (11), gives

$$d_2^j = \frac{\tau_\theta^2}{\gamma}(E(\theta|z^j, y, q_2, q_1) - p_2) \tag{20}$$

and problem (9) gives

$$d_1^i = \frac{1}{\gamma}\left(\frac{\mathbf{e}_2}{\mathbf{b}_2}\right)^2 \tau_q^2 (E(p_2|x^i, y, q_1) - p_1). \tag{21}$$

Note that the forms of equations (20) and (21) differ because I-traders are interested in the next period price, p_2, as opposed to the fundamental value. The term $\left(\frac{\mathbf{e}_2}{\mathbf{b}_2}\right)^2 \tau_q^2$ is the precision of p_2 conditional on the information set of I-traders.

Imposing market clearing and using expressions (18)–(19) and definitions (15)–(16) give expressions for p_2 and p_1 as linear functions of the random variables with coefficients that depend on the primitives and on $\mathbf{b}_2, \mathbf{a}_1, \mathbf{c}_2, \mathbf{c}_1, \mathbf{e}_2, \mathbf{e}_1$, and $\mathbf{g}_2$. For an equilibrium, $\mathbf{b}_2, \mathbf{a}_1, \mathbf{c}_2, \mathbf{c}_1, \mathbf{e}_2, \mathbf{e}_1$, and $\mathbf{g}_2$ must be found that ensure that these price functions are identical to conjectures (13)–(14). The next proposition follows.

Proposition 2. *1. For all parameters, there exists a linear REE. In this equilibrium, demand functions are given as follows:*

$$d_2^j = \mathbf{b}_2 z^j + \mathbf{c}_2 y + \mathbf{g}_2 q_1 - \mathbf{e}_2 p_2, \tag{22}$$

$$d_1^i = \mathbf{a}_1 x^i + \mathbf{c}_1 y - \mathbf{e}_1 p_1, \tag{23}$$

while price functions are given by equations (13)–(14), where

$$\mathbf{b}_2 = \tau_\theta^2 \frac{b_2}{\gamma}, \tag{24}$$

$$\mathbf{c}_2 = \tau_\theta^2 \frac{b_2}{\gamma}\frac{c_2}{b_2 + e_2}, \tag{25}$$

$$\mathbf{e}_2 = \tau_\theta^2 \frac{b_2}{\gamma\,(b_2 + e_2)}, \text{ and} \tag{26}$$

$$\mathbf{g}_2 = \tau_\theta^2 \frac{b_2}{\gamma}\frac{g_2}{e_2 + b_2} \tag{27}$$

and

$$\mathbf{a}_1 = \frac{\tau_q^2}{\gamma}\frac{a_1}{e_2 + b_2}, \tag{28}$$

$$\mathbf{c}_1 = \frac{\tau_q^2}{\gamma}\frac{((b_2 + e_2)c_1 + c_2)a_1}{(e_2 + b_2)((e_2 + b_2)(a_1 + e_1) + g_2)}, \text{ and}$$

$$\mathbf{e}_1 = \frac{\tau_q^2}{\gamma}\frac{a_1}{(e_2 + b_2)((e_2 + b_2)(a_1 + e_1) + g_2)}.$$

Furthermore, all coefficients and equilibrium constants are calculated at $\tau_1 = \tau_1^*$ and $\tau_2 = \tau_2^*$, where $[\tau_1^*, \tau_2^*]$ is the fixed point of the system

$$\delta_2 \tau_\theta^2 \frac{b_2}{\gamma} = \tau_2 \tag{29}$$

$$\delta_1 \frac{\tau_q^2}{\gamma}\frac{a_1}{e_2 + b_2} = \tau_1. \tag{30}$$

2. *When $\nu \to \infty$, there is a unique linear REE, where*

$$\tau_2^* = \alpha \frac{\delta_2}{\gamma}$$

$$\tau_1^* = \alpha^2 \delta_1 \frac{\delta_2^2}{\gamma\,(\gamma^2 + \alpha\delta_2^2)}.$$

3. *When $\frac{\nu^2}{\omega} = \beta$, there exists a unique linear REE, where $\tau_1^* = 0$ and τ_2^* is the unique solution of*

$$\alpha\delta_2(\nu+\omega)\frac{\nu}{\gamma} = \tau_2(\nu^2 + (\nu+\omega)(\alpha+\tau_2^2) + 2\nu\omega).$$

The proposition states that finding an equilibrium is equivalent to finding a fixed point $[\tau_1^*, \tau_2^*]$ of the system equations (29)–(30). Note also that the proposition states existence in general and uniqueness at two particular points of the parameter space. Given previous work, it is not surprising that there is a unique equilibrium in the limit where our information structure converges to the standard information structure. There is also a unique equilibrium at $\beta = \frac{\nu^2}{\omega}$. Recall from Proposition 1 that this is an important point in our parameter space, as second-order expectations are polarized by the public signal if and only if $\beta > \frac{\nu^2}{\omega}$.

3.1.2. Trading volume and information content in trades. I start with a general analysis of traders' equilibrium demand. Note first that rearranging equation (16) for p_2 and substituting into equation (22) gives the first equation in the chain

$$d_2^j = \mathbf{b}_2(z^j - q_2) = \frac{\tau_\theta^2}{\gamma}[b_2(z^j - q_2)] = \mathbf{b}_2\varepsilon^j + u_2. \tag{31}$$

The second equation comes from equation (24), whereas the last one is a consequence of the definition of equation (16). This chain of equations is intuitive. The first expression states that each agent j forms her price contingent demand as follows. She considers the difference between z^j and q_2; her private signal and a noisy measure of the average private signal of other agents as it is aggregated in the given market price. If agent j has a higher private signal than this noisy signal of average private information, she buys the asset; otherwise, she sells the asset. However, the amount she buys or sells also depends on $\mathbf{b}_2$, which I refer to as the agent's trading intensity. The larger the trading intensity, the more aggressively the agent bets on this difference. The second expression decomposes trading intensity. Intuitively, the term in the squared bracket shows how a difference in information translates into a difference in estimated fundamental value. The larger this term, the larger the agent's perceived difference between her estimate and that of the market. The term $\frac{\tau_\theta^2}{\gamma}$ shows how the difference in opinion is translated into positions. The smaller the risk aversion of the agent, γ, and the larger the precision of her fundamental estimation, τ_θ^2, the larger the bet she wants to place for every unit of difference in opinion. Importantly, b_2, $\mathbf{b}_2$, and τ_θ^2 are all functions of the deep parameters and the precision of the price signal, τ_2.

The last expression in equation (31) shows that at the equilibrium prices, agents end up with a position that is a composite of two parts. The second part is just the per-capita supply. I refer to this part as the risk-sharing position. The first part is the trading intensity weighted private noise. I refer to this part as the speculative position. Importantly, agents cannot distinguish these two parts of their own position, as they know neither the supply nor the noise term in their private

signal.[20] This decomposition helps reveal how and why trading volume and other equilibrium objects react to public information.

In the same way that I derived equation (31), I also derive the analogous expressions for I-traders

$$d_1^i = \mathbf{a}_1(x^i - q_1) = \frac{\tau_q^2}{\gamma}\frac{a_1}{e_2 + b_2}(x^i - q_1) = \mathbf{a}_1\varepsilon^i + u_2. \tag{32}$$

For the purposes of this paper, it is also useful to point out how first period demand is related to higher-order expectations. The market clearing condition in Period 2 gives

$$p_2 = \int_0^1 E(\theta|z^j, y, q_2, q_1)dj - \frac{\gamma}{\tau_\theta^2}u_2$$

Thus, I rewrite first period demand, (21), as follows:

$$\frac{1}{\gamma}\left(\frac{\mathbf{e}_2}{\mathbf{b}_2}\right)^2\tau_q^2\left[E\left(\int_0^1 E(\theta|z^j, y, q_2, q_1)dj - \frac{\gamma}{\tau_\theta^2}u_2|x^i, y, q_1\right) - p_1\right]. \tag{33}$$

Note that the term in the squared bracket is the difference between the I-trader's expectation of the average expectation of the J-trader (a second-order expectation) and the first period price. As I will argue, this second-order expectation carries all the intuition built in Section 2. The term $\left(\frac{\mathbf{e}_2}{\mathbf{b}_2}\right)^2\tau_q^2$ is the precision of the I-traders' estimate. This part is endogenously determined in this model, and it can modify the basic intuition of Section 2. Importantly, in a REE, agents do not form expectations about the expectations of others. Still, the logic of the example in Section 2 can be applied in two ways. First, one can interpret expression (33) in an *as if* sense. Traders in the first period form their demand as if they were forecasting the expectation of the average J trader. Second, as I show in online Appendix C, our model is a specific large number limit of a strategic model wherein the agents do form expectations about the strategies of others.

Similar to the decomposition of demands in equation (31), I also define and decompose trading volume as one of the key objects of interest. Given that agents do not hold a position when they enter the market, the expected volume in each period is

$$V_1 \equiv E(|d_1^i|) = \sqrt{\frac{2}{\pi}\left(\frac{1}{\delta_1^2} + \frac{\mathbf{a}_1^2}{\alpha}\right)} \tag{34}$$

$$V_2 \equiv E(|d_2^j|) = \sqrt{\frac{2}{\pi}\left(\frac{1}{\delta_2^2} + \frac{\mathbf{b}_2^2}{\alpha}\right)}.$$

I refer to the first term in the brackets on the left-hand side as the risk-sharing part of volume and the second part as speculative volume. Whereas the risk-sharing part is exogenously given by the variance of the random supply, the speculative part depends on the equilibrium trading intensities $\mathbf{b}_2$, $\mathbf{a}_1$. Note that the volume is influenced by neither the realization of fundamental factors nor the public announcement. As our main interest is the change in that part of the volume that

20. In fact, as explained and clarified in Biais, Bossaerts, and Spatt (2010), the fact that the demand of trader i positively depends on the error term in her private signal, ε^i, is a form of winner's curse. If the trader could distinguish between ε^i and u_1, she would avoid this curse.

is driven by dispersion in private information, I also define speculative volume as the realized volume when aggregate random variables are at their expected values.

$$V_1^S \equiv \frac{1}{2}\int |d_1^i| di|_{u_1=0} = \frac{|\mathbf{a}_1|}{\sqrt{2\alpha\pi}}$$

$$V_2^S \equiv \frac{1}{2}\int |d_2^j| dj|_{u_2=0} = \frac{|\mathbf{b}_2|}{\sqrt{2\alpha\pi}}.$$

It is apparent that, in Case 1, changes in the amount of public information affect expected volume and speculative volume in very similar ways. However, this second measure will turn out to be of independent interest in Case 2.

It is important to point out that neither in this part nor in the rest of the paper do I present arguments against the classic no trade theorems. Just as in any other common prior setup, differential information does not generate trade in itself in this model. To induce trade, prices must be non-fully revealing. Indeed, both the risk sharing and the speculative components of volume and holdings in equations (31) and (34) go to zero as the noise in supply diminishes. However, the decomposition of holdings and volume in equations (31) and (34) also illustrate that dispersion in private information adds to trading volume in a market where prices are not fully revealing. To see this, consider the limiting case $\alpha \to \infty$. This coincides with the symmetric information benchmark. At this limit, the speculative part of equilibrium demand diminishes and only the risk-sharing part remains. Thus, the additional effect on trading of differential information for a given amount of noise is measured by the speculative component in each object. Given that this component depends on the equilibrium objects $\mathbf{b}_2, \mathbf{a}_1$, the way the combination of traders' heterogeneous trading horizon and a weakly correlated information structure influence the speculative component is non-trivial. The analysis of this is the main focus of this paper.

I am also interested in the information content of prices. I define a measure for this as follows:

$$K_1 \equiv \frac{1}{\mathrm{var}(q_1|\theta_I + \theta_K)} = \tau_1^2 = \delta_1^2 \mathbf{a}_1^2, \tag{35}$$

$$K_2 \equiv \frac{1}{\mathrm{var}(q_2|\theta_J + \theta_K)} = \tau_2^2 = \delta_2^2 \mathbf{b}_2^2, \tag{36}$$

where I use equations (28) and (24) for the last equation in each expression, respectively. When this measure is zero, the price does not aggregate any private information. When it is infinity, it aggregates private information perfectly. Note that from equations (34)–trading volume and the information content of prices, it is sufficient to study its effect on the absolute value of trading intensity $|\mathbf{b}_2|, |\mathbf{a}_1|$. When the trading intensity increases in absolute value, our measures of volume and the information content of prices also increase.

I start the analysis with the limit where the importance of group-specific information diminishes, *i.e.* $\nu \to \infty$. As pointed out above, this limit corresponds to the standard information structure. The following proposition shows that public information affects neither trading volume nor the information content of prices in this case.

Proposition 3. *When $\nu \to \infty$, neither trading volume nor the information content of trades is affected by the amount of public information β. That is,*

$$\frac{\partial \mathbf{b}_2}{\partial \beta} = \frac{\partial \mathbf{a}_1}{\partial \beta} = \frac{\partial V_t}{\partial \beta} = \frac{\partial K_t}{\partial \beta} = 0$$

for $t = 1, 2$.

To understand this result, note that the effect of public information on trading intensity in each period can be decomposed as follows:

$$\frac{\partial|\mathbf{b}_2|}{\partial\beta}=\frac{1}{\gamma}\frac{\partial\tau_\theta^2 b_2}{\partial\beta}=\frac{1}{\gamma}\left(\tau_\theta^2\frac{\partial b_2}{\partial\beta}+b_2\frac{\partial\tau_\theta^2}{\partial\beta}\right) \tag{37}$$

$$\frac{\partial|\mathbf{a}_1|}{\partial\beta}=\frac{1}{\gamma}\frac{\partial\frac{\tau_q^2}{e_2+b_2}a_1}{\partial\beta}=\frac{1}{\gamma}\left(\tau_\theta^2\frac{\partial a_1}{\partial\beta}+a_1\frac{\partial\frac{\tau_q^2}{e_2+b_2}}{\partial\beta}\right). \tag{38}$$

The first term in the bracket is public information's effect on the weight of the private signal in each agent's conditional expectation, whereas the second term is public information's effect on the precision of their expectation. It is easy to confirm that, at the limit $\nu\to\infty$, the first term is always negative, whereas the second term is always positive, and their absolute value is the same. Intuitively, more public information decreases disagreement among agents. If an agent knows more from public sources, she will rely less on her private signal. Less disagreement decreases trading intensity. On the other hand, more information makes agents more certain about their estimation of the fundamental value. This increases trading intensity. Proposition 3 states that these two effects exactly cancel out in the standard information structure. As has been pointed out in previous work (*e.g.*, Kim and Verrecchia, 1991; He and Wang, 1995) this result is not robust. Still, the existence of these two opposing forces is a general feature of previous CARA-normal REE models.

In contrast, an important result in this paper is that in our setup the effects of an announcement on precision and conditional expectations not only do not cancel out, but they have the same positive sign, leading to a large increase in trading volume in response to more public information.

Let us turn to the general case when ν is finite so the common factor does not fully dominate the fundamental value. I start the characterization with the following lemma.

Lemma 1. *In every point where* $\frac{\partial V_t}{\partial\beta}$ *exists for both* $t=1,2$

$$\operatorname{sgn}\left(\frac{\partial|\mathbf{a}_1|}{\partial\beta}\right)=\operatorname{sgn}\left(\frac{\partial|\mathbf{b}_2|}{\partial\beta}\right).$$

The lemma states that for any combination of the parameters, public information affects absolute trading intensity (and consequently trading volume and the information content of prices) in the same way across the two periods. The underlying intuition is that, if I-traders trade more aggressively, the price in the first period becomes more informative. Hence, the precision of J-traders' pay-off estimations increases. Consequently, the lemma states that even if decreasing disagreement among J-traders decreases trading intensity, the effect on precision dominates.

Now I turn to the main result of this section. Recall from Proposition 1 that in our example, the public signal polarizes I-traders' expectations regarding the expectation of the average J-trader in a weakly correlated information structure, *i.e.* if and only if $\beta>\frac{\nu^2}{\omega}$. Furthermore, expression (33) shows that the second period price is closely related to the average expectation of the J-traders. Thus, if polarization is indeed the main determinant of the increase in trading volume, the I-traders' volume should increase if and only if $\beta>\frac{\nu^2}{\omega}$. By Lemma 1, J-traders' trading volume should also increase under the same condition. That is, polarization among I-traders increases trading volume among both group of traders even though disagreement about the pay-off among J-traders decreases after the announcement. By previous arguments, the absolute trading intensities of the I- and J-traders $|\mathbf{a}_1|,|\mathbf{b}_2$ and the information content of prices

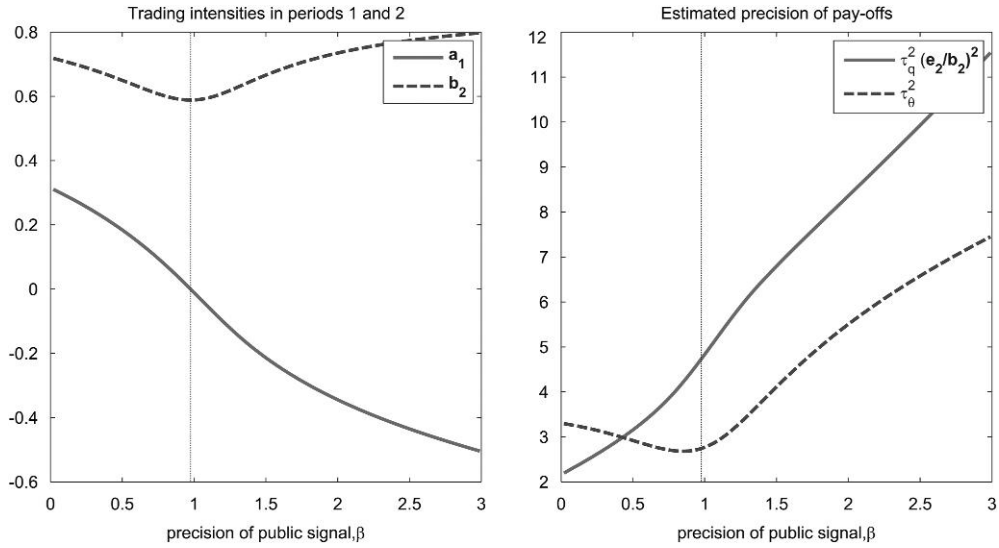

FIGURE 1

Trading intensities and estimated pay-off uncertainty in Case 1. The left panel shows trading intensities $\mathbf{a}_1, \mathbf{b}_2$, in Periods 1 and 2. The right panel shows I-traders' precision of second period price estimate $\tau_q^2\left(\frac{\mathbf{e}_2}{\mathbf{b}_2}\right)^2$ and the precision of J-traders' estimate of the fundamental τ_θ^2. The x-axis is the precision of the public signal β. The vertical line depicts $\beta = \frac{\nu^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information. Parameter values are $\gamma = 1$, $\omega = 4{\cdot}1$, $\nu = 2$, and $\alpha = \delta_1 = \delta_2 = 5$

should change similarly. This is indeed the case as illustrated in Figure 1. The left panel of Figure 1 shows trading intensities that follow the predicted pattern. The right panel of Figure 1 shows the precision of the traders' pay-off estimation, τ_θ^2, $\left(\frac{\mathbf{e}_2}{\mathbf{b}_2}\right)^2\tau_q^2$. As expected, the increase in trading intensity is partially driven by the increase in the precision of the estimates. The left panel of Figure 2 shows that more public information increases trading volume in both periods as long as $\beta > \frac{\nu^2}{\omega}$. In the next proposition, I show that these results are general as long as the trading intensities $\mathbf{a}_1, \mathbf{b}_2$ are continuously differentiable in the amount of public information, β.[21]

Proposition 4. *There are* $\omega_{\min} \in \left[0, \frac{\nu^2}{\beta}\right), \omega_{\max} \in \left(\frac{\nu^2}{\beta}, \infty\right]$ *that as long as* $\omega \in (\omega_{\min}, \omega_{\max})$ *there are corresponding* τ_1^*, τ_2^**, which are continuous in* ω *and continuously differentiable in* β *and* ω*. Furthermore, when* $(\omega_{\min}, \omega_{\max})$ *is the largest such set, as long as* $\omega \in \left(\frac{\nu^2}{\beta}, \omega_{\max}\right)$

$$\frac{\partial|\mathbf{a}_1|}{\partial\beta}, \frac{\partial|\mathbf{b}_2|}{\partial\beta} > 0.$$

That is, in weakly correlated information structures, the absolute values of trading intensity, volume, and information content of prices all increase in both periods.

3.1.3. Volatility of prices. Turning to prices, by definition, the coefficients $\frac{\mathbf{b}_2}{\mathbf{e}_2}, \frac{\mathbf{a}_1}{\mathbf{e}_1}$ show the price effect of the part of fundamentals that agents have private information on, the coefficients $\frac{\mathbf{1}}{\mathbf{e}_2}, \frac{\mathbf{1}}{\mathbf{e}_1}$ show the price effect of supply shocks, whereas $\frac{\mathbf{c}_2}{\mathbf{e}_2}$, $\frac{\mathbf{c}_1}{\mathbf{e}_1}$ show the price effect of

21. Although experiments with a wide range of parameters suggest that this neighbourhood is the whole parameter space, *i.e.* $\omega_{\min} = 0, \omega_{\max} = \infty$, a general proof for this was not found. Hence, the weaker statement.

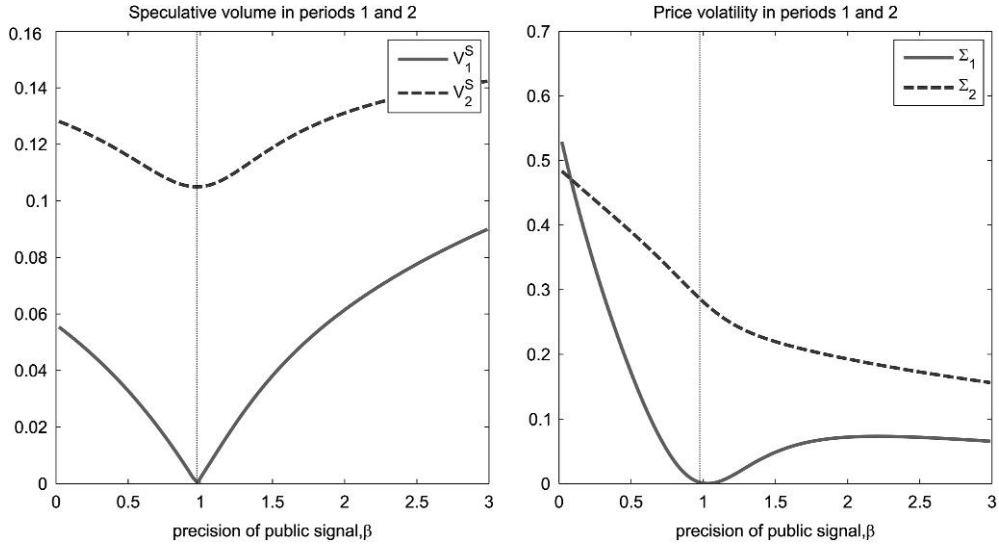

FIGURE 2

Speculative volume and price volatility in Case 1. The left panel shows speculative volume in Periods 1 and 2. The right panel shows the volatility of prices in Periods 1 and 2. The x-axis is the precision of the public signal β. The vertical line depicts $\beta = \frac{\nu^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information. Parameter values are $\gamma = 1$, $\omega = 4{\cdot}1$, $\nu = 2$, and $\alpha = \delta_1 = \delta_2 = 5$

public information. The first two sets of coefficients are particularly important because they determine the relevant measure of price volatility as

$$\Sigma_1 \equiv \text{var}(p_1|y) = \left(\frac{\mathbf{a}_1}{\mathbf{e}_1}\right)^2 \left(\frac{1}{\nu} + \frac{1}{\omega}\right) + \frac{1}{(\mathbf{e}_1)^2 \delta_1} \tag{39}$$

$$\Sigma_2 \equiv \text{var}(p_2|y, p_1) = \left(\frac{\mathbf{b}_2}{\mathbf{e}_2}\right)^2 \left(\frac{1}{\nu} + \frac{1}{\omega}\right) + \frac{1}{(\mathbf{e}_2)^2 \delta_1}. \tag{40}$$

Both measures are conditioned by publicly observed variables in the given period.

Starting again with the standard information structure, the following proposition holds.

Proposition 5. *When $\nu \to \infty$, in each period,*

1. *Prices are positively affected by the average information of traders, and this effect decreases with the precision of public information. That is,*

$$\frac{\mathbf{b}_2}{\mathbf{e}_2}, \frac{\mathbf{a}_1}{\mathbf{e}_1} > 0, \frac{\partial \frac{\mathbf{b}_2}{\mathbf{e}_2}}{\partial \beta}, \frac{\partial \frac{\mathbf{a}_1}{\mathbf{e}_1}}{\partial \beta} < 0.$$

2. *Prices are positively affected by public information, and this effect increases with the precision of public information,*

$$\frac{\mathbf{c}_2}{\mathbf{e}_2}, \frac{\mathbf{c}_1}{\mathbf{e}_1} > 0, \frac{\partial \frac{\mathbf{c}_2}{\mathbf{e}_2}}{\partial \beta}, \frac{\partial \frac{\mathbf{c}_1}{\mathbf{e}_1}}{\partial \beta} > 0.$$

3. *Prices are negatively affected by supply shocks, and this effect decreases with the precision of public information*

$$\frac{\mathbf{1}}{\mathbf{e}_2}, \frac{\mathbf{1}}{\mathbf{e}_1} > 0, \frac{\partial \frac{\mathbf{1}}{\mathbf{e}_2}}{\partial \beta}, \frac{\partial \frac{\mathbf{1}}{\mathbf{e}_1}}{\partial \beta} < 0.$$

4. *Price volatility decreases with the precision of public information*

$$\frac{\partial \Sigma_1}{\partial \beta}, \frac{\partial \Sigma_2}{\partial \beta} < 0.$$

As the last statement in the proposition shows, under the standard information structure, more precise public information decreases price volatility in each period. The result is intuitive. If public information is more precise, agents rely more on public pieces of information and less on every other piece of information. Thus, the price is more sensitive to public information but less sensitive to every other shock. As our volatility measure is only affected by sensitivity to private information and supply shocks, more precise public information decreases price volatility.

As I show in the following proposition, this monotonic pattern generally disappears in a weakly correlated structure.

Proposition 6. *For any set of other parameters,*

1. *There is an interval* $\mathcal{B}_1 \subseteq \left(\frac{\omega}{\nu^2}, \infty\right)$ *such that*

$$\frac{\partial \left|\frac{\mathbf{a}_1}{\mathbf{e}_1}\right|}{\partial \beta}|_{\beta \in \mathcal{B}_1} > 0,$$

i.e. in certain weakly correlated structures, the absolute value of the price effect of the average information of traders in period 1 increases along with the precision of public information.

2. *If the variance of the supply shock,* $\frac{1}{\delta_1}$, *is sufficiently small and* τ_1^*, τ_2^* *are continuous in* β *in* $\left(\frac{\omega}{\nu^2}, \infty\right)$, *then there is an interval* $\mathcal{B}_2 \subseteq \left(\frac{\omega}{\nu^2}, \infty\right)$ *such that*

$$\frac{\partial \left|\frac{\mathbf{1}}{\mathbf{e}_1}\right|}{\partial \beta}|_{\beta \in \mathcal{B}_2} > 0,$$

i.e. in certain weakly correlated structures the absolute price effect of the supply shock in Period 1 increases along with the precision of public information.

This result states that in weakly correlated structures, the price may become more sensitive both to shocks in the average private information and to supply shocks. Especially, when the variance of the supply shock is small, typically there is a set of parameters where both sensitivities increase in precision. Figure 3 shows the equilibrium price coefficients, and the right panel of Figure 2 depicts our volatility measure as a function of the precision of the public information in a typical case. It is apparent that there is a range wherein more public information increases the volatility of price in Period 1. This range is indeed within the interval $\left(\frac{\omega}{\nu^2}, \infty\right)$, *i.e.* it corresponds to a weakly correlated information structure.

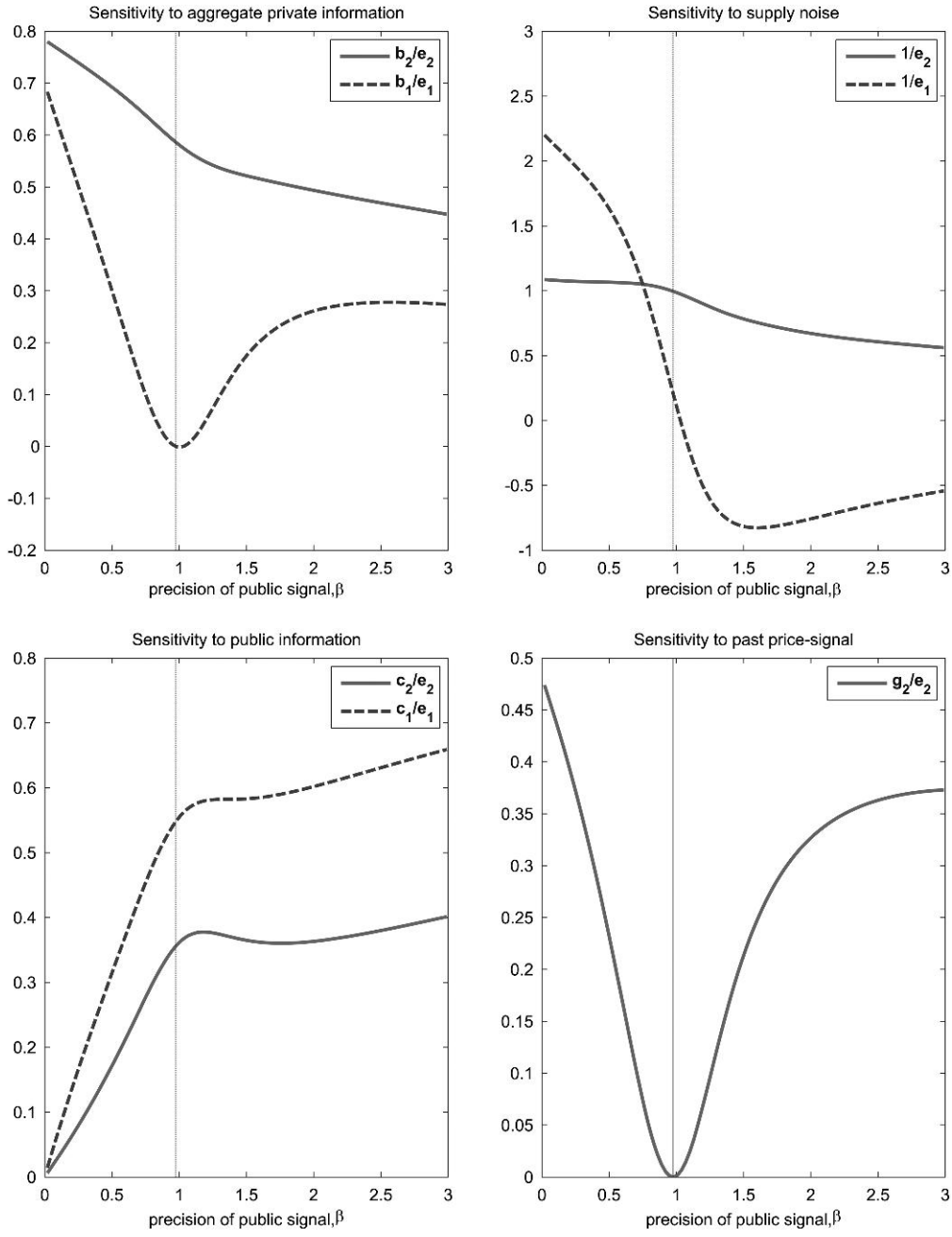

FIGURE 3

Coefficients of the price function in Case 1. Each panel shows a given coefficient of the price function in Periods 1 and 2. The x-axis is the precision of the public signal, β. The vertical line depicts $\beta = \frac{\nu^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information. Parameter values are $\gamma = 1$, $\omega = 4{\cdot}1$, $\nu = 2$, and $\alpha = \delta_1 = \delta_2 = 5$

3.2. *Case 2: Heterogeneous trading horizon*

In this part, I analyse a second variant of the model where J-traders enter in the first period. Thus, I-traders and J-traders can both trade in the first period. Just as before, in the second period, I-traders sell all their holdings to J-traders, leave the market and consume, whereas J-traders only consume in the third period. Also, I normalize the total mass of traders to 1. In particular, the measures of I-traders and J-traders are $(1-\mu)$ and μ, respectively. Although the random supply in the first period is still u_1, to make sure that the supply per capita is independent of μ, I assume that the random supply in the second period is μu_2, where u_1 and u_2 are drawn independently from the distributions

$$u_1 \sim N\left(0, \frac{1}{\delta_1^2}\right), u_2 \sim N\left(0, \frac{1}{\delta_2^2}\right).$$

In every other respect, the setup remains the same. Formally, each I-trader solves problem (9), whereas a J-trader solves problem (11) in the second period and

$$\max_{d_1^j} E[-e^{-\gamma W_J^j} | \mathcal{I}_1^j] \tag{41}$$

$$W_J^j = d_1^j(p_2 - p_1) + d_2^j(\theta - p_2)$$

in the first period, where d_2^j is the optimal strategy in Period 2 and $\mathcal{I}_1^j = \{z^j, y, p_1\}$ is the information set of J-traders in Period 1.

This case might be more well adapted to equity markets where individuals and institutions with different investment horizons co-exist. Whereas some individuals trade very frequently with the explicit purpose of opening and closing positions within a day ("day traders"), others are saving for retirement. It is also an empirically documented fact that the investment horizon of financial institutions varies, perhaps in line with the dispersion in their managers' incentive schemes and the duration of their liabilities.[22] Depending on the groups of long-term and short-term traders being considered, the interpretation of the length of each interval should also vary.

One can think of μ, the fraction of J-traders, as the degree of market integration of the two markets for a particular asset. When this fraction is high, the majority of traders directly trade with all other traders. Thus, the market is well integrated. In contrast, when this fraction is low, the typical trader in the first period is different from the typical trader in the second period. Thus, the market for the given asset is segmented. I focus on the effect of the changing proportions of the two groups on trading activity. I argue that in a weakly correlated information structure, at lower values of μ, (*i.e.* lower proportions of long-horizon traders or, equivalently, less integration of the market for a given asset), the responses of volume, price information content, and (potentially) volatility to public announcements are all more pronounced. As I argue, apart from providing a robustness check, Case 2 also provides testable implications regarding the effects of market integration on trading activity. I will refer to this setup as a model of heterogeneous trading horizon. The drawback of this case is that I have to rely partially on numerical simulations for its analysis.

Given the similarity of the derivation of Case 2, here I only highlight the differences with respect to Case 1. More details on this derivation and on the equilibrium objects are in online Appendix B.

22. See Hotchkiss and Strickland (2003) and Derrien, Thesmar, and Kecskes (2011).

3.2.1. Equilibrium. I follow the same approach used in Case 1 to find a linear REE. I conjecture linear price functions, and under these conjectures, I find traders' optimal demand for any given price. Then, I find the particular coefficients that validate the initial conjecture. The additional complexity compared to Case 1 comes from two sources. First, instead of each agent solving a one-period problem, J-traders solve a two-period problem. Second, the first period demand aggregates the different demand functions of the two groups.

There is little change in the structure of the second period, implying that the price function conjecture (13), the definition of the price signal q_2 and its precision τ_2 remain unchanged. The conjecture for the first period price changes to

$$p_1 = \frac{\mathbf{a}_1(\theta_I + \theta_K) + \mathbf{b}_1(\theta_J + \theta_K) + \mathbf{c}_1 y - u_1}{\mathbf{e}_1} \tag{42}$$

and the definition of the price signal corresponding to Period 1 changes to

$$q_1 \equiv \frac{\mathbf{e}_1 p_1 - \mathbf{c}_1 y}{\mathbf{a}_1 + \mathbf{b}_1} = (1-\phi)\theta_I + \phi\theta_J + \theta_K - \frac{1}{\mathbf{a}_1 + \mathbf{b}_1} u_1, \tag{43}$$

where $\phi \equiv \frac{\mathbf{b}_1}{\mathbf{a}_1 + \mathbf{b}_1}$ is the share of J-traders' private information in the total private information content of the first period price. The conditional precision of q_1 is

$$\tau_1^2 \equiv \frac{1}{\text{var}(q_1 | \theta_I + \theta_K)} = \delta_1^2 (\mathbf{a}_1 + \mathbf{b}_1)^2. \tag{44}$$

The problem of each J-trader in the second period and that of each I-trader in the first period are each very similar to their respective problems in Case 1. The optimal demand of these traders leads to the same formulations of equations (20) and (21). However, J-traders have to solve a two-period problem in Period 1. I derive their demand in online Appendix B. The demand of each trader is still linear in the elements of her information set and can be written as follows:

$$d_1^i = \mathbf{a}_I x^i + \mathbf{c}_I y - \mathbf{e}_I p_1 \tag{45}$$

$$d_1^j = \mathbf{b}_J z^j + \mathbf{c}_J y - \mathbf{e}_J p_1. \tag{46}$$

From market clearing, the coefficients of individual demand functions and the coefficients of first period price are connected as

$$(1-\mu)\mathbf{a}_I = \mathbf{a}_1 \tag{47}$$

$$\mu \mathbf{b}_J = \mathbf{b}_1 \tag{48}$$

$$(1-\mu)\mathbf{c}_I + \mu \mathbf{c}_J = \mathbf{c}_1 \tag{49}$$

$$(1-\mu)\mathbf{e}_I + \mu \mathbf{e}_J = \mathbf{e}_1. \tag{50}$$

In Case 1, I mapped the problem of finding the equilibrium to a fixed-point problem in the space of τ_1 and τ_2. In Case 2, I follow the same procedure, implying a fixed-point problem in the space of τ_1, τ_2, and ϕ. Once we have the equilibrium values of τ_1, τ_2, and ϕ, demand functions give the equilibrium trading intensities $\mathbf{a}_I, \mathbf{b}_J$, and $\mathbf{b}_2$, whereas equations (47)-(50) give the coefficients of the first period price. As I show in the next part, the response of these objects to changes in the amount of public information β and to the degree of market segmentation μ is critical for our analysis.

3.2.2. Information content, trading volume, and price volatility. Our main objects of interest are speculative realized volume[23], which is given by

$$V_1^S = \frac{1}{2}\left(\int |d_1^i| di + \int |d_1^j| dj\right)|_{\theta_I=\theta_J=\theta_K=\eta=u_1=0} = \sqrt{\frac{1}{2\pi}}\frac{1}{\sqrt{\alpha}}\left((1-\mu)\,|\mathbf{a}_I| + \mu\,|\mathbf{b}_J|\right), \quad (51)$$

the information content of prices in each period

$$K_1 \equiv \frac{1}{\mathrm{var}(q_1|\theta_I+\theta_K)} = \tau_1^2 = \delta_1^2(\mathbf{a}_1+\mathbf{b}_1)^2 \quad (52)$$

$$K_2 \equiv \frac{1}{\mathrm{var}(q_2|\theta_J+\theta_K)} = \tau_2^2 = \delta_2^2\mathbf{b}_2^2 \quad (53)$$

and the volatility of prices in Period 1,

$$\Sigma_1 \equiv \mathrm{var}(p_1|y) = \left[\left(\frac{\mathbf{a}_1}{\mathbf{e}_1}\right)^2 + \left(\frac{\mathbf{b}_1}{\mathbf{e}_1}\right)^2\right]\frac{1}{\nu} + \left(\frac{\mathbf{a}_1}{\mathbf{e}_1}+\frac{\mathbf{b}_1}{\mathbf{e}_1}\right)^2\frac{1}{\omega} + \frac{1}{(\mathbf{e}_1)^2\delta_1}.$$

Analogously to Propositions 3, I show in online Appendix B that, in the standard information structure, even if traders with heterogeneous horizons co-exist, the amount of public information still has no effect on trading intensity, the information content of trades or speculative volume. Also, analogously to Propositions 5, more public information always decreases price volatility in the standard information structure.

I analyse the equilibrium in the general case with the help of Figures 4 and 5. In each panel of each figure, the x-axis shows the amount of public information measured by β, and the four curves correspond to different fractions of long-horizon traders in Period 1, μ. The thicker the line is, the larger the fraction of long-horizon traders.[24]

I start the analysis with the response of trading intensities $\mathbf{a}_I, \mathbf{b}_J$, and $\mathbf{b}_2$ to the amount of public information, β, and to the level of market integration, μ. Panel A in Figure 4 shows that the trading intensity of I-traders changes with public information in the same way as in Case 1. It decreases in β in absolute value as long as $\beta < \frac{\nu^2}{\omega}$, and it increases in absolute value when $\beta > \frac{\nu^2}{\omega}$. However, this trading intensity is non-monotonic in μ. As Panel B in Figure 4 illustrates, in the first period, J-traders' trading intensity increases with public information for any β. This is surprising because the intuition shown in Section 2 does not apply to J-traders. If J-traders in Period 1 were to forecast the forecast of the average J-trader in Period 2, the dispersion in their forecasts would decrease with the amount of public information because equation (6) would not hold. The correlation between the private information set of J-traders in Period 1 and that of the average J-trader in Period 2 is high. The reason for $\mathbf{b}_J$ increasing with β is that as public information increases, the second period price is more correlated to the fundamental, so in the first period, all traders can estimate p_2 with more certainty. Although this effect is not sufficient to influence the sign of the derivative of $\mathbf{a}_I$ with respect to public information, it switches that of $\mathbf{b}_J$.[25] Note also that the strength of the response of $\mathbf{b}_J$ to public information

23. As I show in online Appendix B, unlike in Case 1, equilibrium demand does depend on the realization of aggregate random variables because I-traders' and J-traders' demands react differently to each piece of information. This makes the analysis of expected volume more involved. Still, as I show in the same appendix, the main implications for expected volume are similar to the reported implications for speculative volume.

24. The discontinuity in each curve corresponding to $\mu = 0{\cdot}01$ shows the only identified point in the parameter space where a linear equilibrium does not exist. This segment corresponds to a zero-measure set of parameters that are sufficiently close to the set $\phi \to \pm\infty$ as $\mathbf{a}_1+\mathbf{b}_1 \to 0$.

25. Online Appendix B provides further analysis of this result.

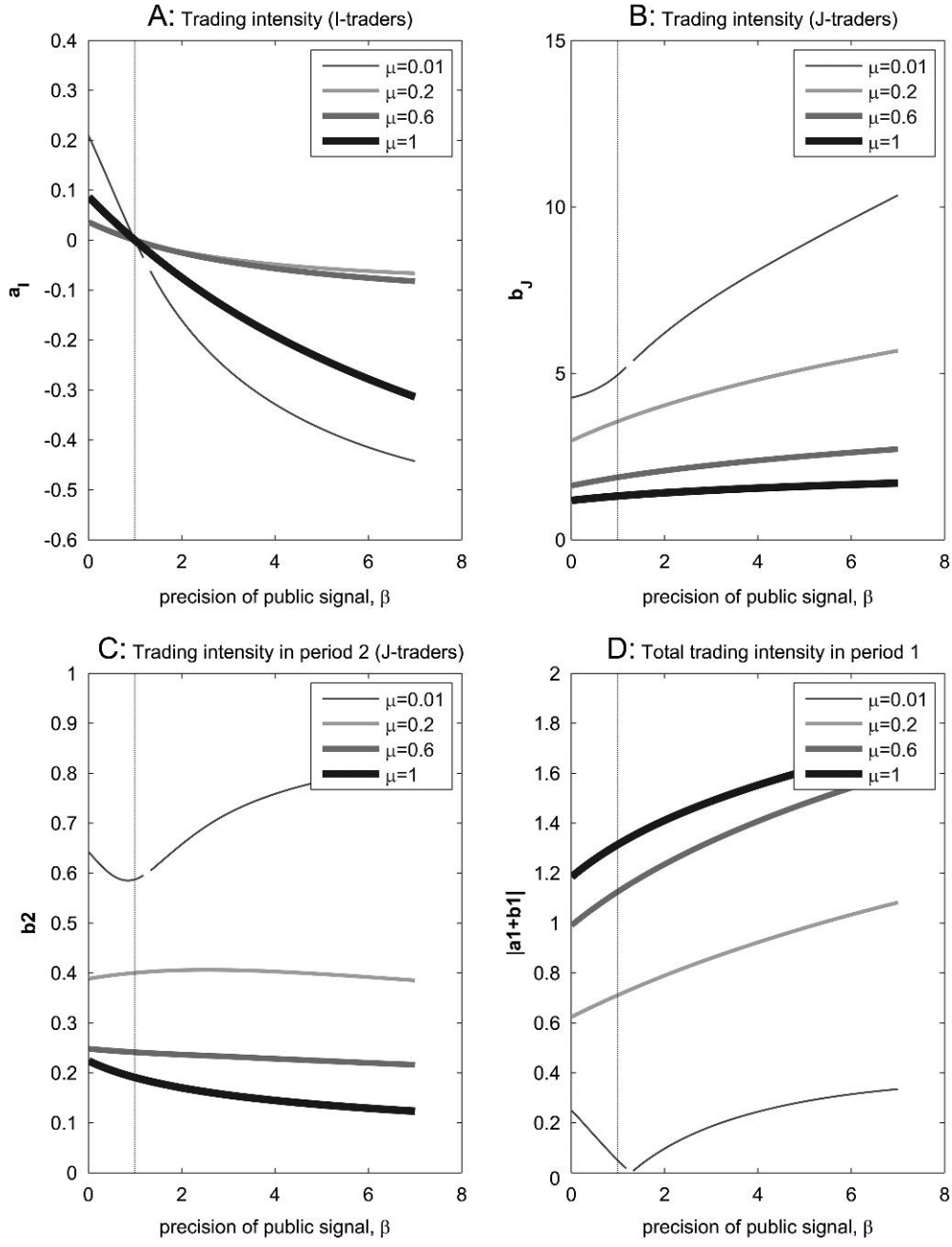

FIGURE 4

Trading intensities in Case 2. Panels A and B show the trading intensities of I-traders and J-traders, and panels C and D show the trading intensity of J-traders in Period 2 and the total trading intensity in Period 1. In each plot, the different curves correspond to a different fraction of J-traders in the market, μ. The thicker the curve is, the larger the fraction. The x-axis is the precision of the public signal β. The vertical line depicts $\beta = \frac{\nu^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information. Parameter values are $\gamma = 1$, $\omega = 4{\cdot}01$, $\nu = 2$, and $\alpha = \delta_1 = \delta_2 = 5$

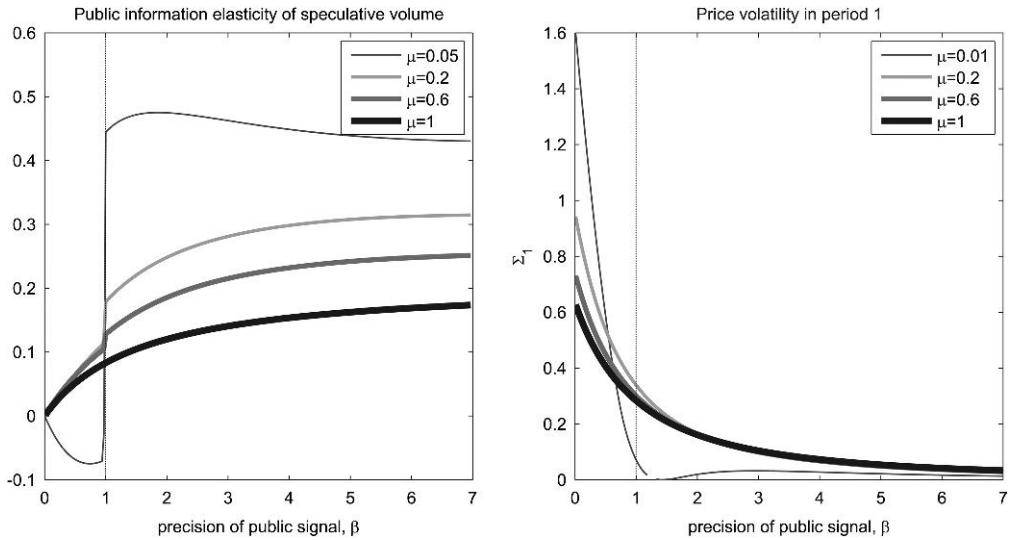

FIGURE 5

Elasticity of volume and the volatility of price in Period 1 in Case 2. The left panel shows the elasticity of speculative volume with respect to the precision of public information, while the right panel shows the volatility of the first period price. In each plot, different curves correspond to different fractions of J-traders on the market, μ. The thicker the curve, the larger the fraction. The x-axis is the precision of the public signal, β. The vertical line depicts $\beta = \frac{\nu^2}{\omega}$, the threshold above which second-order expectations are polarized by more public information. Parameter values are $\gamma = 1$, $\omega = 4{\cdot}01$, $\nu = 2$, and $\alpha = \delta_1 = \delta_2 = 5$

decreases with μ. Finally, Panel C in Figure 4 shows that in the second period, J-traders' trading intensity decreases with public information as long as their fraction in the economy is large. However, when μ is small, as in Case 1, J-traders' trading intensity decreases with β if $\beta < \frac{\nu^2}{\omega}$, and it increases otherwise. This is consistent with the observation that Case 2 is close to Case 1 if μ is small.

Turning to the information content of prices, it is clear that in the second period, information content changes just as trading intensity $\mathbf{b}_2$ does. That is, under a weakly correlated information structure and low market integration μ, more public information increases the private information content of prices. Thus, the result of Case 1 only survives if the economy is sufficiently close to Case 1. If the market is integrated, then public information crowds out private information in the second period. This is in contrast to information content in the first period, which tends to increase with β for any level of market integration under the weakly correlated information structure. This is apparent from Panel D in Figure 4,[26] and it is a consequence of the arguments above implying that, under the given conditions, more public information increases the absolute trading intensity of both types of trader.

In the left panel of Figure 5, I show the elasticity of speculative volume with respect to the amount of public information β. It is apparent that in a weakly correlated information structure, the larger the fraction of short-term traders, the larger the response in speculative volume. That is, even if the strength of the response of $\mathbf{a}_I$ to public information is non-monotonic in μ, the

26. Note that the curve in Panel D corresponding to the less integrated market decreases until the point where $|\mathbf{a}_1 + \mathbf{b}_2| \to 0$ and increases only afterwards. For higher market integration values, there is no such point.

elasticity of speculative volume, which is a weighted average of trading intensities $\mathbf{a}_I$ and $\mathbf{b}_J$, is monotonic. I find this result to be numerically robust to any change in parameters.[27]

The right panel of Figure 5 depicts our volatility measure. It is apparent that only when the share of long-horizon traders is sufficiently low, *i.e.* when the structure is sufficiently close to Case 1, does volatility in Period 1 increase with the amount of public information in any range of the parameter space.

3.2.3. New empirical predictions. The analysis of Case 2 of our model provides additional empirical predictions with which the presented theory can be tested.

First, widely used empirical proxies exist that can be used to measure trading horizon heterogeneity in the investor base of a stock (see Wahal and McConnell, 2000; Gaspar, Massa, and Matos, 2005).[28] If a larger share of short-horizon traders in the investor base was found to correlate with higher volumes and more inflow of information in response to announcements, this would be evidence consistent with the predictions shown in the previous section. I am not aware of any existing studies regarding such a connection.

An alternative empirical strategy is to rely on natural experiments when the characteristics of the investor base of a given asset change abruptly and significantly. Cross-listing of stocks can potentially provide such a natural experiment. As an example, Bailey, Karolyi, and Salva (2006) focuses on the trading volume and return volatility of stocks around earning announcements before and after these stocks were cross-listed on the NYSE. They find that both volatility and volume response increase after the announcement. They find a larger effect for those stocks that were originally listed in the exchange of a developed economy as opposed to an emerging economy exchange. Using a large number of controls, they conclude that this effect must be due to the change in the informational environment due to the cross-listing. However, they cannot explain their findings with the existing theoretical models and call it a puzzle. Although their work is not a direct test of our model, I argue that their finding is consistent with the proposed theory. Consider Case 2 of the model and Figure 5. Although cross-listing changes a range of characteristics of the trading environment of firms, for our purposes think of cross-listing as an increase in the heterogeneity of the investor base or, equivalently, a drop in the level of integration of the market for the asset. That is, μ drops. The left panel of Figure 5 shows that this drop should increase the response of volume to public announcements as long as the prior public information β is sufficiently high. The right panel of Figure 5 shows that this drop might result in an increase in the response of price volatility to earnings announcements. Regarding the difference between emerging market firms and developed market firms, a reasonable assumption is that, although cross-listing increases the amount of available public information prior to the announcement for both firms, emerging market firms are less transparent both before and after cross-listing. Although my model does not provide a clear prediction for this comparative static, it is easy to see that there are scenarios under which it would provide the same results as the empirical evidence.

4. CONCLUSION

In this paper, I show that, in Gaussian information structures where the connection between private signals is sufficiently weak, a public announcement can lead to polarized higher-order

27. Online Appendix B provides further analysis of this result.

28. Recent empirical work has found that the proportion of short-term investors affect the quality of accounting disclosure, mergers and acquisitions, the trade-off between dividends and repurchases, and investment policy. (see Gaspar, Massa, and Matos, 2005; Derrien, Thesmar, and Kecskes, 2011)

expectations regardless of the content of the announcement. I illustrate the economic relevance of these properties using a noisy rational expectations model of financial markets. I show that these properties can explain stylized facts regarding trading patterns associated with announcements such as high trading volume, more informative prices, and more volatile prices.

I believe that the observation that public information might polarize higher-order expectations without polarizing first-order expectations has further economic implications in a wide range of contexts. As another example, in an ongoing project, I analyse a version of the speculative currency attack model of Morris and Shin (1998) wherein the central bank has imperfect knowledge of the state of the economy. To assess the probability of a devaluation, speculators have to second-guess the expectation of the central bank. I show that the fact that a public announcement can polarize higher-order expectations implies that generating and disclosing more public information can destabilize the exchange rate system.

Regarding further research, empirical analyses on the relative effects of announcements on trading patterns and price informativeness across assets and markets with different characteristics (*e.g.*, in the degree of investor base heterogeneity, the frequency of announcements, and the importance of private information) could help to establish the importance of the presented mechanism relative to others.

APPENDIX

A.1. *Proof of Proposition 1*

Note that by the projection theorem

$$\bar{E}(\theta|z^j,y)=\frac{\beta(\nu^2+\alpha\nu+\alpha\omega+2\nu\omega)y+(\theta_J+\theta_K)\alpha\nu(\nu+\omega)}{\alpha\nu^2+\nu^2\beta+\nu^2\omega+\alpha\nu\beta+\alpha\nu\omega+\alpha\beta\omega+2\nu\beta\omega}$$

and

$$E((\theta_J+\theta_K)|x^i,y)=\frac{\beta(\nu+\omega)(\alpha+\nu)y+x^i\alpha(\nu^2-\beta\omega)}{\alpha\nu^2+\nu^2\beta+\nu^2\omega+\alpha\nu\beta+\alpha\nu\omega+\alpha\beta\omega+2\nu\beta\omega},$$

$$E(\bar{E}(\theta|z^j,y)|x^i,y)-\bar{E}(\bar{E}(\theta|z^j,y)|x^i,y)=\frac{\alpha^2\nu(\nu+\omega)(\nu^2-\beta\omega)}{(\alpha\nu^2+\nu^2\beta+\nu^2\omega+\alpha\nu\beta+\alpha\nu\omega+\alpha\beta\omega+2\nu\beta\omega)^2}\varepsilon^i.$$

By the property of folded normal distributions,

$$\frac{\partial\int_j|E(\bar{E}(\theta|z^j,y)|x^i,y)-\bar{E}(\bar{E}(\theta|z^j,y)|x^i,y)|dj}{\partial\beta}$$

$$=\frac{\partial\left|\frac{\alpha^2\nu(\nu+\omega)(\nu^2-\beta\omega)}{(\alpha\nu^2+\nu^2\beta+\nu^2\omega+\alpha\nu\beta+\alpha\nu\omega+\alpha\beta\omega+2\nu\beta\omega)}\right|\sqrt{\frac{2}{\alpha\pi}}}{\partial\beta}=\frac{-\sqrt{\frac{2}{\alpha\pi}}\alpha^2\nu^2(\alpha+\nu)(\nu+\omega)^3}{(\alpha\nu^2+\nu^2\beta+\nu^2\omega+\alpha\nu\beta+\alpha\nu\omega+\alpha\beta\omega+2\nu\beta\omega)^2}\operatorname{sgn}(\nu^2-\beta\omega),$$

which proves the statement.

A.2. *Proof of Proposition 2*

From equations (18) and (20), market clearing implies

$$p_2=b_2(\theta_J+\theta_K)+c_2y+e_2q_2+g_2q_1-\frac{\gamma}{\tau_\theta^2}u_2.$$

From equation (16), this is equivalent to

$$\frac{q_2\mathbf{b}_2+\mathbf{c}_2y+\mathbf{g}_2q_1}{\mathbf{e}_2}=b_2(\theta_J+\theta_K)+c_2y+e_2q_2+g_2q_1-\frac{\gamma}{\tau_\theta^2}u_2$$

or

$$\frac{\mathbf{b}_2(\theta_J+\theta_K)-u_2+\mathbf{c}_2y+\mathbf{g}_2q_1}{\mathbf{e}_2}=b_2(\theta_J+\theta_K)+c_2y+e_2\left((\theta_J+\theta_K)-\frac{u_2}{\mathbf{b}_2}\right)+g_2q_1-\frac{\gamma}{\tau_\theta^2}u_2.$$

This expression has to hold for any realizations of each random variable. This holds for any η , u_1, $\theta_J+\theta_K$, u_2 if and only if

$$\frac{\mathbf{c}_2}{\mathbf{e}_2}=c_2,\ \frac{\mathbf{g}_2}{\mathbf{e}_2}=g_2,\ \frac{\mathbf{b}_2}{\mathbf{e}_2}=b_2+e_2,\ \text{and}\ \frac{1}{\mathbf{e}_2}=\frac{\gamma}{\tau_\theta^2}+\frac{e_2}{\mathbf{b}_2}.$$

Combining these equations give expressions (24)-(27), which in turn imply equation (13). Using the same expressions, I also get equation (22) as follows:

$$d_2^j=\frac{\tau_\theta^2}{\gamma}(b_2z^j+c_2y+e_2q_2+g_2q_1-p_2)=\frac{\tau_\theta^2}{\gamma}\left(b_2z^j+c_2y+e_2q_2+g_2q_1-\frac{q_2\mathbf{b}_2+\mathbf{c}_2y+\mathbf{g}_2q_1}{\mathbf{e}_2}\right)$$

$$=\frac{\tau_\theta^2}{\gamma}\left(b_2z^j+e_2q_2-\frac{q_2\mathbf{b}_2}{\mathbf{e}_2}\right)=\frac{\tau_\theta^2}{\gamma}\left(b_2z^j+\left(e_2-\frac{\mathbf{b}_2}{\mathbf{e}_2}\right)q_2\right)=\mathbf{b}_2(z^j-q_2).$$

Expression (29) is implied by the definition of τ_1 and equation (24). The same steps give all the corresponding expressions for Period 1.

Note that the proposition gives all the equilibrium objects in terms of $b_2,c_2,e_2,g_2,a_1,c_1,e_1,\tau_\theta^2,\tau_q^2$. These coefficients are determined by the projection theorem using the covariance matrix of $[z^j,y,q_1,q_2]$, and the covariance of this vector with the fundamental value, θ. As all these matrices are functions of the primitives and τ_1 and τ_2 only, the same is true for all the equilibrium objects. Consequently, for existence, I only have to prove that equilibrium values for τ_1 and τ_2 exist. Using the explicit expressions for $b_2,e_2,a_1,\tau_\theta^2,\tau_q^2$, equations (29)–(30) define this as a fixed point problem

$$\tau_2=\frac{\delta_2\alpha(\nu+\tau_1^2)(\nu+\omega)}{\gamma(\nu^2+\tau_1^2\tau_2^2+\alpha\nu+\alpha\omega+2\nu\omega+\alpha\tau_1^2+\nu\tau_1^2+\nu\tau_2^2+\omega\tau_1^2+\omega\tau_2^2)} \tag{A.1}$$

$$\tau_1=\frac{\delta_1\frac{\alpha}{\gamma}\frac{\tau_2^2}{\nu+\omega}(\nu^2-\beta\omega)}{(\tau_1^2+\nu)(\tau_2^2+\alpha)}\frac{(\tau_1^2+\tau_2^2+\alpha+\beta+\omega)\nu^2+(2(\tau_1^2\tau_2^2+\beta\omega+\alpha\tau_1^2)+\alpha(\beta+\omega)+(\tau_1^2+\tau_2^2)(\beta+\omega))\nu+(\tau_1^2(\beta+\omega)(\alpha+\tau_2^2)+\beta\omega(\alpha+\tau_1^2+\tau_2^2))}{(\tau_1^2+\tau_2^2+\alpha+\beta+\omega)\nu^2+(2(\tau_1^2\tau_2^2+\beta\omega+\alpha\tau_2^2)+\alpha(\beta+\omega)+(\tau_1^2+\tau_2^2)(\beta+\omega))\nu+(\tau_2^2(\beta+\omega)(\alpha+\tau_1^2)+\beta\omega(\alpha+\tau_1^2+\tau_2^2))}. \tag{A.2}$$

The problem simplifies, if I rewrite this as a fixed point problem in the space of $[\tau_1,\tau_2,Y]$ where equation (A.2) is replaced by

$$\tau_1=\frac{\delta_1(\nu^2-\beta\omega)\alpha\tau_2^2}{\gamma(\nu+\tau_1^2)(\nu+\omega)(\alpha+\tau_2^2)}Y \tag{A.3}$$

$$Y=\frac{(\tau_1^2+\tau_2^2+\alpha+\beta+\omega)\nu^2+(2(\tau_1^2\tau_2^2+\beta\omega+\alpha\tau_1^2)+\alpha(\beta+\omega)+(\tau_1^2+\tau_2^2)(\beta+\omega))\nu+(\tau_1^2(\beta+\omega)(\alpha+\tau_2^2)+\beta\omega(\alpha+\tau_1^2+\tau_2^2))}{(\tau_1^2+\tau_2^2+\alpha+\beta+\omega)\nu^2+(2(\tau_1^2\tau_2^2+\beta\omega+\alpha\tau_2^2)+\alpha(\beta+\omega)+(\tau_1^2+\tau_2^2)(\beta+\omega))\nu+(\tau_2^2(\beta+\omega)(\alpha+\tau_1^2)+\beta\omega(\alpha+\tau_1^2+\tau_2^2))}. \tag{A.4}$$

Let the left-hand side of equations (A.1) and (A.3)–(A.4) be called $F_2(\tau_2,\tau_1)$, $F_1(\tau_2,\tau_1,Y)$, and $F_Y(\tau_2,\tau_1)$, respectively. Also, let $\hat{Y}\equiv F_Y(\tau_2,\tau_1)$ and $\hat{\tau}_2(\tau_1)$, $\hat{\tau}_1(\tau_2,Y)$ implicitly defined as

$$\hat{\tau}_2\equiv F_2(\hat{\tau}_2,\tau_1)\quad\text{and}\quad\hat{\tau}_1\equiv F_1(\tau_2,\hat{\tau}_1,Y).$$

By the implicit function theorem, it is easy to check that $\frac{\partial\hat{\tau}_2}{\partial\tau_1}>0$. Thus, for any τ_1,Y, $\hat{\tau}_2\le\tau_2^{\max}$, where $\tau_2^{\max}$ is defined as

$$\tau_2^{\max}=\lim_{\tau_1\to\infty}\hat{\tau}_2(\tau_1,Y).$$

By simple derivation, $\frac{\partial\hat{Y}}{\partial\tau_1^2}>0$ and $\frac{\partial\hat{Y}}{\partial\tau_2^2}<0$. Thus, for any τ_2, τ_1, $Y^{\min}\le\hat{Y}\le Y^{\max}$, where

$$Y^{\min}=\lim_{\tau_2\to0}\lim_{\tau_1\to\infty}F_Y(\tau_2,\tau_1)=\frac{\nu^2+2\alpha\nu+\alpha\beta+\nu\beta+\alpha\omega+\nu\omega+\beta\omega}{(\nu+\omega)(\nu+\beta)}$$

$$Y^{\max}=\lim_{\tau_1\to0}\lim_{\tau_2\to\infty}F_Y(\tau_2,\tau_1)=(\nu+\omega)\frac{\nu+\beta}{\nu^2+2\alpha\nu+\alpha\beta+\nu\beta+\alpha\omega+\nu\omega+\beta\omega}.$$

Finally, by the implicit function theorem, whenever $\nu^2>\beta\omega$, $\frac{\partial\hat{\tau}_1}{\partial\tau_2},\frac{\partial\hat{\tau}_1}{\partial Y}>0$, while whenever $\nu^2<\beta\omega$, $\frac{\partial\hat{\tau}_1}{\partial\tau_2},\frac{\partial\hat{\tau}_1}{\partial Y}<0$. Thus, if $\tau_1^{\max}$ is defined as the unique solution of

$$Y^{\max}\alpha\frac{\nu^2-\beta\omega}{\gamma(\nu+\omega)(\nu+(\tau_1^{\max})^2)}=\tau_1^{\max},$$

then for any τ_2 and Y, $\hat{\tau}_1\in[0,\tau_1^{\max}]$ when $\nu^2>\beta\omega$ and $\hat{\tau}_1\in[\tau_1^{\max},0]$ when $\nu^2<\omega\beta$.

Consequently, if the space S is defined as

$$[0, \tau_1^{\max}]X[0, \tau_2^{\max}]X[Y^{\min}, Y^{\max}]$$

and as

$$[\tau_1^{\max}, 0]X[0, \tau_2^{\max}]X[Y^{\min}, Y^{\max}]$$

in the case of $\nu^2 > \omega\beta$ and $\nu^2 < \omega\beta$, respectively, then the system (A.1) and (A.3)–(A.4) maps S to itself and S is a closed convex set. Thus, there exist a fixed point, $[\tau_1^*, \tau_2^*, Y^*]$ of the system (A.1) and (A.3)–(A.4), by the Brower fixed point theorem. I conclude that as long as the denominator of the equilibrium objects described in the proposition are not zero the equilibrium exists. It is easy to check that this criteria excludes at most a zero measured set of the parameter space.

A.3. *Proof of Proposition 3*

The result is a consequence of Proposition 2 and the fact that

$$\tau_2^* = \alpha\frac{\delta_2}{\gamma},\ \tau_1^* = \alpha^2\delta_1\frac{\delta_2^2}{\gamma(\gamma^2+\alpha\delta_2^2)}$$

is the fixed point of the system

$$\lim_{\nu\to\infty} F_2(\tau_2, \tau_1) = \tau_2$$

$$\lim_{\nu\to\infty} F_1(\tau_2, \tau_1, F_Y(\tau_2, \tau_1)) = \tau_1.$$

A.4. *Proof of Lemma 1 and Proposition 4*

Substituting in F_Y and reorganizing $F_1(\tau_2, \tau_1, F_Y(\tau_1)) = \tau_1$ and $F_2(\tau_2, \tau_1) = \tau_2$ as polynoms in τ_1 and τ_2, respectively, check that in any equilibrium (τ_1^*, τ_2^*) has to solve

$$0 \equiv G_1 \quad \text{and} \quad 0 \equiv G_2,$$

where

$$G_1 = \gamma\tau_1(\nu+\omega)(\tau_1^2+\nu)(\tau_2^2+\alpha)Z_2 + \alpha\tau_2^2(\beta\omega-\nu^2)Z_1$$

with

$$Z_1 = \nu^2(\alpha+\beta+\omega+\tau_1^2+\tau_2^2) + \alpha\tau_1^2(\beta+2\nu+\omega) + (\tau_1^2+\tau_2^2)(\beta\nu+\beta\omega+\nu\omega)$$

$$+\tau_1^2\tau_2^2(\beta+2\nu+\omega) + \alpha\nu(\beta+\omega) + \beta\omega(\alpha+2\nu)$$

$$Z_2 = Z_1 - \alpha(\tau_1^2-\tau_2^2)(\beta+2\nu+\omega)$$

and

$$G_2 = \gamma(\tau_1^2+\nu+\omega)\tau_2^3 + \gamma(\nu^2+\alpha\nu+\alpha\omega+2\nu\omega+\alpha\tau_1^2+\nu\tau_1^2+\omega\tau_1^2)\tau_2 - (\omega+\alpha\nu\delta_2(\tau_1^2+\nu)).$$

Note that for any fixed τ_1, G_2 is a monotonically increasing function with a single root. Also

$$\left.\frac{\partial G_2(\tau_1,\tau_2)}{\partial\tau_1^2}\right|_{\tau_2=\hat{\tau}_2(\tau_1)} = (\tau_2\gamma(\tau_2^2+\gamma(\alpha+\nu+\omega)) - \alpha\nu\delta_2)$$

$$= (F_2(\tau_2,\tau_1)\gamma(\tau_2^2+\gamma(\alpha+\nu+\omega)) - \alpha\nu\delta_2)$$

$$= \left(\frac{\delta_2\alpha(\nu+\tau_1^2)(\nu+\omega)\gamma(\tau_2^2+(\alpha+\nu+\omega))}{\gamma(\nu^2+\tau_1^2\tau_2^2+\alpha\nu+\alpha\omega+2\nu\omega+\alpha\tau_1^2+\nu\tau_1^2+\nu\tau_2^2+\omega\tau_1^2+\omega\tau_2^2)} - \alpha\nu\delta_2\right)$$

$$= \frac{\alpha\delta_2(\nu+\omega)(\nu+\tau_1^2)(\alpha+\nu+\omega+\tau_2^2)}{\nu^2+\tau_1^2\tau_2^2+\alpha\nu+\alpha\omega+2\nu\omega+\alpha\tau_1^2+\nu\tau_1^2+\nu\tau_2^2+\omega\tau_1^2+\omega\tau_2^2} > 0,$$

where $\hat{\tau}_2(\tau_1)$ is defined as in the proof of Proposition 2.

Also, by the implicit function theorem

$$\frac{\partial \tau_1^*}{\partial \beta} = -\frac{\frac{\partial G_1}{\partial \beta}\frac{\partial G_2}{\partial \tau_2}}{\frac{\partial G_1}{\partial \tau_1}\frac{\partial G_2}{\partial \tau_2} - \frac{\partial G_1}{\partial \tau_2}\frac{\partial G_2}{\partial \tau_1}}. \tag{A.5}$$

$$\frac{\partial \tau_2^*}{\partial \beta} = -\frac{\frac{\partial G_1}{\partial \beta}\frac{\partial G_2}{\partial \tau_1}}{\frac{\partial G_1}{\partial \tau_1}\frac{\partial G_2}{\partial \tau_2} - \frac{\partial G_1}{\partial \tau_2}\frac{\partial G_2}{\partial \tau_1}}. \tag{A.6}$$

Suppose that at a given point $\frac{\partial \tau_1^*}{\partial \beta}$ and $\frac{\partial \tau_2^*}{\partial \beta}$ exists. Clearly, $\frac{\partial \tau_1^*}{\partial \beta} = 0$ is possible only if $\frac{\partial G_1}{\partial \beta} = 0$, but then $\frac{\partial \tau_2^*}{\partial \beta} = 0$ also. If $\frac{\partial \tau_1^*}{\partial \beta} \neq 0$, then

$$\frac{\frac{\partial \tau_2^*}{\partial \beta}}{\frac{\partial \tau_1^*}{\partial \beta}} = \frac{\frac{\partial G_2}{\partial \tau_1}}{\frac{\partial G_2}{\partial \tau_2}} > 0.$$

This proves Lemma 1.

For Proposition 4, consider the next lemma first.

Lemma A.1. *For any* $\beta \geq \frac{\nu^2}{\omega}$, $\frac{\partial G_1}{\partial \beta}\big|_{\tau_1=\tau_1^*} > 0$.

Proof. Consider G_1. It is clear that $Z_1, Z_2, \frac{\partial Z_2}{\partial \beta}, \frac{\partial Z_1}{\partial \beta} > 0$ and

$$\begin{aligned}
\frac{\partial G_1}{\partial \beta} &= \gamma\,\tau_1(\nu+\omega)(\tau_1^2+\nu)(\tau_2^2+\alpha)\frac{\partial Z_2}{\partial \beta} + \alpha\tau_2^2\omega Z_1 + \alpha\tau_2^2(\beta\omega-\nu^2)\frac{\partial Z_1}{\partial \beta} \\
&= \alpha\tau_2^2(\nu^2-\beta\omega)\frac{Z_1}{Z_2}\frac{\partial Z_2}{\partial \beta} + \alpha\tau_2^2\omega Z_1 + \alpha\tau_2^2(\beta\omega-\nu^2)\frac{\partial Z_1}{\partial \beta} \\
&= \left(\frac{\alpha\tau_2^2 Z_1(\nu+\tau_2^2)(\nu+\omega)^2(\alpha+\nu+\tau_1^2)}{(\tau_1^2+\tau_2^2+\alpha+\beta+\omega)\nu^2+(2\tau_1^2\tau_2^2+\alpha\beta+\alpha\omega+2\beta\omega+2\alpha\tau_2^2+\beta\tau_1^2+\beta\tau_2^2+\omega\tau_1^2+\omega\tau_2^2)\nu+(\beta(\tau_1^2\tau_2^2+\alpha\omega+\alpha\tau_2^2+\omega\tau_1^2+\omega\tau_2^2)+\omega\tau_2^2(\alpha+\tau_1^2))}\right) \\
&\quad + \alpha\tau_2^2(\beta\omega-\nu^2)\frac{\partial Z_1}{\partial \beta} > 0
\end{aligned}$$

where I used the equilibrium condition $\tau_1 = F_1$. ‖

Note that $0 \equiv G_2$ has a single solution $\hat{\tau}_2$ for any τ_1^2, and $0 \equiv G_2$ has at least one solution $\hat{\tau}_1$ for given τ_2, but might have more than one. However, when $\beta\omega = \nu^2$, then $\hat{\tau}_1 = 0$ is the only solution of $0 \equiv G_1$. Thus, the system has a unique fixed point where $\tau_1^* = 0, \tau_2^* > 0$.

Note that $\hat{\tau}_2$ is continuous in ω and τ_1. Thus, τ_2^* is also continuous in ω as long as $\hat{\tau}_1$ is continuous in ω. Also, as G_2 is a fifth-order polynomial in τ_1, a necessary condition for $\hat{\tau}_1$ to be discontinuous at a given point is that $\frac{\partial G_1}{\partial \tau_1} = 0$ at that point.

Consider the point $\omega = \frac{\nu^2}{\beta}$ where $\tau_1^* = 0$ and $\tau_2^* > 0$. It is simple to check that at that point $\frac{\partial G_1}{\partial \tau_1} > 0$, $\frac{\partial G_2}{\partial \tau_1} = 2\tau_1\frac{\partial G_2}{\partial \tau_1^2} = 0$, $\frac{\partial G_2}{\partial \tau_2} > 0$ $\frac{\partial G_1}{\partial \tau_2} = 0$. As from Lemma A.1, $\frac{\partial G_1}{\partial \beta} > 0$, at this point $\frac{\partial \tau_1^*}{\partial \beta} < 0$ and $\frac{\partial \tau_2^*}{\partial \beta} = 0$. Also, the fact that $\frac{\partial G_1}{\partial \tau_1} > 0$, $\frac{\partial G_1}{\partial \tau_1}\frac{\partial G_2}{\partial \tau_2} - \frac{\partial G_1}{\partial \tau_2}\frac{\partial G_2}{\partial \tau_1} > 0$ and both are continuous in ω at that point, imply that there is an open set around $\omega = \frac{\nu^2}{\beta}$ that within this set τ_1^*, τ_2^* are continuous functions of ω and continuously differentiable in β. Define $\omega^{\min}, \omega^{\max}$ in a way that the set $(\omega^{\min}, \omega^{\max})$ is the largest such open set around $\omega = \frac{\nu^2}{\beta}$. Then by definition $\frac{\partial G_1}{\partial \tau_1}\frac{\partial G_2}{\partial \tau_2} - \frac{\partial G_1}{\partial \tau_2}\frac{\partial G_2}{\partial \tau_1}$ cannot change sign within this set. Also, as $\beta > \frac{\nu^2}{\omega}$ implies $\tau_1 < 0$, $\frac{\partial G_2}{\partial \tau_1} = 2\tau_1\frac{\partial G_2}{\partial \tau_1^2} < 0$ in this region, while $\frac{\partial G_2}{\partial \tau_2} > 0$, from Lemma A.1, the second statement holds.

A.5. *Proof of Proposition 5*

The result comes from a series of mechanical calculations of the limits of $\frac{\mathbf{b}_2}{\mathbf{e}_2}$, $\frac{\mathbf{a}_1}{\mathbf{e}_1}$, and $\mathbf{e}_1, \mathbf{e}_2$ $\frac{\mathbf{c}_2}{\mathbf{e}_2}$, and $\frac{\mathbf{c}_1}{\mathbf{e}_1}$. As I already showed that τ_1 and τ_2 are insensitive to β in this limit, the partial derivatives of these expressions with respect to β give all the results.

A.6. *Proof of Proposition 6*

I start with the analysis of the relevant equilibrium objects when $\beta = \frac{\nu^2}{\omega}$. At this point, $\tau_1^* = 0$, $\tau_2^* > 0$ and given as the unique root of

$$\delta_2 \alpha \nu \frac{\nu + \omega}{\gamma\,(\nu^2 + \alpha\nu + \alpha\omega + 2\nu\omega + \nu\tau_2^2 + \omega\tau_2^2)} = \tau_2.$$

Also, as I showed in the proof of Proposition 4, at this point τ_1^* and τ_2^* are continuously differentiable in β and $\frac{\partial(\tau_1^*)^2}{\partial\beta} = \frac{\partial(\tau_2^*)^2}{\partial\beta} = 0$. Therefore, at this point, as β changes each equilibrium objects change only by the direct effect of β. I am interested in the properties of $\mathbf{e}_1$ and $\frac{\mathbf{a}_1}{\mathbf{e}_1}$ near this point. As

$$\lim_{\tau_1 \to 0} \frac{\mathbf{a}_1}{\mathbf{e}_1} = \lim_{\tau_1 \to 0} ((e_2 + b_2)(a_1 + e_1) + g_2)$$

$$= \frac{\alpha(\nu^2 - \beta\omega)\nu(\nu+\omega)(\alpha+\tau_2^2)}{(\nu^2\tau_2^2 + \alpha\nu^2 + \nu^2\beta + \nu^2\omega + \nu\beta\tau_2^2 + \nu\omega\tau_2^2 + \beta\omega\tau_2^2 + \alpha\nu\beta + \alpha\nu\omega + \alpha\beta\omega + 2\nu\beta\omega)(\alpha\nu^2 + \nu^2\beta + \nu^2\omega + \alpha\nu\beta + \alpha\nu\omega + \alpha\beta\omega + 2\nu\beta\omega)}$$

and

$$\frac{\partial \frac{(\nu^2 - \beta\omega)\alpha\nu(\nu+\omega)(\alpha+\tau_2^2)}{(\nu^2\tau_2^2 + \alpha\nu^2 + \nu^2\beta + \nu^2\omega + \nu\beta\tau_2^2 + \nu\omega\tau_2^2 + \beta\omega\tau_2^2 + \alpha\nu\beta + \alpha\nu\omega + \alpha\beta\omega + 2\nu\beta\omega)(\alpha\nu^2 + \nu^2\beta + \nu^2\omega + \alpha\nu\beta + \alpha\nu\omega + \alpha\beta\omega + 2\nu\beta\omega)}}{\partial\beta}\Big|_{\frac{\nu^2}{\omega} = \beta} < 0.$$

I conclude that

$$\frac{\partial \left|\frac{\mathbf{a}_1}{\mathbf{e}_1}\right|}{\partial\beta}\Big|_{\frac{\nu^2}{\omega} = \beta} > 0.$$

This implies the first part of the statement. Also, using the expression for $\mathbf{e}_1$ and the observation that $\frac{e_1 + a_1}{a_1} = \frac{\alpha + \tau_1^2}{\alpha}$ and $\frac{g_2}{e_2 + b_2} = \tau_1^2 \frac{\alpha + \nu + \tau_2^2}{(\nu + \tau_1^2)(\alpha + \tau_2^2)}$, I rewrite $\mathbf{e}_1$ as

$$\mathbf{e}_1 = \frac{\tau_q^2}{\gamma} \frac{1}{(e_2 + b_2)} \frac{1}{(e_2 + b_2)\frac{e_1 + a_1}{a_1} + \frac{g_2}{a_1}} = \frac{\tau_q^2}{\gamma\,(e_2 + b_2)^2} \frac{1}{\frac{\alpha + \tau_1^2}{\alpha} + \frac{\tau_1^2}{a_1}\frac{\alpha + \nu + \tau_2^2}{(\nu + \tau_1^2)(\alpha + \tau_2^2)}}.$$

As

$$\frac{\tau_1^2}{a_1} = \frac{\left(\delta_1 \frac{\tau_q^2}{\gamma} \frac{a_1}{(e_2 + b_2)}\right)^2}{a_1} = \left(\delta_1 \frac{\tau_q^2}{\gamma}\right)^2 \frac{a_1}{(e_2 + b_2)^2},$$

$$\mathbf{e}_1|_{\frac{\nu^2}{\omega} = \beta} = \frac{\tau_q^2}{\gamma} \frac{1}{(e_2 + b_2)^2} > 0.$$

Also,

$$\lim_{\beta \to \infty} \left(\frac{\alpha + \tau_1^2}{\alpha} + \frac{\tau_1^2}{a_1} \frac{\alpha + \nu + \tau_2^2}{(\nu + \tau_1^2)(\alpha + \tau_2^2)}\right) = \nu \frac{(-\tau_2^2 - \alpha - \nu - \omega)\tau_1^4 + (-\alpha^2 - 2\alpha\nu - \alpha\tau_2^2 - 2\omega\alpha - \nu^2 - \nu\tau_2^2 - 2\omega\nu - \omega\tau_2^2)\tau_1^2 + (\omega\alpha^2 + \omega\alpha\tau_2^2)}{\alpha\omega(\nu + \tau_1^2)(\alpha + \tau_2^2)},$$

where, for any fixed τ_2, the numerator is a monotonically decreasing function in τ_1^2. As τ_2^* is finite for any τ_1, and $\lim_{\delta_1 \to \infty} \tau_1^2 = \infty$,

$$\lim_{\delta_1 \to \infty} \lim_{\beta \to \infty} \left(\frac{\alpha + \tau_1^2}{\alpha} + \frac{\tau_1^2}{a_1} \frac{\alpha + \nu + \tau_2^2}{(\nu + \tau_1^2)(\alpha + \tau_2^2)}\right) = -\infty$$

in equilibrium. As $\frac{\tau_q^2}{\gamma\,(e_2 + b_2)^2} > 0$, there must be a sufficiently large δ_1 and $\beta \in \left(\frac{\nu^2}{\omega}, \infty\right)$ that $\frac{1}{\mathbf{e}_1}$ is negative. As $\mathbf{e}_1|_{\frac{\nu^2}{\omega} = \beta} > 0$, this implies the second part of the Lemma.

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