THE POLITICS OF PUBLIC PROVISION OF EDUCATION: PROOFS

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In this companion working paper I provide the formal proofs for the results in Levy (2005).

Lemma 1 In the (t, g) policy space:

(i) The ideal policy of p_y is $(t = 1, g = g^*(1))$ and the ideal policy of r_y is $(t = 0, g = 0)$ if $\theta > \frac{w}{w_h}$ and otherwise it is $(t = t^*, g = \dfrac{t^* w}{\theta})$ for some $t^* \in (0, 1)$. The indifference curves of r_y and p_y are weakly concave and differentiable. For all $t' \in [0, 1]$, an indifference curve of $r_y(p_y)$ that goes through $(t', 0)$ lies on or above a line that goes through $(t', 0)$ and has a slope $\dfrac{w_h - w}{1-\theta}(\dfrac{w_l - w}{1-\theta})$.

(ii) The ideal policy of r_o is $(t = 0, g = 0)$, and that of p_o is $(t = 1, g = 0)$. The indifference curves of $r_o(p_o)$ are linear, with a slope $\dfrac{w - w_h}{\theta}(\dfrac{w - w_l}{\theta})$.

Proof of Lemma 1. The derivation of the preferences of r_o and p_o is done in the main paper. Here I focus on r_y and p_y.

Consider first the young rich, r_y. Given any (t, g) they choose s to maximize:

$$\max_{s \geq 0} u((g + s), (w_h(1 - t) + tw - \theta qg - qs))$$

The first order condition is:

$$u_e - qu_x + \lambda = 0$$

where the solution is:

$$\begin{aligned} s &> 0 \text{ if } g \leq g(t) \\ s &= 0 \text{ otherwise.} \end{aligned}$$

By the assumption that the goods are normal, $g(t)$ has a negative slope; when t increases and g is fixed, income decreases for the young rich voters. Thus, for the same g, it must be that one prefers $s = 0$. The slope of the indifference curve, using the indirect utility and the envelope theorem is characterized by:

$$\begin{aligned} (u_e - \theta qu_x)dg + u_x(w - w_h)dt &= 0 \rightarrow \\ \frac{dg}{dt} &= \frac{u_x(w_h - w)}{u_e - \theta qu_x}. \end{aligned}$$

When $s > 0$, $qu_x = u_e$, then the slope is linear and positive:

$$\frac{dg}{dt} = \frac{u_x(w_h - w)}{u_e - \theta qu_x} = \frac{u_x(w_h - w)}{qu_x - \theta qu_x} = \frac{(w_h - w)}{q(1 - \theta)}$$

When $s = 0$, the slope is

$$\frac{u_x(w_h - w)}{u_e - \theta q u_x},$$

which is first positive and greater than $\frac{(w_h - w)}{q(1-\theta)}$, and then becomes negative. For this case of $s = 0$, it is easy to see that since u is strictly quasi-concave, the indifference curves are concave in the (t, g) space. Finally, at the boundary point, when $\lambda = 0$, then $\frac{u_x(w_h-w)}{u_e-\theta q u_x} = \frac{(w_h-w)}{q(1-\theta)}$ and hence the indifference curves are differentiable.

It is also easy to see graphically from Figure II that if

$$\frac{(w_h - w)}{q(1-\theta)} > \frac{w}{q\theta} \Leftrightarrow \theta > \frac{w}{w_h}$$

then the ideal point is $(t = 0, g = 0)$. On the other hand, if this condition is not satisfied, then the ideal point is on the boundaries where

$$g = \frac{tw}{q\theta}$$

at a point where $s = 0$. Thus, equating the mrs with the economy budget constraint:

$$\frac{u_x(w_h - w)}{u_e - \theta q u_x}_{|g=\frac{tw}{q\theta}} = \frac{w}{q\theta}$$

which defines t^*. We now turn to the analysis of the young poor. This is similar to the analysis of the rich, where the solution for the optimal s is:

$$\begin{aligned} s &> 0 \text{ if } g \leq g'(t) \\ s &= 0 \text{ otherwise.} \end{aligned}$$

This implies a threshold function $g'(t)$ with a positive slope such that if g is below this threshold for t, then $s > 0$ whereas above it $s = 0$. The slope of the indifference curve is

$$\frac{dg}{dt} = \frac{-(w - w_l)}{q(1-\theta)}$$

when $s > 0$ (a negative slope), and

$$\frac{u_x(w_l - w)}{u_e - \theta q u_x}$$

when $s = 0$, which is negative and greater than $\frac{-(w-w_l)}{q(1-\theta)}$ (in absolute values) and then positive. It is easy to see graphically that the ideal policy must be on the budget constraint where $t = 1$ and $s = 0$. Hence, it is at the point g which is optimal given $t = 1$, denoted by $g^*(1)$.■

For convenience, I will henceforth use the indirect utility function

$v_{r_y}(g, t) = u(e(g, s^*(t, g, w_h)), x(t, g, w_h, s^*(t, g, w_h)))$ where $s^*(t, g, w_h)$ is the optimal choice of private education given government policies and income. v_{p_y} is analogously defined.

The following Lemma will prove useful later on:

LEMMA A0 When parties cannot form, i.e., in the partition $p_o|p_y|r_o|r_y$:

(i) When the old are a majority, there is a unique equilibrium in which p_o runs for election and wins with his ideal policy $(1, 0)$.

(ii) When the young are a majority, there is a unique equilibrium in which p_y runs for election and wins with his ideal policy $(1, g^*(1))$.

PROOF OF LEMMA A0. First it is easy to see why $p_o(p_y)$ is the unique equilibrium when the old (young) are the majority when one considers equilibria with one platform being offered. Consider for example the case when the old a majority. In this case no one can successfully challenge p_0. If the representative of the young poor challenges the old poor, he is defeated. This is because the old voters, who are the majority, would vote for the old poor since they prefer income redistribution to public provision of education. Moreover, if any of the rich representatives challenges the old poor they are defeated since all the poor vote together and the poor are the majority.[1] A similar reasoning implies that p_y is the unique winner when the young are a majority, in the class of one platform equilibria.

The possibility of two platforms being offered in equilibrium is ruled out by the tie breaking rule, since one platform will generically win. Now consider equilibria with 3 or 4 platforms being offered. In such a case, there exists at least one representative who offers a platforms and loses, and gains only the votes of his own group. In such a case, if he drops from the race he either does not affect the identity of the winner or he is affecting it by getting a better outcome since his voters who share with him the same preferences choose the platform which they like most from the set of the remaining platforms. Thus, such an equilibrium cannot arise.■

LEMMA 2 With endogenous parties:

(i) When $\theta < \frac{1}{2}$:

-The partition $r_o p_y|r_y|p_o$ is stable for all parameters with the party $r_o p_y$ winning with policies in its Pareto set which r_o and p_y prefer to $(1, 0)$. These policies have $g > 0$ and $t < 1$ and are worse for p_o than the ideal policy of p_y. If $\theta > \dfrac{w}{w_h}$, these policies are also preferred by p_o to $(0, 0)$.

-The partition $r_y p_y|r_o|p_o$ is stable for some parameters and more often when $\theta > \dfrac{w}{w_h}$, with $r_y p_y$ winning the election with policies in its Pareto set which r_o and p_y prefer to $(1, 0)$, and are thus characterized by $g > 0$ and $t < 1$. These policies have higher level of g than the winning policies of the party $r_o p_y$.

-The partition $r_o p_y r_y|p_o$ is stable when $\theta > \dfrac{w}{w_h}$, with the party $r_o p_y r_y$ winning with policies in its Pareto set which r_o and p_y prefer to $(1, 0)$, and thus satisfy $g > 0$ and $t < 1$. These policies are also worse for p_o than $(0, 0)$.

(ii) When $\theta > \frac{1}{2}$:

[1] In the case of $\theta > \frac{w}{w_h}$ all the poor - old and young - prefer some redistribution to none at all . In the case of $\theta < \frac{w}{w_h}$ still the poor stick together and the young prefer their fellow old poor to the young rich.

-The partition $r_y p_o | r_o | p_y$ is stable for all parameters with the party $r_y p_o$ winning with policies in its Pareto set which r_y and p_o prefer to the ideal policy of p_y. These policies have $g = 0$ and $0 < t < 1$.

-The partition $r_o p_o | r_y | p_y$ is stable for all parameters with the party $r_o p_o$ winning with policies in its Pareto set which r_y and p_o prefer to the ideal policy of p_y. These policies have $g = 0$ and $0 < t < 1$. If $\theta < \frac{w}{w_h}$, these policies are such that p_y prefers them to the ideal policy of r_y.

Proof of Lemma 2 To find the stable political outcomes, I use several steps. First we have to know which platforms each possible party can offer. In step 1, I therefore identify the Pareto set for each possible party, and prove some preliminary results about these Pareto sets. In step 2, I find the equilibrium winning platforms given each partition on the set of politicians. Finally, step 3 identifies the set of stable political outcomes.

Step 1: Characterization of the Pareto sets.

Given the preferences characterized in Lemma 1, we can now characterize the Pareto set of the different groups in society (this is most easily done using the indifference curves). Once we characterize the Pareto set of any two groups, the rest (i.e., the Pareto set of three groups) follows from the union of all bilateral Pareto sets.

The Pareto set of $r_o p_o$ is $\{t \in [0,1], g = 0\}$. The Pareto set of $p_o p_y$ is $\{t = 1, g \leq g^*(1)\}$. The Pareto set of $r_o p_y$ is as follows. First, note that the indifference curve of r_o is linear and that of p_y is linear for all $s > 0$. Then, if the slope of r_o is less steep (in absolute value) relative to that of p_y, i.e., if:

$$\frac{w_h - w}{q\theta} < \frac{w - w_l}{q(1-\theta)} \Leftrightarrow \frac{w_h - w}{w_h - w_l} < \theta,$$

then the Pareto set is

$$\{t \in [0,1], g = 0\} \cup \{t = 1, g \leq g^*(1)\}.$$

Otherwise there is a part of the Pareto set in which the indifference curves are tangent, when $s = 0$. That is, when:

$$-\frac{\partial v_{p_y}(t,g)}{\partial t} / \frac{\partial v_{p_y}(t,g)}{\partial g} = \frac{w - w_h}{\theta q}$$

This defines an increasing function $\tilde{g}(t)$. The 'interior' Pareto set is therefore

$$\{t \leq t', g = \frac{tw}{q\theta}\} \cup \{t \geq t', g = \tilde{g}(t)\} \text{ where } t' \text{ is defined by } \tilde{g}(t') = \frac{t'w}{q\theta}.$$

The Pareto set of $r_o r_y$ is simple to derive and is:

$$\begin{aligned} \{t &= 0, g = 0\} \text{ if } \theta > \frac{w}{w_h}, \\ \{t &\leq t^*, g = \frac{tw}{q\theta}\} \text{ otherwise.} \end{aligned}$$

Let us now analyze the Pareto set of $r_y p_y$. Let us denote by $g_h^*(t)$ and $g_l^*(t)$ the optimal provision of g given a fixed t, for the young rich and the young poor respectively. Obviously, $g_h^*(t) > g_l^*(t)$ since the rich have higher income. It is easy then to see graphically that this implies that the indifference curves of r_y and p_y can be tangent to one another only when the slope of each is positive.

When $\theta < \dfrac{w}{w_h}$ the Pareto set is fully characterized by the policies (t, g) which satisfy:

$$\frac{\partial v_{p_y}(t,g)}{\partial t} \Big/ \frac{\partial v_{p_y}(t,g)}{\partial g} = \frac{\partial v_{r_y}(t,g)}{\partial t} \Big/ \frac{\partial v_{r_y}(t,g)}{\partial g}.$$

This defines a function $g'(t)$. When $\theta < \frac{w}{w_h}$, let $t' = \max(t | s^*(t, g'(t), w_h) = 0)$ and let $g''(t)$ be defined by

$$-\frac{\partial v_{p_y}(t,g)}{\partial t} \Big/ \frac{\partial v_{p_y}(t,g)}{\partial g} = \frac{(w_h - w)}{q(1-\theta)}.$$

Let t'' be defined by $g''(t) = \frac{tw}{q\theta}$. The Pareto set is therefore

$$\begin{aligned} \{t &\geq t^*, g = g'(t)\} \text{ If } \theta < \frac{w}{w_h} \\ \{t &\in [0, t''], g = \frac{tw}{q\theta}\} \cup \{t \in [t'', t'], g = g''(t)\} \cup \{t \in [t', 1], g = g'(t)\} \text{ otherwise} \end{aligned}$$

Finally, the Pareto set of $r_y p_o$ is as follows. If $\theta > \dfrac{w}{w_h}$ then it is trivially on the $g = 0$ line. If $\theta < \dfrac{w}{w_h}$, there are two possibilities. Either it is on the boundaries of the policy space, which is the case when the slope of the indifference curve of p_o is less steep than that of r_y, or that it is interior:

$$\begin{aligned} \{g &= 0, t \in [0,1]\} \text{ if } \theta > \frac{w}{w_h}, \\ \{g &= 0, t \in [0,1]\} \cup \{t \leq t^*, g = \frac{tw}{q\theta}\} \text{ if } \frac{w - w_l}{w_h - w_l} < \theta < \frac{w}{w_h} \\ \{g &= \bar{g}(t), t \in [t^*, 1]\} \cup \{t = 1, g \leq \bar{g}(1)\} \text{ otherwise,} \end{aligned}$$

where $\bar{g}(t)$ is defined by

$$\frac{\partial v_{r_y}(t,g)}{\partial t} \Big/ \frac{\partial v_{r_y}(t,g)}{\partial g} = \frac{w - w_l}{q\theta}.$$

This is the end of step 1. I now prove some results about the Pareto sets.

LEMMA A1. When $\theta < \frac{1}{2}$, the Pareto set of $r_o p_y$ is interior.

Proof: To see this note that:

$$\frac{1}{2} < \frac{w_h - w}{w_h - w_l}$$

because

$$2\pi w_l + 2(1-\pi)w_h - w_l < w_h \Leftrightarrow w_l < w_h$$

and therefore when $\theta < \frac{1}{2}$ it is also the case that $\theta < \dfrac{w_h - w}{w_h - w_l}$ and the Pareto set of $r_o p_y$ is interior.□

LEMMA A2. When $\theta > \frac{1}{2}$, the Pareto set of $r_y p_o$ is on the boundaries.

Proof: To see this note that

$$\frac{1}{2} > \frac{w - w_l}{w_h - w_l}$$

and hence when $\theta > \frac{1}{2}$, the Pareto set of $r_y p_o$ is on the boundaries.□

LEMMA A3. The Pareto set of $r_y p_y$ is above that of $r_o p_y$ when the latter is interior.

Proof: As established, the Pareto set is the tangency of the indifference curves of r_y and p_y when the slope of the indifference curve of p_y is positive, whereas the Pareto set of $r_o p_y$ has the tangency when the slope is negative (since the slope of r_o is always negative). This implies the above.□

LEMMA A4. When $\dfrac{w}{w_h} < \theta < \frac{1}{2}$, the indifference curves of r_y cross those of p_o only once.

Proof: The slope of the indifference curve of p_o is $\dfrac{w - w_l}{q\theta} < \dfrac{w}{q\theta}$ whereas the slope of the linear part of the indifference curve of r_y is $\dfrac{(w_h - w)}{q(1-\theta)} > \dfrac{w}{q\theta}$. This establishes the result.□

LEMMA A5. When $\theta < \frac{1}{2}$, the Pareto set policies of $r_o p_y$ are worse to p_o relative to that of p_y.

Proof: Suppose not. Consider the indifference curve of p_o that goes through p_y. If some policies on the Pareto set of $r_o p_y$ are better for p_o than the ideal policy of p_y, and given that p_o prefers p_y to r_o, then the Pareto set of $r_o p_y$ must cross this particular indifference curve at least twice. Denote these policies by (t_1, g_1) and (t_2, g_2). But on this indifference curve, the income of the poor, $w_l + I(t, g)$, is fixed. This means that there are two points on the Pareto set of $r_o p_y$ with $I(t_1, g_1) = I(t_2, g_2) \equiv I^*$ and $g_2 > g_1$. These points are on the Pareto set and hence the slope of the indifference curves of p_y that go through these points must be the same for both; by strict concavity however this cannot be the case for different levels of g and fixed income.[2]□

Remark 1 Note that all the results so far hold for all q. It is therefore without loss of generality to assume $q = 1$.

Remark 2 For brevity, steps 2 and 3 are provided below for the case of $\dfrac{w}{w_h} < \theta < \frac{1}{2}$. The proof is similar for the other cases and is available upon request (see also Levy (2004)).

Step 2: Characterization of equilibria for each partition.

[2] If the second point is the corner point of $(1, g^*(1))$ then one can find a point on the Pareto set which is in an ε-ball around $(1, g^*(1))$ and therefore needs to have a fixed slope.

We can now characterize the equilibrium for each partition on the set of politicians. Recall that we focus on the case of $\frac{w}{w_h} < \theta < \frac{1}{2}$, i.e., when the young are a minority and when r_y has the ideal policy $(0,0)$. As shown in Lemma A0, when the partition is $r_o|p_o|r_y|p_y$, then the winner of the election is p_o. We can now analyze other types of partitions:

One party, two members:

In the partition $r_o r_y|p_o|p_y$ it is the case that p_o wins alone since neither player's Pareto set has changed compared to the partition without parties. In the partition $p_o r_o|r_y|p_y$, in equilibria with only one platform the 'old' party wins with all policies better for p_o than p_y (and similarly when the party is $r_y p_o$). To see why others cannot run, note that if p_y runs for election then the party can deviate and run as well and improve the utility of the old, rich and poor, even by offering the ideal policy of p_o. If r_y runs for election, then p_y can run against him and win the votes of p_o. Equilibria with two platforms don't exist by the tie breaking rule. In equilibria in which there are three platforms, then if $p_o r_o$ offer the ideal policy of r_o, then r_y can drop. If not, then r_o votes for r_y. In such a case, if p_y wins then if the old party instead does not offer any platform this would not change the result and hence they can drop. If on the other hand r_y or $p_o r_o$ win, then it means that p_o are not voting with p_y and hence he can drop from the race implying that this is not an equilibrium.

In the partition $p_o p_y|r_o|r_y$, in a one platform equilibrium the poor party wins with all their policies since each of their members prefers this to the ideal policy of the rich. A two platform equilibrium does not exist and in a three-way race equilibrium the poor party must win and hence the others can drop from the race.

I now focus on the partition $r_o p_y|p_o|r_y$. By lemma A1, there are some policies in the Pareto set of $r_o p_y$ such that both r_o and p_y prefer them to p_o. By Assumption 1 it is also the case that some of these policies are such that p_o prefers them to r_y. In an equilibrium of $r_o p_y|p_o|r_y$ with one platform, the party wins therefore the election with the subset of policies described in Levy (2005) $-$ those with relatively high tax rates. Clearly nothing else can be a one-platform equilibrium since the party can win against p_o and p_o can win against r_o. In a three-way race, note that r_o votes with r_y. If $r_o p_y$ or r_y win it means that p_y is not voting with p_o and hence he can drop. If p_o wins then $r_o p_y$ can drop.

Finally, in the partition $r_y p_y|p_o|r_o$ there are two cases of pure strategy equilibrium. If the Pareto set of $r_y p_y$ has no policies that both p_y and r_o prefer to p_o, then p_o wins the election, since no one can contest him successfully. Otherwise, the party can win, in particular with a subset of these policies that p_o prefer to r_o. Again, in a three-way race if p_o does not win he can drop and alternatively if he wins, the party can drop implying that this is not an equilibrium.

Two parties, two members each:

In the partition $r_o r_y|p_o p_y$ the poor always win with all their policies. Consider the partition $r_o p_y|r_y p_o$: for any policy in the Pareto set of $r_o p_y$, then there is a policy in the Pareto set of $r_y p_0$ with $g = 0$ that wins against it (attracts all the rich and p_o). On the other hand, for any such policy with $t > 0$ and $g = 0$, there is a policy in the Pareto set of $r_o p_y$ which can win against it by lemma A1. Thus, the unique equilibrium is that one of the parties offers $g = 0, t = 0$. Finally, in the partition $r_o p_o | r_y p_y$ the party of the old must win. In particular, $g = 0, t = 0$ is a pure strategy equilibrium.

One party, three members:

In the partition $r_o r_y p_y | p_o$, the party wins with policies that are better for p_y and r_o relative to p_o. In the partition $r_o | r_y p_o p_y$, either r_o wins, or the party wins with the policies that are better for p_o than r_o. Finally, in the partition $r_y | r_o p_o p_y$, again r_y can win or the party can win with all policies that are better for p_o than the ideal policy of the rich.

Step 3: Stable political outcomes.

Any party structure with one party and two members such that p_o is one of them is not stable because p_o will break to win alone. The partition in which the only party is $r_o r_y$ is not stable as well because in this partition p_o wins anyhow. Consider now the party $r_o p_y$. If these party members break they get p_o which is worse for both. It is therefore stable. Similarly, if $r_y p_y$ win the election they do not break and otherwise they do.

Consider now two parties, each with two members. The partition $r_o p_o | r_y p_y$ is not stable because p_y can break and get at least the utility from $(1, 0)$ in the equilibrium of $r_o | p_o | r_y p_y$. If it is $r_o p_y | r_y p_o$ it is not stable because p_o will break. In this partition the equilibrium is the ideal policy of r_o whereas if he breaks he gets something which is better than $(0, 0)$. Also $r_y p_y | r_o p_o$ is not stable because in this partition the party of the old wins so p_y can break and get p_o (which is weakly better relative to all other outcomes).

Consider one party with three members. If p_o is together with the rich, it cannot be stable since he breaks to win alone. If $r_o p_o p_y$ or $r_y p_o p_y$ are together or in the grand coalition, then the poor can always weakly improve by deviating together.

Finally, the partition $r_y r_o p_y | p_o$ can be stable. Any equilibrium outcome that can be achieved by $r_y p_y$ or by $r_o p_y$ is not stable since politicians prefer to win in smaller parties. However, consider outcomes that are not achievable by $r_o p_y$, that is, they are better than p_o for both p_y and r_o but worse for p_o than $(0, 0)$. If p_y deviates alone he is worse off since then the outcome is p_o. But also neither r_o nor r_y can profitably deviate alone or with p_y so than these outcomes are stable. If r_o deviates alone then in the partition $r_y p_y | p_o | r_o$ it is a worse outcome for him; either p_o wins or the party wins. When the party $r_y p_y$ wins, it has to win with policies which are better for p_o than r_o. Such a Pareto improvement for p_o must be damaging the utility of r_o then. Also, r_o cannot deviate together with p_y because the current outcome is on their Pareto set. Consider now r_y. A deviation with p_y implies policies which are better for p_o than the current policies (since they are better for p_o than r_o). By the single crossing property identified in Lemma A4, and the fact that the Pareto set of r_y and p_o has policies with $g = 0$, this must be damaging the utility of r_y. The same argument holds for a deviation alone, which implies that policies of $r_o p_y$ are implemented. Thus, the party $r_y r_o p_y$ is stable with policies on the Pareto set of $r_o p_y$, that are better than

p_o for both p_y and r_o but worse for p_o than $(0,0)$.

In a similar way, it is possible to show that this party may for some parameters, be stable when implementing policies in the Pareto set of $r_y p_y$ which are better for all others than p_o but worse for p_o than $(0,0)$.

Finally, note that by Lemmata A3 and A5, all the stable political outcomes provides p_o with a lower utility than the ideal policy of p_y.

This completes the proof of Lemma 2.■

PROPOSITION 1 With endogenous political parties,

(i) When the young are a minority, per capita public provision of education is strictly higher and per capita net income transfer is strictly lower than when the young are a majority.

(ii) All stable political outcomes are characterized by a positive but not a maximum tax rate.

(iii) When the young are a minority, any winning party is composed of the young poor and some rich representatives (young or old). When the young are a majority, then any winning party is composed of the old poor and some rich representatives.

PROOF OF PROPOSITION 1 Lemma 2 establishes that $g = 0$ and $0 < t < 1$ when $\theta > \frac{1}{2}$ and $g > 0$ and $t < 1$ otherwise. The proof also establishes that when $\theta > \frac{1}{2}$, the stable political outcomes provide p_o with a lower utility and hence lower income, than the ideal policy of p_y. On the other hand, when $\theta < \frac{1}{2}$, all the winning policies provide p_o with higher utility and hence higher income than the ideal policy of p_y. Thus, p_o has higher income under $\theta > \frac{1}{2}$. Since income for the poor has a one-to-one mapping with I, it implies that I is higher when $\theta > \frac{1}{2}$ than when $\theta < \frac{1}{2}$. The rest follows from Lemma 2.■

PROPOSITION 2 The rich and the poor are more likely to be equally educated when the young voters are a minority.

PROOF OF PROPOSITION 2 By Lemma 2, then whenever $\theta > \frac{1}{2}$, then $g = 0$ and hence both the rich and the poor buy education privately which implies that for all parameters, the rich are more educated than the poor because $t < 1$ and the hence the rich have a higher income. This implies that the optimal s^* is higher for the rich. On the other hand, when $\theta < \frac{1}{2}$, then $g > 0$ and clearly for some parameters, $s = 0$ for both the rich and the poor. In particular, since the Pareto set of $r_o p_y$ is 'interior' as in Figure IIIa in Levy (2005), then $s = 0$ for the poor and for some parameters also $s = 0$ for the rich. Hence, the rich and the poor can be equally educated.■

PROPOSITION 3 When the old are a majority, higher income inequality may increase tax rates and the level of public education. When the young are a majority, higher income inequality may decrease tax rates.

PROOF OF PROPOSITION 3 When $\theta < \frac{1}{2}$, all policies in the Pareto set of $r_o p_y$ which are better for both factions than $(1,0)$ can be stable when $\theta < \dfrac{w}{w_h}$ whereas the party can implement only a subset of these policies, the ones preferred by p_o to $(0,0)$, when $\theta > \dfrac{w}{w_h}$. This is because when $\theta < \dfrac{w}{w_h}$, the party faces no competition from r_y, whose ideal policy cannot attract p_o. This implies that policies are characterized by higher tax rates and higher income transfers when income inequality is high.

In addition, when $\theta < \frac{1}{2}$, the party $r_y p_y$ or $r_y p_o r_o$ can implement policies in the Pareto set of $r_y p_y$ that are better for r_y, p_o and r_o relative to $(1,0)$. As the Pareto set of $r_y p_y$ lies above that of $r_o p_y$ (see Lemma A3), then such policies prescribe higher levels of public education. However, these parties are more likely to win with these policies when $\theta > \dfrac{w}{w_h}$, since in this case it is more likely that the Pareto set of $r_y p_y$ would include policies that all groups prefer to $(0,0)$.

When $\theta > \frac{1}{2}$, then when $\theta < \dfrac{w}{w_h}$, the stable policies of $r_o p_o$ have to satisfy that p_y prefers them to r_y and this results in higher tax rates compared with the case in which $\theta > \dfrac{w}{w_h}$ and the party's policies do not have to satisfy this constraint.■

References

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