

# Multidimensional Cheap Talk

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## Abstract

In this paper we extend the cheap talk model of Crawford and Sobel (1982) to a multidimensional state space. We provide a characterization of equilibria. Most importantly, we prove that influential equilibria are non-generic when the conflict between the sender and the receiver is too large. Thus, adding more dimensions cannot improve upon information revelation when interests are too divergent.

## 1 Introduction

The seminal work of Crawford and Sobel (1982) on cheap talk, has paved the way for a vast literature applying cheap talk models to various contexts. Cheap talk is, for example, the model for explaining lobbying behavior; lobbies and interest groups are considered to exert political influence merely by convincing politicians, through casual communication, to take the ‘right’ policies.<sup>1</sup> Research in political economy also uses cheap talk to explain the structure of committees in the Congress and the constitutional rules which dictate their activities.<sup>2</sup> In financial economics, cheap talk models explain the behavior of experts in financial markets and in particular the phenomenon of herding.<sup>3</sup> Others explain social behavior such as political correctness.<sup>4</sup>

The work of Crawford and Sobel (1982), however, as most of the applications of cheap talk models, focuses on a policy space of one dimension. In this paper we extend the basic

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<sup>1</sup>On this line of research, see the survey in Grossman and Helpman (2001).

<sup>2</sup>See for example Austen-Smith (1990) and Gilligan and Krehbiel (1987),(1989).

<sup>3</sup>In this literature, investment decisions are cheap talk messages about experts’ abilities, which allow them to acquire good reputation. See for example Trueman (1994) and Levy (2003).

<sup>4</sup>Morris (2001).

cheap talk model to a multidimensional environment. That is, we analyze a cheap talk game between one sender (the informed type) and one receiver (who takes an action following the communication with the sender), when the relevant information as well as the conflict between the sender and the receiver is multidimensional. We characterize the nature of equilibria and focus on the question of feasibility of information transmission, when the conflict of interests between the sender and the receiver is very large.

To be more specific, there are two players in the model, a sender and a receiver. The receiver has to choose a policy in a multidimensional policy space. The appropriate choice of policy depends on the realization of a (multidimensional) state of the world. The receiver initially has a prior distribution on the state of the world. The sender on the other hand is informed about the state, and transmits a message about it to the receiver before the receiver takes her action. The utility of both the sender and the receiver depend on the action and on the state of the world.<sup>5</sup> But they differ in their optimal choice of action given the state of the world, i.e., there is a conflict of interests. We then analyze the scope for information transmission depending on the magnitude of the conflict between the players.

The insight in the unidimensional analysis of Crawford and Sobel (1982) is that information transmission is not feasible when interests are sufficiently divergent. When the conflict between the players is too large, the sender would always prefer the receiver to take the most extreme action. Thus, no matter what is the real state of the world, he would always send the same message - the one which induces the receiver to take the most extreme action with the highest probability. This means that this message, or any other, cannot be informative about the type of the sender.

In the multidimensional world, however, there is some intuition which points otherwise. Some ‘bundling’ of the dimensions may occur in equilibrium, even when the conflict is very large. For example, if the sender prefers a higher action than the receiver on each dimension, he may use in equilibrium messages such as ‘ $x$  is low but  $y$  is high’. The message gains credibility because it is unfavorable for the sender on the  $x$  dimension. It is nonetheless worthwhile for the sender to transmit it, since he gets his way on the  $y$  dimension and, given his type, it is better for him than sending the other equilibrium message that translates as ‘ $y$  is low and  $x$  is high’.

To be phrased more precisely, this intuition relies on the idea that when there are many dimensions, we can always span the space by a dimension (vector) of the conflict, and a dimension on which the sender and the receiver agree on. On this latter dimension, the interests of the sender and the receiver are aligned. The sender should be willing to

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<sup>5</sup>We focus on the case in which the utilities are functions of the Euclidean distances of the state from the actual action on each dimension. This is the common assumption in the literature; we take this assumption in order to give the model its best change of yielding applicable results.

transmit any information on this dimension, disregarding the magnitude of the conflict. It may even be possible for him to transmit all the information on this dimension, implying an equilibrium with infinitely many credible messages.<sup>6</sup>

The following graph in the two-dimensional policy space is helpful for understanding the intuition for the possible existence of an equilibrium with infinitely many messages. Let us focus on Euclidean preferences, i.e., the utility of a player decreases with the distance from his ideal policy. The receiver's ideal policy is the true state of the world whereas the sender's ideal policy is removed from it by the vector  $b$ , for any state of the world. The graph depicts the vector of conflict,  $b$ . The orthogonal line to the vector  $b$  is the dimension on which the sender and the receiver 'agree on', the line  $BB$ . That is, if a sender could 'choose' an action on the line  $BB$ , he would choose the 'most truthful' action; all senders who know that the state of the world is on some line  $AA$ , parallel to the vector  $b$ , would prefer action  $a$  among the points on  $BB$ . Their ideal policy, that is the state of the world removed by the vector  $b$ , is also on the line  $AA$  or on its continuation, and the point  $a$  is closest to them on the line  $BB$ . This is true for any magnitude of the vector  $b$ , and in particular for very large conflicts in which the vector  $b$  could be imagined as stretching to infinity. Similarly, all senders on the line  $A'A'$  would rather choose  $a'$  (ignore  $a''$  for the time being):

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<sup>6</sup>The intuition relies on works by Austen-Smith (1993a) and more recently, Battaglini (2002). These papers are the exception in the literature, since they analyze a multidimensional policy space. However, they analyze models with many informed senders whereas our model extends the basic cheap talk model, with one sender, to a multidimensional environment. We discuss the related literature, and in particular Battaglini (2002), at the end of this section. The intuition about multidimensional analysis is also indicated in Spector (2000). His model is very different from the rest of the literature as he analyzes a repeated game with continuum of agents, divergent priors and discrete policy space.

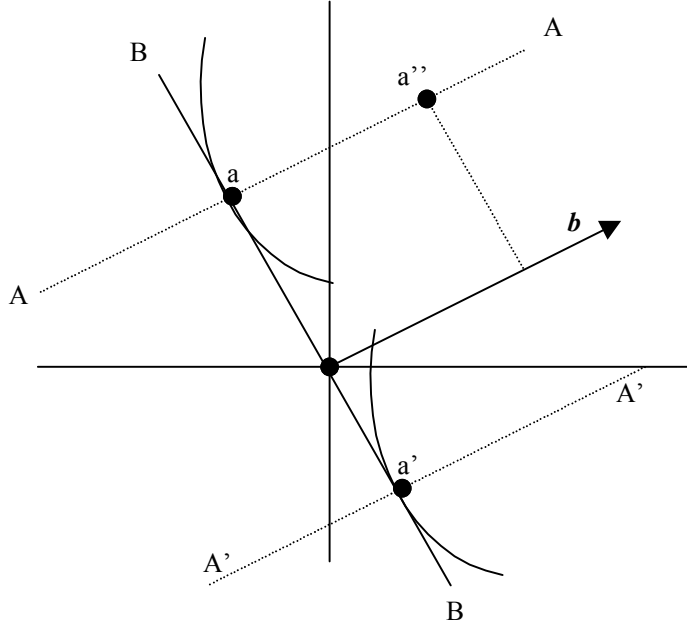


Figure 1: All senders on  $AA$  prefer  $a$  on the line  $BB$  whereas all senders on  $A'A'$  choose  $a'$  on  $BB$ .

Thus, it seems that in equilibrium, the sender may be willing to transmit all information on the line  $BB$ , i.e., that infinitely many credible messages can be transmitted in equilibrium. A closer look at this suggested reporting strategy reveals a problem however in sustaining it in equilibrium. If the receiver observes a message advocating  $a$ , she understands that the real state of the world is somewhere on the  $AA$  line. She then updates her beliefs about the state of the world. These beliefs are surely on the line  $AA$  since these are expectations over this set of states. Her beliefs, however, do not necessarily coincide with the point  $a$ . Suppose that the receiver's expectations over the set of states in  $AA$  coincide with some point  $a''$  on the line  $AA$  and  $a'$  on the line  $A'A'$ .

But then an equilibrium cannot exist; when the conflict is very large, as we show in the paper, the vector of the conflict becomes the *most* important dimension for the sender. This dimension looms large and any two actions whose coordinates differ in the dimension of the conflict,  $b$ , cannot be equilibrium actions. For large conflicts, as in the unidimensional world, the sender would always choose to induce the most extreme action *on the dimension of the conflict*. In the above case, senders both on  $AA$  and on  $A'A'$  would prefer to induce  $a''$ .<sup>7</sup>

In other words, there is indeed a dimension on which the sender and the receiver agree on; but the sender would transmit information on this dimension only under the condition

<sup>7</sup>The coordinate of  $a'$  on the vector  $b$  is at the origin and that of  $a''$  is to the north-east of the origin.

that this information would not affect the action of the receiver on the dimension of the conflict. Generically however, these two dimensions will not be independently distributed. Information about the dimension on which the sender and the receiver agree on, would imply how the state of the world is likely to be distributed on the dimension of the conflict. Thus, the receiver would change his choice on the dimension of the conflict upon observing information on other dimensions and as a result, the sender cannot transmit infinitely many messages in equilibrium.<sup>8</sup>

An equilibrium with infinitely many messages may not exist, but this may be a too ambitious requirement for very large conflicts. It may be the case that an equilibrium with few messages may hold, such as the ‘bundling’ equilibrium described above, with the sender just stating whether ‘ $x$  is low and  $y$  is high’ or the other way around. As we show, however, when the conflict is large enough, the existence of equilibria even with a finite or small number of informative messages are non-generic.

The results are therefore as follows. We first characterize the set of equilibria for one sender in the multidimensional state space. This characterization illustrates that there is a genuine difference between a model with one dimension of conflict and more than one dimension. When the conflict is small, informative equilibria exist and can be characterized by partitions of the space, as in the unidimensional policy space. However, as opposed to the equilibria in the unidimensional model, the elements of the partition are of different shapes, and the partition depends on the shape of the original state space. Moreover, whereas in the unidimensional policy space the existence of an equilibrium with  $n$  informative messages implies the existence of an equilibrium with  $n - k$  informative messages for some integer  $k \in (0, n)$ , this is not true any more in the multidimensional policy space.

We then focus our analysis on very large conflicts. We show that in the limit, when the level of the conflict converges to infinity, the conditions for influential equilibria, i.e., equilibria in which the receiver may take different actions, are too stringent. When the conflict becomes very large, it implies that the distance between typical actions that the receiver may take and the sender’s ideal point, increases. In this case, we show that one dimension of the policy space becomes more and more important. In the limit as this distance goes to infinity, the preferences of the sender become lexicographic; first, alternatives are compared only on this important dimension. If that comparison is inconclusive, a comparison along a different dimension determines the preferences.

This allows us to show that influential equilibria are non-generic for large conflicts. First, we find conditions on the parameters of the model (when essentially the ‘important’ dimensions described above are not distributed independently), so that there is no informative

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<sup>8</sup>This indicates that commitment on behalf of the receiver would have facilitated information transmission. On the issue of commitment in the unidimensional state space, see Dessein (2002).

equilibrium when the conflict is too large. For example, this is the case when the state space is a circle, the prior distribution is uniform, and the sender values each dimension differently. Or, this is the case when the state space is a square, the prior distribution is uniform, the conflict on each dimension is of the same degree, but the sender values each dimension differently.

For all other parameters, we show that even when the conditions for equilibria are satisfied in the limit, then influential equilibria are non-generic, that is, for any equilibrium there is a set of perturbations that upsets the equilibrium. We then establish the continuity of the problem, so that if influential equilibria do not exist in the limit, indeed there exist large enough degrees of the conflict so that influential equilibria cannot be sustained for these parameters as well.

Other features of the work of Crawford and Sobel (1982) have been previously extended, mainly with the purpose of retrieving equilibria with information transmission when the conflict of interests is relatively large. Krishna and Morgan (2001) analyze the case of two senders and one dimension of conflict. They show that more information can be revealed when a receiver communicates with two senders instead of one, as long as the senders' interests are biased in opposite directions. Farrell and Gibbons (1989) show that the existence of two receivers may facilitate information transmission.<sup>9</sup> Aumann and Hart (2003) and Krishna and Morgan (2002) analyze models in which the receiver is also allowed to communicate, even though she is not informed. Her messages allow the players to conduct joint lotteries. These increase the degree of information transmitted, but still cannot induce informative equilibria when the interests are too divergent.

The most related to our paper are works of Battaglini (2002) and of Chakraborty and Harbaugh (2003). Battaglini (2003) also analyzes a model with a multidimensional state space. In his model, there are two senders who both know the state of the world. He shows that in this case all information can be revealed in equilibrium, disregarding the magnitude of the conflict of interests. In the fully revealing equilibrium, the state space is spanned by two vectors; each of these vectors is the dimension on which the receiver has no conflict with one of the senders. Each sender then truthfully tells the receiver the coordinate of the state of the world on this dimension. This information revelation is feasible in a model with two senders, because one sender's message on a particular dimension indeed does not change the action of the receiver on other dimensions, and in particular on the dimension of the conflict. This is because the receiver 'already' knows the exact information on the dimension of the conflict, extracted from the report of the other sender. Our work stresses therefore that such a result cannot in general hold when there is only one sender. In particular, it is not only that full information transmission is not feasible, but that generically, equilibria

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<sup>9</sup>For an application of this idea to international relations see Levy and Razin (2003).

with any information transmission are not robust.

Chakraborty and Harbaugh (2003) study the conditions on utility functions which allow the existence of the two actions ‘bundling’ equilibrium described above, for all degrees of conflict. Our analysis shows that for a subset of the utility functions they identify, that is, the ones that we analyze in this paper, this equilibrium is non generic.

The rest of the paper is organized as follows. In the next section we present the model. In section 3 we characterize the equilibria of the model, as well as illustrate how these differ from the unidimensional model of Crawford and Sobel (1982). Section 4 analyzes equilibria when there is a high degree of conflict between the sender and the receiver. In this section we present Theorem 1, our main result about the non-genericity of influential equilibria. Section 5 concludes by discussing some extensions, notably to the case of an informed receiver and many senders.

## 2 The model

An individual (the receiver) has to choose a policy in a multi-dimensional policy space,  $\mathfrak{R}^d$ . The appropriate choice of policy depends on the realization of a state of the world  $\theta$ , in a compact and convex subset of  $\mathfrak{R}^d$  denoted by  $\Theta$ . The receiver initially holds an atomless and continuous prior distribution on the states in  $\Theta$  denoted by  $F$  with a strictly positive density function  $f$  on  $\Theta$  and expectation at the origin. Assume that the receiver always chooses policy at the expectation of  $\theta$ ,  $E(\theta)$ , according to his posterior.<sup>10</sup>

A sender, who is fully informed about  $\theta$ , has preferences over the actions of the receiver that are represented by his ideal point  $b = (b_1, b_2, \dots, b_d)$  and by a strictly decreasing utility function,  $U(a|\theta) = v(\Delta_\alpha(a, b|\theta))$ , defined over

$$\Delta_\alpha(a, b|\theta) = \sum_{i=1}^d \alpha_i (a_i - (b_i + \theta_i))^2.$$

Note that the vector  $\alpha$  (without loss of generality we assume that all its elements are strictly positive and sum up to one) denotes the relative importance of the different dimensions in the preferences of the sender.

The game that we analyze takes the following form: The sender observes the state of the world,  $\theta$ . He then chooses a message  $m$  in a set of messages  $M = \Theta$ . We analyze (weak) Perfect Bayesian equilibria of this game.

In what follows we focus our analysis on the case of  $d = 2$ . All our results carry over to the case in which  $d > 2$ .

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<sup>10</sup>The assumption that the receiver chooses policy at the expectation is taken for simplicity. Moreover, it is consistent with the receiver maximizing a utility function that is quadratic in the distance of policy from the origin.

### 3 Characterization of equilibria

In this section we characterize the equilibria in the game. We discuss the relation between the Crawford and Sobel (1982) model (henceforth CS) of one-dimensional cheap talk and our multidimensional model. As in CS, we show in Proposition 1 that any equilibrium is almost surely equivalent in terms of outcomes to an equilibrium in ‘partition form’. In such equilibria, the set of types is partitioned into convex sets and all agents in each element of the partition induce, with probability one, the same action.

Unlike the CS model, the partition form of equilibria is not very useful for deriving other applicable results. We illustrate the difference between the one dimensional and the multidimensional models. First, we show that there is no consistency in the shapes of the elements of typical equilibrium partitions. Second, we show that the set  $\Theta$  and in particular its shape plays an important role in determining the set of equilibria. These two observations imply that generally it is difficult to use algorithms to find equilibria, as opposed to the CS equilibria.

Finally, in contrast to the CS model, we illustrate that the non-existence of an equilibrium with  $k$  induced actions, does not imply the non-existence of equilibria with more than  $k$  induced actions. The implication of this observation is that in the multidimensional model it is more difficult to characterize the ‘most informative’ equilibrium.

**Strategies and equilibrium** A strategy of player type  $\theta$  is a probability distribution,  $m_\theta$  over the set of messages  $M$ . For any action  $m \in M$ , let  $a(m)$  denote the action chosen by the receiver.

An equilibrium is a pair of a strategy function  $m : \Theta \rightarrow \Delta(M)$  (denoted by  $m_\theta$ ) for the sender and a belief function for the receiver,  $f(\theta|m)$ , satisfying:

$$(1) \forall \theta \text{ and any } m', m'' \in M \text{ such that } m_\theta(m'), m_\theta(m'') > 0,$$

$$U(a(m')|\theta) = U(a(m'')|\theta) = \max_{m \in M} U(a(m)|\theta)$$

$$(2)$$

$$a(m) = E[\theta|m] = \int_{\Theta} \theta \cdot f(\theta|m) d\theta.$$

$$(3) f(\theta|m) \text{ is updated using Bayes rule whenever possible.}$$

**Remark 1** *An equilibrium always exists (babbling).*

**PROPOSITION 1** *Any equilibrium is almost surely equivalent in outcomes to an equilibrium in which the type space is partitioned into convex sets. All types of senders in the same set induce the same action, and each type induces it with probability one.*

Proposition 1 shows an analogy with the unidimensional model of CS. The rest of this section is devoted to some observations on the equilibria of the game. In particular, using examples, we show how the form of equilibria, although reminiscent of the unidimensional partitions equilibria of the CS model, has different features.

**Comparison with Crawford and Sobel (1982)** Recall that the main features of the equilibria in the unidimensional model of CS are as follows. In any equilibrium, the elements of the partition are intervals of straight lines. Such an informative equilibrium exists if the conflict is small enough, for any ‘shape’ (i.e., length) of the unidimensional state space. Also, an equilibrium with  $k$  induced actions implies the existence of an equilibrium with fewer than  $k$  messages, for any unidimensional state space. As we now show, none of this holds in the multidimensional model.

**OBSERVATION I** *In a typical equilibrium the elements of the partition of the state space have different shapes.*

We illustrate this observation by an example of an equilibrium with two induced actions, for the state space  $\Theta = [0, 1] \times [0, 1]$ . We assume that the prior distribution over the state space (the square) is uniform. In the equilibrium, the square is divided to two groups of senders, each of them sending a different message. We have computed a particular equilibrium for  $(b_1, b_2) = (.4, .3), (\alpha_1, \alpha_2) = (.5, .5)$ . In this equilibrium, all types  $(\theta_1, \theta_2)$  below the line

$$\theta_2 = \lambda\theta_1 + \gamma$$

for  $\{\gamma = -.2705, \lambda = 1.0325\}$ , send the same message and induce the action  $a = (.754, .254)$ , whereas all types  $(\theta_1, \theta_2)$  above this line send a different message and induce the action  $a' = (.4, .596)$ . The equilibrium is depicted in Figure 2:

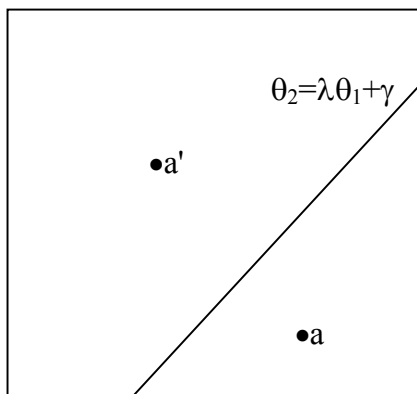


Figure 2: An equilibrium in the square, with two actions.

The actions are the expectations over the set of states represented by each group of senders, whereas it can be easily verified that the dividing line consists of the types who are indifferent between the two actions. This implies that the strategy described constitutes a best response for each sender. Moreover, this equilibrium is robust to small changes in the parameters  $b$  and  $\alpha$ , or in the prior distribution.

As can be seen in this equilibrium, the elements of the partitions differ; the two subsets do not have the same shape. This is in contrast to the unidimensional analysis in which all elements of the partitions are straight lines which differ only in their length.

**Remark 2** *This example also illustrates what we term the ‘bundling’ effect. On each of the dimensions, the conflict has  $b_i \geq \frac{1}{4}$  which implies that a cheap talk game on each dimension separately results in babbling only and no credible information can be transmitted. However, information transmission is feasible once the two dimensions are bundled. In particular, the sender who always wants a higher action than the receiver would take, trades-off unfavourable information on one dimension (admitting it has a low value) with favourable information on the other dimension (that it has a relatively high value).*

OBSERVATION II *The ‘shape’ of  $\Theta$  may determine the characteristics of equilibria.*

To illustrate how the shape of  $\Theta$  may matter, consider now a state space in the form of a circle, i.e.,  $\Theta = \{\theta_1, \theta_2 | \theta_1^2 + \theta_2^2 \leq r\}$  and assume that the prior distribution over the states is uniform. We show that for all conflicts  $(b_1, b_2)$ , generically, there is no equilibrium with two induced actions.

If such an equilibrium exists, by Proposition 1, it must have the following form. There are two convex and closed subsets of senders. Each subset induces a different action. These actions,  $a = (a_1, a_2)$  and  $a' = (a'_1, a'_2)$  are expectations over the two subsets. The two subsets are separated by the set of types who are indifferent between the two actions; in particular, the types of senders  $\theta = (\theta_1, \theta_2)$  who are indifferent between the two actions form a straight line, defined by the following equation:<sup>11</sup>

$$\theta_2 = \theta_1 \frac{\alpha_1(a_1 - a'_1)}{\alpha_2(a'_2 - a_2)} - \frac{\alpha_1(a_1^2 - a'^2_1)}{2\alpha_2(a'_2 - a_2)} + \frac{\alpha_1}{\alpha_2} b_1 \frac{(a_1 - a'_1)}{(a'_2 - a_2)} + \frac{(a'_2 + a_2)}{2} - b_2$$

However, given the straight separating line and the uniform distribution, the expectations over each subset must be on a line which is orthogonal to the line of the indifferent types. This implies that the following has to hold:

$$\frac{\alpha_1(a_1 - a'_1)}{\alpha_2(a'_2 - a_2)} = -\frac{1}{\frac{a'_2 - a_2}{a'_1 - a_1}}$$

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<sup>11</sup>This follows from (weighted) Euclidean preferences. The formal derivation of this equation is in the proof of Proposition 2.

where the left-hand-side is the slope of the line of indifferent types as defined in the previous equation, and the right-hand-side is the slope orthogonal to that between the two induced actions. As seen from this condition, this equation can hold iff  $\alpha_1 = \alpha_2$ . Thus, equilibria with two induced actions generically do not exist in the circle when the prior distribution is uniform.

However, in observation I, we show that if the state space is a square, equilibria with two induced actions hold and are generic. Equilibria are therefore shape-dependent. In particular, the state space can be detrimental regarding whether information revelation exists at all. Again, this marks a difference between the multidimensional and a unidimensional game.

**OBSERVATION III** *If an equilibrium with  $k$  induced actions does not exist, it does not imply that an equilibrium with more than  $k$  induced actions doesn't exist.*

In observation II we show that generically it is impossible to support an equilibrium with two different induced actions when the state space's boundary is a circle and the prior distribution is uniform. This is true for any vector of conflict  $b$ . In particular, it is also true for  $b_1 = b_2 = 0$ . However, note that in this case, i.e., when there is no conflict, there exists an equilibrium with full information transmission, in which the sender reveals the true state and the receiver takes this as her action. This highlights therefore another difference from the unidimensional analysis, in which the non-existence of an equilibrium with  $k$  induced actions implies the non-existence of an equilibrium with more than  $k$  induced actions.

#### 4 Equilibria with high levels of conflicts

In this section we analyze the set of equilibria when the conflict between the sender and the receiver is large. In what follows we increase the distance between the ideal point of the sender and the origin in the direction of the vector  $\mathbf{b}$ , a ray with slope  $\frac{b_2}{b_1}$ . Denote by  $b$  the norm of  $\mathbf{b}$ , i.e.,  $b = \|\mathbf{b}\|$ . Finally, let  $\beta^* = -\frac{\alpha_1}{\alpha_2} \frac{b_1}{b_2}$  and  $\delta^* = -\frac{\alpha_1}{\alpha_2} \frac{1}{\beta^*} = \frac{b_2}{b_1}$ . Assume without loss of generality that  $|\beta^*| \in (0, \infty)$ .

We focus our analysis on influential equilibria:

**DEFINITION 1** An equilibrium is *influential* if there exist at least two messages  $m, m'$  such that  $a(m) \neq a(m')$ .

Our first result is an important building block of what will follow. When the conflict becomes very large, it implies that the distance between typical actions that the receiver may take and the sender's ideal point, increases. In this case, we show that one dimension of the policy space becomes more and more important. In the limit as this distance goes to infinity, the preferences of the sender become lexicographic; first, alternatives are compared only on this important dimension. If that comparison is inconclusive, a comparison along

a different dimension determines the preferences.

In equilibrium, if there are two distinct induced actions, it must be that some types are indifferent between these two actions. Together with the above discussion, this implies that for high degrees of conflict, any two distinct actions induced in equilibrium must both lie approximately on a line with a particular slope. This intuition is now formalized:

**PROPOSITION 2** *For any  $\varepsilon > 0$ , there exists  $\bar{b}$  such that for all  $b > \bar{b}$  and any two distinct actions  $(a'_1, a'_2)$  and  $(a''_1, a''_2)$  induced in equilibrium, (i)  $|\frac{a''_2 - a'_2}{a''_1 - a'_1} - \beta^*| \leq \varepsilon$ ; (ii) The set of sender types who are indifferent between these two actions is a line with slope  $\delta$  such that  $|\delta - \delta^*| \leq \varepsilon$ .*

**PROOF OF PROPOSITION 2:** (i) For any  $b$ , if there are two distinct actions,  $a' = (a'_1, a'_2)$  and  $a'' = (a''_1, a''_2)$ , induced in equilibrium, there must be some sender type  $\theta'$  that is indifferent between inducing either of these actions. For this type we have,

$$\begin{aligned} v(\Delta_\alpha(a', b|\theta')) &= v(\Delta_\alpha(a'', b|\theta')) \Leftrightarrow & (1) \\ \sum_{i=1}^2 \alpha_i (a'_i - (b_i + \theta'_i))^2 &= \sum_{i=1}^2 \alpha_i (a''_i - (b_i + \theta'_i))^2 \Leftrightarrow \\ \alpha_1 (a_1'^2 - a_1''^2) - 2\alpha_1 (b_1 + \theta'_1)(a'_1 - a''_1) &= \alpha_2 (a_2''^2 - a_2'^2) - 2\alpha_2 (b_2 + \theta'_2)(a''_2 - a'_2) & (2) \end{aligned}$$

First suppose that  $a'_1 = a''_1$ . This implies that the left-hand-side of (2) is 0 and therefore to satisfy the equation, it has to be that:

$$\alpha_2 (a''_2 - a'_2)((a''_2 + a'_2) - 2(b_2 + \theta'_2)) = 0$$

But since the actions are distant from one another,  $a''_2 \neq a'_2$ . Thus, for all

$$b_2 > \frac{(a''_2 + a'_2)}{2} - \theta'_2,$$

where  $\frac{(a''_2 + a'_2)}{2} - \theta'_2$  is bounded, equation (2) is violated. Consider then the case of  $a'_1 \neq a''_1$  and let us re-arrange (2):

$$\frac{(a''_2 - a'_2)}{(a'_1 - a''_1)} \left(1 - \frac{\alpha_2 (a''_2 + a'_2)}{2\alpha_2 (b_2 + \theta'_2)}\right) - \frac{\alpha_1 (b_1 + \theta'_1)}{\alpha_2 (b_2 + \theta'_2)} = -\frac{\alpha_1 (a'_1 + a''_1)}{2\alpha_2 (b_2 + \theta'_2)} \Leftrightarrow \quad (3)$$

$$\frac{(a''_2 - a'_2)}{(a'_1 - a''_1)} = \frac{2\alpha_1 (b_1 + \theta'_1) - \alpha_1 (a'_1 + a''_1)}{2\alpha_2 (b_2 + \theta'_2) - \alpha_2 (a''_2 + a'_2)} \quad (4)$$

Note that any induced action must be in (the compact set)  $\Theta$ . Moreover, it must be that both  $b_1$  and  $b_2$  are growing to infinity as  $b$  converges to infinity. This implies that the right hand side of (4) is converging to  $\frac{\alpha_1 b_1}{\alpha_2 b_2}$  when  $b$  converges to infinity. Thus,  $-\frac{(a''_2 - a'_2)}{(a'_1 - a''_1)} = \frac{(a''_2 - a'_2)}{(a''_1 - a'_1)} \rightarrow -\frac{\alpha_1 b_1}{\alpha_2 b_2} = \beta^*$ .

(ii) For any two actions,  $(a'_1, a'_2)$  and  $(a''_1, a''_2)$  induced in equilibrium, look at a typical type,  $\theta'$ , who is indifferent between the two actions. This type should satisfy (1) and therefore by rearranging we get,

$$\theta'_2 = \theta'_1 \frac{\alpha_1(a'_1 - a''_1)}{\alpha_2(a''_2 - a'_2)} - \frac{\alpha_1}{2\alpha_2} \frac{(a'^2_1 - a''^2_1)}{(a''_2 - a'_2)} + \frac{2\alpha_1}{2\alpha_2} b_1 \frac{(a'_1 - a''_1)}{(a''_2 - a'_2)} + \frac{(a''_2 + a'_2)}{2} - b_2, \quad (5)$$

which shows that the set of indifferent sender types is a line with slope  $\frac{\alpha_1(a'_1 - a''_1)}{\alpha_2(a''_2 - a'_2)}$ . By (i),

$\frac{(a'_1 - a''_1)}{(a''_2 - a'_2)} \rightarrow -\frac{1}{\beta^*}$  and so we have that the slope of the set of indifferent sender types converges to  $-\frac{\alpha_1}{\alpha_2} \frac{1}{\beta^*} = \delta^*$ .  $\square$

Proposition 2 puts very stringent conditions on the existence of equilibria. The first part states that all induced actions will tend to be distributed on a particular line of slope  $\beta^*$ . The second part states that the types who support each action will be in a subset of  $\Theta$  with boundaries which tend to be lines with slope  $\delta^*$ .

Note that a sender with a type  $(\frac{a'_1 + a''_1}{2}, \frac{a'_2 + a''_2}{2})$  is indifferent between the two actions for all  $b$ . This is the mid-point between the two actions and is hence equidistant from both. When the line connecting the two actions converges to be with a slope  $\beta^*$ , since the mid-point  $(\frac{a'_1 + a''_1}{2}, \frac{a'_2 + a''_2}{2})$  is trivially on this line, as well as on the line of indifferent types, it implies that the two actions converge to be equidistant from the line of indifferent types in the  $\beta^*$ -dimension.

This implication of Proposition 2 discussed above will play an important role in our analysis. It allows us to translate the preferences of the sender, when the conflict is very large, to *lexicographic preferences*. The relevant dimensions become the lines of slopes  $\beta^*$  and  $\delta^*$ . First, as part (i) of Proposition 1 shows, the sender prefers the action which is on a line with a slope  $\beta^*$  that is closest to him, as if his indifference curves are linear with a slope  $\beta^*$ . If both actions are on the same line with a slope  $\beta^*$ , as we discuss above, the sender prefers the action that is on the closest line with a slope  $\delta^*$  - closest on the  $\beta^*$  direction. Below, we show that this tight structure implies that generically influential equilibria will not exist.

**A reformulation** Proposition 2 established the conditions for the existence of equilibria for very large conflicts. We now use these results to re-construct the mathematical formulation of the problem, which would enable us to prove that generically, influential equilibria do not exist when the conflict is very large.

To make things simpler, we build a new coordinate system with respect to the slopes  $\beta^*$  and  $\delta^*$ . Let us block the set  $\Theta$  with two  $\delta^*$  and two  $\beta^*$  lines that are tangent to  $\Theta$ . We denote one of the points of intersection between a  $\delta^*$  and a  $\beta^*$  line, say the south-west point, as  $(0,0)$  and let the two lines crossing at  $(0,0)$  span the space. Similarly one can denote the

three other intersections by  $(\bar{x}, 0)$  and  $(0, \bar{y})$  and  $(\bar{x}, \bar{y})$ , with the convention that the new  $x - axis$  is the dimension of slope  $\beta^*$  and the  $y - axis$  that of slope  $\delta^*$ . For any  $\theta \in \Theta$ , let  $(x, y)$  be the same point expressed in the new coordinate system and let the set  $\Theta$  be mapped into the set  $\Theta^*$ .

Denote the marginal distribution on the  $y - axis$  as  $f^{\delta^*}(y)$  defined on  $(0, \bar{y})$  and the marginal distribution on the  $x - axis$  as  $f^{\beta^*}(x)$  defined on  $(0, \bar{x})$ . Finally, we term the reaction curve,  $\gamma(x)$ , as the expectations over  $y$  for values of  $y$  for which  $(x, y) \in \Theta^*$ , i.e.,

$$\gamma(x) = E[y | (x, y) \in \Theta^*].$$

Figure 3 illustrates the new coordinate system, and depicts an example of  $\gamma(x)$ . The dashed lines in  $\Theta^*$  are lines with slope  $\delta^*$  and  $\gamma(x)$  is the expectations over each of these lines, i.e., the expectations over  $y$  for a particular value of  $x$  :

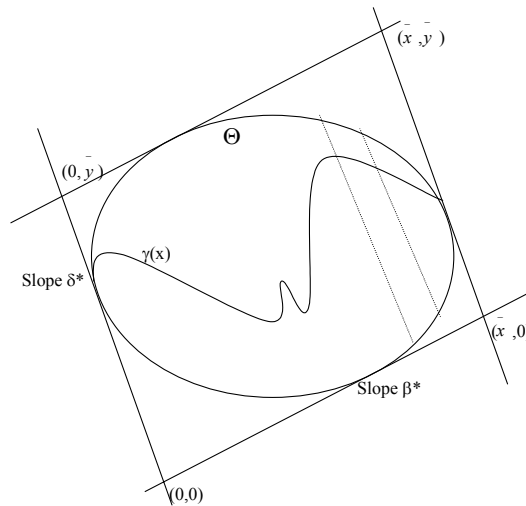


Figure 3: The new coordinate system.

In this new coordinate system, we propose to consider the following two problems. First, think of partitioning  $\Theta^*$  into ‘strips’ with boundaries that are lines with a slope  $\delta^*$ , such that the expectations taken on any two neighboring strips are equidistant (with respect to direction  $\beta^*$ ) from the line that separates them. Figure 4a illustrates a solution to such a problem. It is a vector  $x = (0, x_2, x_3, \bar{x})$ , that induces the expectations  $(a_1, a_2, a_3)$ , which are equidistant from  $x_2$  and from  $x_3$  respectively:

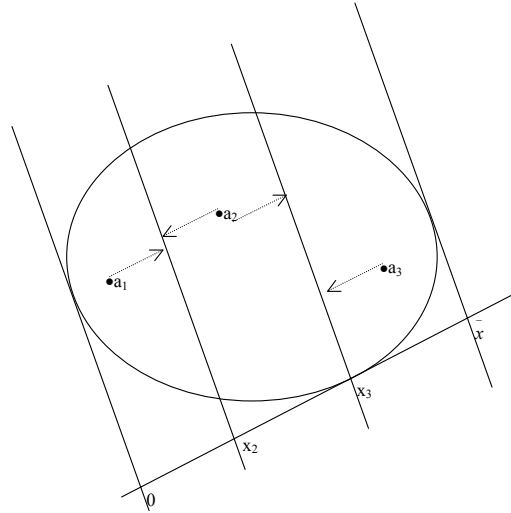


Figure 4a

The second problem that we consider is partitioning  $\Theta^*$  into such strips as well, with boundaries that are lines with a slope  $\delta^*$ , but in a way so that the expectations taken within each strip are all on the same line of slope  $\beta^*$ , that is, they all have the same value of  $y$ . Figure 4b illustrates a solution to such a problem: the vector  $x' = (0, x'_2, x'_3, \bar{x})$  induces the expectations  $(a'_1, a'_2, a'_3)$  which are on a line with slope  $\beta^*$ . In the figure we also illustrate the reaction curve  $\gamma(x)$ , which is helpful in this problem; since  $\gamma(x)$  is the expected value of  $y$  given a particular value of  $x$ , the expectations of the  $y$ -value over some strip  $(x_i, x_{i+1})$  for  $i \in \{1, 2, 3\}$  are essentially the expectations over  $\gamma(x)$  in this strip, where in these expectations each value of  $\gamma(x)$  is weighted by the marginal distribution over  $x$ .

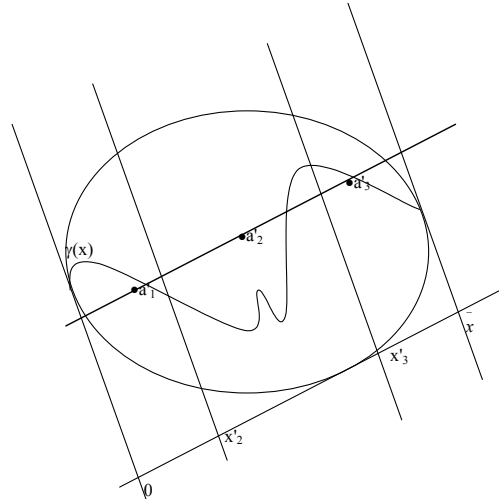


Figure 4b.

We now present formally the two problems discussed above.

PROBLEM A

A vector  $(x_1, \dots, x_k) \in (0, \bar{x})^k$ , such that  $x_i \leq x_{i+1}$  for all  $i \in (1, \dots, k-1)$ , and  $k \geq 3$ , is a solution to problem A if  $x_1 = 0$ ,  $x_k = \bar{x}$  and for all  $i \in (2, \dots, k-1)$ ,

$$\left| \int_{x_{i-1}}^{x_i} x f^{\beta^*}(x) dx - x_i \right| = \left| \int_{x_i}^{x_{i+1}} x f^{\beta^*}(x) dx - x_i \right|$$

#### PROBLEM B

A vector  $(x_1, \dots, x_k) \in (0, \bar{x})^k$ , such that  $x_i \leq x_{i+1}$  for all  $i \in (1, \dots, k-1)$ , and  $k \geq 3$ , is a solution to problem B if  $x_1 = 0$ ,  $x_k = \bar{x}$  and for all  $i \in (2, \dots, k-1)$ ,

$$\int_{x_{i-1}}^{x_i} \frac{\gamma(x) f^{\beta^*}(x)}{\int_{x_{i-1}}^{x_i} f^{\beta^*}(x) dx} dx = \int_{x_i}^{x_{i+1}} \frac{\gamma(x) f^{\beta^*}(x)}{\int_{x_{i-1}}^{x_{i+1}} f^{\beta^*}(x) dx} dx$$

Problems A and B decompose therefore the conditions of Proposition 2 into two problems. In particular, problem A is related to the implication of Proposition 2 that any two actions converge to be equidistant - in the  $\beta^*$  dimension - from the line of the indifferent types. Problem B relates to part (i) of Proposition 2, which establishes that all actions have actually to converge to a line with a slope  $\beta^*$ . Both problems, A and B, rely on part (ii) of Proposition 2, i.e., that the lines of indifferent types converges to be with a slope  $\delta^*$ . Therefore, there is a close relation between the solutions for problems A and B and the existence of equilibria.

In our main result we will show that these two problems, A and B, are independent and that the existence of a solution to both problems is non-generic. This, as we will show, also implies that influential equilibria are non-generic for large conflicts. Even before we prove this formally, one can observe the following: a solution to problem A depends only on the marginal distribution over  $x$ ,  $f^{\beta^*}(x)$ , whereas the solution to problem B depends on  $f^{\beta^*}(x)$  as well as on  $\gamma(x)$ .<sup>12</sup> This implies that the set of solutions is independent. For example, if we alter the reaction curve  $\gamma(x)$ , without changing  $f^{\beta^*}(x)$ , the marginal distribution over  $x$ , then the set of solutions for problem B may change whereas the set of solutions for problem A remains fixed.

Our final building block before presenting the main result is a characterization of the solutions for problems A and B. For any two points,  $x, x' \in [0, \bar{x}]$ , let

$$\mu_{x,x'}^y = E(y | x'' \in [x, x'] \text{ and } (x'', y) \in \Theta^*)$$

and let

$$\mu^y = \mu_{0,\bar{x}}^y$$

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<sup>12</sup>The solution to problem B depends therefore on the marginal on the  $y$  - axis only through the expectations on that dimension.

DEFINITION 2 *The reaction curve  $\gamma(x)$  satisfies the  $l$ -crossing property, if there are  $l$  (finite) solutions to  $\gamma(x) = \mu^y$ .*

PROPOSITION 3

(i) *Problem A: the set of solutions to problem A is isomorphic to the set of equilibrium outcomes of the model with  $\Theta = [0, \bar{x}]$ , a density function  $f^{\beta^*}(x)$  over  $\Theta$  and  $b=0$  (see Crawford and Sobel, 1982). In particular, there exists a countable number of solutions.*

(ii) *Problem B: (a) if  $\gamma(x) = \text{const}$  then any vector  $(x_1, \dots, x_k)$  is a solution to problem B. (b) If the reaction curve  $\gamma(x)$  satisfies the  $l$ -crossing property, then there are at most a finite number of solutions to problem B. (c) If the reaction curve  $\gamma(x)$  satisfies the one-crossing property, then there does not exist a solution to B.*

PROOF OF PROPOSITION 3:

(i) This follows from the analysis of Crawford and Sobel (1982).

(ii) (a) When  $\gamma(x) = c$  for all  $x$ , then for all  $i$ , and  $x_i, x_{i-1}$ ,  $\mu_{x_i, x_{i-1}}^y = c$ . Therefore any vector is a solution for B (note that this implies that if  $x$  and  $y$  are independently distributed then every vector  $x$  is a solution for B).<sup>13</sup>

(ii) (b) We now construct an algorithm to compute the set of solutions for problem B. We will show both that the algorithm provides a finite number of solutions, and that it characterizes all the solutions to problem B.

*Step 1: The algorithm*

1. Start from  $x_1 = 0$  and find the first  $x \in (0, \bar{x}]$  such that  $\mu_{0,x}^y = \mu^y$ . If  $x = \bar{x}$  the algorithm stops with no solution. If  $x < \bar{x}$ , denote  $x_2 = x$ . Suppose we have defined in this way  $x_m$ ,  $m \geq 2$ . Continue this process starting from  $x_m$ . If there exists an  $x \in (x_m, \bar{x})$  such that  $\mu_{x_m,x}^y = \mu^y$  we define  $x_{m+1} = x$  and proceed to the next step in the algorithm starting with  $x_{m+1}$ . If  $x = \bar{x}$ , we define  $x_{m+1} = \bar{x}$  and the algorithm stops. Let  $k$  be the subscript of the last point defined in the algorithm.

By the  $l$ -crossing property, the algorithm stops at a finite time. To see this, note that there cannot be two points  $x_i$  and  $x_{i+1}$  chosen by the algorithm that are between two neighboring solutions of the equation  $\gamma(x) = \mu^y$  (i.e., two ‘crossings’). If there were such two points then either  $\gamma(x) > \mu^y$  or  $\gamma(x) < \mu^y$  in the strip defined by  $(x_i, x_{i+1})$ , in contradiction to the construction of the algorithm. Thus, since there is a finite number of crossings, there must be finite number of solutions in the algorithm.

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<sup>13</sup>In particular, this would imply that when  $x$  and  $y$  are independently distributed, then there exist equilibria with any number of messages (recall that  $x$  and  $y$  are not the original dimensions of the problem but correspond to the  $\beta^*$  and  $\delta^*$  dimensions).

2. The set of solutions to problem B proposed by the algorithm is any (ordered) vector  $(x'_1, \dots, x'_m)$ ,  $3 \leq m \leq k$ , such that  $x'_1 = 0, x'_m = \bar{x}$  and  $x'_i \in \{x_2, \dots, x_{k-1}\}$  for all  $i = 2, \dots, m-1$ .

*Step 2: The solutions proposed by the algorithm are the only solutions for problem B.*

First, it is easy to show that in any solution  $x$  of problem B  $\mu_{x_i, x_{i+1}}^y = \mu^y$ . This follows from the fact that  $\mu_{x_i, x_{i+1}}^y$  are equal valued for any pair  $x_i, x_{i+1}$  in  $x$  and that a weighted average of them must equal to the expectation under the prior,  $\mu^y$ . In particular then, it is also the case that  $\mu_{0, x_i}^y = \mu^y$  for all  $x_i$  which are part of a solution  $x$ .

Second, suppose that we find a solution  $(x'_1, \dots, x'_k)$  but it is not characterized by the algorithm above. In particular, suppose that  $x'_i$  is not part of any solution above. It is therefore in between some  $x_m$  and  $x_{m+1}$  which were characterized by the algorithm. In this case,  $\mu_{0, x_m}^y = \mu^y$  and  $\mu_{0, x'_i}^y = \mu^y$ , which implies that  $\mu_{x_m, x'_i}^y = \mu^y$ . This is however in contradiction to the definition of  $x_{m+1}$ , since  $x_{m+1}$  is the smallest value of  $x > x_m$  so that  $\mu_{x_m, x}^y = \mu^y$ . So  $(x'_1, \dots, x'_k)$  must be part of a solution found by the algorithm.

(ii) (c) If there is a single crossing of  $\gamma(x)$  with  $\mu^y$  this implies that the above algorithm will stop with no solution at the first step. The reason is that for all  $x < \bar{x}$ , either  $\mu_{0, x}^y < \mu^y$  or  $\mu_{0, x}^y > \mu^y$ . To see this, let  $x'$  solve  $\gamma(x') = \mu^y$ . For any  $x < x'$ ,  $\mu_{0, x}^y \neq \mu^y$ , and for any  $x > x'$ ,  $\mu_{x, \bar{x}}^y \neq \mu^y$ , but in the solution  $\mu_{x, \bar{x}}^y = \mu^y$  has to be satisfied. This concludes the proof. ■

The implication of Proposition 3 is that the existence of a solution which solves both problems A and B simultaneously, seems highly unlikely. We investigate this more formally in Theorem 1.

Part (c) of the Proposition already provides sufficient conditions for the non-existence of a solution to problem B. To see that these situations are generic, consider the state space bounded by a circle, where the prior distribution is uniform. In the circle, the reaction curve  $\gamma(x)$ , which is the expectations over lines with slope  $\delta^*$  (as the dotted lines in figure 5), is always orthogonal to  $\delta^*$ .  $\mu^y$  is the  $\beta^*$ -projection of the prior on the  $\delta^*$ -dimension. If  $\beta^* = -\frac{1}{\delta^*}$ , then  $\gamma(x) = \mu^y$  for all  $x \in [0, \bar{x}]$ . In this case any vector of points in  $[0, \bar{x}]$  is a solution of Problem B. However, whenever  $\beta^* \neq -\frac{1}{\delta^*}$ , as depicted in figure 5, then  $\gamma(x)$  crosses the line with slope  $\beta^*$  and has a  $y$ -value of  $\mu^y$  only once. Thus, problem B has no solution:

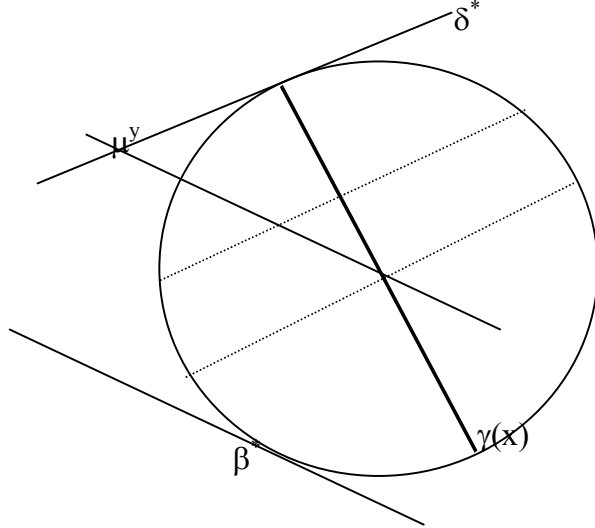


Figure 5: In the circle there are generically no solutions to problem B.

The example of the square, with a uniform distribution, is similar. Let  $\delta^* = 1$ . Then  $\gamma(x)$  is the diagonal with a slope  $-1$  (i.e., it is expectations over lines such as the dotted lines). If  $\beta^* = -1$ , as in the circle example, then  $\gamma(x) = \mu^y$  for all  $x \in [0, \bar{x}]$ . Again, in this case, depicted in figure 6a, any ordered vector of points in  $[0, \bar{x}]$  is a solution to Problem B. However, for any  $\beta^* \neq -1$ , the one-crossing property is satisfied, and there does not exist a solution to problem B (see figure 6b).

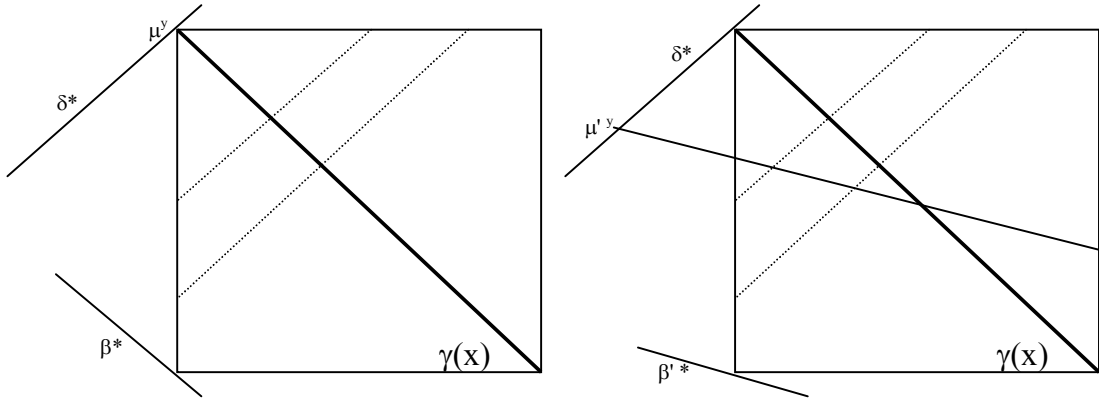


Figure 6a (left) in which every vector is a solution to problem B and figure 6b (right) which shows that generically in the square, problem B has no solution.

Note that the one-crossing property satisfied above, or more generally the  $l$ -crossing property, is a property of the relation between the distribution function  $F(\cdot)$  and the slopes  $\delta^*$  and  $\beta^*$ . That is, it is not a property of a distribution function alone, but a joint property of the prior distribution and the other primitives of the model, the ratio of the conflict on each dimension and the weight that the sender places on each of the dimensions.

**The main result** We can now present the theorem. Let  $Solution(A)$  and  $Solution(B)$  be the sets of solutions of problems  $A$  and  $B$  respectively.

**THEOREM 1** (i) *If  $Solution(A) \cap Solution(B) = \emptyset$  then there exists a  $\bar{b}$  such that for all  $b > \bar{b}$  there exists no influential equilibrium;* (ii) *Suppose that  $Solution(A) \cap Solution(B) \neq \emptyset$ . Then there exists a set of perturbations of the distribution function  $F(\theta^*)$  on  $\Theta^*$  such that  $Solution(A) \cap Solution(B) = \emptyset$ .*

The first part of the Theorem relates problems  $A$  and  $B$  to the existence of an equilibrium for high levels of conflict. That is, it states that if there is no common solution to problem  $A$  and  $B$ , then for high levels of conflict, indeed there is no influential equilibrium. The second part of Theorem 1 establishes that a solution to both problem  $A$  and problem  $B$  is non-generic. In the remaining of this section, we explain the intuition for the proof in two steps, starting with the second part.

### **The non-genericity of a common solution to problems $A$ and $B$**

We establish the non-genericity of common solutions to  $A$  and  $B$  in the following way. We construct perturbations to the distribution function which change the set of solutions for problem  $B$  while maintaining the set of solutions for problem  $A$ , so that none of the new solutions for problem  $B$  can coincide with any of the solutions in  $A$ .

First recall that a solution to problem  $B$  must satisfy that for all  $i$ , and  $x_i, x_{i-1}$ ,  $\mu_{x_i, x_{i+1}}^y = \mu^y$ , and moreover, by the algorithm that we have constructed, it also satisfies that for all  $i$ ,  $\mu_{0, x_{i+1}}^y = \mu^y$ . We now focus on perturbations to  $\gamma(x)$ , whereas we maintained fixed the marginal density over  $x$ ,  $f^{\beta^*}(x)$ . Recall that this type of perturbation will not alter the set of solutions for problem  $A$ .

Let us take for example any (small) local perturbation which alters  $\gamma(x)$  in some strip  $x_i, x_{i+1}$ , which therefore changes the prior in that strip,  $\mu_{x_i, x_{i+1}}^y$ , and as a result changes also  $\mu^y$ . Such a perturbation is described in the following graph:

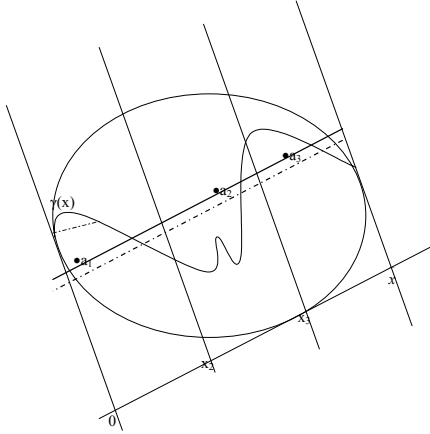


Figure 7: An example of a perturbation.

In the original solution, all the actions  $a_1$ ,  $a_2$  and  $a_3$  are on a line with slope  $\beta^*$  that goes through  $\mu^y$ . There is a small and local perturbation of the reaction curve  $\gamma(x)$  represented by the dashed line, in the strip between 0 and  $x_2$  which changes  $\mu_{0,x_1}^y$  ‘downward’ to  $\hat{\mu}_{0,x_1}^y$ . This perturbation moves  $\mu^y$  ‘downward’, and the dashed line with slope  $\beta^*$  goes through the new prior, denoted by  $\hat{\mu}^y$ . Note that  $\mu^y$  has changed less than the local change in  $\mu_{0,x_1}^y$ .

It is now easy to see that the solution  $(0, x_2, x_3, \bar{x})$  is not a solution any more:  $\hat{\mu}_{0,x_1}^y \neq \hat{\mu}^y$ ,  $\mu_{x_2,x_3}^y \neq \hat{\mu}^y$  and  $\mu_{x_3,\bar{x}}^y \neq \hat{\mu}^y$  since the expectations on these latter strips have not changed. Moreover, neither of the components of this solution can be part of a different solution, since also  $\mu_{0,x_2}^y \neq \hat{\mu}^y$  and  $\mu_{0,x_3}^y \neq \hat{\mu}^y$ . This implies that any former solution to problem B is not a solution any more.

To complete the intuition for part (ii), we note the following. Generically, whenever there is a slight change in the prior  $\mu^y$ , any new solution to problem B is only slightly different from one of the original solutions. This implies that following the perturbations,  $Solution(A) \cap Solution(B)$  is empty, because any previous solution corresponded to a solution in A but since the set of solutions of A is not a continuum, none of the new solutions to B is a solution to problem A.

### The inexistence of influential equilibria for large conflict

The final step of our analysis, is to establish the continuity of the problem. This is complicated by the feature that if an equilibrium with  $k$  messages does not exist, it does not imply that an equilibrium with more than  $k$  messages does not exist (see observation III in section 3). Therefore, one cannot prove that no influential equilibria exist if there is no equilibrium with two different actions (as opposed to the unidimensional model).

In the proof of the first part of the theorem we therefore show that the statement is true for all cases of sequences of equilibria when  $b$  converges to infinity, those in which the

number of induced actions is bounded by some finite number and those in which the number of induced actions converges to infinity. For the case of equilibria with finite actions, we prove that the problem exhibits continuity in the limit, when  $b$  converges to infinity.

We now illustrate the proof for the infinite case; that is, we demonstrate that when  $Solution(A) \cap Solution(B) = \phi$ , then there is no sequence of equilibria with infinitely many induced actions. Assume, by way of contradiction, that such a sequence of equilibria exist. In particular, there must exist a convergent subsequence of these equilibria in which all the actions converge to be on some line with slope  $\beta^*$  (this must also be the line that goes through  $\mu^y$ ). We can then show that this line of actions with a slope  $\beta^*$  must cover  $\Theta$ , that is, the measure of types which support each action must be zero.

We now reach a contradiction to the existence of such a sequence of equilibria in the following way. We look at a strip in which the convex hull of the reaction curve  $\gamma(x)$  and the line of actions with a slope  $\beta^*$  do not intersect, i.e., are distant from one another; we can always find such a strip. If there is no such strip, then essentially  $\gamma(x)$  is the line with a slope  $\beta^*$  which goes through  $\mu^y$ , which implies that any vector is a solution to problem B. This would therefore be in contradiction to the condition in the theorem which demands that  $Solution(A) \cap Solution(B) = \phi$ .

On this strip, we can compute the expectations over  $\theta$ . On the one hand, we can take expectations over the reaction curve  $\gamma(x)$  which intersects this strip. These expectations are therefore in the convex hull of  $\gamma(x)$ . This is simply another way to take expectations; we first take expectations over the lines of slope  $\delta^*$  (which is how we constructed  $\gamma(x)$ ) that intersect this strip, and then weigh each of these expectations by the measure of its relevant line.

On the other hand, we can take expectations in this interval by taking some weighted average of the actions induced in equilibrium. Since all of them must fall on the same line with a slope  $\beta^*$ , their convex combination must be on this line as well, and in the part which intersects the strip.

But now we reach a contradiction, because the expectations cannot be both on the line of slope  $\beta^*$  and in the convex hull of  $\gamma(x)$ , by the way we have constructed this strip. Thus, when  $Solution(A) \cap Solution(B) = \phi$ , there cannot exist an equilibrium with infinitely many actions when the conflict is too large.

### *An Example*

Let us re-visit the state space of a square, i.e.,  $\Theta = [0, 1]^2$ , with the uniform distribution as the prior distribution. Let  $b_1 = b_2 \geq 0$  and  $\alpha_1 = \alpha_2 = \frac{1}{2}$ . In this case,  $\beta^* = -1 = -\frac{1}{\delta^*}$  and  $\gamma(x)$  identifies with a line of slope  $\beta^*$  that goes through the prior expectations. This case is depicted in figure 6a. Indeed, for any  $b_1 = b_2$ , there exists an influential equilibrium

with infinitely many messages; all senders with the type  $\theta_2 = \theta_1 + \lambda$  for  $\lambda \in [-1, 1]$  send the same message  $\lambda$  and the receiver takes an action  $(\frac{1-\lambda}{2}, \frac{1+\lambda}{2})$ . This equilibrium can be approximated by the limit of a sequence of a common solutions for problems A and B, when we increase the size of the vector  $x$ , i.e., when we let  $k$  converge to infinity.

However, when the conflict is very large this equilibrium is non-generic, that is, it is not robust to small changes in the parameters of the model. For example, any local change in the uniform distribution that would slightly change  $\gamma(x)$ , will knock down this equilibrium, as Theorem 1 implies. But moreover, if for example we replace the assumption of  $\alpha_1 = \alpha_2$  by any other values for these parameters, *there is no influential equilibrium at all*. In this case, as in figure 6b,  $\beta^* \neq -1$  and therefore  $\gamma(x)$  satisfies the one-crossing property with respect to the line with a slope  $\beta^* \neq -1$  that goes through the prior. From Proposition 3 we then know that there is no solution to problem B, which by Theorem 1 implies that no informative messages can be sustained in equilibrium.

We find this example important, first, because it demonstrates that any changes to the parameters of the model, not only to the distribution function as presented in Theorem 1, can upset influential equilibria. Thus, there are also other perturbation, on top of those specified in Theorem 1, that induce non-genericity of influential equilibria. This is a consequence of the fact that the  $l$ -crossing property that we have defined, is a property satisfied jointly by all parameters of the model and not by the distribution function alone. Second, it may seem attractive in applications to use the symmetric case of  $b_1 = b_2$  and  $\alpha_1 = \alpha_2$ . We emphasize here that this case may be misleading since results regarding information revelation are completely different for all other parameters.

## 5 Discussion

We have shown that in the limit, influential equilibria are non-generic, that is, for any equilibrium there is a set of perturbations that upsets the equilibrium. We have focused on perturbations to the distribution function over the state space, but one can easily construct many other perturbations, such as to the state space itself, or to the other primitives of the models - the parameters describing the direction of the conflict or the relative weights of the different dimensions in the sender's utility function. In addition, we have established the continuity of the problem, so that if influential equilibria do not exist in the limit, indeed there exist large enough degrees of the conflict so that influential equilibria cannot be sustained for these parameters as well. In what follows, we discuss some extensions of our results.

**Informed receiver** In our model the sender has a perfect private signal about the state of the world, whereas the receiver only knows its prior distribution function. Let us change this now and assume that the receiver also has a private signal about the state of the world,

although, in order to keep things interesting, his signal is not perfect.

The main difference from our basic model now is that the sender, when transmitting his message, perceives the receiver's action *as a lottery*. This is because he does not know the exact signal of the receiver. In an influential equilibrium, different messages will induce lotteries with different support, whereas any two types of senders who send the same message, would face a lottery with the same support but with a different probability distribution over these lotteries (since they view differently the probability distribution over the signals of the receiver).

We conjecture that our results would hold for this environment as well. In particular, it is easy to extend the result in Proposition 2; if there are two different messages in equilibrium, then all types of senders who are indifferent between the two messages, must perceive the *expectations* over the lotteries induced by these two messages as converging to be on a line with a slope  $\beta^*$ .

**Many senders** Another natural way to extend our paper is to increase the number of senders. In particular, it would be interesting to consider the case of the senders being imperfectly informed, i.e., each of them has a conditionally independent signal about the state of the world.<sup>14</sup>

Note that the case of many senders bears strategic resemblance to the case of an informed receiver described above. In particular, when one sender considers which messages to transmit, he perceives the receiver as informed and the receiver's action as a lottery, since other senders may have provided the receiver with some private information. We believe that we can therefore extend our results to this case as well.<sup>15</sup>

**Compactness of the state space** In the model an important assumption is the compactness of the state space,  $\Theta$ . This assumption is crucial in the proof of Proposition 2 and therefore for the results stated in Theorem 1. In particular, when the state space is  $R^2$ , influential equilibria may exist for any degree of conflict.

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<sup>14</sup>If all the senders are equally informed, then as Battaglini (2002) have shown, for any level of conflict there exists an equilibrium with full information revelation. The equilibrium has the particular feature that from the point of view of each sender, any of his messages will indeed induce an action on the same line with a slope  $\beta^*$ .

<sup>15</sup>Several papers have analyzed models with many senders who have imperfect signals. Austen-Smith, (1990) and (1993b), analyzes the case of imperfectly informed senders and compares the information properties of the equilibria with either joint or sequential messages. Wolinsky (2002) shows how allowing communication among the experts may increase information revelation. These papers focus on a unidimensional state space. In Austen-Smith (1993a) the multidimensional state space is analyzed, to study experts' incentives to acquire information. Recently, Battaglini (2003) showed that it is possible to extract information from many senders who have imperfect signal, in a multidimensional environment, for some particular distribution functions. Whether this holds for other assumptions awaits future research.

Our results may be generalized to cover this case also. Assume that the state space is  $R^2$  but that the distribution  $F$  has finite expectations. We are currently working on an extension of Theorem 1 to this new environment. We conjecture that as the degree of conflict increases, the probability that more than one action is induced in equilibrium converges to zero. In contrast to the case of a compact state space, in this environment our result about influential equilibria would be therefore in terms of probabilistic limits.

**Generalizing the utility function** The focus of this paper has been to show why the CS approach to model strategic information transmission, may be limited in a multidimensional state space. In this respect, the assumptions on the utility functions were chosen to correspond to those in the literature. This gives the model its best chance to yield applicable results. Although the results could be extended to more general utility specifications, our analysis proves that this approach may be futile.

The analysis in the paper highlights an interesting point about the CS modelling approach and modeling information transmission in general. It is easy to find specifications of utilities under which even in the one dimensional model it would be difficult to find influential equilibria. The assumptions underling the analysis in CS, which amount to assumptions linking the state space and the policy space, guarantee that this is not the case. One possible direction of future research, in the multidimensional state space, is to find other specifications of utilities, under which, influential equilibria may arise.

## Appendix

**Definitions and notation** For any  $a', a'' \in \mathfrak{R}^d$ , let  $d(a', a'') = \|a' - a''\|$ . For any  $a \in R^d$  and any set  $A \subset R^d$ , define the distance between  $a$  and  $A$ ,

$$d(a, A) = \min_{a' \in A} d(a, a').$$

Let  $a, a' \in \mathfrak{R}^d$  be two actions. We define the set of agents that weakly and strictly prefer  $a$  to  $a'$ , by,  $R(a, a') = \{\theta \in \Theta | U(a|\theta) \geq U(a'|\theta)\}$  and  $P(a, a') = \{\theta \in \Theta | U(a|\theta) > U(a'|\theta)\}$ .

By Proposition 2, the set of indifferent types between  $a$  and  $a'$  is a line. Let  $l^{a, a'}$  denote the line of indifferent types between two induced actions in equilibrium,  $a$  and  $a'$ . We say that two induced actions are *neighbors* if there exists  $\theta \in l^{a, a'} \cap \text{Bdry}(S(a)) \cap \text{Bdry}(S(a'))$ .

Finally, let  $A$  be a ‘well defined’ set, the measure of  $A$  is  $M(A) = \int_A dF$ .

For any induced action  $a$ , we define the *support set* of  $a$  as the set of types that induce  $a$  with a strictly positive density, i.e., for any  $a \in A$ ,

$$S(a) = \{\theta \in \Theta | \exists m \in M \text{ such that } a(m) = a \text{ and } m_\theta(m) > 0\}$$

For any equilibrium with a set of induced actions  $A$ , and any induced action  $a$ , define the *potential support set* of  $a$ ,  $\bar{S}(a)$ , by  $\bar{S}(a) = \bigcap_{a' \neq a, a' \in A} R(a, a')$ . Types that are not in  $\bar{S}(a)$  do not induce  $a$ . Define the *definite support set* for  $a$  by  $\underline{S}(a) = \bigcap_{a' \neq a, a' \in A} P(a, a')$ . Types that are in  $\underline{S}(a)$  will choose to induce  $a$  with probability one.

### Proofs of results PROOF OF PROPOSITION 1:

LEMMA 1 *In any equilibrium and any induced action  $a \in A$ ,*

$$\underline{S}(a) \subseteq S(a) \subseteq \bar{S}(a).$$

Proof: Obviously if  $\theta \in \underline{S}(a)$ ,  $\theta \in S(a)$  as  $a$  is an induced action and it gives type  $\theta$  maximal utility. If  $\theta \notin \bar{S}(a)$  there must exist  $a' \in A$  such that  $U(a'|\theta) > U(a|\theta)$  and therefore by the equilibrium conditions,  $\theta \notin S(a)$ .  $\square$

LEMMA 2 *In any equilibrium and any induced action  $a \in A$ ,  $M(\bar{S}(a)/\underline{S}(a)) = 0$ .*

Proof: For any  $a' \in A$ , we have that  $M(R(a, a')/P(a, a')) = 0$  as it is the set of types who are indifferent between the two actions. As preferences are Euclidean, this set is a line whose measure is zero in  $\Theta$ . Therefore,  $M(\bar{S}(a)/\underline{S}(a)) = M(\bigcap_{\tilde{a} \neq a, \tilde{a} \in A} R(a, \tilde{a}) / \bigcap_{\tilde{a} \neq a, \tilde{a} \in A} P(a, \tilde{a})) = M(\bigcap_{\tilde{a} \neq a, \tilde{a} \in A} (R(a, \tilde{a})/P(a, \tilde{a})) \leq M(R(a, a')/P(a, a')) = 0$ .  $\square$

Finally, Lemma 3 characterizes the sets  $\bar{S}(a)$  and  $\underline{S}(a)$ ,

LEMMA 3 (i)  $\bar{S}(a)$  is a convex, closed set. (ii)  $\underline{S}(a)$  is a convex set.

Proof: (i) Since  $\Theta$  is convex, for any  $\tilde{a} \in A$ ,  $R(a, \tilde{a})$  is convex as it is the intersection of  $\Theta$  and a convex hyperplane. Therefore  $\bar{S}(a) = \bigcap_{\tilde{a} \neq a, \tilde{a} \in A} R(a, \tilde{a})$  is convex. Moreover as  $R(a, \tilde{a})$  is a closed set for any  $\tilde{a} \in Y$ ,  $\bar{S}(a)$  is closed.

(iii) Since  $\Theta$  is convex, for any  $\tilde{a} \in A$ ,  $P(a, \tilde{a})$  is convex as it is the intersection of  $\Theta$  and a convex hyperplane. Therefore  $\bar{S}(a) = \bigcap_{\tilde{a} \neq a, \tilde{a} \in A} R(a, \tilde{a})$  is convex.  $\square$

Lemmata 1,2 and 3 imply Proposition 1 as equilibria may differ from one another in outcomes only by having zero measure sets of types inducing different actions in equilibrium.  $\blacksquare$

PROOF OF THEOREM 1:

PROOF OF PART (i): we are considering a sequence of influential equilibria pertaining to  $\{b_n\}_{n=0}^{\infty}$  with a corresponding set of induced actions  $A_n$ . We prove the following preliminary results:

LEMMA 4 *For any convex set  $C$  with strictly positive measure,  $\eta > 0$ , the distance of the expectation over  $C$  to the boundary of  $C$  is bounded from below by a strictly positive number.*

Proof: Specifically, we show that  $d(E(C), \text{Bdry}(C)) > \frac{\lambda(\frac{\eta}{2})}{2} > 0$ , where  $\lambda(\cdot)$  is an increasing function. Fix  $\eta > 0$ . Look at all possible convex subsets of  $\Theta$  with measure  $\eta$ . For any such set  $C$ , define the width of  $C$  to be the smallest distance between any two parallel lines that are tangent to  $C$ . Note that the width of  $C$  is bounded below by some  $\lambda(\eta) > 0$ , where  $\lambda(\eta)$  is increasing. This is evident by the compactness of  $\Theta$  and the strict positiveness of  $f(\cdot)$ .

Fix a set  $C$  as above and find the closest point,  $p$ , on the border of  $C$  to  $E(C)$ . Let  $\delta^C$  be the slope of the tangent to  $C$  at that point. Now divide  $C$  into two equal measured subsets by a line of slope  $\delta^C$ . Denote the subset that includes  $p$  by  $U$  and the other by  $D$ .  $E[C] = \frac{1}{2}E[U] + \frac{1}{2}E[D]$ . Note that  $U$  and  $D$  are convex sets. The distance of  $p$  from  $D$  must be at least  $\lambda(\frac{\eta}{2})$ . This implies that  $E[C]$  must be distanced from  $p$  by at least  $\frac{\lambda(\frac{\eta}{2})}{2}$ .  $\square$

LEMMA 5 *For any sequence of two neighboring actions,  $\{a_n, a'_n\}$ , either  $\lim M(S(a_n)) = \lim M(S(a'_n)) = 0$  or  $\lim M(S(a_n)) > 0$  and  $\lim M(S(a'_n)) > 0$ .*

Proof: Suppose that along the sequence,  $\lim M(S(a_n)) = 0$  but  $\lim M(S(a'_n)) > 0$ . By Lemma 4 this implies that  $\lim \|a_n - a'_n\| > 0$ . Moreover,  $l^{a_n, a'_n}$  is part of the boundary of  $S(a_n)$  and  $S(a'_n)$ . By proposition 2 the boundaries all converge in slope to  $\delta^*$  and therefore  $S(a_n)$  is converging to be a subset of  $l^{a_n, a'_n}$ . But this implies that  $\lim a_n \in l^{a_n, a'_n}$  and  $\lim d(a'_n, l^{a_n, a'_n}) > 0$  which is a contradiction as  $l^{a_n, a'_n}$  converges to pass through the midpoint between  $a_n$  and  $a'_n$ .  $\square$

**The finite case:**

Suppose that for all  $n$ , the number of actions induced in equilibrium is bounded by some finite  $k$ , i.e.,  $2 \leq |A_n| \leq k$  for any  $n$ . We can then find a convergent subsequence of actions  $A_{n_k}$  in which the actions converge to some set  $A$  such that  $|A| \geq 2$ .

We show that all the actions along the sequence are bounded away from one another.

LEMMA 6 There exists  $K$  such that for all  $k > K$ , the lines of indifferent types between any two neighboring actions do not intersect in  $\Theta$ .

Proof: We first show that no two induced actions, along the sequence, converge to one another. Suppose by way of contradiction that there exist two such actions, i.e., there exists a subsequence of equilibria and two induced actions  $a'_{n_k}, a''_{n_k} \in A_{n_k}$  such that  $\|a'_{n_k} - a''_{n_k}\| \rightarrow 0$ .

Note that for any two neighboring induced actions,  $a$  and  $\tilde{a}$ ,  $a \notin \text{Int}S(\tilde{a})$ . This can be seen as follows.  $a \in S(a)$  and therefore  $a \in R(a, \tilde{a})$ . Therefore,  $a \notin P(\tilde{a}, a)$  and thus  $a \notin \text{Int}(S(\tilde{a}))$ . By Lemma 4 this implies that if  $M(S(a))$  is bounded below by some  $\eta > 0$ , then the two neighboring actions,  $a$  and  $\tilde{a}$ , are bounded away from each other. Then  $M(S(a'_{n_k})), M(S(a''_{n_k})) \rightarrow 0$ . By Lemma 5 this implies that  $M(S(a_{n_k})) \rightarrow 0$  for any  $a_{n_k} \in A_{n_k}$ . To see this note that there are a bounded number of induced actions in equilibrium, and that all induced actions are connected under the “neighboring” relation. This contradicts the supposition that there exist two actions which converge.

We know that any line  $l^{a,a'}$  converges to pass through the midpoint on the line between  $a$  and  $a'$ . Let us focus on any three induced actions  $a, a'$  and  $a''$  such that  $a$  and  $a'$  are neighbors and so are  $a'$  and  $a''$ . By Proposition 2 and the above, the distance between the midpoint between  $a$  and  $a'$  and that between  $a'$  and  $a''$  must be bounded below by some strictly positive number. By the compactness of  $\Theta$  and by Proposition 2, the lines must converge to be parallel and thus, for high enough  $k$ , never cross within  $\Theta$ .

Note also that any  $\theta \in \Theta$  that is strictly between two lines  $l^{a,a'}$  and  $l^{a',a''}$  has the same ordering of preferences over  $A_{n_k}$  and in particular has the same optimal actions. Moreover as any such point is not on any indifference line, it must be that there is a unique induced action that maximizes the types preference.  $\square$

Using the reformulated coordinate system, let  $x = (0, x_2, \dots, x_{|A|-1}, \bar{x})$  represent the collection of corresponding limit-indifference lines between the elements of  $A$ , i.e., each  $x_i$  represents the line  $\{y | (x_i, y) \in \Theta^*\}$ . Let us order the elements of  $|A|$  respectively with the ordering of the  $x$ 's.

LEMMA 7  $x \in \text{Solution}(A) \cap \text{Solution}(B)$ .

PROOF:

We will show that: (i)  $d_{\beta^*}(a^i, x_{i+1}) = d_{\beta^*}(a^{i+1}, x_{i+1})$ , for  $i = 1, \dots, |A| - 1$ , (ii)  $a^i =$

$E[\theta_{|x,y}|x_i, x_{i+1}]$ , and that (iii) All the  $a^i$  lie on a line of slope  $\beta^*$ .

Note that (iii) follows from proposition 1. By Proposition 1 the line of indifference between two actions converges to a slope of  $\delta^*$  and thus converges to the relevant  $x_i$ . By (iii) we know that the two actions are on a line of slope  $\beta^*$  and thus the midpoint between the two actions is on  $x_i$  and indifferent and thus  $\beta^*$ -equidistant from the two actions. This proves (i). We now prove part (ii). Suppose that  $k > K$  (as in Lemma 6). On the sequence of equilibria represented by  $\{A_{n_k}\}_{k=1}^\infty$  and for an induced action  $a_{n_k} \rightarrow a^i$ , define the set  $S^*(a_{n_k})$  to be the largest convex set that is bordered by two lines of slope  $\delta^*$  and the contours of  $\Theta^*$  that is a subset of  $S(a_{n_k})$ . Note that by Lemma 6, this is well defined. We now define a new sequence of actions. For any  $k > K$ ,

$$a_{n_k}^* = E(\theta|\theta \in S^*(a_{n_k})).$$

We first show that  $\lim_{k \rightarrow \infty} \|a_{n_k}^* - a_{n_k}\| = 0$ . First, note that  $M(S(a_{n_k})/S^*(a_{n_k})) \rightarrow 0$  as  $S^*(a_{n_k}) \subset S(a_{n_k})$ ,  $M(S(a_{n_k})) > 0$  and by Proposition 2 the slopes of the boundaries of  $S(a_{n_k})$  converge to  $\delta^*$  and so  $S^*(a_{n_k})$  is converging to  $S(a_{n_k})$ . Finally note that  $S^*(a_{n_k})$  converges to the strip of  $\Theta^*$  bounded by lines of slope  $\delta^*$  passing through the points  $x_i$  and  $x_{i+1}$  on the reformulated  $x$ -axis.  $\square$

This completes the proof of the finite case since we reach a contradiction.

#### **Proof for the infinite case:**

We now prove the theorem for the case of infinite actions. That is, suppose that along the sequence of equilibria  $\{A_n\}_{n=1}^\infty$  the number of induced actions is converging to infinity.

**LEMMA 8** *When the number of induced actions converges to infinity along the sequence of equilibria, then for any induced action,  $a_n$ ,  $M(S(a_n)) \rightarrow 0$ .*

*Proof:* Suppose that along the sequence there exists an induced action  $a_n$  such that  $M(S(a_n)) \not\rightarrow 0$ . This implies, by Lemma 5, that for all its neighboring actions,  $\tilde{a}_n$ , also  $M(S(\tilde{a}_n)) \not\rightarrow 0$  and so on. As a result for all actions induced in equilibrium the measure of the support set is bounded away from 0, a contradiction because then only a finite number of actions can be induced.  $\square$

Now note that by Proposition 2, for high conflicts, all actions induced in equilibria must be close to some line with slope  $\beta^*$ . Let  $l^{\beta^*}$  be the line of slope  $\beta^*$  passing through  $\mu^y$  on the reformulated  $y$ -axis.

Let  $D = \{\theta \in \Theta^* | \delta^* \theta_1 + \gamma' \geq \theta_2 \geq \delta^* \theta_1 + \gamma''\}$  for some  $\gamma', \gamma''$  such that  $|\gamma' - \gamma''| > 0$  and  $d(\theta', Co(Graph(\gamma(x)) \cap D)) > \lambda$  for some  $\lambda > 0$  for all  $\theta' \in D \cap l^{\beta^*}$ . To find such a  $D$  we need to know that there is an interval in which there is no infinite crossing between the line and the reaction curve  $\gamma(x)$  which must follow from the fact that the intersection of the

solution for A and B is empty. This is because first  $\gamma(x)$  is continuous and if it is exactly on a line with slope  $\beta^*$  then there is a solution for A and B together.

Let  $A_n^D = \{a_n \in A_n \cap D\}$ . Note that  $A_n^D$  is not empty since that would be a contradiction to Lemma 8. Recall that these actions must be close to  $l^{\beta^*}$ . Since  $M(S(a_n)) \rightarrow 0$  for any induced  $a_n$ , also  $S(a_n) \subset D$  almost surely for all  $a_n \in A_n^D$  and there are no  $a'_n \notin D$  such that  $S(a'_n) \subset D$ . Denote the expectations over  $D$  by  $E(D)$ . We now take the expectation over actions in  $A_n^D$ . Each action is close to  $l^{\beta^*}$  and is weighted by the measure of its support group. But the difference between this expectations to  $E(D)$  must converge to 0, i.e.,  $d(E(D), l^{\beta^*}) \rightarrow 0$ .

On the other hand,  $E(D)$  must converge to lie within the convex hull of  $Graph(\gamma(x)) \cap D$ .  $Graph(\gamma(x)) \cap D$  is the set of expectations over all lines with slope  $\delta^*$  that go through  $D$ . But then  $d(E(D), Co(Graph(\gamma(x)) \cap D)) \rightarrow 0$ . Taken together with  $d(E(D), l^{\beta^*}) \rightarrow 0$ , this is in contradiction to the choice of  $D$ . Therefore, an equilibrium with infinitely many induced actions cannot be sustained.

This complete the proof of part (i) of Theorem 1.  $\square$

PROOF OF PART (ii):

We define a local perturbation to  $\gamma(x)$  as a perturbation in a strip  $[x', x'']$  such that for any  $x \in Solution(B)$  there exists an  $i$  such that  $[x', x''] \subset [x_{i-1}, x_i]$ . Due to the fact that any solution to B is finite, this definition is not restrictive.

LEMMA 9 *Suppose that  $x \in Solution(A) \cap Solution(B)$ . Following any perturbation of  $F(\cdot)$  which maintains the same  $f^{\beta^*}(x)$  but is a local perturbation to  $\gamma(x)$ , then  $x \notin Solution(A) \cap Solution(B)$ .*

Proof: Denote by  $\hat{\mu}_{x_{i-1}, x_i}^y$  the values after the perturbation and by  $\mu_{x_{i-1}, x_i}^y$  the values before the perturbation. With a local change in  $\gamma(x)$ , then it must be that for any  $x \in Solution(A) \cap Solution(B)$ , then  $\hat{\mu}_{x_{i-1}, x_i}^y \neq \mu^y$  for some unique  $i$  (since the local perturbation is inside this strip). Thus, for any  $x'$  which was part of a solution  $x$  to the original problem, it is now the case that  $\hat{\mu}_{0, x'}^y \neq \hat{\mu}_{x', \bar{x}}^y$  because by the algorithm of constructing solutions to problem B it is either the case that  $x_i \leq x'$ , in which case  $\hat{\mu}_{x', \bar{x}}^y = \mu^y$  but  $\hat{\mu}_{0, x'}^y \neq \mu^y$  or that  $x_i \geq x'$  in which case  $\hat{\mu}_{0, x'}^y = \mu^y$  but  $\hat{\mu}_{x', \bar{x}}^y \neq \mu^y$ . Thus, by the proof of Proposition 3,  $x'$  cannot be a part of a new solution.  $\square$

Denote by  $\gamma'(x)$  the perturbed reaction curve. Let  $\varepsilon = \max d(\gamma'(x), \gamma(x))$  and term the set of perturbations described above as  $\varepsilon$ -perturbations. Let B denote the original problem and B' denote the problem after an  $\varepsilon$ -perturbation. Similarly, let  $x$  denote an original solution and let  $x'$  denote a solution after an  $\varepsilon$ -perturbation.

LEMMA 10 *Following any  $\varepsilon$ -perturbation of  $F(\cdot)$  then generically for any  $x' \in Solution(B')$ , there exists an  $x \in Solution(B)$  such that  $\lim_{\varepsilon \rightarrow 0} |x_i - x'_i| = 0$  for all  $i, x_i \in x$  and  $x'_i \in x'$ .*

Proof: Consider the original solution  $x \in \text{Solution}(B)$ . Generically, for any  $x_j$  which is part of this vector, then  $\frac{d\gamma(x)}{dx}|_{x=x_j} > 0$  or  $\frac{d\gamma(x)}{dx}|_{x=x_j} < 0$ . Given a local perturbation in some strip  $[x_{i-1}, x_i]$ , the new prior is  $\hat{\mu}^y$ . Without loss of generality, assume that  $\hat{\mu}^y > \mu^y$ . Consider now the algorithm of finding a solution to problem B, as outlined in the proof of Proposition 3. In the new problem B', the first value of  $x'$ , defined as  $x'_2$ , has to satisfy  $\hat{\mu}_{0,x'_2}^y = \hat{\mu}^y$ . But when  $\varepsilon \rightarrow 0$ , then  $\hat{\mu}^y \rightarrow \mu^y$  and hence  $\hat{\mu}_{0,x'_2}^y \rightarrow \mu_{0,x_2}^y$  for  $x_2$  which the smallest value of  $x$  which satisfies  $\mu_{0,x_2}^y = \mu^y$  at the original problem. If then  $\frac{d\gamma(x)}{dx}|_{x=x_2} > (<)0$  there is a value  $x'_2 > (<)x_2$ , such that  $\lim_{\varepsilon \rightarrow 0} |x_2 - x'_2| = 0$ , which satisfies that  $\hat{\mu}_{0,x'_2}^y = \hat{\mu}^y$ . The same follows for all solutions of the algorithm.  $\square$

We can now complete the proof of this part. Note that the set of solutions for A does not change with an  $\varepsilon$ -perturbation. Following any such  $\varepsilon$ -perturbation, when  $\varepsilon$  is small enough, then each original solution for B changes infinitesimally. However, the set of solutions for A is not a continuum. Then, no new solution for B can coincide with an original solution for A. This complete part (ii) and the proof of Theorem 1.  $\blacksquare$

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