Endogenous Parties: Cooperative and Non-Cooperative Analysis

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1 Introduction

In Levy (2004) I attempt to analyze whether political parties are effective. That is, whether they change the political outcome relative to the case in which they do not exist and candidates can only run independently. The main result shows that parties are not effective when the policy space is one dimensional but may become effective when the policy space has more than one dimension. To derive this result, restrictions are imposed on how parties can form and which array of parties is defined as stable. This paper shows the robustness of the results in Levy (2004) to the introduction of different stability concepts for endogenous party formation.

In the model, society is formed of N groups, with cardinality n. The size of group i as a share of the population is $p_i, \sum_{i=1,..,n} p_i = 1$. There is a feasible policy set $Q \subseteq R^k$. Each group of voters, i, has a preference ordering $\succeq_i$ on Q represented by a utility function $u_i : Q \to R$ which is continuous and concave.[1] I will focus on preferences which are single-peaked. The ideal policy of group i is denoted for simplicity by i and the utility function admits $u_i(q) = u(q, i)$. These groups form the set of *voters*.

The set of *players* is composed of a representative from each group, i.e., there are n (finite) players, with player i having the same ideological preferences as group i.

In the first phase of the analysis, which I introduce in the next section, the players are organized in coalitions/parties. These coalitions choose platforms on which the voters vote sincerely. The platform that receives the highest number of votes is implemented. In this phase, we analyze the possible equilibria in the platform game for any coalition structure.

In the second phase, we determine which are the equilibria and the coalition structures that are stable. This is introduced in section 3, by using different stability concepts. The aim is to compare the stable political outcomes in the presence of parties, and in the absence of parties, i.e., when

[1] The results hold also for a weakly concave utility function, with a minor modification.

coalitions are composed of one player only. I show that both cooperative and non cooperative concepts rationalize the result about multidimensionality of the policy space as a necessary condition for parties being effective.

2 The Platform Game

There are N players organized in a coalition structure π which is a partition on the set N. The set of all possible partitions is denoted Π. A coalition S is a non-empty subset of players. Denote by π^0 the partition in which $|S| = 1$ for all $S \subset \pi^0$. For a partition π, denote by $\pi_{\setminus K}$ the reduction of π when K players leave the partition. That is, when the other $N \setminus K$ players do not change their positions and stay in their (possibly shrunk) coalitions.

Denote by Q_S the Pareto set of coalition S :

Definition 1: Q_S is the Pareto set of coalition S, i.e.,

$Q_S = \{q \in Q | \nexists\ q'$ s.t. $\forall i \in S$, $u_i(q') \geq_i u_i(q)$ with at least one strict inequality$\}$.

Given the above, define $k^* = \dim(Q_N)$ as the effective number of dimensions of conflict.

The platform game has one strategic stage, in which each coalition S chooses, simultaneously, an action $q_S \in Q_S \cup \{\varnothing\}$. The notation $\varnothing$ means that the coalition offers no policy in the elections, i.e., it chooses not to run.

Denote by $\mathbf{q}(\pi)$ a vector of policy platforms (which may include the null platform $\varnothing$ offered by some coalitions) where each platform is chosen by a coalition $S \subset \pi$. Given the policy vector $\mathbf{q}(\pi)$, elections are held and each voter votes for the policy that she likes most. Voters who are indifferent between several policies, use a fair mixing device. If no policy is offered, a default status quo d is implemented. I assume that $u_i(d) = -\infty$, for all i, so at least one coalition chooses to run in equilibrium. Denote by $V_S(\mathbf{q}(\pi))$ the share of votes that a platform q_S receives when $\mathbf{q}(\pi)$ is offered, $\sum_{q_S \in \mathbf{q}(\pi)} V_S(\mathbf{q}(\pi)) = 1$.

Definition 2: Given $\mathbf{q}(\pi)$, if $\mathbf{q}(\pi) \equiv \varnothing$, d is the elections' outcome. Otherwise, let $W(\mathbf{q}(\pi)) = \{q_S | V_S(\mathbf{q}(\pi)) = \max_{q_S} V_S(\mathbf{q}(\pi))\}$. The expected outcome of the elections is $\frac{1}{\sharp W(\mathbf{q}(\pi))} \sum_{q_S \in W(\mathbf{q}(\pi))} q_S$.

The utility of the players from the platform game is their expected utility from the elections' outcome, that is, for all $i \in N$, $U_i(\mathbf{q}(\pi)) = E(u_i(\mathbf{q}(\pi))) = \frac{1}{\sharp W(\mathbf{q}(\pi))} \sum_{q_S \in W(\mathbf{q}(\pi))} u_i(q_S)$ if $\mathbf{q}(\pi) \neq \varnothing$ and $u_i(d)$ otherwise.

Let Δ_S be the set of probability distributions over $Q_S \cup \{\varnothing\}$. A (mixed) strategy for a coalition S is $\delta_S \in \Delta_S$. A set of strategies for each coalition S in the partition π is $\{\delta_S\}_{S \subset \pi}$. Given a mixed strategy, denote the expected utility of player i by $U_i(\{\delta_S\}_{S \subset \pi})$. Let δ_{-S} denote the strategies taken by all coalitions but S.

Definition 3: Equilibrium in the platform game is a collection $\{\boldsymbol{\delta}_S\}_{S \subset \pi} \equiv \boldsymbol{\delta}(\pi)$ such that for all S there exists no $\delta'_S \in \Delta_S$, $\delta'_S \neq \boldsymbol{\delta}_S$ s.t. for all $i \in S$, $U_i(\delta'_S, \boldsymbol{\delta}_{-S}) \geq U_i(\boldsymbol{\delta}_S, \boldsymbol{\delta}_{-S})$ with at least one strict inequality.

The first proposition assures the existence of equilibria.

Proposition 1 *For all $\pi \in \Pi$, there exists an equilibrium $\boldsymbol{\delta}(\pi)$.*

Proof: see Levy (2004).■

Throughout the analysis, I focus on pure-strategy equilibria whenever they exist, and among them, on 'partisan' equilibria whenever they exist. A 'partisan' equilibrium is an equilibrium in which all party members vote for their party's platform if it offers one.[2] As a tie-breaking-rule, I assume that if all coalition members are indifferent between running and not running, the coalition chooses not to run.[3] Before continuing, I introduce a partial characterization of equilibria in the presence of parties for the unidimensional policy space.

Lemma 2 *When* $k^* = 1$, *for all distributions with a median voter m:*

(i) in π^0, *there exists an equilibrium in which m runs and wins, and there may exist equilibria in which two platforms are offered, where each platform is equidistant from m. No other equilibria may exist.*

(ii) If for all $S \subset \pi$, *for any* $i,j \in S$, *either* $i > j > m$ *or* $i < j < m$, *then any equilibrium in which one, two, or three parties participate, must be an equilibrium in* π^0 *as well. Moreover, it is an equilibrium for any* π' *which is finer than* π. *Any equilibrium with four or more parties running for elections, cannot be a partisan equilibrium.*

(iii) If $\exists S \subset \pi$, *with* $i, j \in S$ *and* $i \leq m < j$ *or* $i < m \leq j$, *then there exist partisan equilibria with an outcome that differs from m.*

(iv) For all π, *there exists an equilibrium in which m wins the elections.*

Proof: see Lemma 1 in Levy (2004).■

3 Endogenous coalitions

The analysis of the platform model have fixed the possible utilities that a player may acquire from each partition. In particular, each partition is characterized by an equilibrium (or a set of equilibria) and the utility from such an equilibrium, $U_i(\boldsymbol{\delta}(\pi))$, is the utility that a player i may accrue from a partition π.

In the second phase of the analysis, we are looking for stable partitions and their respective equilibria, i.e., $(\pi, \boldsymbol{\delta}(\pi))$. Note that $\boldsymbol{\delta}(\pi)$ is defined as an *equilibrium* in the platform game given π and not just a strategy vector. It may be that some partitions are stable for one equilibrium but not for another. If a partition is not stable it means that there is no equilibrium in the platform game that can support it. The analysis of stability pins down therefore not only the stable party structures but also the stable political outcomes. In particular, I am interested in comparing the political outcome in stable partitions in which there exists at least one coalition S', with $|S'| > 1$, to the partition denoted by π^0, in which $|S| = 1$ for all $S \subset \pi^0$.

In Levy (2004), I use a stability concept derived by Ray and Vohra (1997).

[2] Party members are not restricted in their votes if their party is not offering a platform. Focusing on 'partisan' equilibria serves to eliminate equilibria with four or more parties running in election. Also, the non partisan equilibria have an undesirable feature; party members object to deviations to platforms which are closer to their ideal policies.

[3] The assumption that a coalition prefers not to run in case of indifference may be justified by small campaign costs.

In this concept, cooperative in nature, players start from some coalition structure (a partition), and are allowed to fragment coalitions. The players are not allowed to form new coalitions. The deviations can be unilateral or multilateral, i.e., coalitions can deviate as well. The deviators take into account future deviations, both by members of their own coalitions and by members of other coalitions. Credible threats are deviations to finer partitions which are stable themselves. This concept allows for optimistic beliefs in terms of the equilibrium that will be played once a deviation occurs. I then show the following results:

Proposition 3 (Levy (2004)): *When $k^* = 1$, for all distributions of preferences with a median voter, parties are not effective. That is, the outcome in any stable party structure is the median voter's ideal policy which is also an outcome in π^0. When $k^* > 1$, parties may be effective.*

Proposition 4 (Levy (2004): *When $k^*>1$, even if a median voter (a Condorcet winner) exists in π^0, parties may still be effective. In particular, when the Condorcet winner is a unique equilibrium in π^0 and preferences are Euclidean, parties are effective if $k^* = n - 1$.*[4]

In this paper, I analyze the same problem using other stability concepts in order to examine the robustness of these results. Given the above discussion, there are many stability concepts and game forms that we can consider. I restrict the analysis for obvious reasons, and analyze four possible solutions, as defined below.

3.1 Definitions

3.1.1 Cooperative stability concepts

Before we introduce the cooperative concepts, we need one more definition.

Definition 4: A partition $\pi(K)$ is induced from π by a coalition K if $K \subset \pi(K)$ and $\pi_{\setminus K} \subset \pi(K)$.

That is, a group of players K deviates from π by forming its own coalition, while other players do not change their position. The group of deviators can be composed of members from different original coalitions. Thus, this definition allows deviators to form new parties.

The core $(\pi, \boldsymbol{\delta}(\pi))$ is core-stable if there does not exist a coalition K and some $\boldsymbol{\delta}(\pi(K))$, for which $u_i(\boldsymbol{\delta}(\pi(K))) > u_i(\boldsymbol{\delta}(\pi))$ for all $i \in K$.[5]

The bi-core $(\pi, \boldsymbol{\delta}(\pi))$ is bi-core-stable, if:

(i) there exists no $K \subset S \subset \pi$ and some $\boldsymbol{\delta}(\pi(K))$, for which $u_i(\boldsymbol{\delta}(\pi(K))) > u_i(\boldsymbol{\delta}(\pi))$ for all $i \in K$,

(ii) there exists no $K = S \cup S'$ for $S, S' \subset \pi$, and some $\boldsymbol{\delta}(\pi(K))$, for which $u_i(\boldsymbol{\delta}(\pi(K))) > u_i(\boldsymbol{\delta}(\pi))$ for all $i \in K$.

[4] Euclidean preferences imply that the utility of a player is monotonically decreasing with the euclidean distance from her ideal policy.

[5] To be more precise, this concept is termed the $\delta - core$ by Hart and Kurz (1983).

Note that the bi-core restricts the deviations of players relative to those allowed by the core. It allows players to deviate either by breaking a party into two parties, or by forming a party from two existing parties. Thus, whenever $(\pi, \boldsymbol{\delta}(\pi))$ is not bi-core-stable, then it is also not core-stable. On the other hand, when $(\pi, \boldsymbol{\delta}(\pi))$ is core-stable, it is also bi-core-stable. The bi-core stability concept is analyzed here because it may provide stable $(\pi, \boldsymbol{\delta}(\pi))$ in cases in which the core is empty.

Stable sets A set Z with a typical element $(\pi, \boldsymbol{\delta}(\pi))$ is stable, if:

(i) For any $(\pi', \boldsymbol{\delta}(\pi')) \notin Z$, there exists a coalition K and $(\pi'(K), \boldsymbol{\delta}(\pi'(K))) \in Z$, for which $u_i(\boldsymbol{\delta}(\pi'(K))) > u_i(\boldsymbol{\delta}(\pi'))$, $\forall i \in K$,

(ii) For any $(\pi, \boldsymbol{\delta}(\pi)) \in Z$, there exists no coalition K and $(\pi(K), \boldsymbol{\delta}(\pi(K))) \in Z$, for which $u_i(\boldsymbol{\delta}(\pi(K))) > u_i(\boldsymbol{\delta}(\pi))$, $\forall i \in K$.

3.1.2 Non-cooperative games of coalition formation:

A membership game Each player i, announces a coalition S which includes i. $\{S_i\}_{i=1..N} \equiv \mathbf{S}$ is a strategy vector. Define $\pi(\mathbf{S})$ as the outcome of the game. A coalition $S \in \pi(\mathbf{S})$ if and only if $S_i = S$ for all $i \in S$. Otherwise, $i \in \pi(\mathbf{S})$. That is, all players that do not belong to coalitions can only run independently. Then, a strategy vector $\mathbf{S}^*$ and an equilibrium $\boldsymbol{\delta}(\pi(\mathbf{S}^*))$ are Nash-stable if for no $i \in N$, there exists an announcement S_i', that satisfies $u_i(\boldsymbol{\delta}(\pi(S_i^{'}, \mathbf{S}_{-i}^*))) > u_i(\boldsymbol{\delta}(\pi(S_i^*, \mathbf{S}_{-i}^*)))$ for some $\boldsymbol{\delta}(\pi(S_i^{'}, \mathbf{S}_{-i}^*))$.

3.2 Results

Proposition 5 *Parties are not effective in the unidimensional policy space under the Core, the bi-Core, Stable Sets and the membership game. In particular, under all these stability concepts, the only stable equilibrium outcome is the median which is also an outcome in the absence of parties. Moreover, the median is a stable outcome under all these concepts. In the multidimensional policy space, the Core may yield empty predictions. However, parties may be effective under the bi-Core, Stable Sets and the membership game.*

Proof: Consider first the case of $k^* = 1$.

Stable sets I will show that there exist a unique stable set, which is composed of all $(\pi, \boldsymbol{\delta}(\pi))$ for which $\boldsymbol{\delta}(\pi)$ yields m as the political outcome.

Step 1: There exists a stable set which is composed of all $(\pi, \boldsymbol{\delta}(\pi))$ *for which* $\boldsymbol{\delta}(\pi)$ *yields* m *as the political outcome.*

Consider the set Z with all possible $\pi \in \Pi$ and $\boldsymbol{\delta}(\pi) = m$. I will show first that $(\pi, \boldsymbol{\delta}(\pi)) \in Z$ if and only if $\boldsymbol{\delta}(\pi) = m$. Consider $\boldsymbol{\delta}(\pi) \neq m$. If the outcome is biased for example to the right, then $K = \{i|i \leq m\}$ can deviate to $\pi(K)$. By part (iv) of Proposition 2, there exists $\boldsymbol{\delta}(\pi(K)) = m$, $(\pi(K), \boldsymbol{\delta}(\pi(K))) \in Z$ by construction and hence condition (i) is satisfied. If $K \subset \pi$, then $K' = \{i|i < m\}$ can be designated as the deviating coalition. On the other hand, no $(\pi, \boldsymbol{\delta}(\pi))$ with $\boldsymbol{\delta}(\pi) = m$ can be excluded from Z because trivially no $(\pi(K), \boldsymbol{\delta}(\pi(K))) \in Z$ with $\boldsymbol{\delta}(\pi(K)) = m$ can win against it.

Since all partitions in the stable set yield the same outcome, condition (ii) of the definition is satisfied as well.

Step 2: If for all $S \subset \pi$, for any $i, j \in S$, either $i > j > m$ or $i < j < m$, then when $\boldsymbol{\delta}(\pi) = m$, $(\pi, \boldsymbol{\delta}(\pi)) \in Z$ for any stable set Z.

Note that no K with $i, j \in K$ so that either $i \leq m < j$ or $i < m \leq j$ can deviate profitably for all its members. A coalition K with members from one side of the median maintains the structure of only one-sided coalitions, and cannot get more than m according to part (ii) of Proposition 2. Thus, no K and $(\pi(K), \boldsymbol{\delta}(\pi(K)))$ can win against it, implying the above by condition (i).

Step 3: Let $(\pi, \boldsymbol{\delta}(\pi))$ with $\boldsymbol{\delta}(\pi) = m$. If there exists a unique $S \subset \pi$, such that for some $i, j \in S$, either $i \leq j < m$ or $i < j \leq m$, and moreover $m \in S$ and if $i \in S$ and $i < (>)m$ then there is no $j \in S$ with $j > (<)m$, then $(\pi, \boldsymbol{\delta}(\pi)) \in Z$ for any stable set Z.

Assume not. Then there must be some K that prefers to deviate to $\pi(K)$, for some $(\pi(K), \boldsymbol{\delta}(\pi(K))) \in Z$. Obviously, K cannot consist of members from both sides of the median (in the weak sense, that is, including m). Suppose it contains only members from the left (right), that is, $\boldsymbol{\delta}(\pi(K))$ is biased to the left (right). But then it is impossible that $(\pi(K), \boldsymbol{\delta}(\pi(K))) \in Z$; m can deviate from $\pi(K)$ to form his own coalition and induce a partition in which for all $S \subset \pi$, for any $i, j \in S$, either $i > j > m$ or $i < j < m$. Such a partition and the equilibrium in which m wins, belongs to Z by step 2. Hence, $(\pi(K), \boldsymbol{\delta}(\pi(K))) \notin Z$ for all K, implying that $(\pi, \boldsymbol{\delta}(\pi) = m) \in Z$.

Step 4: If $\boldsymbol{\delta}(\pi) \neq m$, then $(\pi, \delta(\pi)) \notin Z$ for any Z.

Consider now the partition $(\pi, \boldsymbol{\delta}(\pi))$ and assume it belongs to the stable set, with a political outcome biased for example to the right. For this to happen, it must be that the partition contains two-sided parties. Then $K = \{i | i \leq m\}$ can deviate to $\pi(K)$ in which m in an equilibrium. $\pi(K)$ contains only one two-sided coalition, in which m is a member and all the other members belong to the same side of m. By step 3, $(\pi(K), m) \in Z$, a contradiction. If $K \subset \pi$, then $K' = \{i | i < m\}$ can deviate instead. $\pi(K)$ contains only one-sided coalitions, and hence by step 2, $(\pi(K'), m) \in Z$.

The above steps showed that no partition with a biased outcome can belong to any stable set and that there exists a stable set which provides m as the political outcome for all partitions.

The core and the bi-core I will now show that there exist core-stable partitions with m as an equilibrium outcome which implies that these partitions are also bi-core-stable. I then show that there are no bi-core-stable partitions with an outcome that differs from m, which implies that these partitions are also not core-stable.

To see that core-stable partitions and equilibria exist, consider π in which all coalitions are one-sided, i.e., for all $S \subset \pi$, and any $i, j \in S$, either $i > j > m$ or $i < j < m$. An equilibrium is that m offers his ideal policy. No coalition K can improve upon this equilibrium; if K includes members from both sides of the median it cannot improve the utility to all of them. If it includes members from one side of the median only, then the partition is as described in part (ii) of **Proposition** 2 and hence the only equilibria yield m

or m in expectations, which means that no profitable deviation exists. This implies also that there exist bi-core-stable partitions with m as an equilibrium outcome.

Assume now a $(\pi, \boldsymbol{\delta}(\pi))$ where $\boldsymbol{\delta}(\pi)$ yields an outcome that differs from m and is biased without loss of generality to the right. By Proposition 2, it implies that there exists an $S \subset \pi$, with $i, j \in S$, and $i < m \leq j$ or $i \leq m < j$. Then $K = \{i | i \leq m, i \in S\}$, the sub-coalition of m and all leftist players, can deviate to $\pi(K)$. In the new partition there exists an equilibrium in which the deviating coalition offers m and wins the elections. This is a profitable deviation for all $i \in K$. If, $K = S$, then the subcoalition $K' = \{i | i < m, i \in S\}$ can deviate to $\pi(K')$ in which m wins and induce a profitable deviation. Hence, any $(\pi, \boldsymbol{\delta}(\pi))$ with an outcome that differs from m is not bi-core-stable and consequently is not core-stable as well.

The membership game Finally, think of the membership game. If the game's outcome is different from m, by part (ii) of Proposition 2 there exists a coalition with members on both sides of the median. By part (iv) of Proposition 2, any partition has m as an equilibrium. If, then, the outcome is biased to the right, a member $i \leq m$ of a two-sided coalition can deviate and announce $S_i = i$. By doing so, the outcome of the game is a different partition in which m is an equilibrium. Thus, no biased outcome is Nash-stable. On the other hand, when $S_i = L$ for $L = \{i | i < m\}$ for all $i < m$, and $S_j = R$ for $R = \{i | i > m\}$, the outcome is a partition in which m is an equilibrium which is Nash-stable, and no player can change the outcome. Any one-player deviation enforces a partition with only one-sided coalitions.[6]

We continue now to prove the Proposition for the case of $k^* > 1$.

Consider the following example. There are 3 players, a, b and c. These players represent 3 groups. Let $p_i < \frac{1}{2}$ for all $i \in \{a, b, c\}$. The groups/politicians have the following preferences: $u_i(x, y) = -\alpha(i_x - x)^2 - (1 - \alpha)(i_y - y)^2$, for $i \in \{a, b, c\}$ and $\alpha = .5$. Let $(a_x, a_y) = (0, 0)$, $(b_x, b_y) = (1, 0)$, $(c_x, c_y) = (1, 1)$. Restrict the policy set to be the triangle formed from their ideal points. If the partition is $a|b|c$, generically b wins the election. If it is $ab|c$, ab win the election with a policy $(x, 0)$, for $x \in (0, 1]$. If it is $bc|a$, then bc win the election with $(1, y)$ for $y \in [0, 1)$. If the partition is $ac|b$, then ac win with a policy (x, x) for $x \in [1 - \sqrt{.5}, \sqrt{.5}]$. If the partition is abc, all feasible policies can be an equilibrium.

The core We first analyze the core. From any policy outcome in the triangle, two players can form a coalition and improve their utility with some winning platform. Thus, the core is empty.

The bi-core Now consider the bi-core. $ac|b$ is stable partition with any of its equilibrium outcome; if the grand coalition is formed, it cannot do any

[6] Note that this is true even if players are allowed to deviate together; no deviation of players from both sides of the median is profitable and any deviation of players from one side of the median ends in a partition with only one-sided coalitions.

better. If a or c break it, they reach the equilibrium in which b wins which is worse. These equilibrium outcomes differ from b, which is the unique outcome when no parties exist. Now consider the grand coalition. We can always find two members that break from it and improve their utility and hence it is not bi-core-stable. A partition in which b is a coalition member is not stable as well, unless they offer b in equilibrium; otherwise b can break it. Finally, the partition $a|b|c$ is not stable because a and c can form and induce $ac|b$.

Stable set Next, we look at the following candidate for a stable set: $\{\{ac|b\}, \{ab|c\}, \{a|bc\}\}$ with the following policies respectively: (x, x), $(z, 0)$ and $(1, y)$ for $(1, y) \sim_c (x, x)$, $(z, 0) \sim_b (1, y)$ and $(z, 0) \sim_a (x, x)$, for $x \in (1 - \sqrt{.5}, \sqrt{.5})$. Any other policy implemented by ab in $ab|c$, is beaten by either a cooperation of bc or of ac and so on. Any policy implemented by abc is beaten by at least two coalitions if not all possible three. None of these partitions is beaten by any partition in the set. This set constitutes therefore a stable set in which parties are effective.

The membership game We now examine the membership game. If a and c both announce ac, and b announces b, then any of the equilibria of the partition $ac|b$ is Nash-stable. If a or c deviates, then b wins and ac yield a (weakly) better outcome for both. Any other structure of parties is not stable. In fact, b has a weakly dominant strategy of announcing b, and staying by himself. By doing so, he can ensure not to be in a coalition with a and c or both. Hence, the only possible stable structures are $a|b|c$ or $ac|b$.[7]■

Proposition 6 *When $k^*>1$, even if a median voter (a Condorcet winner) exists in π^0, parties may still be effective when we use a membership game, bi-core, or stable sets. In particular, when we use membership games, if preferences are Euclidean and the Condorcet winner is a unique equilibrium in π^0, then parties are effective when $k^* = n - 1$.*

Proof: For the first assertion of the proposition, it is sufficient to use the example in the proof of Proposition 5. Consider the second assertion and the membership game. Assume that in π^0 there is a unique equilibrium which identifies with one of the groups, i. Then consider the structure in which $N\backslash i$ form a coalition, while i announces $S_i = i$ and stays by himself. The coalition offers a policy $q' \in Q_{N\backslash i} \cap Q_{N\backslash i}(i)$, where $Q_{N-1}(i)$ denotes all q such that $u_j(q) > u_j(i)$ for all $j \in N\backslash i$.

To see that such a policy exists, we have to show that i does not belong to the Pareto set of $N\backslash i$. This is assured when $k^* = N - 1$ (see Lemma 3 in the proof of Proposition 4 in Levy (2004)).

Since such a policy exists, all such policies constitute a Nash-stable structure. If any of these players deviates, then no coalitions are formed in equilibrium. Hence, the outcome is i. But q' is preferred to i by all coalition members, a contradiction.■

[7] The results of this example under membership game and the bi-core yield the same political outcomes as under Ray and Vohra (1997), as analyzed in Levy (2002).

4 Conclusion

In this paper I showed that the result that multidimensionality is a necessary condition for effectiveness of parties is robust to both cooperative and non cooperative solutions of the coalition formation stage. When the policy space is unidimensional, all the solution concepts that were analyzed showed that parties are not effective. In the multidimensional policy space, non cooperative solutions can provide a prediction that parties are effective, due to the restrictions on player's actions. Cooperative notions may be empty. However, when deviations are restricted (as in stable sets or in Ray and Vohra (1997)), some cooperative solutions do yield stable coalition structures in which parties affect the political outcome. In particular, cooperative and non-cooperative solutions can yield the same outcomes, as is the case for the membership game, the bi-core, and the cooperative notion analyzed in Levy (2004).

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