Does polarisation of opinions lead to polarisation of platforms?
The case of correlation neglect

Gilat Levy and Ronny Razin, LSE

Abstract: In this paper we question the common wisdom that more polarised voters’ opinions imply larger policy polarisation. We analyse a voting model in which the source of the polarisation in voters’ opinions is “correlation neglect”, that is, voters neglect the correlation in their information sources. Our main result shows that such polarisation in opinions does not necessarily translate to policy polarisation; when the electoral system is not too competitive (that is, when there is some aggregate noise in the election’s outcome), then voters with correlation neglect may induce lower levels of policy polarisation compared with rational electorates.

1 Introduction

Recent empirical evidence shows that polarisation between the two major parties in the US is on the rise (see Poole and Rosenthal, 1984, 1985, 2000). While there is a consensus about the “polarising of America”, there are different and competing explanations for the causes of this phenomenon (see a recent survey by Barber and McCarty 2013). One explanation that has been put forward as a cause for polarisation in Congress is polarisation in voters’ opinions. As Barber and McCarty (2013) put it, “...If voters are polarised, re-election motivated legislators would be induced to represent the political ideologies of their constituents, resulting in a polarised Congress.” While direct evidence about polarisation in voters’ opinions is mixed,¹ there are indications that voters are increasingly more attached to political parties on an ideological basis. Abramowitz (2010) finds that voters that are most likely to participate in politics compared to the average party identifier tend to be more extreme, with further polarisation among party activists and donors. But does increased polarisation of voters’ opinions necessarily imply polarisation of policies?

In this paper we explore the link between voter polarisation and policy polarisation. We focus on voters who have more polarised opinions as they neglect the correlation in their information sources. This “correlation neglect” bias is explored in recent literature by, among others, De Marzo et al (2003), Ortoleva and Snowberg (forthcoming), Glaeser and Sunstein

(2009) and Levy and Razin (forthcoming). Correlation neglect implies that voters have more polarised beliefs as they are more confident in their information. We then ask whether increased polarisation in voters’ opinions, arising from correlation neglect, implies more policy polarisation.

To be more precise, we analyse a voting model in which voters have to choose between two platforms, espoused by two strategic, policy-motivated, politicians. Each voter’s ideal policy depends on her political preference parameter as well as on a common, unknown, state of the world. The aggregate vote share along with an aggregate shock determine the political outcome, via majority rule. The probability of winning is therefore increasing in a platform’s expected vote share.

Prior to voting, each voter receives signals about the state of the world. We assume that while the different sources of information which generate the signals might be correlated, voters are not necessarily aware of this. We distinguish between two types of voters. Rational voters understand when the information they receive is correlated. Behavioural voters, on the other hand, are not aware of the correlation, and hence, in expectations, have more polarised ideal policies.

We first illustrate how the more polarised ideal policies of the behavioural voters may imply more informed voting. Intuitively, correlation neglect magnifies the effect of information on individuals’ opinions. Individuals who might otherwise stick with the policy that accords with the direction of their political preferences may be swayed to change their vote if they believe that their information is sufficiently strong in the opposite direction. This implies that behavioural voters base their vote more on their information rather than on their political preference parameters.

We then show that more polarised opinions in the behavioural electorates, and the more informed voting they induce, affect policy polarisation in two ways. One way corresponds to the standard intuition in the literature, and arises through the effect of vote shares on the probability of winning. More informed voting implies that the expected vote share for the right (left) policy in the right (left) state of the world is higher in the behavioural electorate. But a decreasing marginal effect of vote shares on the probability of winning (or a weak aggregate shock) implies that politicians would polarise their platforms more in the behavioural electorate. In other words, a right-wing politician would worry less about deviating to the right as he has a sufficiently high vote share in the right state and, in any

---

2In the context of financial markets, Eyster and Weizsacker (2012) conduct an experiment to show that individuals neglect correlation when choosing a portfolio (see also Kallir and Sonsino 2009 and Enke and Zimmermann 2013).

3In Levy and Razin (forthcoming) we characterize the environments under which this results in better information aggregation.
case, only a small chance of being elected in the left state.\textsuperscript{4} We term this the \textit{vote share effect}.

We find another effect however through which polarised opinions affect policy polarisation and potentially in the opposite direction. This arises through the sensitivity of the vote shares themselves to deviations or the \textit{marginal voter effect}. In the behavioural electorate, voters are more confident in their information, and thus it is the more ideologically extreme voters (in terms of their political preference parameter) who are the marginal voters that will change their vote when a politician polarises. In the rational electorate on the other hand such marginal voters are relatively moderate. The difference between the two electorates arises when information is correlated; if the ideologically extreme voters have a higher degree of correlation in their information sources, then the expected vote shares in the behavioural electorate become more sensitive to deviations. In this case, politicians would be less prone to policy polarisation when facing the behavioural electorate as they would worry about losing too many marginal voters.

We show then that policy polarisation can both increase or decrease when voters have correlation neglect. When the electoral system is very competitive (modelled as a weak aggregate shock), a candidate’s probability of winning is very sensitive to the level of their expected vote shares. In this case the vote share effect dominates and correlation neglect leads to more polarisation of both opinions and policy platforms. When the electoral system is relatively uncompetitive, the marginal voter effect will play a bigger role and thus correlation neglect may lead to more polarisation of opinions along with lower polarisation of platforms.

We next endogenise the information sources that voters are exposed to, by assuming that they can, at a cost, make their information sources more independent. We show that moderates and extremists invest relatively less in the quality of their information compared with intermediate voters.\textsuperscript{5} Endogenising the level of independent information that voters have strengthens the result that polarisation of opinions does not necessarily lead to polarisation of platforms. Specifically, we show that the more extreme voters will invest less in the independence of their information. This, through the marginal voter effect, implies that in behavioural electorates expected vote shares are more sensitive to deviations and thus lower polarisation may arise.

\textsuperscript{4}This intuition is stated in Barber and McCarty (2013), “... the incentives for parties to take positions that appeal to supporters of the other party will diminish. This leads to greater partisan polarization.” This effect is also similar to those derived in Gul and Pesendorfer (2013) and Yuksel (2014), among others.

\textsuperscript{5}Having in mind our particular interpretation, one can relate our results above to the literature on voter sophistication (see Bartels 1996, Weisberg and Nawara 2010). While some find support for why extreme voters are more politically savvy (Sidanius and Lau 1989) others show that it is the ideological centrists who are the most sophisticated in their voting (Tomz and Van Houweling 2008).
Finally we discuss some welfare implications of our results. As shown in Bernhardt, Duggan and Squintani (2009), greater polarisation may be welfare improving as it provides voters with better choice. In our analysis, the welfare benefits of polarisation are increasing in voters’ information. A behavioural electorate behaves as if it is more informed and thus polarisation is desirable. We illustrate using an example how behavioural electorates may have higher welfare compared with rational ones.\(^6\)

Our paper analyses the strategic response of politicians to changes in voters’ ability to process information. In this regard it is in line with Ashworth and Bueno de Mesquita (forthcoming), who provide several examples under which behavioural biases might be beneficial for voters when one takes into account the strategic behaviour of politicians.

Our focus is on correlation neglect, a behavioural bias about which there is a growing literature. In Ortoleva and Snowberg (forthcoming) individuals receive a stream of signals, some correlated. Their model implies that the higher is the level of correlation neglect of an individual, the more extreme his beliefs will be. Glaeser and Sunstein (2009) model a similar behaviour in a group setup, where agents ignore the correlation between theirs’ and others’ information. They show that groups may perform worse than an individual decision maker, and that greater polarisation and overconfidence arises in groups.\(^7\) In Levy and Razin (forthcoming) we characterize distribution functions for which behavioural electorates induce better information aggregation for the case of fixed platforms and exogenously given information. In this paper we analyse instead how each electorate, rational or behavioural, affects the choices of politicians. We also endogenise the level of correlation in the information sources of the voters.

Some recent papers show how platform polarisation can arise when the public is more informed or has more polarised preferences. Feddersen and Gul (2014) analyse a dynamic model in which party polarisation and income inequality are positively correlated. Greater inequality affects the distribution of donors (whose contributions affect the election result) towards those that endorse less redistribution. Yuksel (2014) and Gul and Pesendorfer (2012) show how changes in voters’ sources of information can affect platform polarisation. Yuksel (2014) focuses on specialisation in information gathering while Gul and Pesendorfer (2012) focus on the effects of competition in the media. Both generate an effect which is similar to the vote share effect in our model, where a more informed electorate will encourage politicians to polarise. In contrast, by focusing on polarisation of opinions that arises from a cognitive bias of processing information, we show that the opposite can also arise. That is, in our model, even when voting is more informed, platform polarisation might be lower.

\(^6\)See also Martinelli (2006) and Degan (2006). Degan (2006) finds that polarisation increases information acquisition by voters, as we do.

\(^7\)In Glaeser and Sunstein (2009) information is shared in the group prior to making a decision. In our model information is aggregated through a vote.
2 The Model

Politicians and voters: There are two politicians, $r$ and $l$, who choose platforms $x_r \in [0, 1]$ and $x_l \in [-1, 0]$ to compete in the election. There is a continuum of voters, each characterized by a political preference parameter $v_i$, distributed uniformly on $[1, 1]$. The ideal policy of voter $i$ is $v_i + \omega$, where $\omega \in \{1, 1\}$ is an initially unknown realisation of a state of the world. The common prior is that each realisation occurs with equal probability.

Denote the political outcome by $y$ and let $\hat{y}$ denote the policy that the voter votes for. We assume that a voter maximizes the following utility function:

$$\alpha U(v_i, \omega, y) + U(v_i, \omega, \hat{y})$$

where

$$U(v_i, \omega, z) = - (\omega + v_i - z)^2.$$

The first element implies that a voter derives utility from the political outcome matching her ideal policy, and the second element implies that she also derives utility from her individual vote matching her ideal policy. The parameter $\alpha > 0$ denotes the weight the voter puts on the first term. Note that as we have a continuum of voters, voters are pivotal with zero probability. The second element of their utility function will therefore induce them to vote sincerely.\(^8\) Therefore, a voter $i$ votes for $x_r$ iff

$$E_\omega[-(\omega + v_i - x_r)^2] \geq E_\omega[-(\omega + v_i - x_l)^2].$$

Note that for all voters in $[-1, 1]$, if the platforms are symmetric so that $x_l = -x_r$, then platform $x_r$ is the optimal policy in state $\omega = 1$ and platform $x_l$ is the optimal policy in state $\omega = -1$.

We assume that the two politicians have single-peaked policy preferences, given by a quadratic loss function, which does not depend on $\omega$ (for simplicity). Let the ideal policy of $r$ ($l$) be $z_r = 1$ ($z_l = -1$). The utility of candidate $j \in \{r, l\}$ is then, for some $\mu > 0$:

$$-\mu (z_j - y)^2.$$

The information structure: Voters will base their voting strategy on their political preferences and on their beliefs about $\omega$. We assume that prior to voting, each voter $i$ receives two private signals $s_1^i, s_2^i \in \{-1, 1\}$, each with accuracy $q > \frac{1}{2}$, that is, $\Pr(s_j^i = \omega | \omega) = q$ for $j \in \{1, 2\}$. Furthermore, with probability $\rho(v_i)$, the two signals are fully correlated,

As the focus of this paper is on voters who are unable to process information correctly, we find it more suitable to consider sincere voting and abstract away from complicated strategic voting considerations. Our results also hold with strategic voting. For some recent examples of a similar approach, see Gul and Pesendorfer (2012) or Chan and Suen (2008).
and with probability $1 - \rho(v_i)$, they are conditionally independent. Note that this is the simplest possible environment in which we can consider correlation neglect but our results generalize to more complicated information structures. For now we assume symmetry in the distribution of $\rho$ so that $\rho(v_i) = \rho(-v_i)$. We later on endogenise the function $\rho(v_i)$ and its symmetry properties.

In what follows we compare “behavioural” electorates to rational electorates. We assume that a “behavioural” voter does not understand that the signals might be correlated. Such a voter always believes that she has two (conditionally) independent signals. A rational voter, on the other hand, is aware of the information structure and specifically also fully recognizes when the signals are correlated and when they are independent.\(^9\)

Finally, we assume that at the time of choosing their platforms, the politicians do not have additional information about the state of the world beyond the equal prior.

**The political system:** As in all models with policy polarisation, we need some uncertainty in the election’s outcome in order to induce polarisation (otherwise convergence to the median must arise). We do so in the simplest possible way. Let $V_x$ be the vote share for some policy $x$ (the vote share would depend on the type of the electorate and on the state of the world). We assume that there is some noise in the political system so that policy $x$ is chosen with probability $G(V_x)$, for some continuous and increasing function $G$ which is symmetric around a half (so that $G(V_x) = 1 - G(1 - V_x)$) and concave above a half (and hence convex below a half). Figure 1 below presents examples of $G$:\(^{10}\)

\[\text{Figure 1: Concave and linear } G.\]

\(^9\)The assumption that a rational voter fully recognizes when the signals are correlated is made for simplicity; we can assume instead that he uses the probability $\rho$ in his Bayesian updating.

\(^{10}\)One can extend this family to consider functions which are convex above a half. The concavity of $G$ allows for equilibrium existence and plays a role in polarisation as we show below.
Note that the more concave is \( G \), the more likely it is that a policy \( x \) would be chosen if \( V_x > \frac{1}{2} \). On the other hand, the less concave it is, the less this is sufficient. We can interpret a more concave \( G \) as a more competitive electoral system, that is, one that is less subject to noise and shocks. Below we provide an example that microfound such family of \( G \)'s and will serve as a leading example henceforth:

**Example 1:** Suppose that society is composed of \( N \) (odd) districts, where each district has a continuum of voters. For simplicity assume that districts are identical so that voters are distributed uniformly in each district as described above. This implies that the vote share for platform \( x_j \in \{x_l, x_r\} \), \( V_{x_j} \), would be identical across districts by the law of large numbers. Assume that in each district \( k \), a candidate for the party proposing platform \( x_j \) wins if \( V_{x_j} > V_{-x_j} + \zeta_k \), where \( \zeta_k \) is an idiosyncratic district-noise, distributed uniformly on \([-1,1]\). One example of such shocks is extreme weather events, which affect turnout.\(^{11}\) Platform \( x_j \) (or party \( j \)) wins the overall election if it wins a majority of the districts. The probability that platform \( x_j \) is elected is therefore

\[
G^N(V_{x_j}) = \sum_{k=\frac{N+1}{2}}^{N} \binom{N}{k} (V_{x_j})^k (1 - V_{x_j})^{N-k}. \tag{1}
\]

It is easy to verify that \( G^N(V) \) is a concave function above \( V > 0.5 \), symmetric around a half (and hence convex for \( V < \frac{1}{2} \)) and moreover, that the larger is \( N \), the more concave it is (for \( V > 0.5 \)).\(^{12}\) To see why, note that when \( N = 1 \), the noise \( \zeta \) looms large. Achieving a vote share greater than a half only guarantees a probability of being elected set at \( V \). When there are many districts though, the idiosyncratic district level shocks cancel each other out on the aggregate. For large \( N \), this implies that a vote share larger than a half guarantees winning with probability almost one. A political system with a larger \( N \) can be interpreted therefore as more competitive in the sense that it is more immune to idiosyncratic district shocks.

**Timing and equilibrium:** In the first stage, politicians choose their platforms. The voters then receive their information and vote in the second stage. All voters vote sincerely and the politicians best respond to each other’s choice given the forecasted vote of the electorate.

\(^{11}\)Recent empirical literature in Political Science has shown how weather affects electoral outcomes. Gomez, Hansford and Krauze (2007) examine the effect of weather on voter turnout in 14 U.S. presidential elections by using data from 22,000 U.S. weather stations, and find that rain significantly reduces voter participation by a rate of just less than 1% per inch, while an inch of snowfall decreases turnout by almost .5%. Moreover, they find that poor weather benefits the Republican party’s vote share.

\(^{12}\)When \( N \to \infty \), \( G^N(V) \to 1 \) for \( V > 0.5 \).
Remark 1: Polarisation of opinions. In the above model the beliefs of all voters, rational or not, will have ex ante the same mean. The ex ante variance however will be different; beliefs will differ between rational voters and those with correlation neglect in the second order stochastic dominance sense. To illustrate the polarisation of beliefs, we now plot the ex ante distribution of beliefs in state $\omega = 1$, for the parameters $q = 0.75$ and $\rho(v) = 0.75$ for all $v$. The $x$–axis represents the beliefs that a voter would have, and the $y$–axis represents the probability of having these beliefs. For example, a voter who receives two contrasting signals will have beliefs $0.5$, and this event arises with probability $2(1 − \rho)q(1 − q) = 0.09$, and so on. As can be seen below, the beliefs of behavioural voters are more polarised:\(^{13}\)

![Figure 2: The distribution of beliefs of rational and behavioral voters when $\omega = 1$.](image)

We next consider whether such greater polarisation of beliefs, and hence of expected ideal policies, induces greater polarisation of platforms.

\(^{13}\)Note that in the figure, a rational voter holds the beliefs $0.25$ and $0.75$ with some probability while a behavioural voter doesn’t. This is because a rational voter is sometimes aware of the correlation in the signals and therefore updates based on one signal only. This is not the case for the behavioural voter, who always believes he has observed two independent signals.
3 Voters’ behaviour

We start by using backward induction and analyse the behaviour of voters in the two societies. We will derive two preliminary results that will be important in determining the vote share effect and the marginal voter effect later on. We will show that when the platforms are symmetric, the expected vote share for the optimal policy (e.g., the right-wing policy in the right-wing state), is higher in the behavioural electorate. Moreover, we will show that marginal voters will be more extreme in the behavioural electorate compared with the rational one.

Assume that $x_r = -x_l$. It is easy to see that a voter would vote for $x_r$ if he believes that $\omega = 1$ with a sufficiently high probability and for $x_l$ if he believes that $\omega = -1$ with a sufficiently high probability (where how high this probability has to be depends on the voter’s preference parameter).

Next note that the voting behaviour of rational and behavioural voters differs only when voters receive the two fully correlated signals, (1,1) or (-1,-1). Consider first a behavioural voter, who (wrongly) believes he has two independent signals. Then there exists some cutoff $v_2$, such that a voter would vote for $x_r$ when he has (1,1) and for $x_l$ when he has (-1,-1), if his preference parameter $v$ is in $[-v_2, v_2]$. Note that these voters would vote for $x_r$ with probability $q$ in state $\omega = 1$ for example, which is the probability that the signal (which is repeated) matches the state. Extreme voters, above $v_2$ or below $-v_2$, would vote with their ideology, to $x_r$ and $x_l$ respectively, as even two opposite signals would not convince them otherwise. Their vote is therefore not informative.

On the other hand, consider a rational voter who recognizes that the signals are correlated and thus realizes he really has only one signal. Then there exists a cutoff $v_1$, $v_1 < v_2$, such that he votes for $x_r(x_l)$ if his signal is 1(-1) and his preference parameter $v$ is in $[-v_1, v_1]$. Again, in state $\omega = 1$, voting for $x_r$ would arise for these voters with probability $q$, the probability that the signal matches the state. Rational voters above $v_1$ or below $-v_1$ who have only one signal would vote with their ideology and not use their information.

Finally, it is easy to derive that $v_1 = 2q - 1 < v_2 = \frac{2q - 1}{q^2 + (1-q)^2}$. The key observation is that $v_1 < v_2$ as behavioural voters are more confident in their information.

Figure 3 shows the probability that different voters vote for the right-wing policy in the right-wing state, if $s_i^2$ and $s_i^1$ are correlated:
Figure 3: The probability that different voters vote for \( x_r \) in state \( \omega = 1 \), conditional on the signals being correlated.

There are two implications from the above. First, in the event in which the signals are correlated, the marginal voters in the behavioural electorate are more extreme, at \( v_2 \) and \(-v_2\) instead of at \( v_1 \) and \(-v_1\). These are the cutoffs at which voters start ignoring their information as their preference parameters become more extreme (vote for example to \( x_r \) with probability 1 instead of probability \( q \)). This will play a role in the marginal voter effect below. This is intuitive as the behavioural voters believe that they have two independent signals instead of one and are thus more confident in their information.

Second, the above implies that the expected vote share for the optimal policy is higher in the behavioural electorate. Intuitively, the confidence of behavioural voters in their information implies that more of them use information when voting rather than their political preference parameters. To see this more precisely, consider again Figure 3. As can be seen in the figure, when for example \( \omega = 1 \), and the signals are fully correlated, intermediate voters in \([v_1, v_2]\) vote for \( x_r \) with probability 1 when rational, and with probability \( q \) when behavioural. In \([-v_2, -v_1]\), voters vote for \( x_r \) with probability 0 when rational and with probability \( q \) when behavioural. By symmetry and as \( q > \frac{1}{2} \), the expected aggregate vote share for \( x_r \) in state \( \omega = 1 \) is larger for the behavioural voters.\(^{14}\) Formally, let \( V^J_x(\omega) \) be the vote share for policy \( x \) in state \( \omega \) under electorate \( J \in \{R, B\} \), that is, a rational or a

\(^{14}\)Specifically, it is larger by \( \int_{v_1}^{v_2} \rho(v)(2q - 1) \frac{1}{2} dv > 0 \).
behavioural electorate respectively. We then have:

**Lemma 1:** For all \( x_r = -x_l \), the vote share for the optimal policy is higher in the behavioural electorate in both states, that is, \( V_{x_r}^B(1) > V_{x_r}^R(1) > \frac{1}{2}, V_{x_l}^B(-1) > V_{x_l}^R(-1) > \frac{1}{2}. \)

Note that in both electorates, the expected vote share for the optimal policy is larger than a half as voters base their vote on some information, and those who do not, cancel each other out. Still, different vote shares for the optimal policy in the two electorates will affect politicians’ probability of winning differently, which will play a role in the vote share effect below.

4 Polarisation and correlation neglect

We now derive the equilibrium level of polarisation in the two electorates as chosen by the politicians. We first derive the equilibrium condition and show that the difference between the two electorates will be manifested in how polarising deviations affect the probability of winning. We then show how this change in probabilities of winning can be decomposed into the marginal voter effect and the vote share effect. We then put these two effects together and derive our main result about polarisation.

4.1 Politicians’ optimal choice

Let \( \Delta_r(x_r, x_l) = -\mu(1-x_r)^2 - (-\mu(1-x_l)^2) \). The expected utility of candidate \( r \) in electorate \( J \in \{R, B\} \) can be written as:

\[
Pr^J(r \text{ elected})\Delta_r(x_r, x_l) - \mu(1-x_l)^2,
\]

where

\[
Pr^J(r \text{ elected}) = \frac{1}{2} G(V_{x_r}^J(1)) + \frac{1}{2} G(V_{x_r}^J(-1)).
\]

Given \( x_l \), politician \( r \) chooses \( x_r \) and the first order condition is:

\[
\frac{\partial Pr^J(r \text{ elected})}{\partial x_r} \Delta_r(x_r, x_l) + \frac{\partial(\Delta_r(x_r, x_l))}{\partial x_r} Pr^J(r \text{ elected}) = 0
\]

As is standard in such models, when a politician considers deviating, a trade-off arises between her chances of being elected, and her utility conditional on being elected. If politician \( r \) moves her platform further to the right, her utility will be higher conditional on being elected, while her probability of being elected is reduced. The equilibrium balances these two incentives.\(^{16}\)

\(^{15}\)As we show in Levy and Razin (forthcoming), this arises more generally, and in both states of the world, in societies which are sufficiently balanced between right and left wing voters.

\(^{16}\)Second order conditions will always be satisfied, as the politicians’ utility is concave, \( G \) is concave in \( V \), and \( V \) would be linear in \( x_r \).
Evaluated at a symmetric equilibrium, the first-order condition becomes

$$
\frac{\partial \Pr^J(r \text{ elected})}{\partial x_r} \Delta_r(x_r, x_l) + \frac{\partial (\Delta_r(x_r, x_l))}{\partial x_r} \frac{1}{2} = 0
$$

(2)

It is easy to use (2) to show (the proof of this and all results that follow are in the appendix):

**Proposition 1:** For any electorate \( J \in \{R, B\} \), there exists a unique pair of symmetric equilibrium platforms \((x^J_r, x^J_l)\).

It is clear though from the above that when politicians choose their platforms, the only difference between the two electorates is how each electorate \( J \) affects \( \frac{\partial \Pr^J(r \text{ elected})}{\partial x_r} \). This can be written as:

$$
\frac{\partial \Pr^J(r \text{ elected})}{\partial x_r} = \frac{1}{2} \frac{\partial G(V^J_{x_r}(1))}{\partial V^J_{x_l}(1)} \frac{\partial V^J_{x_l}(1)}{\partial x_r} - \frac{1}{2} \frac{\partial G(V^J_{x_r}(-1))}{\partial V^J_{x_l}(-1)} \frac{\partial V^J_{x_l}(-1)}{\partial x_r}
$$

(3)

Note that as \( G \) is symmetric around a half, and as the symmetry of the model implies \( V^J_{x_r}(1) = V^J_{x_l}(-1) = 1 - V^J_{x_r}(-1) \), we can write it as:

$$
\frac{\partial \Pr^J(r \text{ elected})}{\partial x_r} = \frac{1}{2} \frac{\partial G(V^J_{x_l}(1))}{\partial V^J_{x_l}(1)} \left( \frac{\partial V^J_{x_l}(1)}{\partial x_r} + \frac{\partial V^J_{x_l}(-1)}{\partial x_r} \right)
$$

(4)

As can be seen from (4), the reduction in the probability of winning upon deviation can be decomposed into two effects. The first effect is the derivative of the probability of winning with respect to a change in vote shares, \( \frac{\partial G(V^J_{x_l}(1))}{\partial V^J_{x_l}(1)} \). A higher vote share for the optimal policy, or a more informed voting, may affect differently the actual probability of winning, manifested by \( G \). This effect is denoted as the vote share effect. The second effect is the sensitivity of vote shares to deviations, \( \frac{\partial V^J_{x_l}(\omega)}{\partial x_r} \). This will be negative as some marginal voters would be less inclined to vote for the deviating right-wing politician. This is what we denote as the marginal voter effect.

### 4.2 The vote share effect

We first consider the vote share effect, i.e., that of \( \frac{\partial G(V^J_{x_l}(1))}{\partial V^J_{x_l}(1)} \) in (4). By Lemma 1, evaluated in a symmetric equilibrium, we have that \( V^B_{x_r}(1) > V^R_{x_r}(1) \). This then implies that \( \frac{\partial G(V^B_{x_l}(1))}{\partial V^B_{x_l}(1)} \leq \frac{\partial G(V^R_{x_l}(1))}{\partial V^R_{x_l}(1)} \) as \( G''(V) \leq 0 \) for \( V > \frac{1}{2} \). As illustrated in the figure below, the overall change in the probability of being elected will be smaller when \( r \) deviates if the electorate is behavioural. This implies that this effect induces candidates to polarise more in the behavioural society as they worry less about the loss of the overall probability of being elected:
This effect is driven by the result of Lemma 1: More informed voting in the behavioural electorate implies that a deviation by the politician will not damage much her chances of winning in the correct state of the world, and similarly her chances of losing in the wrong state of the world.

This effect is in line with the intuition expressed in the literature that more polarised beliefs, or a more informed public (which is what, on aggregate, a behavioural society is), will induce more platform polarisation. As shown above, the concavity of the function $G$, which can be interpreted as a feature of the electoral system, can exacerbate this effect. Thus a more competitive electoral system that induces a more concave $G$ would have a stronger such vote share effect, while a less competitive system will have a weaker such effect.

**Example 1 revisited:** When $N = 1$ for example, the differential effect identified above does not arise as in this case $G$ is linear and $\frac{\partial G^N(V_J^J(1))}{\partial V_J^J(1)} = 1$ is fixed for all $V$. Thus when $N = 1$ there will be no such difference between rational and behavioural electorates. More generally, using (1), we have that

$$
\frac{\partial G^N(V_J^J(1))}{\partial V_J^J(1)} = \gamma^N [V_J^J(1)(1 - V_J^J(1))^{\frac{N-1}{2}}]
$$

for some $\gamma^N$ increasing in $N$. As by Lemma 1 $V_J^J(1) > 0.5$, $\frac{\partial G^N(V_J^J(1))}{\partial V_J^J(1)}$ is decreasing in $V_J^J(1)$ as expected. Moreover:

$$
\frac{\partial G^N(V_J^J(1))}{\partial V_J^J(1)} = \frac{V_J^R(1)(1 - V_J^R(1))^{\frac{N-1}{2}}}{V_J^B(1)(1 - V_J^B(1))^{\frac{N-1}{2}}} \geq 1,
$$

for all $N \geq 1$. Specifically, it equals 1 for $N = 1$, greater than 1 otherwise, and goes to infinity for a large enough $N$. 

\[17\] See Barber and McCarty (2013).
4.3 The marginal voter effect

We now consider the marginal voter effect, i.e., how the vote shares change in the two types of electorates. Consider again only the event where the signals are correlated which is what differentiates the two electorates. Note that following a deviation of $r$ to the right, all cutoffs, $v_1, -v_1, v_2$ and $-v_2$, move to the right as voters become less inclined to vote for $x_r$:

Consider the behavioural voters first (and recall that we fix $\omega = 1$). When the signals are correlated, a voter in $-v_2$ moves from voting right with probability $q$ to voting right with probability 0, and a voter in $v_2$ moves from voting to $x_r$ with probability 1 to voting $x_r$ with probability $q$. All changes around the $v_1$ and $-v_1$ cutoffs are neutral as voters in this region behave in the same way below and above the cutoffs. Thus for the behavioural voters the overall change in the vote share -on both the right and the left- amounts to a lower vote share in the order of $-\rho(-v_2)q + \rho(v_2)(q - 1) = -\rho(v_2)$. For the rational voters this change, analogously, amounts to $-\rho(-v_1)q + \rho(v_1)(q - 1) = -\rho(v_1)$. The change for such electorate arises at the point in which voters change their behaviour when they have one signal, which is $v_1$ and $-v_1$.

More generally, in terms of the difference between the two electorates, it is more moderate voters that are sensitive to deviations when we consider the rational voters’ response, and the more extreme voters that are sensitive to a policy deviation when we consider the behavioural voters’ response. The reason is that behavioural voters believe that their information is stronger than what it really is, and thus change their actions only when their preference parameter is strong enough. As the difference between the two electorates is manifested only when the signals are correlated, what is important therefore is the degree of correlation in the information sources of the different marginal voters in each electorate. Formally:

![Figure 5: Cutoffs switch to the right with a deviation of the right-wing politician, so that less voters vote for $x_r$.](image-url)
Lemma 2: \( \frac{\partial V_x^J(\omega)}{\partial x_r} \leq 0 \) and \( \frac{\partial V_x^B(\omega)}{\partial x_r} - \frac{\partial V_x^R(\omega)}{\partial x_r} = \frac{1}{4}(\rho(v_1) - \rho(v_2)). \)

In words, following a deviation to the right, the vote share for \( x_r \) will be reduced less in a rational society compared with a behavioural society if and only if \( \rho(v_1) < \rho(v_2) \).

### 4.4 Policy polarisation

We have identified two effects that differentiate how the probability of a politician to be elected changes when she deviates in a behavioural and a rational electorate. Both effects analysed above relate to the observation that behavioural voters have more polarised opinions. More polarised opinions imply a higher vote share for the policy that matches the state of the world (the vote share effect). More polarised opinions is also what is behind the observation that the voters who are sensitive to deviations are more extreme in the behavioural electorate (the marginal voter effect).

We can now bring these two effects together. If a deviation of the \( r \) politician to the right brings about a stronger reduction in the probability of winning in one electorate, then the politician will be less inclined to polarise, resulting in relative policy moderation in this electorate.

If \( \rho(v_1) > \rho(v_2) \), the informed voting and the marginal voter effects are in line, implying that

\[ \frac{\partial Pr^R(r \text{ elected})}{\partial x_r} < \frac{\partial Pr^B(r \text{ elected})}{\partial x_r} < 0. \]

On the other hand when \( \rho(v_1) < \rho(v_2) \), the opposite result might arise. Consider for example a function \( G \) whose derivative is not too responsive to the different vote shares, for example when \( G(V) = V \). In this case the vote share effect is the same in both electorates (as the derivative does not depend on \( V \)), while the marginal voter effect looms large implying that:

\[ 0 > \frac{\partial Pr^R(r \text{ elected})}{\partial x_r} > \frac{\partial Pr^B(r \text{ elected})}{\partial x_r}. \]

We can then use the above to deduce:

**Proposition 2:** Compared with a rational electorate, (i) the equilibrium in a behavioural electorate has more platform polarisation if \( \rho(v_1) > \rho(v_2) \), (ii) the equilibrium in a behavioural electorate has less platform polarisation if \( \rho(v_1) < \rho(v_2) \) and the political system is not too competitive.

Again, let us use Example 1 to illustrate the result. In this case we have that (for some
constant $\kappa$):

$$
\frac{\frac{\partial \Pr^R(r \text{ elected})}{\partial x_r}}{\frac{\partial \Pr^B(r \text{ elected})}{\partial x_r}} < (> 1) \iff \frac{\kappa}{\kappa + 0.25(\rho(v_1) - \rho(v_2))} > (<) \frac{\frac{V^R_{x_r}(1) - V^B_{x_r}(1)}{V^R_{x_r}(1) - V^B_{x_r}(1)}}^{\frac{N-1}{2}}.
$$

As the RHS is (weakly) larger than 1, if $\rho(v_2) < \rho(v_1)$ polarisation must be smaller in the rational electorate. On the other hand, when $\rho(v_2) > \rho(v_1)$, and as the RHS equals 1 for $N = 1$, and converges to infinity for large $N$, the result as in the Proposition follows.

Thus, polarisation of opinions induced by correlation neglect does not necessarily induce polarisation of platforms by candidates. It is the interaction between how sensitive is the election outcome to the vote shares, and how sensitive is the vote share to deviations, which determines the overall effect on polarisation. The first effect implies that behavioural electorates would induce more polarisation while the latter effect implies that for $\rho(v_1) < \rho(v_2)$, rational electorates might induce more polarisation.

One intuition would imply that it has to be that $\rho(v_1) < \rho(v_2)$: more extreme voters might invest less in their information, or may worry less about the quality of their information. The degree of the correlation in their information sources may therefore be higher. In the next section we provide a model to endogenise $\rho(v)$ and we show that indeed $\rho(v_1) < \rho(v_2)$.

5 Extension: endogenous levels of correlation

Voters take active decisions relating to the sources of information they are exposed to: Voters choose how much time to invest in learning about political issues, they choose whom to speak with, what to read or watch. They can also choose between different news outlets. Some papers, such as the free papers that are distributed in train stations and on buses, tend to reprint bits of news from other sources and have very little original content. Expensive broadsheets or magazines, would typically offer a more investigative approach and might provide an independent source of information to the reader, compared to what he might hear or read in other outlets. In this section we allow voters to invest in the quality of their information which we interpret as the degree of independence of their information sources. While this is interesting in itself, we will also show that this implies that $\rho(v_1) < \rho(v_2)$, which strengthens our result that behavioural electorates can induce less polarisation.

Specifically, we make the following changes to the model. Assume that prior to receiving her information, a voter $i$ can decrease the level of correlation $\rho(v_i)$ or in short $\rho_i$ by investing according to the cost function $c(1 - \rho_i)$, with $c', c'' > 0$. If a voter does not invest, then $\rho_i = 1$. Individuals make their investment decisions conditional on $v_i, x_l, x_r$ and $q$. For simplicity, we assume that when considering how much to invest, a voter with correlation neglect is not
aware that when she will actually observe the signals, she will misinterpret their sources. Our results also hold when voters are aware that they may misinterpret their information.\footnote{In such a case, voters may actually invest in better quality of information (higher degree of independence) in order to protect themselves from being fooled. Thus all such voters invest more (weakly) in information compared with rational voters, which implies that all our results follow.} The rest of the model remains the same.

We start by considering the model with fixed symmetric platforms, $x_r = x$ and $x_l = -x$, and then proceed to endogenise the platforms.

Voters know that given their $\rho_i$, they will receive signals and vote optimally, yielding an ex ante random policy choice $\hat{y}(\rho_i)$. They choose then $\rho_i$ to maximize $E_{\omega} U(v_i, \omega, \hat{y}(\rho_i))-c(1-\rho_i)$. That is, they maximize the probability that they themselves vote for the optimal policy, minus the cost of their investment.

Note that as behavioural voters are not aware of their correlation neglect, and believe that they will recognize when the signals are correlated and when they are not, they will invest in exactly the same level of independent information as rational voters do. When deciding on their degree of independence of information sources, all voters therefore compute the benefit of having two independent signals compared with just one signal and compare it to the cost. We therefore have the following:

**Lemma 3:** Let $x_r = x = -x_l$ and consider all types with $v_i \geq 0$ (types with $v_i < 0$ are characterized symmetrically). For both rational and behavioural voters, the degree of independent information that a voter $i$ has, $1-\rho(v_i)$, increases in $v_i$ for $v_i \in [0, v_1]$, decreases in $v_i$ for $v_i \in [v_1, v_2]$, and is 0 for $v_i \geq v_2$. Also, for all $i$, $\rho(v_i)$ decreases in the degree of polarisation $x$.

Consider first moderate voters (with $v > 0$ but below $v_1$). Getting more independent information implies that they will vote more often for $x_r$, which is the platform that accords more with their political preferences. To see why, note that with one signal they vote to the left whenever this signal indicates that the state is -1, but with two signals they do so only when the two signals are -1.\footnote{When the signals are different, (-1,1), they do not learn anything and hence vote for $x_r$, let alone when the signals are (1,1).} In this region therefore, the stronger is the political preference to the right, the more investing in information is favourable.

Intermediate voters on the other hand will vote to the right when they have only one signal and thus investment in information increases their probability of actually voting to the left. The more ideological they are, the less attractive this is, and hence investment in information is decreasing in their type in this region. Finally, as the second signal does not affect the voting decisions of the most extreme voters, they do not invest at all in the quality
of information. Figure 6 below graphs the $\mathcal{M}$-shaped independent information level $1 - \rho_i$ given the ideology $v_i$:

![Diagram of $1 - \rho_i$ as a function of the ideology $v_i$.](image)

Figure 6: $1 - \rho_i$ as a function of the ideology $v_i$.

We now consider endogenous platforms. The only potential difference in the analysis when the quality of information is endogenous is that individuals will change their investments when candidates change their platforms. In line with Lemma 3, when there is more polarisation, voters invest more because their loss from taking a wrong decision is higher. This implies that when a candidate deviates to a more extreme platform, voting becomes more informative. As we show in the proof however, this effect cancels out as ex-ante, it equally pulls the vote shares in opposing directions for the different states of the world. In other words, the additional votes that a right-wing candidate receives in state $\omega = 1$ (as voters are more informed) equals the reduction in votes that she receives in $\omega = -1$. This implies that the analysis boils down to the effects identified in Section 4.

Moreover, from Lemma 3 we know that $\rho(v_1) < \rho(v_2) = 1$. This arises as the type in $v_2$ finds information of very little use, whereas a moderate voter in $v_1$ changes his behaviour with information and thus finds it useful to invest in the quality of information. Therefore, using similar arguments as in Proposition 2 we can show,

**Proposition 3:** With endogenous information investment $\rho(v_1) < \rho(v_2)$, and therefore if the electoral system is not too competitive, then for any equilibrium in the model with rational voters, there is an equilibrium with less polarisation in the model with behavioural voters. If the electoral system is too competitive, the opposite arises.$^{20}$

6 Discussion: welfare implications

In this section we discuss some of the welfare implications of correlation neglect. As the analysis of this paper centres on polarisation, we start by noting that the level of desired polarisation is related to how informed the electorate is. When voters are fully informed, they would rather have full polarisation at $x_r = 1 = -x_l$. When voters have no information,

---

$^{20}$It is now the case that multiple equilibria may arise as the vote shares will be a function of $\rho(v)$, itself a function of $x_r$ and $x_l$. 

they would not want to take the risk involved in large polarisation. The more information the electorate has, the more polarisation is attractive.

Note that were the platforms fixed, for a high enough \( \alpha \), behavioural electorates would enjoy a higher welfare as the political outcome would be better at information aggregation.\(^{21}\) Once we endogenise platforms, such welfare analysis becomes more complicated as polarisation can potentially increase or decrease in each electorate. When information is endogenous, another complication arises, as a more informed vote such as in the behavioural electorate might induce more polarisation in some cases, while more polarisation will induce more investment in information (that is, lower \( \rho(v) \) for all \( v \)), as shown in Lemma 3. It is not clear however that the higher level of information the voters would have would be sufficient to overcome the risk inherent in polarisation. In addition, with different degrees of polarisation, the cost of information acquisition would also be different for each society.

We now use an example to illustrate the relevant trade-offs for welfare calculations, discussed above. It provides a numerical calculation of the equilibria and its welfare properties for the two electorates when information and platforms are endogenous and we vary the competitiveness of the electoral system (the exact calculations are provided in the appendix).

**Example 1 revisited:** Suppose that \( q = 0.75, c(1 - \rho) = 2(1 - \rho)^2 \), that \( G \) is as in Example 1, that \( N = 1 \) and that platforms are fixed at \( x_r = 0.5 = -x_l \). The vote share for the correct outcome for the behavioural and for the rational electorate is \( 0.7 \) and \( 0.628 \) respectively. The cost of information is the same in both electorates, and as a result the welfare of a behavioural electorate is higher for a high enough \( \alpha \). Specifically, we have that \( \alpha > 0.522 \).

Suppose now that platforms are endogenous and that the politicians utility is \( U_j(y) = -2(z_j - y)^2 \). The unique equilibrium for the behavioural electorate still has \( x_B^r = -x_B^l = 0.5 \) with the correct vote share at 0.7. For rational voters, \( x_R^r = -x_R^l = 0.5265 \) in the unique equilibrium. Rational voters invest slightly more in information but the correct vote share is not much higher at 0.629. Thus the welfare of a behavioural electorate is still higher for a high enough \( \alpha \). Specifically, we need \( \alpha > 0.538 \). Note that we need a higher \( \alpha \) here as the higher polarisation as well as the higher investment in information by rational voters is valued by the voters through their utility from matching their actual individual vote to their ideal policy.

Suppose now that we have more than one district. When \( N = 5 \), we have that \( x_B^r = 0.43 \) and \( x_R^r = 0.4 \) and thus behavioural voters induce more polarisation. Still, similar results arise and we need \( \alpha > 0.23 \). To see why the cutoff for \( \alpha \) can be so low, note that the

\(^{21}\)Recall that \( \alpha \) is the weight in the utility function on the political outcome being the optimal one. Voters have also utility from voting themselves for the optimal policy, which must be higher in a rational society for fixed platforms, as only such voters use correctly their information.
rational individuals have much lower information aggregation as they invest less, and also the difference in the vote shares is magnified with a concave function. Thus the optimal policy is more likely to be chosen in the behavioural electorate compared to the cases above.

7 Conclusion

In this paper we study the relation between polarisation in voters’ opinions to polarisation in the policy choices of politicians. We focus on an environment in which polarisation in voters’ opinions arises do to correlation neglect, i.e., the failure to take into account that different information sources might be correlated. We show that increased voter polarisation doesn’t necessarily imply more policy polarisation. In particular, we show that when the political system is relatively uncompetitive, voter polarisation might even imply policy moderation. These results provide a theoretical critique to some views espoused in the literature that polarisation of policies arises as a result of voter polarisation. Our approach shows that it is important to model the source of polarisation in opinions and the features of the political system in order to derive whether such a link exists.

8 Appendix

Proof of Proposition 1: Note that the equilibrium can be defined by the pair of first order conditions. Note that given the concavity of the politician’s utility and $G$, and the linearity of $V$, we have that the first order conditions are sufficient for a continuous best response function for each politician, defined on a compact set. Moreover, as the cutoffs $v_1$ and $v_2$ do not depend on the degree of polarisation, the best response functions are linear (as the vote shares will not depend on the degree of polarisation). A unique Nash equilibrium therefore exists. Symmetry follows from the symmetric model.

Proof of Lemma 2: The difference between the two voting behaviours occur only in $[v_1 + \hat{x}, v_2 + \hat{x}]$ and in $[-v_2 + \hat{x}, -v_1 + \hat{x}]$ where $\hat{x} = \frac{x_r + x_l}{2}$ is the mid point between the two platforms in some equilibrium. Moreover, in these regions, when signals are independent, voters in the two electorates behave in the same way. Thus, letting $A(1)$ denote equal behaviour in both societies in state $\omega = 1$, we have (recall that $f(v) = \frac{1}{2}$):

\[
V^B_x(1) = A(1) + \int_{-v_2 + \hat{x}}^{-v_1 + \hat{x}} \rho(v)q \frac{1}{2} dv + \int_{v_1 + \hat{x}}^{v_2 + \hat{x}} \rho(v)q \frac{1}{2} dv;
\]

\[
V^R_x(1) = A(1) + \int_{v_1 + \hat{x}}^{v_2 + \hat{x}} \rho(v) \frac{1}{2} dv
\]
and therefore (note that $\frac{\partial x}{\partial x_r} = \frac{1}{2}$ and that we evaluate the derivative at $\hat{x} = 0$):

$$\frac{\partial V^B_{x_r}(1)}{\partial x_r} = \frac{\partial A(1)}{\partial x_r} + \frac{1}{4} q(\rho(-v_1) - \rho(-v_2) + \rho(v_2) - \rho(v_1)) = \frac{\partial A(1)}{\partial x_r};$$

$$\frac{\partial V^R_{x_r}(1)}{\partial x_r} = \frac{\partial A(1)}{\partial x_r} + \frac{1}{4} (\rho(v_2) - \rho(v_1))$$

which implies the result in the Lemma. It is easy to show how the derivation holds also when $w = -1$, as the difference above does not depend on the state of the world. □

**Proof of Lemma 3:** Trivially voters with $v > v_2$ will not change their action when getting more information and will therefore choose $\rho = 1$.

Denote the level of polarisation in a symmetric equilibrium by $x$, that is, $x_r = x = -x_l$. We now look at voters with $v_i < v_1$. Their indirect utility from some $\rho$ is (note that this is the same for rational individuals and behavioural individuals who think they are rational):

$$0.5(-\rho q + (1 - \rho)(1 - (1 - q)^2))(1 + v - x)^2 - (\rho(1 - q) + (1 - \rho)(1 - q)^2)(1 + v + x)^2) + 0.5(-\rho(1 - q) + (1 - \rho)(1 - q^2))(-1 + v - x)^2 - (\rho q + (1 - \rho) q^2)(-1 + v + x)^2$$

The marginal benefit from $\rho$ is:

$$0.5(-(q - 1 + (1 - q)^2))(1 + v - x)^2 - ((1 - q) - (1 - q^2))(1 + v + x)^2) + 0.5(-(1 - q) - (1 - q^2))(-1 + v - x)^2 - (q - q^2)(-1 + v + x)^2) = -4xvq(1 - q)$$

(Note that this is negative as a higher $\rho$ reduces the information value). We then have

$$c'(1 - \rho) = 4xvq(1 - q)$$

with $\rho(v)$ decreasing in $v$ for these types.

We now describe intermediate voters. Their indirect utility from some $\rho$ is:

$$0.5(-\rho(1 - \rho)(1 - (1 - q)^2))(1 + v - x)^2 - (1 - \rho)(1 - q)^2(1 + v + x)^2) + 0.5(-\rho(1 - \rho)(1 - q^2))(-1 + v - x)^2 - (1 - \rho)(1 - q^2)(-1 + v + x)^2)$$

The marginal benefit from $\rho$ is:

$$2x(v - 2q + 2q^2 v - 2qv + 1)$$

we therefore have that

$$c'(1 - \rho) = 2x(v(-1 + 2q(1 - q)) + 2q - 1)$$

21
note that here ρ(v) increases with v and that we have continuity at v₁ and at v₂. Finally, it is easy to see the comparative statics as described in the Lemma.

**Proof of Propositions 2 and 3:**
We first consider an analogue of Lemma 2 for the case of endogenous ρ.

**Lemma A2:**

\[
\begin{align*}
\frac{\partial V^R(1)}{\partial x_r} &= -\frac{1}{4} \rho(v) + \int_{v_1}^{v_2} \frac{\partial \rho(v)}{\partial x_r}(1 - 2q)\frac{1}{2} dv; \\
\frac{\partial V^R(-1)}{\partial x_r} &= -\frac{1}{4} \rho(v) - \int_{v_1}^{v_2} \frac{\partial \rho(v)}{\partial x_r}(1 - 2q)\frac{1}{2} dv; \\
\frac{\partial V^B(1)}{\partial x_r} &= -\frac{1}{4} = \frac{\partial V^B(-1)}{\partial x_r}
\end{align*}
\]

(note that ρ(v₂) = 1 in the case of an endogenous ρ).

**Proof of Lemma A2:** Given Lemma 2, we only need to add the derivative of the vote shares with respect to the endogenous ρ(v), evaluated at the midpoint \( \hat{x} = 0 \).

\[
V^R_{x_r}(1) = \\
\int_{v_1}^{v_1 + \hat{x}} (\rho(v)q + (1 - \rho(v))(1 - (1 - q)^2))\frac{1}{2} dv + \\
\int_{v_1}^{v_1 + \hat{x}} (\rho(v)q + (1 - \rho(v))q^2)\frac{1}{2} dv + \\
+ \int_{v_1 + \hat{x}}^{v_2 + \hat{x}} (\rho(v) + (1 - \rho(v))(1 - (1 - q)^2))\frac{1}{2} dv + \\
\int_{v_2 + \hat{x}}^{v_2 + \hat{x}} (1 - \rho(v))q^2\frac{1}{2} dv + \int_{v_2 + \hat{x}}^{v_2} \frac{1}{2} dv
\]

\[
V^R_{x_r}(1) = \\
\int_{v_1}^{v_1 + \hat{x}} (\rho(v)q + (1 - \rho(v))(1 - (1 - q)^2))\frac{1}{2} dv + \\
\int_{v_1}^{v_1 + \hat{x}} (\rho(v)q + (1 - \rho(v))q^2)\frac{1}{2} dv + \\
+ \int_{v_1 + \hat{x}}^{v_2 + \hat{x}} (\rho(v)q + (1 - \rho(v))(1 - (1 - q)^2))\frac{1}{2} dv + \\
\int_{v_2 + \hat{x}}^{v_2 + \hat{x}} (1 - \rho(v))q^2\frac{1}{2} dv + \int_{v_2 + \hat{x}}^{v_2} \frac{1}{2} dv
\]

First, given the specification in Lemma 2, we find from the above that all common elements are 0, that is, \( \frac{\partial A(1)}{\partial x_r} = \frac{\partial A(-1)}{\partial x_r} = 0 \). We now proceed to take the derivative w.r.t. \( \rho(v) \):

For \( V^R_{x_r}(1) \) this equals:

\[
\int_{v_1}^{v_1} \frac{\partial \rho(v)}{\partial x_r}(q - 1 + (1 - q)^2)\frac{1}{2} dv + \\
\int_{v_1}^{v_1} \frac{\partial \rho(v)}{\partial x_r}(q - q^2)\frac{1}{2} dv + \int_{v_1}^{v_2} \frac{\partial \rho(v)}{\partial x_r}(1 - 2q)\frac{1}{2} dv = \int_{v_1}^{v_2} \frac{\partial \rho(v)}{\partial x_r}(1 - 2q)\frac{1}{2} dv
\]

For \( V^R_{x_r}(-1) \) this equals:

\[
\int_{v_1}^{v_1} \frac{\partial \rho(v)}{\partial x_r}(q - 1 + q^2)\frac{1}{2} dv + \\
\int_{v_1}^{v_2} \frac{\partial \rho(v)}{\partial x_r}(q - q^2)\frac{1}{2} dv - \int_{v_1}^{v_2} \frac{\partial \rho(v)}{\partial x_r}(1 - 2q)\frac{1}{2} dv = -\int_{v_1}^{v_2} \frac{\partial \rho(v)}{\partial x_r}(1 - 2q)\frac{1}{2} dv
\]

and for \( V^R_{x_r}(1) \) and \( V^R_{x_r}(-1) \) this derivative equals 0. In fact, as the accuracies of the two signals are the same, the aggregate vote share for the behavioural agents does not depend
on $\rho$ (in the symmetric distribution $\rho(v)$ case). Putting together the results in Lemma 2 (for exogenous $\rho$) and the derivatives w.r.t. $\rho(v)$ proves the Lemma.\]

**Lemma A3:** For both endogenous and exogenous $\rho(v)$, when $G$ is symmetric, the symmetric equilibrium first order condition for symmetric $x_r, x_l$ is

$$\frac{\partial G(V^J_r(1))}{\partial V^J_r(1)} 2K^J \Delta u_r(x_r, x_l) + \frac{\partial U_r(x_r)}{\partial x_r} \frac{1}{2} = 0$$

where $K^B = -\frac{1}{4}\rho(v_2)$ and $K^R = -\frac{1}{4}\rho(v_1)$.

**Proof of Lemma A3:** This follows from the symmetry of the model in the state of the world, the symmetric equilibrium, and the symmetry of $G$ which imply together that $\frac{\partial G(V^J_r(1))}{\partial V^J_r(1)} = \frac{\partial G(V^J_r(1-))}{\partial V^J_r(1)}$. With the last statement of Lemma A2, Lemma A3 follows.\]

As (5) holds for both the case of an endogenous $\rho$ and the case of an exogenous $\rho$, Proposition 3 will be a special case of Proposition 2. We are now ready to prove a general version of Proposition 2.

Suppose first that $\rho(v_1) > \rho(v_2)$. Consider an equilibrium of the rational model $x^R_r, x^R_l$. Evaluated at these points, we would have $0 > \frac{\partial G(V^B_{x_r}(1))}{\partial V^B_{x_r}(1)} K^B > \frac{\partial G(V^B_{x_l}(1))}{\partial V^B_{x_l}(1)} K^R$ because: (a) $V^B_{x_r}(1) > V^R_{x_r}(1)$ by Lemma 1 implying by the properties of $G$ that $0 \leq \frac{\partial G(V^B_{x_r}(1))}{\partial V^B_{x_r}(1)} \leq \frac{\partial G(V^B_{x_l}(1))}{\partial V^B_{x_l}(1)}$; (b) $0 > K^B > K^R$ as we have that $\rho(v_1) > \rho(v_2)$. Thus we will have more polarisation with behavioural societies.

Suppose now that $\rho(v_1) < \rho(v_2)$ (as is the case for Proposition 3). We then have $K^B < K^R$. Consider $G(V) = V$. We then have that $\frac{\partial G(V^J_r(1))}{\partial V^J_r(1)} = 1$ but $K^B < K^R < 0$ implying that at some equilibrium $x^R_r, x^R_l$ of the rational model, when we evaluate the FOC of the behavioural model at these values, we have that the lhs of (5) is smaller than that of the rational model and thus negative (as for the rational it is zero in the postulated equilibrium). We will have therefore less polarisation with behavioural voters.\[\text{Note that when } G \text{ is not too concave and close to the linear one, the result above would hold insuring robustness.}\]

Note that in the case of Proposition 3, we cannot guarantee uniqueness any more as $\rho$ will be a function of the platforms, and the vote shares are a function of $\rho$. In any case, consider a linear $G$. In this case the derivative of $G$ is fixed at 1 and so $\rho(v_1) > \rho(v_2)$, and for all equilibria we have that the behavioural electorate induces less polarisation. That this arises for other non linear $G$ but still not too concave can be shown as in the example below. When $G$ is sufficiently concave though, it will be the case for any equilibrium as before that $\pi < 1$ (note that this arises as $\rho(v_1) > 0$ always) and thus polarisation will be larger in the behavioural electorate.\[\text{As } K^B \text{ and } K^R \text{ are bounded from zero, we can also find a } G \text{ such that } \frac{\partial G(V^B_{x_r}(1))}{\partial V^B_{x_r}(1)} K^B > \frac{\partial G(V^B_{x_l}(1))}{\partial V^B_{x_l}(1)} K^R \text{ and there will be an equilibrium with more polarisation when voters are behavioural for this } G.\]
Calculations for the example:
Note that for moderate voters:

\[ 2(1 - \rho) = 4xvq(1 - q) \]

and for intermediate voters:

\[ 2(1 - \rho) = 2x(v(-1 + 2q(1 - q)) + 2q - 1) \]

**Case 1: Exogenous platforms** \( x = 0.5 \)

First note for behavioural voters, the vote share for the right option is

\[
\int_{-v_2}^{v_2} (\rho(v)q + (1 - \rho(v))q^2) f(v)dv + \int_0^{v_2} (\rho(v)q + (1 - \rho(v))(1 - (1 - q)^2) f(v)dv + \int_{v_2}^{1} f(v)dv \\
= \int_0^{v_2} (2\rho(v)q + (1 - \rho(v))(1 - (1 - q)^2 + q^2) f(v)dv + \int_{v_2}^{1} f(v)dv \\
= \int_0^{v_2} qdv + \int_{v_2}^{1} 0.5dv = 0.5 + v_2(q - 0.5)
\]

This therefore only depends on \( q \). For \( q = 0.75 \), this is 0.7. Thus for all \( x \) endogenous and exogenous this would be the case.

For rational voters it is

\[
\int_{-v_2}^{v_1} (1 - \rho(v))q^2 f(v)dv + \int_{-v_2}^{v_1} (\rho(v)q + (1 - \rho(v))q^2) f(v)dv \\
+ \int_0^{v_2} (\rho(v)q + (1 - \rho(v))(1 - (1 - q)^2) f(v)dv + \int_{v_1}^{v_2} (\rho(v) + (1 - \rho(v))(1 - (1 - q)^2) f(v)dv \\
+ \int_{v_1}^{1} f(v)dv \\
= \int_0^{v_1} (2\rho(v)q + (1 - \rho(v))(1 - (1 - q)^2 + q^2) f(v)dv \\
+ \int_{v_1}^{v_2} (\rho(v) + (1 - \rho(v))(1 - (1 - q)^2 + q^2) f(v)dv + \int_{v_2}^{1} f(v)dv \\
= \frac{1}{2} + \frac{1}{2}(2q - 1)v_2 - (2q - 1)\frac{1}{2} \int_{v_1}^{v_2} \rho(v)dv
\]

Thus (recall that \( v_1 = 2q - 1 \) and \( v_2 = \frac{2q - 1}{q^2 + (1 - q)^2} \)):

\[
V^{R}_{x_1}(1) = V = \frac{1}{2} + \frac{1}{2}(2q - 1) - \frac{2q - 1}{q^2 + (1 - q)^2} = (2q - 1)\frac{1}{2} \int_{2q-1}^{\frac{2q-1}{q^2 + (1 - q)^2}} \rho dv
\]

\[
V^{B}_{x_1}(1) = U = \frac{1}{2} + \frac{1}{2}(2q - 1) - \frac{2q - 1}{q^2 + (1 - q)^2}
\]
The welfare of the behavioural voters in state $\omega = 1, N = 1$ (so the probability of election is $U$), is:

\[ -\alpha G(U) \int_{-1}^{1} (1 + v - x)^2 0.5 dv - \alpha (1 - G(U)) \int_{-1}^{1} (1 + v + x)^2 0.5 dv \]

\[- \int_{0}^{2q-1} (1 - r)^2 dv - \int_{0}^{2q-1} \frac{2q-1}{q^2 + (1 - q)^2} (1 - \rho)^2 dv \]

\[- \int_{-1}^{\frac{2q-1}{q^2 + (1 - q)^2}} (1 + v + x)^2 0.5 dv - \int_{-1}^{\frac{2q-1}{q^2 + (1 - q)^2}} (1 + v - x)^2 0.5 dv \]

\[- \int_{1}^{-2q} \frac{2q-1}{q^2 + (1 - q)^2} ((\rho q + (1 - \rho) q^2)(1 + v - x)^2 + (\rho(1 - q) + (1 - \rho) (1 - q^2))(1 + v + x)^2) 0.5 dv \]

\[- \int_{0}^{2q-1} (rq + (1 - r) q^2)(1 + v - x)^2 + (r(1 - q) + (1 - r)(1 - q^2))(1 + v + x)^2) 0.5 dv \]

The welfare of rational voters in this case is:

\[ -\alpha G(V) \int_{-1}^{1} (1 + v - x)^2 0.5 dv - \alpha (1 - G(V)) \int_{-1}^{1} (1 + v + x)^2 0.5 dv \]

\[- \int_{0}^{2q-1} (1 - r)^2 dv - \int_{0}^{2q-1} \frac{2q-1}{q^2 + (1 - q)^2} (1 - \rho)^2 dv \]

\[- \int_{-1}^{\frac{2q-1}{q^2 + (1 - q)^2}} (1 + v + x)^2 0.5 dv - \int_{-1}^{\frac{2q-1}{q^2 + (1 - q)^2}} (1 + v - x)^2 0.5 dv \]

\[- \int_{1}^{-2q} \frac{2q-1}{q^2 + (1 - q)^2} ((\rho q + (1 - \rho) (1 - q^2))(1 + v - x)^2 + (\rho (1 - q) + (1 - \rho) (1 - q^2))(1 + v + x)^2) 0.5 dv \]

\[- \int_{0}^{2q-1} (rq + (1 - r)(1 - q^2))(1 + v - x)^2 + (r(1 - q) + (1 - r)(1 - q^2))(1 + v + x)^2) 0.5 dv \]

The welfare calculation remain as above.

The different $\text{FOC}$ is now for behavioural:

\[-2(0.25)4x + 4(1 - x)\frac{1}{2} = 0 \quad (6) \]

and the solution is $x = 0.5$, and so the welfare calculation remain as above.

For rations the FOC is:

\[-2(0.25)(x((2q - 1) (2q (q - 1) + 1) - 2q + 1) + 1)4x + 4(1 - x)\frac{1}{2} = 0 \quad (7) \]

We then have that $x^B_r = 0.526$

Computing as above the welfare of rations it is $-0.91775 - 1.3392\alpha$.

We now consider $N = 5$.

The different FOC is now for behavioural:
\[-2(0.25)(30)U^2(1 - U)^24x + 4(1 - x)\frac{1}{2} = 0 \quad (8)\]

and the solution is \(x^B_R = 0.43\) and for rational:

\[-2(0.25)(30)V^2(1 - V)^2(x ((2q - 1) (2q (q - 1) + 1) - 2q + 1) + 1)4x + 4(1 - x)\frac{1}{2} = 0 \quad (9)\]

with the solution being \(x^R_r = 0.397\).

Computing again the welfare in both cases we get the result on \(\alpha\). Specifically the behavioural welfare is \(-1.018 - 0.93873\alpha\), and the rational welfare is \(-0.97489 - 1.1267\alpha\).}

References


