

Religious Beliefs, Religious Participation and Cooperation

Gilat Levy and Ronny Razin¹

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Abstract:

The paper analyzes the relation between religious beliefs, religious participation and social cooperation. In particular, we focus on religions that instill beliefs about the connection between rewards and punishments and social behaviour. We show how religious organizations arise endogenously, analyze their effect on social interactions in society and identify a "spiritual" as well as a "material" payoff for being religious. We show that religious groups that are more demanding in their rituals are smaller, more cohesive, and are composed of individuals whose beliefs are more "extreme". We use our framework to analyze the response of individual beliefs and religious membership to correlated shocks in society, such as natural disasters or periods of prosperity.

¹Department of Economics, LSE. Email: g.levy1@lse.ac.uk; r.razin@lse.ac.uk. We thank Stephen Hansen for valuable research assistance, and the editor of this journal and two anonymous referees for helpful comments. The ESRC (grant number RES-000-22-1856) and the ERC (grant number 210385) provided financial support.

1 Introduction

The role of religion in society has recently been the subject of renewed empirical and experimental interest. In particular, the literature focuses on the link between religious beliefs and religious practices (Huber 2005) and how such beliefs and practices affect various parameters ranging from micro data such as individuals' well-being and individual behaviour (Gruber 2003, Sosis and Ruffle 2003), to macro parameters such as growth (Barro and McCleary 2003, Guiso, Sapeinza and Zingales 2003) and the public provision of social insurance (Scheve and Stasavage 2006).

Religious beliefs and religious practice are clearly two important and possibly inseparable features of religious organizations and the link between them is yet to be fully understood. In this paper we focus on the role of religious beliefs and analyze a model in which the religion, or a religious organization, is able to instill certain beliefs in individuals. Specifically, we focus on the theology that connects rewards and punishments to different behaviours in the social sphere. Examples of such systems of religious beliefs are abundant in the Anthropological literature. One example is the Nuer religion in Sudan, about which Evans-Pritchard (1956) writes: “...and in any argument about conduct the issue is always whether a person has conformed to the accepted norms of social life....the Nuer are of one voice in saying that sooner or later good will follow right conduct and ill will follow wrong conduct”. Reward and punishment is prevalent in Abrahamic religions as well as in some Eastern religions. According to the Wycliffe Dictionary of Theology, “it is plain from the Bible that sin will be punished”. In Hinduism, a basic concept is that of Karma -the accumulated sum of one's good or bad deeds. More generally, in Buddhism and the Hindu traditions, the state in which one is reincarnated may depend on one's ethical behaviour in the present life.

We consider a society in which individuals are randomly paired to interact socially, modelled by a symmetric Prisoner's Dilemma (PD) game.² Following the strategic interaction, individuals may experience shocks to their well-being. Religious beliefs are defined by a statistical relation between an individual's behaviour in the social context and his likelihood to experience these negative or positive utility shocks. We therefore focus on beliefs about behaviour in the social context, and not vis à vis some supernatural entity. We also consider the simplest such beliefs, where the rewards or punishments only depend on the individual's behaviour and not on his opponent's behaviour or on some average behaviour in society (we discuss such more

²Alternatively one can analyze more general public good games. One difference will be that the larger the social interaction, the smaller the incentives to signal religiosity to others. This will have the effect of lowering the material payoff for being religious.

complicated belief systems in Section 4.3).³

Religious organizations play an active role in shaping beliefs; many invest time and effort in advocating certain kinds of messages while censoring others.⁴ Our main assumption is that an individual's affiliation with a religious organization involves two inseparable consequences. The first is that participation endows the individual with some religious beliefs as described above. The second is that religious participation takes place through rituals that are observable (and potentially costly) activities.⁵ We therefore focus on two particular roles of religious participation or rituals; it is a public activity -such as public attendance in churches or in religious festivals- that allows individuals to identify themselves as belonging to that particular religious organization, and it also triggers religious beliefs, through preaching, indoctrination or other psychological processes.⁶ In contrast, individuals who do not participate in the religion (henceforth seculars) see no statistical connection between rewards and punishment and their actions in the social interaction.

We formalize an equilibrium notion of stable religious organizations which accounts for the social behavior as well as for the affiliation choices of individuals. We therefore explicitly account for the "choice" of beliefs that is involved when someone re-evaluates his religious affiliation. There are two ways to interpret this aspect of our solution concept. It can be taken literally as the considerations of believers who question their faith or of seculars who consider becoming religious. Alternatively, we can think of parents who consider how to raise their children; parents often play an important role in shaping the beliefs of their offsprings through their educational choices. This cultural transmission approach has recently been used in Benabou and Tirole (2006), Bisin and Verdier (2001) and Tabellini (2008).

We first show that stable religious organizations arise endogenously in equilibrium and characterize their properties. Any organized religion always includes, and sometimes exclusively includes, individuals who believe that they are more likely to receive a negative shock when they defect rather than when they cooperate. In addition, religious individuals are more cooperative towards fellow religious members than they are towards seculars.⁷ This is in line with

³Note that there are other theologies which can potentially encourage pro-social behaviour (see for example the Weberian interpretation of predestination, as in Benabou and Tirole 2006 and Levy and Razin 2010).

⁴One example of censorship is the Index Librorum Prohibitorum ("List of Prohibited Books") which, from 1529 to 1966, listed publications prohibited by the Roman Catholic Church.

⁵In Judaism, the public aspect of rituals is even made explicit in the notion of the *Minyan*; no prayer can take place without the presence of at least ten male Jews.

⁶In general, rituals might perform other roles. Also, sometimes rituals are not public as is the case with prayers that are done in private. In this paper we focus on some roles of religious rituals and for simplicity ignore the others.

⁷Our results accord with Johnson and Kruger (2004) who take a different perspective and consider how the

the findings of Guiso *et al* (2003) that religious participation implies a higher level of trust, along with a higher level of intolerance of others. Finally, the model establishes the importance of costly rituals; whenever some in society are secular, religious participation (rituals) must be strictly costly. These results shed light on Barro and McCleary (2003) who find that economic growth responds positively to religious beliefs but negatively to religious participation. Our model implies a positive relation between religious beliefs and cooperative behaviour, which may enhance economic growth. We show why rituals have to be costly, and therefore why religious participation might negatively affect growth.

We identify two motivations for being religious. A spiritual motivation arises as religious individuals believe that cooperation is rewarded and they are relatively more cooperative in equilibrium. A material motivation arises as relative to seculars, religious individuals enjoy in equilibrium a higher level of cooperation from others. The material benefit of religion identified in our model is supported by empirical studies showing a link between religious participation, social ties and mutual assistance.⁸ Normatively, from a material/secular perspective, the spiritual motivation implies that individuals take sub-optimal decisions and thus it reduces material welfare. On the other hand, the material benefit of religion -in the form of enhanced cooperation- might in some cases be sufficient to outweigh the cost of religious participation so that the religion can be beneficial to everyone in society.

When the spiritual benefit of being religious is relatively important, i.e., when utility shocks are significant enough compared to the parameters of the social game, we find that religious groups that are more demanding in their rituals are smaller, more cohesive, and are composed of individuals whose beliefs are more "extreme" about the relation between unsocial behaviour and punishment. Moreover, when several religious organizations arise in equilibrium, the smaller and more conservative religions are less cooperative towards the larger and less conservative ones, thus creating a "hierarchy" of religions (or denominations within religions). These predictions find support in the empirical studies of Iannaccone (1992, 1994, 1998), and in several experimental studies (see Orbell *et al* 1992 and Sosis and Ruffle 2003).⁹

Our explicit focus on religious beliefs implies that our modelling of religions differs from other organizations who might facilitate cooperation (see Dixit 2004, Baron 2010 and Tabellini natural selection of belief in supernatural punishment affects the observed patterns of cooperation in different societies.

⁸See for example Wilson (2002) and Bradley (1995). Ellison and George (1994) find a positive relationship between religious participation and social ties in a survey of 2,956 households in the southeastern United States.

⁹For results on the link between religious participation and multi-denomination societies, see Barros and Garoupa (2002).

2008).¹⁰ The emphasis on beliefs allows us to take the analysis further to a dynamic model and to analyze the effect of information and possibly belief updating on both the spread of beliefs in society and the types of religious organizations which can arise.¹¹ We examine the dynamic evolution of religious organizations in response to correlated utility shocks, such as economic booms, recessions or natural disasters. We find that correlated positive utility shocks cause beliefs to polarize, inducing deconversions and as a result a reduction in the size of the religion. In contrast, negative shocks may yield both conversions and deconversions, with an ambiguous effect on the size of religious organizations. We conclude our dynamic analysis by discussing how, in the context of our model, religions can potentially induce less falsifiable beliefs by using for example the concepts of "afterlife" or "forgiveness".

Our model highlights the role of religious organizations in orchestrating behaviour in the social sphere by influencing beliefs and by creating possibilities for social signalling through participation in rituals. In that, our paper contributes to the growing number of papers on religious organizations. Among the first to formally consider religion as a rational choice decision is the paper by Azzi and Ehrenberg (1975), in which expected afterlife consumption (and possibly some other material benefits) depends on time spent on church related activities.¹² Iannaccone (1992) models religious organizations in which religious activity is a club good, and costly rituals serve to resolve free rider problems in the production of religious goods.¹³ In his model, players derive utility from the collective production of such goods within the group they belong to.

Other papers consider the explicit modelling of religious beliefs; Benabou and Tirole (2006) assume that agents differ in their beliefs with respect to how much hard work is rewarded, in this or in the afterlife, and actively choose to maintain such beliefs. In Scheve and Stasavage (2006) on the other hand, religious beliefs allow for a psychic benefit in bad times, and hence such beliefs negatively correlate with preferences for social insurance.

While we identify some role for religious leaders as possibly determining the content and level of religious rituals which allow for a coordination device and activation of beliefs, our model abstracts away from explicit modelling of the supply side of religion. Some recent papers which attempt to bring together the demand and supply side of religion are Barros

¹⁰While other social organizations besides religious ones may instill pro-social values in its members, religious organizations may be more successful than others as they rely on strong beliefs which are not always verifiable. See Rappaport (1999) or Sosis and Alcora (2003) and the discussion in Johnson and Kruger (2004).

¹¹This naturally relates to belief systems which do not focus only on the afterlife and instead allow for some this-worldly rewards and punishments.

¹²See also Stark (1996) for a theory espousing the rational view of religious participation.

¹³See also Berman (2000).

and Garoupa (2002) and McBride (2010). In Barros and Garoupa (2002) churches choose their religious strictness taking into account the distribution of religious preferences in the population, the value of the religious good it produces (a local public good) and the existence of a rival church. McBride (2010) shows how economic growth produces counteracting effects on religious activity in an open religious market, and that such growth has little effect in markets which are relatively secularized or relatively religious.

The remainder of the paper is organized as follows. In the next section we present the model. In Section 3 we present our main results about religious organizations in equilibrium, including a discussion of welfare properties and comparative statics. In Section 4 we consider a dynamic extension and Section 5 considers relations between several religious groups. An appendix contains all proofs.

2 The model

We consider a society in which individuals are paired to play a Prisoner’s Dilemma (PD) game. Being a member of a religious organization is related to two (inseparable) features: participation in a potentially costly and observable activity, and having certain beliefs about the statistical relation between actions in the PD game and private utility shocks. Given the observable religious activity, individuals can condition their behaviour in the PD game on their opponent’s affiliation. Our notion of equilibrium will involve optimal behaviour in the PD game as well as a stability condition on individuals’ affiliation choices. We now explain the model in more detail.

The social interaction. Individuals are randomly paired to play a one-shot PD game:

| | | |
|---|--------|--------|
| | C | D |
| C | d, d | c, b |
| D | b, c | a, a |

where $b > d > a > c$. We assume strategic complementarities, i.e., that $d - b > c - a$. This assumption is standard in the literature on cooperation. While it is possible to analyze other games within the framework of our model, we find the Prisoner’s Dilemma a natural starting point. In particular, in the context of signalling which is an important aspect of our model, bilateral interactions will provide a more stark analysis than charity or public good games in which an individual’s utility depends only on average outcomes in society.

Social rituals. Being religious is associated with some *observable* action, with a *cost* r (r

can be either positive or negative) to all individuals.¹⁴ An individual that has paid the cost r is “religious”, otherwise he is “secular”. We interpret these observable actions as religious rituals. We consider rituals, such as attendance in religious sermons, which not only allow group members to identify and familiarize themselves with one another but may also have an effect on the beliefs of individuals, as we describe below.¹⁵

The cost of rituals, r , is exogenously given in the model. The content and level of ritualistic practices may be determined to some degree by religious leaders. This is an interesting extension of our analysis that we defer to future research.

Utility shocks. We assume that following the strategic interaction, each individual believes he will receive, in addition to the payoffs of the PD game, either a negative utility shock, $-\varepsilon$, or a positive utility shock, ε . Apart from participation in rituals, we differentiate between seculars and religious individuals also according to their beliefs about the relationship between the shocks and their actions in the PD game, as we now explain.

Beliefs. We assume that religious participation (i.e., paying the cost of rituals) *activates* a set of beliefs about the causality between actions in the PD game and the utility shocks. Specifically, each individual i in the population is endowed with a latent type reflecting his propensity to be affected by religious participation (we soon specify the distribution over these types). Individual i 's type, (q_c^i, q_d^i) , represents his beliefs *when he is religious*. These beliefs are activated when he pays r , and imply that an individual believes that when he cooperates, he receives the negative shock (possibly inflicted by some supernatural entity) with probability $q_c^i \in [0, 1]$ and that when he defects, he receives the negative shock with probability $q_d^i \in [0, 1]$.¹⁶ Individuals receive positive shocks with the remaining probabilities, $1 - q_c^i$ and $1 - q_d^i$ respectively. Note that in the static analysis of the paper, it is possible to interpret these rewards and punishments as either "this-worldly", or shocks in the "afterlife".¹⁷ When an individual is secular and does not pay r , his beliefs are not activated, and he therefore

¹⁴The analysis could easily be extended to accommodate heterogeneous ritual costs.

¹⁵In an alternative model, agents might also choose whom to interact with, conditional on whether others had paid the cost or not. We discuss this possibility in Section 4.

¹⁶Note that this formulation allows for individuals not to be affected by religious participation; a religious individual with $q_c^i = q_d^i$ does not believe that his actions in the social game have any statistical effect on the type of shock he will experience.

¹⁷While many religious texts refer to rewards and punishments in the "afterlife" explicitly, this is sometimes balanced with some real-world motivations. For example, in Evans-Pritchard's (1956) study of the Nuer religion, he finds that: "Nuer avoid so far as possible speaking of death ... they regard it as the most dreadful of all dreadful things... Theirs is a this-worldly religion, a religion of abundant life and the fullness of days, and they neither pretend to know, nor, I think, do they care, what happens to them after death."

believes that there is no relation between actions taken in the PD game and utility shocks (and thus his best response is to defect).

We assume risk neutrality and thus the expected utility of a religious individual who cooperates is $x + \varepsilon(1 - 2q_c^i)$, for $x \in \{c, d\}$ (depending on his rival's action), and similarly, the expected utility of a religious individual who defects is $x + \varepsilon(1 - 2q_d^i)$, for $x \in \{a, b\}$.

It will be sufficient, as will become apparent later on, to characterize the latent types in the population by the parameter $q^i = q_c^i - q_d^i$, where the higher is q^i , the less a religious individual fears that defection will lead to punishment. As types with $q^i > 0$ will play no role in the results, we focus on types in $[-1, 0]$.

Let the distribution over latent types q^i on $[-1, 0]$ be some (exogenous) continuous distribution function $F(\cdot)$, with density f satisfying $0 < f(\cdot) < \infty$ everywhere. We assume that $F(\cdot)$ is common knowledge but that individuals do not observe the belief q^i of their religious opponent i .¹⁸ F captures in some sense the degree to which individuals in society are susceptible to religious preaching.¹⁹

Note that the dominant action of a religious individual with $q^i \geq \bar{q} = \frac{d-b}{2\varepsilon}$ is to defect in the PD game, whereas the dominant action of a religious individual with $q^i \leq \underline{q} = \frac{c-a}{2\varepsilon} < \bar{q}$, is to cooperate, with both \bar{q} and \underline{q} strictly negative. The best response of those with $q^i \in (\underline{q}, \bar{q})$, henceforth “intermediates”, is to cooperate if their opponent does, and defect otherwise. Throughout we will consider a large enough value of the shock ε , which will allow us to derive a unique equilibrium for any r . It will also imply that $\underline{q} = \frac{c-a}{2\varepsilon} > -1$ so that the support of F includes types that always cooperate when religious.

Remark I: Belief activation by rituals. We assume that beliefs are activated by religious rituals instead of simply assuming a pre-determined distribution of beliefs in the population. To understand religious organizations one must model explicitly why some people might convert, while others might lose faith. How individuals re-evaluate their belief systems is therefore an integral part in our understanding of religious participation.

On a more technical note, consider the alternative assumption of a pre-determined distribu-

¹⁸We maintain the assumption that there is no restriction imposed on the personal beliefs of agents given their knowledge of the distribution of beliefs in society at large. This is motivated either by an assumption of non-common priors or by assuming that agents believe that there is no statistical relation between their parameters, q_c^i and q_d^i , and others' parameters.

¹⁹In our model the cost r does not affect the distribution of types F , but does affect the equilibrium distribution of the types who choose to be religious. An alternative assumption is when the cost r can affect the distribution F . For example, a higher level of r may imply more low latent types in the population (i.e., with relatively strong potential religious beliefs).

tion of beliefs in society. Analysis carried out in this alternative model shows that the benefits from being religious vis à vis secular are not monotone in types. The model yields a multiplicity of equilibria with a variety of different organizations and no generally defined characteristics.

Remark II: Beliefs vs. preferences. In the static model one could replace heterogenous beliefs with heterogenous preferences and proceed with a similar analysis. We believe that even in the static model the focus on beliefs is more natural. First, religious organizations are often involved in information dissemination or censorship, which implies that they attempt to affect individuals' beliefs. Second, one of our requirements below will be that individuals contemplate whether to remain religious or not and consider the effect of affiliation on their beliefs. It is more reasonable and more consistent with standard economic analysis that individuals will forecast a change in their beliefs rather than a change in their preferences. Finally, in the dynamic extension that we consider below, belief updating is yet again a more natural process to consider than changes in preferences.

Equilibrium notion. We focus on the case in which there is only one organized religion in society (we relax this assumption in Section 5). For any (r, F) , we look at a configuration of affiliation and PD strategy choices, and check whether it is stable.

Our equilibrium definition consists therefore of two conditions. First, given affiliation choices, individuals must optimally choose how to play in the social interaction. As paying r is observable, the strategy of an individual in the PD game may depend on whether his (randomly matched) opponent is religious or not, as this may help predict his behaviour.

Second, individuals' affiliations are optimal, that is, the religious must prefer to be religious, while seculars must prefer to be secular, given others' behaviour and affiliations. This equilibrium requirement is more subtle, as we need to determine first whether individuals realize that their beliefs will change if they change their affiliation, and second, we need to determine how individuals evaluate counterfactual affiliation choices, i.e., with which system of beliefs.

When individuals forecast their behaviour in other affiliations, the literature offers two extreme alternatives. Individuals might be *naive*, whereby they do not realize that if they switch affiliations their beliefs might change and thus also their behaviour. On the other hand, a *sophisticated* individual realizes that his beliefs and as a result his behaviour will change. For example, one can think of this in terms of parents and their children. Secular parents who consider which school to send their children to may realize that a religious school might instill religious beliefs in their children. Similarly, religious parents, when considering sending their children to a secular school, might suspect an erosion of their childrens' beliefs. For our equi-

librium concept we adopt the sophisticated individuals approach. Using the naive approach will not have any major qualitative impact on the results.²⁰

Second, we assume that an individual evaluates both affiliations given his *current beliefs* (i.e., he uses $q^i \in [-1, 0]$ when religious, and the belief that there is no relation between shocks and actions when secular). This assumption accords with the "partial empathy" approach. Under this approach (see for example Tabellini 2008, Bisin and Verdier 2001 and Benabou and Tirole 2006), adapted to our context, parents' beliefs are fixed, but they choose which beliefs to transmit to their children (religious or secular) and evaluate the future welfare of their children given their *own* beliefs.²¹

Formally, equilibria in the model satisfy the following conditions:

1. (*Optimal behaviour in the social interaction*): Given the affiliation choices, and individual beliefs, the strategies in the PD game are best responses.
2. (*Optimal affiliation*): Given the strategies in the PD game, others' affiliation choices and his current beliefs, a religious individual i prefers to be religious than to be a secular (who defects) and a secular individual i prefers to be secular than to be a religious (who behaves according to q^i).

Note that due to partial empathy there might be types that when secular prefer to stay secular, and when religious prefer to stay religious. This implies multiple equilibria. To avoid this particular type of multiplicity, we will focus on equilibria in which we assign such types to be religious. This equilibrium refinement is not important for our qualitative results and can be motivated by some (unmodelled) incentive of the religious organization to maximize membership.

Notation, preliminaries and a benchmark: Let ρ be the share of the religious in society (with $1 - \rho$ being the share of seculars). Let γ_{RR} be the probability that a randomly chosen religious individual cooperates when he meets another religious opponent. Similarly, let γ_{RS} denote the probability that a randomly chosen religious individual cooperates when he meets a secular opponent. These probabilities (as well as ρ) are determined in equilibrium given the share and the strategies of the types who join the religion.

²⁰Specifically, it will rule out the possibility of sustaining a religion with full cooperation (see Proposition 1) but otherwise will not affect the results.

²¹A full empathy approach will imply that religious agents will have a higher incentive to become secular and the other way around, as current beliefs will not bias their decision as much. One impediment to this analysis is that both q_c^i and q_d^i will play a role and the analysis cannot be reduced to a unidimensional q^i space.

Note that given γ_{RR} , intermediate religious types in $[q, q^*]$ for some $q^* \leq \bar{q}$ will cooperate against religious opponents and types above q^* will defect against them (where a higher γ_{RR} will induce a higher q^*). As all intermediates defect against seculars, we have that $\gamma_{RR} \geq \gamma_{RS}$, where a strict inequality would hold in equilibrium if some religious intermediates cooperate only with fellow religious opponents.

Before proceeding to the equilibrium analysis, we note how the dual role of rituals as activating beliefs and allowing for signalling affects our analysis. If religious rituals exist but they do not instill any beliefs, no cooperation will arise in society. If on the other hand all individuals are already endowed with beliefs, but no rituals exist, then some benchmark level of cooperation will arise. As all individuals below \underline{q} always cooperate, this induces some individuals with $q > \underline{q}$ to cooperate as well, as the probability of facing cooperation in society is strictly positive. Thus, in the autarky equilibrium, all types below some cutoff $q^{aut} \in [\underline{q}, \bar{q}]$ will cooperate and all above will defect, with $F(q^{aut}) = \gamma_{RR}^{aut}$. The taxonomy of religious organizations presented below will highlight the role of costly rituals.

3 Religion: beliefs, rituals and social behaviour

3.1 Equilibrium analysis

To understand how equilibria with religious organizations are determined, consider the benefit for an individual with affiliation $J \in \{R, S\}$ of being religious vs. being secular, $V_J(q^i)$. This relative benefit depends on her current affiliation, or more precisely, on her current beliefs. It is computed as the difference in the expected utility of being in each affiliation, while abstracting from the cost r .

Consider first a religious individual who evaluates whether to stay religious or to become secular, given the equilibrium parameters $\rho, \gamma_{RR}, \gamma_{RS}$. Suppose that this individual has an intermediate q^i , and that her best response is to cooperate against fellow religious opponents and defect against seculars. When religious, her utility from meeting with other religious individuals, which occurs with probability ρ , is $\gamma_{RR}d + (1 - \gamma_{RR})c + \varepsilon(1 - 2q_c^i)$. Meeting with secular agents, which occurs with probability $1 - \rho$, provides a utility of $a + \varepsilon(1 - 2q_d^i)$. If this agent considers being secular, she knows she will always defect (this is the assumption of sophistication), which then, given her current beliefs (due to the assumption of partial empathy), will provide a utility of $\gamma_{RS}b + (1 - \gamma_{RS})a + \varepsilon(1 - 2q_d^i)$ conditional on meeting religious individuals, and a utility of $a + \varepsilon(1 - 2q_d^i)$ conditional on meeting other seculars. We

then have:

$$\begin{aligned}
V_R(q^i) &= & (1) \\
&\rho(\gamma_{RR}d + (1 - \gamma_{RR})c + \varepsilon(1 - 2q_c^i)) + (1 - \rho)(a + \varepsilon(1 - 2q_d^i)) \\
&- \rho(\gamma_{RS}b + (1 - \gamma_{RS})a + \varepsilon(1 - 2q_d^i)) - (1 - \rho)(a + \varepsilon(1 - 2q_d^i))
\end{aligned}$$

As $q^i = q_c^i - q_d^i$, $V_R(q^i)$ can be written as:

$$\begin{aligned}
V_R(q^i) &= \rho(\gamma_{RR}d + (1 - \gamma_{RR})c - \gamma_{RS}b - (1 - \gamma_{RS})a) - 2\varepsilon\rho q^i \\
&= M(q^i) + S(q^i)
\end{aligned}$$

where

$$\begin{aligned}
M(q^i) &= \rho(\gamma_{RR}d + (1 - \gamma_{RR})c - \gamma_{RS}b - (1 - \gamma_{RS})a) \\
S(q^i) &= -2\varepsilon\rho q^i
\end{aligned}$$

For the religious, there may be two reasons for preferring to stay religious. A "material" motivation, $M(q^i)$, arises due to a material gain, as a religious individual may obtain a higher level of cooperation from society if $\gamma_{RR} > \gamma_{RS}$, and a material loss, as a religious individual takes suboptimal actions, namely cooperating with fellow religious individuals. If she were to be secular she would lose the material gain but avoid the material loss from cooperation. The material payoff depends on q^i only through its effect on behaviour.

A "spiritual" motivation, $S(q^i)$, arises when religious individuals anticipate that they will cooperate more often when religious, both because of the beliefs instilled by the religion, but also because others might cooperate with them more often. If they become secular on the other hand, they will defect, a scenario they may wish to avoid given their current beliefs. The spiritual payoff depends directly on q^i and arises because religious participation has an effect on beliefs in our model.

The above decomposition of the religious type's benefit into material and spiritual payoff holds more generally for all types $q^i \in [-1, 0]$. Moreover if we compute the analogous relative benefit of being religious for secular individuals, it would solely consist of the material payoff.

We show in the appendix that all equilibria (with seculars) can be characterized by solving for the fixed point in q' at which the benefit of being religious equates the cost of joining the religion,

$$V_R(q') = r.$$

For example, if the marginal type who joins the religion is, as above, an intermediate type who cooperates against fellow religious agents, then $\rho = F(q')$, $\rho\gamma_{RS} = F(\underline{q})$ and $\gamma_{RR} = 1$. This

implies that the fixed point equation in q' , given (1), becomes:

$$(F(q') - F(\underline{q}))(d - a) + F(\underline{q})(d - b) - 2\varepsilon q' F(q') = r$$

We show in the appendix that, for any F , and as long as r is not too large, solutions exist to sustain different types of religious organizations, which we describe next.

3.2 Taxonomy of religious organizations

The next Proposition describes the different types of religious organizations which may arise in equilibrium for a given r and distribution of types F :²²

PROPOSITION 1 *There exist values $\bar{r} > r^1 > r^2 > \underline{r} > 0$ such that for any $r \in [0, \bar{r}]$ there exists a unique equilibrium with religion, in which all individuals below $q'(r)$ are religious. All equilibria are characterized by one of the following,*

(i) *For any $r \in [r^1, \bar{r}]$ the religion has full cooperation: Only individuals with $q^i \leq q'(r)$ for $q'(r) \leq \underline{q}$ are religious, and they cooperate with all opponents.*

(ii) *For any $r \in [r^2, r^1]$ the religion has selective cooperation: Only individuals with $q^i \leq q'(r)$ for $q'(r) \in [\underline{q}, \bar{q}]$ are religious; religious individuals in $[\underline{q}, q'(r)]$ defect against seculars and cooperate against religious opponents.*

(iii) *For any $r \in [\underline{r}, r^2]$ the religion has free riders: All individuals with $q^i \leq q'(r)$ for some $q'(r) \in [\underline{q}, \bar{q}]$ and in addition some with $q^i > q'(r)$, are religious. Religious individuals in $[\underline{q}, q'(r)]$ defect against seculars and cooperate against religious opponents, whereas religious in $[q'(r), 0]$ defect against all.*

(iv) *For any $r \leq \underline{r}$, society is fully religious; All below $q^{aut} \in [\underline{q}, \bar{q}]$ cooperate against all and those above q^{aut} defect against all.²³*

Proposition 1 states how, as a function of r , religious participation, the level of cooperative behaviour by religious agents, and the actual intensity of their beliefs is determined endogenously in equilibrium.

In the *full cooperation religion*, only individuals below \underline{q} are religious. These agents cooperate indiscriminately, so that $\gamma_{RR} = \gamma_{RS} = 1$. There is no material benefit from being religious, only a material loss. Still, agents are religious because of the spiritual payoff.

²²The cutoffs of r as well as $q'(r)$ depend of course on F . For brevity this index is omitted.

²³The four different types of organizations are the only ones that exist and always exist for some r for any ε . A large enough value of ε implies that each such equilibrium is unique for a particular r and that the equilibria are segregated by r .

In the *selective cooperation religions*, religions are larger and its members are less cooperative towards seculars; intermediates cooperate only against their fellow religious opponents so that $1 = \gamma_{RR} > \gamma_{RS}$. Such selective behaviour is suggested in some of the scriptures and preachings of several religions and its role seems to be to generalize kinship to the larger community. For example, in Judaism, "you shall not hate your brother in your heart" (Leviticus 19:17) and "Do not act vengefully or bear a grudge against members of your nation" (Leviticus 19:19) are based on a concept of "national mutuality" and specifically apply to Jews only. In these equilibria both spiritual and material benefits exist.

In some equilibria "free riders" join the religion and defect against all, so that $\gamma_{RR} < 1$. These free riders become religious to take advantage of the material benefit. They can be thought of as non-believers who nonetheless attend church and participate in religious life. Huber (2005) finds empirical evidence that some church goers are non-believers and that this is more prevalent in less-developed countries which exhibit strong network benefits for religious members.

At the extreme, the whole society can be religious, in which case the material benefit is based on the out of equilibrium beliefs that intermediate types defect against seculars. Note that only this equilibrium can be sustained for negative values of r , i.e., religious rituals that are directly beneficial to one's utility. In all other configurations r has to be strictly positive.

Note that the set of equilibria in which the whole society is religious is equivalent in terms of equilibrium behaviour to the benchmark equilibria in which rituals do not exist, which we specified in the previous Section. Thus, compared with that benchmark, rituals that are too costly might reduce the level of cooperation in society (as in the fully cooperative equilibria), but rituals with lower costs will allow for meaningful signalling and higher levels of mutual cooperation.

In the remainder of the section we provide some additional results. First, note that Proposition 1 had illustrated that globally, larger religions are associated with lower intensities of rituals. We establish that this also holds locally, and explain how this result is implied by the different motivations for being religious.

3.3 Intensity of rituals and religious participation

Recall the two motivations for being religious. The "material" motivation arises as a religious agent obtains a higher level of cooperation from society. This implies that the cost of religion should *increase* when more intermediate types (who cooperate selectively) are religious and thus a positive link should arise between participation and intensity of rituals. On the other

hand, the "spiritual" motivation arises when agents who become religious change their behavior and become more cooperative themselves. This implies that the cost of religion should *decrease* when more intermediate types are religious, as the marginal type is less averse to defection and thus a negative link should arise between religious participation and intensity of rituals. When the shocks are large enough, the "spiritual" motivation becomes more important, inducing the negative monotone relationship stated below (for any fixed F).

PROPOSITION 2: *For all $r \leq \bar{r}$, for ε sufficiently large, religious membership (ρ) decreases in the intensity of rituals (r), whereas internal and external cooperation (γ_{RR} and γ_{RS}) increase in r .*

Our prediction about the relationship between intensity of rituals and the level of cooperation finds some support in recent empirical and experimental studies. Sosis and Ruffle (2007) conduct an experiment involving religious and secular individuals in Israeli Jewish Kibbutzim. They find that religious (Jewish Orthodox) males are more cooperative compared with religious females (who participate in fewer rituals) and with seculars, and that the more frequently religious males attend synagogue, the more cooperative they are. Secular rituals (such as eating in the common dining room) are found to have a weaker effect on cooperation than do religious rituals. In another study, using survey data in the US, Iannaccone (1992) finds that the stricter is the church in its demands, the higher is the level of contributions (in money and time) within the church. Finally, Iannaccone (1998) reports some stylized facts from the US about the relationship between rituals, cooperation, and beliefs. Within religions, more conservative denominations have members who contribute proportionally more income, attend more services, and have more orthodox doctrinal beliefs.

3.4 Religion and welfare

Next we consider some welfare properties of religion. Our model has identified the secular benefits of religion; religious individuals enjoy a higher level of cooperation, and moreover, compared with a fully secular society, religion enhances cooperation overall. There are many documented examples of how religions enhance social cohesion and cooperation, benefiting group members and sometimes non-members through spillovers.²⁴ One is the worship of Dewi Danu, goddess of the waters, in Bali, Indonesia, as studied by Lansing (1991). Lansing describes the technology of the maintenance and allocation of irrigation water in the area. This

²⁴Wilson (2002) discusses several cases of historical and contemporary religions which provide "secular utility" to its members.

technology implies large social benefits from cooperation and it relies on the local religion which connects cooperative behaviour to private welfare; Lansing quotes their beliefs that “*Because the goddess makes the waters, those who do not follow her laws may not possess her rice terraces*”.

When we consider whether religion is potentially welfare-improving in society, such benefits are obviously on the positive side. On the other hand, religion might decrease individuals’ welfare; members of religious organizations might hold wrong beliefs that will lead them to take suboptimal actions. In addition, religious participation involves costly rituals that might be unproductive or wasteful.²⁵ Thus our model illustrates a trade-off between social cohesion on one side and "erroneous beliefs" and costly practice on the other.

Abstracting from the spiritual payoff provided by the religion to its members (which depends on the true distribution governing the shocks), it is easy to find examples in which the existence of a religious organization reduces the utility of some individuals relative to a society with no religion. For example, in small religions with full cooperation, religious individuals take sub-optimal actions as well as pay the cost of rituals and are therefore worse off than in a secular society. We also show below that in some environments, the material benefits of religion can be large enough so that the existence of religion is Pareto-improving. In particular, this can arise with a large enough religion with selective cooperation, whose enhanced internal cooperation is sufficient to outweigh the cost of rituals:

PROPOSITION 3 *(i) For any F , there exist some values of r and equilibria with a religious organization in which some individuals are worse off than in a completely secular society; (ii) There exist distributions F and an equilibrium with religion that provides all individuals in society a higher material utility than that provided in a completely secular society.*

3.5 Religion and scientific progress

We conclude this section with some comparative statics on the implications of "scientific progress"; Weber (1904 and 1922) advanced the secularization hypothesis claiming that as science progresses, the role that religion has to play in people’s life, re-assuring and explaining the world around them, will diminish.²⁶ As more and more scientific knowledge becomes

²⁵The Anthropology literature provides many examples of intense ritualistic societies. One example discussed in Boyer (2001) is the extraordinary number of rituals performed in Melanesian religions for the purpose of protection against witchcraft.

²⁶The evidence on secularization is mixed. For a summary of the empirical evidence on secularization, see Huber (2005).

available it might make individuals less prone to be affected by religious preaching. Thus, we can capture the effects of *scientific progress* by a shift in F , in a first order stochastic sense, where in the limit, when no types with low values of q exist in society, a religious organization cannot arise. We now show that although the arguments above depict a simple global relationship between scientific progress and religious participation, our model indicates that this relationship is more subtle and does not necessarily hold locally.

To assess the effect of F , it is sufficient to analyze the incentives of the marginal religious type.²⁷ Consider such type in a religious organization with selective cooperation. The marginal type, q' is determined by,

$$(F(q') - F(\underline{q}))(d - a) + F(\underline{q})(d - b) - 2\varepsilon q' F(q') = r$$

Suppose we change the distribution of types in society from F to F' where F' first order stochastically dominates F . The first expression represents a material gain from being religious as a share of the religious, namely $F(q') - F(\underline{q})$, cooperates only when their opponent is religious. A shift to $F'(q') - F'(\underline{q})$ can increase or decrease this term so to fix ideas suppose that $F'(q') - F'(\underline{q}) = F(q') - F(\underline{q})$. The second expression represents a material loss; a religious q' cooperates against agents with $q < \underline{q}$, whereas if he were to be secular, he could take advantage of them by defecting. The magnitude of this loss depends on $F(\underline{q})$ so a shift to $F'(\underline{q}) < F(\underline{q})$ reduces this loss. Finally, the spiritual benefit depends on total religious participation, $F(q')$, as this corresponds to the likelihood that the agent will cooperate. A shift to $F'(q') < F(q')$ decreases therefore the spiritual gain. Thus, the spiritual and material motivations are at odds, which in most cases, renders the effect of scientific progress on religious participation ambiguous:

PROPOSITION 4 *A shift from F to F' where F' first order stochastically dominates F decreases the size of the full cooperation religion but has an ambiguous effect on other religious organizations.*

²⁷In the proof of Propositions 1 and 2 we show that for relatively high levels of shocks the benefit of being religious is monotone and thus the change in benefit for the marginal type translates, one-to-one, to the effect on religious participation in the new equilibrium.

4 The dynamics of beliefs

Personal experiences are important in shaping individuals' attitudes and beliefs towards religion, and are often mentioned as motivations for conversions or deconversions.²⁸ We now analyze how realized shocks to the well being of individuals affect their beliefs and what it implies for the dynamics of religious organizations.

To allow for belief dynamics, assume that each individual does not know the exact values of q_c^i and q_d^i but has beliefs (active if religious, latent if secular) about these parameters. We model these beliefs as independently drawn prior density functions, $f^i(q_c^i)$ and $f^i(q_d^i)$, respectively. The definition of q^i is now altered to $q^i = E^i(q_c^i) - E^i(q_d^i)$ and we maintain the assumption that q^i is distributed according to some F . To allow for conversions and de-conversions, we assume that all individuals -whether religious or secular- update their -active or latent- beliefs when confronted with new information.

Specifically, we assume that each individual i updates his beliefs, $f^i(q_c^i)$ and $f^i(q_d^i)$, following his own course of play and the shock he receives.²⁹ If individuals believe that their shocks are independent from those of others, these are indeed the only relevant observations. But even if this is not the case and individuals may realise that shocks are correlated (as in the analysis below), information about their own action may be the only available or credible one. In other words, while individuals may observe others' shocks, they probably have not observed others' behaviour.³⁰

The following Lemma will be useful for our analysis; it establishes that in response to a shock, individuals update their beliefs conditional on the action they have played in the game:

LEMMA 1 *(i) Following a negative shock, an agent who cooperated (defected) will increase (decrease) his q^i . (ii) Following a positive shock, an agent who cooperated (defected) will decrease (increase) his q^i .*

Intuitively, an individual who has been cooperating and experienced a positive shock, believes that on average q_c^i is lower but does not change his beliefs on q_d^i , whereas an individual who

²⁸For example, the 1966 Nobel laureate in Literature, S.Y. Agnon, became religious after in 1924 he lost five years worth of writings in a fire.

²⁹An alternative interpretation is that before the next period, each individual is replaced by a new member of the population (from his "dynasty") who inherits his updated beliefs. Our dynamic process can therefore be interpreted as a process of transmission of beliefs from one generation to another.

³⁰This is similar to the approach taken in Piketty (1995). If individuals will observe others' actions and shocks this will change the speed of long run convergence of beliefs and the type of organizations which can be stable in the long run.

has been defecting and experienced a positive shock, believes that on average q_d^i is lower and does not change his beliefs on q_c^i . Similar intuition holds for negative shocks. An implication of Lemma 1 is that individuals will update their beliefs depending on the particular equilibrium they play, as this determines the actions they take in the PD game.

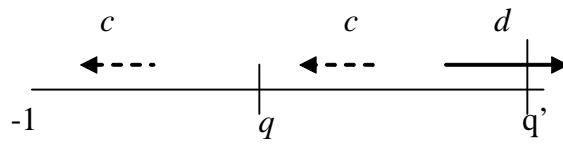
4.1 Religion in "good" and "bad" times

Following natural and other disasters, individuals and religious leaders often re-evaluate, or feel a need to justify, their religious stances (e.g., Jewish theology after the holocaust). In turn, this re-evaluation affects individuals decisions of whether to stay/become religious. For example, in a recent empirical paper, Chen (2010) suggests that following the financial crisis in Indonesia, religious participation had increased.

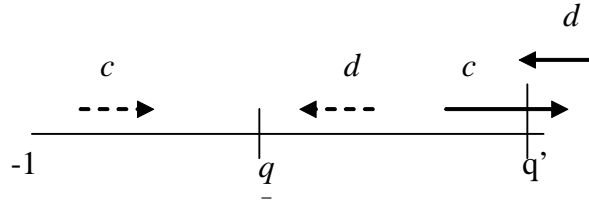
Our formal modeling of religious beliefs and of the “rational” affiliation choice allows us to shed some light on the above effects. In this Section we analyze one-off correlated shocks to individuals in society. We focus on one period, assume that all individuals in society are affected by the same shock, and analyze its effect on belief updating and on religious participation

To fix ideas, we focus in this section on the selective cooperation equilibrium with a fixed r . This equilibrium is characterized by a cutoff $q'(r)$ below which individuals are religious, where some religious individuals (those with $q^i < \underline{q}$) always cooperate, and some religious individuals (with $q^i \in [\underline{q}, q'(r)]$) defect when meeting seculars and cooperate when meeting a fellow religious member. All seculars defect, while their (latent) type is higher than the cutoff $q'(r)$.

Consider the effect of a one-off positive shock experienced by all individuals. By Lemma 1, all individuals with relatively low q^i , who were cooperating, will decrease their q^i even further, and all those with relatively high q^i , who were defecting, will increase their q^i even further. Thus, a correlated positive shock will tend to polarize beliefs in society. Moreover, some religious individuals with intermediate q^i , who happened to meet seculars and were therefore defecting, will actually leave their faith. Therefore, such a shock will not only polarize beliefs but will also decrease the size of the religion. We illustrate this below, where the arrows in the figure mark the direction of the change in beliefs, given the action of the individual, and the bold arrow is the relevant one in terms of (de)conversion:



On the other hand, a negative shock (e.g., a natural disaster) will have the opposite effect. Agents with relatively high q^i who were defecting will reduce their (latent) q^i while agents with low q^i who were cooperating will increase their q^i . The effect of a correlated negative shock on religious participation is therefore not clear-cut. We will observe seculars who convert but also some religious individuals who will leave the faith:



Which effect dominates will generally depend on the exact shapes of $f^i(q_c^i)$ and $f^i(q_d^i)$, but holding these fixed, the size of the religion will increase following a negative shock if the size of the religion is small enough, whereas it will decrease if this size is large. To see why, note that the size of the religion has no effect on the flow into the religion but has a negative impact on the flow out of the religion: The larger is the religion, the more likely it is that intermediate religious types encounter other religious individuals and hence more of them end up cooperating. As the negative shock indicates the inferiority of cooperation, more of these individuals will leave the faith. On the other hand, seculars defect against all individuals so the size of the religion does not affect the rate of religious conversion.

4.2 Maintaining religious beliefs

Church attendance as well as other measures of religious participation in Britain have been steadily falling from around 40% in 1850 to 10% in 1990's, whereas in the U.S. it remained flat during these times. Similarly, while some religious groups show strong persistence over time, some systems of beliefs and practices have vanished.

In the above section we illustrated how religious organizations affect the distribution of beliefs in society whereas our basic model had determined how the distribution of beliefs affects the form of religious organizations. We can now use this reciprocal relationship between beliefs and religious organizations to analyze whether religions can be sustained in the long term.

To analyze the dynamics of religious organizations, we need to specify the true distribution generating the shocks to individuals' utilities, as individuals are allowed to learn and update their beliefs. Denote the true probabilities pertaining to each individual by $(\hat{q}_c^i, \hat{q}_d^i)$ and let the distribution of beliefs in the population (i.e., the collection of $f^i(q_c^i)$ and $f^i(q_d^i)$) be denoted by

G . We say that G is *consistent* with the truth if for any i , $(\hat{q}_c^i, \hat{q}_d^i)$ is in the support of $f^i(q_c^i)$ and $f^i(q_d^i)$ respectively.

DEFINITION 1 *A distribution G and an equilibrium given G is a steady state iff: (i) G is consistent with the truth, (ii) G and the equilibrium do not change after individuals update their beliefs.*³¹

To make the survival of religion more difficult, assume that there is no relation between social actions and shocks. In what follows we assume that $\hat{q}_c^i = \hat{q}_d^i = \lambda$ for all i so that $\hat{q}_i = 0$ for all i . Nonetheless, we now show that religious organizations that are characterized by fully cooperative behaviour will allow their members to maintain erroneous beliefs about the relative benefit of defecting; these members may stick to their beliefs forever as they do not learn anything about defection. In particular, note that religious agents who always cooperate and have $q_c^i = \lambda$ and $E(q_d^i) \in [\lambda - \underline{q}, 1]$, will have types q^i in $[\lambda - 1, \underline{q}]$. We therefore need $\lambda - \underline{q} > 0$ and $\lambda - 1 < \underline{q}$ for such types to arise in equilibrium, as well as a high enough r to support a religion with full cooperation. Thus, for some λ and r , the full cooperation religion can be sustained in the long run, even when the truth is "secular".

PROPOSITION 5 *(i) For some λ , there exists a steady state with a fully cooperative religion. (ii) For any λ , there doesn't exist a steady state with a religion with selective cooperation.*

In the equilibria with selective cooperation, intermediate types experiment, i.e., they sometimes cooperate and sometimes defect. As they learn about all possible actions, in the long run, they (or their offsprings) will converge to hold beliefs which are close to the truth and will thus choose to defect. Thus, no intermediate types can exist in the long run. In other words, religions with selective cooperation cannot survive.³²

4.3 Religions and unverifiable information

Anthropologists such as Rappaport (1999) and Sosis and Alcora (2003) argue that religious rituals are more effective than secular rituals at promoting cooperation because religious beliefs are more stable and less falsifiable. Our analysis above indicated that the level of exposure of

³¹In other words, individuals' beliefs will be "self-confirming", as in Fudenberg and Levine (1993).

³²The equilibrium in which all individuals in society are religious may exist in the long run as well. In both this and the full cooperation equilibria individuals always cooperate or always defect disregarding their opponent affiliation. We find this equilibrium less reasonable as it relies on the off equilibrium path threat in which intermediate types do experiment which may therefore lead to the disappearance of such types.

the religious to information might have implications for the survival of religions in the long-run. Several religious practices indeed allow religions (or religious leaders) to control the extent of such exposure, either by censoring information or by creating systems of beliefs that are immune to it. Our model can shed light on such practices which may have evolved to slow the belief updating process and make beliefs less falsifiable.

First, consider beliefs in the *afterlife*. Focusing beliefs on rewards and punishment in the afterlife implies that such beliefs are not verifiable in this life. Indeed, the Judeo-Christian traditions, bring the afterlife, with its concepts of heaven and hell, to the fore. Similarly, the Eastern religions such as Hinduism, Buddhism and Sikhism, share beliefs of reincarnation, in which the future "self" will pay for the sins or gain from the good deeds of the current one. The analysis above indicates that unfalsifiable beliefs as beliefs in the afterlife will contribute to the stability of religious organizations.

Second, consider the concept of *forgiveness*, with its twin concepts of atonement and repentance, important concepts in the Judeo-Christian tradition. While there are different explanations to the usefulness of forgiveness and its function in religious organizations, our model suggests another way to view this religious practice. In some sense, forgiveness blurs the relation between actions and rewards. While an omnipresent God will surely know that one has sinned, it is not known whether God has forgiven or not, and hence whether punishment will be inflicted upon the individual.

Third, our analysis is simplified by the assumption that the shocks depend only on individual actions; it is often believed that the whole collective is punished for the sins of some, or escapes punishment due to the good deeds of others. Such more *complicated systems of religious beliefs* make it harder for individuals to update their beliefs correctly.

Finally, many religious organizations tend to segregate their members from non-members either physically or in other ways that limit access to information about the experiences of non-members.³³ The benefit of such *religious segregation* can be two-fold. First, religious agents will mostly meet other religious agents and will avoid being taken advantage of. A second benefit is that such isolation might allow the religious to avoid information that is unfavorable for the prevalence of their beliefs.

³³An extension of our model in which individuals who belong to the same affiliation can be matched to play the PD game with members from this affiliation only yields qualitatively similar results.

5 "Hierarchy" of religious denominations

Iannaccone (1994) finds that "Relative to their more mainstream counterparts, members of sectarian groups -both Christian and Jewish- hold fewer memberships in outside groups, contribute less to outside causes, and have fewer outside friends." Farber (2001) shows how Orthodox Jews have been actively hostile towards the less demanding movements, Conservative and Reform Judaism. These studies indicate that the stricter denominations might be less cooperative towards other denominations.

We now extend the model to allow for more than one religious group, in order to explore such asymmetric relations between denominations. For simplicity, we assume that an agent's type q^i determines his beliefs in any religious affiliation so that all religions face the same F in society.³⁴

We say that a religious group is "tolerant" ("intolerant") towards another religious group if all (some) members of the group who cooperate internally, cooperate (defect) when matched with members of the other group. The next result illustrates that whether groups are tolerant towards one another depends on their internal level of cohesiveness, i.e., whether the group has internal free riders or whether it has perfect internal cooperation:

PROPOSITION 6 (i) Whenever intolerance arises between a religion with perfect internal cooperation and another religious group, then the group with the perfect internal cooperation must be intolerant. (ii) For some parameters there exists an equilibrium with one tolerant religion with free riders, and one intolerant group which has perfect internal cooperation. In this equilibrium the group with the perfect internal cooperation has a higher intensity of rituals.

Our result implies a hierarchy of churches or denominations - the stricter church, with the higher intensity of rituals, and the more cohesive behaviour, is less cooperative towards other churches or religions which are less strict in their demands. Some of its members who cooperate internally defect against the other religion, whereas all members of the religion with free riders that actually cooperate internally, also cooperate against the other group.

Note that we have considered "intolerance" as the discriminating behaviour of these group members towards non affiliated members. But intolerance, hostility, or conflicts in society are often more centrally organized and involve extra curricular activities on top of such daily interactions. One way to think about such hostile activities is to embed them within the rituals; the costly observable actions that identify members with the religion and signal their beliefs

³⁴Given this assumption it is straightforward to extend the equilibrium notion presented in Section 2 to several groups.

may include participation in demonstrations, or violent activities that hurt non-members. Such behaviour, if feasible, will render other denominations less attractive. If religious leaders for example value participation they may have an incentive to instruct members to inflict such cost on non-members (and shield their own). In that sense, competition for members between religious and secular groups or more generally between several religious groups can lead simultaneously to a higher religious participation and to more conflicts in society.

6 Conclusion

We have proposed a simple model of religious organizations which relies on the ability of such organizations to alter the beliefs of individuals about the relation between their social actions and shocks to their utility. The model ties together the three most observed aspects of religious organizations: beliefs, social behaviour, and rituals.

We have analyzed the stability of religious organizations and their features, but abstracted away from how they arise. The role of religious leaders as selecting these organizations is an important extension which we leave for future research. Another related topic is "religious governance", i.e., the study of the structure of religious organizations. While some religions are centralized (e.g., the Roman Catholic church), others (such as the Protestants church) are more decentralized. Such differences may depend on features such as the importance of network effects, the level of competition for members, or possibly the particular advocated system of beliefs. A possible direction of future research may be an attempt to link theological differences with governance structure. For example, a free will theology may, in some cases (such as in the case of the Catholic Church) feed well into an hierarchical system in which priests certify which actions will lead to rewards and which to punishments.³⁵

³⁵The case of Islam is at odds with this example, in which free will is to some degree advocated, but the religion is fairly decentralized.

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APPENDIX

PROOFS OF PROPOSITION 1 AND 2: Recall that secular individuals always defect, religious individuals below \underline{q} always cooperate, religious individuals above \bar{q} always defect, and intermediates defect against seculars. It is therefore left to consider how intermediates behave against fellow religious members. A religious individual will defect against another religious agent, who cooperates with probability γ_{RR} , if:

$$\gamma_{RR}b + (1 - \gamma_{RR})a + \varepsilon(1 - 2q_d^i) \geq \gamma_{RR}d + (1 - \gamma_{RR})c + \varepsilon(1 - 2q_c^i)$$

or when $q^i \geq q_{RR}$, for

$$q_{RR} \equiv \frac{1}{2\varepsilon}((d - b)\gamma_{RR} + (c - a)(1 - \gamma_{RR})) = \gamma_{RR}\bar{q} + (1 - \gamma_{RR})\underline{q} \quad (2)$$

where q_{RR} and γ_{RR} are endogenously determined in equilibrium.

We first consider the relative benefit of a religious agent from being religious vs. being secular:

Lemma A1 *For a religious agent, the relative benefit of being religious is monotonically decreasing and continuous in q^i .*

Proof: For a type below \underline{q} , the relative benefit of being religious is given by:

$$\begin{aligned} & \rho(\gamma_{RR}d + (1 - \gamma_{RR})c) + (1 - \rho)c + \varepsilon(1 - 2q_c^i) \\ & - \rho(\gamma_{RS}b + (1 - \gamma_{RS})a) - (1 - \rho)a - \varepsilon(1 - 2q_d^i) \\ = & M^{q^i \leq \underline{q}}(\rho, \gamma_{RR}, \gamma_{RS}) - 2\varepsilon q^i, \end{aligned}$$

where $M^{q^i \leq \underline{q}}(\rho, \gamma_{RR}, \gamma_{RS})$ is fixed for all $q^i \leq \underline{q}$ and represents "material" payoffs from the PD game, while the second term above, $-2\varepsilon q^i$, represents the "spiritual" payoff.

For types above \underline{q} and below q_{RR} , the relative benefit for being religious is given by:

$$\begin{aligned} & \rho(\gamma_{RR}d + (1 - \gamma_{RR})c) + (1 - \rho)a + \varepsilon\rho(1 - 2q_c^i) + \varepsilon(1 - \rho)(1 - 2q_d^i) \\ & - \rho(\gamma_{RS}b + (1 - \gamma_{RS})a) - (1 - \rho)a - \varepsilon(1 - 2q_d^i) \\ = & M^{\underline{q} \leq q^i \leq q_{RR}}(\rho, \gamma_{RR}, \gamma_{RS}) - 2\varepsilon\rho q^i. \end{aligned}$$

Where again the material benefit is fixed for these types and does not change with q^i . It is different though from the material benefit for the lower types as these types defect more often.

Finally for types above q_{RR} :

$$\begin{aligned} & \rho(\gamma_{RR}b + (1 - \gamma_{RR})a) + (1 - \rho)a + \varepsilon(1 - 2q_d^i) \\ & - \rho(\gamma_{RS}b + (1 - \gamma_{RS})a) - (1 - \rho)a - \varepsilon(1 - 2q_d^i) \\ = & M^{q^i > q_{RR}}(\rho, \gamma_{RR}, \gamma_{RS}) = \rho(\gamma_{RR} - \gamma_{RS})(b - a) \geq 0. \end{aligned}$$

Moreover, one can show that $M^{q^i \leq \underline{q}}(\rho, \gamma_{RR}, \gamma_{RS}) - 2\varepsilon \underline{q} = M^{q \leq q^i \leq q_{RR}}(\rho, \gamma_{RR}, \gamma_{RS}) - 2\varepsilon \rho \underline{q}$ and that $M^{q \leq q^i \leq q_{RR}}(\rho, \gamma_{RR}, \gamma_{RS}) - 2\varepsilon \rho q_{RR} = M^{q^i > q_{RR}}$, hence the benefit is continuous in q^i . \square

Given the monotonicity property in Lemma A1, and given some r, ρ, γ_{RS} and γ_{RR} , it is optimal for all religious agents below some q' to be religious.

The expressions for the material payoffs above represent the relative benefit of being religious *as viewed by the seculars*, as these agents consider only the material payoff when evaluating affiliation choices:

Lemma A2 (i) *For seculars, the relative benefit from being religious is (weakly) monotonically increasing in their type q^i .* (ii) *The relative benefit from being religious is equal for a secular type and for a religious type above q_{RR} .*

If q' is high enough such that a religious agents above q_{RR} strictly prefers to be religious, then, together with the refinement, all society is religious in equilibrium. If on the other hand q' is low enough so that such a religious type prefers to be secular, then by Lemma A2, such a secular type also prefers to stay secular, and also all other secular types prefer to stay secular. Thus, all secular agents with $q \geq q'$ will, at least weakly, prefer to be secular. This insures that no individual would prefer to be religious were he to be secular and prefer to be secular were he to have religious beliefs. Together with the refinement, we can then look for equilibria in which all agents with types below q' are religious. Thus there could only be four types of equilibria, depending on the level at which r intersects the religious benefit function outlined in Lemma A1 above:

Full cooperation religion

In the first family of equilibria, $1 = \gamma_{RR} = \gamma_{RS} > 0$, and members are only below \underline{q} . Also, $q_{RR} = \bar{q}$. Let $r_1 = 2\varepsilon F(\underline{q})(\bar{q} - \underline{q})$ and $r_2 = 2\varepsilon(\underline{q} + 1)$. Note that $\min\{r_1, r_2\} > 0$. We now show that these equilibria hold for any $r \in [\min\{r_1, r_2\}, \max\{r_1, r_2\}]$.

What we need to determine is the marginal type, who is indifferent between being religious and being secular, given his beliefs q' . We therefore need to solve:

$$\begin{aligned} M^{q^i \leq \underline{q}}(\rho, \gamma_{RR}, \gamma_{RS}) - 2\varepsilon q' &= r \Leftrightarrow \\ F(q')(d - b) + (1 - F(q'))(c - a) - 2\varepsilon q' &= r \Leftrightarrow \\ F(q')\bar{q} + (1 - F(q'))\underline{q} - q' &= \frac{r}{2\varepsilon} \end{aligned}$$

To see why a solution exists to the above fixed point equation in q' , consider some r in the range described above. If $q' = -1$, then, as $F(q') = 0$, we have that the left-hand-side of the equation is $\underline{q} + 1$ which is greater (smaller) than the right-hand-side when $r_1 < r_2$ ($r_1 > r_2$).

If on the other hand, $q' = \underline{q}$, then the left-hand-side becomes $F(\underline{q})(\bar{q} - \underline{q})$, and it is smaller (greater) than the right-hand-side when $r_1 < r_2$ ($r_1 > r_2$). By continuity, a fixed point exists.

Selective cooperation equilibrium

In the second type of equilibrium, $1 = \gamma_{RR} > \gamma_{RS} > 0$. Let $r_1 = (F(\bar{q}) - F(\underline{q}))(b - a)$ and $r_2 = 2\varepsilon F(\underline{q})(\bar{q} - \underline{q})$. Note that $\min\{r_1, r_2\} > 0$. We now show that these equilibria hold for any $r \in [\min\{r_1, r_2\}, \max\{r_1, r_2\}]$.

Again, as $\gamma_{RR} = 1$, we have that $q_{RR} = \bar{q}$. We need to determine which of the types below q_{RR} is the marginal type, i.e., to solve:

$$\begin{aligned} M^{q \leq q^i \leq q_{RR}}(\rho, \gamma_{RR}, \gamma_{RS}) - 2\varepsilon \rho q' &= r \Leftrightarrow \\ F(q')(d - a) - F(\underline{q})(b - a) - 2\varepsilon F(q')q' &= r \end{aligned}$$

The existence of a fixed point, for the relevant range of r , follows from the same argument as described in the previous section with \underline{q} and \bar{q} as the end points.

Free riders equilibrium

In this family of equilibria, all below q_{RR} and a measure α of types above it are religious, so that $1 > \gamma_{RR} > \gamma_{RS} > 0$. In response to the introduction of religious types who defect, there are fewer types below q_{RR} so that $q_{RR} < \bar{q}$. Such an equilibrium exists for any $r \in [\underline{r}, (F(\bar{q}) - F(\underline{q}))(b - a)]$, for some \underline{r} that will be defined below.

The equilibrium is a solution to the fixed point equation in (2), where $\gamma_{RR} = \frac{F(q_{RR})}{F(q_{RR}) + \alpha}$. Moreover, as the relative benefit to religious types who defect from being religious does not depend on q^i , they must be indifferent so that some of them, but not all, are religious. Thus, given the solution to q_{RR} , we have to find γ_{RR} so that these types are indifferent. For consistency, we let $q_{RR} = q'$:

$$\begin{aligned} q' &= \frac{F(q')}{F(q') + \alpha} \bar{q} + \left(1 - \frac{F(q')}{F(q') + \alpha}\right) \underline{q}; \\ r &= (F(q') - F(\underline{q}))(b - a) \end{aligned}$$

When $\alpha = 0$, q' is set at \bar{q} , and is the limit of the selective cooperation equilibria, when there are no "free riders". When α is higher, and the religion has a larger share of free riders, q' decreases. To determine the lower bound \underline{r} we can find the limit of these equilibria when the whole population is religious (so that $\rho = 1$). Specifically, let $\alpha = 1 - F(q')$. We then have $\underline{r} = (F(q^*) - F(\underline{q}))(b - a)$ where q^* is the solution to $q' = F(q')\bar{q} + (1 - F(q'))\underline{q}$. Note that $\underline{r} > 0$ as for any full-support distribution function F , $q^* > \underline{q}$.

Society with no seculars

Finally, for any $r \leq \underline{r}$, there exists an equilibrium in which the whole society is religious. This equilibrium is as the limit equilibrium described above, which holds for \underline{r} . In this equilibrium, all agents below q^* cooperate, and all agents above q^* defect. If an agent deviates and becomes secular, then a share $F(q^*) - F(\underline{q})$ of the population will defect against him. Obviously, this equilibrium can be sustained for any $r \leq \underline{r}$, including negative values of r .

We now show that for a high enough ε , the religions specified above are segregated in r . In particular, this holds if:

$$(F(\bar{q}) - F(\underline{q}))(b - a) < 2\varepsilon F(\underline{q})(\bar{q} - \underline{q}) < 2\varepsilon(\underline{q} + 1). \quad (3)$$

where in Proposition 1, $r^2 = (F(\bar{q}) - F(\underline{q}))(b - a)$, $r^1 = 2\varepsilon F(\underline{q})(\bar{q} - \underline{q})$ and $\bar{r} = 2\varepsilon(\underline{q} + 1)$.

We now show why (3) holds when ε is high enough. Consider first the left-hand-side inequality:

$$\begin{aligned} (F(\bar{q}) - F(\underline{q}))(b - a) &< 2\varepsilon F(\underline{q})(\bar{q} - \underline{q}) \Leftrightarrow \\ (F(\bar{q}) - F(\underline{q}))(b - a) &< F(\underline{q})(d - c - (b - a)) \Leftrightarrow \\ \frac{F(\bar{q})}{F(\underline{q})} &< \frac{d - c}{b - a} \end{aligned}$$

By strategic complements, $\frac{d-c}{b-a} > 1$. On the other hand, the left-hand-side approaches 1 when ε increases. Consider now the right-hand-side inequality:

$$\begin{aligned} 2\varepsilon F(\underline{q})(\bar{q} - \underline{q}) &< 2\varepsilon(\underline{q} + 1) \Leftrightarrow \\ F(\underline{q})(\bar{q} - \underline{q}) &< \underline{q} + 1 \end{aligned}$$

When ε increases the left-hand-side approaches zero whereas the right-hand-side is positive and bounded away from zero.

We now conduct the local monotonicity analysis (Proposition 2) which also shows uniqueness. In the full cooperation religion, the equilibrium condition is,

$$F(q')\bar{q} + (1 - F(q'))\underline{q} - q' = \frac{r}{2\varepsilon}$$

First note that for high enough shocks, as $(\bar{q} - \underline{q})$ becomes arbitrarily small, the left hand side is decreasing in q' ,

$$\frac{\partial(F(q')\bar{q} + (1 - F(q'))\underline{q} - q')}{\partial q'} = ((f(q')(\bar{q} - \underline{q}) - 1)).$$

Total differentiation of the equilibrium condition implies,

$$\frac{dq'}{dr} = \frac{1}{2\varepsilon((f(q')(\bar{q} - \underline{q}) - 1))}$$

and as shocks grow large we have,

$$\frac{dq'}{dr} < 0,$$

Note that in this equilibrium $\rho = F(q')$ and so is decreasing in r . As for this equilibrium $\gamma_{RS} = \gamma_{RR} = 1$, these are weakly increasing in r .

In the second type of equilibrium, we have a religion where some of its members cooperate selectively. The equilibrium condition is,

$$F(q')(d - a) - F(\underline{q})(b - a) - 2\varepsilon F(q')q' = r$$

First note that for high enough shocks, the left hand side is decreasing in q' . The derivative of the left hand side is given by,

$$\frac{\partial(F(q')(d - a) - F(\underline{q})(b - a) - 2\varepsilon F(q')q')}{\partial q'} = (f(q')((d - a) - 2\varepsilon q') - 2\varepsilon F(q'))$$

where $(d - a) - 2\varepsilon q'$ is bounded by $d - c$ and so negative for high levels of ε . This implies that there is a unique equilibrium.

Total differentiation of the equilibrium condition implies,

$$\frac{dq'}{dr} = \frac{1}{(f(q')((d - a) - 2\varepsilon q') - 2\varepsilon F(q'))}$$

Again as shocks become large this is negative. In this equilibrium $\rho = F(q')$ and so is decreasing in r . In this equilibrium $1 = \gamma_{RR} > \gamma_{RS} = \frac{F(\underline{q})}{F(q')}$, these are weakly increasing in r .

Finally, in the free rider family of equilibria, the equilibrium conditions are,

$$\begin{aligned} q' &= \frac{F(q')}{F(q') + \alpha} \bar{q} + \left(1 - \frac{F(q')}{F(q') + \alpha}\right) \underline{q}; \\ r &= (F(q') - F(\underline{q}))(b - a). \end{aligned}$$

Given r , as $F(\cdot)$ is increasing, there is a unique value of q' satisfying the second equation. Given this value of q' , the value of α satisfying the first equation is unique as the right hand side is decreasing in α . therefore the equilibrium is unique. In this equilibrium $\rho = F(q') + \alpha$.

Total differentiation of the second equation yields,

$$\frac{dq'}{dr} = \frac{1}{f(q')(b - a)}$$

and for the first equation,

$$\frac{d\alpha}{dq'}(q' - \underline{q}) = f(q)(\bar{q} - q) - F(q') - \alpha$$

Putting these together we get,

$$\frac{d(F(q') + \alpha)}{dr} = f(q') \frac{dq'}{dr} + \frac{d\alpha}{dq'} = \frac{1}{(b-a)} + \frac{f(q)(\bar{q} - q) - F(q') - \alpha}{(q - \underline{q})}$$

and once again this becomes negative with high levels of shocks. Finally note that for this equilibrium

$$\begin{aligned}\gamma_{RR} &= \frac{F(q')}{F(q') + \alpha} \\ \gamma_{RS} &= \frac{F(\underline{q})}{F(q') + \alpha}\end{aligned}$$

where by the above γ_{RS} is increasing in r . To see that γ_{RR} is also increasing in r , note that by the above q' is increasing in r . ■

PROOF OF PROPOSITION 3: (i) Consider a small enough religion with full cooperation: religious individuals have a negative material benefit compared to a fully secular society in which all defect, as well as pay the cost to be religious; (ii) First note that the material benefits of types below \underline{q} are the lowest among all types of individuals and we therefore focus on finding equilibria in which they are better off than in a secular world in which all defect and hence utility is a . We now find the equilibrium which offers the best case scenario for these types.

From the proof above, the benefit of these types is increasing in q' , the cutoff in a full cooperation equilibrium. Consider next the selective cooperation equilibrium. The benefit of type below \underline{q} is

$$\begin{aligned}&F(q')d + (1 - F(q'))c - r - a \\ &= (1 - F(q'))(c - a) + F(\underline{q})(b - a) + 2\varepsilon F(q')q'\end{aligned}$$

taking derivative with respect to q' we get,

$$-f(q')(c - a - 2\varepsilon q') + 2\varepsilon F(q') > 0$$

as $q' > \frac{c-a}{2\varepsilon}$.

Consider now equilibria with free riders. The benefit of type below \underline{q} is,

$$\begin{aligned}&F(q')d + (1 - F(q'))c - r - a \\ &= F(q')(d - c - b + a) + F(\underline{q})(b - a) + c - a\end{aligned}$$

This is also increasing in q' and so is maximized at $q' = \bar{q}$ which is the limit of the selective equilibria.

Therefore we consider the selective cooperation equilibrium with the largest religion which satisfies

$$(F(\bar{q}) - F(\underline{q}))(b - a) = r$$

For religions below \underline{q} , the benefit of being in this equilibrium rather than in a secular world is

$$\begin{aligned} & F(\bar{q})d + (1 - F(\bar{q}))c - r - a \\ = & F(\bar{q})(d - b) + (1 - F(\bar{q}))(c - a) + F(\underline{q})(b - a) \end{aligned}$$

Note that this expression is increasing in ε as \bar{q} are \underline{q} increasing in ε and by the strategic complementarities. We can find a $\mu > 0$ and a distribution satisfying $F(\bar{q})(d - b) + F(\underline{q})(b - a) > \mu$ and $(1 - F(\bar{q})) < \frac{\mu}{|c - a|}$ for which this utility difference is strictly larger than zero. ■

PROOF OF PROPOSITION 4: We have illustrated in the text why scientific progress has ambiguous effects on religious participation in religions with selective cooperation. In the full cooperation religion equilibrium conditions are given by,

$$F(q')(d - b) + (1 - F(q'))(c - a) - 2\varepsilon q' = r.$$

In the proofs of Propositions 1 and 2 we have shown that for high enough shocks the left hand side is decreasing in q' implying a unique equilibrium. Note that increased scientific knowledge (a first order stochastic shift in F) imply that the left hand side decreases and hence the resulting new equilibrium will have a higher cutoff q' . □

PROOF OF LEMMA 1: Suppose that an individual cooperated in the game (the analysis for an individual who defected is analogous). Note that such an individual will only update his belief about q_c^i . His updated beliefs satisfy, for any $q_c' \geq q_c''$ (we drop the index i):

$$\frac{f(q_c' | - \varepsilon)}{f(q_c' | \varepsilon)} = \frac{\frac{f(q_c')q_c'}{\int f(q_c)q_c dq_c}}{\frac{f(q_c')(1 - q_c')}{\int f(q_c)(1 - q_c) dq_c}} \geq \frac{\frac{f(q_c'')q_c''}{\int f(q_c)q_c dq_c}}{\frac{f(q_c'')(1 - q_c'')}{\int f(q_c)(1 - q_c) dq_c}} = \frac{f(q_c'' | - \varepsilon)}{f(q_c'' | \varepsilon)}.$$

The MLRP therefore holds and thus the induced $F(q_c | - \varepsilon)$ first order stochastically dominates $F(q_c)$ which first order stochastically dominates $F(q_c | \varepsilon)$. This means that $E(q_c | - \varepsilon) > E(q_c) > E(q_c | \varepsilon)$ implying the result reported in Lemma 1. ■

PROOF OF PROPOSITION 5: (i) Consider some G and a fully cooperative religion equilibrium with some cutoff q^* . We now check whether G can be generated by the individual densities $f^i(q_c^i)$ and $f^i(q_d^i)$, for which the truth is in their support. As all those with $q^i \geq q^*$ defect, they must believe that $q_d^i = \lambda$ with probability one. Let these (latent) types have $E^i(q_c^i) \in (\lambda + q^*, 1)$.

They therefore satisfy $q^i \in [q^*, 1 - \lambda]$. Similarly, as those with $q^i \leq q^*$ put probability one on $q_c^i = \lambda$, we can let these (activated) types have $E^i(q_d^i) \in (\lambda - q^*, 1)$, so that these types satisfy $q^i \in [\lambda - 1, q^*]$.

Note that for the above, we need that $\lambda + q^* \geq 0$ and $\lambda - q^* \geq 0$ implying that we need $\lambda \geq -q^*$ (recall that $q^* < 0$). Therefore, $\lambda > -\bar{q}$. Moreover, to insure that such an equilibrium exists, we need that $\lambda - 1 < \underline{q}$. Thus, for any λ satisfying:

$$-\bar{q} < \lambda < 1 + \underline{q}$$

we can find some G which generates a distribution F over q^i with support on $[\lambda - 1, 1 - \lambda]$ as constructed above, for which there exists a steady state with a fully cooperative religion.

(ii) In a religion in which some agents selectively cooperate, these agents must converge to know the truth with probability one and will therefore converge to have $q^i = 0$. This is a contradiction as such agents will always defect. \square

PROOF OF PROPOSITION 6: (i) Consider a group which has full internal cooperation, group A (which must therefore include only types below \bar{q}), and a group B which has some free riders. If group A is tolerant, all its intermediate types cooperate with group B members, then it must be that all intermediate types of group B will cooperate with members of group A, since it is their best response. Thus there will be no intolerance. Thus, to sustain intolerance between group A and B, it must be that group A is intolerant.

(ii) We now construct an equilibrium with two religions with the following characteristics:

$$1 = \gamma_{R^2 R^2} > \gamma_{R^1 R^2} > \gamma_{R^1 R^1} = \gamma_{R^2 R^1} > \gamma_{R^1 S} > 0.$$

In particular, in this equilibrium, all R^1 agents which cooperate in R^1 also cooperate with agents in R^2 , whereas the opposite is not true, as agents in R^2 enjoy full internal cooperation but some of them defect against members of R^1 .

Let q_{IJ} , where $I \in \{R^1, R^2\}$ and $J \in \{R^1, R^2, S\}$ be the cutoff above which individuals who are in I will defect against people in J . In equilibrium we have $\bar{q} = q_{R^2 R^2} > q_{R^1 R^2} > q_{R^1 R^1} = q_{R^2 R^1} > \underline{q}$. Religious individuals with $q^i \leq \underline{q}$ and will cooperate with all. Religious individuals with $q^i \in (\underline{q}, q_{R^1 R^1})$, will defect against seculars and cooperate otherwise. Religious individuals, in $(q_{R^1 R^1}, q_{R^1 R^2})$, will, when in R^1 or R^2 , cooperate only with R^2 members, and defect otherwise. Religious individuals in $(q_{R^1 R^2}, \bar{q}]$, when in R^2 will cooperate only with R^2 members, and will defect otherwise. Finally, religious individuals with $q^i > \bar{q}$, will defect against all. Note that for all these types, the benefit from joining R^2 is always greater than the benefit from joining R^1 , as they will enjoy a higher level of cooperation from society overall.

We set $r_1 = (F(q_{R^1 R^1}) - F(\underline{q}))(b - a)$ and $r_2 = (F(q_{R^1 R^1}) - F(\underline{q}))(b - a) + (F(q_{R^1 R^2}) - F(q_{R^1 R^1}))(d - c) > r_1$. This implies that religious agents with $q^i < q_{R^1 R^2}$ strictly prefer to be religious than to be secular, but are indifferent with regard to which religion to join. On the other hand, types with $q^i > q_{R^1 R^2}$ are indifferent between joining R^1 and being secular, and strictly prefer it to joining R^2 . We can therefore consider an equilibrium in which R^1 includes a mass α of the types with $q^i \leq \underline{q}$, all types in $(\underline{q}, q_{R^1 R^1})$, and a mass ϕ of types with $q^i > q_{R^1 R^2}$, and in which R^2 includes the remaining share of the types with $q^i \leq \underline{q}$ and all types in $(q_{R^1 R^1}, q_{R^1 R^2})$. The remaining agents are secular. The equations for the cutoff points are:

$$\begin{aligned} q_{R^1 R^1} &= \gamma_{R^1 R^1} \bar{q} + (1 - \gamma_{R^1 R^1}) \underline{q}; \\ q_{R^1 R^2} &= \gamma_{R^1 R^2} \bar{q} + (1 - \gamma_{R^1 R^2}) \underline{q} \end{aligned}$$

this implies,

$$\begin{aligned} q_{R^1 R^1} &= \frac{F(q_{R^1 R^1}) - F(\underline{q}) + \alpha}{F(q_{R^1 R^1}) - F(\underline{q}) + \alpha + \phi} (\bar{q} - \underline{q}) + \underline{q} \\ q_{R^1 R^2} &= \frac{F(\underline{q}) - \alpha}{F(q_{R^1 R^2}) - F(q_{R^1 R^1}) + F(\underline{q}) - \alpha} (\bar{q} - \underline{q}) + \underline{q} \end{aligned}$$

Note that to solve for an equilibrium, we simply need to find α and ϕ such that the solution for the above fixed points equations (note that we first solve for $q_{R^1 R^1}$ and then for $q_{R^1 R^2}$), will satisfy the following conditions:

$$\begin{aligned} 1 &> \gamma_{R^1 R^2} > \gamma_{R^1 R^1} > \gamma_{R^1 S} > 0 \\ 0 &< \alpha < F(\underline{q}); \quad 0 < \phi < 1 - F(q_{R^1 R^2}) \end{aligned}$$

Consider the uniform distribution on $[-1, 1]$, i.e., $F(q) = \frac{1}{2}(1 + q)$. Let $\bar{q} = -0.25$ and let $\underline{q} = -0.5$. Finally, let $\alpha = 0.2$ and let $\phi = 0.3$. The solution is $q_{R^1 R^1} = -0.38446$, $q_{R^2 R^1} = -0.33393$ and it satisfies all the conditions set above. ■