Endogenous Information Acquisition with Sequential Trade

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Abstract

I study how endogenous information acquisition affects financial markets by modelling potentially informed traders who optimally acquire variable information at increasing cost. With a competitive market maker, my model can explain the dynamic behaviour of informed trading and transaction volume. Three proxies for informed trading derived under the exogenous information assumption (spreads, Easley O'Hara's PIN and blockholder interest) may not agree with each other. With a monopolistic market maker, results also deviate from the exogenous benchmark. He can set narrower spreads than a competitive market maker in early periods. On average, spreads can widen over time.

1 Introduction

In modelling financial markets, the theoretical market microstructure literature generally assumes that some subset of traders are exogenously informed: they know the true value of the asset before arriving in the market to trade. While some authors[1] have considered endogenous information acquisition, the empirical literature on informed trading is still based on the exogenous assumption. For example, Dennis and Weston (2001) identify four commonly used measures[2] to capture informed trading and all of them are derived from structural models with exogenously informed traders.

However, investors such as mutual funds, hedge funds and investment banks, clearly rely on costly research to inform their trading decisions. There is also growing evidence to suggest that traders need to exert effort to learn about the effect of news on asset values[3]. To capture this feature, Peng (2005)[4] considers traders who allocate their limited attention between different sources of risk.

I take a more classical approach by modelling endogenously informed traders who can acquire costly information. My model can explain various stylized features of intraday markets which exogenous models cannot. My results also suggest caution when interpreting empirical measures based on the exogenous information assumption.

Building on the sequential trade framework of Glosten and Milgrom (1985), I replace exogenously informed traders with 'potentially informed' ones. These new traders choose how much information to acquire as a function of expected speculative profits from trading, which depend on posted prices. They learn the true value of the asset with a higher probability if they acquire more information and submit a trade only if they successfully learn it, doing nothing otherwise. This setup is also different from other endogenous information acquisition models which follow Grossman and Stiglitz (1980) in only considering a fixed amount of information at a fixed cost.

I start with a market with a competitive market maker which corresponds well to most financial markets we study today. Under a general specification for information acquisition, I derive conditions for the existence of interior prices. Unlike with exogenous information,

[1]Starting with Grossman and Stiglitz (1980), see Section 2 for a more comprehensive review

[2]1) bid-ask spread based on Glosten and Milgrom (1985), Glosten and Harris (1988) and Amihud and Mendelson (1985); 2) adverse selection component of spread based on Huang and Stoll (1997); 3) price impact of trade based on Kyle (1985), Foster and Viswanathan (1993) and Hasbrouck (1991); and 4) probability of informed trading based on Easley et al. (1996).

[3]E.g. Hong et al. (2007), Hou and Moskowitz (2005) and Corwin and Coughenour (2008)

[4]Also Peng and Xiong (2006) and VanNieuwerburgh and Veldkamp (2010)

prices and beliefs do not always converge to the true value in the steady state. If the cost function is discontinuous, potentially informed traders eventually stop acquiring information. Trades are no longer informative and market participants stop updating beliefs, an event I call 'information stoppage'.

I then specify a quadratic information acquisition cost function for information acquisition under which I can characterise costs with a single parameter and solve for prices in closed form. This setup generates two main results[5].

First, my model can capture dynamics for informed trading and transaction volume which exogenous models cannot. At an intraday level, real markets exhibit higher volume and more informed trading after an informational event. I interpret an event as a shock which causes market prices to deviate from their true value. Potentially informed traders can make speculative profits if they learn the true value of the asset so they acquire more information and trade more. Another stylized fact is that informed trading and volume fall over time. In my model, as prices converge to the true value, there are lower speculative profits to incentivise potentially informed traders so they acquire less information and trade less. These dynamics are driven by endogenous information acquisition. Exogenous models assume informed trading and volume are constant.

Empirically, there is also significant variation in informed trading and volume between days. In my model, this corresponds to variations in the size of shocks. Large shocks lead to large price deviations which give potentially informed traders more incentive to acquire information and trade. In exogenous models, informed traders do not respond to prices so this mechanism cannot operate. Instead, the effect is attributed to ad hoc variations in the arrival rate of informed traders. For example, Easley and O'Hara (1992) introduce event days when volume and informed trading is high, and non event days when they are low. This yields two regimes but also requires the strong assumption that the market maker is uncertain about whether an event has occurred[6]. My model can explain more variation without event uncertainty.

Second, I find deviations in three common proxies for informed trading: bid ask spreads, Easley et al. (1996)'s PIN and the proportion of hedge fund or block holder interest. In competitive models, including mine, prices are set as the expected asset value conditional on a trade. Easley et al. (1996)'s PIN is a structural estimator for the probability of an

[5]Although these results quantitatively depend on the quadratic specification, the qualitative features would obtain under other cost functions.

[6]Easley et al. (2008) suggest a GARCH process for the arrival of informed traders. Easley et al. (2012) scale arrival rate by volume.

informed trade. Finally, hedge funds or block holders can be considered informed so their participation is an indicator of informative trading. With exogenous information arrival, the three measures always agree. When there are more informed traders, the probability of an informed trade is high, trades reveal more information and spreads are wider. This is not the case with endogenous information acquisition.

In my model, spreads and the probability of an informed trade may diverge with respect to an increase in the proportion of potentially informed traders in the market. As there are more potentially informed traders, they individually acquire less information because there are lower informational rents available from noise traders. Beyond some point, increasing their proportion actually leads to a fall in aggregate information acquisition. This result is also in contrast to Grossman-Stilgitz in which traders can only acquire a fixed quantity of information. Then more potentially informed traders always mean a higher probability of informed trade.

On the other hand, spreads are monotonically increasing in the proportion of potentially informed traders as in the standard case. Spreads are determined by the ratio of informed to noise trades. While informed trades fall, noise trades are also falling. Thus, spreads can be wide while the probability of an informed trade is low. Trades can be very informative but occur infrequently.

The three proxies suffer from different deficiencies and should not be used interchangeably. Spreads do not capture the frequency of trading. Under my model, the PIN structural estimator would be misspecified and more potentially informed traders do not lead to increase the probability of an informed trade.

Next, I examine endogenous information acquisition with a monopolistic market maker which may be relevant in certain markets. For example Madhavan and Sofianos (1998) find monopoly power among NYSE specialists and Massa and Simonov (2009) in the Italian interdealer bond market. In this section the benchmark is Leach and Madhavan (1993) who consider a monopolistic market maker, price elastic noise traders and exogenous information arrival.

Leach and Madhavan's main insight is that a monopolistic market maker has a new intertemporal trade off not shared by a competitive one. He values information and can influence the information revealed by trades through prices. Therefore he has the incentive to induce more revelation in early periods which he can exploit in later periods. However, inducing information revelation is costly as it entails trading with informed traders. Leach and Madhavan find that a monopolistic market maker sets wider spreads in early periods relative to a competitive one, with them increasing in the total number of trading periods.

In my model, the market information structure determines how prices affect information revelation. Prices now enter both the information acquisition decision of potentially informed traders and the demand of price elastic noise traders. I characterise market information structure by the effect which dominates. If 'information acquisition dominates', narrower spreads increase the information revealed by trades because prices affect potentially informed traders more than noise traders. If 'noise dominates', wider spreads increase the information revealed by trades. This setup generates two main results.

First, if information acquisition dominates, a monopolistic market maker sets narrower spreads in early periods, which are decreasing in the total number of trading periods, contrary to Leach-Madhavan. He makes lower expected profits in those periods but is compensated with higher profits later. He may even set narrower spreads than a competitive market maker, thereby making a loss. If noise dominates, the result is reversed and spreads behave as in the benchmark. Exogenous information acquisition is a special case of the market information structure in which noise dominates.

Second, spreads may widen over time on average. Trades reveal information and the market maker updates beliefs every period. Starting with an uninformative prior, beliefs grow monotonically more certain on average. With exogenous information, this implies that spreads become monotonically *narrower* on average. In my setup, if information acquisition dominates, they may grow monotonically *wider* on average. This result is driven by the interaction of endogenous information acquisition, which determines how prices affect the information revealed by trades, with a monopolistic market maker, who has an intertemporal trade off between information revelation and short term profits.

The rest of the paper proceeds as follows: Section 2 presents related literature. Section 3 introduces the setup with inelastic noise traders and a competitive market maker. Section 4 examines the specific case of quadratic information costs. Section 5 looks at the setup with price elastic noise traders and a monopolistic market maker. Section 6 concludes. Proofs are in the Appendix.

2 Related Literature

2.1 Exogenous Information Acquisition

Amongst the standard exogenous information acquisition literature, my model is most closely related to Glosten and Milgrom (1985). I share their defining features: sequential trade, a unit of asset traded each period,a competitive market maker and price inelastic noise traders drawn from a continuum of two types of traders. This is the exogenous information benchmark against which I set my results.

Another basis for comparison is Easley and O'Hara (1992),the theoretical foundation for Easley et al. (1996)'s PIN measure, one of the most widely used empirical estimators for informed trading. They augment the sequential framework of Glosten-Milgrom with a continuous time arrival process for traders to yield a mixed model which can be estimated using maximum likelihood from transactions data. PIN is of interest because it is widely used in the empirical literature as a measure for informed trading. The similarity of their setup to mine means that my results also affect their measure.

2.2 Endogenous Information Acquisition

The benchmark for endogenous information models is Grossman and Stiglitz (1980). The defining feature of their model is that traders can choose to observe a signal about the return of the risky asset at a constant cost, either becoming informed or staying uninformed. In equilibrium, both traders have the same expected utility. When more traders become informed, the price system becomes more informative and reveals more information to uninformed traders. My model preserves much of the intuition from Grossman-Stiglitz within a sequential trade framework. A drawback of their setup is that traders are homogenous and receive the same signal at a fixed cost.

Verrecchia (1982) considers heterogenous traders who can acquire variable information whose quality is increasing in its cost. In this setup, prices perform an extra role in aggregating heterogenous information. The information acquisition decision of a trader depends on how much information is revealed through prices in equilibrium. My model also features a transmission mechanism from prices to the amount of information acquired although it operates through ex ante expected profits instead of information revelation.

Litvinova and Hui (2003) add variable cost and precision to Grossman-Stiglitz. They find that some of the original results fail to hold with a different form of endogenous information acquisition. Traders exert less effort to acquire information when more of them do so. Thus, the equilibrium price system is not necessarily more informative when more traders acquire information. I also find this feature in my model. Furthermore, they find that more traders may acquire information even if the cost of acquiring information increases and equilibria do not always exist. They conclude that endogenous information acquisition should be taken seriously in the context of asymmetric information models.

Ko and Huang (2007) focus on overconfident traders who face variable information acquisition costs in a Grossman (1976) setup. They find that overconfidence generally improves market efficiency by driving prices closer to true values. While behavioural agents are their main concern, their results rely crucially on the form of endogenous information acquisition. In contrast, Garcia et al. (2007) look at overconfidence with traders who pay fixed costs for information. They reach the opposite conclusion that overconfidence has no effect on market efficiency and prices. These contrasting findings underline the importance of how we model endogenous information acquisition.

My model is also related to Peng (2005) and Peng and Xiong (2006) in which traders have capacity constraints on their ability to process information and face multiple sources of uncertainty. In equilibrium, they endogenously allocate their capacity to learn about these different sources to minimize wealth uncertainty and make intertemporal consumption decisions. This mechanism captures a similar intuition to my model in which rational agents choose the amount of information to acquire at variable cost.

Other notable contributions include Admati and Pfleiderer (1986, 1987, 1988) and Veldkamp (2006). In their series of papers, Admati and Pfleiderer introduce a market for information which is parallel to the standard asset market. They study how prices are set and how traders behave in both marekts. Veldkamp (2006) develops a competitive information production sector that supplies information at an endogenous price. She models information as a non rivalrous good with a novel production technology which increases its output and lowers price following an increase in demand. This setup generates media frenzies and price herding.

2.3 Monopolistic Market Maker

Leach and Madhavan (1993) is the closest paper to mine in structure. They examine price discovery under various market makers in a Glosten-Milgrom framework with elastic noise traders. They show that an optimal monopolistic market maker has an incentive to 'experiment' by setting prices which make trades more informative. They set wider spreads in earlier periods to crowd out elastic noise traders and increase the relative proportion of informed trades. They also find conditions under which having different market makers lead to more robust markets. They present some empirical results to support price experimentation. I generalise their framework so that traders are endogenously informed.

Glosten (1989) is one of the first to analyse the monopolistic market maker. In contrast to Glosten-Milgrom, trades are not restricted to unit amounts and the market maker posts a price schedule over different quantities of the traded asset. They find that if there are not enough noise traders in the market, a competitive market maker is unable to set zero profit prices across all quantities and thus the market breaks down. In contrast, the monopolistic market maker maximises expected profits across quantities so he can subsidise losses from trading in some quantities with profits in others. In some cases, he provides liquidity when a competitive market maker cannot do so. This mechanism is similar to the one in my model except that in my case, the market maker substitutes profits across time instead of quantities.

In general, multi period models with monopolistic market makers are analytically difficult to solve. Das and Magdon-Ismail (2009) approximate beliefs within Glosten-Milgrom by a Gaussian distribution and then solve for the optimal sequential market making algorithm. They find that an optimal monopolistic market maker can provide more liquidity than a perfectly competitive market maker in periods of extreme uncertainty because he is willing to absorb initial losses in order to learn a new valuation rapidly and extract higher profits later. Again, I find a similar intuition.

Madrigal and Scheinkman (1997) considers a market in which traders have private and heterogenous information. The market maker is large and acts strategically because he understands that prices affect first, the information he learns from the order flow, and second, the information he reveals back to other traders. This setup yields a discontinuity in equilibrium prices which Madrigal and Scheinkman interpret as a price crash.

So far market makers have inferred information from anonymous trades. Gammill (1990) lets the market maker learn the identity of traders. He makes small trades with informed traders to extract information and large trades with noise traders to maximise expected profits. This theoretical model finds support in the results of Massa and Simonov (2009) from the Italian interdealer bond market.

3 Potentially Informed Traders and a Competitive Market Maker

3.1 Setup

This section takes the discrete time Glosten-Milgrom trading framework and replaces exogenously informed traders with potentially informed ones who face costly information acquisition. They optimally choose how much information to acquire as a function of their expected profits which depend on prior beliefs and posted ask and bid prices.

There is one traded asset with value $\hat{v}$ which takes two possible terminal values V and 0 where $V > 0$. A unit of the asset is traded every period. At time t the market maker and potentially informed traders have the same prior belief that $\hat{v} = 0$ with probability μ_t, and $\hat{v} = V$ with probability $1 - \mu_t$. The markets contains three agents: the market maker, the potentially informed trader, and the noise trader. The market maker is risk neutral and competitive. He posts ask and bid prices $\{a_t, b_t\}$ from $P \subset \mathbb{R}_+$ which contains the possible values of $\hat{v}$.

A trader is drawn to trade each period from a continuum of traders. A proportion λ of them is potentially informed while the remaining proportion $1 - \lambda$ are noise traders. Noise traders do not maximise profits and trade for exogenous reasons. They submit trades q_t randomly, either a buy, $q_t = +1$, or a sell, $q_t = -1$, with equal probability $\frac{1}{2}$.

Potentially informed traders are constrained in their actions to be either buyers or sellers with equal probability $\frac{1}{2}$. Buyers can only choose to submit a buy trade or no trade, but not a sell, and similarly sellers can only choose to submit a sell trade or no trade, but not a buy. This assumption is nonstandard but does not qualitatively affect any of my results and I relax it in Section 5. I use it here for analytical simplicity because it yields closed form solutions for ask and bid prices.

Like the standard informed trader, potentially informed traders trade for speculative profits. However they learn the true value of the asset $\hat{v}$ with some probability given by the information arrival functions, $X_{a,t}(a_t)$ for buyers and $X_{b,t}(b_t)$ for sellers, defined over prices $a_t \geq b_t$ and beliefs $\mu_t \in [0, 1]$ at time t. The functions are separate for buyers and sellers because buyers only care about the ask price a_t, and sellers the bid price b_t. When potentially informed traders can do both, as in Section 5, there is only one information arrival function and it depends on both prices.

$X_{a,t}$ and $X_{b,t}$ capture how prices affect the amount of information potentially informed traders acquire. I restrict them to be consistent with this intuition. $X_{a,t}$ is weakly decreasing in the ask price a_t because expected profits decrease in a_t so potentially informed traders acquire less information. Similarly, X_b is weakly increasing in the bid price b_t. This specification nests the standard Glosten-Milgrom version of exogenously informed traders. My model is equivalent to theirs when $X_{a,t}$ and $X_{b,t}$ are unity for all prices and beliefs.

Section 4 develops a microfoundation for the information arrival functions $X_{a,t}$ and $X_{b,t}$. In that setup, potentially informed traders see posted prices, a_t and b_t, and choose how much information to acquire at increasing quadratic cost. $X_{a,t}$ and $X_{b,t}$ describe the solutions for the optimal amount of information that potentially informed traders acquire. They are also consistent with other interpretations. For example, potentially informed traders may have private reservation values or be exogenously price elastic. In this paper, I maintain the information acquisition story although my results in this section only require the weak restrictions described above.

The introduction of potentially informed traders creates a transmission channel from prices to the amount of information traders acquire which drives most of my later results. It is intuitively similar to the information acquisition equilibrating mechanism in Grossman-Stiglitz. In their model, a proportion of traders chooses to become informed while the rest remain uninformed depending on the fixed information acquisition cost. In mine, the proportion of uninformed traders is fixed but the increasing information acquisition cost determines how much information potentially informed traders acquire. My specification incorporates this information acquisition decision into Glosten-Milgrom in a tractable way which can be used to investigate multiperiod dynamics.

The timeline for each period t is as follows: 1) the market maker posts ask and bid prices $\{a_t, b_t\}$ based on prior beliefs μ_t; 2) a trader is drawn from the continuum of traders with unit mass, potentially informed buyers with probability $\frac{1}{2}\lambda$, potentially informed sellers with probability $\frac{1}{2}\lambda$, or noise traders with probability $1-\lambda$; 3) the trader submits a unit trade, either a sell, a buy or no trade, $q_t \in \{-1, 0, 1\}$; 4) the market maker completes the trade and forms posterior beliefs $\mu_{t+1}(q_t)$ by Bayes' rule.

3.2 Solving the Model

The market maker solves for zero profit ask and bid prices taking into account the best response of potentially informed traders. Let $B_{V,t}(a_t)$ be the conditional probability that a trader submits a buy order if $\hat{v} = V$ and $B_{0,t}(a_t)$ if $\hat{v} = 0$:

$$B_{V,t}(a_t) = \frac{1}{2}\lambda X_{a,t}(a_t) + \frac{1}{2}(1-\lambda) \tag{1}$$

$$B_{0,t}(a_t) = \frac{1}{2}(1-\lambda) \tag{2}$$

Here I assume that potentially informed buyers always submit a buy order if they learn the true asset value is high, $\hat{v} = V$. This is optimal as long as $a_t < V$. Furthermore, they do not trade if $\hat{v} = 0$. The assumption is equivalent to enforcing that the market is always 'open'.

Definition 1. *The market is open (closed) at time t on the ask side if it allows (excludes) profitable informed trade: $a_t < \theta_2$ ($a_t \geq \theta_2$). The market is open (closed) at time t on the bid side if it allows (excludes) profitable informed trade: $b_t > \theta_t$ ($b_t \leq \theta_1$). The market is open if it is open on at least one side.*

This definition only depends on the participation of informed traders. Noise traders continue to trade even in a 'closed' market. Under a competitive market maker, markets are always open because with price inelastic noise traders, the only way to obtain the zero profit condition is to trade with informed traders. Thus the 'open' assumption holds.

Analogously, let $S_{V,t}(b_t)$ be the conditional probability that a trader submits a sell order if $\hat{v} = V$ and $S_{0,t}(b_t)$ if $\hat{v} = 0$:

$$S_{V,t}(b_t) = \frac{1}{2}(1-\lambda) \tag{3}$$

$$S_{0,t}(b_t) = \frac{1}{2}\lambda X_{b,t}(b_t) + \frac{1}{2}(1-\lambda) \tag{4}$$

The probability that a buy order is submitted in period t is:

$$\mu_t B_{0,t}(a_t) + (1-\mu_t) B_{V,t}(a_t) \tag{5}$$

and the probability that a sell order is submitted in period t is:

$$\mu_t S_{0,t}(b_t) + (1-\mu_t) S_{V,t}(b_t) \tag{6}$$

As in Glosten-Milgrom, and shown in Proposition 12 for a more general setup, under the zero profit condition, the market maker sets the ask price a_t^c as the expected value of the asset conditional on a buy order $q_t = +1$ and beliefs μ_t:

$$\begin{aligned} a_t^c &= E[v|q_t = +1] \\ &= \frac{(1-\mu_t)B_{V,t}(a_t)V}{\mu_t B_{0,t}(a_t) + (1-\mu_t)B_{V,t}(a_t)} \\ &= \frac{(1-\mu_t)[\lambda X_{a,t}(a_t) + 1 - \lambda]V}{(1-\mu_t)\lambda X_{a,t}(a_t) + 1 - \lambda} \end{aligned} \tag{7}$$

Analogously, he sets the bid price b_t^c as the expected value of the asset conditional on a sell order $q_t = -1$ and μ_t:

$$\begin{aligned} b_t^c &= E[v|q_t = -1] \\ &= \frac{(1-\mu_t)S_{V,t}(b_t)V}{\mu_t S_{0,t}(b_t) + (1-\mu_t)S_{V,t}(b_t)} \\ &= \frac{(1-\mu_t)(1-\lambda)V}{\mu_t \lambda X_{b,t}(b_t) + 1 - \lambda} \end{aligned} \tag{8}$$

The market maker completes the trade q_t and forms his posterior belief $\mu_{t+1}(q_t)$ using Bayes' rule. His belief after a buy trade is:

$$\begin{aligned} \mu_{t+1}(+1) &\equiv pr(v = 0 \mid q_t = -1, a_t) \\ &= \frac{\mu_t B_{0,t}(a_t)}{\mu_t B_{0,t}(a_t) + (1-\mu_t)B_{V,t}(a_t)} \\ &= \frac{\mu_t(1-\lambda)}{(1-\mu_t)\lambda X_{a,t}(a_t) + 1 - \lambda} \end{aligned} \tag{9}$$

After a sell trade, it is:

$$\begin{aligned} \mu_{t+1}(-1) &\equiv pr(v = 0 \mid q_t = +1, b_t) \\ &= \frac{\mu_t S_{0,t}(b_t)}{\mu_t S_{0,t}(b_t) + (1-\mu_t)S_{V,t}(b_t)} \\ &= \frac{(1-\mu_t)(\lambda X_{b,t}(b_t) + 1 - \lambda)}{\mu_t \lambda X_{b,t}(b_t) + 1 - \lambda} \end{aligned} \tag{10}$$

Proposition 1. *Zero profit ask and bid prices* (a_t^c, b_t^c) *exist in the range* $[0, V]$ *if the information arrival functions* $X_{a,t}(a_t)$ *and* $X_{b,t}(b_t)$ *are continuous over* $a_t \in [0, V]$ *and* $b_t \in [0, V]$ *respectively.*

The continuity of the information arrival functions $X_{a,t}(a_t)$ and $X_{b,t}(b_t)$ is sufficient to obtain a single crossing property which ensures the existence of prices. The most obvious violation is if potentially informed traders face a fixed cost of information acquisition. Depending on prices, either all potentially informed traders acquire information or none do. Zero profit prices do not exist in general. However, if they do not, the market maker can still open the market by making positive profits.

3.3 Prices, Convergence and Information Stoppage

This subsection presents some static features and convergence results from my general setup. I find a new feature I call 'information stoppage' which may arise with endogenous information acquisition.

Proposition 2. *If zero profit ask price and bid prices (a_t^c, b_t^c) exist, they are monotonically decreasing in the prior belief μ_t. a_t and b_t tend to the true value $\hat{v}$ as μ_t tends to certainty, i.e. $\mu_t = 1$ or 0.*

Proposition 2 gives the standard result that zero profit prices a_t^c and b_t^c are a monotonic function of beliefs and converge to the true value as beliefs tend to certainty. The mid point of prices is the expected value of the asset conditional on beliefs at time t.

Corollary 1. *If zero profit ask and bid prices (a_t^c, b_t^c) exist and potentially informed buyers and sellers acquire information with strictly positive probability, i.e. $X_{a,t}(a_t) \geq 0$ and $X_{b,t}(b_t) \geq 0$ for all beliefs μ_t, then a_t^c and b_t^c converge to the true value in the steady state.*

Corollary 1 obtains the standard convergence result. If zero profit prices exist and potentially informed traders always acquire some information, then trades always reveal information. The market maker can update beliefs after every trade. Over time, in expectation, beliefs update correctly and prices converge to the true value. Prices always converge with exogenous information acquisition. However, with endogenous information acquisition, an information stoppage can occur. The information arrival functions, $X_{a,t}$ and $X_{b,t}$, for potentially informed traders can be 0 and trades stop revealing information.

Definition 2. *An 'information stoppage' occurs if the market is 'open' and the probability that a trader submits an informed trade is 0.*

Corollary 2. *If an information stoppage occurs in any period, prices do not converge to the true value in the steady state.*

If an information stoppage occurs, the market maker stops updating beliefs because he knows that potentially informed traders stop acquiring information. Both prices a_t and b_t are constant until the final period T once a stoppage occurs. By Definition 1, the market can still be open because prices are in the interior of possible asset values $(0, V)$. An informed trader could make a profitable trade if he were drawn into the market but potentially informed traders have no incentive to acquire that information.

An information stoppage in my model is similar to the no trade result of Grossman-Stiglitz. In their model, if the information acquisition cost is too high, no traders acquire information and thus there is no trade. In mine, potentially informed traders may choose to acquire no information for a similar reason. However, unlike in their model, noise traders continue to trade and the market remains open. Also, an information stoppage can occur in any period so a market may start off informative but enter an information stoppage later.

It would be difficult to identify information stoppages empirically because I would need to compare the fundamental value of the asset to a steady state price, neither of which are observable. This model best relates to high frequency markets in which informational events occur frequently and prices do not reach a steady state. However, markets do exhibit periods of low transaction volume with trades having a low price impact which are consistent with an information stoppage.

3.4 Relative to Exogenous Information Acquisition

This subsection compares prices, information revelation and transaction volume of a market with endogenous information acquisition to the Glosten-Milgrom benchmark with exogenous information acquisition.

Proposition 3. *The zero profit spread in a market with potentially informed traders is weakly narrower than in a market with the same proportion of exogenously informed traders.*

The competitive market maker sets zero profit prices by balancing expected profits from noise traders with losses to informed traders. In my model, only a fraction of potentially informed traders acquire information and trade while exogenously informed traders always trade. To meet the zero profit condition, the market maker sets narrower spreads than the exogenous case. Narrower spreads reduce profits from noise traders and increase the participation of potentially informed traders.

With a competitive market makers, spreads measure the information revealed by trades because they are proportional to the change in beliefs conditional on that trade occurring. By the zero profit condition, prices can be written as:

$$a_t^c = (1 - \mu_{t+1}(+1))V \tag{11}$$

$$b_t^c = (1 - \mu_{t+1}(-1))V \tag{12}$$

Since spreads are narrower, trades reveal less information in a market with potentially informed traders compared to one with the same proportion of exogenously informed traders.

Corollary 3. *Equilibrium expected transaction volume in a market with potentially informed traders is weakly lower than in a market with the same proportion of exogenously informed traders.*

With exogenous information acquisition, expected transaction volume is fixed and constant. In my case, potentially informed traders choose how much information to acquire and thus, how often they trade. Since they acquire information with probability weakly less than 1, expected transaction volume must be lower than with exogenous information acquisition. However, a more important feature model of my model is that transaction volume evolves dynamically. I expand on this in the next section.

4 Quadratic cost of information acquisition

This section develops a specific microfoundation for the information arrival function of potentially informed traders $X_{a,t}(a_t)$ and $X_{b,t}(b_t)$. Potentially informed traders choose how much information to acquire at increasing quadratic cost. This setup lets me characterise information acquisition costs with a single parameter and solve for closed form solutions for prices. I then examine the impact of information acquisition cost, beliefs and potentially informed traders on prices, information revealed by trades, expected transaction volume and the behaviour of potentially informed traders.

4.1 Setup

The potentially informed trader can learn the true value of the asset $\hat{v}$ with probability ω by paying the cost $\frac{1}{2}C\omega^2$, where C is a positive parameter which scales the cost of information acquisition. As risk neutral, profit maximising agents, they optimally choose the amount of information ω^* to acquire. Before acquiring information, they have the same prior beliefs as the market maker. A potentially informed buyer acquires the optimal amount of information $\omega^*_{a,t}$ by solving:

$$\max_{\omega_{a,t}} \ (1-\mu_t)\omega_{a,t}(V-a_t) - \frac{1}{2}C\omega^2_{a,t} \tag{13}$$

A potentially informed seller acquires $\omega^*_{b,t}$ by solving:

$$\max_{\omega_{b,t}} \ \mu_t\omega_{b,t}b_t - \frac{1}{2}C\omega^2_{b,t} \tag{14}$$

The solutions determine the information arrival functions $X_{a,t}(a_t)$ and $X_{b,t}(b_t)$ for potentially informed buyers and sellers:

$$\omega^*_{a,t} = X_{a,t}(a_t) = \frac{1}{C}(1-\mu)(V - a_t) \tag{15}$$

$$\omega^*_{b,t} = X_{b,t}(b_t) = \frac{1}{C}\mu b_t \tag{16}$$

After a potentially informed trader acquires information, there is a random draw to determine if he learns the true value. A seller submits a sell trade $q_t = -1$ if he learns that the true value is low, $\hat{v} = 0$, and no trade otherwise. Similarly, a buyer submits a buy trade $q_t = +1$ if he learns that the true value is high, $\hat{v} = V$, and no trade otherwise. From the market maker's perspective, a potentially informed trader submits a trade q_t with probabilities:

$$q_t = \begin{cases} -1 & \text{with probability } \frac{1}{2C}\mu^2 b_t \\ +1 & \text{with probability } \frac{1}{2C}(1-\mu)^2(V-a_t) \\ 0 & \text{with probability } 1 - \frac{1}{2C}\mu^2 b - \frac{1}{2C}(1-\mu)^2(V-a_t) \end{cases}$$

The market maker knows the potentially informed traders' best response functions and sets competitive prices accordingly. The assumption that potentially informed traders are either buyers and sellers means that the zero profit conditions for the ask and bid prices can be solved separately.

Proposition 4. *If $C \geq \frac{1}{4}V$, a competitive market maker posts unique ask and bid prices, a^c_t and b^c_t, given by:*

$$a^c_t = V - \frac{(1-\lambda)C}{2\lambda(1-\mu_t)^2}\left[\sqrt{1 + \frac{4\mu_t(1-\mu_t)^2\lambda V}{(1-\lambda)C}} - 1\right] \tag{17}$$

$$b^c_t = \frac{(1-\lambda)C}{2\lambda{\mu_t}^2}\left[\sqrt{1 + \frac{4\mu_t^2(1-\mu_t)\lambda V}{(1-\lambda)C}} - 1\right] \tag{18}$$

Proposition 4 requires the restriction that the cost of information acquisition C is sufficiently large relative to the maximum value of the asset V. This restriction implies that buyers and sellers acquire information with probabilities weakly less than 1 across the ranges of prior beliefs μ_t and proportions of potentially informed traders λ and thus prices always take the form in the proposition. If the restriction is relaxed, $C < \frac{1}{4}V$, then prices may imply a probability of information acquisition greater than 1. Prices still exist but they are solved like the exogenous case when traders receive information with probability 1. I restrict C since I am interested in cases when potentially informed traders do not acquire full information.

4.2 Spreads

This subsection derives comparative statics for the effect of information acquisition cost C and proportion of potentially informed traders λ on spreads. In any given market, these are fixed exogenous variable, so these statics are for comparisons between different markets with other variables held constant, in particular, prior beliefs μ_t. While μ_t evolves endogenously over time, for now I take them as exogenous.

Proposition 5. *The competitive ask price a_t^c is monotonically decreasing, while the bid price b_t^c is monotonically increasing, in the information acquisition cost C. a_t^c and b_t^c tend to the conditional expected asset value $(1 - \mu_t)V$ as C tends to infinity.*

By Proposition 5, spreads are decreasing in the information acquisition cost C. When C increases, information costs more so, for any set of posted prices, potentially informed traders acquire less. Under the zero profit condition, the market maker sets narrower spreads to give potentially informed traders more incentive to acquire information while reducing expected profits from noise traders.

As the cost of information acquisition C grows to infinity, potentially informed traders stop acquiring information and the model collapses to one without informed traders. Prices are set at the unconditional expected value of the asset. Proposition 4 imposes a lower bound for C: $C \geq \frac{1}{4}V$. If I relax the restriction, as C tends to 0, prices converge those from standard Glosten-Milgrom with no information acquisition costs in which potentially informed agents acquire full information.

As described previously, spreads are proportional to how beliefs are updated and thus measure the information revealed by trades. Therefore, following spreads, trades reveal less information as the information cost C rises, in agreement with Grossman-Stiglitz. In their model, a proportion of traders pay the cost to become informed while the rest remain uninformed. When the cost increases, fewer traders become informed so trades are less informative. The information revealed by trades is directly related to the proportion of informed traders. While the intuition is similar, the transmission mechanism in my model is different. The proportion of potentially informed traders is exogenously fixed but trades reveal less information because each trader acquires less information. Thus there is not necessarily a monotonic relationship between the proportion of potentially informed traders and the information revealed by trades.

The information acquisition cost C also affects the expected profits of traders differently in my model compared to Grossman-Stiglitz. In their case, all traders, informed or uninformed, make the same expected profits in equilibrium. A rise in information acquisition cost lowers profits to all traders equally. In my case, similar to Glosten-Milgrom, potentially informed traders make positive expected profits and noise traders make expected losses. Increasing C lowers expected profits to potentially informed traders but also lowers expected losses to noise traders through narrower spreads.

Grossman-Stiglitz has been tested empirically by comparing the performance of passive index mutual funds, as a proxy for uninformed traders, to actively-managed funds, as a proxy for informed traders. Their model predicts that the two should perform similarly. In general, the literature studying mutual fund performance, such as Wermers (2000), Kosowski et al. (2006) and Banegas et al. (2012), find that actively-managed funds out perform the index. While evidence against Grossman-Stiglitz, it is consistent with the form of information acquisition in my model.

Proposition 6. *The competitive ask price a_t^c is monotonically increasing, while the bid price b_t^c is monotonically decreasing, in the proportion of potentially informed traders λ. a_t^c and b_t^c tend to the conditional expected asset value $(1-\mu_t)V$ as λ tends to 0. They tend to the V and 0 respectively as λ tends to 1.*

By Proposition 6, equilibrium spreads are monotonically increasing in the proportion of potentially informed traders λ. This result is analogous to Glosten-Milgrom's result that spreads are increasing in the proportion of informed traders and is driven by the zero profit condition of a competitive market maker. When there are more potentially informed traders, the market maker sets wider spreads to decrease expected losses to them and increase expected profits from noise traders.

When there are no potentially informed traders, only noise traders, both prices are the unconditional expected value of the asset. No information is revealed by trades and there is no learning. When there are only potentially informed traders, the market maker sets the maximum spread and the standard no trade result obtains. The market maker closes the market because all trades are with informed traders which entail expected losses.

My model predicts the same relationship between the proportion of potentially informed traders λ and the spread as Glosten-Milgrom. However, the empirical support for this prediction is mixed. For example, Dennis and Weston (2001) find that the size of the spread is negatively related to the amount of institutional ownership, a proxy for informed traders while Heflin and Shaw (2000) find the opposite for block owners.

While many empirical studies use the spread as a measure for information based trading, it also includes other components such as the cost of market making and order processing costs. My model suggests another reason why it might be a poor measure. In standard Glosten-Milgrom, the arrival rate of informed traders is fixed. In my case, potentially informed traders enter at different rates depending on how much information they acquire. This yields another measure for information based trading: the probability of informed trade. The two are not equivalent because a trade may cause a large revision in beliefs but happen with low probability. The spread does not capture this dimension of how information enters the market. In the next subsection, I characterise when the two measures deviate.

4.3 Probability of an Informed Trade and Expected Transaction Volume

I define the probability of an informed trade $K_t(a_t, b_t)$ as the unconditional probability that a potentially informed trader, buyer or seller, is drawn into the market and submits an informed trade. It is the sum of the probability of an informed buy trade $G_t(a_t)$ and an informed sell trade $H_t(b_t)$ which are given by:

$$G_t(a_t) = \frac{1}{2}\lambda X_{a,t}(a_t) = \frac{1}{2C}\lambda(1-\mu)(V-a_t) \tag{19}$$

$$H_t(b_t) = \frac{1}{2}\lambda X_{b,t}(b_t) = \frac{1}{2C}\lambda\mu b_t \tag{20}$$

The probability of informed trade $K_t(a_t, b_t)$ is empirically relevant because it is analogous to the widely used PIN measure proposed by Easley et al. (1996). PIN is the probability of an informed trade estimated from a structural model with exogenously informed traders. $K_t(a_t, b_t)$ and PIN are the same if in a market with exogenously informed traders but they can differ once I introduce endogenously informed traders.

Proposition 7. *In equilibrium, the probability of an informed trade $K_t(a_t^c, b_t^c)$ is at its maximum when beliefs μ_t are weakest, i.e. $\mu_t = \frac{1}{2}$. $K_t(a_t^c, b_t^c)$ tends to 0 as beliefs tend to certainty, i.e. $\mu_t = 1$ or 0.*

By Proposition 7, the probability of an informed trade $K_t(a_t^c, b_t^c)$ responds intuitively to beliefs μ_t: it is largest when beliefs are weakest, $\mu_t = \frac{1}{2}$, tending to zero as beliefs tend to certainty, $\mu_t = 0$ or 1. When beliefs are weak, prices are far from their true value. An informed trade yields large speculative profits so potentially informed traders acquire the most information. As beliefs get stronger, prices move toward the true value. Expected profits from trading fall so potentially informed traders acquire less information.

Figure 1 shows this result graphically for some choice of model parameters $V = 10$ and $C = 5$. Figure 1(c) plots the probability of an informed trade $K_t(a_t^c, b_t^c)$ against beliefs μ_t and proportion of potentially informed traders λ. For now, I am interested in μ_t so fix some proportion of λ and look across $K_t(a_t^c, b_t^c)$. The graph is a hump, symmetric about $\mu_t = \frac{1}{2}$. Note that the value of $K_t(a_t^c, b_t^c)$ also depends on λ but across any λ, the shape is the same.

Figures 1(a) and 1(b) plot the probabilities of an informed buy trade $G_t(a_t^c)$ and sell trade $H_t(b_t^c)$ against μ_t and λ. Unlike the aggregate probability $K_t(a_t^c, b_t^c)$, they are not symmetric about $\mu_t = \frac{1}{2}$. One of the advantages of separating potentially informed buyers and sellers is that I can see their different responses to μ_t. Figure 1(a) corresponds to buyers. $G_t(a_t^c)$ is skewed towards $\mu_t = 1$. Potentially informed buyers acquire more information when beliefs tend towards the low asset value because they can make higher profits if they learn that the true value is high. The opposite applies for sellers.

Proposition 7 also has implications for the dynamic behaviour of information acquisition. In expectation, beliefs μ_t converge to certainty about the true value over time. If the market starts with uninformative first period beliefs, $\mu_1 = \frac{1}{2}$, then in expectation, μ_t monotonically increases or decreases to 0 or 1 over time. Therefore, the expected probability of an informed trade $K_t(a_t^c, b_t^c)$ also decreases monotonically over time. With exogenously informed traders, it is constant. Note that while the *expected* paths of μ_t and $K_t(a_t^c, b_t^c)$ are monotonic, they need not be for any given realisation. μ_t may fluctuate over time and thus $K_t(a_t^c, b_t^c)$ may rise and fall.

Corollary 4. *Expected transaction volume $E_t[|q_t|]$ is at its maximum when beliefs μ_t are weakest, i.e. $\mu_t = \frac{1}{2}$. $E_t[|q_t|]$ tends to $1 - \lambda$ as beliefs tend to certainty, i.e. $\mu_t = 1$ or 0.*

Expected transaction volume $E_t[|q_t|]$ follows the probability of informed trades $K_t(a_t^c, b_t^c)$ in response to changes in prior beliefs μ_t. In my setup, $E_t[|q_t|]$ only depends on the participation of potentially informed traders since noise traders always trade. When potentially informed traders acquire less information, they trade less. Again, Corollary 4 also determines the dynamic behaviour of $E_t[|q_t|]$. Starting from an uninformative μ_1, in expectation, $E_t[|q_t|]$ falls over time, in contrast to Glosten-Milgrom.

The dynamic features of the probability of informed trade $K_t(a_t^c, b_t^c)$ and expected transaction volume $E_t[|q_t|]$ with endogenous information acquisition is more empirically appealing

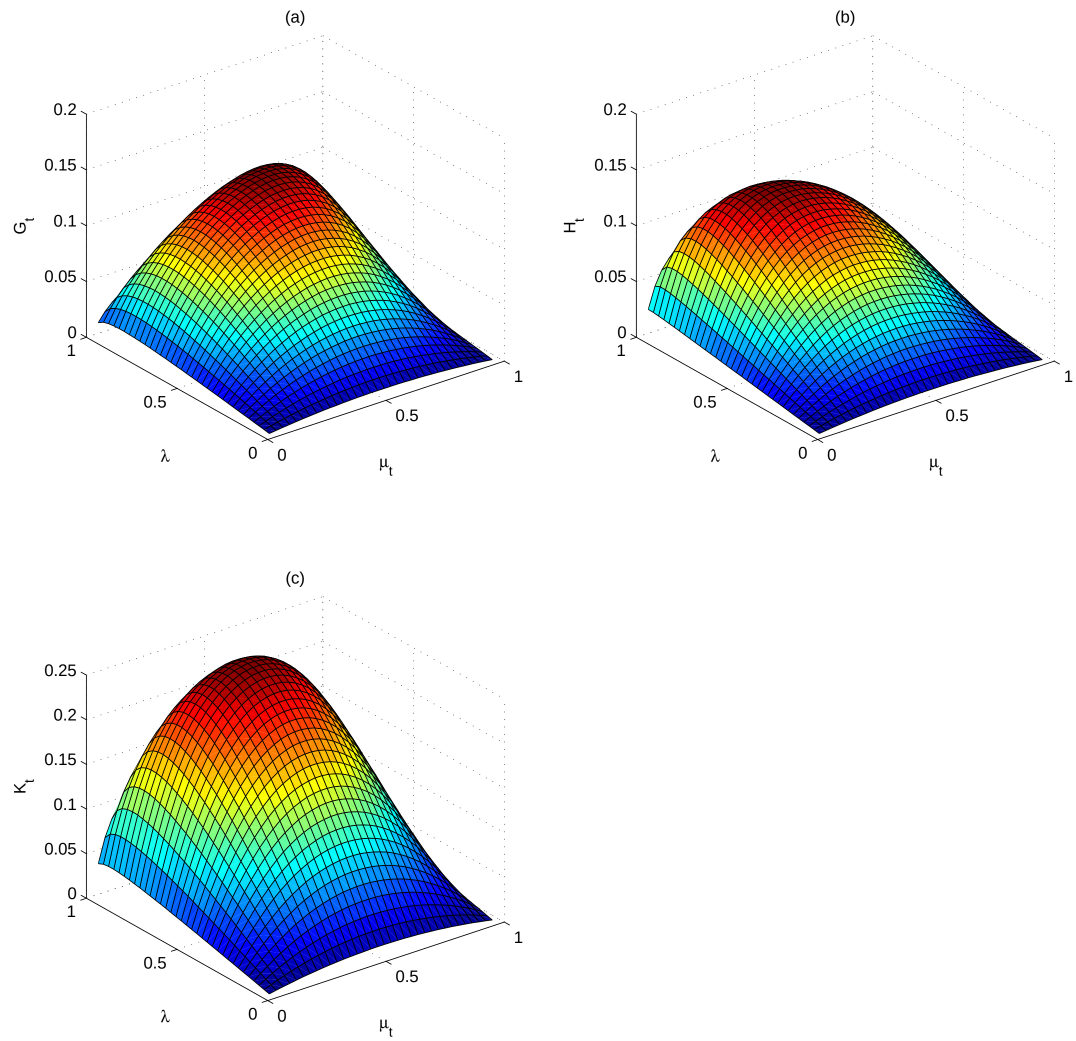

Figure 1: Probabilities of an informed: (a) buy trade $G_t(a_t^c)$; (b) sell trade $H_t(b_t^c)$; or (c) trade of either type $K_t(a_t^c, b_t^c)$; against prior beliefs μ_t and the proportion of potentially informed traders λ for model parameters $V = 10$ and $C = 5$.

than the standard models with exogenously informed traders. A cursory look at high frequency trading data reveals periods of high volume and high participation by institutional traders, often considered informed, which tend to occur after informational events and fall over time. These stylised features are absent from Glosten-Milgrom.

Easley and O'Hara (1992) introduce uncertainty about whether an informational event occurs at the beginning of each trading day. Informed traders only enter the market if it does. This partially accounts for different levels of expected trading volume and informed participation. Easley et al. (1996) then estimate this structural specification. However, this model only allows two trading intensity regimes which last for a whole day. In my model, trading intensity evolves endogenously over time, even within the same day. This seems closer to the stylised features described above. Furthermore, I do not need another dimension of uncertainty. The market maker knows an informational event has occurred. The dynamics are driven by potentially informed traders acquiring different amounts of information over time.

Proposition 8. *In equilibrium, the probability of an informed trade $K_t(a_t^c, b_t^c)$ is monotonically decreasing in the information acquisition cost C.*

By Proposition 5, spreads are decreasing in the information acquisition cost C. By Proposition 8, the probability of an informed trade $K_t(a_t^c, b_t^c)$ responds similarly. Thus, the market is less informative under both measures as C increases.

Corollary 5. *Expected transaction volume $E_t[|q_t|]$ is monotonically decreasing in the information acquisition cost C.*

A higher information cost C leads to lower expected transaction volumes. Like in Corollary 4, expected transaction volume $E_t[|q_t|]$ follows the probability of informed trade $K_t(a_t^c, b_t^c)$. This also agrees with Grossman-Stiglitz. Together with Proposition 5, Proposition 8 and Corollary 5 describe all the effects of C in my model.

Fang and Peress (2009) offer some empirical support for Corollary 5. They find that media coverage affects the returns of some subset of stocks. If I can interpret media coverage as a proxy for information acquisition costs, because it captures the availability of public information information, then this is in line with my predictions.

Proposition 8 offers another testable prediction between information costs and the probability of an informed trade. Ideally I would estimate the probability of an informed trade from a model with endogenous information acquisition and then compare it between assets with different information acquisition costs.

Proposition 9. *The probabilities of an informed buy trade G_t^c and sell trade H_t^c have their maximum at $\widetilde{\lambda_t^G} = \frac{Z_t+1}{2Z_t+1}$ and $\widetilde{\lambda_t^H} = \frac{Y_t+1}{2Y_t+1}$ where $Z_t \equiv (1-\mu_t)\sqrt{\frac{1}{C}\mu_t V}$ and $Y_t \equiv \mu_t\sqrt{\frac{1}{C}(1-\mu_t)V}$. G_t^c and H_t^c tend to* 0 *as the proportion of potentially informed traders λ tends to* 0 *or* 1.

By Proposition 9, the probability of an informed trade $K_t(a_t^c, b_t^c)$ is not monotonically increasing in the proportion of potentially informed traders λ. It is decreasing in λ for $\lambda \geq \max\{\frac{Z_t+1}{2+z1}, \frac{Y_t+1}{2Y_t+1}\}$ where $Z_t \equiv (1-\mu_t)\sqrt{\frac{1}{C}\mu_t V}$ and $Y_t \equiv \mu_t\sqrt{\frac{1}{C}(1-\mu_t)V}$. In this range, increasing the proportion of potentially informed traders leads to less frequent informed trades.

To see this result graphically, return to Figure 1(c) which plots the probability of an informed trade $K_t(a_t^c, b_t^c)$. Fix some prior belief μ_t and look across the proportion of potentially informed traders λ. For $\mu_t = \frac{1}{2}$, $K_t(a_t^c, b_t^c)$ increases with λ until it reaches its maximum at $\widetilde{\lambda_t^K} = \frac{W_t+1}{2W_t+1}$ where $W_t \equiv \frac{1}{2}\sqrt{\frac{1}{2C}V}$. $K_t(a_t^c, b_t^c)$ then falls rapidly to 0 as λ tends to 1. Note that the maximum $\widetilde{\lambda_t^K}$ occurs at larger values as beliefs are more certain, i.e. μ_t closer to 0 or 1.

Proposition 10. *The probabilities that a potentially informed buyer or seller is informed, $X_{a,t}(a_t^c)$ and $X_{b,t}(b_t^c)$, are monotonically decreasing in the proportion of potentially informed traders λ. $X_{a,t}(a_t^c)$ and $X_{b,t}(b_t^c)$ tend to $\frac{1}{C}\mu_t(1-\mu_t)V$ as λ tends to* 0*. They tend to* 0 *as λ tends to* 1.

Recall that the probability that a potentially informed buyer or seller is informed, $X_{a,t}(a_t^c)$ and $X_{b,t}(b_t^c)$ is given by the amount of they choose to acquire. It starts at $\frac{1}{C}\mu_t(1-\mu_t)V$ and decreases monotonically to 0 with the proportion of potentially informed traders λ. The maximum probability is always weakly less than 1 because of the restriction that $C \geq \frac{1}{4}V$. By Proposition 10, increasing λ means each trader acquires less information. However, by Proposition 9, the probability of an informed trade $K_t(a_t^c, b_t^c)$ is not monotonic in λ. To understand the two results, see that λ has two effects on $K_t(a_t^c, b_t^c)$: it 1) increases the number of traders who can choose to acquire information; and 2) decreases the amount of information acquired by each trader.

For low λ, the first effect dominates. An increase in λ outweighs the decrease in the information they acquire individually, as measured by $X_{a,t}(a_t^c)$ or $X_{b,t}(b_t^c)$. Thus the probability of an informed trade $K_t(a_t^c, b_t^c)$ increases. There are sufficiently many noise traders yielding expected profits to the market maker to offset losses from more potentially informed trades. However, for λ larger than $\widetilde{\lambda_t}$, the second effect dominates and the relationship reverses. As

λ continues to increase, $K_t(a_t^c, b_t^c)$ begins to decrease. The market fills with potentially informed traders and the market maker earns lower profits from noise traders so it can support less information acquisition. In the limit, there are no more noise traders and thus no more profits to offer potentially informed traders. Potentially informed traders stop acquiring information.

A large body of empirical literature studies the effect of institutional or block ownership on asset prices and information revelation. For example, Boehmer and Kelly (2009) look at institutional holdings and informational efficiency of prices, measured by deviations from a random walk. There are various theoretical reasons to examine institutional holdings but the asymmetric information literature, to which my model belongs, interprets them as informed traders. This implies that assets with larger institutional holdings should be more informative. If instead institutions are potentially informed and endogenously acquire costly information, Proposition 10 yields conditions when higher institutional holdings leads to less informative markets, as measured by the probability of informed trade. This result might help reconcile the mixed evidence on institutional ownership. It also cautions against using institutional holdings as proxies for informed trading.

Corollary 6. *The probability of an informed trade $K_t(a_t^c, b_t^c)$ is decreasing, while the spread is increasing, in the proportion of potentially informed traders λ for $\lambda \geq \max\{\frac{X_t+1}{2X+1}, \frac{Y_t+1}{2Y_t+1}\}$ where $X_t \equiv (1-\mu_t)\sqrt{\frac{1}{C}\mu_t V}$ and $Y_t \equiv \mu_t \sqrt{\frac{1}{C}(1-\mu_t)V}$.*

Corollary 6 describes the exact conditions when spreads and the probability of an informed trade $K_t(a_t^c, b_t^c)$ deviate from each other. As noted previously, they need not comove and here I show that they respond differently to changing the proportion of potentially informed traders λ. For λ beyond a certain threshold, increasing it further means that trades cause a larger revisions in belief but occurs less frequently.

The empirical literature uses both spreads and the probability of an informed trade as measures for information revelation. Standard theoretical models suggest they can be used interchangeably. I show when they cannot under the quadratic cost function. While this result is not general to all cost functions, I can show it arises for at least some subset of cost functions.

To better understand the divergence, recall that competitive prices, given by Equations (17) and (18), are set as the expected value of the asset conditional on a trade. They are proportional to the ratio of expected informed trades to total trades, both informed and noise. Total expected trades or expected transaction volume, denoted $E_t[q_t = +1]$ for buys,

$E_t[q_t = -1]$ for sells, and $E_t[|q_t|]$ for all trades, are given by:

$$E_t\left[q_t = +1\right] = (1-\mu_t)G_t(a_t^c) + \frac{1}{2}(1-\lambda) \tag{21}$$

$$E_t\left[q_t = -1\right] = \mu_t H_t(b_t^c) + \frac{1}{2}(1-\lambda) \tag{22}$$

$$\begin{aligned} E_t\left[|q_t|\right] &= E_t\left[q_t = +1\right] + E_t\left[q_t = -1\right] \\ &= (1-\mu_t)G_t(a_t^c) + \mu_t H_t(b_t^c) + 1 - \lambda \end{aligned} \tag{23}$$

In Glosten-Milgrom, the only determinant of spreads is the proportion of informed traders because total expected transaction volume $E_t[|q_t|]$ is constant. In my model, $E_t[|q_t|]$ is endogenous.

Corollary 7. *Total expected transaction volume $E_t[|q_t|]$ is decreasing, while spreads are increasing in the proportion of potentially informed traders λ if $\lambda \geq \max\{\frac{Z_t+1}{2Z+1}, \frac{Y_t+1}{2Y_t+1}\}$ where $Z_t \equiv (1-\mu_t)\sqrt{\frac{1}{C}\mu_t V}$ and $Y_t \equiv \mu_t\sqrt{\frac{1}{C}(1-\mu_t)V}$.*

By Corollary 7, when the probability of an informed trade $K_t(a_t^c, b_t^c)$ is decreasing, expected transaction volume $E_t[|q_t|]$ is also decreasing in λ. This drives the deviation between spreads and $K_t(a_t^c, b_t^c)$ from Corollary 6. Although informed trades occur less frequently, they make up a larger proportion of total trades.

Corollary 7 also predicts that, like $K_t(a_t^c, b_t^c)$, $E_t[|q_t|]$ is not monotonic in λ. In contrast, $E_t[|q_t|]$ is constant in Glosten-Milgrom. Easley and O'Hara (1992) has two regimes for $E_t[|q_t|]$ but more informed traders still implies higher $E_t[|q_t|]$.

Again the empirical support for the relationship between institutional holdings and transaction volume is mixed. The difficulty for these studies is the endogeneity of holdings. Institutional traders prefer liquid stocks which have higher expected transaction volumes. These stocks then have more institutional investors so it is difficult to determine causality.

5 Elastic noise traders and a monopolistic market maker

5.1 Setup

This section explores endogenous information acquisition in a model with both price elastic noise traders and a monopolistic market maker. Leach and Madhavan (1993) provide the baseline setup by extending standard Glosten-Milgrom with these two features but they keep exogenously informed traders. Here, I replace them with potentially informed traders.

The model setup is similar to Section 3. There are three agents: potentially informed traders, noise traders and a market maker. Potentially informed traders see posted prices and choose how much information to acquire. They are informed with probabilities given by an information arrival function $X(a_t, b_t, \mu_t)$. For this general setup, I drop the assumption of separate buyers and sellers.

In Section 4, the two information arrival functions are given by the solutions of the maximisation problems for potentially informed buyers and sellers facing quadratic costs. Similarly, you can think of $X(a_t, b_t, \mu_t)$ as the solution to:

$$\max_{\omega_t} \ \omega_t(\mu_t b_t + (1-\mu_t)(V - a_t)) - D(\omega_t) \tag{24}$$

where $D(\omega)$ is some increasing cost function. $X(a_t, b_t, \mu_t)$ describes the optimal ω_t chosen by potentially informed traders. For the rest of this paper, I abstract away from the cost function D and concentrate on X. X preserves the intuition of costly information acquisition under some restrictions: it needs to be bounded $[0, 1]$, decreasing in a_t and increasing in b_t. I use it because it can account for general cost functions and yields a neater characteristion of market information structure, being analogous to the price elasticity of noise traders.

If a potentially informed trader is drawn to the market, he learns the true value of the asset with probability X. He then submits a sell order if the bid price is higher than the true value, a buy order if the ask price is lower than the true value, and no trade otherwise. If he does not learn the true value, he does not trade. He submits a trade:

$$q_t = \begin{cases} -1 & \text{if } b_t > \hat{v} \\ +1 & \text{if } a_t < \hat{v} \\ 0 & \text{otherwise} \end{cases}$$

From the market maker's point of view, a potentially informed trader submits a trade q_t with probabilities:

$$q_t = \begin{cases} -1 & \text{with probability } \mu_t X(a_t, b_t, \mu_t) \\ +1 & \text{with probability } (1-\mu_t) X(a_t, b_t, \mu_t) \\ 0 & \text{with probability } 1 - X(a_t, b_t, \mu_t) \end{cases}$$

Noise traders are now price elastic. While arguably this is more realistic than the price inelastic assumption, it is also essential for deriving interior prices with a monopolistic market maker. If noise traders are price inelastic, a monopolistic market maker maximises profits by setting the maximum spread. Informed traders do not participate and he trades only with noise traders. Such a market is always closed. However, if noise traders are price elastic, he may find it optimal to set interior prices, a more interesting case to analyse.

Following Leach-Madhavan, when a noise trader is drawn into the market, he receives a private reservation value r drawn from a distribution with cumulative density function $F(r)$ where the average $\int r\,\mathrm{d}F(r)$ is in $(0, V)$. He submits a sell order $q_t = -1$ if the bid price is higher than his reservation value, a buy order $q_t = +1$ if the ask price is lower than his reservation value, and no trade $q_t = 0$ otherwise:

$$q_t = \begin{cases} -1 & \text{if } b_t > r \\ +1 & \text{if } a_t < r \\ 0 & \text{otherwise} \end{cases}$$

These occur with probabilities:

$$q_t = \begin{cases} -1 & \text{with probability } F(b_t) \\ +1 & \text{with probability } 1 - F(a_t) \\ 0 & \text{with probability } F(a_t) - F(b_t) \end{cases}$$

As before, after the market maker completes a trade q_t, he forms posterior beliefs $\mu_{t+1}(q_t)$ using Bayes' rule. After a sell, the posterior belief $\mu_{t+1}(-1)$ is:

$$\begin{aligned} \mu_{t+1}(-1) &\equiv pr\left(\widetilde{\theta} = \theta_1 \mid q_t = -1, a_t, b_t\right) \\ &= \frac{\mu_t\left[\lambda X(a_t, b_t, \mu_t) + (1-\lambda)F(b_t)\right]}{\mu_t \lambda X(a_t, b_t, \mu_t) + (1-\lambda)F(b_t)} \end{aligned} \tag{25}$$

After a buy, it is:

$$\begin{aligned} \mu_{t+1}(+1) &\equiv pr\left(\widetilde{\theta} = \theta_1 \mid q_t = +1, a_t, b_t\right) \\ &= \frac{\mu_t(1-\lambda)(1-F(a_t))}{(1-\lambda)(1-F(a_t)) + (1-\mu_t)\lambda X(a_t, b_t, \mu_t)} \end{aligned} \tag{26}$$

And after no trade:

$$\mu_{t+1}(0) \equiv pr\left(\widetilde{\theta} = \theta_1 \mid q_t = 0, a_t, b_t\right)$$
$$= \mu_t \tag{27}$$

Now I define another measure for how much the market learns from a trade. Let the 'informativeness' of a trade $N_t(q_t|a_t, b_t, \mu_t)$ be the expected change in prior belief μ_t after a trade q_t conditional on that prior and a set of posted prices: $N_t(q_t|a_t, b_t, \mu_t) = E[|\mu_{t+1}(q_t) - \mu_t|]$. A trade is more informative if it leads to a larger expected revision of the prior. A buy always leads to a downward revision, $\mu_{t+1}(+1) \leq \mu_t$, while a sell always leads to an upward revision, $\mu_{t+1}(-1) \geq \mu_t$. Under a competitive market maker, $N_t(+1)$ and $N_t(-1)$ are proportional to ask and bid prices. They are analogous to the spread. I define this new measure because with a monopolistic market maker, the spread no longer captures how trades affect beliefs.

5.2 Information and Market Structure

There are two transmission channels from prices to the informativeness of trades: the 'information acquisition' and 'noise' channels. I differentiate 'market structure' by whether wider or narrower spreads increase informativeness. Consider the effect of prices on the informativeness of a sell trade $N_t(q_t = -1|a_t, b_t, \mu_t)$.

First, the 'information acquisition' channel captures the impact of prices on the amount of information acquired by potentially informed traders. Consider the posterior belief after a sell trade $\mu_{t+1}(-1)$ given by Equation (25). Both ask and bid prices, a_t and b_t, enter the information arrival function $X(a_t, b_t, \mu_t)$. Thus, I further distinguish between a 'direct' and 'indirect' information acquisition channel. The 'direct information acquisition' channel affects a sell trade through the bid price b_t. It has a direct effect because it raises profits from selling the asset when its true value is low. Thus potentially informed traders acquire more information when b_t is high, (recall that $X(a, b, \mu_t)$ is increasing in b_t). This channel makes a sell trade more informative with higher b_t. With the assumption of separate buyers and sellers in Section 3, this is the only channel which operates because sellers can only trade at b_t.

The 'indirect information acquisition' channel affects a sell trade through the ask price a_t. While it has no impact on profits from selling the asset, it does affect profits from buying the asset when its value is high. Thus, lowering the ask price raises profits and potentially informed traders acquire more information (recall that the information arrival function $X(a_t, b_t, \mu)$ is decreasing in a_t). This is the 'direct information acquisition' channel of a_t on a *buy* trade. However, it also has an indirect effect on a sell trade because when potentially informed traders acquire more information in expectation of the high asset value, they are also more likely to discover that the true value is low.

Second, the 'noise' channel captures the impact of prices on the participation of noise traders. Returning to the expression for the posterior belief after a sell trade $\mu_{t+1}(-1)$ in Equation (25), the bid price b_t also enters the density function of noise trader reservation values $F(b_t)$. Raising b_t increases the probability that a noise trader has a reservation value below b_t, the criteria to submit a sell order, $F(b_t)$ is increasing in b_t. Thus, raising b_t decreases the proportion of uninformed sell trades in the market and makes a sell trade more informative. The lower bid price crowds out noise traders so a larger proportion of trades are informed.

The two channels operate symmetrically on the informativeness of a buy trade $N_t(q_t = +1|a_t, b_t, \mu_t)$. First, the direct information acquisition channel makes the informativeness of a buy trade decreasing in the ask price a_t and the indirect channel makes it increasing in the bid price b_t. Second, the noise channel makes it increasing in a_t.

Finally, I characterise market structure by how spreads affect the informativeness of trades. The first market structure is characterised by narrower spreads increasing the informativeness of trades. The information acquisition channel makes narrower spreads increase the informativeness of both buy and sell trades. The noise channel does the opposite, decreasing the informativeness of trades. The aggregate impact on informativeness depends on which channel dominates. When the first effect to be stronger, I get the definition of a market structure in which 'information acquisition dominates informativeness'.

Definition 3. *'Information acquisition dominates informativeness' if* $\frac{\partial \mu_{t+1}(-1)}{\partial b_t} - \frac{\partial \mu_{t+1}(-1)}{\partial a_t} > 0$ *and* $\frac{\partial \mu_{t+1}(+1)}{\partial a_t} - \frac{\partial \mu_{t+1}(+1)}{\partial b_t} > 0 \ \forall \ \mu_t \in [0, 1]$, $a_t > b_t$.

The first inequality refers to the informativeness a sell trade. The term $\frac{\partial \mu_{t+1}(-1)}{\partial b_t}$ captures both the direct information acquisition and noise channels of the bid price b_t on a sell trade. The information acquisition channel acts to make it positive while the noise channel, negative, so the net sign depends on the relative sizes of the two channels. The other term $\frac{\partial \mu_{t+1}(-1)}{\partial a_t}$ captures the indirect information acquisition channel of the ask price a_t on a sell trade. This term is always negative. In aggregate, I want narrower spreads to increase informativeness of a sell trade, which is equivalent to increasing the posterior after a sell $\mu_{t+1}(-1)$. Hence the inequality: $\frac{\partial \mu_{t+1}(-1)}{\partial b_t} - \frac{\partial \mu_{t+1}(-1)}{\partial a_t} > 0$. The second inequality captures the analogous effect of prices on the informativeness of a buy trade.

The second market structure is characterised by wider spreads increasing the informativeness of trades. Now, the noise channel is stronger than the information acquisition channel, which yields the definition of a market structure in which 'noise dominates informativeness'.

Definition 4. *'Noise dominates informativeness' if* $\frac{\partial \mu_{t+1}(-1)}{\partial b_t} - \frac{\partial \mu_{t+1}(-1)}{\partial a_t} < 0$ *and* $\frac{\partial \mu_{t+1}(+1)}{\partial a_t} - \frac{\partial \mu_{t+1}(+1)}{\partial b_t} < 0 \ \forall \ \mu_t \in [0,1]$, $a_t > b_t$.

The inequalities are reversed relative to the first market structure. The noise channel must be sufficiently large to overcome both the direct and indirect information acquisition channels. This market information structure nests Leach-Madhavan which corresponds to an information arrival function $X(a,b,\mu)$ of unity. Then, potentially informed traders always receive the true value of the asset and trade, regardless of prices or beliefs. The information acquisition channels do not operate so prices only affect the informativeness of trades through the noise channel. Under my characterisation, noise dominates informativeness and wider spreads increase the informativeness of trades.

Lemma 1. *If X and F are differentiable, information acquisition dominates informativeness if* $F(b_t)(\frac{\partial X}{\partial b_t} - \frac{\partial X}{\partial a_t}) - X(a_t,b_t,\mu_t)\frac{\partial F}{\partial b_t} > 0$, *and* $(1-F(a_t))(\frac{\partial X}{\partial b_t} - \frac{\partial X}{\partial a_t}) - X(a_t,b_t,\mu_t)\frac{\partial F}{\partial a_t} > 0$, $\forall$ $\mu_t \in [0,1]$, $a_t > b_t$.

Lemma 2. *If X and F are differentiable, noise dominates informativeness if* $F(b_t)(\frac{\partial X}{\partial b_t} - \frac{\partial X}{\partial a_t}) - X(a_t,b_t,\mu_t)\frac{\partial F}{\partial b_t} < 0$ *and* $(1-F(a_t))(\frac{\partial X}{\partial b_t} - \frac{\partial X}{\partial a_t}) - X(a_t,b_t,\mu_t)\frac{\partial F}{\partial a_t} < 0$, $\forall$ $\mu_t \in [0,1]$, $a_t > b_t$.

The inequalities in Lemmas 1 and 2 can be separated into the two transmission channels. The first inequality in both Lemmas refer to the informativeness of a sell trade. The term $F(b_t)(\frac{\partial X}{\partial b_t} - \frac{\partial X}{\partial a_t})$ captures the information acquisition channel. It can be further separated into two parts: $F(b_t)\frac{\partial X}{\partial b_t} - F(b_t)\frac{\partial X}{\partial a_t}$. $F(b_t)\frac{\partial X}{\partial b_t}$ captures the direct information acquisition channel of the bid price b_t and $F(b_t)\frac{\partial X}{\partial a_t}$, the indirect information acquisition channel of the ask price a_t. $\frac{\partial X}{\partial b_t}$ is always positive while $\frac{\partial X}{\partial a_t}$ is always negative so the combined term shows the two channels working in the same direction. The level of b_t enters through $F(b_t)$ so the information acquisition channel is stronger when there is greater noise trader participation. Only b_t appears because it has a direct effect on the profitability of a sell trade while a_t has no affect on sells. The term $X(a_t,b_t,\mu_t)\frac{\partial F}{\partial b_t}$ captures the noise channel. $\frac{\partial F}{\partial b_t}$ is always positive so the noise channel always operates counter to the two information acquisition channel. The level of information arrival enters through $X(a_t,b_t,\mu_t)$ so the noise channel is stronger when more potentially informed traders acquire information. Again, the second equality in the Lemmas refer to the informativeness of a buy trade.

This characterisation of market structure is not exhaustive. I concentrate on market structures which are consistent across all prior beliefs $\mu_t \in [0,1]$ and prices $a_t > b_t$. In general, it is possible for one channel to dominate informativeness for some range of prior beliefs and prices while the other dominates for a different range. Such markets exhibit changing informational regimes and prices do not have a consistent effect on the informativeness of trades.

5.3 Market Maker Objective Functions

Following Leach and Madhavan, I consider three types of market maker: an optimal monopolistic, a myopic monopolistic, and a competitive market maker. Price discovery under each market maker is determined by their respective objective functions. In all cases, the market maker's one period expected profit is given by:

$$\begin{aligned}\pi(a_t, b_t; \mu_t) = {} & \lambda X(a_t, b_t, \mu_t)\left[\mu_t(\theta_1 - b_t) + (1-\mu)(a_t - \theta_2)\right] \\ & + (1-\lambda)\left[F(b_t)(\theta_2 - b_t) + (1 - F(a_t))(a_t - \theta_2)\right]\end{aligned} \tag{28}$$

An optimal monopolistic market maker maximises profits from trading over every period up to time T. This is not a static problem because the market maker can influence the information revealed by trades. His belief in later periods depends on the prices he posted earlier. He may forgo some profits from earlier periods to increase information revelation and increase expected profits in later periods. His maximisation yields total expected profits given by:

$$V_n^*(\mu_1) = \sup_{\{a_t, b_t\}} E\left[\sum_{t=T-n+1}^{T} \pi(a_t, b_t; \mu_t)\right] \tag{29}$$

for n remaining trading rounds. The choice variables are a_t and b_t which are history dependent. The prior belief μ_t evolves by Bayes' rule as described previously. The expectation is taken over all random variables. Current period prices are set to extract information optimally. Equation (29) can be written in its Bellman form:

$$V_T^*(\mu_1) = \sup_{\{a_1, b_1\}} \left\{\pi(a_1, b_1; \mu_1) + E\left[V_{T-1}^*(\mu_2(\widetilde{q}))\right]\right\} \tag{30}$$

with terminal condition

$$V_1^*(\mu_T) = \sup_{\{a_T, b_T\}} \left\{\pi(a_T, b_T; \mu_T)\right\} \tag{31}$$

The function $V_n^*(\mu_1)$ is the stochastic dynamic programming problem for the market maker with n periods left to trade before $\widetilde{\theta}$ is revealed. The state variable is the prior belief about the true value of the asset μ_t, the control variables are the ask and bid prices, a_t and b_t, and the transition equation is Bayes' rule.

Proposition 11. *For $\mu_t \in (0,1)$ the optimal monopolist's value function $V_{T-1}^*(\mu)$ is convex and nonnegative.*

Nonnegativity is obvious because the market maker can post prices at which no trades occur. Convexity of the value function is the key property for later results. By the Law of Iterated Expectations, the expected posterior belief under any set of prices must be the prior belief: $E[\mu_{t+1}|\mu_t] = \mu_t$. Together with convexity of the value function, it implies that future information is valuable. For any given prior belief, the market maker expects to be better off in the next period after learning from another trade. Therefore, in non terminal periods, a monopolist market maker never closes the market (by setting $a_t \geq \theta_2$ and $b_t \leq \theta_1$) as trading weakly reveals more information. In non terminal periods, prices which close the market are weakly dominated by those which open it ($a_t < \theta_2$ and $b_t > \theta_1$).

Definition 5. *An optimal monopolistic market maker's first period price choice $p_1^* = (a_1^*, b_1^*) \in P$ is the solution to the Bellman equation, Equation (30).*

An optimal monopolist market maker recognises that learning is endogenous to the prices he posts. He has the incentive to set prices to encourage learning because he trades over multiple periods. Unlike a competitive market maker, he is not constrained in his ability to set prices. Leach Madhavan call this 'active learning' through 'experimentation'. In contrast, a competitive market maker only learns passively.

Definition 6. *A myopic monopolist's first period price choice $p_1^m = (a_1^m, b_1^m) \in P$ is the solution to:*

$$\max \pi(a_1, b_1; \mu_1)$$

A myopic market maker is only concerned with maximising one period profits and does not consider the impact of prices on information revealed by trades. The term may suggest a behavioural story in which the market maker does not recognise the full extent of his actions. However, it is also consistent with a rational market maker facing constraints. Perhaps he only has limited monopoly power because competitors may enter in the next period.

Definition 7. *A competitive first period price choice* $p_1^c = (a_1^c, b_1^c) \in P$ *satisfies:*

$$(a_1^c, b_1^c) = \inf\{a - b \in P : \pi(a,b) \geq 0\}$$

In contrast to a monopolist, a competitive market maker cannot trade at all possible prices. Instead, imagine multiple market makers involved in a Bertrand price setting game in which other traders only submit orders at the narrowest spread. Competitive equilibrium prices minimize expected profits subject to nonnegativity. Under this definition, a competitive market maker can always open the market even if zero profit prices do not exist. By a simple extension of Proposition 1 to include elastic noise traders, zero profit prices do not generally exist if the information arrival function X is not continuous. In that case, a competitive market maker would set the narrowest possible spread while making some non zero profit.

Proposition 12. *If zero profit prices exist, competitive ask and bid prices are given by:*

$$a_t^c = E\left[\widetilde{\theta}\,|q_t = +1, a_t^c, b_t^c; \mu_t\right]$$
$$b_t^c = E\left[\widetilde{\theta}\,|q_t = -1, a_t^c, b_t^c; \mu_t\right]$$

where $a_t^c \geq b_t^c$.

Like Glosten-Milgrom, competitive prices are ex post regret free. The posted ask price is the expected value of the asset conditional on the next trade being a buy while the bid is the expected value conditional on the next trade being a sell. Competition drives expected profits in every period to zero so there is no incentive for the market maker to induce learning. Information still enters the market passively because potentially informed traders acquire information before trading but the value of information does not affect price setting.

5.4 Prices and Volumes

The main result in this section is that endogenous information acquisition affects how an optimal monopolistic market maker sets prices. He sets narrower spreads than the myopic monopolist when information acquisition dominates informativeness, and wider spreads when noise dominates informativeness. The inverse relation holds for expected transactions volume. My model nests Leach-Madhavan as a special case of the second market information structure. In the following propositions, the market maker sets prices in the first of T trading periods. I suppress the subscript for $t = 1$ for clarity.

Proposition 13. *If markets are open under both competitive and monopolistic market makers and information acquisition dominates informativeness, then spreads are narrower under an optimal monopolist than under a myopic monopolist. They are also narrower under a competitive market maker than under a myopic monopolist. Specifically:* $b^* \geq b^m$, $b^c \geq b^m$ *and* $a^* \leq a^m$, $a^c \leq a^m$.

Proposition 14. *If markets are open under both competitive and monopolistic market makers and noise dominates informativeness, then spreads are wider under an optimal monopolist than under a myopic monopolist, which are wider than under a competitive market maker. Specifically:* $b^c \geq b^m \geq b^*$ *and* $a^c \leq a^m \leq a^*$.

The relationship between prices set by a myopic monopolistic market maker and those set by a competitive one is the same in both markets. A myopic monopolistic market maker never sets narrower spreads than a competitive one because by definition, he makes negative expected profits by doing so. However, the prices set by an optimal monopolistic market maker relative to a myopic one depend on the market information structure. If information acquisition dominates informativeness, an optimal monopolist market maker facing multiple trading periods sets narrower spreads while if noise dominates informativeness, he sets wider spreads.

The intuition for this price setting result is that an optimal monopolist market maker who trades over multiple periods has the incentive to increase informativeness of trades in the early periods because stronger beliefs yield higher expected profits in later periods. By Proposition 11, the monopolist market maker's value function is convex in the prior belief μ so increasing the change of that prior, increases the expected profit of the market maker. The next insight is that how the market maker can change prices to increase the informativeness of trades depends on the market information structure.

Propositions 13 and 14 describe the price setting of different market makers *within* a market. However, I cannot use them to compare prices *between* markets because my characterisation of market structure is insufficient to determine price levels. It only describes how price changes affect the informativeness of trades. In general, price levels are determined by the exact relationship between the information arrival function of potentially informed traders X and the density function of noise trader reservation values F.

Corollary 8. *If markets are open and information acquisition dominates informativeness, the first period spreads set by an optimal monopolistic market maker are weakly decreasing in the number of trading periods* T.

Corollary 9. *If markets are open and noise dominates informativeness, first period spreads set by an optimal monopolistic market maker are weakly increasing in the number of trading periods T.*

Increasing the number of trading periods increases the incentive for the market maker to increase informativeness in early periods because by Proposition 11, the market maker has a convex value function. Then the market maker affects prices through spreads depending on the market information structure as described by Propositions 13 and 14.

Corollary 10. *If markets are open with both competitive and monopolistic market makers and information acquisition dominates informativeness, then expected transaction volume is higher under an optimal monopolist than under a myopic monopolist, and it is higher under a competitive market maker than under a myopic monopolist, specifically:* $E[|q^*|] \geq E[|q^m|]$ *and* $E[|q^c|] \geq E[|q^m|]$.

Corollary 11. *If markets are open with both competitive and monopolistic market makers and noise dominates informativeness, then expected transaction volume is lower under an optimal monopolist than under a myopic monopolist, which is lower than under a competitive market maker, specifically:* $E[|q^*|] \leq E[|q^m|] \leq E[|q^c|]$.

The relationship between transaction volume and informativeness of trades is not always the intuitive one. By Corollary 10, when information acquisition dominates informativeness, higher transaction volume corresponds to more informative trades. However, by Corollary 11, when noise dominates informativeness, the opposite holds and lower transaction volume corresponds to more informative trades. With exogenous information acquisition, Leach and Madhavan find only the second case.

Corollary 11 for the general model with elastic noise traders and a monopolistic market maker shares the intuition of Corollary 3 for the model with inelastic noise traders and a competitive market maker. By Corollary 3, for some range of the proportion of potentially informed traders λ, lower expected transaction volume $E[|q|]$ coincides with more informative trades. The two results are driven by the participation of potentially informed traders relative to noise traders. Under the conditions given by the corollaries, trades are more informative despite there being fewer expected trades because a larger proportion of them are informed.

5.5 Dynamics of Prices

In Glosten-Milgrom, spreads always narrow after a trade which strengthens beliefs and widen after a trade which weakens them. I find conditions under which spreads can behave in the opposite direction, widening after a trade which strengthens beliefs and narrowing after a trade which weakens them.

The price schedule is the function of equilibrium ask and bid prices $\{a_t, b_t\}$ set by a market maker at time t for every possible prior belief μ_t. Price schedules are characterised by whether for a given time t, spreads are symmetric and monotonically narrowing or widening in the strength of beliefs, $|\mu_t - \frac{1}{2}|$. I concentrate on the standard case the price schedules imply spreads which narrow in the strength of beliefs although in general, they need not be. The following results refer to the evolution of spreads over time, from time t to $t+1$.

Corollary 12. *If information acquisition dominates informativeness, optimal monopolistic spreads widen after a trade which strengthens beliefs, i.e.* $\mu_t + 1$ *closer to* 0 *or* 1*. If crowding out dominates informativeness, optimal monopolistic spreads may narrow after a trade which weakens beliefs, i.e.* $\mu_t + 1$ *closer to* $\frac{1}{2}$.

Although Corollaries 8 and 9 describe the relationship between first period spreads and the total number of trading periods, this does not translate into the dynamic behaviour of spreads because beliefs evolve endogenously between periods. By Corollary 12, spreads may behave counterintuitively, widening after a trade which strengthens beliefs, or narrowing after a trade which weakens them.

Figure 2 shows an example of each case. It plots the spreads $a_t - b_t$ for periods $t = 1$ and $t = 2$ across beliefs μ_t. In Figure 2(a), information acquisition dominates informativeness so spreads at $t = 1$, drawn with a solid line, are narrower than at $t = 2$, drawn with a broken line, across beliefs μ_t. Consider an uninformative prior belief in period 1, $\mu_1 = 0.5$ and a sell trade which strengthens beliefs in period 2 to $\mu_2 = 0.65$. In this example, the spread in period 2 is wider than in period 1.

In Figure 2(b), noise dominates informativeness so spreads at $t = 1$, solid line, are wider than at $t = 2$, broken line, across beliefs μ_t. Now consider a prior belief in period 1 at $\mu_1 = 0.65$ and a buy trade which weakens beliefs in period 2 to $\mu_2 = 0.5$. In this example, the spread in period 2 is narrower than in period 1. Note that neither of these outcomes are necessary. For example, in Figure 2(a), a larger μ_2 could mean a narrower spread in period 2.

The counterintuitive spread dynamics are driven by the interaction of an optimal monopolistic market maker and market information structure. The optimal monopolistic market maker sets price schedules which change over time and the market information structure determines the direction of the change. The evolution of spreads then depends on relative

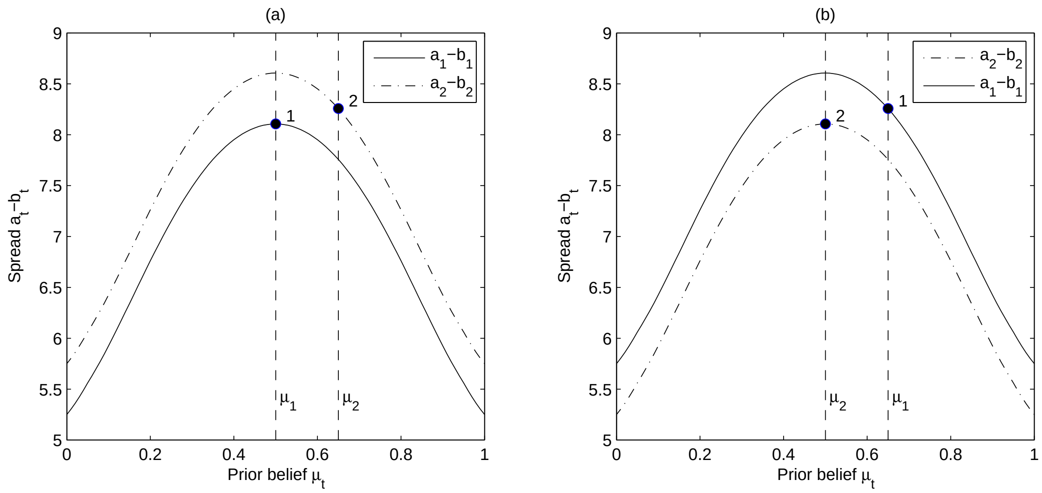

Figure 2: Dynamic behaviour of spread $a_t - b_t$ after a trade which (a) strengthens beliefs when information acquisition dominates informativeness; and (b) weakens beliefs when noise dominates informativeness.

effects of the change in beliefs after a trade and the difference in optimal spreads between each period.

While the examples above apply to realised trades over two periods, the results also apply to expected trades over multiple periods. Starting from an uninformed prior $\mu_1 = \frac{1}{2}$, expected beliefs grow monotonically stronger over time. If each period's price schedule implies sufficiently wider spreads each period, it is possible for expected spreads to widen every period from $t = 1$ to T.

6 Conclusion

This paper makes two main contributions. First, I study the effect of endogenous information acquisition with price inelastic noise traders and a competitive market maker. In this setup, spreads and the probability of an informed trade do not always comove. In particular, over some range, increasing the proportion of potentially informed traders leads to wider spreads but a lower probability of an informed trade. I also find dynamic features for expected transaction volume which better capture the stylised facts.

Second, I take endogenous information acquisition to market with price elastic noise traders and a monopolistic market maker. I find that market information structure determines how the market maker sets prices to influence information revealed by trades. If information acquisition dominates, he sets narrower spreads with more trading periods. If noise dominates, he sets wider spreads with more trading periods. Spreads may also widen over time on average.

In both cases, endogenous information acquisition significantly affects market properties relative to the exogenous information benchmarks. My results suggest caution when interpreting empirical results from structural models under the exogenous assumption. For future work, I aim to follow Easley et al. (1996) in deriving a maximum likelihood estimator for the probability of an informed trade. My model offers a natural extension to their framework because we share the same discrete time setup. By incorporating endogenous information acquisition, I hope to improve on their measure for informed trading.

A Appendix

I suppress the time subscript in proofs of Proposition 1 to 10. I only present proofs for the ask side. The bid side follows analogously. As shorthand, Z^c refers to any function $Z(a, b)$ which takes arguments (a^c, b^c).

Proof of Proposition 1. Let $Z(a) = \frac{(1-\mu)B_V(a)V}{\mu_t B_0(a)+(1-\mu)B_V(a)}$ so that the equilibrium ask price is given by $a^c = Z(a^c)$ from Equation (7). $Z(a)$ is given by:

$$Z(a) = \frac{(1-\mu)(\lambda X_a(a) + (1-\lambda))V}{(1-\mu)\lambda X_a(a) + (1-\lambda)}$$

Let X_a be a continuous, monotonic, increasing in a and bounded $[0, 1]$. Therefore the numerator of Z is continuous. The denominator of Z is also continuous and bounded $[0, 1]$ under the same conditions. Therefore Z is also continuous. Taking the first derivative of Z with respect to a yields:

$$\frac{\partial Z}{\partial a} = \frac{\mu(1-\mu)\lambda(1-\lambda)V}{(\mu\lambda X_a + 1 - \lambda)^2}\frac{\partial X_a}{\partial a}$$

$\frac{\partial X_a}{\partial a} \geq 0$ by earlier assumption. Now I need to show that $Z : \mathbb{R} \to [0, V]$. The lower bound of Z is at $X_a = 0$: $Z = (1-\mu)V : \mathbb{R} \to [0, V]$ when $\mu \in [0, 1]$. The upper bound of Z is at $F = 1$: $Z = \frac{(1-\mu)V}{1-\mu\lambda} : \mathbb{R} \to [0, V]$ when $\mu \in [0, 1]$ and $\lambda \in [0, 1]$. Therefore the function $Y = Z(a)$ must cross $Y = a$ once in the range $[0, V]$ and the solution to Equation (7) must exist in the range $[0, V]$. For uniqueness, I need to rule out the only other alternative: a continuum of solutions. It is easy to see there are no parameter values such that $a = Z(a)\ \forall\ a$. $a = Z(a)$ occurs at a single crossing. ■

Proof of Proposition 2. From Equation (7), differentiate a^c implicitly with respect to μ to obtain:

$$\frac{\partial a^c}{\partial \mu} = \frac{-V(1-\lambda)(1-\lambda(1-X_a^c+\mu(1-\mu)(\frac{\partial X_a^c}{\partial \mu}+\frac{\partial a^c}{\partial \mu}\frac{\partial X_a^c}{\partial a^c})))}{(1-\lambda(1-(1-\mu)X_t^c))^2}$$

Rearrange and see that $\frac{\partial a^c}{\partial \mu} \leq 0$ when $\mu \in [0,1]$, $\lambda \in [0,1]$, $X_a^c \in [0,1]$, $\frac{\partial X_a^c}{\partial \mu} \leq 0$, $\frac{\partial X_a^c}{\partial a^c} \leq 0$ and $a^c \leq V$.

Substituting into Equation (7), $a^c(\mu=0)=V$ and $a^c(\mu=1)=0$. ■

Proof of Proposition 3. Exogenously informed traders have $X_a = 1$ always. Equilibrium ask price in this market is given by:

$$\bar{a}^c = \frac{(1-\mu)V}{\mu(1-\lambda)+1-\mu}$$

Compare with a^c from Equation (7). Let $Z = 1-\lambda(1-g)$. $Z \leq 1$ when $\lambda \in [0,1]$ and $X_a^c : \mathbb{R} \to [0,1]$. Then $\bar{a}^c = \frac{(1-\mu)V}{\mu(1-\lambda)+1-\mu} = \frac{(1-\mu)VX}{\mu(1-\lambda)X+(1-\mu)X} \geq \frac{(1-\mu)VX}{\mu(1-\lambda)+(1-\mu)X} = a^c$ when $Z \leq 1$. ■

Proof of Proposition 4. From Equation (7) the competitive market maker sets the ask price as the expected value of the asset conditional on a buy occuring:

$$a = \frac{(1-\mu)[\lambda\frac{1}{C}((1-\mu)(V-a))+1-\lambda]V}{(1-\mu)\lambda\frac{1}{C}(1-\mu)(V-a)+1-\lambda}$$

The expression is quadratic in a and the two roots are given by:

$$a = V \pm \frac{(1-\lambda)C}{2\lambda(1-\mu)^2}\left[\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}}-1\right]$$

To obtain a unique solution, one of the roots is ruled out. Under the parameter restrictions, $p \in [0,1]$, $\lambda \in [0,1]$, $V > 0$ and $C > 0$, the expressions $\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}}-1$ and $\frac{(1-\lambda)C}{2\lambda(1-\mu_t)^2}$ are both larger than 0 so the first root is larger than V. This is ruled out because the potentially informed trader would always make a loss trading at this price, regardless of the true value.

The remaining root a^c has to satisfy two other restrictions: 1) it implies an information arrival probability, $X_a(a^c)$, which is bounded $[0,1]$, and 2) it is bounded $[0,V]$. For the first part, substitute the root into the expression for X_a:

$$X_a(a^c) = \frac{(1-\lambda)}{2\lambda(1-\mu)}\left[\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}}-1\right]$$

Under the same parameter assumptions, $X_a(a^c) \geq 0$. It also needs to be shown that $X_a(a^c) \leq 1$. Taking partial derivatives, it can be shown that $\frac{\partial X_a(a^c)}{\partial \lambda} < 0$. Since λ is bounded $[0,1]$, $\arg\max_{\lambda} X_a(a^c) = 0$. By l'Hopital's rule, $\lim_{\lambda\to 0} X(a^c) = \frac{1}{C}\mu(1-\mu)V$. The maximum value this can take is $\frac{V}{4C}$ given $\arg\max_{\mu} \frac{1}{C}\mu(1-\mu)V = \frac{1}{2}$. To satisfy $X_a(a^c) \leq 1$ requires the restriction $C > \frac{1}{4}V$. This completes the first part.

For the second part, it remains to be shown that $a^c \geq 0$. $a^c < 0$ is ruled out because the potentially informed trader would always make a profit trading at this price, regardless of the true value. Taking partial derivatives, $\frac{\partial a^c}{\partial \lambda} > 0$ under the parameter restrictions above. Since λ is bounded $[0,1]$, $\arg\min_{\lambda} a^c = 0$. By l'Hopital's rule, $\lim_{\lambda\to 0} a^c = (1-\mu)V$. The minimum value this can take is 0 given $\arg\min_{\mu}(1-\mu)V = 1$ for $\mu \in [0,1]$. Therefore, a^c is the unique root to Equation (7). ■

Proof of Proposition 5. Take the derivative of a^c from Equation (17) with respect to C and simplify to obtain:

$$\frac{\partial a^c}{\partial C} = -\frac{1-\lambda}{2(1-\mu)^2\lambda}\left[\frac{1+\frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}{\sqrt{1+\frac{4}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}}-1\right]$$

Let $Z = \frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V$. $Z \geq 0$ when $\mu \in [0,1]$, $\lambda \in [0,1]$ and $V > 0$. Then $1 + \frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V = 1+Z$ and $\sqrt{1+\frac{4}{(1-\lambda)C}(1-\mu)^2\mu\lambda V} = \sqrt{1+2Z}$. Comparing these two expressions in Z, $1+Z \geq \sqrt{1+2Z}$. Therefore $\frac{1+Z}{\sqrt{1+2Z}} > 1$. $\frac{1-\lambda}{2(1-\mu)^2\lambda} \geq 0$ when $\lambda \in [0,1]$. Therefore $\frac{\partial a^c}{\partial C} \leq 0$.

Take the expression for a^c from Equation (17). Let $Y(C) = \frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}$. The expression $\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}}$ can be written as $(1+Y)^{\frac{1}{2}}$. Using a Taylor expansion: $(1+Y)^{\frac{1}{2}} - 1 \approx (1+\frac{1}{2}Y - \frac{1}{8}Y^2 + ...) - 1 \approx \frac{1}{2}Y - \frac{1}{8}Y^2 + ...$ Also the expression $\frac{(1-\lambda)C}{2\lambda(1-\mu)^2}$ can be written as $\frac{2}{Y}\mu V$. Using this expression with the Taylor expansion, a^c can be written as: $a^c \approx V - \frac{2}{Y}\mu V(\frac{1}{2}Y - \frac{1}{8}Y^2 + ...) \approx V - \mu V + O(Y)$. $\lim_{C\to\infty} Y(C) = 0$. Therefore $a^c \approx (1-\mu)V$. ■

Proof of Proposition 6. Take the derivative of a^c from Equation (17) with respect to λ and simplify to obtain:

$$\frac{\partial a^c}{\partial \lambda} = \frac{C}{2(1-\mu)^2\lambda^2}\left[\frac{1+\frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}{\sqrt{1+\frac{4}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}} - 1\right]$$

$\frac{1+\frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}{\sqrt{1+\frac{4}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}} \geq 0$ as shown in the Proof of Proposition 5. Also, $\frac{C}{2(1-\mu)^2\lambda^2} \geq 0$ when $\mu \in [0,1]$, $\lambda \in [0,1]$ and $C > 0$. Therefore $\frac{\partial a^c}{\partial \lambda} \geq 0$.

Use L'Hopital's rule to evaluate a^c from Equation (17) as $\lambda \to 0$. Let $X(\lambda) = (1-\lambda)C\left[\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}} - 1\right]$ and $Y(\lambda) = 2(1-\mu)^2\lambda$. Then $\frac{\partial X(0)}{\partial \lambda} = -2\mu(1-\mu)^2 V$ and $\frac{\partial Y(0)}{\partial \lambda} = 2(1-\mu)^2$. Therefore $\lim_{\lambda \to 0} \frac{X(\lambda)}{Y(\lambda)} = -\mu V$ and so $\lim_{\lambda \to 0} a^c = (1-\mu)V$. From Equation (17) $a^c(\lambda = 1) = V$. ■

Proof of Proposition 7. To find the maximum of K with respect to μ, solve the first order condition $\frac{\partial K^c}{\partial \mu} = 0$. Taking the derivative of K^c:

$$\begin{aligned}\frac{\partial K^c}{\partial \mu} &= \frac{1-\lambda}{4(1-\mu)^2}\left[1 - \frac{1-\lambda+\frac{2}{C}(1-\mu)^3\lambda V}{\sqrt{1+\frac{4}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}}\right] \\ &\quad - \frac{1-\lambda}{4\mu^2}\left[1 - \frac{1-\lambda+\frac{2}{C}\mu^3\lambda V}{\sqrt{1+\frac{4}{(1-\lambda)C}\mu^2(1-\mu)\lambda V}}\right]\end{aligned}$$

The first order condition is satisfied when $\mu = \frac{1}{2}$.

Use L'Hopital's rule to evaluate $G^c(\mu)$ from Equation (19) as $\mu \to 1$. Let $X(\mu) = (1-\lambda)\left(\sqrt{1+\frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}} - 1\right)$ and $Y(\mu) = 4(1-\mu)$. Then $\frac{\partial X(1)}{\partial \mu} = 0$ and $\frac{\partial Y(1)}{\partial \mu} = 4$. Therefore $\lim_{\mu \to 0} \frac{X(\mu)}{Y(\mu)} = 0$ and $\lim_{\mu \to 0} G^c(\mu) = 0$. From Equation (19) $G^c(\mu = 0) = 0$. Similarly for $H^c(\mu)$. Combining them, I get the result for $K^c(\mu)$. ■

Proof of Proposition 8. Take the derivative of G^c from Equation (19) with respect to C and simplify to obtain:

$$\frac{\partial G^c}{\partial C} = -\frac{\mu(1-\mu)\lambda V}{2C^2\sqrt{1+\frac{4}{(1-\lambda)C}\mu(1-\mu)^2\lambda V}}$$

$\mu(1-\mu)\lambda V \geq 0$ and $1 + \frac{4}{(1-\lambda)C}\mu(1-\mu)^2\lambda V \geq 0$ when $\mu \in [0,1]$, $\lambda \in [0,1]$ and $V > 0$. Therefore $\frac{\partial G^c}{\partial C} \leq 0$. ■

Proof of Proposition 9. Take the derivative of G^c from Equation (19) with respect to λ and solve for the first order condition $\frac{\partial G^c}{\partial \lambda} = 0$. Following some tedious algebra, the two roots to the first order condition are given by: $(1-\mu)(1-2\lambda)\sqrt{\frac{1}{C}\mu V} + 1 - \lambda = 0$ and $(1-\mu)(1-2\lambda)\sqrt{\frac{1}{C}\mu V} - 1 + p = 0$. These roots are $\mu = \frac{Z+1}{2Z+1}$ and $\mu = \frac{Z-1}{2Z-1}$ where $Z = (1-\mu)\sqrt{\frac{1}{C}\mu V}$. $Z \geq 0$ when $\mu \in [0,1]$, $V \geq 0$ and $C \geq 0$. The second root is ruled out because it implies $\mu > 1$ when $\mu \in [0,1]$. Therefore, $\arg\max_{\mu} G^c = \frac{Z+1}{2Z+1}$.

Substituting into the expression for G^c, $G^c(\lambda = 0) = 0$ and $G^c(\lambda = 1) = 0$. ■

Proof of Proposition 10. Take the derivative of X_a^c with respect to λ and simplify to obtain:

$$\frac{\partial X_a^c}{\partial \lambda} = -\frac{1}{2(1-\mu)\lambda^2}\left[\frac{1 + \frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}{\sqrt{1 + \frac{4}{(1-\lambda)C}\mu^2(1-\mu)\lambda V}} - 1\right]$$

$\frac{1+\frac{2}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}{\sqrt{1+\frac{4}{(1-\lambda)C}(1-\mu)^2\mu\lambda V}} \geq 1$ as shown in the Proof of Proposition 5. $2(1-\mu)\lambda^2 \geq 0$ when $\mu \in [0,1]$, $\lambda \in [0,1]$. Therefore $\frac{\partial X_a^c}{\partial \lambda} \leq 0$.

Use L'Hopital's rule to evaluate $X_a^c(\lambda)$ as $\lambda \to 0$. Let $Z(\lambda) = (1-\lambda)\left(\sqrt{1 + \frac{4\mu(1-\mu)^2\lambda V}{(1-\lambda)C}} - 1\right)$ and $Y(\lambda) = 2(1-\mu)\lambda$. Then $\frac{\partial Z(0)}{\partial \lambda} = \frac{2}{C}(1-\mu)^2\mu V$ and $\frac{\partial Y(0)}{\partial \lambda} = 2(1-\mu)$. Therefore $\lim_{\lambda \to 0} X_a^c(\lambda) = \frac{1}{C}(1-\mu)\mu V$. Substituting into the expression for X_a^c, $X_a^c(\lambda = 1) = 0$. ■

Proof of Proposition 11. To establish nonnegativity, I use the fact that the market maker can post prices outside of the interval $[\theta_1, \theta_2]$ in all periods and earn zero expected profits. This sets the lower bound on the value function for all beliefs μ. To establish convexity, consider a $\mu' \in [\mu, \mu'']$ for $\mu, \mu'' \in [0,1]$ and $\mu'' \geq \mu$. I can then write:

$$\mu' = \phi\mu + (1-\phi)\mu''$$

where

$$\phi = (\mu'' = \mu')/(\mu'' - \mu)$$

Starting from the prior μ', suppose there is a set of prices that could induce a posterior of μ with probability ϕ and μ'' with probability $1-\phi$. This is more informative than prices that do not change the posterior because it could lead to a revision in beliefs. Since the market maker cannot be worse off on average by learning the outcome from these prices, it must be that:

$$\phi V_{T-1}^*(\mu) + (1-\phi)V_{T-1}^*(\mu'') \geq V_{T-1}^*(\mu')$$

which is the requirement for convexity. ■

Proof of Proposition 12. By Bayes' rule:

$$\begin{aligned} E\left[\widetilde{\theta}\,|q=+1;a,b\right] &= \frac{\lambda X(a,b)(1-\mu)\theta_2+(1-\lambda)(1-F(a))E_\mu[\widetilde{\theta}]}{\lambda X(a,b)(1-\mu)+(1-\lambda)(1-F(a))} \\ E\left[\widetilde{\theta}\,|q=-1;a,b\right] &= \frac{\lambda X(a,b)\mu\theta_1+(1-\lambda)F(b)E_\mu[\widetilde{\theta}]}{\lambda X(a,b)\mu+(1-\lambda)F(b)} \end{aligned}$$

Solve for $(1-\lambda)(1-F(a))E_\mu[\widetilde{\theta}]$ and $(1-\lambda)F(b)E_\mu[\widetilde{\theta}]$, substitute into Equation (28) and rearrange to obtain:

$$\begin{aligned} \pi(a,b) &= \left(a-E\left[\widetilde{\theta}\,|q=+1;a,b\right]\right)\left[\lambda X(a,b)(1-\mu)+(1-\lambda)(1-F(a))\right]+ \\ &\quad \left(E\left[\widetilde{\theta}\,|q=-1;a,b\right]-b\right)\left[\lambda X(a,b)\mu+(1-\lambda)F(b)\right] \end{aligned}$$

Since $\Pr(q=+1)=\lambda X(a,b)(1-\mu)+(1-\lambda)(1-F(a))$ and $\Pr(q=-1)=\lambda X(a,b)\mu+(1-\lambda)F(b)$, the the market maker's profit in Equation (28) can be written as:

$$\pi(a,b)=\Pr(q=+1)\left(a-E\left[\widetilde{\theta}\,|q=+1;a,b\right]\right)+\Pr(q=-1)\left(E\left[\widetilde{\theta}\,|q=-1;a,b\right]-b\right)$$

and the result follows immediately. ■

Proof of Proposition 13. This proof closely follows the one for Proposition 3 in Leach and Madhavan. First I show that $a^c<a^m$ with all market structures. Suppose the opposite, $a^c>a^m$. By definition of a^c, the expected profits of a^m must be negative, which contradicts the definition of a^m.

The next results hinge on the impact of prices on beliefs with different market structures. I begin with the case when information acquisition dominates informativeness. Then, by Definition 3, $\frac{\partial\mu_2(-1)}{\partial b}>0$ and $\frac{\partial\mu_2(+1)}{\partial a}>0$ for all $\mu\in[0,1]$ when markets are open. Now suppose that a^m is the unique solution to the myopic monopolistic market maker's problem and that $a^m<a^*$. By the definition of a^m, it must be that $\pi(a^m,b,\mu)>\pi(a^*,b,\mu)$. Since μ_2 is increasing in a, the expected value functions yield:

$$E\left[V^*_{T-1}(\mu_2(a^m,b,\widetilde{q}(a^m)))\right]\geq E\left[V^*_{T-1}(\mu_2(a^*,b,\widetilde{q}(a^*)))\right]$$

However, this relation contradicts the definition of a^*. Therefore, if information acquisition dominates informativeness, then $a^c\leq a^m$ and $a^*\leq a^m$. The bid side is analogous and spread implications follow immediately. ■

Proof of Proposition 14. When crowding out dominates informativeness, by Definition 4, $\frac{\partial \mu_2(-1)}{\partial b} < 0$ and $\frac{\partial \mu_2(+1)}{\partial a} < 0$ for all $\mu \in [0,1]$ when markets are open. Fix a b and suppose that $a^c > a^*$. Then $\mu_2(+1, a^*) \geq \mu_2(+1, a^c)$ while $\mu_2(-1)$ is the same for either a. From Proposition 11, convexity of the value function implies:

$$E\left[V^*_{T-1}(\mu_2(a^c, b, \widetilde{q}(a^c)))\right] \geq E\left[V^*_{T-1}(\mu_2(a^*, b, \widetilde{q}(a^*)))\right]$$

for any b. To satisfy the definition of a^*, it must also be that $0 \leq \pi(a^c, b, \mu) \leq \pi(a^*, b, \mu)$. But this means that a^* is lower than a^c and yields non negative profits, contradicting the definition that a^c is the lowest ask price yielding non negative profits. Therefore, it must be that $a^* \geq a^c$.

Now suppose that a^m is the unique solution to the myopic monopolistic market maker's problem and that $a^m > a^*$. By the definition of a^m, $\pi(a^m, b, \mu) > \pi(a^*, b, \mu)$. Since μ_2 is decreasing in a the expected value functions yield:

$$E\left[V^*_{T-1}(\mu_2(a^m, b, \widetilde{q}(a^m)))\right] \geq E\left[V^*_{T-1}(\mu_2(a^*, b, \widetilde{q}(a^*)))\right]$$

However this relation contradicts the definition of a^*. Therefore, if crowding out dominates informativeness, then $a^c \leq a^m \leq a^*$. The bid side is analogous and spread implications follow immediately. ■

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