Deliberation, Single-Peakedness, and the Possibility of Meaningful Democracy: Evidence from Deliberative Polls*

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Abstract

Majority cycling and related social choice paradoxes are often thought to threaten the meaningfulness of democracy. Deliberation can protect against majority cycles—not by inducing unanimity, which is unrealistic, but by bringing preferences closer to single-peakedness. We present the first empirical test of this hypothesis, using data from Deliberative Polls. Comparing preferences before and after deliberation, we find increases in proximity to single-peakedness. The increases are greater for lower- versus higher-salience issues and for individuals who seem to have deliberated more versus less effectively. They are not merely a by-product of increased substantive agreement (which in fact does not generally increase). Our results are important, quite apart from their implications for majority cycling, because single-peakedness can be naturally interpreted in terms of an underlying issue dimension, which can both clarify the debate and allow a majority-winning alternative to be interpreted as a median choice and thus as an attractive "compromise."

Condorcet's and similar paradoxes of social choice have long been seen as serious problems for democracy. Condorcet showed that pairwise majority voting in decisions over three or more alternatives can lead to cyclical majority preferences. If a third of a group (electorate, committee, etc.) prefers *x* to *y* to *z*, another third prefers *y* to *z* to *x*, and the remaining third prefers *z* to *x* to *y*, majorities prefer *x* to *y*, *y* to *z*, and yet *z* to *x*. This illustrates at least three problems with majority rule. First, rational (in particular, transitive) individual preferences may lead to irrational (in particular, cyclical) majority preferences (*collective irrationality*).[1] Second, pairwise majority voting may fail to produce a stable winning outcome (*instability*). Third, when pairwise majority votes are taken sequentially, the winning outcome may depend on the order in which the votes are taken and may thus be manipulable by agenda setters (*path dependence*, leading to *agenda manipulability*). Arrow's theorem (1951/1963) shows that these and related problems generalize well beyond majority rule,[2] and Riker (1982) has influentially argued that they undermine the meaningfulness of democracy.

What may be said against such counsel of despair? A number of authors have recently argued that majority cycles are empirically rare (Miller 2000, List and Goodin 2001, Tsetlin, Regenwetter, and Grofman 2003, Mackie 2004, Regenwetter et al. 2006). That may well be (the literature as a whole has not yet conceded the point), but we should still wish to understand why they are not more frequent, and what helps minimize their frequency. In this light, it is useful to show the existence of realizable conditions under which democratic institutions can robustly protect against cycles, where "robust protection" means that there will be no cycles for a large class of actual and counterfactual circumstances. It is well known that majority cycles are precluded if individual preferences are "single-peaked", in the original, ordinal sense defined by Black (1948) and Arrow (1951/1963). This requires the existence of a left-right ordering of the alternatives such that each individual has (1) a most preferred alternative and (2) a decreasing preference for other alternatives as they get more distant in either direction from it. The median individual's most preferred alternative then prevails. Here we examine the possibility that deliberation can offer robust protection against cycles by moving preferences toward single-peakedness (as suggested by Miller 1992, Knight and Johnson 1994, Dryzek and List 2003; less explicitly, the idea goes back to Arrow 1951/1963, ch. VII).

Deliberation's effect on the extent to which preferences are single-peaked is important, quite apart from its implications for majority cycling. Single-peakedness stands out among structure conditions for the avoidance of majority cycles.[3] It can be naturally interpreted in terms of an underlying issue dimension and regarded as a form of implicit "meta-agreement" (List 2002)—agreement not about the preference ordering of alternatives but about a dimension on which the alternatives can be arrayed. This can both clarify the debate and allow a majority-winning alternative to be interpreted as the choice of the median voter on the agreed dimension and thus as an attractive "compromise."[4] Further, single-peakedness is sufficient not only for the absence of cycles but for the possibility of strategy-proof collective decision making (Moulin 1980).[5]

In sizeable populations, single-peakedness, standardly defined as a binary property—either satisfied or violated by any given combination of preferences across individuals—may be hard to attain. Niemi (1969) and List (2001) have therefore suggested measuring *proximity to single-peakedness*, defined formally below. As proximity to single-peakedness increases, the probability of majority cycles decreases (see Niemi 1969, Feld and Grofman 1986, Gehrlein 2004, and the results below).

But does deliberation actually increase proximity to single-peakedness? And, if so, when and how? This paper presents the first empirical study of these questions. We focus on deliberation in mass publics, using data from *Deliberative Polls* designed to examine deliberation's effects on preferences and other variables (see Fishkin 1997, Luskin, Fishkin, and Jowell 2002, or Fishkin and Luskin 2005 for overviews).[6] The basic design involves interviewing a good-quality random sample; gathering them for a weekend to deliberate in randomly assigned small groups; allowing them to put questions arising from the small group discussions to panels of competing policy experts and policy makers; and then reinterviewing them at the end.[7]

Our central analysis compares measures of proximity to single-peakedness before and after deliberation. We also advance, and examine the evidence for, some hypotheses about when and for whom proximity to single-peakedness increases most. In particular, we contrast the results for high- versus low- to moderate-salience issues, for cases in which there is a more versus less natural left-right ordering of the alternatives, and for participants who seem to have learnt and thought more versus less, judging from the observed knowledge they emerge with. Our results support and refine the hypothesis that deliberation brings preferences closer to single-peakedness.

THEORETICAL BACKGROUND

Individual and Collective Preferences

Consider a set of individuals (voters, committee members, etc.), denoted $N = \{1, 2, \ldots, n\}$, and a set of alternatives (policies, candidates, etc.), denoted $X = \{x, y, z, \ldots\}$. Each individual i has a *preference ordering* over the alternatives, denoted R_i, where xR_iy means that individual i weakly prefers x to y. We write xP_iy if xR_iy and not yR_ix, meaning that individual i strictly prefers x to y. We assume that each individual's preference ordering R_i is *rational* in the following sense. It is *complete* (for every pair of alternatives x and y, xR_iy or yR_ix)[8] and *transitive* (for every triple of alternatives x, y and z, if xR_iy and yR_iz, then xR_iz). A combination of preference orderings across individuals (R_1, R_2, …, R_n), abbreviated as $(R_i)_{i\in N}$, is called a *profile*.

A *decision procedure* is a function that takes a profile $(R_i)_{i\in N}$ as input and produces a *collective preference ordering* R as output, where xRy means that x is weakly collectively preferred to y. In parallel with the individual case, we write xPy (x is strictly collectively preferred to y) if xRy and not yRx. Our concern here is with *pairwise majority voting*, under which, for each profile $(R_i)_{i\in N}$, an alternative *x* is weakly collectively preferred to another alternative *y* (*xRy*) if and only if at least as many individuals weakly prefer *x* to *y* (xR_iy) as weakly prefer *y* to *x* (yR_ix). The collective preference ordering *R* is *rational* if it is complete and transitive. A *Condorcet winner* is an alternative that is weakly collectively preferred to (i.e., beats or ties) every other alternative under pairwise majority voting. A rational collective preference ordering guarantees that there is a Condorcet winner.

Condorcet's paradox, in standard form, lies in the existence of profiles of rational individual preference orderings for which there exists no Condorcet winner. A more general form lies in the possibility that pairwise majority voting may generate irrational—specifically, cyclical and thereby intransitive—collective preference orderings, such as *xPy*, *yPz*, and *zPx*.[9]

The Probability of Cycles under an Impartial Culture

How frequently does Condorcet's paradox occur? The answer depends on the distribution of individual preference orderings. One much studied assumption is that of an *impartial culture*, under which all logically possible preference orderings are equiprobable. Given an impartial culture, the probability that there is no Condorcet winner—a lower bound for the probability of a cycle—increases with both the number of individuals *n* and the number of alternatives *k* (Gehrlein 1983). As *n* tends to infinity, the probability that there exists no Condorcet winner converges to 0.08774 for k=3, to 0.25131 for k=5, and to 0.36918 for k=7. For a reasonably large number of alternatives and a reasonably large group, therefore, cycles should be relatively frequent in an impartial culture. But an impartial culture is the distribution that generally maximizes the probability of cycles (Tsetlin, Regenwetter, and Grofman 2003; see also List and Goodin 2001, appendix 3),[10] and the evidence below argues it is not likely to hold (see also Regenwetter et al. 2006).[11] Empirically, cycles can thus be expected to be less frequent than the impartial culture suggests.

Single-Peakedness and Cycling

Consider some ordering (as distinct from preference ordering) of the alternatives from left to right, for the moment taking the terms "left" and "right" in a purely *geometric* sense. Formally, this is a one-to-one function Ω that assigns to each alternative x in X an integer between 1 and k, where $\Omega(x)=1$ means that alternative x is left-most, $\Omega(x)=2$ means that x is second from left, and so on.[12] Substantively, Ω may (but need not) represent an ordering of the alternatives on some *semantic* issue dimension, where, for example, "left" could be most liberal, most secular, or most environmentalist; "right" most conservative, most religious, or most anti-environmentalist.[13] We shall return to the distinction between geometric and semantic dimensions below.

Individual i's preference ordering R_i is *single-peaked with respect to* Ω if he or she has a most preferred alternative and a decreasing preference for other alternatives as they get more distant in either direction from it (relative to Ω).[14] Collectively, a profile of preference orderings $(R_i)_{i\in N}$ is *single-peaked* if there exists a geometric ordering Ω with respect to which *every* individual's preference ordering R_i is single-peaked. An ordering Ω with this property is called a *structuring dimension*.[15] This is the original, ordinal definition of single-peakedness given by Black (1948) and Arrow (1951/1963)—standard in social choice theory (e.g., Gaertner 2001) and not to be confused with the rather different, cardinal definition in spatial voting theory, which does not apply here.[16]

To illustrate, consider the following profile of preference orderings in a four-member group, where, for each individual, the alternatives are listed in strictly decreasing order of preference:

Individual 1: $x\ y\ z$

Individual 2: $z\ y\ x$

Individual 3: $y\ x\ z$

Individual 4: $y\ z\ x$

This profile is single-peaked, with structuring dimension, in what should be obvious notation, [*x y z*]. By contrast, consider the five-member group consisting of individuals 1-4 and 5:

Individual 5: *x z y*

The profile for this five-member group is not single-peaked. Individual 5's preferences are not single-peaked with respect to the left-right ordering [*x y z*], and there is no left-right ordering of *x*, *y*, and *z* with respect to which all five individuals' preference orderings are single-peaked.

Note the distinction between single-peakedness of an individual preference ordering and single-peakedness of a profile. An individual preference ordering (without ties) is trivially single-peaked with respect to *some* geometric ordering of the alternatives, for instance when these are arrayed in the individual's decreasing order of preference itself: a preference for *x* over *y* over *z* is single-peaked with respect to [*x y z*]. Single-peakedness becomes non-vacuous at the individual level only with respect to a *particular* geometric ordering. A profile, by contrast, is single-peaked if and only if *every* individual's preference ordering is single-peaked with respect to the *same* geometric ordering.

Single-peakedness both constitutes a sufficient condition for avoiding cycles and lends the majority-winning alternative a median interpretation of the sort previously described.[17] For any single-peaked profile $(R_i)_{i\in N}$, pairwise majority voting generates a transitive collective preference ordering, and the median individual's most preferred alternative (relative to a structuring dimension Ω) is a Condorcet winner.[18] This is a version of Black's median voter theorem (Black 1948, Arrow 1951 /1963). The fact that, on domains of single-peaked preferences (relative to some Ω), majority voting becomes a median voting scheme also protects it against strategic voting (Moulin 1980).

Proximity to Single-Peakedness

Single-peakedness in this standard sense is only sufficient, not necessary, for avoiding cycles. A high *proximity* to single-peakedness may be enough:[19] the whole profile of preference orderings across a set of individuals N may not be single-peaked, yet the (sub-)profile across some proper subset of N may be. The existence of a large enough subset with single-peaked preferences may suffice to ensure the existence of a transitive collective preference ordering and a Condorcet winner.

Thus consider the largest (or tied for largest) subset M of N such that the preference orderings of all individuals in M are single-peaked with respect to *some* geometric ordering of the alternatives. Call such an ordering (it need not be unique) a *largest structuring dimension*. The larger the subset M, the closer the original profile is to single-peaked. Define the *proximity to single-peakedness* for the profile $(R_i)_{i\in N}$ as $S=m/n$, where m is the size of the largest (or tied for largest) subset M of N for which the (sub-)profile $(R_i)_{i\in M}$ (note the subscript M) is single-peaked (Niemi 1969, List 2001).

S equals 1 for full single-peakedness ($M=N$) and is typically bounded below by a value strictly above 0. If all individual preferences are strict, the lower bound is $2^{(k-1)}/k!$, which equals 2/3 for $k=3$, 1/3 for $k=4$, 2/15 for $k=5$, and so on, in decreasing fashion.[20] In the example of the five-member group above, the largest subset for which the sub-profile $(R_i)_{i\in M}$ is single-peaked is $M=\{1,2,3,4\}$, and the largest structuring dimension is $[x\,y\,z]$. Thus the proximity to single-peakedness S is 4/5=0.8.

By Black's theorem, $S=1$ is sufficient for the existence of a Condorcet winner. But what about $S<1$? In our five-member group, $S=0.8$, but y is a Condorcet winner, beating both x and z in majority voting. More generally, for any threshold α ($0<\alpha<1$), we can determine the conditional probability—call it p—that there exists a Condorcet winner, given $S>\alpha$ (a deviation from an impartial culture).[21] Analytical and computational results suggest that the higher the α, the higher the p (Niemi 1969, Feld and Grofman 1986, Gehrlein 2004). To illustrate, we have randomly generated 1,000,000 profiles of preference orderings over $k=3$ alternatives for various n.[22] Table 1 reports the relative frequencies (estimating the probabilities) of the existence of a Condorcet winner for given ranges of S for $n=101$. As can be seen, the higher the proximity to single-peakedness, the higher the probability.

(Table 1 about here)

DELIBERATION AND SINGLE-PEAKEDNESS

Deliberation has been variously defined (see, e.g., Cohen 1989, Dryzek 1990, 2000, Fishkin 1991, Knight and Johnson 1994, Gutman and Thompson 1996, Bohman and Rehg 1997, Elster 1998). For present purposes, we define it moderately thinly, as discussion that is substantive, balanced, and civil.[23] It focuses on the policy or electoral alternatives and the reasons for preferring some over others; involves the airing of a broad range of perspectives, arguments, and positions; and takes place in an atmosphere of mutual respect. So defined, deliberation can be expected to have a variety of beneficial effects. Most relevantly for present purposes, it can be expected to induce learning and thinking and thus to produce more considered preferences (for relevant evidence, see, e.g., Luskin, Fishkin, and Jowell 2002, Luskin and Fishkin 2002, Barabas 2004).

Our focal claim is that deliberation shapes preferences in ways that robustly protect against majority cycles. One common but naïve hypothesis is that it does so by creating perfect *substantive agreement*. Elster (1986, p. 112) describes this line of thought as follows:

> "Rather than aggregating or filtering preferences, the political system should be set up with a view to changing them by public debate and confrontation. The input to the social choice mechanism would then not be the raw, quite possibly selfish or irrational, preferences …, but informed and other-regarding preferences … There would [then] not be any need for an aggregation mechanism, since a rational discussion would tend to produce unanimous preferences."

Unanimity, in populations of any size, is unattainable. A more realistic claim would be that deliberation increases substantive agreement. Past Deliberative Polls cast doubt on even this more modest claim,[24] but in any case substantive agreement is unnecessary for avoiding majority cycles. As already noted, single-peakedness, which does not require substantive agreement, precludes cycles, and proximity to single-peakedness, while not precluding them, makes them less probable. Deliberation, in turn, can be expected to increase proximity to single-peakedness (Miller 1992, Knight and Johnson 1994, Dryzek and List 2003). It may do so in either of two broad ways:

Single-Peakedness through Meta-Agreement

One way in which deliberation may increase proximity to single-peakedness is by increasing *meta-agreement*, that is, agreement on a common semantic issue dimension (like liberal/conservative or secular/religious) in terms of which to conceptualize the choice at hand.[25] This involves a three-step process (List 2002). The first step is for the participants to focus on a common semantic issue dimension. The second step is for them to place the alternatives in the same left-right order on it (technically, relating a geometric ordering like [*x y z*] to the semantic issue dimension).[26] The third step is for each individual to identify a most preferred alternative and to adopt a decreasing preference for other alternatives as they get more distant in either direction from it with respect to that common left-right ordering. This process may but need not be conscious.

The process may occur through discussion, with individuals influencing one another. As the participants talk, learn, and think about the alternatives, they consider the reasons for or against them, out of which a semantic issue dimension can be constructed. In deliberating about energy choices, for example, the participants may come to realize that some alternatives are cheap but dirty, others clean but expensive. To the extent that they focus on these attributes in evaluating the alternatives, they will have constructed a semantic issue dimension defined by the trade-off between cost and the environment. That is the first step. As the participants learn more about the attributes of specific alternatives, moreover, they can see how to place them in some geometric order corresponding to the semantic dimension they have constructed. They learn, for example, that coal is cheap but dirty, natural gas cleaner but more expensive, and wind power still cleaner but still more expensive. That is the second step. Then, as each participant considers the semantic issue dimension, the geometric placement of the alternatives on it, and his or her weighing of the underlying reasons, he or she may arrive at a most preferred alternative and rank other alternatives according to their distance in either direction from it. Someone who favors environmental over cost considerations may come to support wind power as a first choice, followed by natural gas and coal (assuming these are the only alternatives). That is the third step. The same three-step process can also occur through excogitation, any discussion aside, with individuals independently arriving at a similar conceptualization of the alternatives.

The semantic issue dimension and geometric ordering in this process may be more or less *natural*, in the sense that the reasons out of which the issue dimension is constructed may be more or less salient for the relevant population, and the geometric placement of the alternatives on it more or less obvious. The salience of given reasons and the obviousness of given placements, of course, will depend on the population in question. The three-step process we have described does not require the existence of any particularly natural issue dimension. All it requires is that some reasons become sufficiently focal to allow the construction of a common issue dimension. The resulting approach to single-peakedness, however, can be expected to be more pronounced to the extent that there is a natural issue dimension and geometric placement of the alternatives on it.

Single-Peakedness without Meta-Agreement

We do not mean to suggest that deliberation can increase proximity to single-peakedness only by increasing meta-agreement. A profile's single-peakedness is a matter of the aggregate coherence or patterning of preferences across individuals, not necessarily the cognitive organization of any individual's preference ordering. Individuals may simply adopt preference orderings they come to recognize as typical of political elites with whom they identify, and to the extent that elite preference profiles are close to single-peaked (presumably due to meta-agreement at the elite level), the resulting public preference profiles will also be close to single-peaked. This second, cue-taking mechanism requires shallower learning and thought than the mechanism involved in meta-agreement.

HYPOTHESES

We expect deliberation's effect on proximity to single-peakedness (*S*) to be nonlinear. Since *S* cannot exceed 1, while deliberation is presumably unbounded above, further increases from high-enough levels of deliberation should bring only small increases in the (correspondingly high) value of *S*. Thus we expect *S* to be an increasing, strictly concave function of deliberation, approaching 1 as deliberation increases. A testable implication is that deliberation's effect should vary with the salience of the issue. For highly salient issues, which have usually already received a good deal of casual deliberation in the participants' everyday environments, preference profiles should already be relatively close to single-peaked, and the effect of the more formal onsite deliberation should be small. It is for issues of low to moderate salience that we expect the onsite deliberation to make an appreciable difference.

As noted above, the effect should be greater to the extent that there is a natural issue dimension and geometric ordering of the alternatives for the participants to adopt (whatever they may have started with). The effect should also be mediated by the learning and thought deliberation induces. The greater the learning and thought, the greater the expected increase in proximity to single-peakedness. This hypothesis is normatively important. It suggests that deliberation's effect stems at least partly from meta-agreement produced by learning and thought, not just from cue-taking.

To sum up, then, our principal hypotheses are:

H1: Deliberation tends to increase proximity to single-peakedness, subject to the constraints in H2, H3, and H4.

H2: The rate of increase diminishes, eventually becoming negligible, at high enough levels of deliberation.

H3: The increase is greatest among those deliberating most "effectively," in the sense of learning and thinking the most.

H4: The increase is greater, *ceteris paribus*, to the extent that there is a natural issue dimension and geometric ordering of the alternatives.

We cannot test H2 directly, but a testable corollary is:

H2′: Deliberation's effect is smaller for higher- than for lower-salience issues.

It should also be noted that we have no quantitative measure of the naturalness of issue dimensions or geometric orderings of the alternatives, but some of the issue dimensions and orderings we shall encounter below do seem distinctly more natural than others, affording at least a rough test of H4.

EMPIRICAL CASES

We have already described the basic design of Deliberative Polls ("DPs"). Members of a random sample are interviewed, then invited to attend a weekend discussing the issues at a common site. Between the first interview and the weekend, they are sent carefully balanced briefing materials laying out arguments for and against policy alternatives. During the weekend, they alternate between discussions in randomly assigned small groups and plenary sessions in which they put questions generated by the small-group discussions to panels of experts or policy makers. The discussions are led by moderators trained to intervene as little and as neutrally as possible, steering the participants neither toward nor away from particular alternatives, and neither toward nor away from consensus. They also try to ensure that no-one dominates, that everyone participates, and that all sides are considered. At the end, the participants answer the same questions as at the beginning. Some DPs also involve control groups.

The "treatment" is thus broadly deliberative, revolving around discussions—in the small groups; in the corridors and over meals, coffees, and drinks; in the dialogues with the plenary session panelists; and in casual exchanges with family, friends, and co-workers in the interval between the initial interview and the onsite proceedings. The discussions also entail and occasion much thinking and learning (as documented, at least with respect to learning, below).

Data and Alternatives

Our particular data come from:

- Six regional DPs commissioned by Texas electric utility companies (SWEPCO, CPL, WTU, Entergy, HL&P, SPS), then regulated monopolies, at the instance of the state's Public Utility Commission, on how to meet future electricity needs (Luskin, Fishkin, and Plane 1999).
- The Australian national DP on the 1999 referendum on making Australia a republic with a parliamentarily appointed president (Luskin, Fishkin, McAllister, Higley, and Ryan 2000).
- The 1996 British national DP on the future of the Monarchy.
- The 2002 regional New Haven DP about revenue sharing and the future of the local airport (Farrar et al. 2010).

Each of these studies contained questions eliciting preference orderings over three or more alternatives.[27] The alternatives were:

- Four ways of meeting electricity needs (in three of the six electric utility DPs): in the SWEPCO and CPL DPs, (1) conservation, (2) building new fossil fuel facilities, (3) building new wind or solar facilities, and (4) buying electricity wholesale from elsewhere; in the WTU DP, (1) building new coal facilities, (2) building new wind or solar facilities, (3) conservation, and (4) building new natural gas facilities.
- Four or five goals to consider in deciding how to meet electricity needs (in all six electric utility DPs): in the WTU, Entergy, HL&P, and SPS DPs, (1) minimizing cost, (2) maintaining environmental quality, (3) ensuring adequate present and future supply, (4) making sure that everyone's basic electricity needs are met, and (5) minimizing outages; in the SWEPCO DP, (1) minimizing cost, (2) maintaining environmental quality, (3) avoiding dependence on any one resource, (4) using renewable resources, and (5) maximizing flexibility to increase or reduce production quickly; in the CPL DP, (1) minimizing cost, (2) maintaining environmental quality, (3) creating jobs, and (4) using renewable resources.

- Three constitutional possibilities for the Australian head of state: (1) the status quo with the Queen as head of state, (2) a republic with a parliamentarily appointed president (the referendum proposal), and (3) a republic with a directly elected president.
- Three possible alterations if there were a change to the British monarchy: (1) a monarchy with a more ordinary royal family, (2) a republic with a head of state with the same duties as the Queen, and (3) a republic with a head of state with the combined duties of Queen and Prime Minister.
- Three possible levels of service to be provided by the New Haven regional airport: (1) maintaining commercial passenger service to nearby cities; (2) expanding commercial passenger service, offering more flights to more places; and (3) ending commercial passenger service.
- Four possible arrangements for sharing property-tax revenues from new commercial development across the fifteen towns of greater New Haven: (1) full local control, (2) voluntary agreements with other towns, (3) state-provided incentives to share, and (4) state requirements to share.

In all, then, we have nine datasets, affording thirteen cases (issues). Four of the datasets afford two issues each: electric utility policies and electric utility goals in the SWEPCO, CPL, and WTU DPs and airport expansion and revenue sharing in the New Haven DP.

Salience and Naturalness

These issues differ both in salience (of the issue as a whole) and in the extent to which there is a natural geometric ordering of the alternatives. Consider first salience. Electric utility policies in Texas and revenue sharing in New Haven had received little public attention before the DP. The Monarchy in Britain, the airport in New Haven, and the Constitutional Referendum in Australia had received incomparably more. These latter three were highly salient. Since values and goals typically receive some thought, the electric utility goals were presumably in between—more salient than the electric utility policies but less salient than the three high-salience issues.

Now consider the extent to which there is a natural ordering of the alternatives. It seems clear that the alternatives in each of the two New Haven cases have a more natural ordering than those in any of the others. The airport alternatives can be obviously ordered from no commercial passenger service to increased service, with the status quo in-between. The revenue-sharing alternatives can be obviously ordered from no revenue-sharing (local control) to mandatory revenue-sharing, with voluntary sharing and incentivized sharing in-between. By contrast, the alternatives in the Australian case include two different departures from the status quo, which cannot be as obviously arrayed on a single dimension. In varying extent, the other non-New-Haven cases resemble the Australian case in this respect. It is of course possible to order the alternatives for the two New Haven cases on dimensions other than the level of commercial passenger service or the compulsoriness of revenue-sharing. Conversely, one could imagine circumstances in which the alternatives in the Australian (or any other) case have a more natural ordering, for instance in terms of the degree to which the citizens can participate directly in choosing the head of state. As matters stand, however, there is a *more* natural ordering of the alternatives for the two New Haven cases than for the rest.[28]

ANALYSIS

Table 2 describes the preferences on these issues before and after deliberation. The first four rows contain the low-salience cases (electric utility policies in Texas and revenue sharing in New Haven), the middle six contain the moderate-salience cases (electric utility goals in Texas), and the last three contain the high-salience cases (the Australian referendum, the British monarchy, and the New Haven airport). The columns give the number of individuals in the sample (n), the number of alternatives (k), the largest structuring dimension (D), the Condorcet winner (C), and the proximity to single-peakedness (S) at both T1 (before deliberation) and T2 (after deliberation).[29] The subscripts "1" and "2" distinguish the T1 and T2 values. A final column gives the change in proximity to single-peakedness from T1 to T2 (S_2 - S_1).

Statistical inference in this context is difficult.[30] While S (at whatever time) is a proportion, specifically of the individuals whose preferences are single-peaked on the largest structuring dimension, its definition depends on the identity of that dimension. Thus neither S_1 nor S_2 has a known

(Table 2 about here)

sampling distribution. As a rough guide, however, we may estimate the standard errors conditional on the obtained structuring dimensions. The difference $S_2 - S_1$ presents an additional difficulty, in that the obtained structuring dimension may change from T1 to T2. For $S_2 - S_1$, therefore, we present worst-case estimates of the standard errors, on the assumption of zero sampling covariance between S_1 and S_2. Table 2 presents these estimated standard errors in parentheses.[31]

Consistent with earlier results on non-ranking attitudinal and choice variables (in, e.g., Luskin, Fishkin, and Jowell 2002, Fishkin and Luskin 1999, and Luskin, Fishkin, and Hahn 2007), deliberation seems to induce considerable preference change. The Condorcet winner changes in eight of the thirteen cases, as does the largest structuring dimension (in not quite the same eight).[32] But our concern here is with proximity to single-peakedness, reported in the rightmost three columns.

Proximity to Single-Peakedness

A first point to note is that the proximity to single-peakedness at T1 is consistent with our sorting of the issues by salience. Among the four low-salience cases, the mean T1 proximity to single-peakedness is 0.421. Among the six moderate-salience cases, it is slightly higher, at 0.441. Among the three high-salience cases, it is vastly higher, at 0.751. Granted, the last three cases are also those for which there were only $k = 3$ alternatives, and S's lower bound, given strict preferences, is highest for $k = 3$.[33] Could this be why the last three cases have the highest S at T1? A rough way of addressing this possibility is to calculate how far S exceeds its minimum as a fraction of how far it could do so. Call this adjusted index $S' = (S - S_{min})/(1 - S_{min})$, where S_{min} is the lower bound. Clearly, S' ranges from 0 (when $S = S_{min}$) to 1 (when $S = 1$). The adjustment is only rough because the formula for S_{min} assumes strict preferences. As many of our respondents are indifferent between given pairs of alternatives, S is in fact occasionally lower than S_{min}. For what it is worth, however, this rough adjustment leaves the three high-salience cases displaying the highest T1 proximity to single-peakedness, the six moderate-salience cases the next highest, and the four low-salience cases by far the lowest. The mean values of S' are 0.268, 0.249, and 0.131, respectively.[34]

More importantly, Table 2 supports H1, H2′, and H4. (We consider the evidence for H3 below.) Regarding H1, the mean increase across all thirteen rows is 0.101. If normally distributed, the observed $S_2 - S_1$ would be statistically significant by conventional standards (by a two-tailed test at the 0.05 level against the null-hypothesis value of 0) for nine of the thirteen cases.

Regarding H2′, the increase is confined to the ten low- and moderate-salience cases, for which the mean increase is 0.134. The mean increase is also larger for the low-salience cases (0.173) than for the moderate-salience cases (0.107). For the three high-salience cases, the mean increase is -0.006. Two of the latter actually show *decreases*. Again if normally distributed, the observed $S_2 - S_1$ would be statistically significant for every low- to moderate-salience issue except electric utility goals in the Entergy DP and statistically insignificant for all three high-salience issues. Here too, this pattern could be an artefact of differences in the number of alternatives. The low-salience cases, coincidentally involving more alternatives, could simply have more room to show increase. But S', roughly controlling for the number of alternatives, shows mean increases of -0.014 for the high-salience cases,[35] 0.149 for the moderate-salience cases, and 0.259 for the low-salience cases—still consistent with H2′. So at least on issues that are not extremely salient, deliberation does seem to increase proximity to single-peakedness.

Regarding H4, the increase is greatest, among both low-to-medium and high-salience cases, for the one case in each set that has a relatively natural left-right ordering of the alternatives (the New Haven revenue-sharing and airport cases, respectively). It is worth noting, however, that at least for low-to-moderate salience cases it is not necessary that there be a relatively natural ordering of the alternatives for there to be a major increase in proximity to single-peakedness. Seven of the other eight low-to-moderate salience cases also show an appreciable increase in proximity to single-peakedness. This supports our earlier claim that shared issue dimensions can be *constructed* (when there is no relatively natural ordering of the alternatives) as well as *discovered* (when there is).

Proximity to Single-Peakedness and Substantive Agreement

Perfect substantive agreement (unanimity) implies single-peakedness, and high levels of substantive agreement may be expected to produce high proximity to single-peakedness. Could the story of our results be simply that deliberation is producing increased substantive agreement, which is in turn producing increased proximity to single-peakedness as a byproduct? That is what Shapiro (2005), for instance, would suppose. But proximity to single-peakedness, as noted, need not rest on substantive agreement.

To examine this question, we adopt the inverse of the Laakso-Taagepera index of fragmentation (Laakso and Taagepera 1979, Taagepera and Grofman 1981) as a simple measure of substantive agreement. We focus on each individual's most preferred alternative. Writing n_j for the number of individuals most preferring the j^{th} alternative (where $j = 1, 2, \ldots, k$), the index of substantive agreement is $A = (n_1/n)^2 + (n_2/n)^2 + \ldots + (n_k/n)^2$. $A = 1$ when all individuals have the same most preferred alternative (perfect substantive agreement), and $A = 1/k$ when equal numbers of individuals most prefer each of the k alternatives (maximum substantive disagreement).

(Table 3 about here)

Table 3 reports the raw frequencies ($n_1, n_2, \ldots, n_k$) of the most preferred alternatives, the index of substantive agreement (A) at T1 and T2 (with the subscripts "1" and "2" again distinguishing the T1 and T2 values), and the change from T1 to T2 ($A_2 - A_1$). It is clear from these results that the observed increases in proximity to single-peakedness are not just a by-product of increased substantive agreement. Indeed, substantive agreement does not generally increase. It increases in only four of the thirteen cases, and the average "increase" (actually a decrease) is -0.046.[36] Furthermore, the changes in substantive agreement are only modestly—and negatively—associated with the changes in proximity to single-peakedness. The Pearsonian correlation between the two across the thirteen cases is -0.308. At least on these particular issues, increased proximity to single-peakedness tends to be associated with *decreased* substantive agreement; the more those deliberating come to disagree, the more they come to agree about what they are disagreeing about.

Learning and Thinking

So far, we have established support for H1, H2′, and H4. But the "how" of deliberation's effect is important. The participants could simply have been taking cues, adopting ready-made preference orderings held by political elites. If that were all they did (as suggested by Aldred 2004), the deliberation behind the increased proximity to single-peakedness would have been quite shallow.

This leads us to consider the mediating effect of learning and thinking about the issues. By "learning" we mean the T1-T2 increase in relevant knowledge—knowledge not just of facts but of arguments for and against the alternatives and of other people's circumstances, beliefs, goals, and capacities, among other things. Of course, knowledge in all this variety is difficult to measure. So is "thinking." Operationally, therefore, we focus on factual knowledge, which is more readily measurable (as the proportion of factual questions answered correctly), is important in its own right, and can serve as a proxy for other sorts of knowledge and thought. Knowledge is positively correlated with the cognitive organization resulting from thought (Neuman 1981, Luskin 1987, Price 1999); knowledge of any one topic in a given domain is positively correlated with knowledge of other topics in the same domain (if for no other reason than that they are all affected by cognitive ability, controlling for interest; see, e.g., Brody 1997); and different sorts of knowledge are positively correlated (as in, e.g., Schneider, Rittle-Johnson, and Star 2011).[37]

For ten of our thirteen cases (all but the three based on the Entergy, HL&P, and SPS DPs), the questionnaires afford enough factual knowledge items to construct a usable index. Both New Haven indices include four general New Haven items, plus two specific to the topic (the airport or revenue sharing). The Australian referendum index includes four general Australian politics items, plus eight specific to the referendum. The SWEPCO index is confined to the five knowledge items shared with the CPL and WTU questionnaires. The indices range from 0 (no items answered correctly) to 1 (all answered correctly). The online Appendix describes the ingredients.

(Table 4 about here)

Table 4 reports the mean T1 and T2 knowledge and the mean knowledge gains from T1 to T2. The table contains only seven rows because three of the electric utility DPs provide six of our original thirteen cases. The first three rows (CPL, WTU, SWEPCO) are a mix of low-salience (for policies) and moderate-salience (for goals), the fourth (New Haven revenue sharing) is low-salience, and the remaining three (the New Haven airport, the Australian referendum, and the British Monarchy) are high-salience.

In every case, the participants learned a great deal. The knowledge gains are a first indication that more is going on than mere cue-taking. The mean increase, across all rows, is a sizable 0.212, akin to the mean score on an exam's increasing by 21 points on the familiar 0 to 100 scale.

Note that Table 4 lends further support to our characterizations of salience. While comparisons of different knowledge indices must be taken with some caution, resting as they do on an implicit assumption of equal average difficulty, the pattern is clear. The mean *T1 knowledge* is highest in the high-salience cases (averaging 0.502) and lowest in the low-salience case (0.362 for New Haven revenue sharing). The mean *knowledge gain* is lowest in the high-salience cases (averaging "only" 0.176) and highest in the low-salience case (0.245 for New Haven revenue sharing).

The next step is to show that the previously observed increases in proximity to single-peakedness are at least partly learning-driven. Since proximity to single-peakedness is intrinsically aggregate, our strategy is to partition the sample into low- and high-learning subsamples and then to perform the same analysis as above separately within each. In practice, this means dividing the sample by observed T2 knowledge, which can be shown, under a broad range of plausible conditions, to be more highly correlated with actual knowledge gain than is observed knowledge gain (T2 observed knowledge minus T1 observed knowledge).[38] Participants who emerge knowing a lot at T2 have typically learned a lot—either observably, if they scored low on knowledge at T1, or unobservably, if they scored high on knowledge at T1 and thus could show little gain. The threshold dividing "high" from "low" knowledge is always drawn so as to divide the sample as equally as possible.[39] Table 4 gives the details. We expect the gain in proximity to single-peakedness to be greater in the high T2 knowledge subsample, in accordance with H3.

(Table 5 about here)

The results, in Table 5, support this hypothesis in spades. Even in the low T2 knowledge subsample, proximity to single-peakedness generally increased. In the high T2 knowledge subsample, it always increased. In every case, moreover, the increase is greater for the high T2 knowledge subsample than for the low T2 knowledge subsample. The mean increase is 0.147 for the former, but only 0.051 for the latter.[40]

Here, too, salience is an important conditioning factor. For the high T2 knowledge subsample, the mean increase in proximity to single-peakedness is 0.209 in the low-salience cases (electric utility policies and New Haven revenue sharing), 0.130 in the moderate-salience cases (electric utility goals), and 0.081 in the high-salience cases (the New Haven airport, the Australian referendum, and the British Monarchy). For the low T2 knowledge subsample, the mean increases in proximity to single-peakedness are similarly consistent with the sorting by salience—but with a twist. While the mean increase is 0.131 in the low-salience cases and 0.103 in the moderate-salience cases, it is not merely smaller but negative—a *decrease*—in the high-salience cases, at -0.106.

This breakdown sheds further light on the failure of overall proximity to single-peakedness to increase in the three high-salience cases. It *does* increase for the high T2 knowledge participants (those who emerge knowing relatively much). There is no overall increase because it *decreases* for the low T2 knowledge participants (those who emerge knowing relatively little). We are unsure of the reason in the British monarchy case, but in the other two high-salience cases, a major reason seems to be that, at least for the low T2 knowledge participants, the largest structuring dimension changes, as Table 6 shows. Recall that S registers only the proportion whose preferences are single-peaked with respect to the largest structuring dimension. If many low T2 knowledge participants continue to hold preferences that are single-peaked with respect to the old largest structuring dimension, the T2 proportion whose preferences are single-peaked with respect to the new one can easily be lower than the T1 proportion whose preferences were single-peaked with respect to the old one—in which event S_2 will be lower than S_1.

(Table 6 about here)

Thus consider the case of the New Haven airport. Among the high T2 knowledge participants, the largest structuring dimension is [2 1 3] at T1 and remains [2 1 3] at T2, and the proportion whose preferences are single-peaked with respect to that dimension increases from 0.789 to 0.859. Among the low T2 knowledge participants, the largest structuring dimension changes from [1 2 3] to [2 1 3]. The low T2 knowledge participants come to acquire the same largest structuring dimension as the high T2 knowledge participants have from the outset. The proportion whose preferences are single-peaked with respect to [1 2 3] at T1 is 0.787, while the single-peaked proportion with respect to [2 1 3] at T2 is only 0.754. Thus S declines. But the proportion whose preferences are single-peaked with respect to [2 1 3] at T1 is 0.754, the same as at T2. The proportion with single-peaked preferences relative to what becomes the largest structuring dimension at T2 does not decline.

In the Australian case, the largest structuring dimension changes from [2 1 3] to [1 2 3] for both the high and low T2 knowledge participants. Among the former, the proportion whose preferences are single-peaked with respect to [1 2 3] at T2 (0.878) exceeds the proportion whose preferences are single-peaked with respect to [2 1 3] at T1 (0.793). Thus *S* increases. Among the latter, however, the proportion whose preferences are single-peaked with respect to [1 2 3] at T2 (0.682) is lower than the proportion whose preferences are single-peaked with respect to [2 1 3] at T1 (0.860). Here *S* declines. But the proportion whose preferences are single-peaked with respect to [1 2 3] at T1 is only 0.363, far lower than the 0.682 at T2. Again, the single-peaked proportion relative to what becomes the largest structuring dimension at T2 does not decline, indeed in this case increases substantially.

DISCUSSION

We have argued that deliberation increases proximity to single-peakedness—at least on low- to moderate-salience issues, where there has not been too much prior deliberation and where proximity to single-peakedness is not already high. We have also argued that deliberation's effect should be greatest among those who are learning and thinking the most, suggesting that it is not just a matter of thoughtless cue-taking. Our analysis, based on Deliberative Polling data, supports these claims.

Controlling for salience, the increases in proximity to single-peakedness are greatest for the issues that have a relatively natural ordering of the alternatives. Where there is a relatively natural ordering of the alternatives, deliberation appears to help people see that. But the increases are appreciable even for the issues that do not have a relatively natural ordering of the alternatives. In those cases, deliberation appears to help people construct a shared ordering.

Our analyses bear some third-cousin-ish relationship to those based on factor-analytic and covariance-structure models of responses to non-ranking policy attitude items (as, e.g., in Judd and

Milburn 1980, Jackson 1983, Peffley and Hurwitz 1993). The dimensionality and fit do not directly reflect attitude organization inside the minds of individual respondents (Luskin 1987, 2002) but do register the degree of aggregate patterning of opinion. The fewer the dimensions and the better the fit, the greater the patterning. Stratifying these analyses by variables like knowledge generally produces results akin to ours: the number of dimensions decreases and the fit increases as knowledge increases (Stimson 1975, Delli Carpini and Keeter 1996). That said, the specific form of patterning we examine here—the proximity to single-peakedness of preference profiles—is distinctive, not only in conceptual detail but in social-choice-theoretic significance.

A skeptic may wonder how much of the increase in proximity to single-peakedness is due to the actual deliberation (and consequent thought and learning) between the initial interview and the post-event questionnaire. But a follow-up experiment enfolded in one of the DPs examined here provides reassurance on this score. The results of the New Haven DP (recounted more fully in Farrar et al. 2010), in which half the participants discussed the airport first and revenue sharing second, and the other half the reverse, suggest that the great bulk of the increase in proximity to single-peakedness stemmed from the onsite deliberation (consisting of both small-group and plenary discussions), rather than from anything between the first interview and the onsite deliberation.[41]

Still, the full story is doubtless more complex than the one told here. The issue's salience and the individuals' learning are probably not the only conditioning or mediating factors. Much may also depend, for example, on the broader quality of the deliberation. The more focused, serious, and reflective the deliberation, the more it should promote proximity to single-peakedness. As the number of DPs with suitable ranking questions increases, we hope to examine these and other hypotheses.

There also remains, as ever, some causal mediation to explore. This is not just a question of the extent to which the movement toward single-peakedness is attributable to the deliberation in the DP treatment (as discussed in Farrar et al. 2010), but of what it is about deliberation that is responsible.

Two sorts of disaggregation are of interest: the first to apportion deliberation's effect as between various possible intervening social and psychological mechanisms, the second to apportion its effect as between the various components of the DP treatment. To what extent, for example, is it thinking versus learning that increases proximity to single-peakedness? To what extent is it the small-group discussions versus the plenary sessions with experts and policy makers? The argument above suggests some possible answers, as do the results regarding knowledge, whose effect may be mediating as well as conditioning. Closer examination must await future studies, with finer measurement or experimental manipulation aimed at disaggregation of either sort. But for now it is an important advance to show, as we have done here, that there *is* an effect to be parsed—that, at least under widely prevalent conditions, deliberation does move preferences closer to single-peakedness.

This implies that deliberation can robustly protect against majority cycles—in the sense explained above—by moving preferences toward single-peakedness. Ironically, it was Riker (1982, p. 128) who first raised this possibility, writing that "[i]f, by reason of discussion, debate, civic education, and political socialization, voters have a common view of the political dimension (as evidenced by single-peakedness), then a transitive outcome is guaranteed." He immediately added, lest too much hope be drawn from this remark, that he expected this only for "issues of minor importance".

Here we see that deliberation has the posited effect only on issues of low to moderate *salience*. But salience and importance are hardly the same. Many issues are important, but only a few, at any moment, are salient.[42] Thus the domain of the deliberation's effect on proximity to single-peakedness is broad. And when unattended but important issues become more salient, the "discussion, debate, civic education, and political socialization"—in short, the deliberation—they then receive will move preferences toward single-peakedness, thus helping to ensure that "a transitive outcome is guaranteed."

REFERENCES

Aldred, J. 2004. “Social Choice Theory and Deliberative Democracy: A Comment.” *British Journal of Political Science* 34(4, October): 747-752.

Arrow, K. 1951/1963. *Social Choice and Individual Values*. New Haven: Yale University Press.

Barabas, J. 2004. “How Deliberation Affects Policy Opinions.” *American Political Science Review* 98(4, November 2004): 687-701.

Bennett, S.E. 2001. “‘Reconsidering the Measurement of Political Knowledge’ Revisited: A Response to Jeffery Mondak.” *American Review of Politics* 22(3, Fall): 327-48.

Black, D. 1948. “On the Rationale of Group Decision-Making.” *Journal of Political Economy* 56(1, February): 23-34.

Bohman, J., and W. Rehg, eds. 1997. *Deliberative Democracy: Essays on Reason and Politics*. Cambridge, MA: MIT Press.

Brams, S.J., M.A. Jones, and D.M. Kilgour. 2002. “Single-Peakedness and Disconnected Coalitions.” *Journal of Theoretical Politics* 14(3): 359-383.

Bransford, J.D., and M.K. Johnson. 1972. “Contextual prerequisites for understanding: Some investigations of comprehension and recall.” *Journal of Verbal Learning and Verbal Behavior* 11(6): 717-726.

Brody, N. 1997. “Intelligence, Schooling, and Society.” *American Psychologist* 52: 1046-1050.

Cohen, J. 1989. “Deliberation and Democratic Legitimacy.” In A. Hamlin and P. Pettit, eds. *The Good Polity: Normative Analysis of the State*. Oxford: Blackwell.

Converse, Philip E. 2000. ‘‘Assessing the Capacity of Mass Electorates.’’ *Annual Review of Political Science* 3: 331-353.

Delli Carpini, M.X., and S. Keeter. 1996. *What Americans Know About Politics and Why It Matters*. New Haven: Yale University Press.

Dietrich, F., and C. List. 2010. “Majority voting on restricted domains.” *Journal of Economic Theory* 145(2): 512-543

Dryzek, J. 1990. *Discursive Democracy*. New York: C.U.P.

Dryzek, J. 2000. *Deliberative Democracy and Beyond*. Oxford: O.U.P.

Dryzek, J., and C. List. 2003. “Social Choice Theory and Deliberative Democracy: A Reconciliation.” *British Journal of Political Science* 33(1, January): 1-28.

Elsholtz, C., and C. List. 2005. “A Simple Proof of Sen's Possibility Theorem on Majority Decisions.” *Elemente der Mathematik* 60(2): 45-56.

Elster, J. 1986. “The Market and the Forum.” In J. Elster and A. Hylland, eds., *Foundations of Social Choice Theory*, Cambridge: C.U.P.

Elster, J., ed. 1998. *Deliberative Democracy*. New York: C.U.P.

Farrar, C., J.S. Fishkin, D.P. Green, C. List, R.C. Luskin and E.L. Paluck. 2010. “Disaggregating Deliberation’s Effects: An Experiment within a Deliberative Poll.” *British Journal of Political Science* 40 (2): 333-347.

Feld, S.L. and B. Grofman 1986. “Partial single-peakedness. An extension and clarification.” Public Choice 51(1): 71-80.

Fishkin, J.S. 1991. *Democracy and Deliberation*. New Haven: Yale University Press.

Fishkin, J.S. 1997. *The Voice of the People: Public Opinion and Democracy*. New Haven: Yale University Press.

Fishkin, J.S., and R.C. Luskin. 1999. “Bringing Deliberation to the Democratic Dialogue: The NIC and Beyond.” In M. McCombs and A. Reynolds, eds., *A Poll with a Human Face: The National Issues Convention Experiment in Political Communication*. Mahwah, NJ: Erlbaum.

Fishkin, J.S., and R.C. Luskin. 2005. “Experimenting with a Democratic Ideal: Deliberative Polling and Public Opinion.” *Acta Politica* 40 (3, September): 284-298.

Gaertner, W. 2001. *Domain Conditions in Social Choice Theory*. Cambridge: C.U.P.

Gehrlein, W.V. 1983. “Condorcet’s Paradox.” *Theory and Decision* 15(2, June): 161-197.

Gehrlein, W.V. 2004. “Probabilities of Election Outcomes with Two Parameters: The Relative Impact of Unifying and Polarizing Candidates.” Working paper, Department of Business Administration, University of Delaware.

Gibbard, A. 1973. “Manipulation of Voting Schemes: A General Result.” *Econometrica* 41(4, July): 587-601.

Gutman, A., and D. Thompson. 1996. *Democracy and Disagreement*. Cambridge, MA: Harvard University Press.

Jackson, J.E. 1983. “The Systematic Beliefs of the Mass Public: Estimating Poling Preferences with Survey Data.” *Journal of Politics* 45(4, November): 840-65.

Judd, C.M., and M.A. Milburn. 1980. “The Structure of Attitude systems in the Public: Comparisons of a Structural Equation Model.” *American Sociological Review* 45(4, August): 627-43.

Hambrick, Donald Z. 2003. “Why are some people more knowledgeable than others? A longitudinal study of knowledge acquisition.” *Memory and Cognition* 31: 902-917.

Knight, J., and J. Johnson. 1994. “Aggregation and Deliberation: On the Possibility of Democratic Legitimacy.” *Political Theory* 22(2, May): 277-296.

Laakso, M., and R. Taagepera. 1979. “‘Effective’ Number of Parties: A Measure with Application to West Europe.” *Comparative Political Studies* 12(1, April): 3-27.

List, C. 2001. *Mission Impossible? The Problem of Democratic Aggregation in the Face of Arrow’s Theorem*. DPhil thesis, University of Oxford.

List, C. and R.E. Goodin. 2001. “Epistemic Democracy: Generalizing the Condorcet Jury Theorem.” *Journal of Political Philosophy* 9(3, September): 277-306.

List, C. 2002. “Two Concepts of Agreement.” *The Good Society* 11(1): 72-79.

Luskin, R.C. 1987. "Measuring Political Sophistication." *American Journal of Political Science* 31(4, November): 856-899.

Luskin, R.C. 2002. "From Denial to Extenuation (and Finally Beyond): Political Sophistication and Citizen Performance." In J. H. Kuklinski (ed.), *Thinking about Political Psychology*. New York: C.U.P.

Luskin, R.C., and J.S. Fishkin. 2002. "Deliberation and 'Better Citizens'." Paper presented at the annual Joint Sessions of Workshops of the European Consortium for Political Research, Turin, Italy, 3/2002.

Luskin, R.C., J.S. Fishkin, and K.S. Hahn. 2007. "Deliberation and Net Attitude Change." Paper presented at the ECPR General Conference, Pisa, Italy, September 6-8, 2007.

Luskin, R.C., J.S. Fishkin, and R. Jowell. 2002. "Considered Opinions: Deliberative Polling in Britain." *British Journal of Political Science* 32 (3, July): 455-487.

Luskin, R.C., J.S. Fishkin, I. McAllister, J. Higley, and P. Ryan. 2000. "Information Effects in Referendum Voting: Evidence from the Australian Deliberative Poll." Paper presented at the annual meeting of the European Consortium for Political Research, Copenhagen, 4/2000.

Luskin, R.C., J.S. Fishkin, and D.L. Plane. 1999 "Deliberative Polling and Policy Outcomes: Electric Utility Issues in Texas." Paper presented at the meetings of the MPSA, Chicago, 4/1999, and of the Association for Public Policy Analysis and Management, Washington DC, 11/1999.

Luskin, R.C., J.S. Fishkin, I. McAllister, J. Higley, and P. Ryan. 2000. "Information Effects in Referendum Voting: Evidence from the Australian Deliberative Poll." Paper presented at the annual meeting of the European Consortium for Political Research, Copenhagen, 4/2000.

Luskin, R.C., A. Helfer, and G. Sood. 2011. "Measuring Learning in Informative Processes." Presented at the annual meetings of the Society for Political Methodology, July 28-30.

Mackie, G. 2004. *Democracy Defended.* Cambridge: C.U.P.

McKelvey, R.D. 1979. “General conditions for global intransitivities in a formal voting model.” *Econometrica* 47 (5, September): 1085-1112.

Mendelberg, T. 2002. “The Deliberative Citizen: Theory and Evidence.” *Political Decision Making, Deliberation and Participation* 6: 151-193.

Miller, D. 1992. “Deliberative Democracy and Social Choice.” *Political Studies* 40 (special issue): 54-67.

Miller, N. 2000. “The Geometry of Voting Cycles: Theoretical Developments.” Paper presented at the Workshop on Mathematical Models of Individual and Public Choice, Institute for Mathematical Behavioral Sciences, University of California, Irvine, July 2000.

Mondak, J.J. 2001. “Developing Valid Knowledge Scales.” *American Journal of American Journal of Political Science* 45 (1, January): 224-38.

Moulin, H. 1980. “On strategy-proofness and single peakedness.” *Public Choice* 35 (4, January): 437-455.

Neuman, W.R. 1981. “Differentiation and Integration: Two Dimensions of Political Thinking.” *American Journal of Sociology* 86 (6, May): 1236-68.

Niemi, R.G. 1969. “Majority Decision-Making with Partial Unidimensionality.” *American Political Science Review* 63 (2, June): 488-497.

Penn, E.M., J.W. Patty and S. Gailmard. 2011. “Manipulation and Single-Peakedness: A General Result.” *American Journal of Political Science* 55(2): 436-449.

Peffley, M., and J. Hurwitz. 1993. “Models of Attitude Constraint in Foreign Affairs.” *Political Behavior* 15 (1, March): 61-90.

Price, V. 1999. “Political Information.” In J. P. Robinson, P. R. Shaver, and L. S. Wrightsman, eds., *Measures of Political Attitudes*. San Diego, CA: Academic Press.

Recht, D.R., and L. Leslie. 1988. “Effect of prior knowledge on good and poor readers’ memory of text.” *Journal of Educational Psychology* 80: 16-20.

Regenwetter, M., B. Grofman, A.A.J. Marley and I. Tsetlin. 2006. *Behavioral Social Choice* Cambridge: C.U.P.

Riker, W.H. 1982. *Liberalism Against Populism*. San Francisco: W. H. Freeman.

Satterthwaite, M. 1975. “Strategy-Proofness and Arrow’s Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions.” *Journal of Economic Theory* 10 (2, April): 187-217.

Schneider, M., B. Rittle-Johnson, and J.R. Star. 2011. “Relations Among Conceptual Knowledge, Procedural Knowledge, and Procedural Flexibility in Two Samples Differing in Prior Knowledge.” *Developmental Psychology* 47: 1525-1538.

Schofield, N. 1976. “Instability of simple dynamic games.” *Review of Economic Studies* 45 (3, October): 575-594.

Sen, A.K. 1966. “A Possibility Theorem on Majority Decisions.” *Econometrica* 34 (2, Apr.): 491-499.

Shapiro, I. 2005. "The State of Democratic Theory: A Reply to James Fishkin." *Critical Review of International Social and Political Philosophy* 8 (1, March): 79-83.

Steiner, J., A. Bächtiger, M. Spörndli and M.R. Steenbergen. 2005. *Deliberative Politics in Action*: *Analyzing Parliamentary Discourse*. Cambridge: C.U.P.

Stimson, J.A. 1975. Belief Systems: Constraint, Complexity, and the 1972 Election. *American Journal of Political Science* 19 (3, August): 393-417.

Taagepera, R., and B. Grofman. 1981. “Effective Size and Number of Components.” *Sociological Methods and Research* 10 (1): 63-81.

Tsetlin, I., M. Regenwetter, and B. Grofman. 2003. “The Impartial Culture Maximizes the Probability of Majority Cycles.” *Social Choice and Welfare* 21 (3, December): 387-398.

Table 1: Relative Frequency of the Existence of a Condorcet Winner for Given Ranges of *S* (for *n* = 101)

Range of *S*	**Relative Frequency**
[0.67,0.68)	0.7610
[0.68,0.69)	0.8030
[0.69,0.7)	0.8535
[0.7,0.71)	0.9011
[0.71,0.72)	0.9362
[0.72,0.73)	0.9593
[0.73,0.74)	0.9738
[0.74,0.75)	0.9846
[0.75,0.76)	0.9921
[0.76,0.77)	09954
[0.77,0.78)	0.9974
[0.78,0.79)	0.9978
[0.79,0.80)	0.9995
[0.80,0.81)	0.9997
[0.81,1)	1

Note: *S*'s being in the range [*a*,*b*) means that $a \leq S < b$. Recall that 2/3 is the theoretical minimum of *S* when $k = 3$.

Table 2: Before-After Results: Largest Structuring Dimension, Condorcet Winner, and Proximity to Single-Peakedness

Issue	DP	*n*	*K*	D_1	D_2	C_1	C_2	S_1	S_2	$S_2 - S_1$
Electric Utility Policies	**SWEPCO**	232	4	[2314]	[2314]	3	1	0.405 (0.032)	0.556 (0.033)	0.151 (0.046)
	CPL	216	4	[2314]	[3124]	3	1	0.389 (0.033)	0.519 (0.034)	0.130 (0.047)
	WTU	230	4	[2314]	[2134]	3	3	0.374 (0.032)	0.496 (0.033)	0.122 (0.046)
Revenue Sharing	**New Haven**	132	4	[1234]	[1234]	3	2	0.515 (0.043)	0.803 (0.035)	0.288 (0.056)
Electric Utility Goals	**SWEPCO**	232	5	[21435]	[12345]	4	3	0.237 (0.028)	0.362 (0.032)	0.125 (0.042)
	CPL	216	4	[1423]	[3124]	4	2	0.444 (0.034)	0.579 (0.034)	0.135 (0.048)
	WTU	230	5	[12435]	[13425]	4	3	0.243 (0.028)	0.330 (0.031)	0.087 (0.042)
	Entergy	175	4	[3124]	[3214]	1	2	0.640 (0.036)	0.691 (0.035)	0.051 (0.050)
	HL&P	192	4	[3124]	[3124]	1	1	0.521 (0.036)	0.677 (0.034)	0.156 (0.049)
	SPS	222	4	[3214]	[1234]	2	2	0.559 (0.033)	0.649 (0.032)	0.090 (0.046)
Airport Expansion	**New Haven**	132	3	[213]	[213]	2	2	0.773 (0.036)	0.811 (0.034)	0.038 (0.050)
Australian Head of State	**Australian Referendum**	343	3	[213]	[123]	1	2	0.828 (0.020)	0.776 (0.023)	-0.052 (0.030)
Changing the British Monarchy	**British Monarchy**	258	3	[213]	[213]	1	1	0.651 (0.030)	0.647 (0.030)	-0.004 (0.042)

Note: *n* is the sample size; *k* the number of alternatives, numbered as in the text; D_1 and D_2 the largest structuring dimensions at T1 and T2; C_1 and C_2 the Condorcet winners at T1 and T2; and S_1 and S_2 the proximity to single-peakedness at T1 and T2. The parenthetical entries in the S_1 and S_2 columns are estimated standard errors.

Table 3: Before-After Substantive Agreement

Issue	DP	$n_1, \ldots, n_k$		A_1	A_2	$A_2 - A_1$
		T1	T2			
Electric Utility Policies	**SWEPCO**	25, 16, 103, 5	115, 29, 65, 15	.519	.369	-0.150
	CPL	16, 16, 99, 15	96, 61, 32, 17	.494	.336	-0.158
	WTU	11, 18, 112, 15	69, 37, 79, 40	.543	.276	-0.267
Revenue Sharing	**New Haven**	43, 14, 48, 15	23, 51, 44, 12	.318	.308	-0.010
Electric Utility Goals	**SWEPCO**	53, 31, 34, 83, 22	45, 37, 104, 37, 9	.247	.291	.044
	CPL	45, 29, 41, 94	63, 73, 19, 58	.306	.287	-0.019
	WTU	59, 24, 34, 93, 11	46, 36, 76, 60, 11	.286	.246	-0.040
	Entergy	99, 29, 18, 13	65, 88, 16, 4	.440	.409	-0.031
	HL&P	110, 30, 21, 13	112, 39, 33, 5	.450	.425	-0.025
	SPS	40, 107, 52, 16	31, 107, 63, 14	.346	.359	.013
Airport Expansion	**New Haven**	33, 82, 4	30, 83, 13	.553	.501	-0.052
Australian Head of State	**Australian Referendum**	170, 69, 91	67, 209, 51	.385	.475	.090
Changing the British Monarchy	**British Monarchy**	136, 41, 33	128, 46, 24	.482	.487	.005

Note: $n_1, \ldots, n_k$ (at T1 and T2) are the numbers of respondents with first preferences for alternatives 1, …, k, numbered as in the text; and A_1 and A_2 the index of substantive agreement at T1 and T2.

Table 4: Before-After Knowledge Measures

Issue	DP	# Items	K_1	K_2	$K_2 - K_1$	Threshold for "High K_2"	"Low K_2" n	"High K_2" n
Electric Utility Policies/Goals	**SWEPCO**	5	.435	.638	.203***	≥ 0.80	122	110
	CPL	5	.360	.580	.219***	≥ 0.80	144	72
	WTU	5	.399	.687	.288***	≥ 0.80	102	128
Revenue Sharing	**New Haven**	6	.362	.607	.245***	≥ 0.66	56	76
Airport Expansion	**New Haven**	6	.407	.588	.182***	≥ 0.66	61	71
Australian Head of State	**Australian Referendum**	9	.452	.651	.199***	≥ 0.83	179	164
Changing the British Monarchy	**British Monarchy**	9	.646	.794	.148***	≥ 0.88	122	136

Note: #Items is the number of knowledge questions in the knowledge index, K_1 and K_2 the knowledge index at T1 and T2, Threshold for "High K_2" the threshold for subdividing the sample into the high and low T2 knowledge subsamples, and "Low K_2" n and "High K_2" n the numbers of respondents in each subsample. *** indicates $p < 0.01$.

Table 5: Before-After Single-Peakedness Conditional on T2 Knowledge

		"Low K_2"		"High K_2"		$S_2 - S_1$	
Issue	**DP**	S_1	S_2	S_1	S_2	"Low K_2"	"High K_2"
Electric Utility Policies	**SWEPCO**	.393	.525	.418	.591	.132	.173
	CPL	.382	.469	.431	.597	.087	.166
	WTU	.382	.490	.367	.508	.108	.141
Revenue Sharing	**New Haven**	.518	.714	.513	.868	.196	.355
Electric Utility Goals	**SWEPCO**	.246	.361	.236	.364	.115	.128
	CPL	.410	.535	.514	.667	.125	.153
	WTU	.235	.304	.281	.391	.069	.110
Airport Expansion	**New Haven**	.787	.754	.789	.859	-0.033	.070
Australian Head of State	**Australian Referendum**	.860	.682	.793	.878	-0.178	.085
Changing the British Monarchy	**British Monarchy**	.582	.475	.713	.802	-0.107	.089

Note: "Low K_2" S_1 and S_2 are the proximity to single-peakedness at T1 and T2 for the low T2 knowledge subsample, "High K_2" S_1 and S_2 the proximity to single-peakedness at T1 and T2 for the high T2 knowledge subsample.

Table 6: The Largest Structuring Dimension in the High-Salience Cases Conditional on T2 Knowledge

		"Low K_2"		"High K_2"	
Issue	**DP**	D_1	D_2	D_1	D_2
Airport Expansion	**New Haven**	[1 2 3]	[2 1 3]	[2 1 3]	[2 1 3]
Australian Head of State	**Australian Referendum**	[2 1 3]	[1 2 3]	[2 1 3]	[1 2 3]
Changing the British Monarchy	**British Monarchy**	[2 1 3]	[2 1 3]	[2 1 3]	[2 1 3]

Note: "Low K_2" D_1 and D_2 are the largest structuring dimension at T1 and T2 for the low T2 knowledge subsample; "High K_2" D_1 and D_2 the largest structuring dimension at T1 and T2 for the high T2 knowledge subsample. The alternatives are numbered as in the text.

ONLINE APPENDIX: KNOWLEDGE INDICES

SWEPCO, CPL, SPS

The knowledge items ask (1) whether coal, wind, natural gas, fuel oil, nuclear power, or solar power produces the largest share of the service area's electricity, (2) whether residential, commercial, or industrial customers consume the largest share of the service area's electricity, (3) whether the service area's residential, commercial, or industrial customers are charged the highest rates, (4) whether nuclear-, coal-, or natural gas-powered electric facilities produce the most air emissions, and (5) what state agency sets electric rates. The correct answers, varying in some cases according to the service area, are (1) coal for SWEPCO and natural gas for CPL and WTU, (2) residential for WTU and commercial for SWEPCO and CPL, (3) residential for SWEPCO and WTU and commercial for CPL, (4) coal, and (5) the Public Utilities Commission.

New Haven Airport

The four general New Haven knowledge items ask (1) whether the population of the greater New Haven region is closest to 250,000, 350,000, 550,000, or 750,000, (2) whether the rate of job growth in the New Haven region during the 1990's was more, about the same, or less than the rest of the United States, (3) whether the population of the city of New Haven increased, stayed the same, or decreased during the 1990's, (4) whether sales taxes, property taxes, direct state subsidies, or direct federal subsidies are the major source of revenue for most town governments in the region. The airport-specific items ask (5) whether the FAA would describe Tweed New Haven Airport as a major hub, medium hub, minor hub, or a non-hub, and (6) whether maintaining the airport at its current level of service would require any significant investment. The correct answers are (1) 550,000, (2) less, (3) decreased, (4) property taxes, (5) non-hub, and (6) yes.

New Haven Revenue Sharing

The revenue sharing knowledge index contains the same four general New Haven items, plus two revenue sharing-specific items asking (5') whether Connecticut law allows communities to share property tax revenues and (6') whether those communities with the most valuable property tend to have the lowest property tax rates, average property tax rates, or the highest property tax rates. The correct answers are (5') yes and (6') lowest.

Australian Constitutional Referendum

The first four referendum-specific knowledge items ask (1) whether the role of the proposed president would most resemble that of the British Prime Minister, the Australian Prime Minister, the American President, or the Australian Governor General; (2) whether the Prime Minister could remove the president at any time without reporting to the House, at any time with subsequent House approval, after a fair trial in Parliament, or not at all; (3) whether the Queen currently chooses the Governor General, appoints him/her on the recommendation of Parliament, appoints him/her on the advice of the Prime Minister, or plays no role in selecting him/her; (4) whether the Governor General currently controls the government, acts on the Queen's Instructions, performs only ceremonial duties, or can decide to dismiss the government. The remaining four ask whether each of four things would definitely change if the referendum were to pass: (5) the Australian flag, (6) the national anthem, (7) the word "royal" in the names of the "Royal Australian Navy" and the "Royal Australian Air Force," and (8) Australia's participation in the Commonwealth games. The general political knowledge items ask (9) whether the Liberal party is more concerned, the Labour party is more concerned, or the two are equally concerned about social and welfare issues; (10) whether, on the whole, the Liberal party is closer, the Labour party is closer, or the two are equally close to business interests; (11) whether Aden Ridgeway was currently the leader of the Federal opposition, the

Leader of the Democrats, a Justice of the High Court, or an Aboriginal Senator in Parliament; (12) whether Jennie George was currently the Secretary of the Teacher's Federation, a Labour MP, the leader of a worker's union, or the President of the Australian Council of Trade Unions. The correct answers are (1) the Governor General, (2) at any time with subsequent House approval, (3) appoints him/her on the advice of the Prime Minister, (4) can decide to dismiss the government, (5) no, (6) no, (7) yes, (8) no, (9) Labour party more, (10) Liberal party closer, (11) Aboriginal Senator in Parliament, and (12) President of the Australian Council of Trade Unions.

British monarchy

The knowledge items ask (1) whether Princess Anne is next in line to the throne after Prince Charles; (2) whether the Prime Minister rather than the Queen is Britain's Head of State; (3) whether the Queen still heads the Commonwealth; (4) whether it is the Queen's duty to decide the date of all General Elections; (5) whether prime ministers cannot take office without being asked to by the Queen; (6) whether the Queen heads the Church of England; (7) whether the Queen heads the Church of Scotland; (8) whether Britain has an unwritten constitution; and (9) whether the brother or sister, eldest brother, eldest child, eldest son, or eldest daughter is next in line to the throne when a British monarch dies. All but (9) are in true-false format. The correct answers are (1) false, (2) false, (3) true, (4) false, (5) true, (6) true, (7) false, (8) true, and (9) the eldest son.

NOTES

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[1] By "rational," we mean complete and transitive, as defined formally below.

[2] Any procedure satisfying "universal domain," the "weak Pareto principle," "independence of irrelevant of alternatives," and "non-dictatorship" fails to guarantee rational collective preferences. Although "independence" is more controversial than the other conditions, its violation may open the door to agenda manipulation (Riker 1982) and strategic voting (Gibbard 1973, Satterthwaite 1975). Other generalizations of Condorcet's paradox include McKelvey (1979) and Schofield (1976).

[3] Other conditions include *value restriction* (Sen 1966) and some necessary and sufficient conditions (e.g., Miller 2000, Elsholtz and List 2005, Regenwetter et al. 2006). Since these other condi-

tions are less demanding than single-peakedness, the hypothesis that deliberation induces structure in individual preferences is logically stronger and more interesting when formulated in terms of single-peakedness, rather than some weaker condition. For general discussions of domain restrictions in social choice, see Gaertner (2001) and Dietrich and List (2010).

[4] More formally, single-peakedness lends majority voting a natural interpretation as a median voting scheme (Black 1948, Arrow 1951/1963).

[5] A decision procedure is *strategy-proof* if truthful expression of preferences is a dominant strategy for every individual. Moulin (1980) famously shows that, on domains of single-peaked preferences, median voting schemes like majority voting are strategy-proof. Penn, Patty and Gailmard (2011) have recently challenged this claim, though there is no contradiction between the classic result and their findings, since the opportunities for manipulation they identify involve departures from the single-peaked domain.

[6] For studies of legislative and jury deliberation, see Steiner et al. (2005) and Mendelberg (2002).

[7] The use of random samples of the mass public sets this study apart from most research on the occurrence of cycles, which typically examines narrower and more homogeneous groups like professional associations, clubs, or committees (for a critical overview, see Mackie 2004).

[8] This implies that R_i is also *reflexive* (for every alternative x, xR_ix).

[9] When the cycles are "top-cycles," we have an instance of the standard paradox, where there exists no Condorcet winner. This paradox implies the more general one, but not vice-versa when there are more than three alternatives. On the various forms of the paradox, see Gehrlein (1983).

[10] There are exceptions. Highly symmetrical distributions can be constructed so as to reproduce the conditions under which Condorcet's paradox occurs with a high probability (List and Goodin 2001,

appendix 3). Tsetlin, Regenwetter and Grofman's results specify the precise (large) class of possible distributions among which the impartial culture maximizes the probability of cycles.

[11] An impartial culture implies a low probability that a preference profile exhibits a high proximity to single-peakedness *S*, as defined below. Computer simulations, conducted partly with Susan Holmes, show that *S* tends to be near its theoretical minimum under an impartial culture (see also Niemi 1969). But the empirical values of *S* are often much higher. See Table 2 below.

[12] Alternative *y* is *between* *x* and *z* with respect to Ω if $\Omega(x) < \Omega(y) < \Omega(z)$ or $\Omega(z) < \Omega(y) < \Omega(x)$.

[13] Formally, a *left-right ordering* of the alternatives is a one-to-one function $\Omega : X \rightarrow \{1, 2, ..., k\}$ that assigns to each alternative *x* in *X* a number between 1 and *k*, where $\Omega(x) = 1$ means that alternative *x* is left-most, $\Omega(x) = 2$ means that *x* is second from left, and so on. An alternative *y* lies *between* alternatives *x* and *z* with respect to Ω if $\Omega(x) < \Omega(y) < \Omega(z)$ or $\Omega(z) < \Omega(y) < \Omega(x)$.

[14] Formally, R_i is *single-peaked with respect to* Ω if, for any triple *x*, *y*, *z* in *X* such that *y* is between *x* and *z* with respect to Ω, xR_iy implies xP_iz. The clause "xR_iy implies xP_iz" rules out the possibility that both xR_iy and zR_ix hold, which would imply a "cave" between *x* and *z*, at *y*.

[15] A structuring dimension is unique at most up to mirror image; e.g., a profile is single-peaked with respect to structuring dimension [*x y z*] if and only if it is single-peaked with respect to [*z y x*].

[16] On the distinction, see, e.g., Brams, Jones and Kilgour (2002). In spatial voting theory, preferences are a function of the alternatives' Euclidean (or other) distances from the individual's ideal point, which crucially requires identifying the alternatives with points in a Euclidean space, a needlessly restrictive stipulation in the present case. This spatial notion of single-peakedness is sufficient for acyclical majority preferences only if the underlying Euclidean space is one-dimensional, but that condition is more demanding than single-peakedness in the classical, ordinal sense.

[17] Recall that we are referring to the ordinal notion of single-peakedness, à la Black and Arrow.

[18] The *median individual* is the one with an equal number of individuals to the left and to the right, where the individuals are arrayed on the structuring dimension according to their most preferred alternatives. To avoid (harmless) ties, we assume that the number of individuals n is odd.

[19] Other conditions are discussed in an earlier note.

[20] The formula is stated without proof by Niemi (1969). A proof for k =3 is available on request.

[21] Taking an impartial culture as a Bayesian prior, p is the probability we would obtain through Bayesian updating given that $S > \alpha$. This probability coincides with the proportion of profiles for which there exists a Condorcet winner among all possible profiles satisfying $S > \alpha$.

[22] More extensive simulations, in collaboration with Susan Holmes, show similar results, if anything more dramatic, for $k > 3$, where cycles are more frequent.

[23] For a slightly longer but similar list, see Fishkin and Luskin (2005). Note that deliberation may commonly entail a good many other, non-definitional properties. For example, we should expect deliberation defined as it is here, to increase the frequency of appeals to the common good, for reasons sketched by Mill, Rawls, and others.

[24] See Luskin, Fishkin, and Jowell (2002).

[25] On the distinction between substantive and meta-agreement, see List 2002.

[26] They need not arrive at the same preference ordering, nor therefore agree on the most preferred alternative. This is meta-agreement, not substantive agreement.

[27] The questions asked the respondents for their most-preferred to $(k\text{-}1)^{st}$ most preferred alternatives, from which their k^{th} most preferred alternative could be inferred.

[28] It is the (fairly explicit) phrasing of the alternatives as matters of degree, rather than in merely categorical terms, that makes a geometric ordering (as a matter of degree) more obvious for the New Haven airport and for revenue sharing than for the other issues.

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[29] There need not generally be a Condorcet winner, but in all these cases there happens to be one.

[30] The desired inference, in each DP, would be to what we should see if the whole population hypothetically received the same treatment as the sample.

[31] As for any two random variables, $V(S_2 - S_1)$ is $V(S_2) + V(S_1) - 2C(S_2, S_1)$, where V and C represent variance and covariance. The potential change of largest structuring dimension may complicate the calculation or at least the interpretation of the usual estimate of the covariance, but we do know that it is almost certainly well above zero, making $V(S_2 - S_1)$ less than $V(S_2) + V(S_1)$. As we have argued and shown, $S_2 - S_1$ is a decreasing function of S_1 (because the latter is higher for already-salient issues), but a given difference in S_1 will not generally produce anything close to an equal (oppositely signed) difference in $S_2 - S_1$. Thus in the bivariate, linear relation $S_2 - S_1 = \alpha_0 + \alpha_1 S_1 + u$ (where u is a disturbance), we have $-1 < \alpha_1 < 0$, which implies that the slope of the corollary relation $S_2 = \alpha_0 + (\alpha_1 + 1)S_1 + u$ is positive ($0 < (\alpha_1 + 1) < 1$). This is certainly the pattern in Table 2, from which the OLS estimates of α_0, α_1, and $(\alpha_1 + 1)$ are 0.229,

-0.253, and 0.747, and the correlation between S_1 and S_2 is 0.890. Thus assuming a 0 covariance—taking $V(S_2 - S_1)$ to be just $V(S_2) + V(S_1)$—is extremely conservative.

[32] The case for which we have quasi-control groups (the Australian Constitutional Referendum DP) suggests that these observed changes can indeed be attributed to the deliberative treatment. In the treatment group, the Condorcet-winning alternative changes from a popularly elected president (1) to a parliamentarily appointed one (2), and the structuring dimension changes from [2 1 3] to [1 2 3], where 3 is the status quo. Among the original interviewees who did not participate in the deliberations but were later reinterviewed (n=227), the Condorcet-winning alternative remains 1, and the structuring dimension remains [2 1 3]. Similarly, among a fresh random sample questioned by the

Australian national election study in the period immediately after the referendum (n=3439), the Condorcet-winning alternative is 1, and the structuring dimension [2 1 3].

[33] Recall that the lower bound for S is 2/15≈0.133 for k=5, 1/3≈0.333 for k=4, and 2/3≈0.667 for k=3.

[34] The 0.263 figure counts the British monarchy issue, where S fell slightly short of S_{min}, as 0. If instead we count S' as -0.047, the 0.268 falls to 0.252, still the highest of these figures.

[35] The figure is -0.018 if we do not cut S' off at 0.

[36] Similarly, the number of first preferences for the alternative drawing the most first preferences (the maximum among n_1, n_2, …, n_k) increases in only six of the thirteen cases.

[37] We write of "knowledge," the aptest, in our view, of several related concepts, including "information," "sophistication," and "expertise" (see Luskin 2002).

[38] Three phenomena underlie this result: (1) learning (knowledge gain) is facilitated by, and thus an increasing function of, prior knowledge (e.g., Bransford and Johnson 1972, Recht and Leslie 1988, Hambrick 1993); (2) observed knowledge, defined as the proportion of the survey's knowledge items answered correctly, is ceilinged at 1.0; (3) the knowledge items on any survey are far from a random draw from the universe of potential such items, including many that only experts, if anyone, would know (Converse 2000, Luskin, Helfer, and Sood 2011). The selection is biased toward easy items, lest the observed knowledge index have almost no real variance, departing from zero only via lucky guessing. Under assumptions formalizing these phenomena, (1) the correlation between true and observed knowledge gain can be negative; (2) the correlation between true knowledge gain and observed T2 knowledge is always positive; (3) the former is less than the latter for most plausible configurations of parameters. For proofs, see Luskin, Helfer, and Sood (2011). In analyses relating learning to other variables, observed T2 knowledge can therefore yield sharper results, although in the present case we get essentially the same results if we use observed knowledge gain.

[39] This would be a bad idea at T1, as in ordinary surveys, where the distribution of knowledge is severely right-skewed, so that dichotomizing at or near the median would yield a "high knowledge" group containing many respondents not much more knowledgeable than the members of the "low knowledge" group. Here, after deliberation, it is reasonable.

[40] Even in this learning context, to be sure, T1 and T2 information are correlated: those high at T2 tend already to have been high—and thus already to have had preferences closer to being single-peaked—at T1. But the difference in proximity to single-peakedness between the high- and low-information subsamples widens from 0.025 at T1 to 0.129 at T2.

[41] On the airport, an extremely salient issue, there is no increase in S to apportion, but on revenue sharing, the timing of the increase differs tellingly between treatment groups. In each group, the increase occurs largely or entirely during the interval bracketing the group's discussion of revenue sharing (earlier in the one, later in the other). See Farrar et al. (2010).

[42] This is not to say that salient issues are a proper subset of important ones. It is not unknown for relatively frivolous issues to be salient.