Collective Economic Decisions and the Discursive Paradox*

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Abstract

Collective economic decisions – like most economic decisions – rely on judgments. We show that collective economic judgments may be subject to a 'discursive paradox' where the group's aggregate judgments are inconsistent. This happens despite each individual having consistent judgments. This result has important consequences for economic decisions, as the decisions will depend on whether the group uses a premise- or a conclusion-based decision-making procedure. Furthermore, it has implications for the design of decision-making institutions. The current literature, primarily within jurisprudence and philosophy, focuses on the aggregation of qualitative judgments on propositions. Most economic decisions, however, also involve quantitative judgments on economic variables. We develop a framework that is suitable for analyzing the relevance of the discursive paradox when the judgments are quantitative.

Keywords: Collective economic decisions, Judgement aggregation, Inconsistency

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1 Introduction

Many economic decisions are made by groups rather than individuals. Governments decide fiscal policies, monetary policy committees set interest rates, corporate boards make investment decisions, and families choose their mortgage. Like most other economic decisions, collective decisions are often based on imperfect information and must rely on judgments. For example, interest rate decisions rely on judgments about inflationary pressures and financial fragility, corporate investment decisions rely on judgments of future cash flows and cost of capital, and so on.

Aggregating individual judgments to a 'group judgment' is not straight forward. Recent research in philosophy, jurisprudence, and political science, shows that group judgments mey be subject to a 'discursive paradox', see e.g. List (2004) and Dietrich (2004).[1] The paradox can be illustrated by the following (fictitious) example: Suppose that George Bush, Colin Powell, and Donald Rumsfeld came together some day in 2002 to decide whether the US should invade Iraq. They agreed that the premises for an invasion are that the following two two propositions were judged to be true: (i) Iraq hides weapons of mass destruction, and (ii) the war can be won with 'acceptable' military losses. This logical link between the judgments on (i) and (ii) and the conclusion is denoted the *rule of inference.* Suppose the individual judgments were as in Table 1.

Table 1

	Weapons of mass destruction?	Acceptable losses?	Invasion?
Bush	Yes	No	No
Powell	No	Yes	No
Rumsfeld	Yes	Yes	Yes
Majority	Yes	Yes	No

As the bottom row shows, the group's aggregate conclusion (No) is inconsistent with its aggregate judgments on the propositions (Yes, Yes) and the rule of inference. Furthermore, it makes the group's decision depend, not only on the policymakers' judgments and aggregation method (majority, consensus etc.), but also on its decision procedure. If the policymakers vote on (i) and (ii) separately, and then let the rule of inference dictate the conclusion, there would be an invasion. If they instead voted directly on the conclusion, there would not be an invasion.

The focus of the existing literature on the discursive paradox is, as in the example above, on binary judgment aggregation.[2] The aggregation is binary

[1] It is also known as the 'discursive dilemma' or the 'doctrinal paradox'.

[2] Guilbaud (1966), Kornhauser and Sager (1986), and Kornhauser (1992) are early contributors to this literature. List and Pettit (2002) have recently provided an impossi-

because the premises and the conclusion are yes/no judgments on propositions. Such aggregation of yes/no judgments is relevant for many types of decisions in groups. However, most *economic* decisions (and many others) are not binary. Rather, the typical economic decision-making problem is to find the correct or optimal level of a continuous variable. Furthermore, the premises for the conclusion are typically judgments on continuous variables. Generally, the rule of inference for many economic decisions may be written as

$$c = f(p_1, p_2, ..., p_n),$$

where c is a continuous conclusion variable (e.g., the interest rate, the tax rate, the level of investments, etc), $p_1, p_2, ..., p_n$ are continuous premise variables, i.e., the information set on which the economic decision is based, and $f(\cdot)$ is some continuous function. As many decisions in economics are collective and based on judgments on the premise variables, it might also make a difference for this type of decisions whether the group aggregates judgments directly on the conclusion variable or indirectly through deciding on the premise variables. A very simple example illustrates that there can be a discursive paradox also in this case. Consider a group of three policymakers who decide on the size of a policy variable c, the 'conclusion variable'. They all agree that c should depend on the judgments on two premise variables, p_1 and p_2, and the 'rule of inference' $c = p_1 + p_2$. Suppose the individual judgments are as in Table 2, and that the aggregation method is majority voting, where the outcome of a vote on a variable is the median judgment on that variable.

Table 2

	p_1	p_2	$c = p_1 + p_2$	c
Individual 1	2	3	Agree	5
Individual 2	4	1	Agree	5
Individual 3	1	2	Agree	3
Majority	2	2	Agree	5

As the first three rows show, the individual conclusions are consistent with the judgments on the premise variables and the rule of inference. However, the aggregate judgments are clearly not, since $2+2 \neq 5$ (bottom row of Table 2). Furthermore, and as a consequence of this inconsistency, a direct vote on the conclusion gives $c = 5$, while separate votes on p_1 and p_2 give $c = 4$. Thus, a conclusion-based decision procedure, where the group votes on the conclusion directly, gives a different decision from a premise-based procedure. The example is intentionally simplistic, but captures the essence of the paradox.

bility theorem on the aggregation of judgments on interrelated propositions.

The binary decision framework studied in the existing liturature is not suitable for studying the relevance of the discursive paradox for judgments on continuous variables. The relevance of the discursive paradox for collective economic decisions, which are typically decisions on continuous variables, has not been investigated. In this paper, we develop a framework for analysing this and present conditions under which the discursive paradox can and cannot apply.[3]

Although the analysis of the discursive paradox within a continuous framework is new, it builds on the literature on the aggregation of judgments on interconnected propositions. Guilbaud (1966), Kornhauser and Sager (1986), and Kornhauser (1992) are early contributors to this literature. Recently, List and Pettit (2002) have provided an impossibility theorem on the aggregation of judgments on interrelated propositions. The theorem has been generalized by Pauly and van Hees (2003) (see also Dietrich (2003), Nehring and Puppe (2004), van Hees (2004) and Dietrich (2004)). The impossibility theorem states that there is no non-dictatorial aggregation method that generally produces consistent collective judgments on interconnected propositions and satisfies some minimal conditions. Since a finite set of judgments on continuous variables can be translated into judgments on a set of interconnected propositions, the theorem applies to contiuous variables. However, the impossibility theorem is too general to be useful for investigating whether a *specific* inconsistency, such as the discursive paradox, applies (see Section 4 for a further discussion of this). As we show in the paper, the existence of the discursive paradox hinges crucially on the functional form $f(\cdot)$ of the rule of inference. In the existing literature on the discursive paradox, the rule of inference is not a mathematical function, but a rule that states whether the conclusion should be 'Yes' or 'No' depending on the judgments of the validity of a set of propositions, cf. the Iraq example above. It is thus not possible on the basis of the binary decision-making framework to derive general conditions for the existence of the paradox in the type of judgment aggregation relevant for most economic decisions.[4]

It should be noted that there is a difference between judgment aggregation, as studied here, and the more traditional discipline of social choice, which was sparked off by Arrow's seminal work (Arrow (1951/1963)). Traditional social choice concerns the problem of aggregating individual preference orderings over several alternatives into an aggregate preference ordering

[3] In the existing literature the term 'discursive dilemma' or 'discursive paradox' labels situations where the aggregation method is majority voting and there is a difference between the outcome of a premise- and a conclusion-driven decision procedure. We use the term 'discursive paradox' to label situations where the group's decision depends on the decision procedure, regardless of the aggregation method.

[4] Although we focus on continuous variables, our framework can also be used for non-continuous variables as long as the rule of inference can be stated as function.

over these alternatives. Applied to our judgment aggregation setting, traditional social choice concerns the problem of aggregating individual orderings on alternative judgments on *one* variable into a corresponding aggregate ordering over the judgments on this variable (see e.g. Hylland and Zeckhauser (1979)). In table 2, for example, traditional social choice would concern the problem of aggregating the three individuals' orderings over the three judgments on one variable (p_1, p_2, or c) into a collective judgment on the *same* variable. In contrast, the type of judgment aggregation we study concerns the consistency between judgments on *different* variables, i.e. between judgments on the premise variables and the judgments on the conclusion (decision) variable.[5]

In Section 2, we introduce the analytical framework and present the general results. We present some applications of our results to specific economic decisions in Section 3, and provide a discussion of the assumptions and topics for further research in Section 4. Section 5 concludes.

2 Analytical framework

2.1 Model

Consider a group where N denotes the set of members and where $|N| = n$ is odd, finite and greater than 1. The group, which could be a government, a monetary policy committee, a corporate board, an expert panel, etc., has to make a conclusion on the size of a policy parameter $c \in \mathbb{R}$. The policy parameter could be the level/size of a tax or a tariff, the interest rate, the optimal size of a plant, etc.

The members of the group agree that their conclusion should depend on the judgments on k premise variables $p_1, p_2, ..., p_k$. Each member $i \in N$ has a separate judgment p_{ij} on each premise variable p_j where $j \in J$, $J = (1, 2, ..., k)$. The judgments on premise variable p_j, $j \in J$ can take any value in the interval $\left[p_j^-, p_j^+\right]$ where $p_j^- < p_j^+$, and $p_j^-, p_j^+ \in \mathbb{R}$. Thus, the set of possible judgments on all premise variables is a Cartesian product of possible judgments for each premise variable.[6] Formally,

Assumption 1 The *set of possible judgments on the premise variables* is $Q = \prod_{j \in J} \left[p_j^-, p_j^+\right]$ where $J = (1, 2, ..., k)$ and $p_j^- < p_j^+$ for all $j \in J$, $p_j^-, p_j^+ \in \mathbb{R}$.

[5] See List and Pettit (2004) for a discussion of the links and differences between the impossibility results on the aggregation of judgments on interconected propositions and the impossibility result on the aggregation of preferences.

[6] This means that the set of possible judgements on all premise variables is given by the interval $\left[p_j^-, p_j^+\right]$ when $k = 1$, a square defined by $\left[p_1^-, p_1^+\right] \times \left[p_2^-, p_2^+\right]$ when $k = 2$, a box defined by $\left[p_1^-, p_1^+\right] \times \left[p_2^-, p_2^+\right] \times \left[p_3^-, p_3^+\right]$ when $k = 3$, and so on.

Individual i's vector of judgments is denoted $\mathbf{p}_i = (p_{i1}, p_{i2}, \ldots, p_{ik})$, where $\mathbf{p}_i \in Q$ and $i \in N$. The sets of premise and conclusion judgments for the whole group are denoted $P = [p_{ij}]_{n\times k}$and $C = (c_1, c_2, \ldots, c_n)$, respectively. We think of P and C as the judgments that exist after the members of the group have shared the information they possess.

A 'rule of inference' establishes the logical link between judgments on the premise variables and the conclusion. The rule may, for example, be an explicit formula like $c = p_1 + p_2$ in Table 2, or the Taylor rule in monetary policy (c.f. Sect. 3). It can also be a more complicated economic model, or an approximation of essential facets of the group's thinking about how premises and the conclusion are logically linked.

Definition 1 *A rule of inference* $f_i(\mathbf{p})$ *is a continuous function that for each set of judgments* $\mathbf{p} = (p_1, p_2, ..., p_k) \in Q$ *and for each* $i \in N$ *specifies a conclusion* c*:*

$$c = f_i(\mathbf{p}) : Q \to \mathbb{R}$$

We abstract from judgment aggregation problems that arise because the individuals have different rules of inference. Hence,

Assumption 2 The individuals have the same rule of inference $c = f(\mathbf{p})$; i.e.,

$$f(\mathbf{p}) = f_i(\mathbf{p}) = f_m(\mathbf{p}), \forall i, m \in N$$

In line with most of the literature on the aggregation of judgment on interconnected propositions, we abstract from strategic behavior. The individuals are assumed to report their true judgments.

Assumption 3 *Sincere behavior*. All members of N always report their true judgments and reveal all relevant information they possess.[7]

Denote the vector of the group's aggregate judgments $\mathbf{p}^A = \left(p_1^A, p_2^A, ..., p_k^A\right)$ and the aggregate judgment on the conclusion c^A. Then, if the group aggregates the conclusion directly, for example by voting directly on the conclusion, the aggregate conclusion is c^A. Call such a decision procedure a conclusion-based decision-making procedure (CBP). If the group aggregates the judgments on the premise variables and uses the rule of inference to generate a conclusion, the aggregate conclusion (decision) is $f\left(\mathbf{p}^A\right)$. Call such a decision procedure a premise-based decision procedure (PBP). We say there is a 'discursive paradox' if the CBP gives a differnet decision (conclusion) frin the PBP. Hence,

[7] Assumption 3 and our interpretation of P and C as the set of judgments that exists after the members of the group have shared all relevant information, imply that for $\mathbf{p}_i \neq \mathbf{p}_j$ for some $i, j \in n$, there have to be (i) some imperfections in the information transmission within the group, or (ii) differences between the individuals that make them form different judgments for the same set of information.

Definition 2 *There is a discursive paradox if* $c^A \neq f\left(\mathbf{p}^A\right)$

Generally, groups may aggregate their judgments in many ways. The existing literature on the discursive paradox fucusses on voting. Recently, the literature on monetary policy committees has also considered 'averaging' as an aggregation procedure, where the group's aggregate judgment is the average of the individual judgments, see Munnich, Maksa and Mokken (1999), (Blinder and Morgan (2000), and Gerlach-Kristen (2003), and therefore we analyze this type of judgment aggregation in addition to majority voting. Note also that under certain assumptions, decisions based on consensus can be expressed as an average of the initial judgments, see DeGroot (1974), Chatterjee and Seneta (1977), and Berger (1981).

To model majority voting, we assume that the individuals' ordering on the judgments on each variable p_j $j \in J$, and the ordering on the conclusion c, are single-peaked. Denote the median judgments on premise j for p_j^m, and the median judgment on the conclusion c^m. With single-peaked orderings, p_j^m will beat any other judgment in a pair-wise vote over the judgments on p_j. Similarly c^m will beat any other alternative in a pair-wise vote over the judgments on c. Hence, if majority voting is used to aggregate judgments, the aggregate judgments are given by $p^m = (p_1^m, p_2^m, ..., p_k^m)$ and c^m.

Under averaging, the vector of aggregate judgments on the premises and conclusion is given by $\mathbf{p}^{avg}$ and c^{avg} where $\mathbf{p}^{avg} = (p_1^{avg}, p_2^{avg}, ..., p_k^{avg})$, and $p_j^{avg} = \sum_{i=1}^{n} \frac{1}{n} p_{ij}$ and $c^{avg} = \sum_{i=1}^{n} \frac{1}{n} c_i$.

2.2 Results

2.2.1 Majority voting

Definition 2 and the way we model voting decisions imply that when judgments are aggregated by majority voting there will be a discursive paradox if $c^m \neq f(\mathbf{p}^m)$. We start by looking at the simpler situation where $k = 1$.

2.2.2 $k = 1$

Let $\theta(p_i)$ be the numerical position of $p_i \in P$ when the elements of P are arranged in an increasing order. Similarly, let $\theta(c_i)$ be the numerical position of $c_i \in C$ when the elements of C are arranged in increasing order. A necessary condition for a paradox under majority voting is that $\theta(p_i) \neq \theta(c_i)$ for some q and $s \in N$ where $p_q \neq p_s$. The necessary condition can only be fulfilled if $f(p)$ is non-monotonic on Q. If not, $\theta(c_i)$ is determined entirely by $\theta(p_i)$. Generally, when $k = 1$, the numerical position of an element $c_i \in C$ depends on two factors: (i) the numerical position of $p_i \in P$, and (ii) the functional form of the rule of inference. Thus:

Proposition 1 *If* N *aggregates judgments by majority voting and* $k = 1$, *then* *(i)* $c^m = f(p^m)$ *for all* $P \subset Q$ *if* $f(p)$ *is monotonic for* $p \in Q$,

(ii) there exists a $P \subset Q$ *such that* $c^m \neq f(p^m)$ *if* $f(p)$ *is non-monotonic for* $p \in Q$.

Proof. As indicated before the proposition. ■

Proposition 1 states that if the group aggregates judgments by majority voting, a discursive paradox cannot be ruled out if the rule of inference is non-monotonic on its domain. It can only be ruled out if the rule of inference is monotonic in its domain.

A general proposition for when there will be a paradox does not exist, since the existence of a paradox depends both on the functional form of the rule of inference and the particular set of judgments. With assumptions 1–3 there always exist sets of judgments (P) where all elements are in the monotonic parts of a rule. Furthermore, even if the set of judgments covers also the non-monotonic part of the rule, there may still be a set of judgments that generates a linear relationship between the judgments and the conclusion.[8] However, for non-monotonic rules with only one local maximum or minimum we can reach a stronger conclusion. Let $p^{\max} \equiv \max P$ and $p^{\min} \equiv \min P$, and $p^* \equiv \arg\max f(p)$ if $f(p)$ is non-monotonic with one local maximum, and $p^* \equiv \arg\max -f(p)$ if $f(p)$ is non-monotonic with one local minimum. Call the set of judgments P *dispersed* if it has elements in both the increasing and decreasing parts of the rule of inference, i.e.

Definition 3 *The set of judgments* P *is dispersed if* $p^{\min} < p^*$, *and* $p^{\max} > p^*$.

We then have the following result.

Corollary 1 *If* N *aggregates judgments by majority voting,* $k = 1$, *and* $f(p)$ *has either one local maximum or one local minimum, then there will be a discursive paradox iff* P *is dispersed and*

(a) $f(p)$ *has one local maximum and* $f(p^{\max}) < f(p^m)$ *and* $f(p^{\min}) < f(p^m)$, *or*

(b) $f(p)$ *has one local minimum and* $f(p^{\max}) > f(p^m)$ *and* $f(p^{\min}) > f(p^m)$.

Proof. See appendix. ■

Corollary 1 provides sufficient conditions for a paradox. The corollary says that in order for there to be a paradox, there must be judgments on both the increasing and decreasing parts of the rule. Furthermore, if the rule of inference has one local maximum, the highest and lowest judgments on the premise variable must imply a lower conclusion than the one that follows from the median judgment. Similarly, if the rule of inference has one

[8] For example, $k = 1$, $n = 3$, $Q = \mathbb{R}$, $f(p) = \sin p$, and $P = (-\pi, 0, \pi)$

local minimum, the highest and lowest judgments on the premise variable must imply a higher conclusion than the one that follows from the median judgment.

We may also analyze the way that the decision is skewed, depending on which decision procedure is used.

Corollary 2 *If N aggregates judgments by majority voting, $k = 1$, and $f(p)$ has either one local maximum or one local minimum, then*

(a) $c^m \leq f(p^m)$ if $f(p)$ is non-monotonic with one local maximum, and

(b) $c^m > f(p^m)$ if $f(p)$ is non-monotonic with one local minimum.

Proof. See proof of Corollary 1 in Appendix. ■

Corollary 2 says that the CBP tends to yield a lower (higher) c than the PBP when $f(p)$ is concave (convex).

2.2.3 $k > 1$

If $k > 1$, the numerical position of an element $c_i \in C$ depends on the shape of the rule of inference and the numerical position of the judgments on *two* different premise variables. Thus, as the example in the introduction shows, with $k > 1$, a linear rule of inference does not rule out a paradox. Our second proposition generalizes this insight.

Proposition 2 *If N aggregates judgments by a simple majority rule, and $k > 1$, then there exists a $P \subset Q$ such that $c^m \neq f(\mathbf{p}^m)$.*

Proof. See appendix. ■

Proposition 2 states that if the group aggregates judgments by majority voting, and there is more than one premise variable, then a discursive paradox cannot be ruled out.

As in the case when $k = 1$, it is not possible to specify a general theorem for when there *will be* a paradox. Nor do there exist specific functional forms $f(\mathbf{p})$ for which there will never be a paradox.

2.2.4 Averaging

Definition 2 implies that when judgments are aggregated by averaging, there will be a discursive paradox if $c^{avg} \neq f(\mathbf{p}^{avg})$.

If the rule of inference is linear, there can never be a paradox since then $c^{avg} = f(\mathbf{p}^{avg})$ for any $\mathbf{p}$. If $f(\mathbf{p})$ is strictly concave or convex and $k = 1$ there must be a paradox if the individuals have different judgments on the premise variable (which also follows from Jensen's inequality). Thus, if $k = 1$ and the rule of inference is non-linear on Q, then there exist sets of judgments with a discursive paradox. Our third proposition generalizes this result.

Proposition 3 *If N aggregates judgments by averaging, then*

(i) $c^{avg} = f(\mathbf{p}^{avg})$ for all $P \subset Q$ if $f(p)$ is a linear function for all $\mathbf{p} \in Q$,

(ii) there exists a $P \subset Q$ such that $c^{avg} \neq f(\mathbf{p}^{avg})$ if $f(p)$ is a non-linear function for some $\mathbf{p} \in Q$.

Proof. See Appendix. ■

Proposition 3 states that if the group aggregates judgments by averaging, a discursive paradox cannot be ruled out if the rule of inference is non-linear on its domain. It can be ruled out if the rule of inference is a linear function.

If the rule of inference is *strictly* concave or convex we can make two corollaries. The first regards a situation that may very well prevail.

Corollary 3 *If N aggregates judgments by averaging, $k = 1$, $f(p)$ is strictly concave or convex for $p \in Q$, and $p_i \neq p_s$ where $i \neq s$ and $i, s \in N$, then $c^{avg} \neq f(p^{avg})$.*

Proof. Jensen's inequality ■

Corollary 3 says that with averaging there will always be a paradox if at least two individuals have different judgments on the same premise variable, and the rule is strictly concave or convex.

The second corollary regards how the decision will be skewed depending on the decision procedure.

Corollary 4 *If N aggregates judgments by averaging and $k = 1$, then $c^{avg} \leq f(p^{avg})$ when $f(p)$ is strictly concave in Q, and $c^{avg} \geq f(p^{avg})$ when $f(p)$ is strictly convex in Q.*

Proof. Jensen's inequality ■

The corollary says that the CBP tends to give a lower(higher) c than the PBP when the rule of inference is strictly concave(convex).

3 Applications

3.1 Linear policy rules

Monetary policy decisions are usually taken by a group, often called a monetary policy committee (MPC), and involve judgments on many variables.

It has become popular to specify interest rate decisions in terms of an 'interest rate rule', for example a Taylor rule (Taylor (1993)). Suppose that all the members of the MPC specify their interest rate proposals according to the following (classic) Taylor rule

$$i_t = r_t^* + \pi^* + a(\pi_t - \pi^*) + by_t, \tag{1}$$

where i_t is the nominal interest rate in period t, r_t^* is the neutral/natural real interest rate, which is assumed to vary over time, π^* is the desired rate of inflation (inflation target), π_t is actual inflation, and y_t is the output gap. In practice, the neutral real interest rate r_t^* and the output gap y_t cannot be observed and are subject to judgment. It is therefore reasonable to assume that the MPC members will, to some extent, disagree on the estimates of these variables. We assume that π_t can be observed perfectly, and with no loss of generality we consider a situation where inflation is equal to the target, i.e. $\pi_t = \pi^* = 2$. Moreover, we set $b = 0.5$ as in Taylor's (1993) classic specification. In order to keep the example as simple as possible, suppose the MCP consists of three members. Members each have their own estimates of r_t^* and y_t, represented in Table 3.

Table 3

	r_t^*	y_t	i_t
Member 1	2.1	2.6	5.4
Member 2	3.0	1.0	5.5
Member 3	2.2	1.2	4.8

In voting, the possibility of a paradox can never be excluded, c.f. Proposition 2. From Table 3 we see that $i_t^m = 5.4$, while $r_t^{*m} + 2 + 0.5y_t^m = 4.8$, and the discursive paradox therefore applies in the example. With averaging, there will never be a paradox when the rule is linear, c.f. 3. In the example in table 3 we have $i_t^{avg} = 5.2$ and $r_t^{*avg} + 2 + 0.5y_t^{avg} = 5.2$.

3.2 Non-linear rules

Linear rules are often approximations of non-linear rules, and one could argue that non-linear rules are more relevant for economic decisions. One type of premise variable that typically yields a non-linear rule of inference is the effects of the policy instrument. When deciding the appropriate level of the policy instrument, one has to take into account how the instrument affects the target variable(s). In many situations there will be disagreement about the exact effects of the policy instrument.

The difference between linear and non-linear rules of inference has its counterpart in the difference between additive and multiplicative uncertainty. Differences in individual judgments are caused by uncertainty, and it is natural to relate judgment aggregation problems to the literature on policymaking under uncertainty. We will therefore illustrate the discursive paradox within the framework of the classic model by Brainard (1967). Suppose that the relationship between the target variable y and the policy instrument z is given by

$$y = \alpha z + x, \tag{2}$$

where x represents exogenous variables that affect the policymakers' target variable. Equation (2) may represent a wide range of policy effects, for example, the monetary policy transmission mechanism, the effect of unemployment benefits on equilibrium unemployment, the effect of tariffs on the trade balance, the effect of fiscal expenditures on GDP, and so on. In many theoretical models, one can log-linearize the reduced form and yield an expression equivalent to (2).

We assume that α cannot be observed by the policymakers and is perceived as stochastic. Committee members each have their own estimate/judgment of α, denoted α_i, $i = 1, .., n$. For simplicity, we assume that each committee member perceives α to be equally uncertain, represented by the variance σ_α^2, which therefore has no subscript for committee member. The policymakers' objective is to set the policy instrument such that the target variable y is brought as close as possible to the target level y^*. The objective function is quadratic and given by

$$U = -(y - y^*)^2. \tag{3}$$

Due to uncertainty about α, the committee seeks to maximize $E(U)$ with respect to z. Member i's policy proposal is based on maximizing $E_i(U)$, where E_i is the expectations operator based on member i's estimate of α. Straight-forward optimization gives the following level for the policy instrument proposed by member i,

$$z_i = \frac{\alpha_i}{\alpha_i^2 + \sigma_\alpha^2}(y^* - x). \tag{4}$$

We will denote $\frac{\alpha_i}{\alpha_i^2+\sigma_\alpha^2}$ the 'policy response coefficient', as it says how strongly the policy instrument responds to the exogenous variables. Without loss of generality, we normalize $(y^* - x)$ to one, so that the rule of inference can be written as

$$f(\alpha) = \frac{\alpha}{\alpha^2 + \sigma_\alpha^2} \tag{5}$$

Figure 1 shows the shape of the policy response coefficient when $\alpha > 0$.[9] The rule is clearly non-monotonic.

[9]The figure for $\alpha < 0$ is the mirror image.

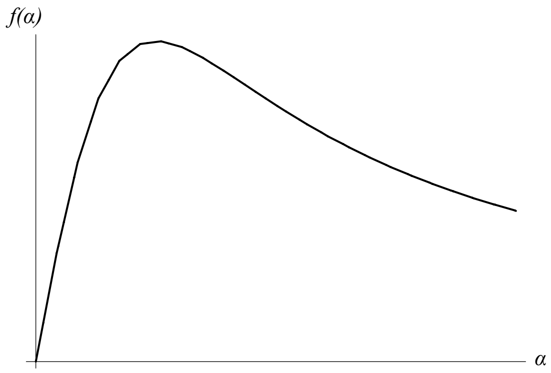

Figure 1

Consider first *voting*. We have from Proposition 1 that with one premise variable, one cannot exclude a paradox if the rule of inference is non-monotonic. Thus, a discursive paradox cannot be ruled out if α can take values on both the increasing and decreasing part of $f(\alpha)$. Whether there actually will be a paradox depends on the distribution of estimates/judgments. According to Corollary 1, a necessary and sufficient condition for the paradox when the rule of inference is given by (5) is

$$f(\alpha_m) > f(\alpha_{\min}) \text{ and } f(\alpha_m) > f(\alpha_{\max})$$

If the discursive paradox applies, a conclusion-based decision procedure will always give rise to a more cautious policy response than a premise-based procedure (corollary 2). An extreme case is when the distribution of estimates is such that $f'(\alpha_m) = 0$. In that case the premise-based decision procedure will give a policy response that is based on the most extreme value of the members' response coefficients.

Consider next *averaging*, and note that the rule of inference is single-peaked and globally non-linear. It thus satisfies the conditions in Corollary 3, so that decisions based on averaging will generally yield a discursive paradox. An important question is whether a premise-based procedure would result in a more or a less cautious policy (in addition to the cautiousness due to multiplicative uncertainty). From Corollary 4, we know that a premise-based procedure would give a weaker policy response if the rule of inference is strictly concave in α_i, while it will give a stronger policy response if it is strictly convex. We know that the rule of inference is strictly concave when $0 < \alpha_i < \sqrt{3}\sigma$ and strictly convex when $\alpha_i > \sqrt{3}\sigma$. The sign of the discursive paradox is therefore ambiguous. However, the higher the degree of uncertainty relative to the point estimate, the more likely it is that the conclusion-based procedure will yield a weaker policy response than a premise-based procedure.

4 Discussion

We have assumed that the premises are continuous variables (Assumption 1). This assumption is not necessary for Proposition 3 and Proposition 1, but is convenient, as it rules out particular combinations of Q and $f(\mathbf{p})$ for which there will never be a paradox.[10] The proof of Proposition 2 builds on Assumption 1, but it is sufficient for Proposition 2 that Assumption 1 holds for a subset of the set of possible judgements. Furthermore, it is easy to construct examples with a paradox even if Assumption 1 does not hold even for a subset.[11] As long as the premise variables are not perfectly correlated our results also hold true if the domain is more restricted than a Cartesian product (Assumption 1).[12]

Assumption 2 may seem very restrictive. However, it does *not* mean that the individuals have to agree on a specific policy rule (e.g., a Taylor rule). $f(\mathbf{p})$ represents what all members of the group can subscribe to. For example, consider the following 'policy rule': $c = \alpha x$, where c is the decision variable (e.g., the central bank's key interest rate), x is an economic variable (e.g., the rate of underlying inflation), and α is a parameter that says how much a change in x should affect c. If the individuals disagree on both x and α, the rule of inference has two premise variables; x and α. One may easily generalize this example to show that each individual may have a different policy rule for their decisions – even policy rules with different right-hand side variables and functional forms – but yet it will be possible to formulate a rule of inference that all agree on.

We have also assumed that the individuals report their true judgments (Assumption 3). Our results hinge on this assumption. Suppose, for example, that $k = 1$, $f(p)$ is non-monotonic with one local maximum and $n = 3$. Then, if the decision procedure is majority voting, the individual with the median conclusion judgment (c^m) can always report a false premise judgment so that the conclusion under a premise-based decision procedure will be c^m (c.f. figure 1). If the decision procedure is averaging, each member can report a judgement that affects the average judgement so that it coincides with her own true judgement. Consequently there is no pure strategy Nash equilibrium.[13] However, there are good reasons for making Assumption 3.

[10] Example: $p(\mathbf{x}) = ax_1 + b\sin x_2$, and $Q := \{x_1 \in \mathbb{R},\ x_2 = -2\pi, 0, 2\pi\}$

[11] Example: $p(\mathbf{x}) = x_1 + x_2$, $Q := \{x_1 \in 1,2,3; x_2 \in 1,2,3\}$.

[12] The shape of the domain Q will have a strong bearing on the likelihood of there being a paradox.

[13] List (2004) discusses strategic voting in the aggregation of judgments on interconnected propositions (the binary case), and notes that if all individuals act strategically under majority voting, a (formal) premise-based procedure will give a decision that is

The first is methodological. In order to analyze strategic behavior, one must first understand the equilibria without strategic behavior. Second, sincere behaviour is a reasonable assumption for expert panels and policy committees like (some) MPCs. Such groups are supposed to pool information and judgment, not to aggregate preferences. The members of such groups are supposed not to let their preferences over outcomes influence their behavior.

There already exist impossibility theorems for the aggregation of judgments on interconnected propositions (binary decisions). Since judgments on variables that can take many values can be mirrored in a set of judgments on interconnected propositions, our exercise may therefore seem superfluous.[14] However, the impossibility theorems for the aggregation of judgments on interconnected propositions do not necessarily imply a discursive paradox in the aggregation of judgments on variables. A simple example shows this.

Suppose $k = 1$, $N = \{A, B, C\}$ and $f(\mathbf{p}) = p$. Let $\mathbf{P} = (p_A, p_B, p_C)$, and the ordering on this set of judgments be

$A : p_A \succ p_B \succ p_C,$

$B : p_B \succ p_C \succ p_A,$

$C : p_C \succ p_B \succ p_A.$

It follows that $p^m = p_B$, and $c^m = c_B$. Thus, there is no discursive paradox as we have defined it (c.f. definition 2).

Now, let the propositions ρ_1, ρ_2 be defined as $\rho_1 : p_B \succ p_A$ and $\rho_2 : p_B \succ p_C$. Let ρ_3 be the proposition that $\rho_1$and ρ_2 are true: $(\rho_3 \leftrightarrow \rho_1 \wedge \rho_2)$. We can then summarize the individuals' judgments on these propositions as in the three first rows of Table 4.

Table 4

	ρ_1	ρ_2	ρ_3
Individual A	No	Yes	No
Individual B	Yes	Yes	Yes
Individual C	Yes	No	No
Majority	Yes	Yes	No

There is clearly an aggregation problem (bottom row). This is an aggregation problem of the type that the impossibility theorems of List and Pettit (2002) and others describe. Thus, there is no discursive paradox, as the decision procedure does not matter. However, there is a judgment aggregation problem as it is defined in the literature on the aggregation of judgments on interconnected propositions.

identical to that of a conclusion-based procedure.

[14] Any ordering $\succ$ on a set of mutually exclusive judgments, $\{p', p'', p'''\}$ can be expressed as a set of propositions of the type $\boldsymbol{\rho} = \{p' \succ p'',\ p'' \succ p''',\ p' \succ p'''\}$. Interlinkages between the propositions in $\boldsymbol{\rho}$ are determined by the rule of inference and more general consistency requirements like$(p' \succ p'' \wedge p'' \succ p''' \rightarrow p' \succ p''')$ etc.

Another important argument for our approach is that the translation of quantitative judgments into judgments on propositions hides the crucial element that determines the existence of a discursive paradox for economic decisions of continuous kind, namely the *functional form* of the rule of inference. When the number of individuals in the group increases and there are judgments on more than 2 variables, the number of propositions required to cover all pair-wise comparisons and logical interdependences becomes too large to handle. The analysis of collective judgments on variables that can take many values therefore requires a model that does not hide the function form of the rule of inference. Moreover, it is hard to see how the binary framework can be used to study averaging (consensus decisions).

5 Conclusion

In this paper we have developed a model to study an inconsistency that may arise when individual judgments on a set of continuous premise variables and a continuous conclusion variable are aggregated into group judgments on these variables. We have looked at two aggregation methods: majority voting and averaging. We have shown that in both cases the group's conclusion is prone to be inconsistent with its aggregate judgments on the premise variables. This inconsistency arises even though each individual reaches a conclusion consistent with his or her judgments on the premise variables. The aggregate inconsistency makes the decision depend on the group's *decision procedure*: a conclusion-based decision procedure, where the group aggregates the conclusion directly, gives another decision than a premise-based decision procedure, where the group first aggregates the judgments on the premise variables and then lets these aggregate judgments dictate the decision. We find that the possibility of an inconsistency depends on the combination of two factors: (i) the functional form of a 'rule of inference', which represents the logical link between the conclusion and the judgments on premise variables, and (ii) the set of possible judgments on the conclusion variable and the premise variables.

Although we are particularly interested in collective *economic* decisions, our findings are relevant for many other collective decisions. In medicine, for example, a team of doctors deciding how much of a drug to give a patient will face potential aggregation inconsistencies. In courts, juries deciding the duration of a prison sentences face similar potential aggregation inconsistencies, and their decision may depend on the decision procedure. Generally, the results apply to any collective decision that depends on the judgments on a set of premise variables.

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Appendix. Proofs

Proof of Corollary 1

Let $P^{<m} := \{p \in P \mid p < p^m\}$ and $P^{>m} := \{p \in P \mid p > p^m\}$

Part (a):

Step 1:

Suppose $f(p^{\max}) > f(p^m)$. Then $p^m < p^*$. Consequently $f(p) < f(p^m)$ for $p \in P^{<m}$ and $f(p) > f(p^m)$ for $p \in P^{>m}$ which implies that $c^m = f(p^m)$.

Suppose $f(p^{\min}) > f(p^m)$. Then $p^m > p^*$. Consequently $f(p) < f(p^m)$ for $p \in P^{>m}$ and $f(p) > f(p^m)$ for $p \in P^{<m}$ which implies that $c^m = f(p^m)$.

Suppose $f(p^{\max}) < f(p^m)$ and $f(p^{\min}) < f(p^m)$. Then $p^{\min} < p^m < p^{\max}$. Consequently $f(p) < f(p^m)$ for $p \in P^{<m}$ and at least one $p \in P^{>m}$, or $f(p) < f(p^m)$ for $p \in P^{>m}$ and at least one $p \in P^{<m}$ which imply that $c^m < f(p^m)$.

Step 2:

Suppose $f(p^{\max}) = f(p^m)$. If $f(p^{\max}) = f(p^m)$ because $p^m = p^{\max}$, then $c^m = f(p^m)$. If $f(p^{\max}) = f(p^m)$ and $p^m \neq p^{\max}$, then $f(p) < f(p^m)$ for $p \in P^{<m}$ and $f(p) \geq f(p^m)$ for $p \in P^{>m}$, and consequently $c^m = f(p^m)$. The proof for $f(p^{\min}) = f(p^m) \Longrightarrow c^m = f(p^m)$ is parallel.

Part (b): Parallels the proof of (a).

Proof of Proposition 2.

Assumption 1 implies that there generally exists at least one set of judgments $P^* \subset Q$ such that one premise can be expressed as a function of another premise, i.e. $p_j = g(p_s)$ where $g(p_s)$ is a continuous function.

Suppose $k = 2$ and $f(\mathbf{p}) = f(p_j, p_s)$. Then, if the set of judgments is P^*, we may express $f(\mathbf{p})$ as $f(g(p_s), p_s)$.

From Proposition 1 we know that a discursive paradox cannot be ruled out if $f(g(p_s), p_s)$ is non-monotonic for $p_s \subset Q$.

With Assumption 1 it is always possible to construct a $g(p_s)$ such that $f(g(p_s), p_s)$ is strictly concave or convex for some compact subset of Q. Thus, the discursive paradox cannot be ruled out if $k = 2$.

If $k > 2$ a possible P is one where all individuals judge all premises except p_j and p_s to be the same. Then the proof also holds true for the case when $k > 2$.

Proof of Proposition 3.

Part *(i)*: Property of linear functions.

Part *(ii)*: Let $P' := \{P \in Q \mid p_{ij} = p_{zj}$ for $i, z \in N$ and $j \in J \setminus \{s\}\}$. Then, since $f(\mathbf{p})$ is a non-linear function for $p_s \in Q$, there exists a set of judgments $P \in P'$ where $p_{is} \neq p_{zs}$ such that $c^{avg} \neq f(\mathbf{p}^{avg})$ (Jensen's inequality).