Can Vagueness Cut Out At Any Order?

Abstract

Could a sentence be, say, 3rd order vague, but 4th order precise? In Williamson [1999] we find an argument that seems to show that this is impossible: every sentence is either 1st order precise, 2nd order precise, or infinitely vague. The argument for this claim is unpersuasive, however, and this paper explains why.

Introduction

My target is Williamson's argument that every sentence is either 1st order precise, 2nd order precise, or infinitely vague [Williamson 1999]. Williamson's argument presupposes a framework for analysing higher-order vagueness, and I begin by sketching this framework (Section I). I then give a brief overview of Williamson's argument (section II), before turning to my criticism (section III).

Section I: The Vagueness Framework

The vagueness framework is analogous to the possible worlds framework. Rather than a set of possible worlds, however, a model on the vagueness framework contains a set of points each corresponding to a sharp interpretation of the language.[1] To say that a sentence α is true at some point is to say that α is true under that interpretation. The truth-value of a sentence may vary from point to point in a model – not because different points represent different precise states of affairs – but because different points can interpret the same sentence differently. Williamson writes: 'We can think of each model as corresponding to a fixed precise state of the world, including for instance the number of hairs on Jack's head; what vary from point to point in the model are the vague facts, for instance whether Jack is bald' [Williamson 1999: 130].

On the vagueness framework the 'definitely' operator (Δ) plays a role analogous to that played by the 'necessarily' operator on the possible worlds framework. On the possible worlds framework, 'necessarily α' is true at some possible world w_x iff α is true at every world accessible from w_x; roughly, a world w_y is accessible from a world w_x iff w_y would be possible were w_x actual. On Williamson's vagueness framework, 'definitely α' is true at some point *i* iff α is true at every point accessible from *i*. How should the accessibility relation on the vagueness framework be understood? Supervaluationists and epistemicists will answer this question differently: Williamson's vagueness framework is designed to be available to both theorists [Williamson 1999: 128]. In this paper I focus on how a follower of Williamson's own brand of epistemicism should understand the framework.[2]

On Williamson's epistemic view, there is one uniquely correct sharp interpretation of the language. The correct interpretation of any given vague term is fixed by our use, but in such a

[1] An interpretation of a language is 'sharp' iff under that interpretation, bivalence holds.
[2] The supervaluationist may be able to give a response to Williamson similar to the response I recommend in this paper to the epistemicist. The supervaluationist may be able to reject Williamson's argument – as (I show) the epistemicist can – provided that the supervaluationist can coherently claim that 'definitely' is a vague term on her account.

way that a slight, unnoticeable and easily possible shift in use would have resulted in a slightly different interpretation. Thus though there is a single correct sharp interpretation of the language, there is also a range of alternative sharp interpretations any one of which could easily have obtained without our noticing any difference. If I utter some sentence α, and α is false under some interpretation in this range, then (according to Williamson) my utterance is too unreliable to count as knowledge, for I could easily have uttered the very same words falsely [Williamson 1994: 230-1]. For α to be knowable, α must be true under every interpretation in the range. On the epistemic view, there is a close link between definiteness and knowability: α is definitely true iff α meets the requirement for knowability outlined above. Thus α is definitely true iff α is true under every interpretation that could easily have obtained without our noticing any difference. This gives us the range of interpretations accessible from the actual point – i.e. from the point corresponding to the actual interpretation of the language. More generally, the points accessible from some point *i* correspond to those interpretations that – from the standpoint of *i* – could easily have obtained without our noticing any difference[3].

Having outlined how the epistemicist should understand the accessibility relation, I turn now to Williamson's definition of precision in his framework. For the sort of models that we are interested in[4], α is 1^{st} order precise iff α is either true at every point in the model, or false at every point in the model [Williamson 1999: 131]. Williamson defines precision at higher-orders using classifications. A classification is a set of sentences closed under all truth-functors. $C_1\{\alpha\}$ is the smallest classification containing α: every sentence in $C_1\{\alpha\}$ is truth-

[3] Williamson introduces the concept of the 'standpoint' of a given point [1997: 262]. I take it that 'from the standpoint of *i*' means something like 'if the interpretation at *i* were the correct interpretation of the language'.
[4] By 'the sorts of models we are interested in', I mean 'generated' models. A generated model contains a point that can reach every other point through a series of accessibility relations. Williamson states that 'formally we lose nothing by restricting our attention to generated models' [Williamson 1999: 131].

functionally equivalent to either α, $\neg\alpha$, $\top$ or $\bot$. $C_2\{\alpha\}$ is the smallest classification containing a definitization of every member of $C_1\{\alpha\}$: every sentence in $C_2\{\alpha\}$ is truth-functionally equivalent to either $\Delta\alpha$, $\Delta\neg\alpha$, $\neg\Delta\alpha$, $\neg\Delta\neg\alpha$, $\top$ or $\bot$. α is 2^{nd} order precise iff every member of $C_2\{\alpha\}$ is precise. In general, α is *n*th order precise iff every member of $C_n\{\alpha\}$ is precise, where $C_n\{\alpha\} = C\{\Delta\alpha: \alpha \in C_{n-1}\{\alpha\}\}$ [Williamson 1999: 132].

Section II: Williamson's Argument

Williamson's argument depends on the following three rules [Williamson 1999: 129]:

Rule K: If $\models\Delta(\alpha \rightarrow \beta)$ then $\models (\Delta\alpha \rightarrow \Delta\beta)$

Rule RN: If $\models\alpha$ then $\models\Delta\alpha$

Rule B: $\models\alpha\rightarrow\Delta\neg\Delta\neg\alpha$

'$\models\alpha$' means that α is valid. For our purposes, we can assume that $\models\alpha$ holds if and only if α is true at every point in the relevant model.[5]

Rules K and RN hold automatically on Williamson's framework [Williamson 1999: 128]. Rule B follows from the assumption that the accessibility relation is symmetric. For consider that if α is true at point *i*, and the accessibility relation is symmetric, then every point that *i* can access will be able to access a point where α is true (i.e., point *i*). Thus $\neg\Delta\neg\alpha$ will be true at every point that *i* can access, and so $\Delta\neg\Delta\neg\alpha$ will be true at *i*. Thus $\alpha\rightarrow\Delta\neg\Delta\neg\alpha$ is true at every point.

[5] Each model corresponds to a fixed precise state of the world. A 'class' of models contains, for every possible precise state of the world, a model corresponding to that state. Williamson claims that $\models\alpha$ is true just in case α is true at every point in every model in the relevant class [1999: 129]. Following Williamson, however, we can restrict our attention to sentences that are necessarily true or necessarily false. For example, rather than considering sentences like, 'Bruce Willis is bald', we can focus on sentences like 'any possible person with 5,383 hairs is bald'. If such a sentence is true at every point in a given model, it will be true at every point in every model in the relevant class. Thus, for our purposes, $\models\alpha$ is true just in case α is true at every point in a given model.

In section III, I challenge the assumption that the accessibility relation is symmetric. Here I briefly consider why it would seem natural for the epistemicist to assume that the accessibility relation is symmetric.[6] Recall that for the epistemicist, point *i* can access point *j* iff from the standpoint of *i*, the interpretation at *j* could easily have obtained without our noticing any difference. It is natural to suppose that if from the standpoint of *i* the interpretation at *j* could easily have obtained, then from the standpoint of *j* the interpretation at *i* could easily have obtained: if a slight shift in reference is an easy possibility, then why wouldn't a slight shift back be an easy possibility too? And it is natural to suppose that if, from the standpoint of *i* the interpretation at *j* is not noticeably different, then from the standpoint of *j* the interpretation at *i* is not noticeably different: this is not the sort of case where we might expect epistemic accessibility to be asymmetric [Williamson 1999: 137-8]. Williamson does not attempt to *prove* that the accessibility relation is symmetric, but he understandably takes symmetry as the default position, and finds no convincing reason to abandon it. And from the natural assumption that the accessibility relation is symmetric, rule B follows.

In addition to rules K, RN and B, to construct Williamson's argument we also need the following claims which I label (X), (Y) and (Z). (X) is the claim that if α is *n*th order precise, then α is (n+1)th order precise [Williamson 1999: 134].[7] (Y) is the claim that if α is semantically equivalent to β, then α is vague iff β is vague [Williamson 1999: 131]. And (Z) is the claim that we cannot create vague sentences by combining precise sentences with truth-functors [Williamson 1999: 132]. I do not repeat the reasoning behind each of these claims here.

[6] Williamson suggests that this would also be a natural assumption for the supervaluationist [Williamson 1999: 130], but in this paper I focus only on how a follower of Williamson's brand of epistemicism should understand the framework.

[7] To see this, suppose then that sentence α is *n*th order precise, and so that every sentence in $C_n\{\alpha\}$ is precise. Take some sentence β from $C_n\{\alpha\}$. Given that β is precise, it follows that $\Delta\beta$ is precise. For (on the sort of models that we are interested in – see footnote 3), if β is precise then either β is true at every point (in which case $\Delta\beta$ is also true at every point), or β is false at every point (in which case $\Delta\beta$ is false at every point). $\Delta\beta$ is thus either true at every point or false at every point – and so precise. Thus for any sentence β in $C_n\{\alpha\}$, the definitization of that sentence (i.e. $\Delta\beta$) is precise. $C_{n+1}\{\alpha\}$ consists of definitizations of the sentences in $C_n\{\alpha\}$, combined with truth-functors. As vague sentences cannot be created by combining precise sentences with truth-functors (Rule Z above, Williamson 1999: 132), $C_{n+1}\{\alpha\}$ consists of precise sentences, and so α is n+1th order precise. In general, if a sentence is precise at the *n*th order, then it is precise at the n+1th order: a sentence cannot be lower-order precise and higher-order vague [Williamson 1999: 134].

Now for Williamson's argument. Williamson demonstrates that for any $n \geq 1$, if a sentence has $n+2$ order precision, then it will also have $n+1$ order precision. Williamson begins by arguing that $\Delta\alpha$ and $\Delta\neg\Delta\neg\Delta\alpha$ are semantically equivalent. This can be inferred as follows:

1	$\models \Delta\alpha \rightarrow \Delta\neg\Delta\neg\Delta\alpha$	(Special case of B)
2	$\models \neg\alpha \rightarrow \Delta\neg\Delta\alpha$	(Special case of B)
3	$\models \neg\Delta\neg\Delta\alpha \rightarrow \alpha$	(2 contraposed)
4	$\models \Delta(\neg\Delta\neg\Delta\alpha \rightarrow \alpha)$	(3, RN)
5	$\models \Delta\neg\Delta\neg\Delta\alpha \rightarrow \Delta\alpha$	(4, K)
6	$\models \Delta\alpha \leftrightarrow \Delta\neg\Delta\neg\Delta\alpha$	(1,5)

Now suppose that α is $n+2$ order precise. It follows from this that α is $n+3$ order precise – for by claim (X) α cannot be lower-order precise and higher-order vague. It follows that every member of $C_{n+3}\{\alpha\}$ is precise.

Let sentence β be a member of $C_n\{\alpha\}$.[8] The sentence $\Delta\neg\Delta\neg\Delta\beta$ will therefore be a member of $C_{n+3}\{\alpha\}$. As every member of $C_{n+3}\{\alpha\}$ is precise, it follows that $\Delta\neg\Delta\neg\Delta\beta$ is precise. However, $\Delta\neg\Delta\neg\Delta\beta$ is semantically equivalent to $\Delta\beta$, and therefore (by (Y)) if one of these expressions is precise, then they both are. It follows that $\Delta\beta$ is precise.

Let us generalise: β simply represents any sentence in $C_n\{\alpha\}$. Thus for any sentence (β) in $C_n\{\alpha\}$, we can infer that the definitization of that sentence (i.e. $\Delta\beta$) is precise. The classification $C_{n+1}\{\alpha\}$ consists of definitizations of the sentences in $C_n\{\alpha\}$, closed under all truth-functors. Thus the classification $C_{n+1}\{\alpha\}$ consists of precise sentences, closed under all truth-functors. As (by (Z)) we cannot create vague sentences by combining precise sentences with truth-functors, every member of $C_{n+1}\{\alpha\}$ is precise. Thus α has $n+1$ order precision.

We have argued from the claim that α has $n+2$ order precision to the claim that α has $n+1$ order precision, for $n \geq 1$. Therefore, if any sentence has 3rd, or higher-order precision, then it

[8] This is where the condition that $n \geq 1$ becomes relevant. If n is 0 or less, then there will be no classification $C_n\{\alpha\}$.

has 2nd order precision. Every sentence is thus either 1st order precise, 2nd order precise, or infinitely vague.

Section III: My Objection to Williamson's Argument

Williamson's argument depends on rule B. The main motivation Williamson gives for rule B is that B is guaranteed if the accessibility relation is symmetric [1999: 130, 137-8]. Williamson can find no good reason to doubt that the accessibility relation is symmetric, and concludes as follows:

> For the time being, the possibility in principle of second-order vagueness without *n*th-order vagueness for all *n* must remain highly problematic. For both epistemicists and supervaluationists, a more securely fine-grained understanding of accessibility than we presently possess is required to resolve the matter satisfactorily. The question is left open here. [Williamson 1999: 138]

In this paper I provide the epistemicist with a better understanding of the accessibility relation on Williamson's framework. This will give her good grounds to doubt that the accessibility relation is symmetric. In turn, this will undermine the motivation for rule B, and so will allow her to dismiss Williamson's argument as unpersuasive. As far as the epistemicist is concerned, then, the possibility of second-order vagueness without *n*th-order vagueness for all *n* may be unproblematic.

My reasoning consists of these four steps:

A. The term 'definitely' is itself vague.

B. The models that we are interested in include models where the interpretation of 'definitely' varies across points.

C. If two points interpret ‘definitely’ differently, then one may have a wider accessibility range than the other.

D. Thus on the models that we are interested in, some points may have wider accessibility ranges than others. This gives us good reason to doubt that the accessibility relation is symmetric, and so undermines the motivation for the B-schema.

I now develop each of these steps in more detail:

A. The term ‘definitely’ is itself vague

On Williamson’s epistemic account, whether a sentence is definitely true depends on whether it is true under a certain range of interpretations. To fall within this range, an interpretation must be such that it *could easily* have been the right interpretation: in other words, there must be a *sufficiently similar case* where it is the right interpretation. But this notion of *sufficient similarity* between pairs of cases is itself vague. It may be indefinite whether two cases are sufficiently similar to each other. Williamson seems to accept this point, as can be seen from the following quote:

> If one believes *p* truly in case α, one must avoid false belief in other cases sufficiently similar to α in order to count as reliable enough to know *p* in α. The vagueness in ‘sufficiently similar’ matches the vagueness in ‘reliable’ and in ‘knows’ [Williamson 2000: 100].

Thus on Williamson's epistemic account the term 'definitely' is itself vague. The sentence 'definitely α' is true iff α is true under every interpretation that stands in a certain relation (roughly, sufficient similarity) to the actual interpretation. Not only may we be unable to know which sharp interpretation of α is correct; we also do not know which sharp interpretation of 'sufficiently similar' is correct. Thus the sentence 'definitely α' is vague in a way that is not simply due to the vagueness of α.

Of course, the epistemicist would claim that all terms – even vague terms such as 'definitely' – are sharply bounded. Thus there is one uniquely correct sharp interpretation of 'definitely'. But there are also other slightly different interpretations of 'definitely' that could easily have obtained without our noticing any difference. We can suppose, for example, that without our noticing any difference 'definitely' could easily have meant definitely$_{+1}$, where it is slightly harder for something to be definitely$_{+1}$ true than it is for something to be definitely true. We can suppose that for a sentence to be definitely true, it must be true under every interpretation in a certain range R, but for a sentence to be definitely$_{+1}$ true, it must be true under every interpretation in a slightly wider range, R_{+1}. Similarly, we can suppose that 'definitely' could easily have meant definitely$_{-1}$ where it is slightly easier for something to be definitely$_{-1}$ true than it is for something to be definitely true: the sentence need only be true under every interpretation in range R_{-1}.

B. *The models that we are interested in include models where the interpretation of 'definitely' varies across points.*

A point corresponds to a sharp interpretation of the language. The 'definitely' operator is part of the object language [Williamson 1999: 128]. There is thus nothing incoherent in the idea of a model where the interpretation of 'definitely' varies across points. The question is: need we consider such models? Or can we restrict our attention to models where every point interprets 'definitely' in the same way? I claim that we cannot: the models that we are interested in include models where the interpretation of 'definitely' varies across points.

To see this, consider that we should be able to apply Williamson's framework to *all* vague sentences. And some vague sentences contain the term 'definitely'. Take for example the sentence 'definitely Jack is bald'. On the epistemic view, there is a single correct precise interpretation of this sentence, but there are also alternative interpretations of this sentence that could easily have obtained without our noticing any difference. A model of this sentence would need to contain points corresponding to each of these interpretations. There would need to be points on the model where the interpretation of 'bald' is different, and also points on the model where the interpretation of 'definitely' is different. To apply Williamson's vagueness framework to the sentence 'definitely Jack is bald', then, we need a model where the interpretation of 'definitely' varies across points.

Williamson's framework is supposed to apply to all vague sentences, and I have shown that to apply Williamson's framework to some vague sentences – those containing the term 'definitely' – we need a model where the interpretation of 'definitely' varies across points. But in fact we need such a model even for sentences that do not contain the term 'definitely'.

To see this, consider that on Williamson's account, higher-order vagueness in any given sentence is defined as lower-order vagueness in complex sentences containing the term 'definitely'. For example, the sentence 'Jack is bald' is 2nd order precise only if the sentence 'definitely Jack is bald' is 1st order precise. And whether 'definitely Jack is bald' is 1st order precise depends on whether it is true under a variety of alternative interpretations – including both alternative interpretations of 'bald' and alternative interpretations of 'definitely'. A model of the vagueness in the sentence 'Jack is bald' is automatically also a model of the vagueness in the sentence 'definitely Jack is bald' – and so will need to contain points that interpret 'definitely' in a variety of ways.

The models that we are interested in, then, include models where the interpretation of 'definitely' varies across points.

C. If two points interpret 'definitely' differently, one may have a wider accessibility range than the other.

Let us define the 'distance' between two points *i* and *j*, as the distance between the closest pair of possible worlds at which the facts about language use are such as to make the interpretations associated with *i* and *j* correct.[9] Intuitively, the distance between two points is a measure of the (dis)similarity of their interpretations. Each point in a given model has an accessibility range – i.e. a range of points that it can access. We can define the 'width' of a point's accessibility range as the distance between itself and the most distant point that it can

[9] I would like to thank an anonymous reviewer for suggesting this way of defining the distance between two points.

access. In this section I claim that the width of a point's accessibility range will depend on the interpretation of 'definitely' at the point in question.

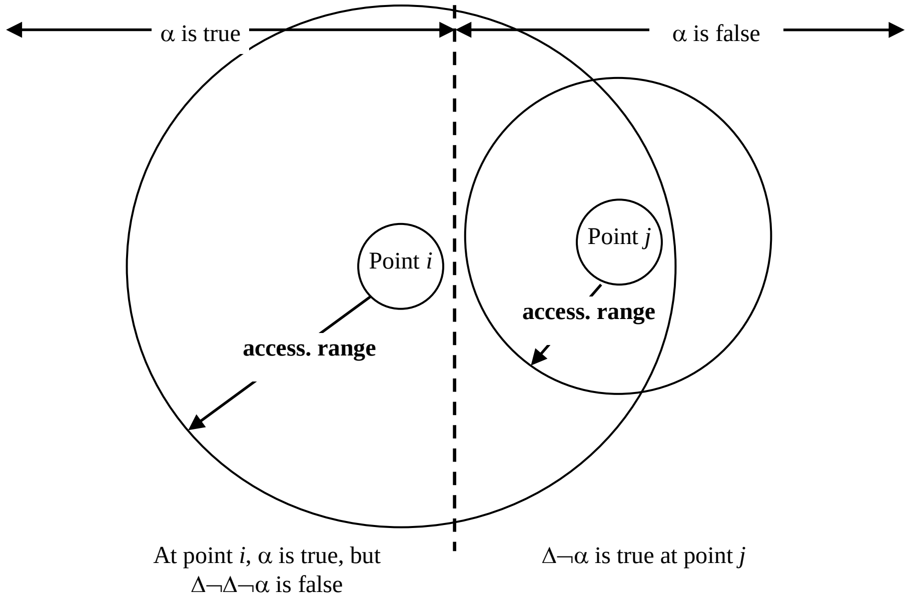
To see this, recall that on Williamson's framework, a sentence is true at a given point iff the sentence is true under the interpretation at that point. So whether 'definitely α' is true at a given point depends (amongst other things) on the interpretation of 'definitely' at that point. Now suppose that at some point *i*, 'definitely' is interpreted in such a way that it is hard for a sentence α to be 'definitely' true: 'definitely α' is true at point *i* iff α is true under a wide range of interpretations – including some interpretations that are quite dissimilar to the interpretation at *i*. Point *i* must therefore be able to access some quite distant points, and so will have a wide accessibility range. Now consider point *j*, where 'definitely' is interpreted in such a way that it is quite easy for a sentence to be 'definitely' true: 'definitely α' is true at point *j* iff α is true under a narrow range of interpretations, each of which is very similar to the interpretation at *j*. Point *j* will therefore have a narrower accessibility range than point *i*. In general, if two points interpret 'definitely' differently, one may have a wider accessibility range than the other.

D. *Thus on the models that we are interested in, some points may have wider accessibility ranges than others. This gives us good reason to doubt that the accessibility relation is symmetric.*

The models that we are interested in include models where the interpretation of 'definitely' varies across points. If two points interpret 'definitely' differently, one may have a wider accessibility range than the other. If one point has a wider accessibility range than another, then it may be that the point with the wider accessibility range can access the point with the narrower accessibility range, but not vice-versa. We should thus not expect the accessibility relation to be symmetric.

Here is an analogy. Consider the two place relation 'lives near'. We can assume that this relation is symmetric: if A lives near B, then B lives near A. Now consider the two place relation 'would agree that (s)he lives near'. This relation may be nonsymmetric, because not everyone has the same idea of what 'living near' involves. Perhaps A would agree that she 'lives near' anyone within a 10 mile radius, whereas B would only agree that she 'lives near' people within a 1 mile radius. Then if A and B live, say, 5 miles apart, A would agree that she lives near B, but B would not agree that she lives near A. My claim is that the accessibility relation is analogous to the relation 'would agree that (s)he lives near'. Each point determines its own accessibility range, based on its interpretation of 'definitely'. Whether i can access j depends on whether j falls within i's accessibility range; whether j can access i depends on whether i falls within j's accessibility range. The root of the nonsymmetry is that i and j might interpret 'definitely' differently, and so one may have a wider accessibility range than the other.

Once we see that the accessibility relation need not be symmetric, the motivation for accepting rule B is undermined. We can see how rule B might fail using the following example. Suppose that point j has a fairly narrow accessibility range and cannot access point i, but point i has a wider accessibility range and can access point j. Suppose also that at point j, α is false – and in fact point j can only access points where α is false: thus $\Delta\neg\alpha$ is true at point *j*. Suppose however that at point *i*, α is true. Given that *i* can access *j*, Δ¬Δ¬α does not hold at point *i*, even though α holds there. Thus rule B ($\models \alpha \rightarrow \Delta\neg\Delta\neg\alpha$) fails.



Conclusion

We have looked at how a follower of Williamson's brand of epistemicism should understand Williamson's vagueness framework. For such a theorist, the term 'definitely' is vague, and its interpretation varies across points on at least some of the models that we are interested in. If two points interpret 'definitely' differently, one may have a wider accessibility range than the other. Thus on the models that we are interested in, some points may have wider accessibility ranges than others. We should therefore not expect the accessibility range to be symmetric, and so there is no reason to accept rule B. Without rule B, Williamson's argument does not go through. Thus a follower of Williamson's own brand of epistemicism can reject the conclusion that every sentence is either 1st order precise, 2nd order precise or infinitely vague. For such a theorist, the possibility of 2nd order vagueness without *n*th order vagueness for all *n* may be unproblematic.

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