Outsized Arbitrage

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ABSTRACT

The paper studies incentives and trading decisions of an arbitrageur who can take concentrated bets in an illiquid market and who cares about interim as well as long-term performance. By scaling up his position and using price impact, the arbitrageur can prop up the value of his position, helping him weather periods of low valuation and successfully complete the arbitrage. But that approach also can trap him into building an outsized arbitrage position, which can cause persistent mispricing in the market, even in the presence of other arbitrageurs, and lead to large losses to investors.

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A crucial characteristic of any money manager is the ability to identify and exploit arbitrage opportunities. Famous trades, such as Soros' British pound short bet or Paulson's bet against the U.S. subprime mortgage market, instantly become legendary and propel the arbitrageurs behind them into a Hall of Fame. But finding such bets is not easy. According to the acclaimed hedge fund manager Stanley Druckenmiller, "only maybe one or two times a year do you see something that really, really excites you." And when you see it, you should "bet the ranch on it."[1]

At first glance, the idea that active money managers should double down on exceptional bets might appear convincing. Given the widespread availability of mutual funds and exchange-traded funds today, investors can easily form diversified portfolios themselves. Hence, they should only be willing to pay for alpha—i.e., excess returns—that are otherwise not available to them. However, this view neglects the agency problems that shape investor-manager relationships in money management.

By taking a concentrated bet, a manager makes his future career depend on the performance of the trade. But few arbitrage trades are truly riskless. Often, there is a risk that, in the short term, fundamentals can move against the manager. If investors are uncertain about a manager's skills, they may negatively update their beliefs and withdraw their funds (see Shleifer and Vishny (1997)). Thus, in practice the manager is under pressure to show good performance not just when he completes his arbitrage but also in the interim period. But what can the manager do to show good performance when fundamentals move against him?

In a perfectly liquid market, the answer would be "nothing." However, in markets that are not perfectly competitive and liquid, and therefore in which arbitrage opportunities are more likely to exist, the manager may be tempted to exploit the price impact of his trades to inflate the marked-to-market value of his arbitrage position.[2] Inflating the marked-to-market value of his arbitrage position can help the manager weather periods of low valuation and, therefore, successfully complete the arbitrage. But it can also trap him in outsized arbitrage: Every time the arbitrageur scales up his position in an attempt to support its marked-to-market value, he gets more exposed to the value of this position; which, in turn, creates even stronger incentives to support it. Eventually, the position can become so large that the manager may find it optimal to prop up the asset value even above its fundamental level. This can cause persistent mispricing in the market and lead to large losses to investors.

This paper describes a new framework I developed to study incentives and trading decisions of an arbitrageur who can make concentrated bets in an illiquid market. The

[1] Presentation at the Lone Tree Club, North Palm Beach, Florida, January 18, 2015, http://covestreetcapital.com/wp-content/uploads/2015/03/Druckenmiller-_Speech.pdf.

[2] Price impact is co-movement of prices in the direction of trades.

basic setup considers a single fund manager who learns about an arbitrage opportunity and can trade on it. The arbitrage closes at a random time, at which point the market becomes perfectly liquid, the manager liquidates his arbitrage position, and the game ends. Prior to this time, the manager's trades result in price impact. The fund's assets are marked to market and are subject to random valuation shocks. The manager cares about both the final profit from the arbitrage trades and the interim valuation of the fund's assets. Price impact limits the arbitrage profits but also allows the manager to mitigate the effect of negative shocks. By trading in the direction of the existing position, the manager increases marked-to-market value. The larger the position, the higher the benefit of propping up the asset's value. For sufficiently large positions, the manager finds it optimal to acquire assets even above their fundamental value: Trading at a loss is compensated by the increased valuation of the overall position.

My model shows that the position tends to grow over time. Therefore, the longer the arbitrage stays open, the higher the chance that the manager accumulates an outsized arbitrage position and trades at a loss, trying to support it. As a result, fund returns are negatively skewed, and investors can realize large losses. Furthermore, by trading at a loss and propping up the value of his position, the manager increases the wedge between the market price and the asset's fundamental value, making the market less and less efficient.

One might hypothesize that it would be impossible for the manager to push the price above the fair value if other arbitrageurs were present in the market and ready to correct the mispricing. Surprisingly, I show that the mispricing can persist, and even become stronger, in the presence of other arbitrageurs. To demonstrate this, I consider an extension of the main setup wherein I allow other arbitrageurs to trade in the market. Unlike the manager, who is a "whale" and can establish a very large position in the risky asset, I assume that other arbitrageurs are restricted in their risk-taking capacity.

I first assume that arbitrageurs do not have to worry about their interim performance. I study two cases: stealth and open trading. The two cases exhibit very different equilibrium dynamics.

In the stealth trading case, arbitrageurs secretly trade against the manager. In this case, arbitrageurs not only fail to eliminate the mispricing, but their trading leads to even larger mispricing. This happens because the manager, oblivious of the presence of arbitrageurs, keeps defending the value of his position until other arbitrageurs exhaust their risk-taking capacity.

In the open trading case, the manager and arbitrageurs are aware of each other and take each other's trading strategy as a given. In this case, I focus on a scenario of one arbitrageur and the manager. In the closed-loop equilibrium, faced with the prospect of trading against the arbitrageur, the manager revises his strategy and scales down his risky position. Interestingly, the arbitrageur front-runs the manager, quickly establishing an oversized short position and then gradually reducing it. The eventual outcome is the reduction in mispricing.

Which scenario is more likely? I show that the arbitrageur's profit is higher if she trades secretly from the manager. By revealing her presence, the arbitrageur makes the manager quickly scale down his risky position. As a result, the arbitrage spread gets reduced, which has a negative effect on the arbitrageur's profit. Thus, the arbitrageur, who only cares about the final profit, would always enter the market and never want to reveal herself.

The above conclusion, however, crucially depends on the ability of the arbitrageur to maintain her arbitrage position over the course of the arbitrage. The marked-to-market profit of the arbitrageur if she trades secretly from the manager stays negative over the course of the arbitrage, except for a short initial period. Thus, the arbitrageur realizes her profits only when the arbitrage is closed. If the arbitrageur draws her capital from outside investors who are unwilling to tolerate losses, then trading secretly against the manager is not a viable strategy.

In contrast to stealth trading, the marked-to-market profit stays positive for all periods in the open trading case. It is tempting to conclude that the arbitrageur who has to worry about her interim performance will have incentives to reveal her presence before entering the market. This view, however, overlooks the fact that the incentives to show good interim performance weaken the market power of the arbitrageur.

Whenever the arbitrageur enters the market and starts trading, the manager is worse off, compared to when he is alone in the market. Therefore, the manager is better off if he can commit to trading strategies that deter the arbitrageur's entrance. If the arbitrageur does not need to be concerned about her interim performance, there is little the manager can do to prevent her from entering, since every trade against a nonzero arbitrage spread eventually results in a profit for the arbitrageur. But if the arbitrageur cannot tolerate interim losses and learns about the arbitrage after the manager has established an outsized position, she can succeed only if she can "out-trade" the manager, which may not be possible if she is restricted in her risk-taking capacity. Thus, if arbitrageurs have to worry about their interim performance, the prospect of trading against the whale can deter them from entering the market, and the mispricing can persist for a long time.

The model's predictions fit well with the narrative of dramatic trades by Bruno

Iksil, dubbed the "London Whale" for the enormous size of his position.[3] As an arbitrageur at JPMorgan Chase, Iksil built a risky position in credit derivatives with a notional of more than $150 billion between January 2011 and March 2012. This outsized position had a substantial impact on the prices of underlying securities. A number of arbitrageurs tried to correct the apparent mispricing but were unable to do so until some frustrated industry insiders leaked the news about Mr. Iksil's large positions to the media.[4] Confronted by the media, JPMorgan Chase had to wind down the position, which eventually resulted in a $6.2 billion loss.

While credit derivatives may be viewed as specialized markets, a recent giant bet by SoftBank was focused on some of the most liquid stocks in the world: Amazon, Microsoft, Netflix, and Tesla. Many observers believe that SoftBank's bet contributed to the vertigo-inducing rise of technology stocks, leading to valuations that seem to be far removed from fundamentals.[5]

Together, the "London Whale" and the SoftBank cases highlight the potential danger of outsized arbitrages in a world in which large amounts of capital are concentrated in the hands of a few lightly regulated hedge funds and other investment vehicles. While it is too early to assess the impact of the SoftBank case, the "London Whale" case received a lot of public attention and scrutiny, not the least because it occurred inside a major public financial institution subject to strict regulation and controls. The case has had a long-lasting negative impact on the public perception of the entire financial sector—a cost that arguably extends far beyond the realized trading losses.

In addition to anecdotal evidence, the model predictions also are consistent with the results of Carhart, Kaniel, Musto, and Reed (2002) and Ben-David, Franzoni, Landier, and Moussawi (2013), who show that some mutual funds and hedge funds systematically attempt to inflate stock prices by purchasing stocks already held in the last minutes of trading on their reporting dates, and that price inflation increases with stock illiquidity.

Finally, the model is able to generate negative skewness in fund returns, one of the most salient features of hedge fund performance. Getmansky, Lo, and Makarov (2004) show that hedge funds that invest in illiquid securities have negative skewness, which is opposite to funds that invest in liquid instruments. In the model, skewness

[3] Report by the Permanent Subcommittee on Investigations Majority and Minority Staff entitled "JPMorgan Chase Whale Trades: A Case History of Derivatives Risks and Abuses," March 15, 2013, http://online.wsj.com/public/resources/documents/JPMWhalePSI.pdf.

[4] "London Whale Rattles Debt Market," Wall Street Journal, G. Zuckerman and K. Burne (4/6/2012), http://online.wsj.com/article/SB10001424052702303299604577326031119412436.html.

[5] "SoftBank's Bet on Tech Giants Fueled Powerful Market Rally," Wall Street Journal, S. Said, L. Hoffman, G. Benerji, and P. Dvorak (4/9/2020), https://www.wsj.com/articles/softbanks-bet-on-tech-giants-fueled-powerful-market-rally-11599232205.

increases with the expected horizon of arbitrage. The long horizon increases the chance of adverse movements in the value of the arbitrage position in the interim period, and therefore increases the probability that the manager can accumulate an outsized arbitrage position.

This paper is related to several lines of research in the literature. Price impact and practice of marking-to-market positions play an important role in Brunnermeier and Pedersen (2005) and Attari, Mello, and Ruckes (2005) who study strategic interaction between a financially constrained institution and its competitors. They show that the practice of marking-to-market positions gives competitors incentives to engage in predatory trading, whereby in selling assets held by an institution, competitors can force the institution to liquidate its assets below their fundamental values. In both of those papers, however, analysis starts when the institution has already established a large position. In contrast, in this paper, I show how agency problems can lead to building an excessively large position, and how the threat of aggressively defending this position can deter the entrance of other arbitrageurs.

A motive of booking profits and concealing losses because of balance sheet constraints is also present in Milbrandt (2012). He shows that an institution may suspend trading in illiquid Level 3 assets if the trade results in a low price at which the existing assets will have to be marked to market. The incentives to manipulate a fund's performance are also studied in Acharya, Pagano, and Volpin (2016); DeMarzo, Livdan, and Tchistyi (2013); Dasgupta, Prat, and Verardo (2010); Makarov and Plantin (2015); and Moreira (2019).

This paper also aligns with a common theme in the literature on price manipulation (see Allen and Gale (1992), Chakraborty and Yilmaz (2004), Van Bommel (2003), Kyle and Viswanathan (2008), and Spatt (2014)). As in the literature cited, the manager uses price impact of his trades to manipulate prices. However, in a departure from the literature, the reason the manager finds it profitable to manipulate prices is a function of agency problems between him and his investors, and not the possibility of making trading profit at the expense of other arbitrageurs.

Finally, on a general level, this paper is related to literature on the limits of arbitrage. As with this literature, I assume that investors have imperfect knowledge of arbitrage trades, and I show that asset prices may not be equal to their fundamental values in the presence of arbitrageurs. Most of the literature on the limits of arbitrage assumes that arbitragers have limited capital and is focused on the role of capital constraints—see Shleifer and Vishny (1997) and Gromb and Vayanos (2002). In contrast, in my model, the main reason why arbitrage is limited and investors can suffer large losses is because the manager has access to unlimited capital.

The rest of the paper is organized as follows: First, I describe the basic setup, with a single arbitrageur; Section 2 provides analysis of the basic setup; Section 3 considers an extension of the setup and allows for multiple arbitrageurs; and Section 4 provides my conclusions.

1. Basic Setup

Consider a risk-neutral money manager who can trade in two assets: a riskless asset and a risky asset available for trading at dates $t = 0, \ldots, \tau$. The trading dates are evenly spaced over time. Denote the length of the period between two consecutive dates by h.

The riskless asset is in perfectly elastic supply with the rate of return r being a nonnegative constant. For simplicity, I assume $r = 0$. Shares of the risky asset are infinitely divisible. Each share of the risky asset pays a liquidation value of v at the random final date $\tau \geq 1$, which is geometrically distributed with the parameter δh, $\delta > 0$.

Initially, the manager has a zero position in the risky asset, so all the fund's assets under management, W_0, are invested in the riskless asset. At time zero, the manager learns the liquidation value and starts trading in the risky asset. Denote the realization of the liquidation value v by V and the price of the risky asset at time t by P_t. Trading in the risky asset prior to realization of the liquidation value generates price impact. If the manager submits a trading order θ_t in period $t < \tau$, then it is executed at the price

$$P_{t+1} = P_t + \lambda(\theta_t + \varepsilon_{t+1}), \tag{1}$$

where ε_{t+1} represents a random fluctuation in the price, which is outside the manager's control. All ε_t are identically and independently distributed over time, according to the normal distribution with zero mean and standard deviation σ_ε. At time τ, when the liquidation value is realized, the manager liquidates his position in the risky asset at price V, and the game ends.

The price dynamics (1) can be rationalized in a number of ways. For example, one could follow Brunnermeier and Pedersen (2005) and assume that in addition to the manager, the market is populated by two types of agents: the long-term traders and the noise traders. At each period $t < \tau$, the noise traders submit a trading order of ε_{t+1}. The long-term traders take the price as given and have an aggregate demand of $D(P) = (E(v) - P)/\lambda$. Then, the equilibrium price evolves according to (1).

The parameter λ is a measure of price impact generated by trading in the risky asset. For simplicity, price impact is modelled to be permanent, linear, and constant over time.[6] Constant λ is a common assumption in the literature that studies costs associated with trading pressure (Carlin, Lobo, and Viswanathan (2007), Bertsimas and Lo (1998), and Obizhaeva and Wang (2006)). Huberman and Stanzl (2004) show that a permanent and time-independent price impact must be linear to rule out arbitrage.

The manager's objective consists of two parts. First, when the liquidation value is realized the manager receives a fraction ψ_π of the realized profit from all his trades prior to this date. The second part comes from the manager's interim performance over the course of arbitrage. The large body of literature that studies limits of arbitrage documents that few investors are willing to tolerate losses, and they withdraw their funds at first sight of negative returns (see e.g., Shleifer and Vishny (1997)). Faced with funds outflows, the manager then can be forced to liquidate his arbitrage position prematurely, which can lead to even larger losses. To model the impact of the interim performance, I assume that at every trading date prior to the liquidation date, fund investors audit the performance of the manager, with probability ph, $p > 0$. An audit reports the current value of the fund's assets under management at the prevailing market price P_t. Each audit report has an additive effect, $\psi_\omega(W_t - W_0)$, $\psi_\omega \geq 0$, on manager's objective function. Thus, at any time t prior to the realization of the liquidation value, the manager solves the following problem:

$$
\begin{aligned}
J(P_t, X_t) = \max_{\{\theta_s\}} E_t[\sum_{s=t}^{\tau-1} \psi_\pi(V - P_{s+1})\theta_s + \psi_\omega ph(W_{s+1} - W_t)], \qquad (2) \\
\text{s. t.} \quad P_{s+1} &= P_s + \lambda(\theta_s + \varepsilon_{s+1}), \\
X_{s+1} &= X_s + \theta_s, \\
W_{s+1} &= W_s + X_s(P_{s+1} - P_s),
\end{aligned}
$$

where X_t is the time-t fund's position in the risky asset, and W_t is the time-t marked-to-market value of the fund.

In formulation (2), the manager's objective is linear in his profit and his interim performance. In particular, the assets under management and payments to the manager can take negative values. The main friction comes from the fact that the interim performance of the manager is evaluated at the prevailing market prices, which he can influence. A higher market price leads to a higher valuation of assets under management. In practice, the manager is protected by the limited liability, and the effect of interim performance may not be linear. For example, the fund can be closed following a sufficiently negative performance. Incorporating the limited liability or fund termina-

[6] The results in the presence of temporary price impact are similar and available upon request.

tion in the analysis gives stronger incentives to the manager to inflate the value of the fund's assets, but it also significantly complicates the model. Therefore, to make the model and its insights as clear as possible, in the basic setup, I use objective (2) as a means of providing the manager with incentives to care about his interim performance, and study the impact of fund termination in Section 3.

2. Analysis

Because of price impact, objective (2) is a linear-quadratic optimal control problem. The solution is summarized in Proposition 1.

Proposition 1: *Suppose*

$$\varphi = \frac{\psi_\omega p}{\psi_\pi \delta} < 1 \tag{3}$$

and the manager solves problem (2). Then his optimal trading strategy is

$$\theta_t^* = \frac{V - P_t + \lambda \varphi X_t}{\lambda \left(1 + \sqrt{\frac{1-(1-\delta h)\varphi^2}{\delta h}}\right)}. \tag{4}$$

Proof: See the Appendix.

The optimal strategy does not depend on the initial state and random fluctuations in the price, which is a well-known result in the linear quadratic control theory (Anderson, Hansen, McGrattan, and Sargent (1995)). The parameter φ, defined by equation (3), plays an important role in the subsequent analysis. It quantifies the benefits associated with inflating the value of the existing arbitrage position in the risky asset. The incentives to inflate the value increase with ψ_ω and p, and decrease with δ and ψ_π. By the property of the geometric distribution, $1/\delta$ is the expected horizon of the arbitrage trade. Thus, p/δ is the expected number of audits during the life of the arbitrage. A higher $\psi_\omega p/\delta$ or a lower ψ_π means a higher relative weight of interim performance in the manager's objective function.

The assumption $\varphi < 1$ is necessary for a well-defined solution. If $\varphi \geq 1$, the manager has too strong an incentive to inflate the price, which leads to an unbounded solution. If $\varphi = 0$, the manager only cares about his final profit, and trading rule (4) reduces to

$$\theta_t^{**} = \frac{V - P_t}{\lambda \left(1 + \sqrt{\frac{1}{\delta h}}\right)}.$$

In this case, the manager always trades in the direction implied by the current arbitrage spread, $V - P_t$. Because of the price impact of trades, the manager smooths his trades over time. His trading intensity is inversely proportional to price impact multiplied by the square root of the expected number of trading periods.

A nonzero φ affects trading decisions in two ways. First, it makes trading more aggressive, because more aggressive trading allows the manager to accelerate booking of trading gains. Second, and more importantly, it introduces a bias into the manager's trading decisions. The bias is proportional to the current position in the risky asset.

The position in the risky asset is endogenous and is jointly determined with the dynamics of the arbitrage spread $V - P_t$. If the manager follows his optimal trading strategy (4), then the arbitrage spread and the manager's position in the risky asset evolve according to the first-order autoregressive process:

$$\begin{pmatrix} V - P_{t+1} \\ X_{t+1} \end{pmatrix} = \Gamma \begin{pmatrix} V - P_t \\ X_t \end{pmatrix} + \Omega \varepsilon_{t+1}, \tag{5}$$

where

$$\Gamma = \frac{1}{1-\varphi} \begin{pmatrix} \nu - \varphi & -\lambda\left(1-\nu\right)\varphi \\ (1-\nu)/\lambda & 1-\nu\varphi \end{pmatrix}, \; \Omega = \begin{pmatrix} -\lambda \\ 0 \end{pmatrix}, \; \nu = \left(1 - \frac{1-\varphi}{1+\sqrt{\frac{1-(1-\delta h)\varphi^2}{\delta h}}}\right).$$

Iterating (5) we obtain

$$\begin{pmatrix} V - P_t \\ X_t \end{pmatrix} = \Gamma^t \begin{pmatrix} V - P_0 \\ X_0 \end{pmatrix} + \sum_{s=0}^{t-1} \Gamma^s \Omega \varepsilon_{t-s},$$

where

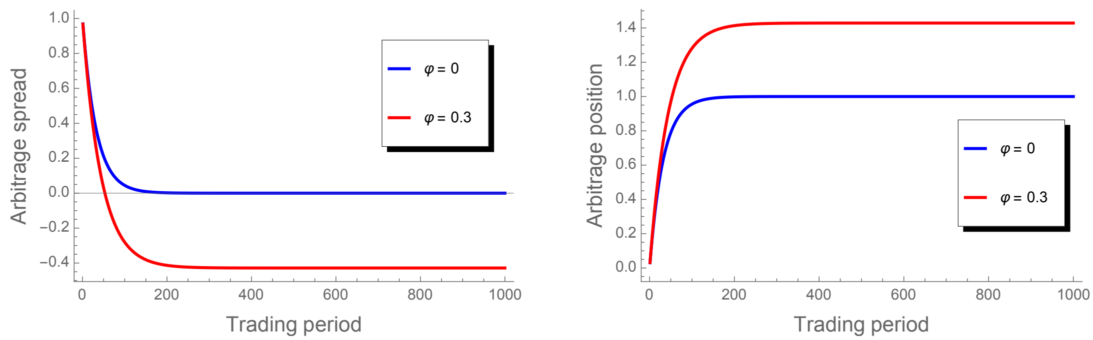
$$\Gamma^t = \frac{1}{1-\varphi} \begin{pmatrix} \nu^t - \varphi & -\lambda\left(1-\nu^t\right)\varphi \\ (1-\nu^t)/\lambda & 1-\nu^t\varphi \end{pmatrix}.$$

The assumption $\varphi < 1$ ensures that $\nu < 1$. If the manager has a zero initial position in the risky asset, then the expected arbitrage spread and his position in the risky asset at time t are given by the following expression:

$$E_0 \begin{pmatrix} V - P_t \\ X_t \end{pmatrix} = \frac{V - P_0}{1-\varphi} \begin{pmatrix} \nu^t - \varphi \\ (1-\nu^t)/\lambda \end{pmatrix}. \tag{6}$$

Figure 1 plots the expected arbitrage spread and the manager's arbitrage position in trading period t, conditional on $t < \tau$ for the two cases $\varphi = 0$ and $\varphi = 0.3$. The blue line corresponds to the case of $\varphi = 0$. The red line depicts the case of $\varphi = 0.3$. The cases $\varphi = 0$ and $\varphi > 0$ stand in stark contrast to each other. If $\varphi = 0$, the expected arbitrage spread gradually converges to zero. However, if $\varphi > 0$, it changes its sign and stays negative, which means that as the manager's position grows, he eventually acquires the risky asset at a price that is above its fair value.

Figure 1: *Expected arbitrage spread and arbitrage position*



(a) (b)

Panel (a) shows the expected period-t arbitrage spread, $(V - P_t)$, $t = 1, \ldots, 1000$. Panel (b) shows the expected period-t arbitrage position in the risky asset, X_t. Other parameters are set as follows: $V - P_0 = 1$, $\delta = 1$, $h = 0.001$, and $\lambda = 1$. The blue line corresponds to the case of $\varphi = 0$, the red line to $\varphi = 0.3$.

By acquiring the asset at a price above its fair value, the manager potentially exposes investors to large losses. The contribution of the period-t trade to the total profit is

$$\pi_t = (V - P_{t+1})\theta_t.$$

Suppose the liquidation value is realized in trading period $T \geq 2$. Since trading in the last period results in zero profit, the total trading profit comes from all trades prior to the last period, that is:

$$\Pi_T = \sum_{t=0}^{T-2} \pi_t.$$

Consider first the case of $\sigma_\varepsilon = 0$, in which the price changes only because of the manager's trades. Direct computations show that

$$\Pi_T = (V - P_0) \frac{\left(1 - \nu^{T-1}\right)\left(\nu\left(1 + \nu^{T-1} - \varphi\right) - \varphi\right)}{\lambda(1+\nu)(1-\varphi)^2}. \tag{7}$$

Figure 2 shows the total trading profit Π_T for the three values of φ ($\varphi = 0$, $\varphi = 0.3$, and $\varphi = 0.6$) and different values of T. Because higher values of φ lead to more aggressive trading, the total arbitrage profit for small T is higher for higher values of φ. However, higher values of φ also lead to building an outsized arbitrage position at inflated prices. Therefore, for large values of T, the total trading profit declines with φ, and even can become negative.

Figure 2: *Trading profit* ($\sigma_\varepsilon = 0$)

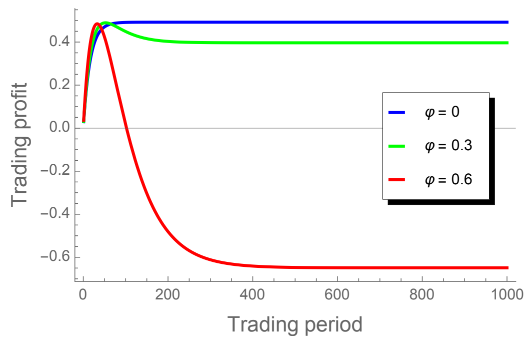


Figure 2 shows the total trading profit if the liquidation value is realized in trading period T, $T = 2, \ldots, 1000$ for the three values of φ : $\varphi = 0$, $\varphi = 0.3$, and $\varphi = 0.6$. Other parameters are set as follows: $V - P_0 = 1$, $\delta = 1$, $h = 0.001$, and $\lambda = 1$.

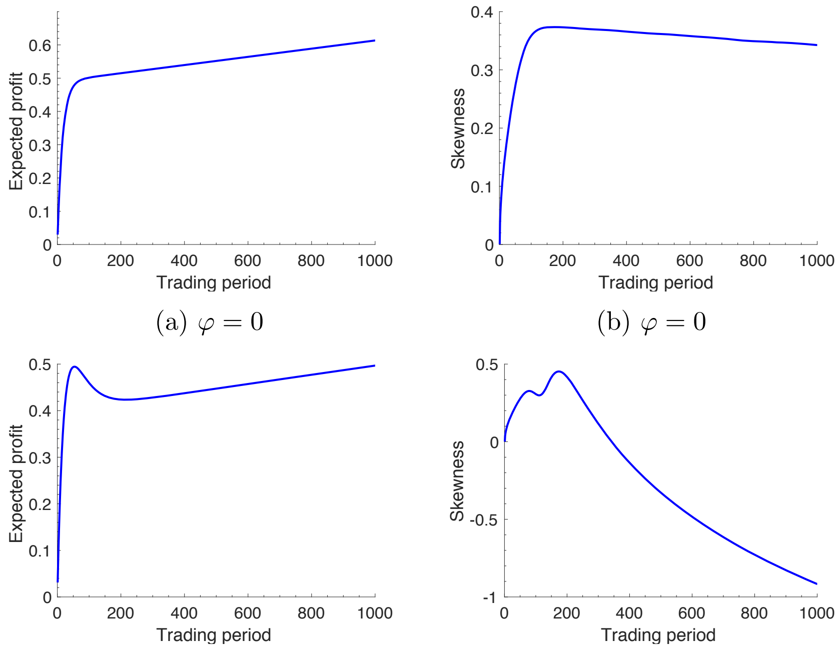
Next, consider a more realistic case of $\sigma_\varepsilon \neq 0$, in which some fluctuation in price is outside the manager's control. Because the manager has the option of when to trade, he benefits from random fluctuations in price.[7] When $\varphi = 0$, and the manager's compensation depends only on his final profit, fluctuations in price also benefit his investors. Figure 3, Panel (a) shows that the expected trading profit increases with the number of trading periods. This is in contrast with the case of $\sigma_\varepsilon = 0$, in which the total profit converges to a constant. When $\sigma_\varepsilon \neq 0$, a higher number of trading periods implies a higher chance that the arbitrage spread will widen. In the absence of borrowing constraints and constraints on the position size, a wider arbitrage spread represents a better trading opportunity. Since the manager always trades in the direction of the arbitrage spread, every trade increases the total trading profit. As a result, the distribution of the total profit is positively skewed, as shown in Figure 3, Panel (b).

When the manager's value function depends on his interim performance, price fluctuations have two effects. On the one hand, as before, they contribute positively to the expected total trading profit by giving the manager the option to trade against the arbitrage spread. On the other hand, they increases the chance that the manager accumulates a large arbitrage position and gets trapped in outsized arbitrage, which, in turn, can lead to large losses. Figure 3, Panel (d) shows that in contrast to the case of $\varphi = 0$, the skewness of the total trading profit now decreases and becomes negative with the number of trading periods.

The above results are consistent with empirical evidence. The negative skewness in hedge fund returns is one of the most salient features of hedge fund performance.

[7] This would not necessarily be the case if investors could withdraw their investments and terminate the fund following its negative performance.

Figure 3: *Trading profit* ($\sigma_\varepsilon \neq 0$)



(a) $\varphi = 0$ (b) $\varphi = 0$

(c) $\varphi = 0.3$ (d) $\varphi = 0.3$

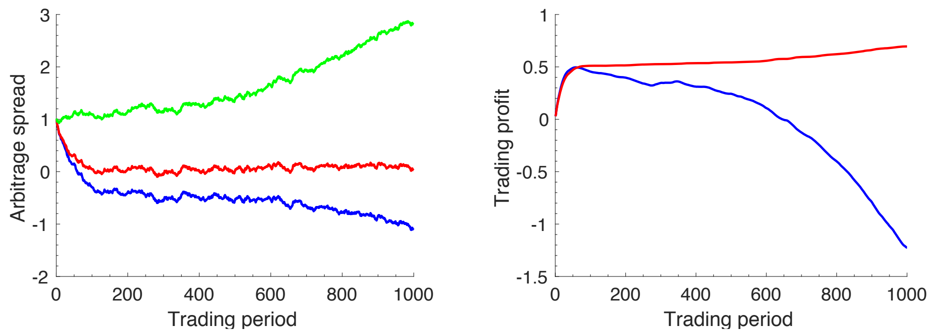
Panels (a) and (c) show the expected total trading profit if the liquidation value is realized in trading period T, $T = 2, \ldots, 1000$. Panel (b) and (d) show the skewness of the total trading profit. The parameter φ is set to 0 in Panels (a) and (b), and to 0.3 in Panels (b) and (d). Other parameters are set as follows: $V - P_0 = 1$, $\delta = 1$, $h = 0.001$, $\lambda = 1$, and $\sigma_\varepsilon = 0.5\sqrt{h}$.

Getmansky, Lo, and Makarov (2004) show that hedge funds that invest in illiquid securities have negative skewness, as opposed to funds that invest in liquid instruments.

To develop an understanding of the negative skewness, consider Figure 4, which shows the evolution of the arbitrage spread and total trading profit for a particular realization of ε_t, $t = 1, \ldots, 1000$. The green line in Panel (a) shows the path of arbitrage spread that would prevail if the manager did not trade in the market. In this scenario, the spread keeps increasing over time. The red lines in Panels (a) and (b) show the arbitrage spread and total trading profit when the manager trades in the market and cares only about his final profit. In this case, the spread closes in early trading rounds and stays around zero thereafter. Each trading period contributes to the total profit, which therefore gradually increases over time.

Finally, the blue lines in Panels (a) and (b) show the arbitrage spread and total trading profit when the manager trades in the market and cares about his interim performance. In contrast to the case of $\varphi = 0$, the arbitrage spread gets more and more negative with the number of trading rounds. After establishing a large position

Figure 4: *Simulation example*



(a) (b)

Figure 4 shows the results from a particular realization of ε_t, $t = 1, \ldots, 1000$.
The green line in Panel (a) shows the path of arbitrage spread if the manager does not trade. The red and blue lines in Panels (a) and (b) show the arbitrage spread and the total trading profit when the manager trades, and $\varphi = 0$ and $\varphi = 0.3$, correspondingly. Other parameters are set as follows: $V - P_0 = 1$, $\delta = 1$, $h = 0.001$, $\lambda = 1$, and $\sigma_\varepsilon = 0.5\sqrt{h}$.

in initial trading rounds, the manager defends it against adverse price movements. That leads to even larger position in the risky asset, and even stronger incentives to defend it later. As a result, total trading profit declines over time and becomes negative in later trading periods. Figure 5 shows the probability of loss as a function of arbitrage horizon. The probability increases, with the horizon with the unconditional probability being 4.5%.

Figure 5: *Loss probability*

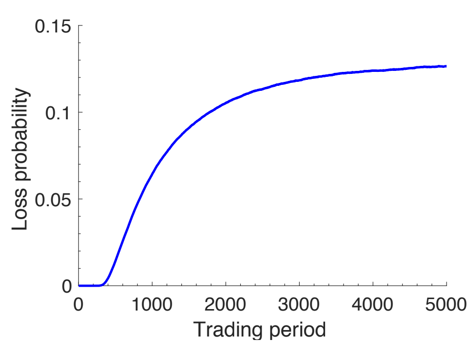


Figure 5 shows the probability of total trading profit to be negative if the liquidation value is realized in trading period T, $T = 2, \ldots, 5000$. Other parameters are set as follows: $V - P_0 = 1$, $\delta = 1$, $h = 0.001$, $\lambda = 1$, $\sigma_\varepsilon = 0.5\sqrt{h}$, and $\varphi = 0.3$.

A necessary condition to realize negative total profit is that the manager acquires the risky asset at a price above its fair value. One might hypothesize that if other arbitragers were present in the market, they would trade as to correct the mispricing. To investigate if this is indeed the case, the next section considers an extension of the model wherein the manager trades against other arbitrageurs.

3. Trading Against Other Arbitrageurs

Suppose there is another arbitrageur who also learns the liquidation value and starts trading at some time t. For ease of exposition, I will continue to refer to the incumbent arbitrageur as the manager and the new arbitrageur simply as the arbitrageur.

Unlike the manager, who is a "whale" and can establish a very large position in the risky asset, I assume the arbitrageur incurs quadratic holding costs per unit of time, which constrain the size of the risky position she can take. The holding costs can be viewed as a reduced-form specification of the risk-aversion or collateral costs. Similar preferences are used in Du and Zhu (2017), Vives (2011), and Malamud and Rostek (2017). I consider first the case where the arbitrageur does not have to worry about her interim performance, and then show how preferences over interim performance affect the results.

One can imagine different informational scenarios for trading between the manager and arbitrageur. Because of the difference in size, it is likely that the arbitrageur is aware of the manager, but the manager may or may not be aware of the presence of the arbitrageur. Section 3.1 studies the case wherein the manager is oblivious of the presence of the arbitrageur and therefore, follows his previously derived trading strategy (4). The arbitrageur, in contrast, is fully aware of the manager's trades and adjusts her trades accordingly. Alternatively, Section 3.2 studies the case in which both manager and arbitrageur are aware of the other.

3.1. Stealth Trading

In this scenario, the arbitrageur is aware of the existence of a "whale" in the market and she can trade secretly against him without revealing herself. Because the manager is unaware of the arbitrageur's presence, he continues to follow his previously derived trading strategy θ_s^* given by equation (4). Thus, the arbitrageur solves

$$J_a = \max_{\{\theta_{a,s}\}} E_t[\sum_{s=t}^{\tau-1}(V - P_{s+1})\theta_{a,s} - \gamma h X_{a,s}^2], \tag{8}$$

$$\text{s. t} \quad P_{s+1} = P_s + \lambda(\theta_s^* + \theta_{a,s} + \varepsilon_{s+1}),$$
$$X_{a,s+1} = X_{a,s} + \theta_{a,s},$$

where $X_{a,t}$ is the time-t arbitrageur's position in the risky asset, and $\gamma > 0$ is the holding cost parameter. The first term is the expected profits from arbitrage when the liquidation value of realized. The second term is the quadratic holding costs. The solution is summarized in Proposition 2:

Proposition 2: *The objective (8) can be written as a linear quadratic optimization problem:*

$$J_a = \max_{\{\theta_{a,s}\}} \sum_{s=t}^{\infty} \beta^{s-t+1} \left[Z_s^T \;\; \theta_{a,s} \right] \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} Z_s \\ \theta_{a,s} \end{bmatrix}, \tag{9}$$
$$s.\ t. \quad Z_{s+1} = AZ_s + B\theta_{a,s} + \Omega\varepsilon_{s+1},$$

where $Z_s = (V - P_s, \theta_s^*, X_{a,s})^T$, $\beta = 1 - \delta h$, and matrices A, B, Ω, R, S, and Q are defined in the Appendix. The optimal strategy $\theta_{a,s}$ is given by

$$\theta_{a,s}^* = FZ_t,$$

where

$$F = -\left(R + \beta B^T PB\right)^{-1} \left(\beta B^T PA + S^T\right),$$

and P is the unique stabilizing solution of the algebraic Riccati equation:

$$P = \beta A^T PA - \left(\beta A^T PB + S\right) \left(R + \beta B^T PB\right)^{-1} \left(\beta B^T PA + S^T\right) + Q. \tag{10}$$

Proof: See the Appendix.

Riccati equation (10) is highly nonlinear in terms of matrix elements. A closed-form solution is only available in special cases, so the system must be solved numerically. When solving (10) numerically, I set the parameter γ to 1. Other parameters are as in Section 1: $\lambda = 1$, $\delta = 1$, $\varphi = 0.3$, and $h = 0.001$. With these parameter values the arbitrageur's strategy takes the following form:

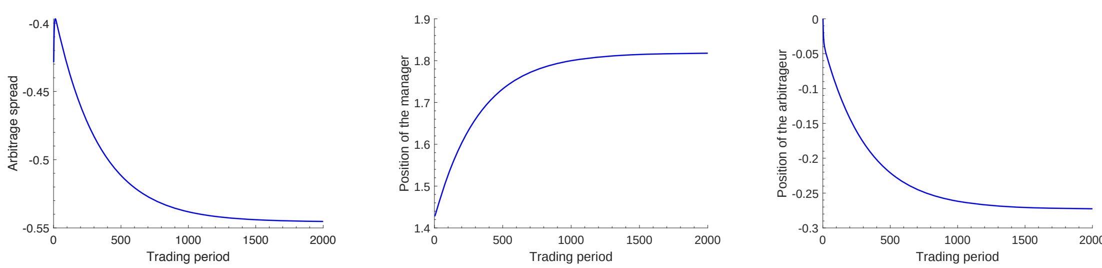
$$\theta_{a,t}^* = 0.019(V - P_t) + 4.668\theta_t^* - 0.039X_{a,t}. \tag{11}$$

The arbitrageur's strategy increases with the arbitrage spread and decreases with the arbitrageur's position in the risky asset. This is intuitive, as a larger arbitrage spread implies better trading opportunities to make profit, and holding costs increase in the position in the risky asset. Inspecting (11), we can see that the arbitrageur's strategy is also positively linked to the trading behavior of the manager. To understand this result, notice that in the absence of random shocks ε_t and arbitrageur's orders, the manager's trading strategy follows a first-order autoregressive process: $\theta_{t+1}^* = \nu\theta_t^*$, where $\nu > 0$ is defined by equation (5). If the manager is buying the risky asset today, he is also expected to buy it in the future. Therefore, the future price is expected to rise, which makes it rational for the arbitrageur to buy the risky asset today.

Figure 6 shows the resulting dynamics, in the absence of random shocks, when the arbitrageur learns about the arbitrage opportunity and starts trading after the manager's position reaches the steady-state studied in Section 2. Because the arbitrage spread is negative, the arbitrageur establishes a short position, which reduces the absolute value of the arbitrage spread. The reduction in the arbitrage spread, however, prompts the manager to defend the value of his existing position. As a result, the arbitrage spreads widens, giving incentives to the arbitrageur to take an even larger short position. This tug-of-war continues until the arbitrageur exhausts her risk-taking capacity, and the arbitrage spread reaches its new steady-state level. Panel (a) shows that in the new steady state, the arbitrage spread becomes even more negative compared to its level before the arrival of the arbitrageur.

Thus, by trading secretly against a "whale," an arbitrageur with limited risk capacity not only fails to correct mispricing but actually makes the market even more inefficient. I now turn to the scenario in which both the manager and the arbitrageur are aware of each other.

Figure 6: *Dynamics* ($\sigma_\varepsilon = 0$)



(a) (b) (c)

Figure 6 shows the equilibrium dynamics of the arbitrage spread and positions in the risky asset of the manager and the arbitrageur when the arbitrageur learns V after the manager and can secretly trade against him. The parameters are set as follows: $V - P_0 = 1$, $\delta = 0.001$, $\lambda = 1$, $\gamma = 0.001$.

3.2. Open Trading

If the manager and the arbitrageur are aware of each other and take each other's trading strategy as given, then the manager solves

$$J_m = \max_{\{\theta_{m,s;s\geq t}\}} E_t[\sum_{s=t}^{\tau-1} \psi_\pi (V - P_{s+1})\theta_{m,s} + \psi_\omega ph(W_{m,s+1} - W_{m,t})], \tag{12}$$

$$\begin{aligned} \text{s. t.} \quad & P_{s+1} = P_s + \lambda(\theta_{m,s} + \theta^*_{a,s} + \varepsilon_{s+1}), \\ & X_{m,s+1} = X_{m,s} + \theta_{m,s}, \\ & W_{m,s+1} = W_{m,s} + X_s(P_{s+1} - P_s), \end{aligned}$$

where $\theta^*_{a,s}$ is an equilibrium strategy of the arbitrageur; and the arbitrageur solves

$$J_a = \max_{\{\theta_{a,s}\}} E_t[\sum_{s=t}^{\tau-1} (V - P_{s+1})\theta_{a,s} - \gamma h X^2_{a,s}], \tag{13}$$

$$\begin{aligned} \text{s. t} \quad P_{s+1} &= P_s + \lambda(\theta^*_{m,s} + \theta_{a,s} + \varepsilon_{s+1}), \\ X_{a,s+1} &= X_{a,s} + \theta_{a,s}, \end{aligned}$$

where $\theta^*_{m,s}$ is the equilibrium strategy of the manager.

Problems (12) and (13) define a linear quadratic game. The assumptions ensure that the trading game is stationary, so it is natural to focus on a solution where optimal trading strategies do not depend on time and are linear functions of state variables. The state variables are the arbitrage spread $V - P_t$ and the position in the risky asset of the manager and the arbitrageur, $X_{m,t}$ and $X_{a,t}$, respectively. Denote the vector of the state variables by $Z_t = (V - P_t, X_{m,t}, X_{a,t})^T$. Using (12) and (13), the dynamics of Z_t can be written as

$$Z_{t+1} = Z_t + B_1\theta_{m,t} + B_2\theta_{a,t} + \Omega\varepsilon_{t+1}, \tag{14}$$

where $B_1 = (-\lambda, 1, 0)^T$, $B_2 = (-\lambda, 0, 1)^T$, and $\Omega = (-\lambda, 0, 0)^T$. Therefore, if $\theta^*_{m,s} = F_1 Z_t$ and $\theta^*_{a,s} = F_2 Z_t$, from the manager's perspective, the vector of the state variables evolves according to

$$Z_{t+1} = A_1 Z_s + B_1\theta_{m,s} + \Omega\varepsilon_{s+1}, \tag{15}$$

where $A_1 = I + B_2F_2$. Similarly, from the arbitrageur's perspective, the dynamics of the state variables are

$$Z_{t+1} = A_2 Z_s + B_2\theta_{a,s} + \Omega\varepsilon_{s+1}, \tag{16}$$

where $A_2 = I + B_1F_1$. The next proposition characterizes a linear, closed loop Nash equilibrium.

Proposition 3: *The objective of the manager and the arbitrageur can be written as*

$$J_m(Z_t) = \max_{\{\theta_{m,s}\}} \sum_{s=t}^{\infty} \beta^{s-t+1} \begin{bmatrix} Z_s & \theta_{m,s} \end{bmatrix} \begin{bmatrix} Q_1 & S_1 \\ S_1^T & R \end{bmatrix} \begin{bmatrix} Z_s \\ \theta_{m,s} \end{bmatrix}, \tag{17}$$
$$s.\ to \quad Z_{s+1} = A_1 Z_s + B_1 \theta_{m,s} + \Omega \varepsilon_{s+1},$$

and

$$J_a(Z_t) = \max_{\{\theta_{a,s}\}} \sum_{s=t}^{\infty} \beta^{s-t+1} \begin{bmatrix} Z_s & \theta_{a,s} \end{bmatrix} \begin{bmatrix} Q_2 & S_2 \\ S_2^T & R \end{bmatrix} \begin{bmatrix} Z_s \\ \theta_{a,s} \end{bmatrix}, \tag{18}$$
$$s.\ to \quad Z_{s+1} = A_2 Z_s + B_2 \theta_{a,s} + \Omega \varepsilon_{s+1},$$

where matrices R, S_i, Q_i, $i = 1, 2$ are defined in the Appendix. Strategies $\theta^*_{m,s} = F_1 Z_t$ and $\theta^*_{a,s} = F_2 Z_t$ constitute a linear, closed loop Nash equilibrium if and only if there is a solution to the following system equations:

$$P_1 = \beta A_1^T P_1 A_1 - \left(\beta A_1^T P_1 B_1 + S_1\right)\left(R + \beta B_1^T P_1 B_1\right)^{-1}\left(\beta B_1^T P_1 A_1 + S_1^T\right) + Q_1, \tag{19}$$
$$P_2 = \beta A_2^T P_2 A_2 - \left(\beta A_2^T P_2 B_2 + S_2\right)\left(R + \beta B_2^T P_2 B_2\right)^{-1}\left(\beta B_2^T P_2 A_2 + S_2^T\right) + Q_2, \tag{20}$$
$$F_1 = -\left(R + \beta B_1^T P_1 B_1\right)^{-1}\left(\beta B_1^T P_1 A_1 + S_1^T\right), \tag{21}$$
$$F_2 = -\left(R + \beta B_2^T P_2 B_2\right)^{-1}\left(\beta B_2^T P_2 A_2 + S_2^T\right). \tag{22}$$

Proof: See the Appendix.

The system (19)–(22) is known as a system of coupled Riccati equations. A closed-form solution is not available, so the system must be solved numerically. For consistency, the parameter values are kept the same: $\lambda = 1$, $\delta = 1$, $\varphi = 0.3$, $\gamma = 1$, and $h = 0.001$. With these parameters, the equilibrium strategies of the manager and arbitrageur take the following form:

$$\theta_{m,t} = 0.2209(V - P_t) + 0.0219 X_{m,t} - 0.1228 X_{a,t}, \tag{23}$$
$$\theta_{a,t} = 0.2357(V - P_t) + 0.0215 X_{m,t} - 0.1585 X_{a,t}.$$

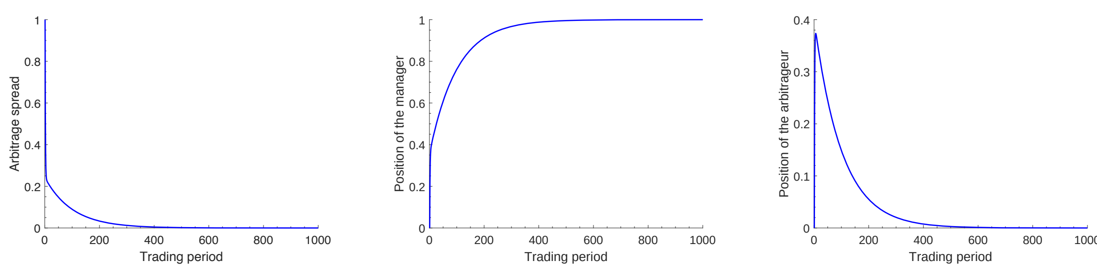
To understand the intuition behind this solution, it is instructive to consider first the solution when $\varphi = 0$, where the manager does not have to worry about his interim performance. In this case, the trading strategies take the following form:

$$\theta_{m,t} = 0.2180(V - P_t) - 0.1157X_{a,t}, \tag{24}$$
$$\theta_{a,t} = 0.2287(V - P_t) - 0.1479X_{a,t}.$$

Because the manager does not need to worry about his interim performance, and does not have any holding costs, the size of his position is no longer a state variable.

Figure 7 shows the equilibrium dynamics, in the absence of random shocks, when the manager and arbitrageur learn V at the same time, $t = 0$, and follow strategies (24). Unlike the case when the manager trades alone, now the arbitrage spread decreases to zero almost instantaneously as both the manager and the arbitrageur compete with each other for the trading profits. Inspection of (24) reveals that the arbitrageur is more aggressive at eliminating the arbitrage spread than the manager. This might seem surprising, since it is the arbitrageur who has the holding costs; therefore, it is she who might be expected to trade more cautiously. The dynamics of the arbitrage positions provide an explanation for this apparently surprising result. While the manager's arbitrage position monotonically increases over time, the arbitrageur's position exhibits a reversal. After quickly building up the position in the risky asset, the arbitrageur gradually sells it to the manager. Exploiting predictability in the manager's large trading program, the arbitrageur is able to effectively front-run the manager's trading orders.

Figure 7: *Nash equilibrium* ($\varphi = 0$)

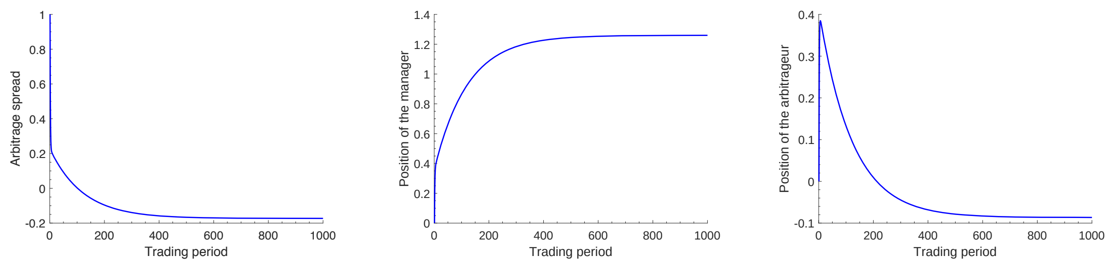


(a) (b) (c)

Figure 7 shows the equilibrium dynamics of the arbitrage spread and positions in the risky asset of the manager and the arbitrageur when both learn V at the same time, $t = 0$. The parameters are set as follows: $V - P_0 = 1$, $\delta = 0.001$, $\lambda = 1$, and $\gamma = 0.001$.

When $\varphi > 0$, so the manager's compensation depends on his interim performance, the size of his risky position becomes a state variable. Similar to what happens when he is alone in the market, incentives to show good interim performance make the manager trade more aggressively to accelerate booking of trading gains. More aggressive trading by the manage, in turn, leads to more aggressive trading by the arbitrageur. Figure 8 shows the equilibrium dynamics, in the absence of random shocks, when the manager and arbitrageur learn V at the same time, $t = 0$, and follow strategies (23). Similar to the case of $\varphi = 0$, the arbitrageur front-runs the manager in the beginning, only to unload her risky position to the manager later. But unlike the case of $\varphi = 0$, the steady-state arbitrage spread is now negative, and the arbitrageur is short the risky asset.

Figure 8: *Nash equilibrium ($\varphi = 0.3$)*

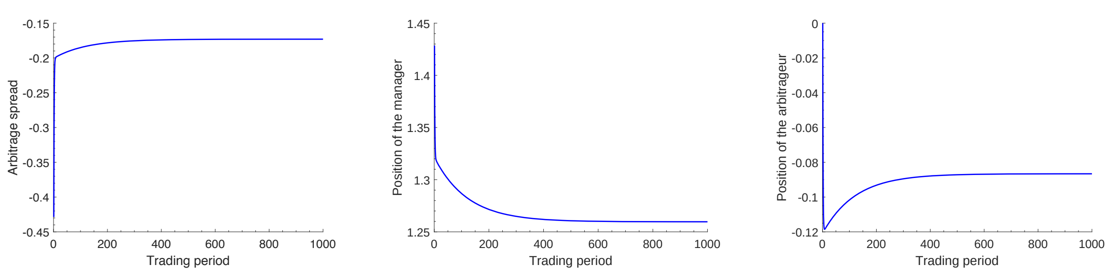


(a) (b) (c)

Figure 8 shows the equilibrium dynamics of the arbitrage spread and positions in the risky asset of the manager and the arbitrageur when both learn V at the same time, $t = 0$. The parameters are set as follows: $V - P_0 = 1$, $\delta = 0.001$, $\lambda = 1$, and $\gamma = 0.001$.

Because price impact is permanent, in the absence of random shocks, the sum $V - P_t + \lambda(X_{m,t} + X_{a,t})$ stays constant over time. Hence, one eigenvalue of matrix $I + B_1F_1 + B_2F_2$ that governs the evolution of the vector of state variables Z_t is equal to 1. It can be verified that the other two eigenvalues of matrix $I + B_1F_1 + B_2F_2$ are less than 1 in modulus. Therefore, the steady state is the same for all initial states that have the same value of $V - P_0 + \lambda(X_{m,0} + X_{a,0})$. In particular, it is the same for the case in which the arbitrageur learns V at the same time as the manager, or when she learns it sometime after the manager.

Figure 9: *Equilibrium*



(a) (b) (c)

Figure 9 shows the equilibrium dynamics of the arbitrage spread and positions in the risky asset of the manager and the arbitrageur when the arbitrageur learns V after the manager. The parameters are set as follows: $V - P_0 = 1$, $\delta = 0.001$, $\lambda = 1$, and $\gamma = 0.001$.

Figure 9 shows the equilibrium dynamics when the arbitrageur learns V after the manager's position and arbitrage spread converge to their steady-state values when the manager is alone in the market. As before, there is a short initial stage in which the absolute value of the arbitrage spread sharply decreases. During this stage the arbitrageur quickly establishes a short position. Faced with an aggressive trading program by the arbitrageur, the manager revises his strategy and scales down his risky position. Again, faced with predictable changes in the price, the arbitrageur front-runs the manager—she quickly establishes an oversize short position and then gradually reduces it.

3.3. Stealth vs. Open Trading

Figures 6 and 9 show that the equilibrium dynamics are very different in the cases of stealth and open trading. In the stealth trading case, the manager's position grows, and the arbitrage spread widens. This is in contrast to the case of open trading, in which the manager scales down his position and the arbitrage spread gets significantly reduced. Hence, the two cases have very different implications for market efficiency. In the case of stealth trading, mispricing can persist for a long time; whereas in the open trading case, the market gets more and more efficient as more arbitrageurs find out about the mispricing. Because the arbitrageur can always reveal her presence to the manager, the question becomes whether she will ever find it to her advantage to trade secretly.

To answer this question, I compare the value function of the arbitrageur in the two cases. The value function is given by

$$J_a = Z_0^* P_a Z_0 + \frac{\beta\sigma_\varepsilon^2}{1-\beta}\Omega^* P_a \Omega, \tag{25}$$

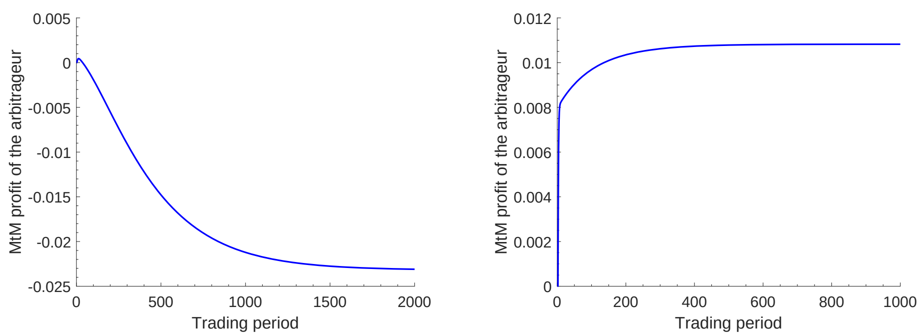
where Z_0 is an initial vector of state variables, and P_a is the solution to Riccati equation (10) when the arbitrageur trades in the stealth mode, and to coupled Riccati equation (19) when the manager and arbitrageur are aware of each other. The respective solution matrices are given below:

$$\begin{pmatrix} 0.411 & 0.127 & 0.022 \\ 0.127 & 0.061 & 0.156 \\ 0.022 & 0.156 & -0.003 \end{pmatrix}, \qquad \begin{pmatrix} 0.153 & 0.017 & -0.061 \\ 0.017 & 0.053 & 0.043 \\ -0.061 & 0.043 & -0.247 \end{pmatrix}.$$

It can be directly verified that for any initial state where the arbitrageur starts with zero holdings of the risky asset, her utility is positive in both cases and is higher than in the case when the manager is oblivious of her trading. By revealing her presence to the manager, the arbitrageur makes the manager quickly scale down his risky position. As a result, the arbitrage spread gets reduced, which has a negative effect on the arbitrageur's profit. Thus, the arbitrageur who only cares about the final profit would always enter the market and would never want to reveal herself.

The above conclusion, however, crucially depends on the ability of the arbitrageur to maintain her arbitrage position over the course of the arbitrage. Figure 10, Panel (a) shows the marked-to-market profit of the arbitrageur when she learns V after the manager and trades secretly from the manager in the absence of random shocks. The marked-to-market profit stays negative over the course of the arbitrage, except for a short initial period. Thus, the arbitrageur realizes her profits only when the arbitrage is closed. If the arbitrageur draws her capital from outside investors who are unwilling tolerate losses, then the arbitrageur will have to liquidate her position prematurely, and her trades would result in losses. Thus, trading secretly against the manager may not be a viable strategy for the arbitrageur in these circumstances.

Figure 10: *Marked-to-market profit*



(a) (b)

Figure 10 shows the marked-to-market (MtM) profit of the arbitrageur when she trades secretly from the manager (Panel (a)) and when both the manager and arbitrageur are aware of each other (Panel (b)). The trading starts after the manager reaches the steady state studied in Section 2. The parameters are set as follows: $V - P_0 = 1$, $\delta = 1$, $h = 0.001$, $\lambda = 1$, and $\gamma = 1$.

Panel (b) shows the marked-to-market profit of the arbitrageur in the open trading case. In contrast to what occurs in stealth trading, the marked-to-market profit stays positive for all periods. The above result may therefore suggest that the arbitrageur who has to worry about her interim performance will have incentive to reveal her presence before entering the market. This view, however, neglects to consider the fact that the incentive to show good interim performance weakens the arbitrageur's market power.

Whenever the arbitrageur enters the market and starts trading, the manager is worse off compared when he is alone in the market. Therefore, the manager is better off if he can commit to trading strategies that deter the arbitrageur's entrance. If the arbitrageur does not need to be concerned about her interim performance, there is little the manager can do to prevent her from from entering, since every trade against nonzero arbitrage spread eventually results in profit for the arbitrageur. But if the arbitrageur cannot tolerate interim losses, she becomes vulnerable to the manager's aggressive trading strategies.

To show that this is indeed the case, suppose that there are no random fluctuations in the price. As before, the arbitrageur incurs holding costs and maximizes her final profit, but with an additional constraint that the arbitrageur's fund is terminated whenever her interim profit turns negative. While this is obviously a strong assumption that can be relaxed, it significantly simplifies the analysis and makes interpretation as clear as possible. Also, for simplicity, suppose that upon termination, the fund liquidates its entire position at once, and the manager incurs liquidation costs $\chi > 0$.

Let τ_a be a stopping time when the fund is terminated:

$$\tau_a = \min\{s \leq \tau : W_{a,s} < 0\}, \tag{26}$$

where $W_{a,s}$ is marked-to-market profit of the arbitrageur. The manager solves

$$J_m = \max_{\{\theta_{m,s;s\geq t}\}} E_t[\sum_{s=t}^{\tau-1} \psi_\pi (V - P_{s+1})\theta_{m,s} + \psi_\omega ph(W_{m,s+1} - W_{m,t})] \tag{27}$$

$$\text{s. t.} \quad P_{s+1} = \begin{cases} P_s + \lambda(\theta_{m,s} + \theta^*_{a,s}), & \text{if} \quad W_{a,s} \geq 0, \\ P_s + \lambda(\theta_{m,s} - X_{a,s}), & \text{if} \quad W_{a,s} < 0, \end{cases}$$

$$X_{m,s+1} = X_{m,s} + \theta_{m,s},$$

$$W_{m,s+1} = W_{m,s} + X_s(P_{s+1} - P_s),$$

and the arbitrageur solves

$$J_a = \max_{\{\theta_{a,s}\}} E_t \left[1_{(\tau_a \geq \tau-1)} \sum_{s=t}^{\tau-1} [(V - P_{s+1})\theta_{a,s} - \gamma h X^2_{a,s}] - 1_{(\tau_a < \tau-1)}\chi \right], \tag{28}$$

$$\text{s. t} \quad P_{s+1} = \begin{cases} P_s + \lambda(\theta_{m,s} + \theta^*_{a,s}), & \text{if} \quad W_{a,s} \geq 0, \\ P_s + \lambda(\theta_{m,s} - X_{a,s}), & \text{if} \quad W_{a,s} < 0, \end{cases}$$

$$X_{a,s+1} = X_{a,s} + \theta_{a,s},$$

$$W_{a,s+1} = W_{a,s} + X_{a,s}(P_{s+1} - P_s), \quad W_{a,0} = 0.$$

This lead us to Proposition 4.

Proposition 4: *Suppose the manager and and arbitrageur solve problems (27) and (28), and the arbitrageur learns V after the manager reaches his steady state. Suppose, too, that*

$$(1-\delta h)\chi > \max_{\theta} h\left[\delta(V-P_0-\lambda\theta)\theta-\gamma\theta^2\right]. \tag{29}$$

Then, the only pure strategy equilibrium is where the arbitrageur does not enter the market.

Proof: See the Appendix.

In the corresponding proof, I show that it is always in the interest of the manager to trade aggressively and ensure that the arbitrageur's fund is liquidated right after she enters the market. Because the manager does not know when the arbitrageur arrives to the market, he becomes aware of the arbitrageur and reacts to her actions only after she enters. Therefore, the arbitrageur avoids fund liquidation only if the arbitrage closes in the next period after she enters the market. The probability of this event is δh. The expected profit of the arbitrageur, net of holding costs, is therefore

$$\delta h(V-P_0-\lambda\theta)\theta-\gamma h\theta^2. \tag{30}$$

Condition (29) ensures that the expected profit over one trading period is less than the expected cost of liquidation—a mild condition. As the length of the period goes to zero, so does the expected profit.

4. Conclusions

This paper studies incentives and trading decisions of a whale fund manager who can take concentrated bets in illiquid markets. The paper challenges a popular view that active money managers should double down on exceptional bets. It argues that this view neglects the agency problems that shape investor-manager relationships in money management, where managers care about both interim and long-term performance.

In illiquid markets, a manager may be tempted to use the price impact of his own trades to mitigate the effect of negative shocks on the value of his position. These "stabilizing" trades can help him complete the arbitrage, but can also trap him into building an outsized arbitrage position. Trying to defend his position, he may find it optimal to acquire assets above their fundamental value, which can ultimately lead to large losses to investors and prolonged mispricing in the market. Other arbitrageurs, facing the prospect of trading against the whale, may be unable to eliminate the mispricing caused by the whale, and may contribute to even larger mispricing.

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Appendix. Proofs

Proof of Proposition 1: Because τ follows geometric distribution the manager's problem (1) can be written as

$$\begin{aligned}
&\max_{\{\theta_s\}} E_t \sum_{s=t}^{\infty} \beta^{s-t+1} \left(\psi_\pi (V - P_{s+1})\theta_s + \psi_\omega p h (W_{s+1} - W_t)\right), \\
\text{s. t.} \quad & P_{s+1} = P_s + \lambda(\theta_s + \varepsilon_{s+1}), \\
& X_{s+1} = X_s + \theta_s, \\
& W_{s+1} = W_s + X_s (P_{s+1} - P_s),
\end{aligned}$$

where $\beta = 1 - \delta h$. Note that

$$E_t \sum_{s=t}^{\infty} \beta^{s-t+1} (W_{s+1} - W_t) = E_t \sum_{s=t}^{\infty} \beta^{s-t+1} \left(\lambda \sum_{i=t}^{s} X_i \theta_i\right) =$$

$$\frac{1}{1-\beta} E_t \left(\lambda \sum_{s=t}^{\infty} \beta^{s-t+1} X_s \theta_s\right).$$

Thus, we can write the manager's problem as

$$\left(\max_{\{\theta_s\}} \psi_\pi E_t \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \left((V - P_s)\theta_s + \lambda \varphi X_{s-1} \theta_s\right)\right]\right),$$

where

$$\varphi = \frac{\psi_\omega p}{\psi_\pi \delta}.$$

Define $J(P_t, X_t)$ as:

$$J(P_t, X_t) = \max_{\{\theta_s\}} E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left((V - P_{s+1})\theta_s + \lambda \varphi X_s \theta_s\right)\right].$$

The optimal trading strategy $\{\theta_s^*\}_{s=t}^{\tau-1}$ is a solution to the following Bellman equation:

$$J(P_t, X_t) = \max_{\theta_t} \left\{(V - P_t + \lambda \varphi X_t)\theta_t - \lambda \theta_t^2 + \beta E_t [J(P_t + \lambda(\theta_t + \varepsilon_{t+1}), X_t + \theta_t)]\right\}. \quad \text{(A1)}$$

Conjecture that:

$$J(P_t, X_t) = a(V - P_t + \lambda \varphi X_t)^2 + b.$$

The first-order condition implies that

$$\theta_t = \frac{1 - 2a\beta\lambda(1-\varphi)}{2\lambda\left(1 - a\beta\lambda(1-\varphi)^2\right)}(V - P_t + \lambda\varphi X_t). \tag{A2}$$

Substituting (A2) back into (A1), we obtain equations for a and b:

$$a = \frac{1 + 4a\beta\lambda\varphi}{4\lambda\left(1 - a\beta\lambda(1-\varphi)^2\right)}, \tag{A3}$$

$$b = \frac{a\beta\lambda^2\sigma_\varepsilon^2}{1-\beta}. \tag{A4}$$

Equation (A3) has two solutions, but only one solution results in a stable control:

$$a = \frac{1}{2\lambda\left(1 - \beta\varphi + \sqrt{(1-\beta)\left(1-\beta\varphi^2\right)}\right)}. \tag{A5}$$

Substituting (A5) back into (A2), we obtain the optimal trading strategy θ^*:

$$\theta_t^* = \frac{V - P_t + \lambda\varphi X_t}{\lambda\left(1 + \sqrt{\frac{1-\beta\varphi^2}{1-\beta}}\right)}.$$

Q.E.D.

Proof of Proposition 2: Following similar steps as in the proof of Proposition 1, one can write the objective of the arbitrageur as:

$$\begin{aligned} J_a = \max_{\{\theta_{a,s}\}} \quad & E_t\left[\sum_{s=t}^{\infty} \beta^{s-t+1}\left((V - P_{s+1})\theta_{a,s} - \gamma X_{a,s}^2\right)\right] \\ \text{s. t.} \quad & P_{s+1} = P_s + \lambda(\theta_s^* + \theta_{a,s} + \varepsilon_{s+1}), \\ & X_{s+1} = X_s + \theta_s^*, \\ & X_{a,s+1} = X_{a,s} + \theta_{a,s}. \end{aligned} \tag{A6}$$

Since

$$\theta_s^* = \frac{V - P_s + \lambda\varphi X_s}{\lambda\left(1 + \sqrt{\frac{1-\beta\varphi^2}{1-\beta}}\right)},$$

the objective (A6) can be written as follows:

$$J_a = \max_{\{\theta_{a,s}\}} \sum_{s=t}^{\infty} \beta^{s-t+1}\left[Z_s^T \;\; \theta_{a,s}\right]\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}\begin{bmatrix} Z_s \\ \theta_{a,s} \end{bmatrix}, \tag{A7}$$

$$\text{s. t.} \quad Z_{s+1} = AZ_s + B\theta_{a,s} + \Omega\varepsilon_{s+1},$$

where $Z_s = (V - P_s, \theta_s^*, X_{a,s})^T$, $B = (-\lambda, -\lambda, 1)^T$, $\Omega = (-\lambda, -\lambda, 0)^T$, $R = -\lambda$, $S = \frac{1}{2}(1, -\alpha, 0)^T$, and

$$A = \begin{pmatrix} 1 & -\alpha & 0 \\ 0 & 1 - \alpha(1-\varphi) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\gamma h \end{pmatrix}, \quad \alpha = \frac{1}{1 + \sqrt{\frac{1-\beta\varphi^2}{1-\beta}}}.$$

Since all eigenvalues of matrix $\beta^{\frac{1}{2}} A$ lie inside the unit circle, a standard result in linear quadratic control (see Anderson, Hansen, McGrattan, and Sargent (1995)) is that the optimal strategy $\theta_{a,s}$ is given by

$$\theta_{a,s}^* = F Z_t,$$

where

$$F = -\left(R + \beta B^T P B\right)^{-1} \left(\beta B^T P A + S^T\right),$$

and P is a unique stabilizing solution of the algebraic Riccati equation:

$$P = \beta A^T P A - \left(\beta A^T P B + S\right)\left(R + \beta B^T P B\right)^{-1}\left(\beta B^T P A + S^T\right) + Q. \tag{A8}$$

Q.E.D.

Proof of Proposition 3: Following similar steps as in the proof of Proposition 1, one can write the objective of the manager as

$$\begin{aligned} \max_{\{\theta_{m,s}\}} \quad & \psi_\pi E_t \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \left((V - P_{s+1})\theta_{m,s} + \lambda\varphi X_s(\theta_{m,s} + \theta_{a,s}^*) \right) \right] \\ \text{s. t.} \quad & P_{s+1} = P_s + \lambda(\theta_{m,s} + \theta_{a,s}^* + \varepsilon_{s+1}), \\ & X_{m,s+1} = X_{m,s} + \theta_{m,s}, \\ & X_{a,s+1} = X_{a,s} + \theta_{a,s}^*. \end{aligned} \tag{A9}$$

and the objective of the arbitrageur as

$$\begin{aligned} J_a = \max_{\{\theta_{a,s}\}} \quad & E_t \left[\sum_{s=t}^{\infty} \beta^{s-t+1} \left((V - P_{s+1})\theta_{a,s} - \gamma X_{a,s}^2 \right) \right] \\ \text{s. t.} \quad & P_{s+1} = P_s + \lambda(\theta_{m,s}^* + \theta_{a,s} + \varepsilon_{s+1}), \\ & X_{m,s+1} = X_{m,s} + \theta_{m,s}^*, \\ & X_{a,s+1} = X_{a,s} + \theta_{a,s}. \end{aligned} \tag{A10}$$

If $\theta^*_{a,t} = F_2 Z_t$, where $F_2 = (f_{21}, f_{22}, f_{23})$, then objective (A9) can be written as

$$\begin{aligned} \max_{\{\theta_{m,s}\}} \psi_\pi \sum_{s=t}^{\infty} \beta^{s-t+1} \begin{bmatrix} Z_s^T & \theta_{m,s} \end{bmatrix} \begin{bmatrix} Q_1 & S_1 \\ S_1^T & R \end{bmatrix} \begin{bmatrix} Z_s \\ \theta_{m,s} \end{bmatrix}, \\ \text{s. to} \quad Z_{s+1} = A_1 Z_s + B_1 \theta_{1,s} + \Omega \varepsilon_{s+1}, \end{aligned} \tag{A11}$$

where $\beta = 1 - \delta$, $A_1 = I + B_2 F_2$, $B_1 = (-\lambda, 1, 0)^T$, $B_2 = (-\lambda, 0, 1)^T$, $\Omega = (-\lambda, 0, 0)^T$, $S_1 = \frac{1}{2}[(1, \lambda\varphi_1, 0) - \lambda F_2]^T$, $R = -\lambda$, and

$$Q_1 = \lambda\varphi_1 \begin{pmatrix} 0 & f_{21}/2 & 0 \\ f_{21}/2 & f_{22} & f_{23}/2 \\ 0 & f_{23}/2 & 0 \end{pmatrix}.$$

Similarly, if $\theta^*_{m,t} = F_1 Z_t$, where $F_2 = (f_{11}, f_{12}, f_{13})$, then objective (A10) can be written as

$$\begin{aligned} \max_{\{\theta_{a,s}\}} \sum_{s=t}^{\infty} \beta^{s-t+1} \begin{bmatrix} Z_s^T & \theta_{a,s} \end{bmatrix} \begin{bmatrix} Q_2 & S_2 \\ S_2^T & R \end{bmatrix} \begin{bmatrix} Z_s \\ \theta_{a,s} \end{bmatrix}, \\ \text{s. to} \quad Z_s = A_2 Z_{s-1} + B_2 \theta_{1,s} + \Omega \varepsilon_s, \end{aligned} \tag{A12}$$

where $A_2 = I + B_1 F_1$, $S_2 = \frac{1}{2}[(1, 0, \lambda\varphi_2) - \lambda F_1]^T$, and

$$Q_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 - \gamma h \end{pmatrix}.$$

Suppose a linear, closed loop Nash equilibrium exists, with F_1 and F_2 being equilibrium strategies of the manager and the arbitrageur. Then,

$$F_1 = -\left(R + \beta B_1^T P_1 B_1\right)^{-1} \left(\beta B_1^T P_1 A_1 + S_1^T\right), \tag{A13}$$

where P_1 solves Riccati equation

$$P_1 = \beta A_1^T P_1 A_1 - \left(\beta A_1^T P_1 B_1 + S_1\right) \left(R + \beta B_1^T P_1 B_1\right)^{-1} \left(\beta B_1^T P_1 A_1 + S_1^T\right) + Q_1, \tag{A14}$$

and

$$F_2 = -\left(R + \beta B_2^T P_2 B_2\right)^{-1} \left(\beta B_2^T P_2 A_2 + S_2^T\right), \tag{A15}$$

where P_2 solves Riccati equation

$$P_2 = \beta A_2^T P_2 A_2 - \left(\beta A_2^T P_2 B_2 + S_2\right)\left(R + \beta B_2^T P_2 B_2\right)^{-1}\left(\beta B_2^T P_2 A_2 + S_2^T\right) + Q_2. \quad \text{(A16)}$$

Alternatively, any solution to (A13)-(A16) constitutes a closed-loop equilibrium.

Q.E.D.

Proof of Proposition 4: Suppose there exists a pure strategy equilibrium where the arbitrageur enters. For ease notation, denote this period by 0. By assumption, the arbitrageur arrives after the steady state described in Section 2 is reached. At this point,

$$\varphi X_0 = -(V - P_0), \quad \text{(A17)}$$

and if the arbitrageur does not enter the manager's continuation utility is zero.

Denote the equilibrium strategy of the arbitrageur by $\theta_0^a, \theta_1^a, \theta_2^a \ldots$ and that of the manager by $\theta_1^m, \theta_2^m, \ldots$. Because the manager does not know when the arbitrageur arrives to the market he becomes aware of the arbitrageur and reacts to her actions only after she enters.

To prove the proposition it is enough to show that it is always in the interest of the manager to ensure that the arbitrageur's fund is liquidated in period 2 following her entrance. Indeed, if the fund is going to be liquidated at time 2 then the only case when the arbitrageur avoids the liquidation if the arbitrage closes at time 2. Hence, the expected utility of the arbitrageur is

$$\delta h(V - P_0 - \lambda\theta_0^a)\theta_0^a - \gamma h(\theta_0^a)^2 - (1-\delta h)\chi, \quad \text{(A18)}$$

which is less than zero by the assumption of the proposition. Therefore, the arbitrageur does not enter the market.

Without loss of generality, one can assume that both $\theta_0^a < 0$ and $\theta_1^a \leq 0$. If $\theta_0^a > 0$, then the arbitrageur acquires the asset at the price above its fundamental level, and therefore trades at a loss. If $\theta_1^a > 0$, then even in the absence of the manager's response, the marked-to-market value of the arbitrageur's position at time 2 is negative, and therefore the fund is liquidated.

If $\theta_1^a < 0$, to ensure liquidation, the manager has to buy $\theta_1^m = -\theta_1^a + \varepsilon$, where $\varepsilon > 0$ is an arbitrary small number. In what follows, I assume that $\theta_1^m = -\theta_1^a$. If the arbitrage closes at time 2, then the manager's actions have no effect on his profit. Therefore, consider the manager's utility conditional on the arbitrage not closing at time 2.

Suppose the manager plays $\theta_1^m = -\theta_1^a$ and $\theta_2^m = -\theta_1^m = \theta_1^a$. Then, in period 3, with probability δh the arbitrage closes. In this case, the trade submitted at time 2 is executed at the fair price V so the only trade that has a nontrivial impact on the manager's profit is his trade at time 1. In this trade, the manager buys $-\theta_1^a$ units of the risky asset at the price $P_0 + \lambda(\theta_0^a + \theta_1^a + \theta_1^m) = P_0 + \lambda\theta_0^a$. Thus, the profit from this trade is

$$(V - P_0 - \lambda\theta_0^a)\theta_1^m. \tag{A19}$$

If the arbitrage does not close at time 3, which happens with probability $1-\delta h$, then the manager sells his position at time 3 at the price $P_0+\lambda(\theta_0^a+\theta_1^a+\theta_1^m+\theta_2^m-(\theta_0^a+\theta_1^a)) = P_0$ (because the arbitrageur's fund is liquidated). Hence, the manager's profit from the trade is

$$(P_0 - P_0 - \lambda\theta_0^a)\theta_1^m = \lambda\theta_0^a\theta_1^a. \tag{A20}$$

Therefore, the total expected profit from the trade is

$$\psi_\pi \left(\delta h(V - P_0 - \lambda\theta_0^a)\theta_1^m + (1 - \delta h)\lambda\theta_0^a\theta_1^a\right). \tag{A21}$$

To compute the total impact of the trade on the manager's utility one also needs to consider its contribution to the interim profit. The gain from buying extra θ_1^m units is

$$\psi_w phX_0\lambda\theta_1^m. \tag{A22}$$

Since

$$\psi_w phX_0\lambda = -\psi_\pi\delta h(V - P_0) \quad \Leftrightarrow \quad \varphi X_0 = -(V - P_0) \tag{A23}$$

the total effect of the manager's trade on his utility is

$$\psi_\pi\lambda\theta_0^a\theta_1^a \geq 0. \tag{A24}$$

Thus, if the arbitrage does not close at time 2, the manager can always ensure his continuation utility to be nonnegative if he forces fund liquidation at time 2. Hence, in any equilibrium, conditional on the arbitrage not closing at time 2, the arbitrageur's total expected profits from trade should be nonpositive (since any profits to the arbitrageur results in a loss of utility for the manager).

This, in turn, implies that the expected trading profit of the arbitrageur at period 3, and all subsequent periods should be nonpositive as well. Otherwise, the arbitrageur, following periods of positive expected profits, will be tempted to deviate, facing a prospect of negative expected profits. But the only way for the arbitrageur to have nonpositive expected profits in period 3 is if she liquidates her short position in the asset. This leads to a negative marked-to-market profit and triggers the fund liquidation. Thus, the arbitrageur is better off not entering the market.

Finally, the no-entry equilibrium can be supported if, following entry by the arbitrageur, the manager commits to prop up the price by buying $X(\gamma)$ units of the risky asset, where $X(\gamma)$ is such that the arbitrageur with holding costs γ will never want to sell more than $X(\gamma)$ units.

Q.E.D.