Sequential Credit Markets

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ABSTRACT

Entrepreneurs who seek financing for projects typically do so in decentralized markets where they need to approach investors sequentially. We study how well such sequential markets allocate resources when investors have expertise in evaluating investment opportunities, and how surplus is split between entrepreneurs and financiers. Contrary to common belief, we show that the introduction of a credit registry that tracks the application history of a borrower leads to more adverse selection, quicker market break down, and higher rents to investors which are not competed away even as the number of investors grows large. Although sequential search markets lead to substantial investment inefficiencies, they can nevertheless be more efficient than a centralized exchange where excessive competition may impede information aggregation. We also show that investors who rely purely on public information in their lending decisions can out-compete better informed investors with soft information, and that an introduction of interest rate caps can increase the efficiency of the market.

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The main role of primary financial markets is to channel resources to firms with worthwhile projects. This process requires information about investment opportunities, technological feasibility, management ability, current industry and macroeconomic conditions, as well as expertise in using such information. No single investor typically possesses all relevant pieces of information. Therefore, the efficiency of the capital allocation process depends on how well markets aggregate this information, which in turn depends on how they are organized.

At least until very recently, the majority of primary capital markets for small- and medium sized firms operate as decentralized search markets. This is true whether firms are seeking capital from banks or from equity investors such as business angels and venture capitalists. Historically, transparency of these markets has been limited but advances in technology over the last decades have paved the way to many fintech solutions that made these markets more transparent and even brought some market activity to centralized market places such as peer-to-peer and crowdfunding platforms.

In this paper, we ask which markets are socially optimal and which markets are better for entrepreneurs. Do more transparent markets lead to better investment decisions and a lower cost of capital for entrepreneurs, and what are the implications for regulation? We develop a general but tractable decentralized search model of credit markets to study these questions, and contrast the results with the ones we have developed in a companion paper on centralized markets (Axelson and Makarov (2020)). We show that contrary to common intuition, decentralized search markets can be more efficient at aggregating information than centralized markets. Even more surprisingly, we show that increased transparency can lead to worse lending decisions and lower surplus for entrepreneurs. Finally, policies such as interest rate caps can lead to more efficient decentralized markets but higher rent for investors.

We consider a setting in which an entrepreneur with a project idea searches for credit by approaching potential financiers sequentially (one-by-one). There is uncertainty about whether the project is worthwhile or not. Each investor, if approached, can do due diligence which results in a private signal about the prospects of the project. The search continues until the entrepreneur either finds an investor who is willing to accept her terms for financing the project or runs out of options and abandons the project.

In this paper, we want to abstract from the friction introduced by the cost of finding a counterparty and therefore assume that the entrepreneur is infinitely patient and has no search cost. Instead, we focus on the friction introduced by dispersed information and sequential interactions. There are two problems that impede information aggregation. First, whenever the entrepreneur comes to an agreement with an investor, information aggregation stops although there is potentially valuable information held by investors who have not yet been approached. Second, when the entrepreneur does not come to an agreement with an investor and continues her search, not all of the information held by the investor is passed on to the next investor she meets. In particular, each new investor faces an adverse selection problem created by the fact that the entrepreneur failed to receive financing in previous interactions with informed investors.

This adverse selection depends on what is observed by investors. We study two cases of market transparency: with and without a credit registry. In general, a credit registry may perform several functions. First, it may produce information about credit quality of the project such as identification data, business owner data, and payment and loan history data. This function is well understood and has been extensively studied in the prior literature.[1] Therefore, in this paper we focus on the second function. Namely, that a credit registry can record how many credit checks have been performed on the entrepreneur in the past. This information allows investors to deduce how many times the entrepreneur has applied for financing previously and can be particularly important in the VC or PE space for the valuation of early stage projects and firms. We refer to the case where application history is observable as the credit registry case.

In the no credit registry case, an investor does not know how many other investors the entrepreneur has visited before. This is commonly the case in less developed countries, in informal lending markets, and in non-bank markets such as when an entrepreneur seeks angel- or venture capital financing. Importantly and consistent with practice, irrespective of whether there is a credit registry or not, investors do not observe financing terms at which the entrepreneur was rejected.

As a result, the impact of a rejection on the beliefs of remaining investors depends on the terms at which they believe the entrepreneur was rejected. If investors believe the entrepreneur asked for financing at favorable terms (a low interest rate), a rejection may not be very bad news. But if investors believe the entrepreneur offered a high interest rate, a rejection is really bad news, and can locked out the entrepreneur of the market—a situation when the entrepreneur cannot get financing because even an investor with the most optimistic signal will think that the project is negative NPV.

In equilibrium, beliefs should be consistent with actual financing offers. This puts constraints on the offers that can be supported in equilibrium. We show that with a credit registry in place, the entrepreneur cannot credibly ask for favorable terms. Because financing terms are not observable, the entrepreneur cannot affect the beliefs of investors and improve her prospects in future rounds by asking for more favorable terms in the current round. Asking for favorable terms without the ability to affect the beliefs of future investors, however, is costly for the entrepreneur because it means

[1] See, for example, Jappelli and Pagano (1993) and Padilla and Pagano (1997).

high probability of rejection. As a result, the entrepreneur is biased towards offering less favorable terms.

When there is no credit registry, an investor cannot verify how many times an applicant has been rejected previously. This is potentially bad for an entrepreneur who has not been rejected, since she might be pooled with rejected entrepreneurs with worse credit quality. A first-time applicant therefore has an incentive to signal her type, and we show that she will always be able to do so by asking for more favorable financing terms. This is a credible signal, because a request for more favorable terms has a higher probability of rejection, and rejection is less costly for a first-time applicant who has many investors left to visit. This logic extends to all rounds, leading to a fully separating equilibrium, in which the need for signalling creates a credible way for the entrepreneur to ask for favorable terms.

Asking for favorable financing terms has two consequences. First, it reduces the rents to investors. We show that as the number of potential investors grows large, investors' rent is competed away in the case of no credit bureau. In contrast, with a credit registry, investors continue to earn significant rents even though the entrepreneur has all bargaining power. The rent can be so high that uninformed investors who can commit to use only public information are sometimes able to out compete investors with both public and private information. The reason is that uninformed investors never earn any rents, which for high credit quality entrepreneurs can make them more attractive despite the lower surplus created.

Second, asking for favorable financing terms leads to more financing rounds relative to the case with a credit registry because credit quality deteriorates slower with each rejection. In the case of no credit registry, the entrepreneur can visit all available investors. In contrast, in the case of a credit registry, the entrepreneur might get locked out of the market even after a single rejection.

The benefits of having extended search depend on the informational content of the signal distribution. The way many financing rounds are sustained is by asking for offers that only the most optimistic investor would accept, while less optimistic information is never incorporated in the financing decision. As a result, extended search is desirable in situations where the informational content of the signal distribution is concentrated towards the top. We show that for these situations, as the number of potential investors grows large, the social surplus without a credit registry approaches that attained in a large first-price auction, which is also the maximal possible one.

However, extended search can lead to less informative financing decisions in situations where the informational content of the signal distribution is not concentrated towards the top. In these situations, the market with a credit registry and few financing rounds is more efficient and can dominate even a centralized auction market.

Finally, we show that the sequential market with a credit registry can have multiple equilibria, due to the feedback effect of equilibrium beliefs. When investors believe that rejected borrowers have low credit quality, rejection is more costly for the entrepreneur. Therefore, the entrepreneur is more likely to ask for unfavorable financing terms in early rounds to avoid rejection, which means that rejection is a signal of worse quality—a self-fulfilling prophesy. Hence, equilibria with few financing rounds and equilibria with more financing rounds can coexist. The equilibria with few financing rounds are often worse for entrepreneurs because of the unfavorable financing terms, but can be good for social surplus. This gives the surprising implication that social welfare can be improved if the government imposes an interest rate cap. An interest rate cap can eliminate "sub-prime" markets for rejected borrowers, and hence can eliminate the socially inefficient equilibria with many financing rounds.

Our paper is related to several bodies of work. The efficiency of investment decisions in our model depends on the extent to which information is aggregated. Starting with Hayek (1945) and Grossman (1976) there is a large literature that studies information aggregation in financial markets. The closest papers in this literature are those that study herding and informational cascades in sequential decision making, see e.g., Bikhchandani, Hirshleifer and Welch (1992), Welch (1992), and Avery and Zemsky (1998). Bikhchandani, Hirshleifer and Welch (1992) and Welch (1992) consider nontradable assets; Avery and Zemsky (1998) focus on tradable assets. Our setup is closer to that in Bikhchandani, Hirshleifer and Welch (1992) and Welch (1992) but unlike Bikhchandani, Hirshleifer and Welch (1992) and Welch (1992) who assume the same exogenous offers in all rounds, we allow the entrepreneur to adjust her offers in different rounds. Therefore, in our setup, herding does not always occur in equilibrium as in Bikhchandani, Hirshleifer and Welch (1992) and Welch (1992), and whether it exists or not depends on the signal distribution.

Similar to us, Bulow and Klemperer (2009), Roberts and Sweeting (2013), and Glode and Opp (2017) study relative efficiency of sequential and centralized markets. However, the economic mechanism in our paper is very different from those in the above papers. First, all three papers study selling mechanisms of an existed asset. Because information generated in a selling mechanism has no value for production information aggregation plays no role in their models. In their settings, having as many potential buyers as possible is always good for a seller, which is not necessarily the case in our setup. Second, Bulow and Klemperer (2009) and Roberts and Sweeting (2013) assume nonzero participation costs, and Glode and Opp (2017) assume nonzero costs of information acquisition. These cost are the main sources of inefficiency in their models. In contrast, these costs are not present in our model, in which the main cause of inefficiency is imperfect information aggregation.

Another related work is Lauermann and Wolinsky (2016), who study a decentralized search setup with a seller searching for buyers. Lauermann and Wolinsky (2016) consider the case of an infinite number of buyers and assume that search history is not observable. In their analysis, Lauermann and Wolinsky (2016) focus on pooling equilibria and conclude that search markets are worse at aggregating information than the centralized markets. In contrast, we show that with finite but arbitrary large number of buyers there is a separating equilibrium, which can be as efficient at aggregating information as centralized markets. In addition, we consider the case in which search history is observable and show that search markets with a credit registry can be more efficient than centralized markets. Our work is also related to Zhu (2012) who considers a model of opaque over-the-counter markets. In his model, it is buyers and not the seller who make take-it-or-leave it offers. Similar to Lauermann and Wolinsky (2016), Zhu (2012) considers a sale of an existing asset, assumes that search history is not observable, and studies only pooling equilibria. As a result, both the focus and analysis of Zhu (2012) are significantly different from those in our paper.

More broadly, we also relate to large literature on search markets. Many papers in this literature focus on the friction introduced by the cost of finding a counter-party in private value environments (see, e.g., Duffie, Garleanu and Pedersen (2005), Lagos and Rocheteau (2009), Vayanos and Weill (2008), Weill (2008)). We differ from this literature by focusing on the consequences of sequential interactions in the common-value environment, where the entrepreneur is infinitely patient and has no search cost.

Our paper is also related to the literature on relationship lending started with a seminal paper by Rajan (1992). In common with papers in this literature, when an informed lender refuses credit in our model he creates adverse selection for other borrowers, but in the context of a first-time borrower rather than an existing borrower.

Finally, similar to Fishman and Parker (2015) we show that there could be multiple equilibria with different amounts of screening. However, the economic mechanism that leads to multiple equilibria in our paper is different from theirs. Fishman and Parker (2015) assume that information acquisition is costly and that informed investors have full bargaining power. Higher amounts of screening lead to lower average prices of rejected and unscreened projects, and therefore, to greater returns to screening, making it possible for multiple equilibria to exist. In our setting, there are no costs of information acquisition and it is the entrepreneur who has full bargaining power. Multiple equilibria can be supported by different investors' beliefs about financing terms offered by the entrepreneur.

1. Setup

We consider a penniless entrepreneur seeking financing to start a new project from a set of $N < \infty$ investors. All agents are risk neutral. The project requires one unit of investment, and can be of two types: good ($\theta = G$) or bad ($\theta = B$). The good project pays $1 + X$, while the bad projects returns 0. The ex ante probability that the project is good is π_0.

No one knows the project type but investors have access to a screening technology. When an investor makes an investigation, he gets a private signal S_i about the project type. Conditional on the project type θ, signals are drawn identically and independently on $[0, 1]$ with conditional densities $f_G(s)$ and $f_B(s)$ satisfying the stict maximum likelihood ratio property (MLRP):

Assumption 1:

$$\forall s > s', \quad \frac{f_G(s)}{f_B(s)} > \frac{f_G(s')}{f_B(s')}.$$

Assumption 1 ensures that higher signals are better news than lower signals.[2] Without loss of generality, we assume that $f_G(s)$ and $f_B(s)$ are left-continuous and have right limits everywhere. We also assume that $f_B(1) > 0$, and that the likelihood ratio $f_G(1)/f_B(1)$ at the most optimistic signal realization $s = 1$ is bounded. These assumptions ensure that the observation of a single signal can never rule out the possibility of the project being bad, while an observer of all signals will be able to learn the project type perfectly as the number of investors goes to infinity.

To exclude the trivial case when the project is never financed we assume that the project is positive NPV conditional on the top signal of a single investor:

Assumption 2: $\Pr(G|S_i = 1)X > \Pr(B|S_i = 1)$.

To streamline the exposition, we also assume that the project is negative NPV conditional on the lowest signal of a single investor:

Assumption 3: $\Pr(G|S_i = 0)X > \Pr(B|S_i = 0)$.

Assumption 3 is not essential for our results, what matters is that the investment decision is non-trivial conditional on observing a sufficient number of signals, which is already guaranteed by Assumption 1.

[2]The assumption of strict MLRP is for simplicity. It allows us to focus on pure strategy equilibrium. All results go through under the weaker assumptions that signals satisfy weak MLRP: $\forall s \geq s',\ f_G(s)/f_B(s) \geq f_G(s')/f_B(s')$.

The entrepreneur contacts investors sequentially in a random order indexed by $i \in \{1, ..., N\}$. When contacting investor i the entrepreneur makes an exclusive take-it-or-leave-it offer, in which she asks for the loan size of one to start the project in exchange for the repayment of $1 + r_i$ in case the project is successful. If the project is unsuccessful, both the entrepreneur and the investor receive zero payoffs. If there is no credit registry, investors rely solely on their own signal and any information volunteered by the entrepreneur when making their financing decision. With a credit registry in place, investors access any information collected by the registry by performing a credit check. In particular, they see how many credit checks have been performed on the entrepreneur in the past and therefore deduce how many times the entrepreneur has applied for financing previously.

Based on his signal and other available information, investor i decides whether to accept the offer or not. If the offer is rejected the entrepreneur goes to investor $i + 1$. For simplicity, we assume that the entrepreneur commits not to visit the same investor twice. It is clearly in the interest of the entrepreneur to commit not to re-visit the same investor when there is only one investor available. With many investors the situation is less clear. We show that our main results still hold if we allow multiple visits.

If the offer is accepted the project is financed and production starts. We do not allow the entrepreneur to "shop around" by showing an accepted offer to other investors in the hope of getting better financing terms. This assumption of exclusivity is important and should be viewed as one of the defining properties of sequential markets. Since we abstract from any search costs and costs of generating information allowing the entrepreneur to take accepted offers to other investors without losing them would make the resulting mechanism look similar to a competitive centralized market place, which we study in detail in Axelson and Makarov (2020).

2. Maximal social surplus

In any of the settings we study, a strategy for the entrepreneur is a set of interest rates $\{r_i\}_{i=1}^N$ offered in sequence to investors $i \in \{1, ..., N\}$ until some investor accepts. As a benchmark, we first derive the maximal social surplus achievable by a social planner who can publicly commit to a set of interest rate offers and a sequence in which investors are approached.

We first make the observation that picking a vector of offers $\{r_i\}_{i=1}^N$ is equivalent to picking a set of screening thresholds $\{s_i^*\}_{i=1}^N$ such that the project gets started if only if there is an investor i with signal S_i above threshold s_i^*. To see this, consider investor i who is approached with an offer of financing the project at interest rate r_i.

His expected profit from accepting to finance the project given his own signal $S_i = s$ is

$$\Pr(G|\Omega_i, S_i = s)r_i - \Pr(B|\Omega_i, S_i = s),$$

where Ω_i is the information that all previous investors $j < i$ have rejected the project at interest rates r_j. Hence, the investor accepts the offer if and only if

$$r_i \geq \frac{\Pr(B|\Omega_i, S_i = s)}{\Pr(G|\Omega_i, S_i = s)} = \frac{\Pr(B|\Omega_i)}{\Pr(G|\Omega_i)} \frac{f_B(s)}{f_G(s)}, \tag{1}$$

where the last equality follows from Bayes' rule and the mutual independence of signals conditional on the project type. MLRP implies that the right-hand side in Equation (1) decreases in s. Therefore, the project is either rejected for any signal, or there is a unique screening level s_i^* such that the offer is accepted if and only if $S_i \geq s_i^*$.

Denote the vector of screening thresholds $\{s_j^*\}_{j=1}^{i-1}$ used prior to round i by $\mathbf{s}_{\mathbf{i-1}}^*$. Equation (1) together with strict MLRP imply that the interest rate offer $r_i(s_i, \mathbf{s}_{\mathbf{i-1}}^*)$ in round i that implements screening threshold s_i is given by

$$r_i(s_i, \mathbf{s}_{\mathbf{i-1}}^*) = \frac{\Pr(B|S_1 < s_1^*, ..., S_{i-1} < s_{i-1}^*, S_i = s_i)}{\Pr(G|S_1 < s_1^*, ..., S_{i-1} < s_{i-1}^*, S_i = s_i)}. \tag{2}$$

We can now write the social planner's surplus maximization problem as a choice of screening thresholds $\{s_i^*\}_{i=1}^N$. The project is started whenever there is at least one signal S_i above screening threshold s_i^*. Therefore, social surplus is equal to

$$\max_{\{s_i^*\}_{i=1}^N} \pi_0 \overline{\Pr}(S_1 < s_1^*, ..., S_N < s_N^*|G)X - (1 - \pi_0)\overline{\Pr}(S_1 < s_1^*, ..., S_N < s_N^*|B), \tag{3}$$

where

$$\overline{\Pr}(S_1 < s_1^*, ..., S_N < s_N^*|\theta) = 1 - \Pr(S_1 < s_1^*, ..., S_N < s_N^*|\theta), \quad \theta = \{B, G\}.$$

Proposition 1: *(i) If* $\frac{f_G(s)}{F_G(s)}/\frac{f_B(s)}{F_B(s)}$ *is a strictly increasing function then social surplus strictly increases with* N*. The socially optimal screening policy is to use the same screening threshold* s^* *in all rounds. The optimal screening threshold is the lowest signal at which investor* N *breaks even at the maximal interest rate* X*:*

$$\Pr(G|\max_{i\leq N} S_i = s^*)X - \Pr(B|\max_{i\leq N} S_i = s^*) \geq 0. \tag{4}$$

(ii) If $\frac{f_G(s)}{F_G(s)}/\frac{f_B(s)}{F_B(s)}$ *is a decreasing function on* $[\hat{s}_n, 1]$*, where* $\hat{s}_n$ *is the lowest signal at*

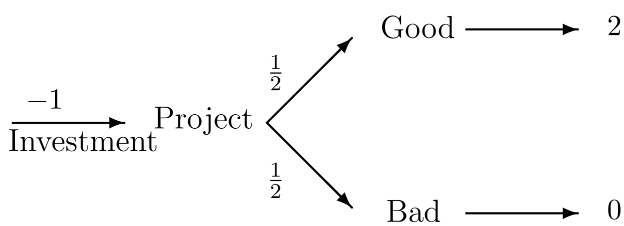
which investor $n \leq N$ breaks even at the maximal interest rate X:

$$\Pr(G|\max_{i\leq n} S_i = \hat{s}_n)X - \Pr(B|\max_{i\leq n} S_i = \hat{s}_n) \geq 0 \tag{5}$$

then the maximal social surplus is achieved with no more than n screenings.

Proof: See the Appendix.

Proposition 1 shows that the social planner may find it optimal to restrict the number of screening rounds. To understand this result, consider first a particular example, where $X = 1$ and $\pi_0 = 1/2$.



Suppose investors can get a high or a low signal about the project, with $\Pr(H|G) = 1$, $\Pr(H|B) = 1/2$. Consider first the case with a single investor. If the investor gets a low signal, he learns that the project is bad, since good projects never generate low signals. Therefore, he will not finance the startup. If the investor gets a high signal, he updates the probability that the project is good to $2/3$:

$$\Pr(G|H) = \frac{\Pr(H|G)\Pr(G)}{\Pr(H|G)\Pr(G) + \Pr(H|B)\Pr(B)} = \frac{2}{3}.$$

Therefore, conditional on a high signal, the project is positive NPV:

$$V^H = \frac{2}{3} \times 1 - \frac{1}{3} \times 1 = \frac{1}{3},$$

and the expected surplus is

$$\Pr(H) \times V^H = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}.$$

Now suppose there are two investors. If the first investor invests whenever he gets a high signal then the second investors will never finance the project since the rejected project is sure to be bad. Suppose the social planner prescribes the first investor to play a mixed strategy, that is upon receiving a high signal to finance the project with some probability $0 \leq \mu \leq 1$. What will happen to the surplus?

For states where the first investor finances the project, there is no change since the project is still financed. We therefore compare investment efficiency on the set of projects the first investor now passes on, which consists of all projects he gets a low signal on and a fraction $(1-\mu)$ of projects he gets a high signal on.

Projects from this pool will now be financed if and only if the second investor gets a high signal. He therefore finances a good project from the pool with probability $\Pr(H|G) = 1$ and a bad project with probability $\Pr(H|B) = 1/2$. The original investor, when he was the only investor, also financed all good projects out of the pool. Of the bad projects in the pool, he invested in the ones where he erroneously received a high signal, which consists of a fraction

$$\frac{\Pr(H|B)(1-\mu)}{\Pr(H|B)(1-\mu)+\Pr(L|B)} = \frac{\frac{1}{2}(1-\mu)}{\frac{1}{2}(1-\mu)+\frac{1}{2}} = \frac{1-\mu}{2-\mu},$$

which is lower than 1/2. Therefore, the screening of the first investor on this pool when he is alone is more efficient, so surplus goes down whenever the first investor plays a mixed strategy.

In the above example, increasing screening threshold of the first investor creates a negative externality on the quality of the remaining projects: the updated likelihood that the project is good is multiplied by the factor $F_G(s^*)/F_B(s^*)$. In a general case, increasing screening threshold also has a positive effect on the quality of the marginal project: the updated likelihood that the project is good is multiplied by the factor $f_G(s^*)/f_B(s^*)$. The net effect depends on the behavior of $\frac{f_G(s)}{f_B(s)}/\frac{F_G(s)}{F_B(s)}$. If $f_G(s)/f_B(s)$ increases faster than $F_G(s)/F_B(s)$ then increasing existing screening thresholds and having an extra screening leads to a more efficient outcome. If, on the contrary, $f_G(s)/f_B(s)$ increases slower than $F_G(s)/F_B(s)$ then the ability to have extra screening does not lead to any efficiency gains.

Proposition 1 shows that in the case when surplus increases with the number of investors it is best to set all screening thresholds to the same level s^* so that the project just breaks even when $\max\{S_1, S_2, \ldots, S_N\} = s^*$. In Axelson and Makarov (2020) we show that this is also the investment outcome realized in a first-price auction with N bidders. Thus, no sequential credit market can generate higher surplus than a first-price auction in this case.

The situation is different in the case when having large number of investors does not lead to efficiency gains. First, the social planner may choose different screening thresholds across investors.[3] Second, if the entrepreneur is unable to commit to restrict the number of investors the market size may be inefficiently large[4]. This happens

[3]It can be verified that $s_1^* = 0.32$ and $s_2^* = 0.664$ maximize social surplus if $X = 1$, $\pi_0 = 1/1.35$, $f_B(s) = 1$, and $f_G(s)$ is as defined in equation (13).

[4]Restricting the set of potential investors may be difficult in practice because it is ex post optimal for the entrepreneur to consider any offer he receives, even if the offer is unsolicited.

because the marginal investor does not internalize the negative externality he imposes on allocational efficiency when he enters the market.

In the next section we show that a sequential market without a credit registry will always lead to a maximum number of screenings, which is optimal when the social planner prefers large markets but reduces social surplus when the planner prefers small markets. In Section 4, we show that the introduction of a credit registry endogenously limits the size of the market, which can increase surplus when the social planner prefers small markets. However, the introduction of a credit registry will always reduce the fraction of surplus going to the entrepreneur.

3. Equilibrium without a credit registry

We now turn to the least transparent case in which neither previous offers nor rejections are observed by an investor who is approached for financing. The only information available to an investor in this case is the interest rate he is being offered. An important implication of the fact that the entrepreneur earns nothing unless the project is good is that her optimal strategy is independent of her information about the success probability—she always acts to maximize her payoff conditional on the project being successful. However, the interest rate the entrepreneur asks may provide useful information about how many times she has been rejected previously and on which terms.

Our main result in this section is to show that under suitable restrictions on out-of-equilibrium beliefs, only fully separating equilibria exist and that as the number of investors increases, the entrepreneur extracts all the surplus, and the surplus converges to that realized in a first-price auction. Separation obtains because entrepreneurs with few rejections find it profitable to separate from entrepreneurs who has been rejected many times. They do so by increasing their interest rate offers after each rejection. The probability of acceptance increases with the interest rate offer. This makes it increasingly costly for an entrepreneur with many rejections and only few investors left to visit to ask for a low interest rate.

Consider a candidate separating equilibrium in which $\{r_i\}_{i=1}^N$, $r_i \neq r_j$ for $i \neq j$ is a set of interest rate offers made by the entrepreneur. In a separating equilibrium, having observed an interest rate offer, investors correctly infer how many times the entrepreneur has been rejected and what interest rates were offered in previous rounds. Hence, equilibrium screening thresholds $\{s_i^*\}_{i=1}^N$ associated with interest rates offers are as defined in Equation (2).

We now formulate the incentive compatibility constraints that must hold so that the entrepreneur will not find it profitable to deviate in round i and ask for interest rate r_j, $j \neq i$. Let V_i denote the expected surplus of the entrepreneur in the beginning of financing round i conditional on the project being good. The expected surplus V_i is the sum of two terms: the expected surplus if the offer is accepted

$$U_i = (1 - F_G(s_i)) \left(X - r_i(s_i, \mathbf{s}^*_{\mathbf{i-1}}) \right),$$

and continuation value $F_G(s_i)V_{i+1}$ if the offer is rejected. Thus, $V_N = U_N$ and

$$V_i = U_i + F_G(s_i)V_{i+1}, \quad i = N-1, ..., 1. \tag{6}$$

To be incentive compatible, a set of interest rate offers must be such that the entrepreneur in financing round i would not be tempted to deviate and quote a different interest rate:

$$V_i \geq U_j + F_G(s_j)V_{i+1}, \quad j \neq i. \tag{7}$$

The incentive compatibility constraints (7) together with (6) imply that for any $i > j$

$$(F_G(s_j) - F_G(s_i))V_{j+1} \geq U_i - U_j \geq (F_G(s_j) - F_G(s_i))V_{i+1}. \tag{8}$$

Since $V_{i+1} < V_{j+1}$, for inequalities (8) to hold it must be that $s_j > s_i$. In other words, the probability of receiving financing must increase with the number of rejections. Because the entrepreneur always prefers lower interest rate for a given probability of being financed, interest rate offers must increase with the number of rejections.

Further inspection of (8) reveals that if the incentive compatibility (IC) constraints (7) hold for all adjacent financing rounds i and $j = i + 1$ then they hold for any rounds i and j. Finally, since entrepreneurs with few rejections would like to separate from entrepreneurs with more rejections the entrepreneur in round i is never tempted to ask for the interest rate r_{i+1}. Thus, only the IC constraints that ensure that the entrepreneur in round $i + 1$ is not tempted to ask for the interest rate r_i can be binding in equilibrium. We can now state our main result in this section.

Proposition 2: *Any equilibrium that survives the Cho and Kreps intuitive criterion must be separating. In any separating equilibrium, interest rates strictly increase and screening thresholds strictly decrease with the number of rejections. The screening* *thresholds s_i^* solve*

$$V_N = \max_{s_N}(1 - F_G(s_N))\left(X - r_N(s_N, \mathbf{s^*_{N-1}})\right),$$

$$V_i = \max_{s_i}(1 - F_G(s_i))\left(X - r_i(s_i, \mathbf{s^*_{i-1}})\right) + F_G(s_i)V_{i+1}, \quad i = N-1, ..., 1 \quad (9)$$

$$s.t.\ (1 - F_G(s_i))\left(X - r_i(s_i, \mathbf{s^*_{i-1}})\right) + F_G(s_i)V_{i+2} \leq V_{i+1}, \quad (10)$$

where interest rates $r_i(s_i, \mathbf{s^*_{i-1}})$ are as defined in Equation (2). As N goes to infinity the entrepreneur extracts all the surplus and the surplus converges to that generated in a first-price auction with N investors.

Proof: See the Appendix.

The proof that the surplus converges to that generated in a first-price auction is based on the fact that screening threshold in the last round converges to one. Since screening thresholds decrease with the number of rejections it also implies that all screening thresholds converge to one as the number of investors increases without bound. As a result, investors earn no rent and the entrepreneur extracts all the surplus in the limit. Notice that screening threshold in the last round can converge to one only if the interest rate offer in the last round converges to the maximal interest rate X. Otherwise, the entrepreneur is better off lowering screening threshold by offering a slightly higher interest rate. Thus, both in a first-price auction and in the sequential setting without credit registry the last investor just breaks even at the maximal interest rate conditional on other investors refusing to finance the project. This implies that the probabilities of financing the project in the two setups converge to each other, which in turn implies the convergence of surplus values.

To see why screening threshold in the last round must converge to one notice that because after each rejection the perception of the project's quality deteriorates, for the entrepreneur to have a chance of obtaining financing in the last round only a bounded number of screening thresholds can stay away from one as N goes to infinity. Therefore, if screening threshold in the last round does not go to one with N then interest rates offers in all rounds are bounded away from X and there will exist an i such that screening threshold in round i converges to one but the screening threshold in round $i+1$ stays bounded away from one. Screening threshold in round i can go to one only if the IC constraint (10) is binding. Otherwise, the entrepreneur again would be better off lowering the screening threshold by offering a slightly higher interest rate. The binding constraint means that $U_i - U_{i+1} = (F_G(s_{i+1}) - F_G(s_i))V_{i+2}$. The right-hand side of this equation is positive. But the left-hand side is negative since U_{i+1} is bounded away from zero and U_i goes to zero. Thus, the IC constraint cannot be binding, which shows that screening threshold in the last round must converge to one.

4. Equilibrium with a credit registry

In general, a credit registry may perform several functions. First, it may produce information about credit quality of the project such as identification data, business owner data, and payment and loan history data. This function is well understood and has been extensively studied in the prior literature.[5] Therefore, in this paper we focus on the second function. Namely, that a credit registry can record how many credit checks have been performed on the entrepreneur in the past. This information allows investors to deduce how many times the entrepreneur has applied for financing previously and can be particularly important in the VC or PE space for the valuation of early stage projects and firms. We refer to the case where the sequence is observable as the "credit registry" case.

We assume that with a credit registry in place investors can learn how many times the entrepreneur has been rejected previously but not the terms on which she has been rejected. Therefore, as in the case of no credit registry studied in Section 3, investors have to form beliefs about the terms at which the entrepreneur has been rejected previously. However, now there is no need for the entrepreneur to signal how many times she was rejected. As a result, there is no reason for an investor to change his beliefs about previous offers if he is offered an out-of-equilibrium interest rate offer.

Proposition 3 characterizes an equilibrium. The equilibrium screening thresholds solve the same maximization problem as in the case without a credit registry but without incentive compatibility constraints (10). The proposition shows that without the need to signal her application history the entrepreneur cannot credibly make low interest rate offers and this biases her towards asking for higher interest rates in early rounds in equilibrium.

Proposition 3: *Suppose rejections are publicly observable. Then equilibrium screening thresholds solve*

$$\begin{aligned} V_N &\equiv \max_{s_N}(1-F_G(s_N))\left(X-r_N(s_N,\mathbf{s}^*_{\mathbf{N-1}})\right), \\ V_i &\equiv \max_{s_i}(1-F_G(s_i))\left(X-r_i(s_i,\mathbf{s}^*_{\mathbf{i-1}})\right)+F_G(s_i)V_{i+1}, \quad i=N-1,...,1, \end{aligned} \tag{11}$$

where interest rates $r_i(s_i,\mathbf{s}^*_{\mathbf{i-1}})$ are as defined in Equation (2). There is an $\varepsilon>0$ such that for any number of investors N screening threshold in the first round is less than

[5] See, for example, Jappelli and Pagano (1993) and Padilla and Pagano (1997).

$1-\varepsilon$.

Proof: See the Appendix.

With strict MLRP, the first contacted investor breaks even if he finances the project when his signal is equal to s_1^* and makes a positive expected profit if his signal is above s_1^*. Proposition 3 shows that s_1^* is bounded away from one. Therefore, the expected profit does not vanish in the limit as N goes to infinity. Thus we have:

Corollary 1: *In equilibrium with a credit registry investors earn strictly positive rent, which is not competed away as the number investors grows large.*

The rent collected by investors can be so large that it can outweigh the benefits of informed lending. To illustrate, suppose that $X = 1$, $N = 10$, $f_B(s) \equiv 1$, and $f_G(s) = 2s$. Figure 1 shows social surplus and entrepreneur's profit in two cases: when the entrepreneur's obtains financing from uninformed investors and from investors with private signals. The uninformed investors finance the project if and only if the project is positive NPV:

$$\pi_0 - (1-\pi_0) \geq 0 \quad \Leftrightarrow \quad \pi_0 \geq 1/2.$$

We assume that uninformed investors are competitive so they just break even on their investment and the entrepreneur captures all expected surplus, $2\pi_0 - 1$. We can see that social surplus is always higher in the case of informed financing since more information leads to better investment decisions. In contrast, the entrepreneur can be better off if she raises financing from uninformed investors when the project's quality is high enough. Uninformed investors are able to outcompete informed investors because they do not earn rent. While using both public and private information results in better investment decisions the benefits for high quality projects are not sufficient to compensate the rent that must be surrendered to privately informed investors.

The immediate consequence of Propositions 2 and 3 is that in the case in which it is best to have as large markets as possible the introduction of a credit registry can actually reduce market efficiency and lead to lower profits for the entrepreneur. Consider next the case when there are many potential investors but it is best to have small markets. Proposition 2 shows that without a credit registry the entrepreneur is never excluded from the market and visits potentially all available investors, which is inefficient. Proposition 3 shows that with a credit registry, screening thresholds in the first rounds are lower than they would be if there was no credit registry. While low interest rate offers mean that some rent is left for investors, they also mean that the negative impact of rejections is stronger and can lead to the exclusion of the entrepreneur from the market. When small markets are efficient restricting competition among investors and allowing them to utilize their information more efficiently can lead to higher social surplus and entrepreneur's profit.

To illustrate, consider our example from Section 2. We show in Lemma 1 in the Appendix that social surplus is maximized with a single investor who finances the project if and only if he gets a high signal. This optimal outcome can also be achieved in a sequential market with a credit registry. Suppose the entrepreneur asks in the first round for the interest rate that corresponds to the threshold $s_1^* = 1/2$. This generates the maximal surplus, and all surplus is captured by the entrepreneur. There will be no second round, because if the project is rejected by the first investor, the updated credit quality is so low that no investor would be willing to finance the project at any interest rate. Thus, the market with a credit registry creates more social surplus and more profits for the entrepreneur than the market without a credit registry and centralized auction market. Proposition 4 shows that our example is not an isolated case. When it is best to have small markets a credit registry endogenously restricts the number of investors the entrepreneur can visit. This contrasts with the case of no credit credit in which the entrepreneur can always visit all potential investors.

Proposition 4: *If* $\frac{f_G(s)}{F_G(s)}/\frac{f_B(s)}{F_B(s)}$ *is a strictly increasing function then the entrepreneur is never excluded from the market. If* $\frac{f_G(s)^2}{F_G(s)}/\frac{f_B(s)^2}{F_B(s)}$ *is a strictly decreasing function at some neighborhood of* $s = 1$ *then for large* N *the entrepreneur visits strictly less than* N *investors.*

Proof: See the Appendix.

We conclude this section by noticing that the negative effect of the credit registry comes from the fact that it reveals only partial information about the application history of the entrepreneur. In practice, it might not be feasible for a credit registry to record terms on which an entrepreneur is rejected. However, if it is feasible, the entrepreneur would always prefer the market with a credit registry over that without one because she can always replicate surplus and profits generated in the market without credit registry by offering the same sequence of interest rates.

4.1. Equilibrium existence and uniqueness

So far we have abstracted from the questions of equilibrium existence and uniqueness. In this section, we provide sufficient conditions for equilibrium existence and show that there can be multiple equilibria. We have

Proposition 5: *Suppose that $f_B(s)/f_G(s)$ is a continuous function and for any y*

$$(1 - F_G(s)) \left(y - \frac{f_B(s)}{f_G(s)} \right) \tag{12}$$

is a quasi-concave function of s. Then there exists a pure-strategy equilibrium in sequential markets with any number of investors with or without a credit registry.

Proof: See the Appendix.

Condition (12) is satisfied for a wide range of distributions. For instance, it holds in our examples: $f_B(s) = 1$ and $f_G(s) = \lambda s^{\lambda - 1}$, $\lambda > 1$, or in the case when signals take two values. Notice that condition (12) is only a sufficient condition for the existence of equilibrium. We have not been able to find a case where equilibrium does not exist. Next, we present an example of multiple equilibria.

Suppose $X = 1$, $f_B(s) \equiv 1$, and $f_G(s)$ is given by the following equation:

$$f_G(s) = 0.25 + \frac{1}{\exp\left(-100\left(s - \frac{1}{3}\right)\right) + 1} + \frac{0.25}{\exp\left(-100\left(s - \frac{2}{3}\right)\right) + 1}. \tag{13}$$

The density $f_G(s)$ is plotted in Figure 2 Panel (a). It is a continuous version of the case when investors' signals take three values: low, medium and high as depicted in Figure 2 Panel (b). If the project is bad then any of the values is equally likely. If the project is good then the respective probabilities of low, medium and high signals are 1/12, 5/12, 1/2.

Figure 3 plots the expected profit of the entrepreneur as a function of the screening threshold s when there is only one investor. Panels (a), (b), and (c) correspond to the three values of π_0: 0.47, 0.486, and 0.5. We can see that two flat areas of $f_G(s)$ lead to two humps in the expected surplus. The probability of receiving financing increases with the ex-ante quality of the project: At high values of π_0 the entrepreneur's profit is maximized at low screening thresholds while at low values of π_0 the profit is maximized at high screening thresholds.[6] When $\pi_0 = 0.486$ the same expected surplus is achieved at two different values $s^* = 0.39$ and $s^* = 0.69$. These two values correspond to two equilibria in the case with a single investor.

Even though there is a unique equilibrium with a single investor when $\pi_0 \neq 0.486$ nonmonotocity of the entrepreneur's profit leads to multiple equilibria with two investors and a credit registry in place. In the first equilibrium, the second investor believes that the entrepreneur offers the first investor a high interest rate. This makes rejection very costly for the entrepreneur: If she is rejected she can no longer obtain

[6] The fact that probability of financing increases with ex-ante project's quality is a general result that does not depend on particular specification of signals.

financing from the second investor even if he receives the most optimistic signal. Therefore, the entrepreneur has no choice but to offer a high interest rate to the first investor. In the second equilibrium, the second investor believes that the entrepreneur offers the first investor a low interest rate. In this case, the cost of rejection is relatively low: even if the entrepreneur is rejected by the first investor she has still a chance to obtain financing from the second investor. As a result, it makes optimal for the entrepreneur to ask the first investor for a low interest rate.

For the two equilibria to exist the entrepreneur's choice of screening thresholds must be consistent with investors' beliefs. This happens if initial prior π_0 lies in the interval $(0.486, 0.507)$. If π_0 is just below 0.95 then only the second equilibrium with two financing rounds exists because even with a single investor the entrepreneur is better off by offering the first investor a low interest rate. If π_0 is just above 0.507 then only the first equilibrium with one financing round exists because even if the second investor believes that the interest rate in the first round is low the entrepreneur finds it profitable to deviate and offer a high interest rate. As a result, the entrepreneur can no longer take advantage of two investors.

Figure 4, panel (a) plots the entrepreneur's expected profit in the two equilibria as a function of the ex-ante project's quality π_0; panel (b) plots social surplus. We can see that the entrepreneur is better off in equilibrium with two financing rounds but social surplus is higher in equilibrium with one financing round. The ability to ask for a lower interest rate in the first round reduces investors' rent so much that it compensates for a decrease in total surplus.

Panel (a) illustrates that the entrepreneur's profit can be non-monotone in the ex-ante project's quality. If $\pi_0$is just above 0.507 the entrepreneur is unable to commit to offer a low interest to the first investor. As a result, investors get higher rent and the entrepreneur is worse off compared to the equilibrium with two financing rounds when π_0 is just below 0.507.

Since social surplus is higher in equilibrium with one financing round a policy that imposes an interest rate cap can be welfare improving. If the entrepreneur is rejected by the first investor then she will have to offer a higher interest rate to the second investor. If there is an interest rate cap the rejected entrepreneur might no longer be able to obtain financing in the second round. As a result, the second equilibrium with two financing rounds will no longer be sustainable. Thus, an interest rate cap can eliminate socially inefficient equilibria with many financing rounds.

5. Conclusion

We have developed a sequential credit market model to analyze the efficiency of primary capital markets for new projects. We compare two cases with different level of transparency: A sequential market where lenders have no information about the search history of an entrepreneur, a sequential market where lenders can observe the search history via a credit registry. None of these markets lead to first-best investment decisions, even when the number of potential investors grows so large that the aggregate information in the market allows for perfect investment decisions, and even when entrepreneurs are infinitely patient and there are zero search costs. Moving to a more transparent market via the introduction of a credit registry tends to increase rents to investors at the expense of entrepreneurs, leads to shorter search for financing by the entrepreneur, and has ambiguous effects on the efficiency or resource allocation.

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Appendix. Proofs

Proof of Proposition 1:

Consider maximization problem (3). Let $n \leq N$ be the largest n such that the expected surplus generated with n screenings is strictly higher than that generated with $n-1$ screenings. Then for all $i > n$, $s_i = 1$. Suppose that the first n screening thresholds are not the same. Without loss of generality, assume that $s_1 = s_*$ and $s_2 = s^*$, $s_* \neq s^*$. Let

$$\Lambda = \frac{(1-\pi_0)}{\pi_0 X} \prod_{i>2} \frac{F_B(s_i)}{F_G(s_i)}.$$

Screening thresholds $s_1 = s_*$ and $s_2 = s^*$ maximize social surplus if and only if they solve

$$\max_{s_1, s_2} \Lambda F_B(s_1) F_B(s_2) - F_G(s_1) F_G(s_2). \tag{A1}$$

Consider x and $y(x)$ such that $F_B(x)F_B(y(x)) = F_B(s_*)F_B(s^*)$. We have

$$y'(x) = -\frac{f_B(x)}{f_B(y)} \frac{F_B(y)}{F_B(x)}. \tag{A2}$$

Since $s_1 = s_*$ and $s_2 = s^*$ solve (A1) $F_G(x)F_G(y(x))$ should be minimized at $x = s_*$. Hence, it must be that

$$(F_G(x)F_G(y(x)))' = \frac{f_B(x)F_G(x)F_G(y)}{F_B(x)} \left(\frac{f_G(x)}{f_B(x)} \frac{F_B(x)}{F_G(x)} - \frac{f_G(y)}{f_B(y)} \frac{F_B(y)}{F_G(y)} \right) = 0,$$

and therefore,

$$\frac{f_G(s_*)}{f_B(s_*)} \frac{F_B(s_*)}{F_G(s_*)} = \frac{f_G(s^*)}{f_B(s^*)} \frac{F_B(s^*)}{F_G(s^*)}.$$

Thus, we showed that all interior screening thresholds are solutions of the following equation

$$H(s) \equiv \frac{f_G(s)}{f_B(s)} \frac{F_B(s)}{F_G(s)} = c,$$

where c is some constant. In particular, if H is a strictly increasing function then all screening thresholds are the same.

To prove that the expected surplus strictly increases with the number of screenings if $H(s)$ is a strictly increasing function of s we need to show that for any N the solution to the maximization problem (3) is interior. Suppose on the contrary that for some N it is optimal to set s_N to one. Let N be the lowest number of screenings when this happens. The optimal screening threshold level is the same in all $N-1$ screenings and solves the F.O.C.

$$\pi X f_G(s) F_G(s)^{N-2} = (1-\pi) f_B(s) F_B(s)^{N-2}.$$

Taking the derivative of the surplus with respect to s_N at $s_N = 1$ we have

$$(1-\pi) f_B(1) F_B(s)^{N-q} - \pi X f_G(1) F_G(s)^{N-1} = f_B(1) F_B(s)^{N-1} \left(1 - \lambda \frac{F_G(s)}{F_B(s)} \frac{f_B(s)}{f_G(s)}\right) < 0,$$

where we have used the F.O.C and where the last inequality follows from the fact that $\frac{F_G(s)}{F_B(s)} \frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s and therefore takes the lowest value λ^{-1} at $s = 1$. As a result, it is suboptimal to set s_N to 1 and the solution must be indeed interior.

Finally, we will prove that the maximal expected surplus can be achieved with no more than n screenings if $H(s)$ is a strictly decreasing function on $s \in [s_n^*, 1]$. Notice that MLRP implies that Equation (5) has a unique solution, s_n^*, which is strictly increasing in n. Suppose on the contrary that social surplus is strictly higher with $n+1$ screenings. Each screening threshold s_j has to satisfy the F.O.C.:

$$\frac{f_G(s_j) \prod_{i \le n, i \ne j} F_G(s_i)}{f_B(s_j) \prod_{i \le n, i \ne j} F_B(s_i)} = \frac{(1-\pi_0)}{\pi_0 X}, \quad j = 1, \ldots, n+1. \tag{A3}$$

By MLRP and from the definition of s_n^* there should be at least two screening thresholds above s_n^*. Since $H(s)$ is a strictly decreasing function on $s \in [s_n^*, 1]$ these two thresholds must be the same, and must maximize (A1). Notice, however, that (A1) can be increased if one increases one of the thresholds and decreases the other so that to keep the product $F_B(x) F_B(y(x))$ constant. Therefore, (A1) is maximized by setting one of the thresholds to 1. Hence, we arrived to contradiction.

Q.E.D.

Proof of Proposition 2:

First, we show that any equilibrium that survives the Cho and Kreps intuitive criterion must be separating. For this, we need to show that the entrepreneur who has been rejected i times would always like to separate herself from those who have been rejected more than i times. Denote the entrepreneur who has been rejected $i-1$ times by E_i, and her expected surplus (conditional on the project being good) by V_i, $i = 1, ..., N$. Suppose contrary to the statement of the proposition that there is some pooling in equilibrium. Let i be the first instance such that E_i pools with entrepreneurs rejected more than i times. Let $j = \min\{k > i : E_k \quad \text{pools with} \quad E_i\}$.

Let s^* be a screening threshold asked by E_i and E_j. Let π^* be an investor's belief that the project is good if the entrepreneur asks for screening threshold s^* before the investor observes his private signal. We have

$$\begin{aligned} V_i &= (1 - F_G(s^*))\,(X - r(\pi^*, s^*)) + F_G(s^*)V_{i+1}, \\ V_j &= (1 - F_G(s^*))\,(X - r(\pi^*, s^*)) + F_G(s^*)V_{j+1}, \end{aligned}$$

where

$$r(\pi^*, s^*) = \frac{1-\pi^*}{\pi^*}\frac{f_B(s^*)}{f_G(s^*)}.$$

Let $\hat{\pi}$ be the investor's belief that the project is good if the investor believes that the entrepreneur is of type E_i. Clearly, $\hat{\pi} > \pi^*$. Let $\hat{s}$ be such that

$$V_j = (1 - F_G(\hat{s}))\,(X - r(\hat{\pi}, \hat{s})) + F_G(\hat{s})V_{j+1}. \tag{A4}$$

Suppose that investors believe that the entrepreneur is of type E_i if she asks for screening threshold $\hat{s}$. Then the type E_j entrepreneur is indifferent between asking for s^* and $\hat{s}$. Note that $V_{i+1} > V_{j+1}$ because the type E_i entrepreneur can always follow the strategy of the type E_j entrepreneur. Therefore, Equation (A4) implies that

$$V_i < (1 - F_G(\hat{s}))\,(X - r(\hat{\pi}, \hat{s})) + F_G(\hat{s})V_{i+1}.$$

Hence, E_i is better off by deviating and asking for a screening threshold, which is slightly above $\hat{s}$. At the same time, E_j is worse off by deviating to this threshold. Thus, no pooling equilibrium survives the Cho-Kreps intuitive criterion.

Next, we prove that if MLRP holds strictly then as N goes to infinity the entrepreneur extracts all the surplus and his surplus converges to that generated in the first-price auction. The proof is done in two steps. First, we show that if $r_N(1, \mathbf{s}^*_{\mathbf{N-1}})$ goes to X as N goes to infinity then the entrepreneur's surplus converges to that generated in the first-price auction. Then, we show that in equilibrium $r_N(1, \mathbf{s}^*_{\mathbf{N-1}})$ must go to X.

Step 1. Suppose that $\lim_{N\to\infty} r_N(1, \mathbf{s}^*_{\mathbf{N-1}}) = X$, where $r_N(1, \mathbf{s}^*_{\mathbf{N-1}})$ is defined as in Equation (2). The expression for social surplus (3) implies that if $\prod_{i=1}^N F_G(s_i^*) \to F_G^N(s^*)$ and $\prod_{i=1}^N F_B(s_i) \to F_B^N(s^*)$, where s^* is a screening threshold in the first-price auction, then surpluses generated in a sequential credit market and in a first-price auction are asymptotically the same.

Using equation (2) for the interest rate $r_N(1, \mathbf{s^*_{N-1}})$ we can see that

$$\lim_{N\to\infty} r_N(1, \mathbf{s^*_{N-1}}) = X \Leftrightarrow \lim_{N\to\infty} \lambda \frac{\pi_0 X}{1-\pi_0} \prod_{i=1}^{N-1} \frac{F_G(s_i)}{F_B(s_i)} = 1. \tag{A5}$$

If the entrepreneur is rejected $N-1$ times then in the last round she solves

$$V_N = \max_{s_N} (1 - F_G(s_N)) \left(X - r_N(s_N, \mathbf{s^*_{N-1}}) \right).$$

If $\lim_{N\to\infty} r_N(1, \mathbf{s^*_{N-1}}) = X$ and the strict MLRP holds then $\lim_{N\to\infty} s^*_N = 1$. We showed in Section 3 that for the IC constraints to hold it must be that $s^*_i > s^*_N$. Therefore for any i, $\lim_{N\to\infty} s^*_i = 1$. Let $\Delta s_i = 1 - s^*_i$. Taking the Taylor's series of (A5) we have

$$\sum_{i=1}^{N-1} \Delta s_i = \tau + O(\Delta s_N), \quad \tau = \frac{\ln(\lambda\pi_0 X/(1-\pi_0))}{\lambda - 1}. \tag{A6}$$

Therefore,

$$\begin{aligned} \prod_{i=1}^{N} F_G(s_i) &= e^{-\lambda\tau} + O(\Delta s_N), \\ \prod_{i=1}^{N} F_B(s_i) &= e^{-\tau} + O(\Delta s_N). \end{aligned}$$

We show in Axelson and Makarov (2020) that $F_G^N(s^*)$ and $F_B^N(s^*)$ converge to the same corresponding limits.

Step 2. We now show $r_N(1, \mathbf{s^*_{N-1}})$ goes to X in equilibrium. Suppose to the contrary that there exists $\varepsilon > 0$ such that for all N $r_N(1, \mathbf{s^*_{N-1}}) < X - \varepsilon$ for some . Note that only a bounded number of screening thresholds can stay away from one as N goes to infinity. Otherwise, the entrepreneur would not be able to obtain financing in the last round. Let M be the maximal index such that $\limsup_{N\to\infty} s_{N-M} = 1$ but $\limsup_{N\to\infty} s_{N-M+1} < 1$. Consider the problem of the entrepreneur who has been rejected $N - M - 1$ times. She solves problem (9):

$$\begin{aligned} V_{N-M} \equiv \max_{s_{N-M}} (1 - F_G(s_{N-M})) \left(X - r_i(s_{N-M}, \mathbf{s^*_{N-M-1}}) \right) + F_G(s_{N-M}) V_{N-M+1}, \\ \text{s.t. } V_{N-M+1} \geq (1 - F_G(s_{N-M})) \left(X - r_i(s_{N-M}, \mathbf{s^*_{N-M-1}}) \right) + F_G(s_{N-M}) V_{N-M+2}. \end{aligned}$$

As in the proof of Proposition 3 below, one can show that the unconstrained solution to the above problem entails s_{N-M} to be bounded away from one. Since by assumption, s_{N-M} goes to one it must be that the incentive compatibility constraint is binding. However, with s_{N-M+1} being away from one, s_{N-M} converging to one, and $r_i(s_{N-M}, \mathbf{s^*_{N-M-1}}) < X - \varepsilon$, the incentive compatibility constraint cannot be binding.

Q.E.D.

Proof of Proposition 3: Consider the maximization problem of the entrepreneur in the first round:

$$\max_{s_1} (1 - F_G(s_1)) (X - r_1(s_1)) + F_G(s_1) V_2, \tag{A7}$$

where V_2 is the entrepreneur's continuation value. The entrepreneur cannot affect investors' beliefs with his choice of s_1. Therefore, she takes V_2 as given. Since $F_G(s)$ is an increasing function of s the equilibrium choice of s_1^* is an increasing function of continuation value V_2.

Lemma 1 below provides an upper bound on the maximal expected surplus that can be achieved with a screening technology with finite λ. Therefore, V_2 is less than $X - (1-\pi_0)/(\pi_0\lambda)$. In the proof of Lemma 1 we actually show that if MLRP holds strictly then the bound is strict, that is there exists $\delta > 0$ such that $V_2 = X-(1-\pi_0)/(\pi_0\lambda)-\delta$. Using Equation (2) for $r_1(s_1)$ we can rewrite the maximization problem (A7) as

$$\max_{s_1} (1 - F_G(s_1)) \left(\delta - \frac{1-\pi_0}{\pi_0} \left(\frac{f_B(s_1)}{f_G(s_1)} - \frac{1}{\lambda} \right) \right) + V_2.$$

It is clear that the solution s_1^* to the above problem is strictly less than one.

Q.E.D.

Lemma 1: *The maximal expected social surplus in a sequential market with a screening technology that satisfies* $f_G(1)/f_B(1) = \lambda$ *is no larger than* $\max(\pi_0 X - (1 - \pi_0)/\lambda), 0)$.

Proof: We first observe that the maximal expected surplus respects the order induced by MLPR on the space of signal distributions. Consider two cases of informative signals. Suppose that in both cases if the project is bad the signal is drawn from the same distribution $F_B(s)$. At the same time, if the project is good then in the first case, the signal is drawn from a distribution F_{G_1} with density f_{G_1}, and in the second case, from a distribution F_{G_2} with density f_{G_2}. Suppose that for all $s > s'$

$$\frac{f_{G_1}(s)}{f_{G_2}(s)} \geq \frac{f_{G_1}(s')}{f_{G_2}(s')},$$

then the maximal surplus in the first case is no less than that in the second case. This follows from the fact that MLRP implies the monotone probability ratio (Milgrom (1981)).

Suppose for now that $f_B(s) \equiv 1$. Then given λ, the maximal expected surplus is achieved with $f_G(s) = 0$ for $s \in [0, 1-\lambda^{-1})$ and $f_G(s) = \lambda$ for $s \in [1-\lambda^{-1}, 1]$. Setting a screening threshold level to $1-\lambda^{-1}$ ensures that good projects are always financed and bad projects are financed with probability λ^{-1}. Thus, with a single screening the expected surplus is $\pi_0 X - (1-\pi_0)/\lambda$. Direct computations show that $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is an increasing function for $s \in [1-\lambda^{-1}, 1]$. Thus, by Proposition 1, $\pi_0 X - (1-\pi_0)/\lambda$ is in fact the maximal expected surplus. Finally, notice that the assumption that $f_B(s) \equiv 1$ is innocuous. For an arbitrary $f_B(s)$ the maximal surplus is achieved with $f_G(s) = 0$ for $s \in [0, \bar{s})$ and $f_G(s) = \lambda f_B(s)$ for $s \in [\bar{s}, 1]$, where $\bar{s}$ is determined by the condition that $\int_{\bar{s}}^1 \lambda f_B(s) ds = 1$. Hence, $\int_0^{\bar{s}} f_B(s) ds = 1 - \lambda^{-1}$.

Q.E.D.

Proof of Proposition 4:

Consider first the case when $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s. Suppose that there is a round $i < N$ such that $s_i^* < 1$ and the entrepreneur is unable to contact another investor after being rejected in round i, that is $s_{i+1}^* \geq 1$. In this round i, the entrepreneur solves

$$\max_{s_i^*} (1 - F_G(s_i^*)) \left(X - r_i(s_i, \hat{\mathbf{s}}_{\mathbf{i-1}})\right), \tag{A8}$$

where $r_i(s_i, \hat{\mathbf{s}}_{\mathbf{i-1}})$ is given by (2). Since $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s we have

$$\frac{F_B(s_i^*)}{F_G(s_i^*)}\frac{f_G(s_i^*)}{f_B(s_i^*)} < \frac{F_B(1)}{F_G(1)}\frac{f_G(1)}{f_B(1)} = \lambda.$$

Therefore, there exists $s_{i+1}^* < 1$ such that

$$r_{i+1}(s_{i+1}^*, \hat{\mathbf{s}}_{\mathbf{i}}) = r_i(s_i, \hat{\mathbf{s}}_{\mathbf{i-1}}) \times \frac{f_G(s_i^*)}{f_B(s_i^*)}\frac{F_B(s_j^*)}{F_G(s_j^*)} \times \frac{f_B(s_{i+1}^*)}{f_G(s_{i+1}^*)} < X.$$

Hence, the entrepreneur has a chance to get financing if she approaches another investor and therefore, round i cannot be the last round.

Suppose now that $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)^2}{f_G(s)^2}$ is a strictly increasing function of s in some neighbourhood of $s = 1$. We first show that this implies a bound on the derivative of the likelihood ratio at $s = 1$. For simplicity, we assume that $f_B(s) \equiv 1$. Note that

$$\left(\frac{F_G(s)}{F_B(s)}\right)'_{s=1} = \lambda - 1.$$

Since

$$\left(\frac{F_G(s)}{F_B(s)}\frac{1}{f_G(s)^2}\right)' = \left(\frac{F_G(s)}{F_B(s)}\right)'\frac{1}{f_G(s)^2} + \frac{F_G(s)}{F_B(s)}\left(\frac{1}{f_G(s)^2}\right)'$$

the fact that $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)^2}{f_G(s)^2}$ is a strictly increasing function at $s = 1$ implies that

$$0 \leq f_G'(1) < \frac{\lambda(\lambda - 1)}{2}. \tag{A9}$$

The idea of the proof is to show that relative flatness of the likelihood ratio leads to large screening thresholds. The entrepreneur can contacts all available investors when N goes to infinity only if the number of screening thresholds bounded away from one is uniformly bounded. Suppose for a moment that round i is the last round. The entrepreneur then solves problem (A8). To simplify notation, let

$$z = \frac{\pi_0}{1-\pi_0}\Pi_{j=1}^{i-1}\frac{F_G(\hat{s}_j)}{F_B(\hat{s}_j)}.$$

Then

$$r_i(s_i, \hat{\mathbf{s}}_{\mathbf{i-1}}) = \frac{1}{zf_G(s_i)}.$$

The F.O.C. to the above problem is

$$-(1 - F_G(s_i))\left(\frac{1}{f_G(s_i)}\right)' = f_G(s_i)zX - 1. \tag{A10}$$

Let $\Delta s = 1 - s_i^*$, where s_i^* is a solution to (A10). Taking the Taylor's series of (A10) at $s_i = 1$ we have

$$\frac{f_G'(1)\Delta s}{\lambda} = \lambda zX - 1 - f_G'(1)zX\Delta s + o(\Delta s).$$

Hence,

$$\Delta s = \frac{\lambda(\lambda zX - 1)}{f_G'(1)(1 + \lambda zX)} + o(\lambda zX - 1). \tag{A11}$$

Therefore,

$$F_G(s_i^*)/F_B(s_i^*) = (1-(\lambda-1)\Delta s)+o(\lambda zX-1) = \left(1 - \frac{\lambda(\lambda-1)(\lambda zX - 1)}{f_G'(1)(1+\lambda zX)}\right)+o(\lambda zX-1).$$

Inequality (A9) implies that

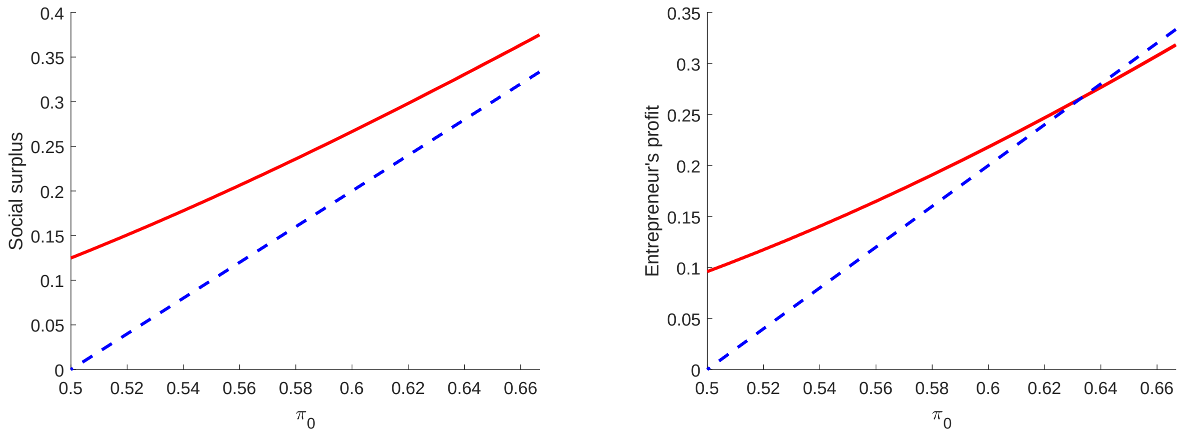
$$\left(1 - \frac{\lambda(\lambda-1)(\lambda zX - 1)}{f_G'(1)(1+\lambda zX)}\right) < \frac{1}{\lambda zX}.$$

Therefore, if rejected, the entrepreneur is unable to contact another investor.

Q.E.D.

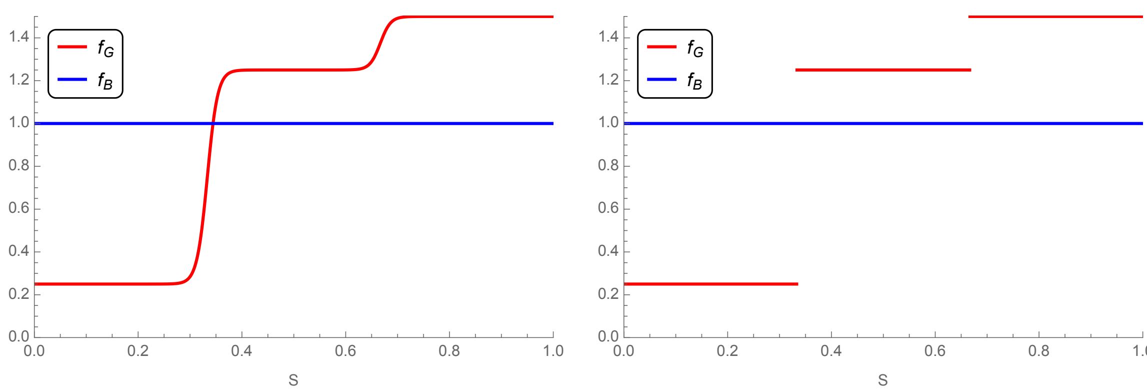
Proof of Proposition 5: First, consider the case of with a credit registry. We can view the optimization problem in each round i as if it is done by a fictitious agent i. Each fictitious agent i takes decisions of other agents as given and solves (11), which is the same as maximizing (12) with respect to s with an appropriately chosen y. By assumption the payoff of each agent i is quasi-concave in his own action and continuously depends on the actions of other agents. Therefore, by Theorem 1.2 of Fudenberg and Tirole (1991) there exists a pure-strategy equilibrium. The proof is similar if there is no credit registry. The quasi-concavity of the payoff ensures that the action space of every agent that satisfies the incentive compatibility constraint (10) is a concave set. Therefore, Theorem 1.2 of Fudenberg and Tirole (1991) still applies.

Q.E.D.



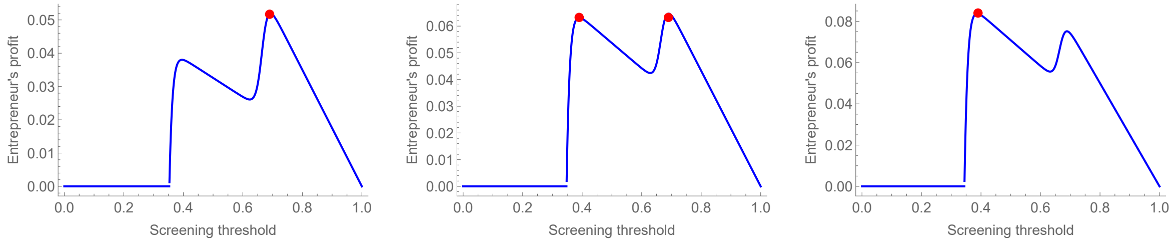
(a) (b)

Figure 1. Uninformed vs informed financing. Panels (a) and (b) show social surplus and entrepreneur's profit for the cases of informed financing with a credit registry (solid red line) and unformed financing (dashed blue line) as a function of the ex-ante project's quality π_0. The parameters are as follows: $X = 1$, $N = 10$, $f_B(s) = 1$, and $f_G(s) = 2s$.



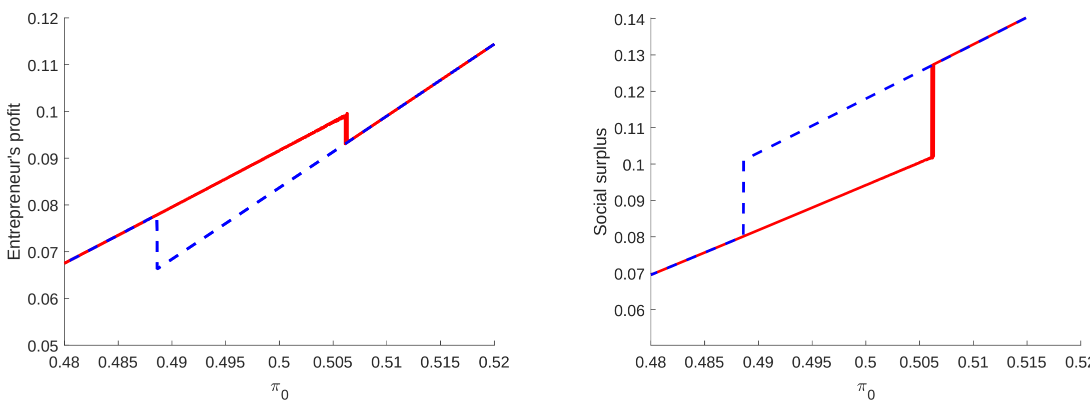
(a) (b)

Figure 2. Signal densities. Figure 2, panel (a) plots densities $f_B(s)$ and $f_G(s)$, where $f_G(s) \equiv 1$ and $f_G(s)$ is as defined in equation (13). Densities in panel (a) are smoothed versions of the densities shown in panel (b).



(a) (b) (c)

Figure 3. Entrepreneur's profit. Panels (a), (b), and (c) show the entrepreneur's expected profit with a single investor as a function of screening threshold for three values of π_0: 0.47, 0.486, and 0.5. Other parameters are as follows: $X = 1$, densities f_B and f_G are displayed in Figure 2, panel (a).



(a) (b)

Figure 4. Multiple equilibria. Figure 4 Panel (a) plots the entrepreneur's expected profit in the two equilibria described in Section 4.1 as a function of the ex-ante project's quality π_0. Panel (b) plots social surplus. The dashed blue line corresponds to the equilibrium with one financing round; the solid red line - to the equilibrium with two financing rounds.