How Much Do Financial Markets Matter? Cole-Obstfeld Revisited

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Abstract

Cole and Obstfeld (1991) asked, "How much do financial markets matter?" Emphasizing the importance of intratemporal relative price adjustment as a risk-sharing mechanism that operates even in the absence of financial asset trade, their answer was: not much. I revisit their question and find that in calibrations featuring rare disasters that generate reasonable risk premia without implausibly high risk aversion, the cost of shutting down trade in financial assets is on the order of 3 to 20 per cent of wealth.

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This paper addresses a question posed by Cole and Obstfeld (1991): assuming *intratemporal* trade in *goods* is permitted, what is the welfare cost of shutting down *intertemporal* trade in *financial assets*? How much do international financial markets matter? In giving their answer—not much—Cole and Obstfeld emphasized the importance of terms-of-trade[1] movements as a risk-sharing mechanism. I consider a world consisting of two countries, each featuring a representative agent. Both representative agents view the countries' outputs as imperfect substitutes, so changes in their relative supply lead to fluctuations in the terms of trade.

As an extreme example, if the elasticity of substitution between home and foreign goods, η, is as low as one (the Cobb-Douglas case), then relative prices move so strongly in response to shocks that full risk-sharing is achieved even in the absence of financial markets. At the opposite extreme, models in the finance literature typically feature just one good or, equivalently, goods that are perfect substitutes, $\eta = \infty$.[2] I start by considering this case, which is simple in that there are no terms-of-trade effects, and hence no gains from intratemporal trade. Nonetheless, the welfare analysis is surprisingly involved.

Despite financial economists' attachment to the perfect-substitutes case, the international finance literature argues for imperfect substitution between goods produced in different countries. Obstfeld and Rogoff (2000) propose an elasticity of substitution η in the range 2–6; Obstfeld and Rogoff (2007) suggest that $\eta = 2$ or 3 are plausible values. Here I consider the case $\eta = 2$, in which the algebra is tractable. In addition to its empirical plausibility, this case is of theoretical interest because it combines the two risk-sharing mechanisms that are present separately in the cases $\eta = 1$ and $\eta = \infty$. Since $\eta = 2$ is at the low end of the range of suggested values, the resulting estimates can be thought of as lower bounds on the cost of shutting down financial markets.

The main contribution of the paper is that it allows for the possibility that the two countries may be afflicted by rare output disasters: in this respect, it is to Cole and Obstfeld

[1] That is, the *intra*temporal price of home goods relative to foreign goods.

[2] An exception is Piazzesi, Schneider and Tuzel (2004).

(1991) as Barro (2009) and Martin (2008) are to Lucas (1987, chapter 3). I do so because it is important to explore questions regarding the benefits of risksharing in a model that generates a reasonable risk premium without relying on implausibly high risk aversion.[3] I find that calibrations with rare disasters imply substantially larger welfare benefits of international trade in assets. Moreover, since investment is absent from the analysis—as it was in Cole and Obstfeld (1991)—an important motive for financial asset trade is ignored, so presumably these calculations provide a lower bound on its importance.

I solve the model analytically, providing integral formulas for welfare costs, and so remove the need for simulations, since the formulas can be calculated to very high accuracy almost instantaneously.[4] It is particularly desirable to do so because of this paper's focus on rare disasters, which makes it important to avoid the Monte Carlo techniques that Cole and Obstfeld used in their calibrations. Methodologically, the key ingredients are to translate expected utility in each of the four regimes (with and without asset trade; $\eta = 2$ and ∞) into a form in which it can be thought of as a function of asset prices; and then to extend a result in Martin (2009a) so that these prices, and hence welfare, can be expressed in terms of integral formulas.[5] The end result is expressions for welfare costs that are valid for a wide range of possible output processes for each country.

The paper does not address the puzzling facts of home bias or of low cross-country correlations in aggregate consumption growth, nor does it address issues such as information asymmetry, or the role of nontradables or of nonseparabilities between consumption and leisure that have been brought up in connection with it.[6] Implicitly, it restricts attention

[3] Several authors have recently argued for the importance of accounting for rare disasters in understanding the level of risk premia (for example, Rietz (1988), Barro (2006), Gabaix (2009)).

[4] In the lognormal case, the integrals can be evaluated in closed form, up to hypergeometric functions.

[5] Note that the economics of this paper are very different from Martin (2009a), since that paper assumes the existence of a single globally diversified representative agent, while here each country has its own representative agent so that the no-asset-trade case is meaningful.

[6] See, for example, Backus, Kehoe and Kyland (1992), Backus and Smith (1993), Baxter and Jermann (1997), Baxter, Jermann and King (1998), Brennan and Solnik (1989), Devereux, Gregory and Smith (1992),

to the subset of investors who *are* globally diversified, with highly correlated consumption, and who *would* lose out if asset markets were closed.

1 Setup

There are two goods, two agents, and two countries, indexed $i = 1, 2$. At time t, country i produces Y_{it} units of good i; agent 1 consumes D_{1t} units of good 1 and D_{2t} units of good 2; and agent 2 consumes D_{1t}^* units of good 1 and D_{2t}^* units of good 2.

Agent 1 has power utility, with risk aversion $\gamma > 0$, over a CES consumption aggregator, and therefore maximizes

$$\mathbb{E} \int_0^\infty e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma} \, dt, \tag{1}$$

where C_t is the consumption aggregator

$$C_t \equiv \left[D_{1t}^{\frac{\eta-1}{\eta}} + D_{2t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} .$$

Here, η is the elasticity of intratemporal substitution between the goods of the two assets. Agent 2 has exactly the same utility function, up to the obvious changes in notation: C_t becomes C_t^*, and so on. At time 0, agent i owns a claim to the output of country i.

Output growth in each country is i.i.d. over time, though shocks may be correlated across countries. Formally, $(\log Y_{1,t}/Y_{1,0}, \log Y_{2,t}/Y_{2,0})_{t \geq 0}$ is a two-dimensional Lévy process. The technological side of the model can be summarized by the cumulant-generating function (CGF) of $(\log(Y_{1,t+1}/Y_{1,t}), \log(Y_{2,t+1}/Y_{2,t}))$, defined by

$$\boldsymbol{c}(\theta_1, \theta_2) \equiv \log \mathbb{E} \left[\left(\frac{Y_{1,t+1}}{Y_{1,t}} \right)^{\theta_1} \left(\frac{Y_{2,t+1}}{Y_{2,t}} \right)^{\theta_2} \right] . \tag{2}$$

The i.i.d. assumption ensures that $\boldsymbol{c}(\theta_1, \theta_2)$ is independent of t. This framework allows for a flexible specification of jumps in output: jumps can occur simultaneously in both countries, or at independent times, and to the extent that jumps are synchronized, their sizes can be

Hassan (2010), Heathcote and Perri (2009), Lewis (1996), Tesar (1993), and Van Nieuwerburgh and Veldkamp (2009).

correlated or uncorrelated; in fact, an almost arbitrary joint size distribution of jumps is possible, subject to the requirement that expected utility must remain finite.

I make two assumptions that simplify the analysis by ensuring that the two agents start out with equal wealth. First, the outputs of the two countries are equal at time 0 (though subsequently they will differ in general). Second, the world is symmetric in the sense that $\boldsymbol{c}(\theta_1, \theta_2) = \boldsymbol{c}(\theta_2, \theta_1)$. This implies, for example, that the means and volatilities of output growth are equal across the two countries. Similar assumptions were made by Lucas (1982) and Cole and Obstfeld (1991).

For parametric cost calculations, I assume that the log outputs follow correlated Brownian motions with drifts μ, volatilities σ, $i = 1, 2$, and correlation κ, on top of which are layered two kinds of jumps. Jumps of the first kind affect country i idiosyncratically, $i = 1, 2$. Jumps of the second kind occur simultaneously in each country. Jump arrival times follow Poisson processes with arrival rates ω for the idiosyncratic jumps, and ω_s for the simultaneous jumps. The sizes of these jumps in log outputs are Normally distributed. Idiosyncratic jumps have mean μ_J and volatility σ_J. When there is a simultaneous jump, the sizes of jumps in the two countries are jointly Normal, with means ν, volatilities τ and correlation ξ. The resulting CGF, which satisfies the symmetry assumption $\boldsymbol{c}(\theta_1, \theta_2) = \boldsymbol{c}(\theta_2, \theta_1)$, is

$$\begin{aligned}
\boldsymbol{c}(\theta_1, \theta_2) \quad = \quad & \mu\theta_1 + \mu\theta_2 + \frac{1}{2}\sigma^2\theta_1^2 + \kappa\sigma^2\theta_1\theta_2 + \frac{1}{2}\sigma^2\theta_2^2 + \\
& + \omega\left(e^{\mu_J\theta_1 + \frac{1}{2}\sigma_J^2\theta_1^2} - 1\right) + \omega\left(e^{\mu_J\theta_2 + \frac{1}{2}\sigma_J^2\theta_2^2} - 1\right) + \\
& + \omega_s\left(e^{\nu\theta_1 + \nu\theta_2 + \frac{1}{2}\tau^2\theta_1^2 + \xi\tau^2\theta_1\theta_2 + \frac{1}{2}\tau^2\theta_2^2} - 1\right). \qquad (3)
\end{aligned}$$

I will calibrate the model so that the jumps correspond to rare (low ω and ω_s) disasters (negative and reasonably large μ_J), but the theoretical results apply equally well to the case with small, frequently-occurring, jumps.

2 Equilibrium without asset trade

When trade in financial assets is not possible, the agents may wish to trade away from their endowment, Y_{it}, at each time t, via an intratemporal exchange of goods. Agent 1 solves the static problem[7]

$$\max_{D_{1t},D_{2t}} \left(D_{1t}^{\frac{\eta-1}{\eta}} + D_{2t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \qquad \text{s.t. } D_{1t} + p_t D_{2t} = Y_{1t}, \tag{4}$$

taking p_t, the relative price of good 2 in units of good 1, as given. Agent 2 solves the problem

$$\max_{D_{1t}^*,D_{2t}^*} \left(D_{1t}^{*\frac{\eta-1}{\eta}} + D_{2t}^{*\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \qquad \text{s.t. } D_{1t}^* + p_t D_{2t}^* = p_t Y_{2t}.$$

In equilibrium, p_t adjusts so that $D_{1t} + D_{1t}^* = Y_{1t}$, which clears the market. Equivalently, in the absence of asset trade goods trade must be balanced, $p_t D_{2t} = D_{1t}^*$.

2.1 The perfect substitutes case, $\eta = \infty$

If the goods are perfect substitutes, the relative price p_t is fixed at 1. There are no gains from trade, so each agent's consumption equals his endowment: $C_t = D_{1t} + D_{2t} = Y_{1t}$, and $C_t^* = D_{1t}^* + D_{2t}^* = Y_{2t}$. (In fact, in the absence of asset trade, goods trade could be shut down completely without any effect on welfare.)

2.2 The intermediate case, $\eta = 2$

Agent 1 chooses D_{1t} and D_{2t} to maximize (4) or, equivalently, to solve

$$\max_{D_{1t},D_{2t}} \sqrt{D_{1t}} + \sqrt{D_{2t}} \qquad \text{s.t. } D_{1t} + p_t D_{2t} = Y_{1t}.$$

The first-order conditions for this problem and its analogue for agent 2 are that

$$D_{1t} = p_t^2 D_{2t} \qquad \text{and} \qquad D_{1t}^* = p_t^2 D_{2t}^*. \tag{5}$$

[7] In the Cobb-Douglas case, $\eta = 1$, the agents maximize $C_t = \sqrt{D_{1t} D_{2t}}$ and $C_t^* = \sqrt{D_{1t}^* D_{2t}^*}$ respectively.

Substituting back into the budget constraints, we find

$$p_t(1+p_t)D_{2t} = Y_{1t} \qquad \text{and} \qquad (1+p_t)D_{2t}^* = Y_{2t}. \tag{6}$$

The market-clearing (or balanced-trade) condition requires that $D_{1t}^* = p_t D_{2t}$, which implies, in combination with (5) and (6), that

$$p_t = \sqrt{\frac{Y_{1t}}{Y_{2t}}}.$$

So agent 1 ends up with

$$D_{1t} = \frac{\sqrt{Y_{1t}/Y_{2t}}}{1+\sqrt{Y_{1t}/Y_{2t}}} Y_{1t} \qquad \text{and} \qquad D_{2t} = \frac{\sqrt{Y_{1t}/Y_{2t}}}{1+\sqrt{Y_{1t}/Y_{2t}}} Y_{2t},$$

and has consumption aggregator

$$C_t = \left(\sqrt{D_{1t}} + \sqrt{D_{2t}}\right)^2 = Y_{1t} + \sqrt{Y_{1t}Y_{2t}}.$$

2.3 The Cobb-Douglas case, $\eta = 1$

This case was considered by Cole and Obstfeld (1991). The agents each spend a constant fraction of their wealth, each period, on each good. Symmetry implies that they each spend half their wealth on their own good—so $D_{1t} = Y_{1t}/2$ and $D_{2t}^* = Y_{2t}/2$. It follows from market-clearing that $D_{2t} = Y_{2t}/2$ and $D_{1t}^* = Y_{1t}/2$.

3 Equilibrium with asset trade

When trade in financial assets is allowed, the fact that agents have equal wealth and identical preferences means that they will trade in such a way as to equate their consumption aggregators, $C_t = C_t^*$. The symmetry of the setup implies that $D_{1t} = D_{1t}^* = Y_{1t}/2$ and $D_{2t} = D_{2t}^* = Y_{2t}/2$.

Asset trade permits the value of a country's consumption bundle to differ from the value of its output: i.e., permits countries to run current account surpluses and deficits. Country 1's current account is $CA_{1t} \equiv Y_{1t} - D_{1t} - p_t D_{2t}$, using its own good as numeraire.

This implies that $CA_{1t} = (Y_{1t} - Y_{2t})/2$ in the perfect substitutes case and $CA_{1t} = (Y_{1t} - \sqrt{Y_{1t}Y_{2t}})/2$ if $\eta = 2$. (Since the global current account must balance, country 2's current account satisfies $CA_{2t} = -CA_{1t}$, using the same numeraire.) In either case, countries run current account deficits when their output is low in relative terms, and surpluses when their output is high. In the Cobb-Douglas case, $CA_{1t} = 0$.

Dividing through by Y_{1t}, country 1's current account relative to GDP is

$$\frac{CA_{1t}}{Y_{1t}} = \begin{cases} \frac{1}{2}\left(1 - \frac{Y_{2t}}{Y_{1t}}\right) & \text{if } \eta = \infty \\ \frac{1}{2}\left(1 - \sqrt{\frac{Y_{2t}}{Y_{1t}}}\right) & \text{if } \eta = 2 \\ 0 & \text{if } \eta = 1 \end{cases} . \tag{7}$$

Now define $S_t \equiv Y_{1t}/(Y_{1t} + p_t Y_{2t})$ to be country 1's share of world output (measured in units of good 1). Since the world is symmetric at time 0, $S_0 = 1/2$. At subsequent times, $S_t = Y_{1t}/(Y_{1t} + Y_{2t})$ if $\eta = \infty$, $S_t = Y_{1t}/(Y_{1t} + \sqrt{Y_{1t}Y_{2t}})$ if $\eta = 2$, and $S_t \equiv 1/2$ if $\eta = 1$. This definition unifies the three cases above, since in each of them

$$\frac{CA_{1t}}{Y_{1t}} = \frac{1}{2}\left(\frac{1}{S_0} - \frac{1}{S_t}\right). \tag{8}$$

This equation provides another way to understand the effect of imperfect substitution: to the extent that movements in the terms of trade offset shocks to output, country 1's share of world output is stabilized. As a result, country 1's current account is less volatile and hence the role of asset trade is diminished. In the Cobb-Douglas case considered by Cole and Obstfeld (1991), the terms of trade perfectly offset shocks to output so country 1's share of world output is constant. So, by (8), both countries have balanced current accounts, and there is no role for asset trade.

We can use (7) to get a rough sense of the quantitative implications of the model. Consider a scenario in which $Y_{1t} = Y_{2t} = 1$, so both countries have a balanced current account. Now suppose that country 1's output experiences a 30% decline to 0.7. Its current account deficit, relative to GDP, will be -21.4% in the perfect substitutes case, -9.8% in the $\eta = 2$ case, and 0% in the Cobb-Douglas case. In the first two cases, the substantial current account deficit permits the country to cushion the decline in its consumption following the disaster.

4 The cost of shutting down asset trade

I now explore how significant this cushioning effect is in welfare terms, in the three cases $\eta = \infty$, $\eta = 2$, and $\eta = 1$.

4.1 The perfect substitutes case, $\eta = \infty$

With asset trade, agent 1 has consumption aggregator $C_t = D_{1t} + D_{2t} = (Y_{1t} + Y_{2t})/2$. Expected utility (1) can be rewritten as

$$\frac{(Y_{10}+Y_{20})^{-\gamma}}{2^{1-\gamma}(1-\gamma)} \left[\mathbb{E} \int_0^\infty e^{-\rho t} \left(\frac{Y_{1t}+Y_{2t}}{Y_{10}+Y_{20}} \right)^{-\gamma} Y_{1t}\, dt + \mathbb{E} \int_0^\infty e^{-\rho t} \left(\frac{Y_{1t}+Y_{2t}}{Y_{10}+Y_{20}} \right)^{-\gamma} Y_{2t}\, dt \right]. \quad (9)$$

The point of rewriting (1) in this form is that, for $i = 1, 2$,

$$P_{i0} \equiv \mathbb{E} \int_0^\infty e^{-\rho t} \left(\frac{Y_{1t}+Y_{2t}}{Y_{10}+Y_{20}} \right)^{-\gamma} Y_{it}\, dt$$

can be thought of as an asset price: it is the price an agent with power utility and consumption stream $Y_{1t}+Y_{2t}$ would attach to (a marginal unit of) an asset with dividend stream Y_{it}. Temporarily postponing the issue of pricing this asset, and writing V for the price-dividend ratios P_{i0}/Y_{i0}, which are equal by symmetry, expected utility (9) simplifies to

$$\begin{aligned} EU(C_{10}; \eta = \infty, \text{asset trade}) &= \frac{(Y_{10}+Y_{20})^{-\gamma}}{2^{1-\gamma}(1-\gamma)} \left[P_{10} + P_{20} \right] \\ &= \frac{(Y_{10}+Y_{20})^{1-\gamma}}{2^{1-\gamma}(1-\gamma)} V \\ &= \frac{C_0^{1-\gamma}}{(1-\gamma)} \cdot V \,. \end{aligned} \quad (10)$$

The price-dividend ratio V is given in Proposition 1 of Martin (2009a):[8]

$$V \equiv 2^\gamma \cdot \int_{-\infty}^{\infty} \frac{\mathscr{F}_\gamma(v)}{\rho - \boldsymbol{c}(1-\gamma/2 - iv, -\gamma/2 + iv)}\, dv,$$

where $\mathscr{F}_\gamma(v) = \frac{1}{2\pi} \cdot \Gamma(\gamma/2 - iv)\Gamma(\gamma/2 + iv)/\Gamma(\gamma)$ is defined in terms of the Gamma function. For this equation to apply, we require that $\rho - \boldsymbol{c}(1-\gamma/2, -\gamma/2) > 0$, which ensures that expected utility is finite.

If asset trade is not allowed, things are simpler: $C_t = Y_{1t}$, so expected utility is

$$\begin{aligned} EU(C_{10}; \eta = \infty, \text{no asset trade}) &= \mathbb{E} \int_0^\infty e^{-\rho t} \frac{Y_{1t}^{1-\gamma}}{1-\gamma}\, dt \\ &= \frac{C_0^{1-\gamma}}{1-\gamma} \cdot V_\times \,, \end{aligned} \tag{11}$$

where the quantity

$$V_\times \equiv \frac{1}{\rho - \boldsymbol{c}(1-\gamma, 0)}$$

can also be interpreted as a price-dividend ratio (of the claim to country 1's output, if the representative agent consumes only country 1's output). In this case, finiteness of expected utility requires that $\rho - \boldsymbol{c}(1-\gamma, 0) > 0$.

Different technological assumptions—on the mean, correlation, and volatility of output growth in each country, on the size and frequency of disasters, and so on—feed into (10) and (11) via the cumulant-generating function $\boldsymbol{c}(\cdot, \cdot)$.

We can now follow Cole and Obstfeld (1991), and ask how large a (permanent) decline in output in each country—from Y_{i0} to $(1-\delta_\infty)Y_{i0}$—would correspond, in welfare terms, to closing down asset trade. Using (10) and (11), δ_∞ satisfies

$$\delta_\infty = 1 - \left(\frac{V}{V_\times}\right)^{\frac{1}{\gamma-1}} . \tag{12}$$

[8]As discussed in Martin (2009a), this integral formula—together with the others that appear in the paper—can be calculated effectively instantly on a computer because the integrand decays to zero exponentially fast.

4.2 The intermediate case, $\eta = 2$

With asset trade, the consumption aggregator of agent 1 is $C_t = \widehat{C}_t/2$, where $\widehat{C}_t \equiv \left(\sqrt{Y_{1t}} + \sqrt{Y_{2t}}\right)^2$. The expected utility (1) can again be rewritten, this time as

$$\frac{\widehat{C}_0^{1/2-\gamma}}{2^{1-\gamma}(1-\gamma)} \mathbb{E} \int_0^\infty e^{-\rho t} \left(\frac{\widehat{C}_t}{\widehat{C}_0}\right)^{1/2-\gamma} \left(Y_{1t}^{1/2} + Y_{2t}^{1/2}\right) dt \,.$$

Now write

$$W = \mathbb{E} \int_0^\infty e^{-\rho t} \left(\frac{\widehat{C}_t}{\widehat{C}_0}\right)^{1/2-\gamma} \left(\frac{Y_{it}}{Y_{i0}}\right)^{1/2} dt \,;$$

by symmetry this price-dividend ratio is the same whether i equals 1 or 2. With this definition, we have, along the lines that led to (10),

$$\begin{aligned} EU(C_{10}; \eta = 2, \text{asset trade}) &= \frac{\widehat{C}_0^{1/2-\gamma}}{2^{1-\gamma}(1-\gamma)} \left[Y_{10}^{1/2} W + Y_{20}^{1/2} W\right] \\ &= \frac{C_0^{1-\gamma}}{1-\gamma} \cdot W \,. \end{aligned} \tag{13}$$

In the Appendix, I show how to modify the techniques of Martin (2009a) to derive the following integral formula for W:

$$W \equiv 2^{2\gamma-1} \int_{-\infty}^\infty \frac{\mathscr{F}_{2\gamma-1}(v)}{\rho - \boldsymbol{c}(3/4 - \gamma/2 - iv/2, 1/4 - \gamma/2 + iv/2)} \, dv$$

whenever $\gamma > 1/2$ and $\rho - \boldsymbol{c}(3/4 - \gamma/2, 1/4 - \gamma/2) > 0$.

In the absence of asset trade, the consumption aggregator of agent 1 is $C_t = Y_{1t} + \sqrt{Y_{1t} Y_{2t}}$. In this case, we can use a trick to simplify the expected utility calculation. Introduce the "pseudo-outputs" $X_{1t} = Y_{1t}$ and $X_{2t} = \sqrt{Y_{1t} Y_{2t}}$, so $C_t = X_{1t} + X_{2t}$. Expected utility can be rewritten in terms of X_{1t} and X_{2t} as

$$\frac{(X_{10} + X_{20})^{-\gamma}}{1-\gamma} \left[\mathbb{E} \int_0^\infty e^{-\rho t} \left(\frac{X_{1t} + X_{2t}}{X_{10} + X_{20}}\right)^{-\gamma} X_{1t} \, dt + \mathbb{E} \int_0^\infty e^{-\rho t} \left(\frac{X_{1t} + X_{2t}}{X_{10} + X_{20}}\right)^{-\gamma} X_{2t} \, dt\right] . \tag{14}$$

Much as in equation (9), the expectations inside the brackets can be interpreted as price-dividend ratios. Now, though, the relevant price-dividend ratios must be calculated for a pseudo-economy with two trees whose (perfectly substitutable) dividends are X_{1t} and X_{2t}.

The important feature of X_{1t} and X_{2t} is that they inherit the i.i.d. growth property from Y_{1t} and Y_{2t}. We can therefore define a new CGF

$$\boldsymbol{c}^{+}(\theta_1, \theta_2) \equiv \log \mathbb{E}\left[\left(\frac{X_{1,t+1}}{X_{1,t}}\right)^{\theta_1}\left(\frac{X_{2,t+1}}{X_{2,t}}\right)^{\theta_2}\right].$$

It is easy to check that $\boldsymbol{c}^{+}(\theta_1, \theta_2) \equiv \boldsymbol{c}(\theta_1 + \theta_2/2, \theta_2/2)$. To calculate the price-dividend ratios that appear in (14), we do all calculations with respect to the cumulant-generating function $\boldsymbol{c}^{+}(\theta_1, \theta_2)$. In doing so, it is important to note that due to the asymmetry in X_{1t} and X_{2t}—$\boldsymbol{c}^{+}(\theta_1, \theta_2) \neq \boldsymbol{c}^{+}(\theta_2, \theta_1)$—the price-dividend ratios of the two pseudo-trees are no longer equal. Some algebra along the lines of (10) gives

$$EU(C_{10}; \eta = 2, \text{no asset trade}) = \frac{C_0^{1-\gamma}}{(1-\gamma)} \cdot W_{\times}\,, \tag{15}$$

where

$$\begin{aligned} W_{\times} \quad \equiv \quad & 2^{\gamma-1}\left[\int_{-\infty}^{\infty} \frac{\mathscr{F}_{\gamma}(v)}{\rho - \boldsymbol{c}^{+}(1-\gamma/2 - iv, -\gamma/2 + iv)}\, dv + \right. \\ & \left. + \int_{-\infty}^{\infty} \frac{\mathscr{F}_{\gamma}(v)}{\rho - \boldsymbol{c}^{+}(-\gamma/2 - iv, 1-\gamma/2 + iv)}\, dv\right], \end{aligned}$$

assuming $\rho - \boldsymbol{c}^{+}(1-\gamma/2, -\gamma/2) > 0$ and $\rho - \boldsymbol{c}^{+}(-\gamma/2, 1-\gamma/2) > 0$. The cost of shutting down asset trade, δ_2, can then be calculated from (13) and (15):

$$\delta_2 = 1 - \left(\frac{W}{W_{\times}}\right)^{\frac{1}{\gamma-1}}. \tag{16}$$

4.3 The Cobb-Douglas case, $\eta = 1$

The equilibrium outcome is the same whether or not trade in financial assets is permitted, so the welfare cost of banning trade in financial assets is zero.

4.4 Numerical results

I now use equations (12) and (16) to explore the cost of shutting down asset trade in a number of different calibrations.

	$\eta = 2$	$\eta = \infty$	$\eta = 2$	$\eta = \infty$
$\gamma = 2$	0.0347	0.1722	0.1458	0.5818
$\gamma = 4$	0.0619	0.2760	0.1549	0.6226
$\gamma = 6$	0.0752	0.3284	0.1613	0.6525
$\gamma = 8$	0.0829	0.3605	0.1673	0.6809
$\gamma = 10$	0.0882	0.3841	0.1733	0.7100
$\gamma = 30$	0.0987	0.4851	0.2596	1.1835

Table 1: Left: Costs estimated by Cole and Obstfeld (1991) in numerical simulations, measured in percentage points. Right: Costs computed in a lognormal model with the same mean, volatility and correlation across countries.

Table 1 reproduces, on its left-hand side, the numbers calculated by Cole and Obstfeld (1991) in simulations. They estimated the cost of shutting down financial markets by conducting simulations over 50-year time horizons in a 4-state Markov chain calibrated to 1.8% mean output growth, 2.7% standard deviation of output growth, 0.102 first lagged autocorrelation of output growth, and cross-country correlation of 0.375.

To generate a similar example, I assume that output growth is lognormal, picking μ and σ so that the mean and standard deviation of output growth are 1.8% and 2.7%, as in Cole and Obstfeld (1991), and set $\kappa = 0.375$ and $\omega = \omega_s = 0$ (so there are no jumps). They assumed that $e^{-\rho} = 0.98$, so I do too. I compute the cost of shutting down asset trade using formulas (12) and (16). The right-hand side of Table 1 shows the resulting numbers. Like Cole and Obstfeld, I find low costs of shutting down asset trade both in the $\eta = 2$ case and in the $\eta = \infty$ case, with plausible values of risk aversion leading to costs below 1 per cent.

However, this calibration comes up against the equity premium puzzle: in the the absence of asset trade and with $\eta = \infty$, for example, the equity premium in each country is $\gamma \times 0.027^2$, which is on the order of 0.0007γ. With γ in a reasonable range (say, between 2 and 10), this is counterfactually low, which raises the concern that it is unsatisfactory to answer a question about risk-sharing mechanisms in an environment in which there is little risk to be shared. Motivated by the equity premium puzzle, Barro (2006) has revived Rietz's (1988) idea that the high equity premium may reflect concerns about the possibility of rare disasters, so I now use the results of the previous section to compute the costs of shutting down asset trade in environments with such disasters.[9]

	Disaster 1	Disaster 2	Disaster 3	Lognormal
μ	0.0200	0.0200	0.0200	0.0171
σ	0.0280	0.0280	0.0280	0.0393
κ	0.375	0.375	0.375	0.375
ω	0.01	0	0	0
μ_J	−0.30	—	—	—
σ_J	0.15	—	—	—
ω_s	0	0.01	0.01	0
ν	—	−0.30	−0.30	—
τ	—	0.15	0.15	—
ξ	—	0	0.5	—

Table 2: Parameters in calibrations. Mean (standard deviation) of output growth is 1.8% (4%) in each case. Dashes indicate irrelevant parameters.

In choosing the parameters that govern the size distribution and frequency of disas-

[9] An alternative potential resolution of the equity premium puzzle is that "γ is higher than you think". If so, new problems arise. The cost of shutting down asset trade, even in the lognormal example, becomes high and sensitively dependent on γ as γ increases: at $\gamma = 51.8$ (corresponding to a 3.6% equity premium) the cost of shutting down asset trade is over 10 per cent; at $\gamma = 51.8655$, the cost is over 20 per cent; at $\gamma = 51.87$, the equilibrium with asset trade is *incomparably* more appealing, in the sense that agents would be prepared to sacrifice any amount of current consumption less than 100 per cent in order to be able to trade financial assets. Such extreme parameter sensitivity is problematic, aside from the fact that high values of γ are implausible for other reasons.

ters, I am guided by empirical evidence presented by Barro (2006) and Barro, Nakamura, Steinsson and Ursua (2009). For simplicity, I assume that log disaster sizes are Normally distributed. Barro (2006) finds a disaster arrival rate of 0.017, and uses an empirical distribution of GDP disasters whose mean and standard deviation (in logs) are -0.39 and 0.25, respectively. Barro, Nakamura, Steinsson and Ursua (2009) estimate a more complicated model in which disasters have both permanent and transitory effects. They suggest that in a simple i.i.d. model (such as that considered here) their parameter estimates would correspond roughly to a disaster arrival rate of 0.0138 coupled with a constant disaster (log) size of -0.30.[10] Here, I assume that disasters arrive at rate 0.01, and have mean (log) size equal to -0.30 and standard deviation 0.15. (This calibration was also used as a representation of rare disaster models by Backus, Chernov and Martin (2010).) The first column of Table 2 reports parameter values for a calibration in which disasters arrive independently. The second and third columns report parameters for the case in which disasters arrive simultaneously in the two countries. Conditional on the arrival of a disaster, the (log) disaster sizes are jointly Normal with means -0.30 and standard deviation 0.15 in each country. Columns 2 and 3 differ only in the assumed correlation between disaster sizes across countries. In Column 2, disaster sizes are uncorrelated. In Column 3, disaster sizes have correlation 0.5, so that an unusually severe disaster in country 1 increases the likelihood that country 2 also experienced an unusually severe disaster. Since it is hard to estimate the correlation in sizes of rare disasters, precisely *because* they are rare, it is reassuring that (as it will turn out) the Disaster 2 and Disaster 3 calibrations deliver broadly similar results.

In all the calibrations, I pick μ and σ so that the mean and standard deviation of output growth in each country are 1.8% and 4%, respectively, and set the correlation between Brownian motions $\kappa = 0.375$. These parameter choices imply that the standard deviation of output growth, conditional on disasters not occurring in sample, is 2.86%—which is very close both to the standard deviation assumed by Cole and Obstfeld (1991) and to the

[10] Note, however, that their estimates are based on consumption data rather than output data.

empirical evidence from post-war data presented by Barro and Ursua (2008)[11]—and that mean output growth, conditional on disasters not occurring, is 2.06%. The fourth column of Table 2 reports parameters for a calibration without disasters, as a benchmark; it differs from the example considered in Table 1 only in that the standard deviation of output growth is higher, at 4%, to match the standard deviation in the disaster calibrations.

$\eta = \infty$	Disaster 1	Disaster 2	Disaster 3	Lognormal
$\gamma = 2$	0.207	0.350	0.374	0.212
$\gamma = 4$	0.456	0.974	1.09	0.424
$\gamma = 6$	0.759	2.22	2.67	0.636
$\gamma = 8$	1.12	4.91	6.54	0.849
$\gamma = 10$	—	—	—	1.06
$\gamma = 30$	—	—	—	—
$\eta = 2$	Disaster 1	Disaster 2	Disaster 3	Lognormal
$\gamma = 2$	0.265	0.374	0.392	0.236
$\gamma = 4$	0.551	1.01	1.12	0.448
$\gamma = 6$	0.925	2.30	2.73	0.660
$\gamma = 8$	1.43	5.12	6.69	0.872
$\gamma = 10$	2.12	—	—	1.08
$\gamma = 30$	—	—	—	3.21

Table 3: Equity premium, in percentage points, with asset trade.

Table 3 shows how the equity premium varies with risk aversion, γ, in each of these

[11]Over the period 1870–2006, Barro and Ursua (2008) report that the standard deviation of real per capita GDP growth in the USA was 4.98%, considerably higher than in the sample period over which Cole and Obstfeld calibrated their data (1968–1987). Across 21 OECD countries over the same time period, the average standard deviation was 5.44%, while over the more recent period 1948–2006, the average standard deviation was lower, at 2.84%. The corresponding means of output growth are: 2.17% (US, 1870–2006); 2.05% (average of 21 OECD countries, 1870–2006); 2.87% (average of 21 OECD countries, 1948–2006).

calibrations. The right-hand column makes the familiar point that lognormal calibrations cannot deliver an equity premium close to what is observed in the data without assuming an implausibly high level of risk aversion. The left-most column shows that even the Disaster 1 calibration (with independent disasters) leads to an equity premium puzzle. But the Disaster 2 and 3 calibrations generate plausible risk premia with $\gamma = 8$.

Utility cost	Disaster 1	Disaster 2	Disaster 3	Lognormal
$\gamma = 2$	2.47	1.08	0.876	1.29
$\gamma = 4$	4.10	1.75	1.29	1.44
$\gamma = 6$	7.94	3.74	2.56	1.59
$\gamma = 8$	30.1	22.5	18.1	1.76
$\gamma = 10$	—	—	—	1.95
$\gamma = 30$	—	—	—	—
Change in risk premium	**Disaster 1**	**Disaster 2**	**Disaster 3**	**Lognormal**
$\gamma = 2$	0.240	0.097	0.073	0.097
$\gamma = 4$	0.864	0.346	0.230	0.193
$\gamma = 6$	2.59	1.13	0.680	0.290
$\gamma = 8$	7.73	3.94	2.31	0.381
$\gamma = 10$	—	—	—	0.480
$\gamma = 30$	—	—	—	—

Table 4: Two measures of the effect of shutting down financial markets: the utility cost and change in risk premium, in percentage points, $\eta = \infty$.

Tables 4 and 5 report, for each of these calibrations, two measures of the cost of shutting down asset trade. The first measure is the utility cost discussed previously. The second is the amount by which the risk premium would increase in each country if asset markets were shut down. The two measures line up fairly closely, though not perfectly.

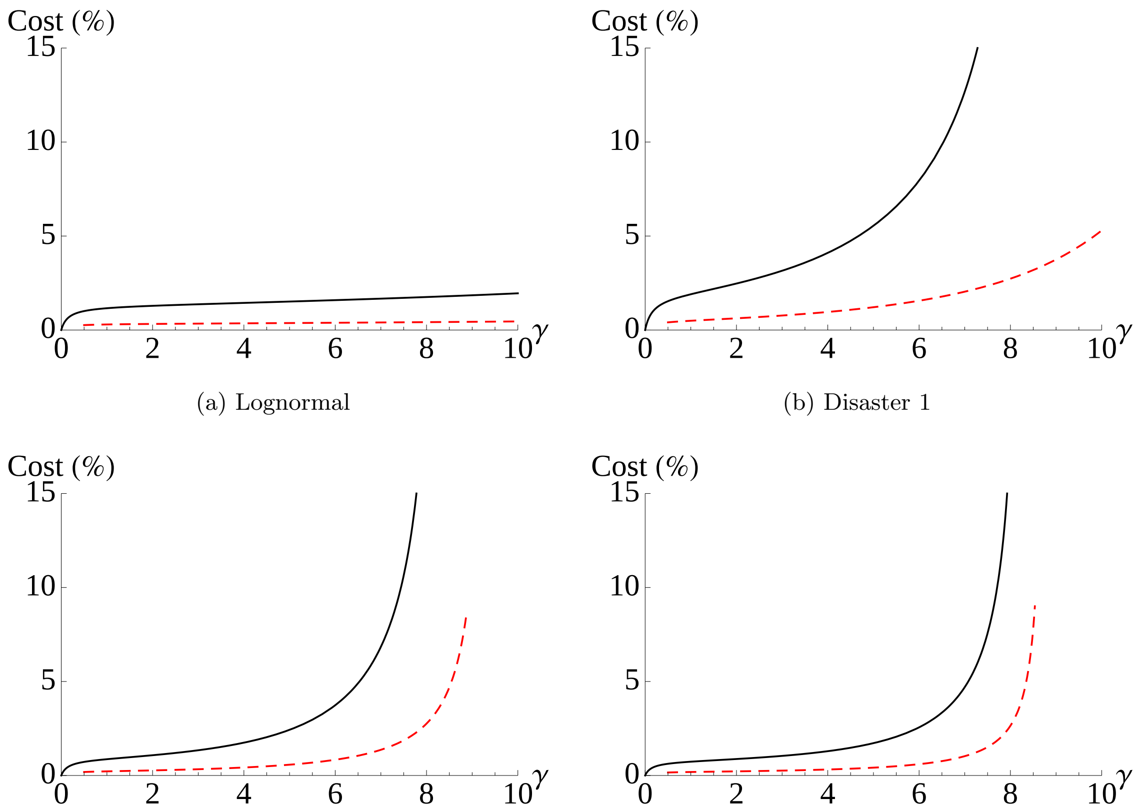
Figure 1 makes the point graphically, plotting the utility cost of shutting down asset

Utility cost	Disaster 1	Disaster 2	Disaster 3	Lognormal
$\gamma = 2$	0.624	0.271	0.220	0.324
$\gamma = 4$	0.961	0.425	0.319	0.357
$\gamma = 6$	1.56	0.836	0.602	0.387
$\gamma = 8$	2.73	2.76	2.65	0.420
$\gamma = 10$	5.30	—	—	0.457
$\gamma = 30$	—	—	—	3.18
Change in risk premium	**Disaster 1**	**Disaster 2**	**Disaster 3**	**Lognormal**
$\gamma = 2$	0.055	0.023	0.018	0.024
$\gamma = 4$	0.263	0.110	0.070	0.073
$\gamma = 6$	0.745	0.340	0.220	0.123
$\gamma = 8$	1.94	1.11	0.710	0.178
$\gamma = 10$	5.11	—	—	0.230
$\gamma = 30$	—	—	—	1.56

Table 5: Two measures of the effect of shutting down financial markets: the utility cost and change in risk premium, in percentage points, $\eta = 2$.

trade against γ for each calibration.[12] The figure and the tables show that the cost is increasing in γ, and is higher in the $\eta = \infty$ case than in the $\eta = 2$ case. In the perfect substitution case, the cost is higher when jumps occur at independent times in the two countries than when jumps are synchronized. (This is not always true when $\eta = 2$, due to the intratemporal price adjustment effect emphasized by Cole and Obstfeld (1991).) In the two calibrations that feature synchronized jumps, the cost is higher when disaster sizes are uncorrelated across countries (as in the Disaster 2 calibration) than when they are positively

[12]Dashes in Table 4 indicate when one of the inequalities $\rho - \boldsymbol{c}(1-\gamma/2, -\gamma/2) > 0$, $\rho - \boldsymbol{c}(1-\gamma, 0) > 0$, $\rho - \boldsymbol{c}(3/4-\gamma/2, 1/4-\gamma/2) > 0$, $\rho - \boldsymbol{c}^{+}(1-\gamma/2, -\gamma/2) > 0$, or $\rho - \boldsymbol{c}^{+}(-\gamma/2, 1-\gamma/2) > 0$ fails to hold. In Figure 1, I only plot the cost for values of γ such that all these inequalities are satisfied.



(a) Lognormal

(b) Disaster 1

(c) Disaster 2

(d) Disaster 3

Figure 1: Cost of shutting down asset trade, against γ. Solid: $\eta = \infty$. Dashed: $\eta = 2$.

correlated (as in the Disaster 3 calibration). Using the Disaster 2 and 3 calibrations and setting $\gamma = 8$ to match the equity premium, the cost of shutting down asset trade is on the order of 20% (approximately 56 times the corresponding number in Cole and Obstfeld (1991)) when $\eta = \infty$, and on the order of 3% (approximately 34 times the corresponding number) when $\eta = 2$.

Note, finally, that the fact that asset *prices* in different countries sometimes experience (near-)simultaneous crashes does not necessarily mean that jumps are synchronized in the sense used here: the jumps here are jumps in *fundamentals*. In fact, one of the *signatures* of asynchronous disasters in this model is that strong "contagion" effects are seen across countries: a disaster affecting output in one country will cause simultaneous crashes in the prices of the claims to other countries' outputs, especially if the originally affected country is large. In this context, it is sometimes argued—for example, during the 2008-9 crisis—that "contagion" substantially reduces the diversification benefits of international risk-sharing. In the present framework, contagion should be viewed as a symptom of *good* international risk-sharing, in the sense that it emerges when markets are perfectly integrated, and disappears when asset trade is not allowed. This issue is discussed further in Martin (2009a, 2009b).

4.5 A comparison with the Brandt, Cochrane and Santa-Clara (2006) risk-sharing index

Brandt, Cochrane and Santa-Clara (2006) propose an index of international risk-sharing

$$1 - \frac{\operatorname{var}(\log m^f_{t+1} - \log m^d_{t+1})}{\operatorname{var}(\log m^f_{t+1}) + \operatorname{var}(\log m^d_{t+1})}$$

that they show is very close to 1 in the data. Within the present model, the SDFs m^f_{t+1} and m^d_{t+1} are identical if there is asset trade, so the index is exactly equal to 1.

In the absence of asset trade, on the other hand, the index is strictly less than 1, and in the analytically tractable case $\eta = \infty$ is given by

$$1 - \frac{\operatorname{var}(\Delta \log Y^*_{t+1} - \Delta \log Y_{t+1})}{\operatorname{var}(\Delta \log Y^*_{t+1}) + \operatorname{var}(\Delta \log Y_{t+1})} = \operatorname{corr}(\Delta \log Y^*_{t+1}, \Delta \log Y_{t+1}) .$$

This suggests an alternative measure of the cost of shutting down asset trade: we can calculate, within the model, how much the Brandt–Cochrane–Santa-Clara (BCS-C) index would drop if asset trade were not allowed. The lower the correlation between output processes, the greater the potential gain from risk-sharing, and the more costly it is to shut down asset trade. This is reflected in a large drop in the BCS-C index. In the lognormal calibration, the index declines to 0.375. With independently occurring disasters (Disaster 1) the index drops to 0.154. If disasters occur simultaneously, the index equals 0.625 if disaster sizes are independent, or 0.684 if disaster sizes are positively correlated.

	Disaster 1	Disaster 2	Disaster 3	Lognormal
$\lvert\Delta$BCS-C$\rvert$	0.846	0.375	0.316	0.625
welfare cost	0.301	0.225	0.181	0.018

Table 6: Two measures of the cost of shutting down asset trade, in the $\eta = \infty$ case.

The first line of Table 6 reports the absolute value of the *change* in this index, $\lvert\Delta$BCS-C$\rvert$, when we move from asset trade to no asset trade. The second line reproduces the welfare cost in the case $\gamma = 8$ from Table 4, which measures the true cost of shutting down asset markets. We can now compare the ordering the two measures put on the four calibrations. (The numerical values cannot meaningfully be compared across the two rows.) The measures agree on one thing: comparing Disaster 1 to the lognormal calibration, it is a much worse idea to close asset markets in the presence of independent disasters, when the benefits of risk-sharing are very large, than to close them when there are no disasters. The BCS-C index generates this result because the correlation of output processes drops in the presence of independent disasters.

But the measures disagree on the importance of asset markets in the presence of simultaneous disasters, for the following reason: the BCS-C index only takes second moments of *log* SDFs into account. As a result, it is appropriate for lognormal or diffusion-based models, but not for measuring risk-sharing in calibrations with disasters.[13] When disasters are simultaneous, this drives the correlation in output processes *up*, so the BCS-C index declines (relatively) little when asset markets are closed. In contrast, the welfare cost measure,

[13] Analogously, it is reasonable to approximate $\sigma(M)/\mathbb{E}M$, the Hansen-Jagannathan (1991) measure of stochastic discount factor variability, by $\sigma(\log M)$ in a lognormal model—as Campbell and Cochrane (1999) do, for example. (Consider a lognormal model with SDF $M = e^{\mu+\sigma Z}$, where Z is Normal. Then $\sigma(M)/\mathbb{E}(M) = \sqrt{e^{\sigma^2} - 1} \approx \sigma = \sigma(\log M)$.) But the approximation fails badly in highly non-lognormal models, e.g. if there are disasters.

This observation applies to Brandt, Cochrane and Santa-Clara's (2006) equation (2); of course, none of this invalidates the logic behind their equation (1), which is an identity if asset trade is allowed.

which takes into account the effects of higher moments on expected utility, demonstrates that there is a significant cost of closing asset markets even if disasters are synchronized.

5 Conclusion

How much do financial markets matter? In calibrations similar to those proposed by Barro, Nakamura, Steinsson and Ursua (2009) and by Backus, Chernov and Martin (2010), I find that the cost of shutting down trade in financial assets is on the order of 3% of wealth if $\eta = 2$ and of 20% if $\eta = \infty$. These figures, which are, respectively, roughly 30 and 60 times larger than Cole and Obstfeld (1991) found in their corresponding calibrations, suggest that financial asset markets are of first-order importance for international risk-sharing. An important difference between the calibrations considered in this paper and the Cole-Obstfeld calibrations is that my calibrations deliver a realistic equity premium.

There is a parallel with the literature on the welfare costs of uncertainty: Lucas (1987) found that the welfare benefit of removing all consumption uncertainty—were that possible—would be negligible, while Barro (2009) and Martin (2008) observed that this welfare benefit would be orders of magnitude larger in a rare disaster model that generates a reasonable equity premium. In each case, the conclusions that should be drawn are order-of-magnitude conclusions; and they are important because of their implications for where the attention of economists and policy-makers should be directed.

A Derivation of the integral formula for W

Recall the definition of W:

$$W = \mathbb{E} \int_0^\infty e^{-\rho t} \left(\frac{\widehat{C}_t}{\widehat{C}_0} \right)^{1/2-\gamma} \left(\frac{Y_{it}}{Y_{i0}} \right)^{1/2} dt \, ,$$

where $\widehat{C}_t \equiv \left(\sqrt{Y_{1t}} + \sqrt{Y_{2t}} \right)^2$. By the symmetry assumption, W is independent of i, but to be concrete, I will fix $i = 1$ in what follows.

Write $y_{jt} = \log Y_{jt}$, and $\widetilde{y}_{jt} = \log(Y_{jt}/Y_{j0})$; thus $Y_{jt} = e^{y_{j0}+\widetilde{y}_{jt}}$, $\widetilde{y}_{j0} = 0$, and

$$W = \mathbb{E} \int_0^\infty e^{-\rho t} \left\{ \frac{\left[e^{(y_{10}+\widetilde{y}_{1t})/2} + e^{(y_{20}+\widetilde{y}_{2t})/2}\right]^2}{\left[e^{y_{10}/2} + e^{y_{20}/2}\right]^2} \right\}^{1/2-\gamma} e^{\widetilde{y}_{1t}/2}\, dt\,.$$

By the symmetry assumption, the two countries initially have the same outputs, so $y_{10} = y_{20}$. Cancelling terms in y_{10} and y_{20}, and taking the expectation inside the integral,

$$W = 2^{\widehat{\gamma}} \int_0^\infty e^{-\rho t} \cdot \mathbb{E}\left[\frac{e^{\widetilde{y}_{1t}/2}}{\left(e^{\widetilde{y}_{1t}/2} + e^{\widetilde{y}_{2t}/2}\right)^{\widehat{\gamma}}} \right] dt\,,$$

where I write $\widehat{\gamma}$ for $2\gamma - 1$. The main challenge is now to simplify the expectation

$$\begin{aligned} E_t &\equiv \mathbb{E}\left[\frac{e^{\widetilde{y}_{1t}/2}}{\left(e^{\widetilde{y}_{1t}/2} + e^{\widetilde{y}_{2t}/2}\right)^{\widehat{\gamma}}} \right] \\ &= \mathbb{E}\left[\frac{e^{(1/2-\widehat{\gamma}/4)\widetilde{y}_{1t}-(\widehat{\gamma}/4)\widetilde{y}_{2t}}}{\left[e^{(\widetilde{y}_{2t}-\widetilde{y}_{1t})/4} + e^{-(\widetilde{y}_{2t}-\widetilde{y}_{1t})/4}\right]^{\widehat{\gamma}}} \right]. \end{aligned}$$

To do so, I exploit a Fourier transform result used in Martin (2009a): for $\omega \in \mathbb{R}$ and $\widehat{\gamma} > 0$,

$$\frac{1}{\left(e^{\omega/2} + e^{-\omega/2}\right)^{\widehat{\gamma}}} = \int_{-\infty}^\infty e^{i\omega v} \mathscr{F}_{\widehat{\gamma}}(v)\, dv\,,$$

where i is the complex number $\sqrt{-1}$, and $\mathscr{F}_{\widehat{\gamma}}(v) \equiv \frac{1}{2\pi} \cdot \Gamma(\widehat{\gamma}/2 + iv)\Gamma(\widehat{\gamma}/2 - iv)/\Gamma(\widehat{\gamma})$ defines $\mathscr{F}_{\widehat{\gamma}}(v)$ in terms of the Gamma function. Applying this with $\omega = (\widetilde{y}_{2t} - \widetilde{y}_{1t})/2$, we find

$$\begin{aligned} E_t &= \mathbb{E}\left[e^{(1/2-\widehat{\gamma}/4)\widetilde{y}_{1t}-(\widehat{\gamma}/4)\widetilde{y}_{2t}} \cdot \int_{v=-\infty}^\infty e^{iv(\widetilde{y}_{2t}-\widetilde{y}_{1t})/2} \mathscr{F}_{\widehat{\gamma}}(v)\, dv \right] \\ &= \int_{-\infty}^\infty \mathscr{F}_{\widehat{\gamma}}(v)\, e^{\boldsymbol{c}(1/2-\widehat{\gamma}/4-iv/2,-\widehat{\gamma}/4+iv/2)t}\, dv\,. \end{aligned}$$

Substituting this back into the expression for W,

$$\begin{aligned} W &= 2^{\widehat{\gamma}} \int_0^\infty e^{-\rho t} \cdot E_t\, dt \\ &= 2^{\widehat{\gamma}} \int_{-\infty}^\infty \int_0^\infty \mathscr{F}_{\widehat{\gamma}}(v)\, e^{-[\rho-\boldsymbol{c}(1/2-\widehat{\gamma}/4-iv/2,-\widehat{\gamma}/4+iv/2)]t}\, dt\, dv \\ &= 2^{\widehat{\gamma}} \int_{-\infty}^\infty \frac{\mathscr{F}_{\widehat{\gamma}}(v)}{\rho - \boldsymbol{c}(1/2-\widehat{\gamma}/4-iv/2, -\widehat{\gamma}/4+iv/2)}\, dv\,. \end{aligned}$$

The final equality requires that $\mathrm{Re}\ \rho - \boldsymbol{c}(1/2-\widehat{\gamma}/4-iv/2, -\widehat{\gamma}/4+iv/2) > 0$ for all $v \in \mathbb{R}$. Martin (2009a) shows that a necessary and sufficient condition for this inequality to hold is that $\rho - \boldsymbol{c}(1/2-\widehat{\gamma}/4, -\widehat{\gamma}/4) > 0$, i.e. $\rho - \boldsymbol{c}(3/4-\gamma/2, 1/4-\gamma/2) > 0$.

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