Debt and Deficits:
Fiscal Analysis with Stationary Ratios

John Y. Campbell   Can Gao   Ian W. R. Martin*

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Abstract

We introduce a measure of a government's fiscal position that exploits cointegrating relationships among fiscal variables. The measure is a loglinear combination of tax revenue, government spending and the market value of government debt that—unlike the debt-GDP ratio—appears stationary in long historical data from the US and the UK and in postwar data from these and 14 other developed countries. The fiscal position must forecast either government debt returns or fiscal adjustment (a combination of high tax growth and low spending growth), or both. We find that fiscal adjustment, particularly through changes in spending, is the empirically relevant channel.

*John Campbell: Harvard University and NBER, https://scholar.harvard.edu/campbell/home. Can Gao: University of St. Gallen and Swiss Finance Institute, https://sites.google.com/view/can-gao/home. Ian Martin: London School of Economics, https://personal.lse.ac.uk/martiniw. We acknowledge helpful comments from Henning Bohn, John Cochrane, Bernard Dumas, Martin Ellison, Hanno Lustig, Greg Mankiw, Casey Mulligan, Emi Nakamura, Jon Steinsson, Andrew Scott, Tuomo Vuolteenaho, and seminar participants at Copenhagen Business School, Harvard, London School of Economics, University of British Columbia, University of Washington, Collegio Carlo Alberto, University of Turin, ESCP Business School, University of Hamburg, University of St. Gallen, Stockholm School of Economics, University of Cambridge, BIS, University of Lugano, NHH School of Economics, NBER Summer Institute, CEPR Paris Symposium, Macro Finance Society Oslo Meeting, Adam-Smith Workshop, CICF, EFA.

If a government is in a weak fiscal position, then over the long run taxes must rise, or spending must fall, or the returns to the holders of government debt must adjust; or some combination of all three possibilities must occur. As we will show, this follows essentially as a matter of accounting. But which channel is most important empirically?

Any answer to this question requires a suitable definition of the "fiscal position." We will argue that some seemingly natural definitions are problematic. Certainly the primary surplus of a government is an essential ingredient. The primary surplus—the excess of tax revenue over government expenditure—is the flow of resources that the government devotes to servicing its debt. When it is positive, the debt grows at a slower rate than the return on the debt. When it is negative—that is, when the government runs a primary deficit—the debt grows at a faster rate than the return on debt. In this paper we assume that both the growth rate of debt and the return on debt are stationary time series that have well defined unconditional means. It is traditional to assume that the unconditional mean return on the debt exceeds the unconditional mean growth rate of the debt (i.e., that "$r > g$"). Our framework allows for this possibility, but as it has been challenged by economists such as Ball, Elmendorf and Mankiw (1998), Blanchard (2019), Mehrotra and Sergeyev (2021) and Mian, Straub and Sufi (2022), and as it is inconsistent with sample averages in US data, we do not *require* that the assumption holds—and our main results are not sensitive to whether it holds or not.

To be useful in fiscal analysis, the primary surplus must be scaled in some way so that the resulting ratio is stationary. A common approach is to divide both the primary surplus and the value of debt by GDP to create the surplus-GDP and debt-GDP ratios. Given stationary debt growth and returns, stationarity of either of these two ratios implies the stationarity of the other because of the identity that links the primary surplus with the evolving value of the debt. Many papers treat both ratios as stationary and ask what forces return the debt-GDP ratio to its unconditional mean (see, for example, Bohn (1998, 1991, 2008), Cochrane (2001, 2022, 2023), Blanchard (2019), and Jiang et al. (2021*b*)).

Contrary to this approach, we find that the debt-GDP ratio does not behave like a stationary time series in US and UK data. Figure 1 plots the debt-GDP ratio over the period 1841-2022 in the US (panel (a)) and the period 1727-2022 in the UK (panel (b)).[1] The debt-GDP ratio has drifted persistently up and down

[1] Internet appendix IA.1 describes our data sources.

for long periods of time in each country, showing no strong tendency to return to a constant mean. Unit root tests do not reject, either in logs or levels, the null hypothesis that the debt-GDP ratio has a unit root, and cointegration tests fail to find statistically significant evidence that government debt is cointegrated with GDP. This nonstationarity helps to explain the (at first sight puzzling) finding in this literature that the debt-GDP ratio is not a successful predictor of fiscal outcomes.

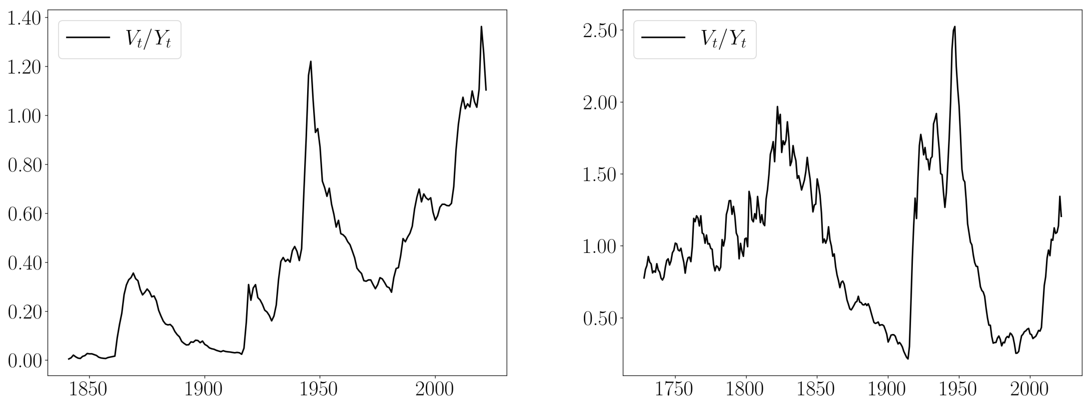
From a theoretical perspective, the nonstationarity of debt-GDP is not particularly surprising: for example, Barro (1979) writes, "There is no force that causes the ratio of debt to income to approach some target value". Even if one believes that economic forces act to make the primary surplus-GDP ratio and the debt-GDP ratio truly stationary in the very long run, the persistence of these time series implies that it is inadvisable to model them using the standard techniques of stationary time-series analysis (Campbell and Perron, 1991).

It would however be surprising to find an unconditionally positive or negative drift in debt-GDP, because that would imply arbitrarily large or small values for this ratio in the distant future. While the US data plotted in panel (a) of Figure 1 do convey a visual impression of an unconditionally positive (upward) drift, this drift is not statistically significant—equivalently, the mean difference between the growth rate of debt and the growth rate of GDP is statistically insignificant—and our analysis will assume that debt and GDP have the same unconditional mean growth rates so that the unconditional drift of the debt-GDP ratio is zero. The UK data plotted in panel (b) of Figure 1 are also consistent with this assumption.

If the debt-GDP ratio is nonstationary with zero unconditional drift, is there some other way to scale fiscal variables to make them stationary? One possibility is to scale the primary surplus by the value of debt, and work with the primary surplus-debt ratio. Under our maintained assumptions that the growth rate of debt and the return on the debt are stationary, the primary surplus-debt ratio should also be stationary.[2] The primary surplus-debt ratio is analogous in the fiscal context to the dividend-price ratio on a stock. Just as a corporation pays dividends to the owners of its stock, so the government pays primary surpluses to the owners of its

[2] Indeed, in the US and UK data standard unit root tests reject the null hypothesis that the primary surplus-debt ratio has a unit root in favor of the alternative that it is stationary (see Tables IA.14 and IA.18). However, this is also true of the primary surplus-GDP ratio. Primary surpluses are noisy enough that nonstationary dynamics in scaled surplus are hard for standard tests to detect. For this reason we do not emphasize unit root test results for ratios with the primary surplus in the numerator.

Figure 1: The debt-GDP ratio is nonstationary in US and UK data.



(a) US debt-GDP ratio.

(b) UK debt-GDP ratio.

debt. This suggests the possibility of analyzing the primary surplus-debt ratio using a Campbell and Shiller (1988*a*) loglinearization to relate it to future log returns on debt and log growth rates of primary surpluses.

Two problems arise in doing so, and both result from the fact that the primary surplus can be negative. First, the log growth rate of the primary surplus is ill-defined when the surplus is negative. Second, an exogenous increase in the debt can either raise or lower the primary surplus-debt ratio depending on whether the primary surplus is currently positive or negative. Thus, the effect of a given shock to the primary surplus-debt ratio depends on the sign of the ratio at each point in time. Both these problems also afflict the standard analysis of the primary surplus-GDP ratio.

In this paper we develop an alternative loglinear analysis that solves these problems. Our approach is to define the fiscal position of the government as a linear combination of the log ratios of tax revenue to debt and government spending to debt. Two free parameters in our fiscal position measure are chosen so that the fiscal position has the same mean as a logarithmic variant of the primary surplus-debt ratio, and so that it well approximates the stationary time-series variation of that ratio. We show that our measure can be loglinearly related to the growth rates of tax revenue and government expenditure. Since both revenue and expenditure are always positive, their log growth rates are well defined. The effect of debt on our fiscal position measure does not depend on whether the primary surplus is currently positive or negative, but only on the long-run average value of the primary surplus (or equivalently, the relationship between the unconditional means of the growth rate of and return on the government debt).

The approximations developed by Giannitsarou, Scott and Leeper (2006) and Berndt, Lustig and Yeltekin (2012) are similar in spirit but rely on the assumption that the tax revenue-debt and government expenditure-debt ratios are stationary, so that one can approximate around their means. The evidence regarding this assumption is mixed: we can reject the presence of a unit root for both series in long-run US data but not in postwar US data, nor in either long-run or postwar UK data. We find, however, that their logs are cointegrated (with a cointegrating vector that is close to but not equal to a unit vector). We use this finding to develop an approximation, related to the work of Gao and Martin (2021), that does not rely on questionable stationarity assumptions.

As our measure of the fiscal position is stationary, it is a useful predictor variable for fiscal analysis. We use it to explore the dynamics of debt, tax revenue, and government expenditure in the long-run US and UK data. We estimate both local projections and a vector autoregression (VAR) that includes the return on debt, the growth rate of tax revenue, the growth rate of output (which we include as a fundamental determinant of tax revenue and spending), and our measure of the fiscal position. The VAR system includes one extra lag of the fiscal position to ensure that the information set and hence our empirical results are identical (up to approximation error) whether we include tax revenue or spending growth in the VAR.

Our main empirical findings are as follows. First, expected returns on government debt, while time-varying, are not variable or persistent enough to contribute importantly to the dynamics of the fiscal position. Instead, fiscal adjustment—changes in the growth rates of tax revenue and spending—accounts for the mean reversion of the fiscal position. Second, spending and taxes contribute roughly equally to fiscal adjustment in the US, but in the UK spending is the primary driver of fiscal adjustment. Third, the response of the fiscal position to shocks in tax revenue and government expenditures is determined almost entirely by mean reversion in the growth rates of taxes and expenditures. Expected returns on government debt again have little importance, and the same is true for unexpected returns on debt contemporaneous with tax and expenditure shocks.

We repeat the analysis for post-World War II data from the US, the UK, and

14 other developed countries: Canada, Japan, Switzerland, and 11 countries in the eurozone (Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, the Netherlands, and Portugal). While the sample periods are much shorter in these other countries, reducing the power of unit root tests, the surplus-debt ratio appears stationary and the debt-GDP ratio nonstationary, as in the long-run US and UK data. Our main findings hold up well across countries. Returns make only a minor contribution to the dynamics of the fiscal position, and in most countries fiscal adjustment is driven primarily by spending growth rather than by the growth rate of tax revenue.

Two points should be kept in mind when interpreting our results. First, because we conduct a reduced-form time-series analysis, we cannot make causal statements about fiscal dynamics. The starting point for our analysis is an identity also exploited in the literature on the fiscal theory of the price level (Sargent and Wallace, 1981; Leeper, 1991; Sims, 1994; Woodford, 1995; Cochrane, 2001, 2023). Our finding that shocks to expected and unexpected returns on debt make only a minor contribution to resolving variations in the fiscal position cuts against a channel often emphasized in that literature, but we cannot offer a causal interpretation. Although variations in the fiscal position are largely resolved by adjustments to surpluses in the developed countries we study, we do not provide a theory of the mechanisms underlying that fact; and it might be that shocks to expected returns play a more substantial role in some emerging economies that have experienced more turbulent political and economic conditions.

Second, we take the returns on government debt as given, measuring them in the data without requiring them to satisfy the restrictions of any particular asset pricing model or even ruling out the existence of a bubble in the value of the debt. Our framework accommodates the potential presence of a convenience yield in government debt that investors value separately from its return (Greenwood, Hanson and Stein, 2015; Jiang et al., 2021*a*; Krishnamurthy and Vissing-Jorgensen, 2012; Reis, 2022; Mian, Straub and Sufi, 2022). If such a convenience yield is present, it will drive down the average measured financial return on the debt, but the identities we exploit continue to hold. Indeed, we can even allow for the possibility that convenience yield considerations place us in an "$r < g$" regime in which the mean return on the debt is lower than its mean growth rate.

The organization of the paper is as follows. In Section 1 we lay out our framework for fiscal analysis, beginning with a simple steady-state analysis that motivates our subsequent dynamic model. We apply the framework empirically to long-run historical US and UK data in Section 2, which decomposes the variability of the fiscal position and of the responses to tax, spending, and return shocks. We examine post-World War II data in the US, the UK, and 14 other developed countries in Section 3. Section 4 concludes. An online appendix (Campbell, Gao and Martin, 2025) presents supplementary details.

1 A framework for fiscal analysis

By definition, the gross return on government debt is

$$R_{t+1} = \frac{V_{t+1} + T_{t+1} - X_{t+1}}{V_t}. \tag{1}$$

Here R_{t+1} is the return on debt from time t to $t+1$, V_t is the total market value of the debt in period t, T_{t+1} is tax revenue and X_{t+1} is expenditure. All variables are defined in real terms.

We define the primary surplus as $S_t = T_t - X_t$ and assume throughout that the gross return R_{t+1} is strictly positive: this rules out the possibility of a total default on all government debt obligations with zero recovery. Note that the debt return R_{t+1} should only be interpreted as a riskless interest rate in the special case in which all government debt is short-term real debt. We allow debt to be risky: the realized return on debt is low if, for example, real yields rise, or if there is an explicit default. Inflation can also resolve fiscal difficulties by driving down real returns. It can do so in two ways: either via an unexpected inflation, which reduces the value of all nominal debt, or via increases in expected inflation that reduce the value of long-dated nominal debt by driving up nominal yields.

1.1 The primary surplus-debt ratio in steady state

As a first step toward a simple benchmark, let us suppose that the conditional expectations of growth in tax revenue, spending, and the debt are all equal to some constant, G.[3] Similarly, let us suppose that the conditionally expected return on

[3] This assumption is not unreasonable for unconditional expectations. Table IA.20 shows that we cannot reject equality of mean growth rates for tax, spending, debt and GDP in either the US or the UK.

the debt equals R. Equation (1) then implies

$$R = \mathbb{E}_t \frac{V_{t+1}}{V_t} + \mathbb{E}_t \frac{T_{t+1}}{T_t}\frac{T_t}{V_t} - \mathbb{E}_t \frac{X_{t+1}}{X_t}\frac{X_t}{V_t} = G\left(1 + \frac{S_t}{V_t}\right). \tag{2}$$

It follows that the primary surplus-debt ratio is a constant:

$$\frac{S_t}{V_t} = \frac{R - G}{G}. \tag{3}$$

This expression resembles the classic Gordon formula for the value of a stock paying dividends that grow at a constant rate G. Importantly, however, there is no presumption in this context that S is positive or that R exceeds G. When R is lower than G, the government runs a primary deficit (a negative S_t) in steady state. For future reference, notice that equation (3) can also be written as

$$\log\left(1 + \frac{S_t}{V_t}\right) = \log R - \log G. \tag{4}$$

Within this steady-state model, any increase in debt V_t must change S_t in the same proportion. Equivalently, the impact of an extra dollar of debt V_t is to change the surplus S_t by $(R-G)/G$ dollars. In the case where R exceeds G, additional debt requires taxes to increase relative to spending; but in the case where R is lower than G, additional debt allows taxes to fall relative to spending because the steady-state primary deficit increases in proportion to the value of the debt.

It is illuminating to calculate the ratio of spending to taxes in this steady-state model. We have

$$\frac{X_t}{T_t} = 1 - \frac{S_t}{T_t} = 1 - \frac{S_t/V_t}{T_t/V_t} = 1 - \frac{(R-G)/G}{T_t/V_t}. \tag{5}$$

The ratio of spending to taxes depends both on the primary surplus-debt ratio (or equivalently the difference between R and G), and on the size of the government relative to the size of the debt as captured by the tax-debt ratio T_t/V_t. In the case where R exceeds G, the government must run a primary surplus so the spending-tax ratio is less than one; but when the government uses taxes to finance a large quantity of current spending as well as to service its debt, both tax revenue and spending are large relative to the debt so the spending-tax ratio need not vary far from one to generate the required primary surplus-debt ratio.

The steady-state model allows shocks to move taxes, but if debt is held constant any shock to the level of taxes must be offset by an equal shock to the level of spending so that the primary surplus remains unchanged. In proportional terms, at a constant level of debt any proportional change in taxes must equal the spending-tax ratio times the proportional change in spending. In the case where R exceeds G, the spending-tax ratio is less than one so the proportional change in spending must be greater than the proportional change in taxes; the reverse is true in the case where R is less than G. These properties of the steady-state model will help us understand the loglinear approximation that we develop in the next subsection.

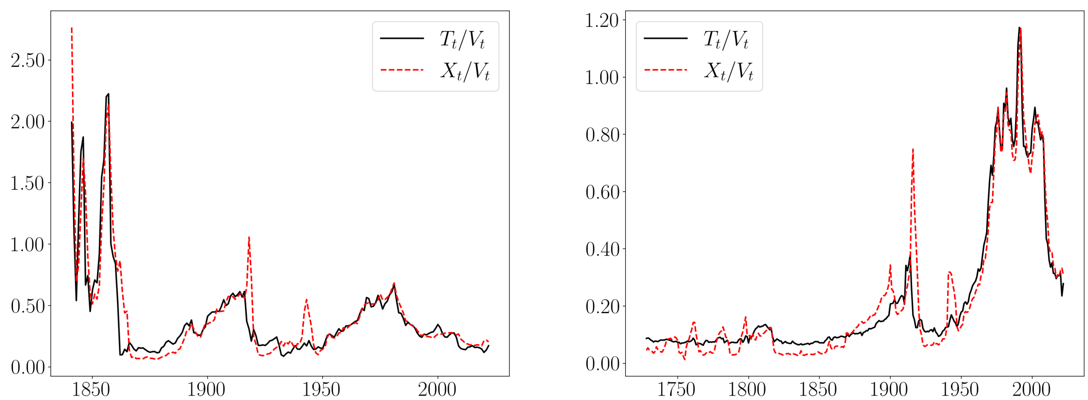
1.2 The dynamics of tax-debt and spending-debt ratios

Our goal is to relate a measure of the government's fiscal position to adjustments in tax growth, spending growth, and debt returns within a linear model. Writing $\tau_t = \log(T_t)$, $x_t = \log(X_t)$, and $r_t = \log(R_t)$, we want to relate the fiscal position to $\Delta\tau_t$, Δx_t, and r_t. Our framework does not restrict the time-series behavior of conditional expectations, but as noted above we assume that the *unconditional* means of the growth rates of tax revenue, spending, and debt are all equal to each other (and to the unconditional mean of GDP growth) so that tax-debt, spending-debt, and debt-GDP ratios do not trend upwards or downwards over time. We write g for this shared growth rate, $\mathbb{E}\,\Delta\tau_{t+1} = \mathbb{E}\,\Delta x_{t+1} = \mathbb{E}\,\Delta v_{t+1} = g$, and r for the unconditional expected log return on the debt, $\mathbb{E}\,r_{t+1} = r$.

For our model to be linear, we need our fiscal position measure fp_t to be linear in τ_t, x_t, and v_t. Since the fiscal position is homogeneous of degree zero in these inputs (reflecting the arbitrary units used to measure tax revenue, spending, and debt), we can without loss of generality write it as linear in the two ratios $\tau v_t = \log T_t/V_t$ and $xv_t = \log X_t/V_t$.

We require that our fiscal position measure is stationary, since it will be linearly related to the stationary variables $\Delta\tau_t$, Δx_t, and r_t. In long-term US and UK data the ratios τv_t and xv_t do not appear stationary, reflecting long-term changes in the size of the government's currently funded activities relative to the size of its debt. We illustrate this by plotting in Figure 2 the levels of the ratios, T_t/V_t and X_t/V_t, with US data in panel (a) and UK data in panel (b). The figures show persistent variation in the size of government relative to the size of the debt: in the US, the federal government had little debt before the US Civil War and was therefore large

Figure 2: The spending-debt and tax-debt ratios in US and UK.



(a) US

(b) UK

relative to its debt, while in the UK the government's tax revenues and expenditures grew relative to the debt with the creation of the modern welfare state after World War II. The figures also show that the tax-debt and spending-debt ratios tend to move together, with transitory deviations between the two reflecting events such as major wars that temporarily drive up spending. Both visual inspection and standard tests for cointegration indicate that the two ratios are cointegrated, in other words that a stationary linear combination of the two ratios exists.[4] This raises the hope that we can parameterize our fiscal position measure to be stationary despite the persistent variation of τv_t and $x v_t$.

1.3 The logarithmic primary surplus-debt ratio

We choose our fiscal position measure to approximate a logarithmic version of the primary surplus-debt ratio, $\log(1 + S_{t+1}/V_{t+1})$. This ratio is analogous to the dividend-price ratio measure, $\log(1+D_{t+1}/P_{t+1})$, studied by Gao and Martin (2021). It allows the primary surplus to go negative; moreover, the measure is in natural units, in the sense that $\log(1 + S_{t+1}/V_{t+1})$ is approximately equal to S_{t+1}/V_{t+1} if surplus-debt is small.

[4]Johansen tests (in both the trace and eigenvalue form) reject the null hypothesis that there is no cointegrating relationship between τv_t and $x v_t$ at the 99% level.

To motivate this approach, rewrite equation (1) as

$$R_{t+1} = \frac{V_{t+1}}{V_t}\left(1 + \frac{S_{t+1}}{V_{t+1}}\right). \tag{6}$$

Taking logs of (6), and using lower-case letters to denote logarithms of variables written with upper-case letters, we have

$$\log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right) = r_{t+1} - \Delta v_{t+1}. \tag{7}$$

Since returns and debt growth are stationary, the logarithmic surplus-debt ratio should also be stationary; and a fiscal position measure that approximates the left-hand side of this equation will be linearly related to future returns and debt growth. Unconditionally, equation (7) implies the Gordon-growth-type relationship

$$\mathbb{E}\log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right) = r - g. \tag{8}$$

To illustrate the empirical practicability of this approach, Figure 3 shows the evolution of the measure of the fiscal position introduced below, together with the logarithmic primary surplus-debt ratio, $\log(1 + S_t/V_t)$, over the sample period 1841-2022 in the US and 1727-2022 in the UK. As the surplus can take negative values, we plot the series on a linear scale. Although the surplus-debt ratio is not constant as it would be in a Gordon-growth-type model, it does appear to be stationary.[5]

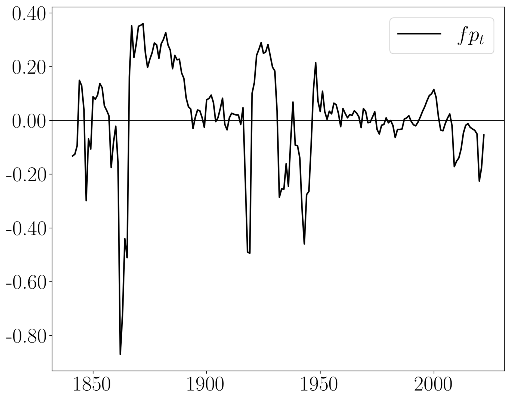
1.4 A loglinear measure of the fiscal position

To construct a tractable measure of the fiscal position, we linearize the logarithmic primary surplus-debt ratio in τv_t and xv_t. Specifically, we use

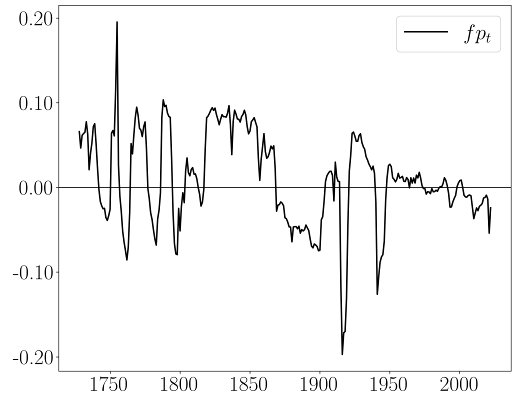
$$\log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right) = \log\left(1 + e^{\tau v_{t+1}} - e^{xv_{t+1}}\right) \tag{9}$$

[5]This impression is supported by an augmented Dickey–Fuller (ADF) test, reported in Table IA.14, which rejects for each of our sample periods with p-values that never exceed 0.04. Although unit root tests can have poor finite-sample properties for ratios with noisy numerators such as the primary surplus, this finding, together with the theoretical presumption that the surplus-debt ratio should be stationary, gives us confidence to base our analysis on a stationarity assumption.

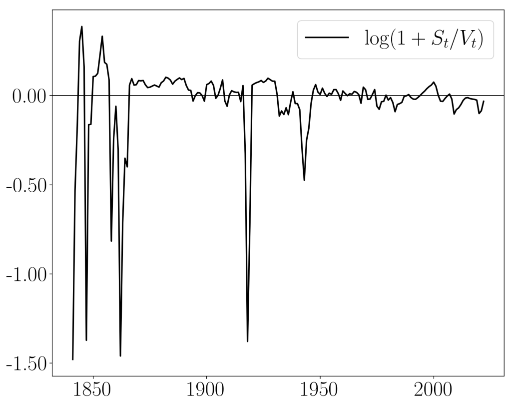
Figure 3: The fiscal position and surplus-to-debt are stationary in the US and UK.



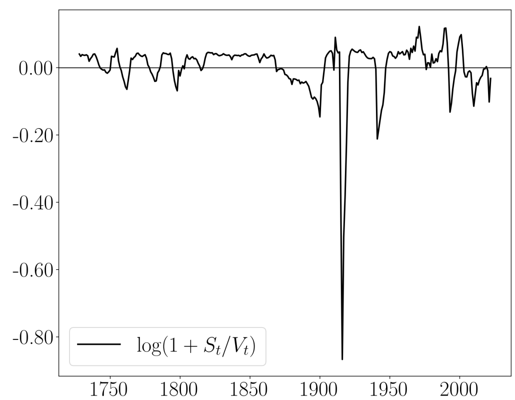
(a) US fiscal position.



(b) UK fiscal position.



(c) US surplus-GDP ratio, $\log(1 + S_t/V_t)$.



(d) UK surplus-GDP ratio, $\log(1 + S_t/V_t)$.

and linearize the right hand side of this equation around $(\tau v_{t+1}, xv_{t+1}) = (\log a, \log b)$, where the linearization points a and b are both positive. We have

$$\log\left(1 + e^{\tau v_{t+1}} - e^{xv_{t+1}}\right) = k + \frac{1}{1 + a - b}\left(a\,\tau v_{t+1} - b\,xv_{t+1}\right) \tag{10}$$

up to higher order terms in τv_{t+1} and xv_{t+1}, where

$$k = \log\left(1 + a - b\right) + \frac{b\log b - a\log a}{1 + a - b}\,. \tag{11}$$

We choose the linearization points a and b to satisfy two conditions. First, we want to linearize around the unconditional mean of $\log(1 + S_{t+1}/V_{t+1})$: that is, we require

$$\log(1 + a - b) = \mathbb{E} \log \left(1 + \frac{S_t}{V_t}\right) . \tag{12}$$

This assumption will ensure that our chosen measure of the fiscal position behaves like $\log(1 + S/V)$ in deterministic or steady-state models.

We write

$$\mathbb{E} \log(1 + S_t/V_t) = - \log \rho \, , \tag{13}$$

where the parameter ρ lies between zero and one if $\mathbb{E} \log(1 + S_t/V_t) > 0$ and is greater than one if $\mathbb{E} \log(1 + S_t/V_t) < 0$. From equation (8), these two possibilities correspond to the cases where $r > g$ and $r < g$, respectively. In this notation, equation (12) becomes

$$1 + a - b = \frac{1}{\rho} \, . \tag{14}$$

Thus when $\mathbb{E} \log(1 + S_t/V_t) > 0$ (the "$r > g$" case), a is greater than b, reflecting the fact that taxes must exceed spending on average so that proportional changes in taxes have a greater dollar impact than the same proportional changes in spending. When $\mathbb{E} \log(1 + S_t/V_t) < 0$ (the "$r < g$" case), a is less than b, reflecting taxes that are lower than spending on average and whose proportional changes therefore have a lower dollar impact than the same proportional changes in spending.

Second, we want the right-hand side of (10) to be stationary, as the left-hand side is. Given the cointegrating relationship between τv_t and $x v_t$, this requires that

$$\frac{b}{a} = \beta \, , \tag{15}$$

where β is the cointegrating coefficient such that $\tau v_t - \beta \, x v_t$ is stationary. The parameter β is less than one in the $r > g$ case and greater than one in the $r < g$ case, again reflecting the unequal dollar impact of proportional changes in taxes and spending when taxes are greater or less than spending on average. The boundary between these two cases, where $r = g$, has $\rho = \beta = 1$.

Equations (14) and (15) jointly determine a and b in terms of β and ρ. We have

$$a = \frac{1}{1 - \beta} \frac{1 - \rho}{\rho} \quad \text{and} \quad b = \frac{\beta}{1 - \beta} \frac{1 - \rho}{\rho} \, . \tag{16}$$

Plugging the expressions for a and b back into (10), we have our linearization

$$\log\left(1+\frac{S_{t+1}}{V_{t+1}}\right)=\log\left(1+e^{\tau v_{t+1}}-e^{xv_{t+1}}\right)=\underbrace{k+\frac{1-\rho}{1-\beta}\left(\tau v_{t+1}-\beta\, xv_{t+1}\right)}_{fp_{t+1}}, \tag{17}$$

where the first equality follows from the definition of surplus. Here k is as in equation (11) with a and b given by (16).

We will refer to the quantity on the far right-hand side of equation (17) as fp_{t+1} and will use it as our measure of the government's fiscal position. That is, we define

$$fp_t=k+\frac{1-\rho}{1-\beta}\left(\tau v_t-\beta\, xv_t\right)=k+\frac{1-\rho}{1-\beta}\left(\tau_t-\beta x_t-(1-\beta)v_t\right), \tag{18}$$

where

$$k=\rho\log\rho+(1-\rho)\log\frac{1-\rho}{1-\beta}-\frac{1-\rho}{1-\beta}\beta\log\beta. \tag{19}$$

The fiscal position is a linear combination of log tax revenue, log spending, and the log value of the government debt. The value of the debt has an effect proportional to $(1-\beta)$. As r approaches g, β approaches one and the value of the debt becomes less important for the fiscal position.

In the case where $r=g$, $\beta=1$ and debt drops out. In this case, the linearization points a and b are equal to one another and the fiscal position takes a particularly simple form:

$$fp_t=\tau_t-x_t, \tag{20}$$

the log ratio of tax revenue to spending. The level of debt is irrelevant when $r=g$ because in this case any amount of debt can be rolled over forever without requiring fiscal adjustment in the future.

Although fp_t equals $\log(1+S_t/V_t)$ in a deterministic or steady-state model, it differs in an important way from $\log(1+S_t/V_t)$ away from steady state. Consider first the case $r>g$ in which ρ and β are less than one. As the level of debt rises with surplus held fixed, fp_t declines whether the surplus is currently positive or negative. Combining this property with the standard positive response of fp_t to tax revenue and negative response to spending, we can think of fp_t as a measure of the fiscal position: it is high when the government is in a strong fiscal position, and low when the government is in a weak fiscal position. By contrast, the more conventional measures S_t/V_t and $\log(1+S_t/V_t)$ are harder to interpret: as the debt grows, they go *down* if the primary surplus is currently positive, but *up* if the surplus is currently negative. Conversely, in the $r < g$ case, with ρ and β greater than one, the fiscal position fp_t is increasing with the size of the debt, regardless of the sign of the current surplus; and in the boundary case where $r = g$, debt has no effect on the fiscal position, again regardless of the sign of the current surplus.

The irrelevance of debt for the fiscal position in the case $r = g$ is also important for understanding the accuracy of the linear approximation (10). That approximation uses fixed linearization points a and b corresponding to given levels of the tax-debt and spending-debt ratios. Since these ratios are nonstationary, one might be concerned that the accuracy of the approximation will be poor when they drift far away from the points a and b. Equivalently, cointegration of the log ratios can only be approximate if the primary surplus-debt ratio is stationary in levels: when tax and spending increase relative to debt, as happened for example in the UK after World War II, the log ratios of tax-debt and spending-debt must approach one another to maintain stationarity of the primary surplus-debt ratio in levels, and this will violate the assumption that the log ratios are cointegrated with a fixed cointegrating vector. Fortunately, this effect disappears when $r = g$, because then $\rho = \beta = 1$ for any level of debt; and the effect is very weak when r is close to g, because then ρ and β are very close to one and relatively insensitive to the level of debt. Empirically, in the US we estimate r slightly smaller than g and in the UK we estimate it slightly greater than g, but in both cases r and g are sufficiently close to one another that log tax-debt and log spending-debt appear cointegrated over long periods of time and our loglinear framework has only modest approximation error.

1.5 A present value model for the fiscal position

The linearity of fp_t allows us to relate it to fundamentals in a linear present value framework. Inserting the linearization (17) into the exact identity (7), we have

$$r_{t+1} = \Delta v_{t+1} + fp_{t+1} \,. \tag{21}$$

Taking differences of (18) and rearranging, we have

$$(1-\rho)\Delta v_{t+1} = \frac{1-\rho}{1-\beta}\Delta\tau_{t+1} - \beta\frac{1-\rho}{1-\beta}\Delta x_{t+1} - \Delta fp_{t+1} \,. \tag{22}$$

We use (22) to eliminate Δv_{t+1} from (21), giving, after some rearrangement,

$$fp_t = (1-\rho)\left[r_{t+1} - \frac{1}{1-\beta}\Delta\tau_{t+1} + \frac{\beta}{1-\beta}\Delta x_{t+1}\right] + \rho\, fp_{t+1}\,. \tag{23}$$

The unconditional implications of equation (23) are straightforward. Taking unconditional expectations and rearranging, we find that

$$\mathbb{E}\, fp_t = \mathbb{E}\, r_t - \frac{1}{1-\beta}\,\mathbb{E}\,\Delta\tau_t + \frac{\beta}{1-\beta}\,\mathbb{E}\,\Delta x_t\,. \tag{24}$$

Moreover, we must have equal unconditional growth rates of tax, spending, and debt so that fiscal ratios do not trend upwards or downwards over time. Writing $\mathbb{E}\,\Delta\tau_t = \mathbb{E}\,\Delta x_t = \mathbb{E}\,\Delta v_t = g$ and $\mathbb{E}\, r_t = r$, as before, equation (24) shows that the unconditional Gordon growth model (8) continues to hold under our approximation,

$$\mathbb{E}\, fp_t = r - g\,. \tag{25}$$

To characterize conditional dynamics, we solve (23) forward in the usual way to find that

$$fp_t = (1-\rho)\sum_{j=0}^{T-1}\rho^j\left[r_{t+1+j} - \frac{1}{1-\beta}\Delta\tau_{t+1+j} + \frac{\beta}{1-\beta}\Delta x_{t+1+j}\right] + \rho^T fp_{t+T}\,. \tag{26}$$

At this point it is tempting to take a limit as $T\to\infty$, and indeed if $\rho<1$ we can. But as we want to allow for the possibility that $\rho>1$, we do not—yet—do so. Instead we take conditional expectations and subtract unconditional expectations, giving

$$\hat{fp}_t = (1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\left[\hat{r}_{t+1+j} - \frac{1}{1-\beta}\hat{\Delta\tau}_{t+1+j} + \frac{\beta}{1-\beta}\hat{\Delta x}_{t+1+j}\right] + \rho^T\,\mathbb{E}_t\,\hat{fp}_{t+T}\,, \tag{27}$$

where a hat on a variable indicates a deviation from unconditional mean, $\hat{y}_t = y_t - \mathbb{E}\, y_t$.

For the right-hand side of this equation to converge appropriately as T tends to infinity, we require that the conditional expectations of the various hatted quantities converge to zero faster than ρ^T tends to infinity. This always holds in the $r>g$ case, with $\rho<1$; in the $r<g$ case, with $\rho>1$, we impose this as an assumption.

(In our empirical work, we will use a VAR to calculate conditional expectations. In this case the assumption will hold so long as the maximal eigenvalue of the VAR matrix has magnitude less than $1/\rho$. This property ensures that all variables in our VAR mean-revert sufficiently quickly. Empirically, we find that it holds in our applications.) When it holds, we have

$$\hat{fp}_t = (1-\rho)\sum_{j=0}^{\infty}\rho^j\,\mathbb{E}_t\left[\hat{r}_{t+1+j} - \frac{1}{1-\beta}\hat{\Delta\tau}_{t+1+j} + \frac{\beta}{1-\beta}\hat{\Delta x}_{t+1+j}\right]. \tag{28}$$

This is the dynamic generalization of the static present value formula that we were seeking. In words, if the government is in a strong fiscal position ($\hat{fp}_t$ is high), then either the holders of government debt will earn unusually high log returns (in the case where $\rho < 1$) or unusually low log returns (in the case where $\rho > 1$), or taxes will grow slowly, or government expenditure will grow rapidly, or some combination of the above will occur, at some point in the future. Note that only the effect of debt returns switches sign when the parameter ρ crosses one, because high debt returns increase the value of debt which weakens the fiscal position when $\rho < 1$ but strengthens it when $\rho > 1$. Tax and spending growth, by contrast, are related to the fiscal position with the same sign regardless of the value of ρ, because the coefficient β is always the same side of one as the coefficient ρ.

Three further points about equation (28) are worth noting. First, the discounting of the future with discount factor ρ implies that when $\rho < 1$, the longer fiscal adjustment through tax or spending growth is delayed, the larger it must ultimately be. When $\rho > 1$, by contrast, delayed adjustment is more powerful because the changes in debt that occur along the adjustment path reduce the adjustment that is ultimately needed. Both these effects are weak, however, when ρ is close to one as we estimate to be the case in both US and UK data.

Second, the multiplication of tax growth by $1/(1-\beta)$ and of spending growth by $\beta/(1-\beta)$—both large numbers, given that β is close to one—reflects the fact that the average primary surplus is small relative to the average levels of tax revenue and government expenditure, so that small percentage changes in either taxes or spending have large proportional effects on the primary surplus and hence on our measure of the fiscal position. The coefficient on tax growth is slightly larger when ρ and β are less than one, because then taxes are higher on average than spending so proportional changes in taxes have a larger dollar impact than the same proportional changes in spending. The reverse is true when ρ and β are greater than one.

Another way to understand this point is to use equation (16) to express β in terms of ρ and the loglinearization parameters a and b. We could express β either as a function of ρ and a or as a function of ρ and b; to keep things symmetrical, we do both and then average the resulting identities. This allows us to rewrite (28) as

$$fp_t = \sum_{j=0}^{\infty} \rho^j \left[(1-\rho) \left(r_{t+1+j} - \frac{\Delta\tau_{t+1+j} + \Delta x_{t+1+j}}{2} \right) + \rho\phi \left(\Delta x_{t+1+j} - \Delta\tau_{t+1+j} \right) \right], \tag{29}$$

where $\phi = (a+b)/2$.

Writing the identity in this way allows us to emphasize two conceptually distinct factors which matter for the interplay between debt and deficits. The first is captured by the parameter ρ, which one can think of as measuring the burden of debt: it is linked to the average size of the surplus that is required to service the debt, as discussed in the steady-state example of Section 1. When ρ is low, a large surplus is required to service each dollar of market value of the debt; at the other end of the spectrum, when $\rho = 1$ there is no debt burden at all because the debt need never be paid off. The second, captured by ϕ, measures the scale of tax and of spending in gross terms: it captures the overall size of the government relative to the value of its debt. The first term on the right hand side of equation (29) corresponds to the standard Gordon growth model, where growth is measured using the average of tax and spending, and the second term captures the effect of changing the growth rate of spending relative to the growth rate of tax revenue. When the government is large, as captured by the parameter ϕ, small changes in the relative growth rates of spending and taxes can have a large impact on the fiscal position.

Finally, we should bear in mind that it is expected *log* returns that dictate whether $r > g$ or $r < g$, and that enter into the identity (28).[6] As Gao and Martin

[6]Related, Kocherlakota (2023) shows, in models driven by a discrete-time time-homogeneous Markov process, that infinite debt rollover can be sustained if the yield on an infinitely long-term zero-coupon bond is sufficiently low. Martin and Ross (2019) show, in the finite-state Markov chain setting, that the infinitely long yield equals the unconditional expected log return on the long bond; in this case infinite debt rollover is possible if the expected log return on debt is sufficiently low. For us, the relevant quantity is the expected log return on the debt as a whole, as the government does not in practice finance itself through long-horizon zero-coupon borrowing.

(2021) note, we can write

$$\mathbb{E}_t\, r_{t+1+j} = \log \mathbb{E}_t\, R_{t+1+j} - \frac{1}{2}\operatorname{var}_t r_{t+1+j} - \sum_{n=3}^{\infty} \frac{\kappa_t^{(n)}(r_{t+1+j})}{n!}\,, \tag{30}$$

where $\kappa_t^{(n)}(r_{t+1+j})$ is the nth conditional cumulant of the log return. If debt returns are conditionally lognormal, then the higher cumulants $\kappa_t^{(n)}(r_{t+1+j})$ are zero for $n \geq 3$, but even in this case, low expected log returns—a potential resolution of poor fiscal health in the case where $\rho < 1$—may be consistent with *high* expected simple returns if returns are volatile (that is, the second cumulant is large); and the gap between the two may be wider still if log returns are right-skewed (so that the third cumulant is large) or fat-tailed (so that the fourth cumulant is large); and so on.

Some of our results below analyze the importance of tax and spending separately. We also find it useful to define a combination of the two that we call *fiscal adjustment* and write as fa:

$$fa_{t+1} = \Delta\tau_{t+1} - \beta\Delta x_{t+1}. \tag{31}$$

Fiscal adjustment is the change in the stationary linear combination of τ_{t+1} and x_{t+1} defined by the cointegrating coefficient β. As β tends to one, fiscal adjustment approaches the difference between the growth rates of taxes and of spending, $\Delta\tau_{t+1} - \Delta x_{t+1}$. With this definition, the identity (26) becomes

$$fp_t = (1-\rho)\sum_{j=0}^{T-1} \rho^j \left[r_{t+1+j} - \frac{1}{1-\beta} fa_{t+1+j} \right] + \rho^T fp_{t+T}, \tag{32}$$

and identity (28) becomes

$$\hat{fp}_t = (1-\rho)\sum_{j=0}^{\infty} \rho^j\, \mathbb{E}_t \left[\hat{r}_{t+1+j} - \frac{1}{1-\beta} \hat{fa}_{t+1+j} \right]. \tag{33}$$

This equation highlights the distinction between returns on the debt and fiscal adjustment of taxes and spending as responses to the government's fiscal position.

2 Debt and deficits in long-term US and UK data

To implement our approach to fiscal analysis, we begin by using these results—notably the identities (28) and (33)—to study debt and deficits in long-term data from the US and the UK, the series illustrated in Figures 1, 2, and 3.

2.1 Data

To measure tax revenue and spending of the US federal government, we use annual data on total receipts, outlays, and interest payments reported by the Office of Management and Budget (OMB). These data are available on the FRED website for recent years (since 1901 for receipts and outlays, and 1940 for interest payments), and we augment them with hand-collected data for earlier years. We use total receipts to measure T_t, and the difference between total outlays and interest payments to measure X_t.[7]

We treat social security taxes as taxes and social security benefit payments as outlays, so we are consolidating the Social Security trust fund with the federal government. For this reason we do *not* include the nonmarketable debt held by the Social Security trust fund in our measure of government debt. Conversely, as our measures of tax receipts and expenditures do include transfers between the Federal Reserve and the US Treasury, we are treating the Federal Reserve as being outside the federal government. We therefore include the marketable debt held by the Federal Reserve in our measure of the debt.

Our framework requires that we measure the market value of the government debt, not the more readily available face value of the debt. We use data on the market value of marketable federal debt provided by the Federal Reserve Bank of Dallas from 1942, and splice this series together with data from Hall and Sargent (2021) for years before 1942.[8] The earliest data available are from the 1790s, but

[7]Receipts include taxes and other collections from the public. See table 17.1 in https://www.whitehouse.gov/wp-content/uploads/2023/03/ap_17_receipts_fy2024.pdf for details. Outlays are payments that liquidate obligations. Details are given in the chapter on outlays in https://www.whitehouse.gov/wp-content/uploads/2023/03/ap_15_concepts_fy2024.pdf. The US federal government also collects income from the public through market-oriented activities. Collections from these activities are subtracted from gross outlays, rather than being added to taxes and other governmental receipts. See table 18.1 in https://www.whitehouse.gov/wp-content/uploads/2023/03/ap_18_offsetting_fy2024.pdf for details.

[8]The Hall and Sargent data align closely with the Federal Reserve Bank of Dallas data in the period where both series are available.

there is a break in the available data between 1838 and 1840, so we begin our series in 1841. To calculate real returns on the debt we apply the accounting identity (1) to the time series of debt, tax revenue, and spending and adjust for inflation.[9] We measure inflation using the GDP deflator from the Bureau of Economic Analysis since 1929, spliced to the historical deflator calculated by Louis Johnston and Samuel H. Williamson and available on the Measuring Worth website for earlier years.[10] We use the same two data sources to measure US GDP.

We proceed in a similar fashion to obtain historical data from the UK. To measure tax revenue and spending of the UK central government, we obtain data from the historical public finances database of the UK Office for Budget Responsibility. To measure the market value of central government debt, we splice together Bank of England data for the period 1727-2016 with BIS data from 2017-2022. We obtain data on UK nominal GDP and the GDP deflator from the Measuring Worth website.

Summary statistics of these data are provided in section IA.5.

2.2 Unit root tests and linearization parameters

In section IA.6 of the internet appendix we report unit root test statistics and sample autocorrelations for the major time series: government debt returns, the growth rates of tax revenue, spending, and output, the ratio of debt to GDP, the ratios of taxes and spending to output and debt, the primary surplus-debt ratio and the logarithmic primary surplus-debt ratio, and finally our loglinear measure of the fiscal position. We report these statistics for the full historical sample and for the postwar subsample beginning in 1947. As we have already discussed, the results indicate that returns and the growth rates of tax, spending, and GDP are all stationary; debt-GDP, tax-GDP, spending-GDP, tax-debt, and spending-debt are all nonstationary; and the primary surplus-debt ratio, the logarithmic primary surplus-debt ratio, and the fiscal position are stationary.

With these data in hand, the first task is to fix the linearization parameters ρ and β. We choose ρ by using the relationship $-\log\rho = \mathbb{E}\, r_{t+1} - \mathbb{E}\, g_{t+1}$, where

[9] In the postwar period we have confirmed the plausibility of the implied return series by regressing it on contemporaneous variables that explain the returns on short-term and long-term government debt: the short-term realized real interest rate and the change in the long-term bond yield. These regressions have high explanatory power and coefficients with the right sign and strong statistical significance. Results are reported in Section IA.3 of the internet appendix.

[10] See https://www.measuringworth.com/ for details.

r_{t+1} is the log real return on government debt and g_{t+1} is the growth rate of real GDP. In the full US sample, $\mathbb{E}\, r_{t+1} = 2.9\%$ and $\mathbb{E}\, g_{t+1} = 3.5\%$, so $\rho = 1.006$. We also estimate ρ greater than one in shorter subsamples of US data. In the full UK sample, $\mathbb{E}\, r_{t+1} = 2.1\%$ and $\mathbb{E}\, g_{t+1} = 1.8\%$, so $\rho = 0.997$. We also estimate ρ less than one in shorter subsamples of UK data.

In our baseline analysis we use these values of ρ, 1.006 for the US and 0.997 for the UK, which illustrate the two possible cases where $R < G$ and where $R > G$. For consistency with equation (25) and the surrounding discussion, we demean the growth rates of real tax revenue and spending using the theoretical restriction that their means are equal to the mean real GDP growth rate (3.5% for the US and 1.8% for the UK), and we demean the fiscal position using the theoretical restriction that its mean equals $-\log \rho$ (-0.006 for the US and 0.003 for the UK).

Finally, we choose β so that our measure of the fiscal position, fp_t, optimally approximates $\log(1 + S_t/V_t)$ in a least-squares sense. That is, β is chosen to solve the problem

$$\min_{\beta} \sum_t \left(\log\left(1 + S_t/V_t\right) - \underbrace{\left[k + \frac{1-\rho}{1-\beta}\left(\tau v_t - \beta\, x v_t\right)\right]}_{fp_t} \right)^2, \tag{34}$$

where k is given in equation (19). With $\rho = 1.006$, this procedure sets $\beta = 1.014$ in the US. With $\rho = 0.997$, $\beta = 0.975$ in the UK. As required by our theory, ρ and β are on the same side of one: both greater than one in the US, and both less than one in the UK.

2.3 Local projections

As a first way to understand the drivers of the variation in fp_t shown in Figure 3, we implement a local projections approach following Jordà (2005), Plagborg-Møller and Wolf (2021), and Li, Plagborg-Møller and Wolf (2022). This approach imposes minimal structure on the multivariate dynamic system for the fiscal position and its determinants, but it does require us to truncate the horizon T we can consider in equation (26) and rules out the use of the infinite-horizon identity (28).

We begin by regressing future returns, future fiscal adjustment, and the future fiscal position onto the current US fiscal position, for each future year from 1 to 10 years ahead, and plotting the coefficients along with two-standard-error bands in panels (a), (b), and (c) of Figure 4. Panel (a) shows that the US fiscal position has only short-term predictive power for future returns on the government debt. The one-year ahead return coefficient is positive and statistically significant at the 5% level, but subsequent estimates are all close to zero and statistically insignificant. We note also that because we estimate $\rho > 1$ in the US, the positive return coefficient in the first year has the wrong sign for returns to contribute to mean reversion in the fiscal position. Panel (b) shows that the fiscal position does predict fiscal adjustment, with positive and statistically significant coefficients up to five years ahead and a hump-shaped response that rises after the first year and then gradually declines. Panel (c) illustrates the mean reversion of the fiscal position, with coefficients declining from about 0.80 one year ahead to below 0.40 three years ahead, and becoming insignificantly different from zero thereafter. Panel (d) looks at tax and spending growth separately, showing a strong short-run response of tax growth that dies off quickly, and a hump-shaped response for spending growth that is close to zero in the first year, rises for the next three years, and dies off thereafter.

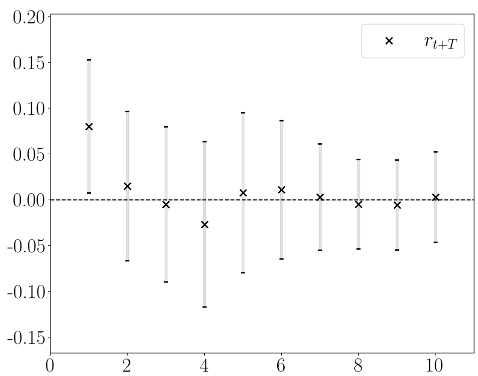
The patterns in Figure 4 are not attributable to our use of a linearization parameter ρ that exceeds one. In the internet appendix we show the same figure for a case where we set $\rho = 0.98$, and find visually indistinguishable results.

In Figure 5 we repeat this analysis for the UK and find very similar results. The UK fiscal position is a statistically significant predictor of returns up to three years ahead, with positive coefficients that contribute to mean reversion in the fiscal position given our estimate that $\rho < 1$ in the UK. The UK fiscal position predicts fiscal adjustment with a strongly hump-shaped response that is statistically significant up to eight years ahead. It predicts its own future values with coefficients that start around 0.90 one year ahead and decline to about 0.30 five years ahead, remaining statistically significant throughout this period. It predicts a weak and gradually decaying response of tax growth that is statistically significant for the first three years, and a strong hump-shaped response of spending growth that is statistically significant up to eight years ahead.

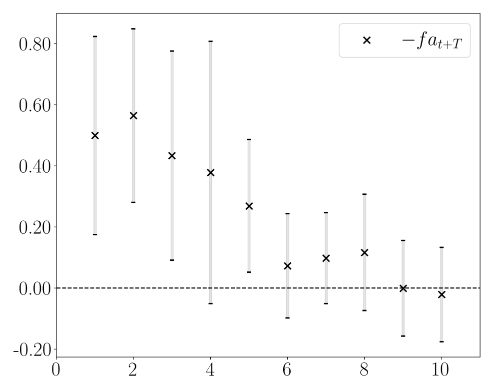
Figures 6 and 7 illustrate the predictive power of the US and UK fiscal position measures in a different way. These figures aggregate the first ten years, plotting the history of the fiscal position fp_t against subsequent realized ten-year fiscal adjustment in panel (a) and ten-year returns in panel (b). The coefficient ρ is used to discount fiscal adjustment and returns, but since ρ is very close to one this makes no visible difference to the figures. All series in Figures 6 and 7 are demeaned and

Figure 4: What does the US fiscal position predict?

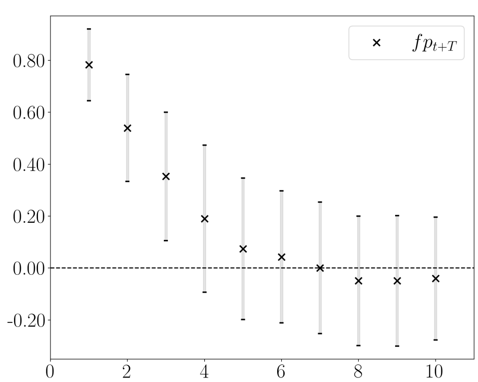
This figure plots estimated slope coefficients (with ±2 Newey–West standard error bands) from regressions $\theta_{t+T} = \alpha + \beta_{\theta,T}\, fp_t + \varepsilon_{\theta,t+T}$, for $T = 1, \ldots, 10$, where the variables θ_{t+T} are indicated in the legend of each subfigure. US data 1841-2022, $\rho = 1.006$, $\beta = 1.014$.



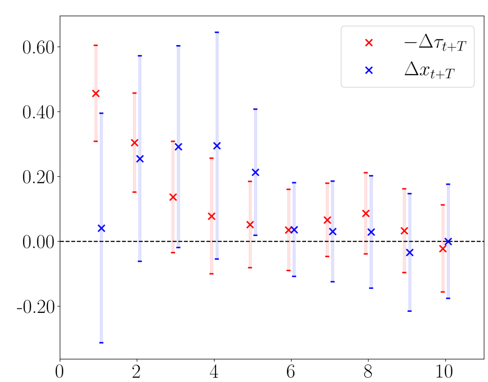
(a) Return



(b) Fiscal Adjustment



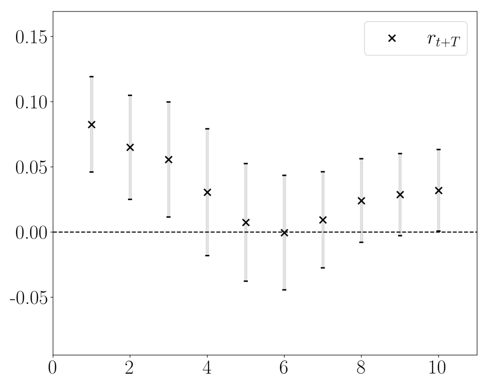
(c) Future fp



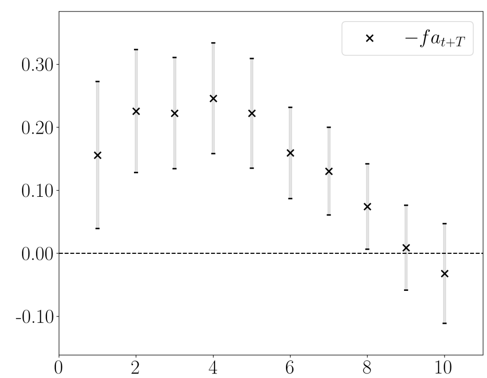
(d) Tax and spending growth

Figure 5: What does the UK fiscal position predict?

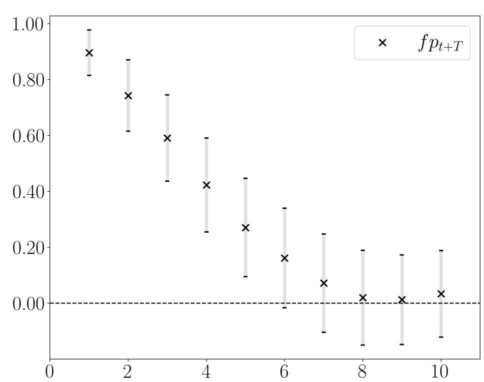
This figure plots estimated slope coefficients (with ±2 Newey–West standard error bands) from regressions $\theta_{t+T} = \alpha + \beta_{\theta,T}\, fp_t + \varepsilon_{\theta,t+T}$, for $T = 1, \ldots, 10$, where the variables θ_{t+T} are indicated in the legend of each subfigure. UK data 1727-2022, $\rho = 0.997$, $\beta = 0.975$.



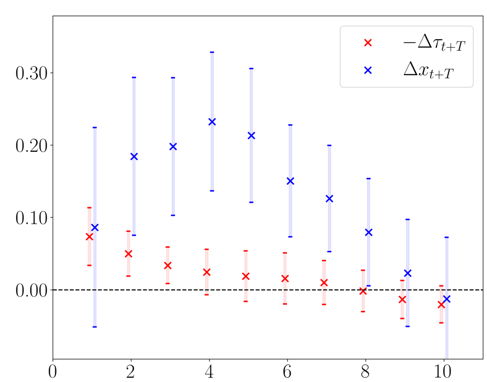
(a) Return



(b) Fiscal Adjustment

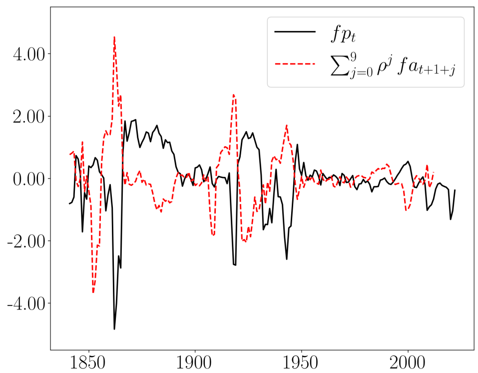


(c) Future fp

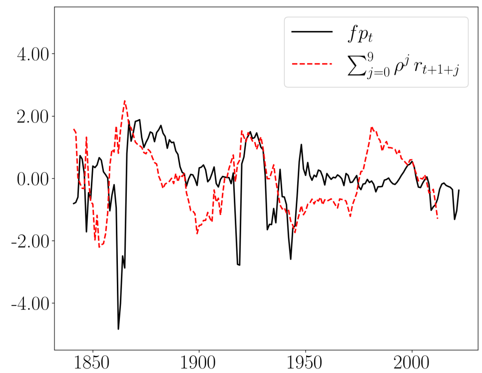


(d) Tax and spending growth

Figure 6: The fiscal position and subsequent fiscal adjustment and returns. US data 1841-2022, $\rho = 1.006$, $\beta = 1.014$.

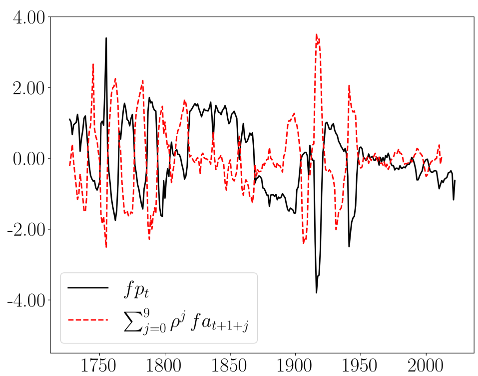


(a) Fiscal Adjustment

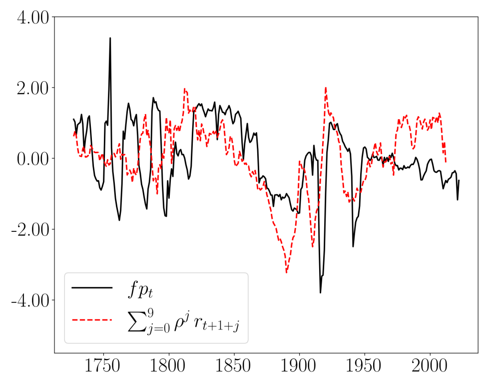


(b) Return

Figure 7: The fiscal position and subsequent fiscal adjustment and returns. UK data 1727-2022, $\rho = 0.997$, $\beta = 0.975$.



(a) Fiscal Adjustment



(b) Return

divided by their standard deviation to eliminate scale effects (including the effect of scaling returns by $(1-\rho)$ and fiscal adjustment by ρ in equation (26)).

In each figure, panel (a) shows visually how the fiscal position is associated with subsequent ten-year fiscal adjustment that brings the fiscal position back to its mean. In both the US and the UK wars (the Civil War for the US, wars with France in the 18th and early 19th Centuries for the UK, and World Wars I and II in both countries) are associated with a weak fiscal position and subsequent ten-year fiscal adjustment. Panel (b) shows a much weaker and less consistent relationship between the fiscal position and subsequent returns on the government debt.

Finally, we explore the contributions of fiscal adjustment and returns to the identity (26) by running regressions

$$(1-\rho)\sum_{j=0}^{T-1}\rho^j r_{t+1+j} = \alpha_{r,T} + \beta_{r,T} fp_t + \varepsilon_{r,t+T}, \tag{35}$$

$$-(1-\rho)\sum_{j=0}^{T-1}\rho^j \frac{1}{1-\beta} fa_{t+1+j} = \alpha_{f,T} + \beta_{fa,T} fp_t + \varepsilon_{f,t+T}, \tag{36}$$

$$\rho^T fp_{t+T} = \alpha_{fp,T} + \beta_{fp,T} fp_t + \varepsilon_{fp,t+T} \tag{37}$$

at horizons $T = 1, 3$, and 10 years. The estimated coefficients $\beta_{r,T}$, $\beta_{f,T}$ and $\beta_{fp,T}$ are shown in the second, third, and fourth columns of Table 1. The quantities reported in square brackets are Newey–West standard errors calculated using 2, 5, and 15 lags, respectively. The loglinearization (32) implies that the coefficients satisfy the restriction

$$\beta_{r,T} + \beta_{fa,T} + \beta_{fp,T} = 1 \tag{38}$$

at each horizon, up to a small approximation error.

We assess the forecasting power of the fiscal position for tax and spending separately by running the regressions

$$-(1-\rho)\sum_{j=0}^{T-1}\rho^j \frac{1}{1-\beta}\Delta\tau_{t+1+j} = \alpha_{\tau,T} + \beta_{\tau,T} fp_t + \varepsilon_{\tau,t+T},$$

$$(1-\rho)\sum_{j=0}^{T-1}\rho^j \frac{\beta}{1-\beta}\Delta x_{t+1+j} = \alpha_{x,T} + \beta_{x,T} fp_t + \varepsilon_{x,t+T}. \tag{39}$$

Table 1: Local projections, US and UK.

The table reports Newey–West standard errors with lags of 2, 5, and 15, respectively, at horizons $T = 1, 3$ and 10. The standard error for the spending ratio is computed by the delta method using the Newey–West standard errors of $\beta_{\tau,T}$ and $\beta_{x,T}$.

US: 1841 to 2022, $\rho = 1.006$, $\beta = 1.014$

horizon	return	fiscal adjustment	future fp	spending ratio
1	-0.1	21.4	78.8	9.3
	[0.0]	[10.0]	[6.6]	[30.2]
3	-0.1	64.0	36.4	40.0
	[0.0]	[20.9]	[12.6]	[17.1]
10	-0.0	104.6	-4.2	49.0
	[0.2]	[35.0]	[18.0]	[16.8]

UK: 1727 to 2022, $\rho = 0.997$, $\beta = 0.975$

horizon	return	fiscal adjustment	future fp	spending ratio
1	0.1	10.4	89.5	52.8
	[0.0]	[5.6]	[3.8]	[26.1]
3	0.3	39.9	59.7	73.9
	[0.1]	[9.8]	[7.5]	[9.2]
10	0.5	94.3	4.9	86.2
	[0.4]	[22.4]	[16.0]	[6.7]

The rightmost column of Table 1 reports the spending ratio $\beta_{x,T}/(\beta_{x,T} + \beta_{\tau,T})$, with standard errors calculated using the delta method and the Newey–West standard errors of $\beta_{\tau,T}$ and $\beta_{x,T}$. This ratio tells us what fraction of fiscal adjustment is accounted for by spending adjustments as opposed to tax adjustments. Full details are provided in Section IA.7.1 of the Appendix.

Table 1 shows several important results. First, returns contribute negligibly to the fiscal position at any horizon (and with the wrong sign in the US given our estimated $\rho > 1$). The trivial contribution of returns is due in part to the fact that ρ is close to one, the limiting case where debt can be rolled over forever, so that the level of debt does not impose a burden on the government which simply has to ensure that primary deficits are zero on average. But in addition, returns are not highly correlated with the fiscal position as we saw in Figures 6 and 7, so even values of ρ that are far from one do not deliver a substantial role for returns.[11]

Second, the contribution of fiscal adjustment rises steadily with the horizon while the contribution of the future fiscal position declines. At a one-year horizon, just over 20% of the variance of the US fiscal position and 10% of the variance of the UK fiscal position is accounted for by one-year fiscal adjustment, leaving the rest to the future fiscal position. After three years, fiscal adjustment accounts for 64% of the variance of the US fiscal position and 40% of the variance of the UK fiscal position, and after ten years it accounts for almost all the variance. This pattern corresponds to the fact we have previously noted, that the fiscal position follows a stationary process that gradually reverts to its mean.

Finally, the spending ratio increases over time, starting below 10% in the US and rising to about 50% at a ten-year horizon, and starting above 50% in the UK and rising to over 85% at a ten-year horizon. These patterns make sense given the response patterns illustrated in Figures 4 and 5.

2.4 VAR estimation

The local projections approach is appealingly direct, requiring no assumptions about the dynamics of fiscal variables, but it does not allow us to consider what happens in the long run. Recall that the approximate identity (28) relates our measure of the fiscal position, fp_t, to the expected present value of debt returns, tax growth, and spending growth appropriately cumulated over the infinite future. When the government is in a weak fiscal position (i.e., fp_t is low) we must subsequently predict some combination of low debt returns, high tax growth, and low spending growth.

To determine which of these channels is most important empirically, we now assume that fiscal variables can be described by a low-order vector autoregression (VAR) that allows us to calculate dynamics over the infinite future. We estimate a first-order VAR in the variables r_t, $\Delta\tau_t$, Δy_t, and fp_t. We include r_t, $\Delta\tau_t$, and fp_t for obvious reasons, given our interest in the identity (28). We include GDP growth, Δy_t, because of its importance for forecasting the other variables in the VAR: for example, we expect a larger economy to be able to raise a larger amount of tax revenue.

We do not include Δx_t as it is mechanically related to fp_t, fp_{t-1}, r_t and $\Delta\tau_t$ via the approximate identity (23). Indeed, we treat the identity as holding exactly, so

[11]For more detail, see Table IA.33 in the internet appendix.

that we can infer Δx_t using variables included in the VAR,

$$\frac{\beta}{1-\beta}\Delta x_t = \frac{fp_{t-1} - \rho fp_t}{1-\rho} - r_t + \frac{1}{1-\beta}\Delta\tau_t \,. \tag{40}$$

Note however that inferring Δx_t is possible only if we include an additional lag of the fiscal position, fp_{t-1} as well as fp_t, in the system. We include this additional lag so that (except for approximation error) our results are invariant to the decision to include $\Delta\tau$ in the VAR rather than Δx.[12]

The estimated VAR for US data is shown in the first four columns of Table 2. The US fiscal position fp_{t+1} is relatively predictable, with an R^2 of almost 65%, and is strongly predicted by its lag. A strong fiscal position (high fp_t) forecasts high returns for debt holders and low tax growth. US GDP growth (Δy_t) is a highly significant forecaster of tax growth with a coefficient above one, consistent with the presence of increasing marginal tax rates. The last two columns of Table 2 show imputed coefficients for spending growth and fiscal adjustment $fa_{t+1} = \Delta\tau_{t+1} - \beta\Delta x_{t+1}$. A strong fiscal position predicts high spending growth, but the effect operates with a lag. When the effects on tax and spending are combined in the fiscal adjustment measure, rapid growth (high Δy_t) and a poor fiscal position (low fp_t and fp_{t-1}) forecast large fiscal adjustment.

The estimated VAR for UK data is shown in the first four columns of Table 3. The UK fiscal position fp_{t+1} is even more highly predictable than in the US, with an R^2 above 80%, and is strongly predicted by its lag. As in the US, a strong fiscal position (high fp_t) forecasts high returns for debt holders and low tax growth. By contrast with the US, GDP growth (Δy_t) is only a weak and statistically insignificant predictor of tax revenue growth. As in the US, a strong fiscal position predicts high spending growth with a lag. When the effects on tax and spending are combined in the fiscal adjustment measure, the fiscal position predicts positive fiscal adjustment one year ahead, but negative fiscal position two years ahead. The latter effect is stronger, so fiscal adjustment generates mean reversion in the fiscal position.

[12]See Engsted, Pedersen and Tanggaard (2012) for a thoughtful discussion of this issue. Our approach can also be understood as an extension of the approach of Cochrane (2008). Working on the topic of equity market predictability, Cochrane estimates a model whose only predictor variable is a valuation ratio analogous to our fp_t. He emphasizes the linkage between predictions of returns, cash flow growth, and future valuation ratios in that model. We extend his model by adding lagged dependent variables while continuing to include one additional lag of the valuation ratio (fp_t, in our context).

Table 2: VAR coefficient estimates. US data, 1841–2022, $\rho = 1.006$, $\beta = 1.014$. OLS standard errors are reported in square brackets. The last two columns show the imputed coefficients for spending growth and $fa_{t+1} = \Delta\tau_{t+1} - \beta\Delta x_{t+1}$.

	r_{t+1}	$\Delta\tau_{t+1}$	Δy_{t+1}	fp_{t+1}	Δx_{t+1}	fa_{t+1}
r_t	0.054	0.191	0.040	0.049	0.068	0.122
	[0.075]	[0.094]	[0.033]	[0.085]	[0.215]	[0.215]
$\Delta\tau_t$	−0.047	0.143	−0.019	−0.019	0.188	−0.047
	[0.058]	[0.073]	[0.026]	[0.066]	[0.167]	[0.168]
Δy_t	0.123	1.120	0.304	0.129	0.787	0.323
	[0.176]	[0.222]	[0.078]	[0.199]	[0.507]	[0.507]
fp_t	0.219	−0.573	−0.028	0.943	−0.436	−0.131
	[0.067]	[0.084]	[0.029]	[0.075]	[0.192]	[0.192]
fp_{t-1}	−0.177	0.190	0.033	−0.209	0.706	−0.526
	[0.077]	[0.097]	[0.034]	[0.087]	[0.221]	[0.221]
R^2	6.03%	45.20%	10.55%	63.50%	9.15%	13.88%

Table 3: VAR coefficient estimates. UK data, 1727—2022, $\rho = 0.997$, $\beta = 0.975$. OLS standard errors are reported in square brackets. The last two columns show the imputed coefficients for spending growth and $fa_{t+1} = \Delta\tau_{t+1} - \beta\Delta x_{t+1}$.

	r_{t+1}	$\Delta\tau_{t+1}$	Δy_{t+1}	fp_{t+1}	Δx_{t+1}	fa_{t+1}
r_t	0.163	−0.172	0.065	0.031	−0.427	0.244
	[0.059]	[0.057]	[0.026]	[0.020]	[0.164]	[0.156]
$\Delta\tau_t$	0.096	0.118	−0.013	−0.045	0.480	−0.350
	[0.060]	[0.058]	[0.027]	[0.020]	[0.167]	[0.160]
Δy_t	−0.421	0.173	−0.058	−0.077	0.810	−0.617
	[0.13]	[0.124]	[0.058]	[0.043]	[0.359]	[0.342]
fp_t	0.504	−0.664	−0.164	1.136	−1.749	1.040
	[0.171]	[0.164]	[0.077]	[0.057]	[0.475]	[0.453]
fp_{t-1}	−0.133	0.454	0.231	−0.291	2.802	−2.278
	[0.171]	[0.164]	[0.076]	[0.057]	[0.474]	[0.452]
R^2	16.87%	15.41%	6.68%	83.30%	17.35%	16.06%

2.5 Decomposing the variance of the fiscal position

We can use the VAR to understand what fluctuations in the fiscal position, fp_t, imply about the subsequent evolution of debt returns, tax growth, and spending growth. Stacking the variables into a vector $\boldsymbol{z}_{t+1} = (r_{t+1}, \Delta\tau_{t+1}, \Delta y_{t+1}, fp_{t+1}, fp_t)'$, we can arrange the entries of Table 2 into a coefficient matrix $\boldsymbol{A}$ such that $\mathbb{E}_t\, \boldsymbol{z}_{t+j} = \boldsymbol{A}^j \boldsymbol{z}_t$. If we write $\boldsymbol{e}_n$ for a vector with one in the nth entry and zeroes elsewhere, we therefore have $\mathbb{E}_t\, r_{t+j} = \boldsymbol{e}_1' \boldsymbol{A}^j \boldsymbol{z}_t$, $\mathbb{E}_t\, \Delta\tau_{t+j} = \boldsymbol{e}_2' \boldsymbol{A}^j \boldsymbol{z}_t$, and so on.

We use the identity (32) to derive finite-horizon variance decompositions in the form

$$1 = \frac{\operatorname{cov}(fp_t, (1-\rho)\sum_{j=0}^{T-1} \rho^j\, \mathbb{E}_t\, r_{t+1+j})}{\operatorname{var}(fp_t)} + \frac{\operatorname{cov}(fp_t, -(1-\rho)\sum_{j=0}^{T-1} \rho^j\, \mathbb{E}_t\, \frac{1}{1-\beta} fa_{t+1+j})}{\operatorname{var}(fp_t)} + \\ + \frac{\operatorname{cov}(fp_t, \rho^T\, \mathbb{E}_t\, fp_{t+T})}{\operatorname{var}(fp_t)}\,. \qquad (41)$$

This decomposition can be derived by taking time-t conditional expectations of both sides of (32), computing covariances with fp_t and, finally, scaling by the variance of fp_t so that the three terms on the right-hand side of (41) add up to 100%. It allows us to formalize the statement with which we began: given that the fiscal position varies, it must, for any given horizon T, forecast some combination of future returns on the debt, future fiscal adjustment, and/or persistent variation in the future fiscal position. As we let the horizon increase, the contribution of the future fiscal position declines to zero and we are left with a two-variable infinite-horizon variance decomposition for the fiscal position.

Table 4 reports the results of this exercise over various different horizons T, for the US in panel (a) and the UK in panel (b). At each horizon, we report the three terms on the right-hand side of (41) in the columns labelled "return", "fiscal adjustment", and "future fp". These are measured in percent, and the three columns add up to approximately 100% at each horizon.[13]

Bootstrapped 95% confidence intervals for these estimates are shown in square brackets under the point estimates. Each bootstrap sample is computed by first drawing a new VAR coefficient matrix using the point estimates and the covariance matrix of the estimated coefficients. Using this VAR coefficient matrix, we generate the news series and do the variance decomposition. We repeat this procedure 2,000

[13] If the loglinear approximation were exact, the three columns would add up to exactly 100%.

Table 4: Variance decomposition for the fiscal position, fp_t, in US and UK data. All quantities are measured in percent. Bootstrapped 95% confidence intervals are reported in square brackets.

US: 1841 to 2022, $\rho = 1.006$, $\beta = 1.014$

horizon	return	fiscal adjustment	future fp	spending ratio
1	-0.1	21.1	79.5	10.2
	[-0.1, -0.0]	[10.3, 28.4]	[72.2, 90.3]	[-70.9, 36.7]
3	-0.1	63.6	37.0	39.6
	[-0.2, 0.0]	[34.3, 81.0]	[19.5, 66.4]	[-32.8, 65.7]
10	-0.1	99.1	1.6	51.0
	[-0.3, 0.1]	[76.3, 101.6]	[-1.0, 24.4]	[-18.8, 77.6]
∞	-0.1	100.6	0.0	51.4
	[-0.3, 0.1]	[100.4, 100.9]	[-0.0, 0.0]	[-15.5, 77.6]

UK: 1727 to 2022, $\rho = 0.997$, $\beta = 0.975$

horizon	return	fiscal adjustment	future fp	spending ratio
1	0.1	10.1	90.2	52.5
	[0.1, 0.2]	[4.2, 14.7]	[85.6, 96.0]	[6.2, 72.5]
3	0.3	39.6	60.5	74.9
	[0.2, 0.5]	[22.4, 52.8]	[47.2, 77.7]	[50.4, 88.3]
10	0.5	91.0	8.9	84.7
	[0.2, 1.0]	[69.6, 100.6]	[-0.7, 30.1]	[63.2, 96.1]
∞	0.6	99.8	0.0	85.6
	[0.2, 1.3]	[99.1, 100.1]	[-0.0, 0.0]	[65.1, 96.5]

times and report the 2.5% and 97.5% quantiles.

At short horizons, variation in fp_t is largely reflected in short-run future fp_t: if the fiscal position is weak this year, it probably will be next year too. But the component explained by future fp_t decays at long horizons, and reaches zero in the long run; and, at all horizons, there is essentially no relationship between the fiscal position and expected real returns. (This last fact contrasts with the evidence that dividend yields do forecast returns on the stock market.)

As a result, the fiscal position fp_t must in the long run forecast fiscal adjustment. Specifically, we find that a poor fiscal position (low fp_t) is associated with high expected tax growth and/or low expected spending growth over the medium and long run.

As fiscal adjustment can be split into the contribution of tax increases and expenditure cuts, $fa_{t+1+j} = \Delta\tau_{t+1+j} - \beta\Delta x_{t+1+j}$, the dominant second term in (41) can be decomposed further, as

$$\frac{\operatorname{cov}(fp_t, -(1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\,\frac{1}{1-\beta}fa_{t+1+j})}{\operatorname{var}(fp_t)} = \frac{\operatorname{cov}(fp_t, -(1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\,\frac{1}{1-\beta}\Delta\tau_{t+1+j})}{\operatorname{var}(fp_t)} + \\ + \frac{\operatorname{cov}(fp_t, (1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\,\frac{\beta}{1-\beta}\Delta x_{t+1+j})}{\operatorname{var}(fp_t)}. \tag{42}$$

The fourth column of Table 4, labelled "spending ratio", reports the share of the contribution of fiscal adjustment that reflects adjustments in spending rather than tax: that is, it reports the ratio of the second term on the right-hand side of (42) to the term on the left-hand side. At medium and long horizons, the point estimates are that around half of the variance accounted for by US fiscal adjustment, and at least three quarters of the variance accounted for by UK fiscal adjustment, is adjustment in spending rather than in taxes. These estimates are consistent with the local projection results reported in Table 1. The confidence intervals are fairly wide, however, particularly in the US.

2.6 The impact of the average surplus-debt ratio

When $\rho = 1$, the returns on the government debt are irrelevant for the dynamics of the fiscal position which just equals the log ratio of tax revenue to spending. Given this fact, one might be concerned that our results follow mechanically from our estimates of ρ which are close to one in both the US and the UK. However, in fact our major conclusions are not sensitive to the choice of ρ within a reasonable range. We demonstrate this in two ways. First, in appendix Section IA.7 we reproduce all our US results setting $\rho = 0.980$ to demonstrate that our US results do not depend on setting $\rho > 1$ as we do in our base case.

Second, in appendix Section IA.7.5 we reproduce the US and UK variance decompositions of Sections 2.5 and 2.7 for a range of values between $\rho = 1.01$ and $\rho = 0.96$. These different values of ρ represent different assumptions about the true unconditional population expectation of the logarithmic surplus-debt ratio, $\mathbb{E}\log(1 + S_t/V_t)$, ranging from -1.0% when $\rho = 1.01$ to 4.2% when $\rho = 0.96$. Lower values of ρ are associated with higher values of $\mathbb{E}\log(1 + S_t/V_t)$; loosely speaking, lower ρ represents higher "$r - g$", so that issuing debt is more burdensome. The unconditional mean of the log return on US government debt is 2.5% when $\rho = 1.01$ and 7.6% when $\rho = 0.96$. We do not consider the lowest values of ρ in the table to be reasonable: we include them merely to show how our results would change in a world in which r is much higher than g.

As ρ influences the choice of β in problem (34) and the linearized variable fp_t in our VAR, we recalculate β and reestimate the VAR for each value of ρ. As in our baseline VAR, we impose consistency on our model by de-meaning with theoretical means, as discussed in Section 2.2. The appendix also shows the effect of varying β away from the estimated values, while fixing ρ. We consider two boundary values of β associated with the lowest and highest values of T_t/V_t observed in the sample.

In each table in appendix Section IA.7.5 the first five columns report the various values of ρ together with the associated implied unconditional mean return on government debt, the estimated value of β, the approximation error in (34), and the maximum eigenvalue of the coefficient matrix which must be smaller than $1/\rho$ in magnitude in order that equation (27) is well-defined in the limit as $T \to \infty$.[14] The rightmost three columns report the VAR-based contributions of debt returns and fiscal adjustment to the variance of the fiscal position at an infinite horizon, and the estimated spending ratio.

In these tables the contribution of returns remains trivially small for almost all parameter configurations. It increases above 10% only when both ρ and β are set to

[14]As the theoretically imposed means move further from the observed sample means, the variables appear increasingly persistent. As a result the maximal eigenvalues increase, but they always remain below $1/\rho$.

implausibly low values. Accordingly the contribution of fiscal adjustment remains close to 100% in almost all cases. The estimates of the spending ratio are also quite stable, typically close to 50% for the US and 85% for the UK.

2.7 Tax shocks vs. spending shocks

As our framework allows us to analyze the behavior of tax and spending separately, we can also ask whether deficits driven by shocks to taxes look different from deficits driven by shocks to spending.

We address this question by using the identity (26) to explore the implications of unexpected shocks to taxes or spending. Applying the "news operator", $\Delta\,\mathbb{E}_{t+1} = \mathbb{E}_{t+1} - \mathbb{E}_t$, to both sides of (26) and rearranging, we have

$$\underbrace{\Delta\,\mathbb{E}_{t+1}\,\tau_{t+1}}_{\substack{\text{short-run tax news:}\\ N_{\text{SR tax},t+1}}} = (1-\beta)\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j r_{t+1+j}}_{\text{return news: } N_{\text{return},t+1}} - \underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=1}^{T-1}\rho^j \Delta\tau_{t+1+j}}_{\substack{\text{long-run tax news:}\\ N_{\text{LR tax},t+1}}} + \beta\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j \Delta x_{t+1+j}}_{\text{spending news: } N_{\text{spending},t+1}} + \frac{1-\beta}{1-\rho}\underbrace{\Delta\,\mathbb{E}_{t+1}\,\rho^T fp_{t+T}}_{\substack{\text{future fiscal position news:}\\ N_{\text{future fp},t+1}}}. \quad (43)$$

This identity allows us to trace out the consequences of an unexpected shock to taxes. We refer to such a shock as short-run tax news, $N_{\text{SR tax},t+1} = \Delta\,\mathbb{E}_{t+1}\,\tau_{t+1}$. A positive short-run tax shock must be reflected in some combination of (i) news about returns, $N_{\text{return},t+1} = \Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j r_{t+1+j}$; (ii) news about declines in long-run tax growth, $N_{\text{LR tax},t+1} = \Delta\,\mathbb{E}_{t+1}\sum_{j=1}^{T-1}\rho^j\Delta\tau_{t+1+j}$; (iii) news about spending growth, $N_{\text{spending},t+1} = \Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j\Delta x_{t+1+j}$; and (iv) news about the future fiscal position, $N_{\text{future fp},t+1} = \Delta\,\mathbb{E}_{t+1}\,\rho^T fp_{t+T}$. This last term becomes negligible once the horizon, T, is sufficiently long.

Taking covariances of both sides of (43) with short-run tax news, $N_{\text{SR tax},t+1} =$

$\Delta\,\mathbb{E}_{t+1}\,\tau_{t+1}$, and rearranging, we have

$$1 = \frac{\operatorname{cov}\left((1-\beta)N_{\text{return},t+1}, N_{\text{SR tax},t+1}\right)}{\operatorname{var}(N_{\text{SR tax},t+1})} + \frac{\operatorname{cov}\left(-N_{\text{LR tax},t+1}, N_{\text{SR tax},t+1}\right)}{\operatorname{var}(N_{\text{SR tax},t+1})} + \frac{\operatorname{cov}\left(\beta N_{\text{spending},t+1}, N_{\text{SR tax},t+1}\right)}{\operatorname{var}(N_{\text{SR tax},t+1})} + \frac{\operatorname{cov}\left(\frac{1-\beta}{1-\rho}N_{\text{future fp},t+1}, N_{\text{ SR tax},t+1}\right)}{\operatorname{var}(N_{\text{SR tax},t+1})}. \quad (44)$$

When there is an unanticipated tax cut, either bond holders must feel the impact (experiencing returns over the long run that are worse than expected prior to the tax cut if $\beta < 1$, or better than expected if $\beta > 1$), or future taxes must increase, or future spending must decrease. Which is it?

Table 5 reports US and UK results for a range of horizons, T, using the VAR systems estimated in Tables 2 and 3. For comparability with previous tables, we collect the second and third terms on the right-hand side of (44), which capture adjustments to taxes and to spending, into a single column labelled "fiscal adjustment", and report the share of fiscal adjustment accounted for by the spending component in the column labelled "spending ratio." Again, the first three terms in each row would add up to precisely 100% if our loglinear approximation were exact. Bootstrapped 95% confidence intervals, calculated in the same way as in Table 4, are shown in square brackets.

In both the US and the UK, returns play almost no role in the response to tax shocks. At the shortest horizon—that is, in the year the shock occurs—fiscal adjustment is by definition entirely a contemporaneous adjustment in spending: it accounts for 59% of the response in the US and 67% in the UK, with the one-year-ahead fiscal position accounting for the remainder of the response. Over subsequent years, the stationarity of the fiscal position implies that its contribution declines (reaching zero at an infinite horizon) and the contribution of fiscal adjustment therefore increases. But at all horizons, the point estimate of the spending ratio remains close to 100% in both countries, implying that there is very little mean reversion in tax revenue: shocks to revenue are almost entirely permanent. The lower bound of the spending ratio confidence interval at an infinite horizon is close to 70% in the US and close to 90% in the UK, so the data rule out anything more than modest mean reversion in tax revenue.

We can carry out a similar exercise for spending rather than taxes, rewriting the identity (43) as

$$\underbrace{\Delta\,\mathbb{E}_{t+1}\,x_{t+1}}_{\substack{\text{short-run spending news:}\\ N_{\text{SR spending},t+1}}} = -\frac{1-\beta}{\beta}\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j r_{t+1+j}}_{\text{return news: } N_{\text{return},t+1}} + \frac{1}{\beta}\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j \Delta\tau_{t+1+j}}_{\text{tax news: } N_{\text{tax},t+1}} + \\ -\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=1}^{T-1}\rho^j \Delta x_{t+1+j}}_{\substack{\text{long-run spending news:}\\ N_{\text{LR spending},t+1}}} - \frac{1-\beta}{\beta(1-\rho)}\underbrace{\Delta\,\mathbb{E}_{t+1}\,\rho^T fp_{t+T}}_{\substack{\text{future fiscal position news:}\\ N_{\text{future fp},t+1}}}. \tag{45}$$

We write $N_{\text{tax},t+1}$ for the tax news term that appears on the right-hand side of identity (45). This is the sum of short-run tax news and long-run tax news, as defined in (43): $N_{\text{tax},t+1} = N_{\text{SR tax},t+1} + N_{\text{LR tax},t+1}$. Similarly, we write $N_{\text{SR spending},t+1}$ for short-run spending news and $N_{\text{LR spending},t+1}$ for long-run spending news, so that $N_{\text{spending},t+1}$ as defined after identity (43) is equal to the sum $N_{\text{SR spending},t+1} + N_{\text{LR spending},t+1}$.

We can now decompose the variance of short-run spending news as the sum of its covariances with news about returns, about tax growth, about long-run spending growth, and about the long-run fiscal position:

$$1 = \frac{\operatorname{cov}\left(-\frac{1-\beta}{\beta}N_{\text{return},t+1}, N_{\text{SR spending},t+1}\right)}{\operatorname{var}(N_{\text{SR spending},t+1})} + \frac{\operatorname{cov}\left(\frac{1}{\beta}N_{\text{tax},t+1}, N_{\text{SR spending},t+1}\right)}{\operatorname{var}(N_{\text{SR spending},t+1})} \\ + \frac{\operatorname{cov}\left(-N_{\text{LR spending},t+1}, N_{\text{SR spending},t+1}\right)}{\operatorname{var}(N_{\text{SR spending},t+1})} + \frac{\operatorname{cov}\left(-\frac{1-\beta}{\beta(1-\rho)}N_{\text{future fp},t+1}, N_{\text{SR spending},t+1}\right)}{\operatorname{var}(N_{\text{SR spending},t+1})}. \tag{46}$$

Table 6 reports results. We use the same format as before, collecting the second and third terms on the right-hand side of (46) into the single column labelled "fiscal adjustment" and reporting the share of spending in fiscal adjustment in the column labelled "spending ratio".

In both the US and the UK, returns play almost no role in the response to spending shocks. This finding is similar to the pattern reported for tax shocks in Table 5. Other results, however, are different. In the year a spending shock occurs, fiscal adjustment (which at this horizon is, by definition, entirely a contemporaneous

Table 5: Variance decomposition for short-run tax news, US and UK data.

US: 1841 to 2022, $\rho = 1.006$, $\beta = 1.014$

horizon	return	fiscal adjustment	future fp	spending ratio
1	-0.2	58.6	43.2	100.0
	[-0.2, -0.1]	[52.9, 64.4]	[37.5, 48.7]	[100.0, 100.0]
3	-0.2	72.3	29.5	122.6
	[-0.4, 0.0]	[31.8, 110.7]	[-8.9, 69.9]	[97.0, 163.9]
10	-0.2	100.5	1.3	107.0
	[-0.5, 0.1]	[89.7, 103.3]	[-1.4, 11.8]	[72.4, 145.8]
∞	-0.2	101.8	0.0	106.6
	[-0.5, 0.1]	[101.5, 102.1]	[0.0, 0.0]	[69.4, 145.9]

UK: 1727 to 2022, $\rho = 0.997$, $\beta = 0.975$

horizon	return	fiscal adjustment	future fp	spending ratio
1	0.7	66.6	30.9	100.0
	[0.7, 0.8]	[62.6, 71.0]	[26.5, 35.0]	[100.0, 100.0]
3	1.2	97.7	-0.6	103.7
	[0.8, 1.7]	[58.8, 138.6]	[-41.5, 38.1]	[90.6, 117.4]
10	1.2	98.2	-1.1	104.4
	[0.6, 1.8]	[85.7, 109.9]	[-12.3, 11.2]	[89.0, 121.5]
∞	1.2	97.1	0.0	104.5
	[0.6, 1.8]	[96.5, 97.7]	[0.0, 0.0]	[89.0, 124.4]

Table 6: Variance decomposition for short-run spending news, US and UK data.

US: 1841 to 2022, $\rho = 1.006$, $\beta = 1.014$

horizon	return	fiscal adjustment	future fp	spending ratio
1	0.1	10.8	89.5	0.0
	[0.0, 0.1]	[9.6, 11.8]	[88.5, 90.7]	[0.0, 0.0]
3	-0.1	38.2	62.2	-42.4
	[-0.2, 0.0]	[16.1, 53.8]	[46.6, 84.3]	[-258.0, 5.5]
10	-0.1	97.5	2.9	22.3
	[-0.3, 0.1]	[83.0, 102.4]	[-2.0, 17.5]	[-27.1, 45.8]
∞	-0.1	100.4	0.0	23.8
	[-0.3, 0.1]	[100.2, 100.7]	[0.0, 0.0]	[-21.2, 47.1]

UK: 1727 to 2022, $\rho = 0.997$, $\beta = 0.975$

horizon	return	fiscal adjustment	future fp	spending ratio
1.0	0.1	8.6	91.7	0.0
	[0.0, 0.1]	[8.1, 9.2]	[91.2, 92.3]	[0.0, 0.0]
3.0	0.4	-2.7	102.8	1012.8
	[0.2, 0.6]	[-20.1, 11.2]	[88.9, 120.0]	[-4206.9, 4235.4]
10.0	0.8	80.6	19.0	55.7
	[0.4, 1.3]	[55.8, 98.0]	[1.7, 43.8]	[15.7, 74.3]
∞	0.9	99.6	0.0	63.1
	[0.4, 1.7]	[98.7, 100.0]	[0.0, 0.0]	[41.1, 76.8]

adjustment in tax revenue) accounts for only 11% of the response in the US and 9% in the UK, with the one-year-ahead fiscal position accounting for the remainder of the response. Over subsequent years, stationarity implies that the contribution of the fiscal position declines towards zero, and the contribution of fiscal adjustment increases. By contrast with the results for tax shocks, the point estimate of the spending ratio remains low in the US, reaching only 24% at an infinite horizon (with an upper bound for the confidence interval at 47%). It is higher in the UK, reaching 63% at an infinite horizon (with a confidence interval from 41% to 77%).[15]

We can also use our VAR system to decompose the variance of return shocks into components attributable to news about future returns and to news about future fiscal adjustment. This is analogous to the Campbell (1991) decomposition of the variance of aggregate US stock returns, although returns on the government debt are considerably less volatile, with an annualized standard deviation around 10% in the US and 7% in the UK, than returns on the aggregate stock market. In section IA.7.6 of the internet appendix we find that in both the US and the UK, the variance decomposition for government debt returns has a large positive contribution from future fiscal adjustment and a smaller negative contribution from future returns. The latter component reflects positive autocorrelation of government debt returns: since an upward revision in expected future returns is associated with a downward shock in current returns, positive autocorrelation dampens the variance of current returns so that more than 100% of this variance is attributed to news about future fiscal adjustment. These findings contrast with those reported by Campbell (1991) and others for the aggregate US stock market, where long-run mean reversion in returns contributes positively to an explanation of return variance.

3 The postwar experience

We now focus on the decades since World War II. Over this period, data becomes available for a wider range of countries; and a richer set of potentially informative state variables is available for the US.

[15] There is one very large and imprecise entry for the spending ratio at the three-year horizon in the UK. This reflects the fact that at this horizon the contribution of the UK fiscal position—the denominator of the spending ratio—is very close to zero.

Table 7: Variance decomposition of the fiscal position fp_t for 16 countries at horizon $T = 10$, based on the VAR system $(r_t, \Delta\tau_t, \Delta y_t, fp_t, fp_{t-1})$

country	return	fiscal adjustment	future fp	spending ratio
US	-0.7	99.7	2.3	160.1
	$[-2.9, 1.5]$	$[86.5, 102.8]$	$[0.0, 14.5]$	$[106.8, 285.2]$
UK	-1.8	91.6	11.5	63.2
	$[-3.2, -0.5]$	$[59.0, 105.7]$	$[-2.5, 44.9]$	$[31.7, 110.4]$
Canada	3.7	97.8	1.6	78.1
	$[0.2, 8.4]$	$[50.9, 104.0]$	$[-4.3, 47.0]$	$[51.1, 163.8]$
Japan	8.3	33.5	60.4	92.2
	$[-0.5, 18.5]$	$[-29.0, 68.3]$	$[27.8, 118.1]$	$[-456.3, 659.1]$
Switzerland	2.6	104.6	-2.7	78.5
	$[-0.1, 6.4]$	$[84.9, 131.5]$	$[-29.7, 16.5]$	$[34.4, 140.1]$
Austria	1.8	105.9	-3.2	25.9
	$[0.6, 8.6]$	$[60.6, 157.7]$	$[-54.9, 40.2]$	$[-165.6, 56.5]$
Belgium	-0.3	101.1	3.8	68.5
	$[-0.7, -0.0]$	$[45.0, 116.9]$	$[-11.9, 59.8]$	$[14.5, 190.0]$
Germany	0.8	109.5	-5.7	75.8
	$[-1.4, 4.5]$	$[68.0, 152.5]$	$[-49.1, 34.7]$	$[52.9, 172.0]$
Spain	-0.1	105.8	-1.2	81.6
	$[-1.7, 1.3]$	$[42.0, 160.5]$	$[-55.0, 62.3]$	$[17.6, 294.1]$
Finland	15.3	86.1	3.2	98.0
	$[6.5, 29.8]$	$[9.0, 106.2]$	$[-13.4, 66.6]$	$[44.1, 563.3]$
France	1.2	103.2	0.2	26.4
	$[-0.1, 3.6]$	$[31.4, 132.7]$	$[-30.1, 71.7]$	$[-30.6, 94.8]$
Greece	2.8	90.9	10.9	108.7
	$[-2.7, 9.3]$	$[7.7, 154.1]$	$[-51.4, 92.1]$	$[-101.6, 393.3]$
Ireland	-0.3	104.8	0.1	106.8
	$[-3.8, 4.5]$	$[55.0, 124.3]$	$[-16.9, 45.6]$	$[59.2, 302.7]$
Italy	1.6	102.1	0.9	42.4
	$[-1.7, 6.8]$	$[40.5, 136.3]$	$[-32.8, 63.1]$	$[-92.4, 109.5]$
Netherlands	-0.3	104.2	0.6	80.0
	$[-0.8, 0.6]$	$[79.4, 115.4]$	$[-10.2, 24.7]$	$[44.9, 172.6]$
Portugal	-0.6	86.9	18.3	70.3
	$[-4.8, 2.7]$	$[22.0, 110.4]$	$[-5.4, 84.1]$	$[-37.8, 179.3]$

3.1 International debt and deficits

We first repeat the analysis using a larger cross-section of countries for which we have been able to obtain appropriate data in the decades since World War II. For comparison, we include our previous data from the US and the UK, using the post-war sample period starting in 1947. We add Canada, Switzerland, Japan, and a collection of European countries that share the euro as currency.

The key challenge in international data is obtaining a time series for the market value of the government debt. Standard sources often report the face value of the debt instead. We have market value data from 1989 in Canada, 1980 in Japan, and 1999 in Switzerland, as well as data for 11 countries in the eurozone (Austria, Belgium, Germany, Spain, Finland, France, Greece, Ireland, Italy, the Netherlands, and Portugal) since the creation of the euro in 1999. See appendix IA.1 for details. A secondary challenge is confirming the plausibility of imputed debt returns; we conduct this exercise in the appendix IA.3.

We follow the same procedure as before to estimate the parameters ρ and β. We estimate $\rho = 1.010$ for the US, 0.991 for the UK, 0.955 for Canada, 0.912 for Japan, and 0.994 for Switzerland; ρ estimates for the eurozone countries are reported in appendix section IA.4. We choose β, conditional on ρ, to achieve the best least-squares fit of our fiscal position measure fp_t to $\log(1+S_t/V_t)$ which it approximates. As before, and as required by our theory, ρ and β always lie on the same side of one.

Section IA.2 of the internet appendix plots the data for all these countries. The top left panels of each figure show the history of debt-GDP ratios. The nonstationarity of these ratios is visually apparent, and confirmed by unit root tests in internet appendix section IA.6. The top right panels of each figure show the surplus-debt ratio S_t/V_t, the bottom left panels show the tax-output and spending-outpout ratios T_t/Y_t and X_t/Y_t, and the bottom right panels show the tax-debt and spending-debt ratios T_t/V_t and X_t/V_t. Section IA.7.2 reports VAR estimates for the US and the UK in the shorter post-World War II period, and section IA.8.1 reports VAR estimates for the other countries.

Table 7 reports the implications of these estimates for the variance decomposition of the fiscal position at a 10-year horizon for each country. Most of the patterns we saw in the long-sample US and UK data appear again in this table. In all countries except Japan and Finland the returns on government debt have a minimal influence on the dynamics of the fiscal position, and in all countries except Japan the fiscal position mean-reverts quickly enough that almost all its variability is accounted for by ten-year fiscal adjustment. The point estimates of the spending ratio exceed 50% in all countries except Austria, France, and Italy, although confidence intervals are wide given the relatively short sample periods in this table.

Section IA.7.1 in the Appendix reports post-World War II results for the US and the UK based on local projections at 1-, 3-, 10-year horizons. (Given the 10-year horizon, we require a long sample period for the local projections approach to be feasible. The US and the UK are the two countries for which we observe post-World War II data over a sufficiently long period.) In the US, the fiscal position mean-reverts substantially over 10 years, so that the future fiscal position contributes 21% of the variance of the fiscal position at this horizon. Mean-reversion is slower in the UK, where the future fiscal position contributes 49% of the variance at a 10-year horizon. Consistent with our other results, returns contribute very little to the variance of the fiscal position, and fiscal adjustment is dominated by spending (although the standard error for the spending ratio is large given the short sample period).

3.2 Other state variables

In principle, the VAR analysis can be augmented by including any state variables relevant for forecasting fiscal outcomes or debt returns.

3.2.1 Implications of a stationary US tax-GDP ratio

In the postwar US, for example, the log tax-GDP ratio appears to be stationary, as is visually apparent in Figure IA.2. Table IA.13 shows that we can reject a unit root in postwar data, with a p-value of 0.000 (though not in the longer sample, and not for the log spending-GDP ratio over either sample period). What conclusions would be drawn by an economist prepared to accept that we are in a new, stable, postwar political equilibrium with stationary log tax-GDP ratio?

Table IA.95 in Internet Appendix IA.9 reports results for a VAR, estimated over the 1947–2022 sample period, that includes τy_t, and so takes into account the stationary postwar relationship between tax and output. The tax-GDP ratio is quite predictable, notably by its own lag and by tax growth, and in turn it predicts high returns on debt and low future tax growth.

Table 8: Variance decomposition of fiscal position sv_t, based on a VAR that includes the tax-GDP ratio, τy_t, in postwar US data.
All quantities are measured in percent. Bootstrapped 95% confidence intervals are reported in square brackets.

horizon	return	fiscal adjustment	future sv	spending ratio
1	0.0	24.5	76.8	66.8
	$[0.0, 0.0]$	$[11.7, 36.8]$	$[64.5, 89.6]$	$[42.7, 86.5]$
3	0.1	73.4	27.9	74.7
	$[0.0, 0.1]$	$[38.5, 102.7]$	$[-1.4, 62.8]$	$[55.4, 97.6]$
10	0.0	100.6	0.7	102.1
	$[-0.1, 0.1]$	$[77.4, 107.5]$	$[-6.2, 24.0]$	$[90.5, 131.9]$
∞	0.0	101.3	0.0	101.1
	$[-0.1, 0.2]$	$[101.2, 101.4]$	$[0.0, 0.0]$	$[89.4, 137.7]$

We can use this VAR to conduct a variance decomposition analogous to the one reported in Table 4. The results are shown in Table 8. As before, we find that variation in the government's fiscal position reflects expected future fiscal adjustment rather than expected future bond returns. What is new, relative to the earlier results, is that in this system fiscal adjustment takes place almost entirely through changes in expected spending growth as opposed to expected tax growth; and the confidence intervals for the contribution of spending are much smaller. This is the case because the fiscal position has little ability to forecast GDP growth (as shown in Table IA.95). It must therefore also have little ability to forecast tax growth, given the stationarity of the tax-GDP ratio.

3.2.2 Yield curve forecasting variables

In the postwar period, yield curve data are available that might potentially help to forecast returns on the debt. We have therefore also considered VAR systems that include bond return forecasting variables based on the yield curve. Unlike the tax-GDP ratio, however, these variables have very little effect on our conclusions. In the interest of space, we only report results for the US.

Table IA.96, in the Internet Appendix IA.9, adds the real short rate (calculated as the nominal 1 yr yield minus lagged inflation), $yr_{1,t+1}$, and 1-10 year yield spread, $spr_{1\to 10,t+1}$, to the variables considered in our baseline VAR. These variables help to forecast returns, but the variance decomposition using this expanded system (shown in Table IA.98) is almost identical to the baseline variance decomposition reported in Table 4. This is consistent with the visual impression given by Figure 6: as realized returns exhibit little comovement with the fiscal position, an improved ability to forecast returns has very little effect on our conclusions.

Table IA.97 reports results when both the tax-GDP ratio, τy_t, and the yield curve variables $yr_{1,t+1}$ and $spr_{1\to10,t+1}$ are included in the VAR. The resulting variance decomposition (Table IA.99) is very similar to that shown in Table 8, so once again our conclusions are unaltered by inclusion of the bond return forecasting variables.

4 Conclusion

Conventional tests do not reject the presence of a unit root in the debt-GDP ratio in long-run data for the US and UK, nor (albeit in shorter time series) for 14 other countries. We have presented a framework for fiscal analysis that takes this uncomfortable fact into account by making the surplus-debt ratio—which does appear to be stationary—the central object of interest.

Our framework considers not only what one might call the burden of the debt—that is, the size of the surplus that is required to service the debt—but also the size of the government relative to the debt. Both tax revenue and government spending are typically very large relative to the primary surplus, which is the difference between these two numbers. Thus, say, a 1% change in the level of spending can have a very large proportional impact on the primary surplus. This has important implications for fiscal adjustment.

We analyze the contributions of taxes and spending to surplus separately, and so, for example, we can distinguish between deficits associated with declines in tax revenue and those associated with increases in government expenditure. There are good economic reasons to analyze these two variables separately: in a recession, tax revenue declines at a faster rate than GDP in the presence of increasing marginal tax rates, whereas spending increases, but there is no particular reason to expect tax and spending to adjust symmetrically.

We organize our empirical work by deriving a loglinear approximation to the surplus-debt ratio that summarizes the fiscal position of the government. Our key identity relates the fiscal position to future returns on government debt and to future tax and spending growth rates, just as the identities derived by Campbell and Shiller (1988*a*) relate the dividend yield on a security to that security's future returns and dividend growth rates. A weak fiscal position must be followed by some combination of low long-run returns on government debt, high long-run tax growth, and low long-run spending growth.

We use this identity to interpret variation in the fiscal position over time in long-run data from the US and the UK and from these and 14 other developed countries in the post-World War II period. In all these countries the fiscal position has limited forecasting power for future debt returns over the long run; instead, it forecasts long-run future fiscal adjustment, i.e., changes in the growth rates of tax revenue and government spending. In the US fiscal adjustment occurs roughly equally in taxes and in spending; in the UK and in many other countries we study, it occurs more through spending than through taxes.

These findings contrast with the results of papers that study the ratio of debt to GDP, a nonstationary ratio that has little ability to predict fiscal adjustment and mostly predicts its own future value (Jiang et al. (2021*b*)). They also contrast with the findings of Campbell and Shiller (1988*a*) and Campbell and Shiller (1988*b*) that the dividend yield and cyclically adjusted price-earnings (CAPE) ratio on the aggregate US stock market primarily predict future stock returns rather than future growth rates of dividends or earnings. Other studies have found a greater role for predictable cash flow growth when noisier measures of corporate cash flows, such as aggregate net payouts, are used to scale the market value of corporate assets (Larrain and Yogo (2008)). One way to understand our results is that primary surpluses, like net corporate payouts but unlike aggregate dividends or smoothed earnings, have important transitory variation which our framework relates to changing growth rates of tax revenue and government spending.[16]

We also use our identity to analyze long-run responses to tax and spending shocks. Again we find that debt returns, both unexpected returns at the time the shocks occur and subsequent predictable returns, play almost no role in these responses. Instead, mean-reverting tax and spending growth follow stochastic processes that allow debt value to remain stable. While our framework does not allow us to say which variables are exogenous and which are endogenous, this pattern does tell us that governments in the countries we study have chosen fiscal policies that

[16] Returns on government debt are also far more stable than returns on aggregate stock indexes. Thus, the dividend-price ratio can be thought of as a way to remove a persistent component from stock prices, while the primary surplus-debt ratio and our loglinear approximation to it can be thought of as a way to remove a persistent component from primary surpluses. We are grateful to John Cochrane for suggesting this interpretation of our findings.

avoid large predictable or unpredictable returns to debtholders.

One reason for these policy choices could be that large swings in the value of the debt are politically risky for incumbent policymakers. James Carville, a political adviser to US President Bill Clinton, is reported to have said, "I used to think that if there was reincarnation, I wanted to come back as the president or the pope or as a .400 baseball hitter. But now I would like to come back as the bond market. You can intimidate everybody." An illustration of this principle was recently provided by the market reaction to unexpectedly large tax cuts in the September 2022 "mini-budget" in the United Kingdom, which led to the rapid departure of both Chancellor of the Exchequer Kwasi Karteng and Prime Minister Liz Truss.

It is possible, perhaps even probable, that our framework would attribute a more significant role to debt returns in countries that have experienced turbulent macroeconomic crises. A priority for future research should be to apply our analysis to other countries, including emerging markets, where data are available on the market value (as opposed to the face value) of the public debt.

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