Debt and Deficits: Fiscal Analysis with Stationary Ratios

John Y. Campbell   Can Gao   Ian W. R. Martin

March, 2023

Introduction

- What happens when a government's fiscal position deteriorates?
  - Poor returns for bond holders? Rises in tax revenue? Cuts in spending?
- What is the fiscal position, anyway?
  - Some seemingly natural definitions are problematic. We suggest an alternative
- We derive an identity that relates the fiscal position to tax and spending growth and debt returns, and use it to do variance decompositions for the US and UK
  - A deterioration in the fiscal position forecasts a decline in spending over the long run
  - It does not forecast increases in tax revenue
  - Nor does it forecast low returns for bond holders

Health warning

- This project develops a loglinear intertemporal accounting system to understand the historical dynamics of government debt and deficits
- There is no attempt to identify structural shocks
- There are no causal statements
- There are no counterfactuals
- Any impression I may give to the contrary is unintentional and misleading!

How should we measure a government's fiscal position?

- The debt is serviced by the primary surplus: tax revenue minus expenditure
  - If surplus is positive (negative), debt grows more slowly (faster) than the return on debt
  - If the return on debt, R, is greater than its growth rate, G, then the value of the debt is the expected discounted value of primary surpluses
- But how should we scale the surplus?
  - Conventional approach (Cochrane 2001, 2022, 2023; Jiang, Lustig, van Nieuwerburgh and Xiaolan 2021): scale by GDP and work with surplus-GDP and debt-GDP ratios
  - Problem: these ratios appear nonstationary in post-WW2 US and UK data
  - Even if they are stationary in the very long run, their persistence makes it inadvisable to model them using standard stationary time-series analysis (Campbell and Perron, 1991)

The debt-GDP ratio appears to be nonstationary

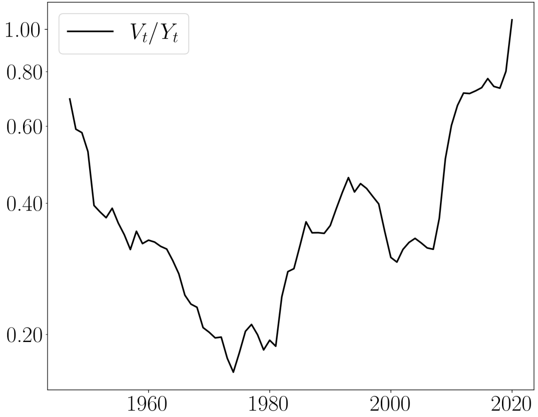

Figure: Debt at market value from Dallas Fed, GDP from NIPA via FRED. Log scale

The debt-GDP ratio appears to be nonstationary

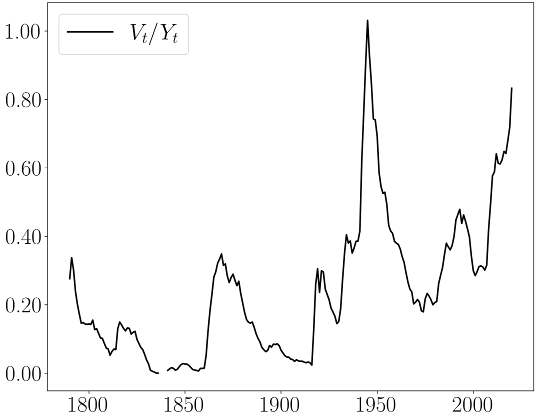

Figure: Data from Hall and Sargent (2021) and Johnston and Williamson (2023). Linear scale

But the surplus-debt ratio appears to be stationary

- While the government can expand or shrink relative to the size of the economy, it must pay off its debt
- Just as a corporation pays dividends to owners of its stock, so the government pays the primary surplus to owners of its debt
- This suggests an analogy in which the surplus-debt ratio plays the role of the dividend-price ratio
- Good news: In postwar US and UK data, standard unit root tests reject nonstationarity for the surplus-debt ratio (though not for the surplus-GDP ratio)

But the surplus-debt ratio appears to be stationary

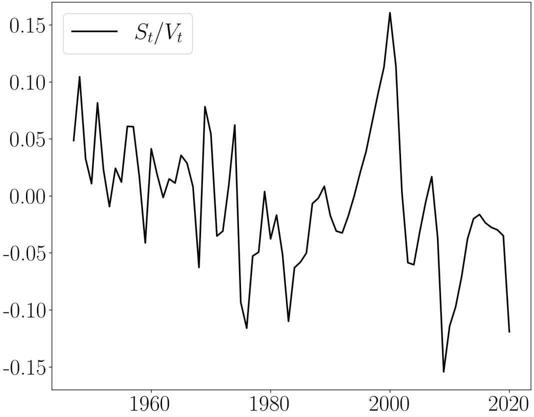

Figure: The surplus-to-debt ratio is stationary in postwar data. NIPA data, 1947–2020. Linear scale

And yet . . . the surplus-debt ratio is also a flawed measure

- The surplus-debt ratio has two problems as a measure of the fiscal position
- Both are related to the fact that the surplus can be either positive or negative

1. An exogenous increase in debt, with unchanged surplus, should worsen the fiscal position. But it *increases* the surplus-debt ratio if surplus is negative
2. The analogy with the dividend-price ratio suggests a Campbell–Shiller-like approximation relating the log surplus-debt ratio to expected future debt returns and surplus growth rates. But the analogy fails: log surplus cannot be defined, as surplus can go negative

A way forward

- Instead of surplus growth rates, we work with tax and spending growth rates, and log tax-debt and log spending-debt ratios
- Giannitsarou, Scott and Leeper (2006) and Berndt, Lustig and Yeltekin (2012) use this approach
- They assume that log tax-debt and log spending-debt ratios are stationary, then do a loglinear approximation around their means
  - Empirical problem: neither of these ratios appears to be stationary in the data
  - Conceptual problem: there is no reason to expect either to be stationary. A government's activities can be large or small relative to its debt
  - Good news: If surplus-debt is stationary, then tax-debt and spending-debt are cointegrated in levels and approximately cointegrated in logs

The tax-debt and spending-debt ratios appear to be nonstationary

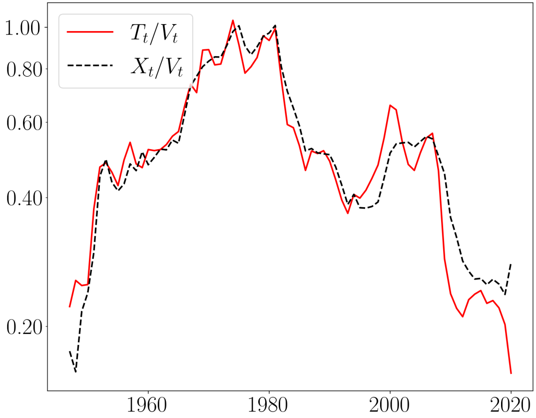

Figure: Tax-debt and spending-debt ratios appear to be nonstationary in postwar data. Log scale

One more stationary ratio

- We look for other cointegrating relationships, and find one
- The tax-GDP ratio is stationary
  - This may reflect political economy considerations that limit the extent to which tax revenue can vary as a fraction of GDP
  - Jiang, Sargent, Wang and Yang (2022) cite Keynes (1923) arguing that tax-GDP has an upper bound that is politically supportable
- No other fiscal variables are so closely related to GDP: spending-GDP, surplus-GDP, and debt-GDP ratios are all nonstationary
- Cochrane (2001, 2022, 2023) argues for scaling by consumption rather than GDP
  - We have tried this but it does not alter our conclusions

The tax-GDP ratio appears to be stationary

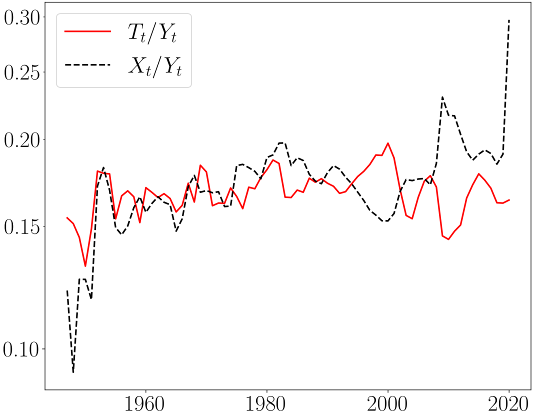

Figure: Spending-to-GDP is also nonstationary, but tax-to-GDP is stationary. Log scale

A framework for fiscal analysis

The Gordon growth model benchmark

- The gross return on government debt is

$$R_{t+1} = \frac{V_{t+1} + T_{t+1} - X_{t+1}}{V_t}$$

  - V_t the value of debt; T_{t+1} tax income; X_{t+1} expenditure; surplus is $S_t = T_t - X_t$
- If taxes, spending, and the debt all grow at constant rate (G) and the expected return on debt is constant (R), then $R = \mathbb{E}_t \frac{V_{t+1}}{V_t} + \mathbb{E}_t \frac{T_{t+1}}{T_t}\frac{T_t}{V_t} - \mathbb{E}_t \frac{X_{t+1}}{X_t}\frac{X_t}{V_t} = G\left(1 + \frac{S_t}{V_t}\right)$
- So

$$\log\left(1 + \frac{S_t}{V_t}\right) = \log R - \log G$$

- Assuming $R > G$, market value of debt is present value of future surpluses

A framework for fiscal analysis

- For the general case, rewrite the gross return on government debt as

$$R_{t+1} = \frac{V_{t+1}}{V_t}\left(1 + \frac{S_{t+1}}{V_{t+1}}\right)$$

- Linearize $\log(1 + S_{t+1}/V_{t+1})$, following Gao and Martin (2021):

$$r_{t+1} = \Delta v_{t+1} + \log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right)$$

- The log surplus-debt ratio is a function of $\tau v_t = \log(T_t/V_t)$ and $xv_t = \log(X_t/V_t)$:

$$\log\left(1 + \frac{S_{t+1}}{V_{t+1}}\right) = \log\left(1 + e^{\tau v_{t+1}} - e^{xv_{t+1}}\right)$$

Loglinearization (1)

- We linearize around $(\tau v_{t+1}, x v_{t+1}) = (\log a, \log b)$, where a and b are each positive
- We obtain

$$\log\left(1 + e^{\tau v_{t+1}} - e^{x v_{t+1}}\right) \approx k + \frac{1}{1+a-b}\left(a\,\tau v_{t+1} - b\,x v_{t+1}\right)$$

where

$$k = \log(1+a-b) + \frac{b\log b - a\log a}{1+a-b}$$

Loglinearization (2)

We choose a and b to

1. Match the unconditional mean:

$$\log(1 + a - b) = \mathbb{E} \log \left(1 + \frac{S_t}{V_t}\right) = -\log \rho$$

   where $0 < \rho < 1$ when the mean surplus is positive

2. Enforce stationarity: If τv_t and xv_t are cointegrated, $\tau v_t - \beta xv_t$ is stationary for some constant β, where $0 < \beta < 1$ when the mean surplus is positive. To make the approximated surplus-debt ratio stationary, we need

$$\frac{b}{a} = \beta$$

Loglinearization (3)

- These conditions determine a and b in terms of β and ρ:

$$a = \frac{1}{1-\beta}\frac{1-\rho}{\rho} \quad \text{and} \quad b = \frac{\beta}{1-\beta}\frac{1-\rho}{\rho}$$

- The resulting approximation is

$$\log\left(1 + \frac{S_t}{V_t}\right) \approx \underbrace{k + \frac{1-\rho}{1-\beta}\left(\tau v_t - \beta x v_t\right)}_{sv_t}$$

- sv_t is our proposed measure of the fiscal position
  - It is stationary
  - It falls when tax falls, spending rises, or debt rises
  - It satisfies the approximate identity $r_{t+1} = \Delta v_{t+1} + sv_{t+1}$

Choosing the linearization parameters

- In our 1947–2020 data, the sample mean surplus-debt ratio is negative. This contradicts the theory we are using which requires a positive population mean. We set $\rho = 0.999$ to come close to the data while remaining consistent with the theory
- We then determine $\beta = 0.997$ by solving

$$\min_{\beta} \sum_t \left(\log\left(1 + S_t / V_t\right) - k - \frac{1-\rho}{1-\beta} \left(\tau v_t - \beta x v_t\right) \right)^2$$

- When estimating dynamics of the data, we impose theoretically motivated means $\mathbb{E}\, r_t = 0.031$, $\mathbb{E}\, \Delta \tau_t = 0.03$, and $\mathbb{E}\, s v_t = 0.001$ rather than using sample means

Our measure of the fiscal position, sv_t, and the surplus-debt ratio

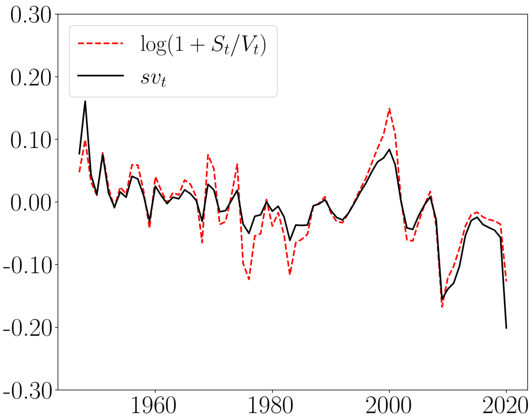

Figure: sv_t and $\log(1 + S_t/V_t)$.

An identity

- Rearranging the approximate identity, we have

$$sv_t = (1-\rho)\left[r_{t+1} - \frac{1}{1-\beta}\Delta\tau_{t+1} + \frac{\beta}{1-\beta}\Delta x_{t+1}\right] + \rho\, sv_{t+1}$$

- Solving forward, we have generalized "$S/V = R - G$":

$$sv_t = (1-\rho)\sum_{j=0}^{\infty}\rho^j\left[r_{t+1+j} - \frac{1}{1-\beta}\Delta\tau_{t+1+j} + \frac{\beta}{1-\beta}\Delta x_{t+1+j}\right]$$

  - A strong fiscal position implies some combination of high returns on debt, low tax growth, and high spending growth over the infinite future

- We use this (approximate) identity to do variance decompositions

- Table reports ADF test statistics and p-values associated with the null hypothesis of a unit root
- Lags chosen to minimize AIC
- Red entries where we can reject unit roots
  - Returns, spending growth, tax growth, surplus-debt ratio, tax-GDP ratio
  - We estimate a VAR based on the stationary variables r_t, $\Delta\tau_t$, sv_t, τy_t
  - We do not include Δx_t as it is linked to r_t, $\Delta\tau_t$ and sv_t by the approximate identity
- Black entries where we do not reject a unit root
  - Tax-debt, spending-debt, spending-GDP, surplus-GDP, and debt-GDP ratios

Table: Summary of unit root tests

Variable	test stat	p-value
r_t	−7.62	0.000
Δx_t	−9.47	0.000
$\Delta\tau_t$	−5.51	0.000
τv_t	−0.80	0.820
xv_t	−1.95	0.306
sv_t	−3.15	0.022
S_t/V_t	−3.62	0.005
$\log(1+S_t/V_t)$	−3.63	0.005
T_t/Y_t	−4.63	0.000
X_t/Y_t	−1.37	0.595
S_t/Y_t	−1.71	0.425
V_t/Y_t	1.50	0.997
τy_t	−4.67	0.000
xy_t	−2.16	0.219
vy_t	−0.23	0.934

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Table: Summary statistics

Variable	mean	std
r_t	0.023	0.057
Δx_t	0.033	0.118
$\Delta\tau_t$	0.029	0.067
τv_t	−0.751	0.460
xv_t	−0.730	0.440
sv_t	−0.009	0.054
S_t/V_t	−0.008	0.060
$\log(1+S_t/V_t)$	−0.010	0.060
T_t/Y_t	0.168	0.012
X_t/Y_t	0.173	0.026
S_t/Y_t	−0.005	0.028
V_t/Y_t	0.391	0.186
τy_t	−1.787	0.074
xy_t	−1.765	0.154
vy_t	−1.036	0.433

The VAR

Table: VAR coefficient estimates for $(r_t, \Delta\tau_t, sv_t, \tau y_t)$, US data 1947–2020.

	r_{t+1}	$\Delta\tau_{t+1}$	sv_{t+1}	τy_{t+1}
r_t	0.060	−0.259	−0.220	−0.324
	[0.110]	[0.115]	[0.080]	[0.103]
$\Delta\tau_t$	−0.072	0.355	−0.037	0.367
	[0.092]	[0.096]	[0.066]	[0.085]
sv_t	−0.102	−0.136	0.763	−0.221
	[0.127]	[0.133]	[0.092]	[0.119]
τy_t	0.287	−0.419	0.003	0.676
	[0.091]	[0.095]	[0.066]	[0.084]
R^2	17.11%	40.58%	60.87%	63.58%

A variance decomposition for sv_t

- Recall that

$$sv_t = (1-\rho)\sum_{j=0}^{\infty}\rho^j\left[r_{t+1+j} - \frac{1}{1-\beta}\Delta\tau_{t+1+j} + \frac{\beta}{1-\beta}\Delta x_{t+1+j}\right]$$

- Hence, over an infinite horizon

$$1 = \frac{\operatorname{cov}(sv_t, (1-\rho)\sum_{j=0}^{\infty}\rho^j\,\mathbb{E}_t\,r_{t+1+j})}{\operatorname{var} sv_t} + \frac{\operatorname{cov}(sv_t, -(1-\rho)\sum_{j=0}^{\infty}\rho^j\,\mathbb{E}_t\,\frac{1}{1-\beta}\Delta\tau_{t+1+j})}{\operatorname{var} sv_t} + \frac{\operatorname{cov}(sv_t, (1-\rho)\sum_{j=0}^{\infty}\rho^j\,\mathbb{E}_t\,\frac{\beta}{1-\beta}\Delta x_{t+1+j})}{\operatorname{var} sv_t}$$

A variance decomposition for sv_t

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- Hence, over a finite horizon T

$$1 = \frac{\operatorname{cov}(sv_t, (1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\,r_{t+1+j})}{\operatorname{var} sv_t} + \frac{\operatorname{cov}(sv_t, -(1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\,\frac{1}{1-\beta}\Delta\tau_{t+1+j})}{\operatorname{var} sv_t} + $$
$$+ \frac{\operatorname{cov}(sv_t, (1-\rho)\sum_{j=0}^{T-1}\rho^j\,\mathbb{E}_t\,\frac{\beta}{1-\beta}\Delta x_{t+1+j})}{\operatorname{var} sv_t} + \frac{\operatorname{cov}(sv_t, \rho^T\,\mathbb{E}_t\,sv_{t+T})}{\operatorname{var} sv_t}$$

A variance decomposition for sv_t

Panel A: Variance decomposition for sv_t				
Horizon	return	tax	spending	future sv
1	-0.0%	4.2%	14.5%	82.7%
3	-0.0%	19.4%	32.4%	49.5%
10	-0.1%	3.4%	85.3%	12.7%
30	-0.1%	0.4%	100.9%	0.2%
∞	-0.1%	0.3%	101.2%	0.0%

A variance decomposition for sv_t

Panel B: Bootstrap intervals				
Horizon	return	tax	spending	future sv
1	[-0.0%, 0.0%]	[-1.2%, 27.8%]	[3.6%, 43.3%]	[37.7%, 90.8%]
3	[-0.1%, 0.1%]	[0.5%, 36.6%]	[9.4%, 65.9%]	[9.3%, 80.9%]
10	[-0.3%, 0.1%]	[-26.3%, 19.2%]	[52.8%, 101.8%]	[-0.1%, 61.1%]
30	[-0.5%, 0.1%]	[-69.6%, 18.7%]	[82.0%, 146.5%]	[-0.0%, 29.7%]
∞	[-0.7%, 0.1%]	[-108.6%, 18.7%]	[82.8%, 210.5%]	[-0.0%, 0.0%]

Interpretation

- Variation in the fiscal position is primarily resolved by long-run predictable changes in government spending
  - A weak fiscal position is typically resolved by declines in the growth rate of spending, rather than by increases in tax revenue or poor returns for bondholders
- In historical US data, government debt returns have modest variability and limited persistence
  - Hence returns play little role at any horizon
  - Contrast with the Campbell–Shiller (1988) finding that returns are the dominant driver of fluctuations in the market dividend-price ratio
- Over the long run, taxes are linked to GDP and fiscal variables do not strongly predict GDP growth
  - Hence taxes play little role in the long run

The critical role of the tax-GDP ratio

- If we drop the tax-GDP ratio from the system the stabilizing force on tax growth is missing so the framework suggests a much larger role for long-run tax adjustment

Table: Variance decomposition for sv_t based on a VAR in $(r_t, \Delta\tau_t, sv_t)$

Variance decomposition				
Horizon	return	tax	spending	future sv
1	-0.0%	4.6%	14.1%	82.7%
3	0.0%	25.2%	27.0%	49.2%
10	0.0%	56.3%	36.8%	8.3%
30	0.0%	62.6%	38.7%	0.1%
∞	0.0%	62.7%	38.7%	0.0%

What happens when there is a tax shock?

- We can rearrange the identity to describe the correlates of a tax shock:

$$\underbrace{\Delta\,\mathbb{E}_{t+1}\,\tau_{t+1}}_{\text{short-run tax news}} = \underbrace{(1-\beta)\,\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j r_{t+1+j}}_{\text{return news}} - \underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=1}^{T-1}\rho^j\Delta\tau_{t+1+j}}_{\text{long-run tax news}} +$$

$$+\underbrace{\beta\,\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j\Delta x_{t+1+j}}_{\text{spending news}} + \frac{1-\beta}{1-\rho}\underbrace{\Delta\,\mathbb{E}_{t+1}\,\rho^T sv_{t+T}}_{\text{future fiscal health news}}$$

- Higher taxes today must be associated with some combination of (i) higher returns on the debt, (ii) lower tax growth in the long run, (iii) higher spending now, or in the long run, or both, and (iv) a stronger fiscal position

A variance decomposition for short-run tax news

Panel A: Variance decomposition for short-run tax news				
T	return	LR tax	spending	future sv
1	-0.0%	—	-37.6%	139.2%
3	0.1%	43.8%	-16.3%	74.0%
10	0.0%	75.7%	9.3%	16.5%
30	0.0%	77.2%	24.1%	0.3%
∞	0.0%	77.1%	24.4%	—

- These describe responses to a tax shock with typical contemporaneous responses of returns and spending, not an exogenous structural shock

A variance decomposition for short-run tax news

Panel B: Bootstrap intervals				
T	return	LR tax	spending	future sv
1	[-0.1%, -0.0%]	[-0.0%, 0.0%]	[-45.9%, -29.5%]	[131.0%, 147.5%]
3	[-0.0%, 0.1%]	[11.2%, 69.5%]	[-61.2%, 23.9%]	[25.2%, 135.2%]
10	[-0.2%, 0.2%]	[43.9%, 97.7%]	[-33.0%, 33.3%]	[-0.4%, 70.0%]
30	[-0.4%, 0.2%]	[16.9%, 98.7%]	[1.8%, 68.5%]	[-0.1%, 29.9%]
∞	[-0.5%, 0.1%]	[-18.3%, 98.6%]	[2.7%, 120.5%]	[-0.0%, 0.0%]

- These describe responses to a tax shock with typical contemporaneous responses of returns and spending, not an exogenous structural shock

What happens when there is a spending shock?

$$\underbrace{\Delta\,\mathbb{E}_{t+1}\,x_{t+1}}_{\text{short-run spending news}} = -\frac{1-\beta}{\beta}\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j r_{t+1+j}}_{\text{return news}} + \frac{1}{\beta}\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=0}^{T-1}\rho^j \Delta\tau_{t+1+j}}_{\text{tax news}} +$$

$$-\underbrace{\Delta\,\mathbb{E}_{t+1}\sum_{j=1}^{T-1}\rho^j \Delta x_{t+1+j}}_{\text{long-run spending news}} - \frac{1-\beta}{\beta(1-\rho)}\underbrace{\Delta\,\mathbb{E}_{t+1}\,\rho^T sv_{t+T}}_{\text{future fiscal health news}}$$

- Higher spending today must be associated with some combination of (i) lower returns on the debt, (ii) higher tax growth now, or in the long run, or both, (iii) lower spending growth in the long run, and (iv) a weaker fiscal position

A variance decomposition for short-run spending news

Panel A: Variance decomposition for short-run spending news				
T	return	tax	LR spending	future sv
1	-0.0%	-15.9%	—	117.4%
3	-0.1%	-14.3%	37.1%	78.6%
10	-0.1%	-25.3%	107.7%	19.1%
30	-0.1%	-29.9%	131.2%	0.3%
∞	-0.1%	-30.0%	131.6%	—

- These describe responses to a spending shock with typical contemporaneous responses of returns and tax revenue, not an exogenous structural shock

A variance decomposition for short-run spending news

Panel B: Bootstrap intervals				
T	return	tax	LR spending	future sv
1	[-0.1%, -0.0%]	[-0.0%, 0.0%]	[-0.0%, 0.0%]	[113.7%, 120.8%]
3	[-0.1%, -0.0%]	[-15.6%, 18.7%]	[13.0%, 59.0%]	[50.5%, 110.6%]
10	[-0.4%, -0.0%]	[-38.1%, 9.2%]	[61.0%, 127.3%]	[0.4%, 82.1%]
30	[-0.7%, 0.0%]	[-91.2%, 8.8%]	[107.8%, 182.1%]	[-0.0%, 39.5%]
∞	[-0.9%, 0.0%]	[-140.5%, 8.8%]	[108.7%, 258.6%]	[-0.0%, 0.0%]

- These describe responses to a spending shock with typical contemporaneous responses of returns and tax revenue, not an exogenous structural shock

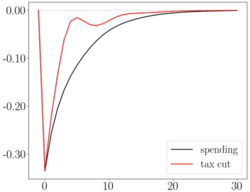
Interpretation

- The limited variability and persistence of government debt returns makes the return component small at all horizons
  - Contrast with the fiscal theory of the price level, which postulates large changes in real debt valuation in response to exogenous shocks to taxes or spending
  - It remains possible that the FTPL holds, but the US government has chosen not to change taxes or spending in a way that requires volatile real debt returns
- In historical US data, typical tax cuts have been 3/4 reversed by subsequent tax growth and 1/4 accommodated by slower long-run spending growth
- Typical spending increases have been associated with subsequent tax *declines* in the long run, and hence with more than one-for-one declines in long run spending

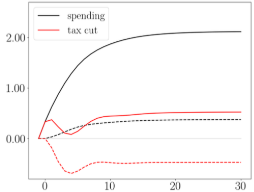
Taxes vs. spending

- To understand the interaction of debt and deficits, it is important to distinguish between tax and spending, rather than working directly with surplus
- Debt-financed tax cut $\neq$ debt-financed spending increase
- And a tax-financed spending increase can cause a deterioration in the fiscal position despite having no contemporaneous impact on surplus

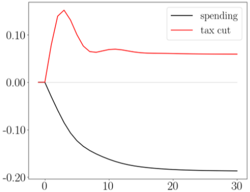
Debt-financed spending increase or tax decline



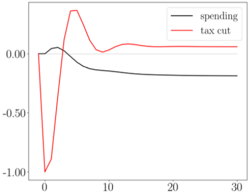
(a) sv_t



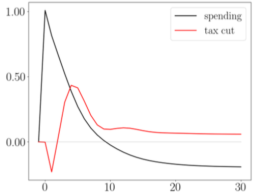
(b) v_t



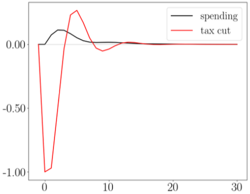
(c) y_t



(d) τ_t

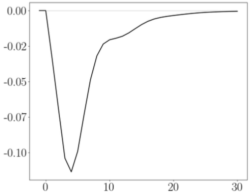


(e) x_t

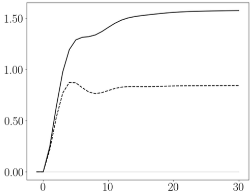


(f) τy_t

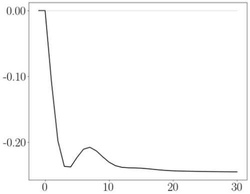
Tax-financed spending increase



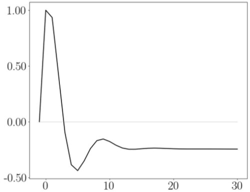
(a) sv_t



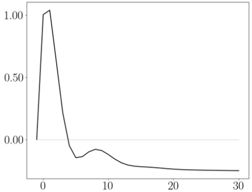
(b) v_t



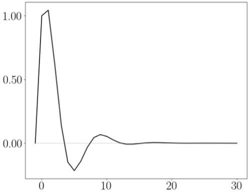
(c) y_t



(d) τ_t



(e) x_t



(f) τy_t

We find very similar results for the UK

- Debt-GDP ratio is nonstationary
- Surplus-debt ratio is stationary
  - But tax-debt and spending-debt ratios are each nonstationary
- Mixed evidence for tax-GDP ratio: p-value under the null of a unit root is 0.114
  - Given our results for US, we treat it as stationary
- Similar coefficient estimates in the VAR
- Same bottom line: variation in the fiscal position is resolved, in the long run, by adjustments in spending

The debt-GDP ratio appears to be nonstationary

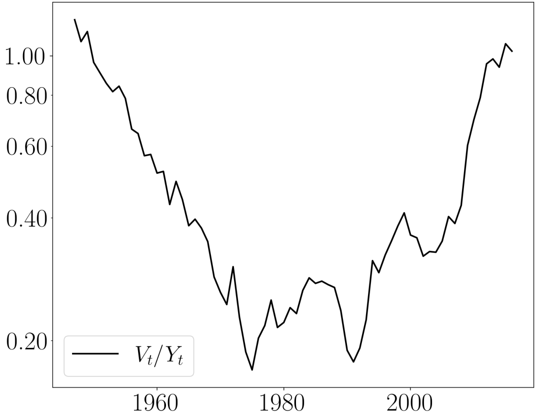


Figure: Debt-GDP ratio, UK data, 1947–2016. Log scale

The surplus-debt ratio appears to be stationary

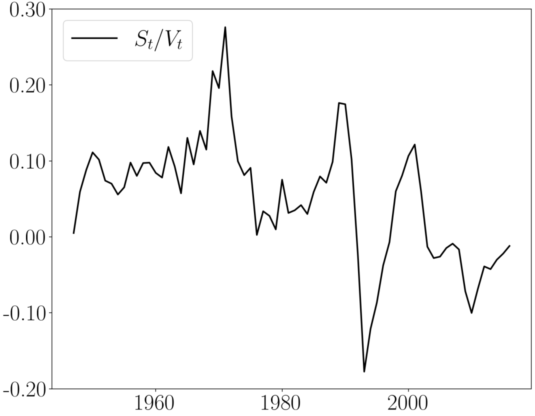


Figure: Surplus-debt ratio, UK data, 1947–2016. Linear scale

The tax-debt and spending-debt ratios appear to be nonstationary

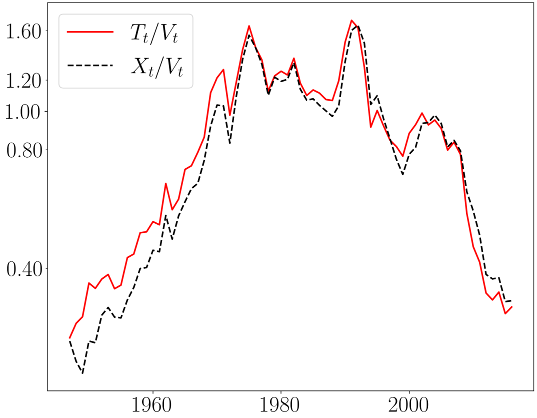


Figure: Tax-debt and spending-debt ratios, UK data, 1947–2016. Log scale

We treat tax-GDP ratio as stationary. . . evidence is mixed

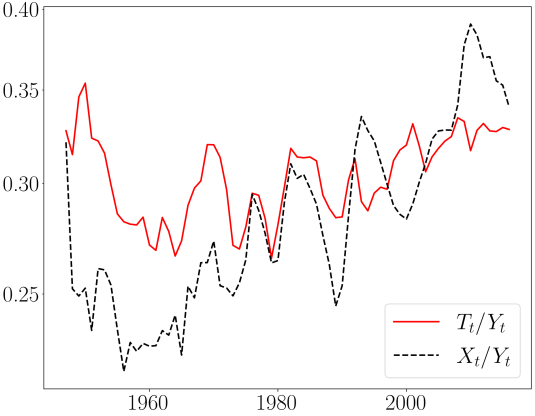


Figure: Tax-debt and spending-debt ratios, UK data, 1947–2016. Log scale

The VAR for the UK

Table: VAR coefficient estimates for $(r_t, \Delta\tau_t, sv_t, \tau v_t)$, UK data 1947–2016.

	r_{t+1}	$\Delta\tau_{t+1}$	sv_{t+1}	τy_{t+1}
r_t	−0.228	−0.064	−0.037	−0.131
	[0.119]	[0.037]	[0.047]	[0.040]
$\Delta\tau_t$	0.600	0.446	0.102	0.342
	[0.328]	[0.103]	[0.129]	[0.110]
sv_t	0.024	−0.084	0.873	−0.113
	[0.147]	[0.046]	[0.058]	[0.049]
τy_t	0.166	−0.201	−0.072	0.827
	[0.176]	[0.055]	[0.069]	[0.059]
R^2	9.04%	30.83%	79.94%	77.84%

A variance decomposition for sv_t in the UK

Panel A: Variance decomposition for sv_t				
Horizon	return	tax	spending	future sv
1	0.2%	-0.1%	14.1%	87.2%
3	-0.1%	3.1%	34.3%	64.1%
10	-0.6%	-11.8%	86.1%	27.7%
30	-0.8%	-30.1%	129.3%	3.0%
∞	-0.8%	-32.3%	134.6%	0.0%

A variance decomposition for sv_t in the UK

Panel B: Bootstrap intervals				
Horizon	return	tax	spending	future sv
1	[-1.9%, 2.0%]	[-11.5%, 15.6%]	[10.9%, 41.8%]	[57.1%, 89.0%]
3	[-3.7%, 3.2%]	[-20.4%, 25.9%]	[18.9%, 68.8%]	[32.4%, 78.1%]
10	[-7.4%, 6.0%]	[-51.6%, 21.4%]	[52.8%, 125.0%]	[5.3%, 52.6%]
30	[-11.4%, 9.1%]	[-96.3%, 17.9%]	[84.4%, 184.8%]	[-0.0%, 19.3%]
∞	[-12.7%, 10.0%]	[-114.6%, 17.3%]	[90.2%, 215.7%]	[0.0%, 0.0%]

Summary

- Our framework uses identities to organize the time-series analysis of historical data
- We have not identified structural shocks and cannot make causal statements or explore counterfactuals
- However, the identities in our paper are in a convenient form to be combined with typical loglinear macro models, whether in the DSGE tradition or the NK tradition
- We think it is important to distinguish the separate influences of taxes and spending
  - Consistent with the distinction drawn by Alesina, Favero and Giavazzi (2020) between tax-driven and spending-driven austerity
- In the US and UK, shocks to the fiscal position are associated with adjustment in spending over the long run, rather than with taxes or returns