Sustainability in a Risky World

John Y. Campbell   Ian W. R. Martin

Harvard and LSE

May 26, 2021

Is impatience ethical?

Three views:

- Preferences must be respected: impatience, aka rate of time preference, is whatever it is
- Any positive time preference is unethical, at least across generations (Ramsey 1928)
  - Influence on the low discount rates used in the Stern Report (2006)
- Ethical considerations impose a sustainability constraint on the rate of time preference

What is sustainability?

- World Commission on Environment and Development (1987): a sustainable consumption plan

  "meets the needs of the present without compromising the ability of future generations to meet their own needs."

- Solow (1993):

  "A sustainable national economy is one that allows every future generation the option of being as well off as its predecessors. The duty imposed by sustainability is to bequeath to posterity not any particular thing . . . but rather to endow them with whatever it takes to achieve a standard of living at least as good as our own and to look after their next generation similarly."

Ethics as a constraint

- Expressing ethics as a constraint, rather than incorporating ethics in preferences, is unfamiliar to many economists who tend to be philosophical utilitarians
- However, it is consistent with the positions of many moral philosophers including Rawls (*A Theory of Justice*, rev. ed. 1999):

"The principles of right, and so of justice, put limits on which satisfactions have value; they impose restrictions on what are reasonable conceptions of one's good. In drawing up plans and deciding on aspirations men are to take these constraints into account."

Formalizing sustainability in a riskless world

- In a riskless world, sustainability is straightforward (Arrow et al. 2004)
- Assume a single form of capital with a constant riskless rate of return
- Sustainability constraint: the social rate of time preference cannot exceed the riskless interest rate
- When the constraint binds, time preference equals the riskless rate so society consumes the return on its capital and leaves wealth unchanged over time
- Then consumption, utility of current consumption, marginal utility of current consumption, and social value (the discounted present value of utility) are all constant over time

Sustainability allows more impatience than Ramsey

- Sustainable consumption is only feasible if the riskless rate is positive
- Then, positive time preference is sustainable so the sustainability constraint is not as tight as Ramsey's zero-time-preference constraint
- Sustainability responds to the Koopmans (1960, 1967) critique of Ramsey:

> *"One cannot adopt ethical principles without regard to . . . the anticipated technological possibilities. Any proposed optimality criterion needs to be subjected to a mathematical screening, to determine whether it does indeed bear on the problem at hand, under the circumstances assumed. More specifically, too much weight given to generations far in the future turns out to be self-defeating. It does nobody any good. How much weight is too much has to be determined in each case."*

What about risk?

- In a risky economy the return on capital is uncertain so it is infeasible to guarantee that social value remains constant over time
- Instead, we impose a weaker constraint, suggested but not analyzed by Howarth (1995), that social value is not expected to decline over time
- Our constraint is related to, but different from, the “sustainable spending” constraints analyzed by Campbell and Sigalov (2021), which require that consumption or log consumption are not expected to decline over time

Outline

- Unconstrained model of consumption and portfolio choice with a riskless and a risky asset
- Imposing the sustainability constraint
  - Comparing the constrained and unconstrained solutions
  - Sustainable drifts in wealth and marginal utility
  - Solutions and numerical examples
- Sustainability without a riskless asset
- Sustainability with population growth
- Conclusion

Model ingredients

- Two assets:
  - Safe log return r_f
  - Risky asset has expected excess return μ, Brownian volatility σ, and jumps of size L that arrive at rate ω
  - If no jumps occur, expected excess return is $\widehat{\mu} = \mu + \omega \mathrm{E}L$
- Representative investor chooses consumption-wealth ratio $\theta = C_t/W_t$ and risky portfolio share α to maximize

$$U_0 = \mathrm{E} \int_0^\infty e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma} \, dt$$

where

$$\frac{dC}{C} = \frac{dW}{W} = \left[r_f + \alpha \underbrace{(\mu + \omega \mathrm{E}L)}_{\widehat{\mu}} - \theta \right] dt + \alpha\sigma \, dZ - \alpha L \, dN$$

Microfoundations

- We take the representative investor's objective function as given
- One microfoundation could be as in Blanchard (1985):
  - Individual agents have a constant probability of death
  - No population growth, so each deceased agent is replaced by a newborn agent
  - Wealth of deceased agents is reallocated to newborns
- Then all agents alive at a given time are identical, and ρ reflects
  - True time preference within each individual's lifetime
  - The probability of death (which raises ρ)
  - Any degree of intergenerational altruism (which lowers ρ)
- The sustainability constraint protects the interests of future generations vs those currently alive
- The framework can accommodate population growth, but we discuss this later

Unconstrained solution

$$U_0 = \frac{W_0^{1-\gamma}}{1-\gamma} \frac{\theta^{1-\gamma}}{\rho - (1-\gamma)\left(r_f + \alpha\widehat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2 - \theta\right) - \omega \mathrm{E}\left[(1-\alpha L)^{1-\gamma} - 1\right]}$$

- Expected utility is a function of current wealth W_0
- Maximize with respect to the optimal portfolio decision, α, and consumption-savings decision, θ
- The optimal consumption-wealth ratio is

$$\theta_{\text{unc}} = \frac{\rho + (\gamma - 1)\left(r_f + \alpha\widehat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2\right) - \omega \mathrm{E}\left[(1-\alpha L)^{1-\gamma} - 1\right]}{\gamma}$$

- Impatient (high ρ) $\implies$ high consumption-wealth ratio
- If $\rho > \theta_{\text{unc}}$ then expected utility is expected to decline without limit

Unconstrained solution

$$U_0 = \frac{W_0^{1-\gamma}}{1-\gamma} \frac{\theta^{1-\gamma}}{\rho - (1-\gamma)\left(r_f + \alpha\widehat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2 - \theta\right) - \omega \mathrm{E}\left[(1-\alpha L)^{1-\gamma} - 1\right]}$$

- The optimal risky portfolio share is defined implicitly by

$$\widehat{\mu} - \alpha\gamma\sigma^2 = \omega \mathrm{E}\left[L(1-\alpha L)^{-\gamma}\right]$$

  - Gives the classic Merton formula, $\alpha = \mu/\gamma\sigma^2$, if there are no jumps

Imposing the sustainability constraint

- Zero drift in expected utility requires zero drift in $X_t = W_t^{1-\gamma}$
- The stochastic process for X_t is

$$\begin{aligned}\frac{dX}{X} &= (1-\gamma)\left(r_f + \alpha\widehat{\mu} - \theta - \frac{1}{2}\gamma\alpha^2\sigma^2\right)dt + (1-\gamma)\alpha\sigma dZ \\ &\quad + \left[(1-\alpha L)^{1-\gamma} - 1\right]dN\end{aligned}$$

- When the constraint binds, we have

$$\theta_{\text{con}} = r_f + \alpha\widehat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2 + \omega\frac{\mathrm{E}\left[(1-\alpha L)^{1-\gamma} - 1\right]}{1-\gamma}$$

- This allows us to eliminate θ from the objective function
- Maximizing with respect to α gives the same solution as before: the sustainability constraint does not distort portfolio choice

Sustainable C/W exceeds the riskless interest rate

- We can use the solution for α to rewrite the constrained consumption-wealth ratio as

$$\theta_{\text{con}} = r_f + \frac{1}{2}\gamma\alpha^2\sigma^2 + \omega\frac{\mathrm{E}\left[(1-\alpha\gamma L)(1-\alpha L)^{-\gamma} - 1\right]}{1-\gamma}$$

- The second and third terms are positive
- Hence, the sustainable consumption-wealth ratio exceeds the riskless interest rate
  - Risk is critical for this result!

Sustainability constraint as an inequality

- The constraint is one-sided. We allow expected utility to drift up, not down. Equivalently, we allow the consumption-wealth ratio to be lower than the constrained level:

$$\theta_{\text{sus}} = \min\{\theta_{\text{unc}}, \theta_{\text{con}}\}$$

- Comparing the constrained and unconstrained solutions for θ,

$$\theta_{\text{unc}} = \frac{1}{\gamma}\rho + \left(1 - \frac{1}{\gamma}\right)\theta_{\text{con}}$$

  - The constraint binds if and only if $\rho > \theta_{\text{con}}$, or equivalently $\rho > \theta_{\text{unc}}$
  - The constraint has a smaller effect when γ is very large (because then the unconstrained consumption path is close to flat)
  - The constraint can be implemented by imposing an adjusted time preference rate $\widehat{\rho} = \min\{\rho, \theta_{\text{con}}\}$

Sustainable drift in wealth

- Although utility (equivalently, $X = W^{1-\gamma}$) has zero drift when the sustainability constraint binds, wealth W has a positive drift
- The positive drift in wealth implies that sustainable utility is a tighter constraint than the sustainable spending constraint imposed by Campbell and Sigalov (2021)
  - Sustainable spending distorts portfolio choice, sustainable utility does not
- Intuitively, risk cumulates over time so later generations are exposed to more of it. To compensate them, they must have higher wealth in expectation

Sustainable drift in marginal utility

- Although utility (equivalently, $X = W^{1-\gamma}$) has zero drift when the sustainability constraint binds, marginal utility $M = W^{-\gamma}$ has a positive drift
- This is another way to understand the result that $\theta_{\text{con}} > r_f$, as the FOC for optimal investment in the riskless asset implies that

$$\mathrm{E}\frac{dM}{M} = \theta_{\text{con}} - r_f$$

- How can $X = MW$ have zero drift if M and W have positive drift?
  - The answer is that M and W covary negatively, and

$$\frac{dX}{X} = \frac{dM}{M} + \frac{dW}{W} + \frac{dM}{M}\frac{dW}{W}$$

  - Once again risk is critical!

Explicit solution for Brownian motion

- If we turn off jumps, we get an explicit solution for the case of pure Brownian risk
- Using the Merton portfolio rule, $\alpha = \mu / \gamma\sigma^2$, we have

$$\theta_{\text{con}} = r_f + \frac{1}{2}\frac{\mu^2}{\gamma\sigma^2} = r_f + \frac{\alpha\mu}{2}$$

- The sustainable consumption-wealth ratio is the average of the riskless interest rate and the return on the optimally invested portfolio

Numerical examples for Brownian motion

Table: Numerical examples in the Brownian case

Baseline calibration sets $r_f = 1\%, \mu = 8\%, \sigma = 20\%$

γ	θ_{con}	α	$\mathrm{E}\,\frac{dW}{W}$	$\mathrm{E}\,d\log W$	$\mathrm{E}\,\frac{dW^{-\gamma}}{W^{-\gamma}}$
1	0.09	2	0.08	0	0.08
2	0.05	1	0.04	0.02	0.04
5	0.026	0.4	0.016	0.0128	0.016
10	0.018	0.2	0.008	0.0072	0.008

Numerical examples with jumps

Baseline calibration sets $r_f = 2\%, \mu = 4\%, \sigma = 10\%, \omega = 4\%, L = 0.4, \gamma = 2$

	γ	r_f	μ	σ	ω	L	θ_{con}	α	$\mathrm{E}\,\frac{dW}{W}$	$\mathrm{E}\,d\log W$	$\mathrm{E}\,\frac{dW^{-\gamma}}{W^{-\gamma}}$
Baseline	2	0.02	0.04	0.10	0.02	0.40	0.045	1.11	0.019	0.010	0.025
High γ	5						0.031	0.49	0.009	0.007	0.011
Low γ	1						0.065	1.83	0.028	0	0.045
High r_f		0.04					0.065	1.11	0.019	0.010	0.025
Low r_f		0					0.025	1.11	0.019	0.010	0.025
High μ			0.08				0.10	1.56	0.045	0.026	0.080
Low μ			0.02				0.027	0.66	0.006	0.003	0.007
High σ				0.15			0.035	0.72	0.014	0.007	0.015
Low σ				0.06			0.055	1.36	0.019	0.011	0.035
High ω					0.04		0.040	0.88	0.016	0.008	0.020
Low ω					0		0.060	2.00	0.040	0.020	0.040
High L						0.60	0.037	0.72	0.012	0.007	0.017
Low L						0.20	0.056	1.73	0.034	0.017	0.036
Negative L						-0.40	0.054	1.74	0.036	0.017	0.034

Equilibrium without a riskless asset

- If the riskless asset is in zero net supply, then $\alpha = 1$ in equilibrium
- The Brownian solution becomes

$$\theta_{\text{con}} = r_f + \frac{1}{2}\gamma\sigma^2 = r_f + \frac{\mu}{2}$$

  - The sustainable consumption-wealth ratio is the average of the riskless interest rate and the risky return

Equilibrium without a riskless asset

- With jumps, we have

$$\theta_{\text{con}} = r_f + \frac{1}{2}\gamma\sigma^2 + \omega\left[\boldsymbol{m}(\gamma) - \boldsymbol{m}(\gamma-1) - \frac{\boldsymbol{m}(\gamma-1) - \boldsymbol{m}(0)}{\gamma - 1}\right]$$

  where $\boldsymbol{m}(x) = \mathrm{E}\left[(1-L)^{-x}\right]$ summarizes the jump size distribution
- This can be rewritten

$$\theta_{\text{con}} = r_f + \mu - \frac{1}{2}\gamma\sigma^2 - \omega\left[\frac{\boldsymbol{m}(\gamma-1) - \boldsymbol{m}(0)}{\gamma - 1} - \frac{\boldsymbol{m}(0) - \boldsymbol{m}(-1)}{1}\right]$$

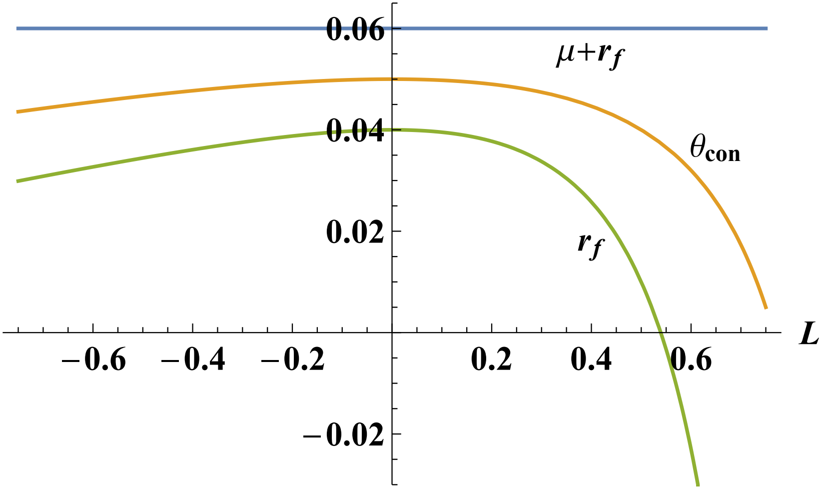
- As $\boldsymbol{m}(x)$ is convex, these imply bounds on θ_{con}:

$$r_f + \frac{1}{2}\gamma\sigma^2 \leq \theta_{\text{con}} \leq r_f + \mu - \frac{1}{2}\gamma\sigma^2$$

  - Upper bound can be $\approx$ tight if jumps are bad news
  - Lower bound can be $\approx$ tight if jumps are good news

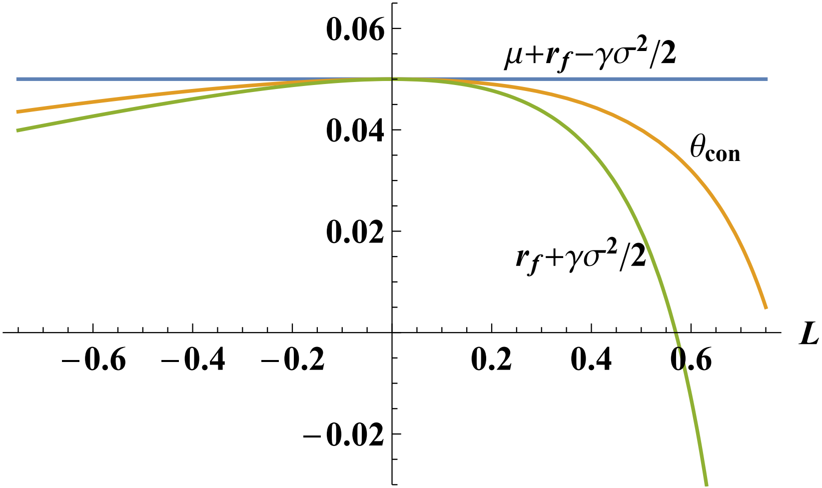
Equilibrium without a riskless asset: illustration (1)

$\gamma = 2$, $\sigma = 0.1$, $\omega = 0.02$, $\mu + r_f = 0.06$, L deterministic



Equilibrium without a riskless asset: illustration (2)

$\gamma = 2$, $\sigma = 0.1$, $\omega = 0.02$, $\mu + r_f = 0.06$, L deterministic



Numerical examples with no riskless asset

Baseline calibration sets $\mu + r_f = 6\%, \sigma = 10\%, \omega = 2\%, L = 0.4, \gamma = 2$ with $\alpha = 1$

	γ	$\mu + r_f$	σ	ω	L	θ_{con}	r_f	μ	$\mathrm{E}\,\frac{dW}{W}$	$\mathrm{E}\,d\log W$	$\mathrm{E}\,\frac{dW^{-\gamma}}{W^{-\gamma}}$
Baseline	2	0.06	0.10	0.02	0.40	0.045	0.026	0.034	0.015	0.008	0.019
High γ	5					0.009	-0.085	0.145	0.051	0.043	0.094
Low γ	1					0.053	0.045	0.015	0.007	0	0.008
High $\mu + r_f$		0.10				0.085	0.066	0.034	0.015	0.008	0.019
Low $\mu + r_f$		0.02				0.005	-0.014	0.034	0.015	0.008	0.019
High σ			0.15			0.032	0.001	0.059	0.028	0.014	0.031
Low σ			0.06			0.051	0.039	0.021	0.009	0.005	0.012
High ω				0.04		0.039	0.012	0.048	0.021	0.011	0.028
Low ω				0		0.050	0.040	0.020	0.010	0.005	0.010
High L					0.50	0.040	0.010	0.050	0.020	0.011	0.030
Low L					0.30	0.047	0.034	0.026	0.013	0.006	0.014
Negative L					-0.40	0.048	0.036	0.024	0.012	0.006	0.012

Population growth

- Population growth is notoriously challenging for intertemporal ethics, particularly when population is endogenous (Parfit 1984, Dasgupta 2001)
- But we can accommodate exogenous deterministic population growth, if we assume that the utility of a representative newborn individual must have zero drift
- Since society's wealth must be shared among a growing number of people, we need to increase saving to compensate
- The sustainable consumption-wealth ratio declines by the population growth rate g
  - It can be lower than the riskfree rate
  - But for realistic parameters, it will remain higher than the riskfree rate as our numerical examples show

Conclusion

- In a risky world, there is no necessary inconsistency between a substantial rate of time preference and the ethical criterion of sustainability
  - High risky returns can make it attractive for society to save for the future even if people are sufficiently impatient that they would dissave if only a safe asset with a low return were available
- In a risky world, there is no unique discount rate for investments
  - The low riskfree interest rate should be used to discount safe investments, and this is a lower discount rate than the sustainable rate of time preference
- We have made these points using a deliberately simple model
  - We have ignored parameter uncertainty (Weitzman 2001)
  - We have ignored time-varying expected returns and the resulting term structure of discount rates (Gollier 2002, Bansal and Yaron 2004)