What is the Expected Return on the Market?

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Returns on the stock market are predictable

\[
\text{return}_{t+1} = \frac{\text{price}_{t+1} + \text{dividend}_{t+1}}{\text{price}_t} = \frac{\text{price}_{t+1}}{\text{price}_t} \left(1 + \frac{\text{dividend}_{t+1}}{\text{price}_t}\right) = \frac{\text{price}_{t+1}}{\text{price}_t} + \left(\frac{\text{dividend}_{t+1}}{\text{price}_t}\right)
\]

- **Naive investor**: If I buy when the dividend yield is high, I will have a high return on average
- ‘Sophisticated’ investor: No! The high dividend yield—that is, low price—is a sign that the market anticipates that future dividends will be disappointing. I therefore expect that a low capital gain will offset the high dividend yield
- Empirically, it appears that the naive investor is right
S&P 500 Price / 10-Year Average of Earnings

Average P/E = 16.34

P/E in 9/09 = 18.77
The equity premium
Figure from John Campbell’s Princeton Lecture in Finance
Motivation

Find an asset price that forecasts expected returns

- without using accounting data
- without having to estimate any parameters
- imposing minimal theoretical structure
- and in real time
A lower bound on the equity premium

1 year horizon, in %
A lower bound on the equity premium

1 month horizon, annualized, in %
Outline

1. A volatility index, SVIX, gives a lower bound on the equity premium

2. SVIX and VIX

3. SVIX as a predictor variable

4. What is the probability of a 20% decline in the market?
Outline

1. A volatility index, SVIX, gives a lower bound on the equity premium
2. SVIX and VIX
3. SVIX as a predictor variable
4. What is the probability of a 20% decline in the market?
**Notation**

- $S_T$: level of S&P 500 index at time $T$
- $R_T$: gross return on the S&P 500 from time $t$ to time $T$
- $R_{f,t}$: riskless rate from time $t$ to time $T$
- $M_T$: SDF that prices time-$T$ payoffs from the perspective of time $t$
- We can price any time-$T$ payoff $X_T$ either via the SDF or by computing expectations with risk-neutral probabilities:
  \[
  \text{time-}t \, \text{price of a claim to } X_T = \mathbb{E}_t(M_T X_T) = \frac{1}{R_{f,t}} \mathbb{E}^*_t X_T
  \]
- Asterisks indicate risk-neutral quantities
Risk-neutral variance and the risk premium

- As an example, we can write conditional risk-neutral variance as

\[ \text{var}^* R_T = \mathbb{E}_t^* R_T^2 - (\mathbb{E}_t^* R_T)^2 = R_{f,t} \mathbb{E}_t (M_T R_T^2) - R_{f,t}^2 \]  

(1)

- We can decompose the equity premium into two components:

\[ \mathbb{E}_t R_T - R_{f,t} = \left[ \mathbb{E}_t (M_T R_T^2) - R_{f,t} \right] - \left[ \mathbb{E}_t (M_T R_T^2) - \mathbb{E}_t R_T \right] \]

\[ = \frac{1}{R_{f,t}} \text{var}^* R_T - \text{cov}_t (M_T R_T, R_T) \]

- The first line adds and subtracts \( \mathbb{E}_t (M_T R_T^2) \)

- The second exploits equation (1) and the fact that \( \mathbb{E}_t M_T R_T = 1 \)
Risk-neutral variance and the risk premium

$$\mathbb{E}_t R_T - R_{f,t} = \frac{1}{R_{f,t}} \var^*_t R_T - \text{cov}_t(M_T R_T, R_T) \leq 0, \text{ under the NCC}$$

- The decomposition splits the risk premium into two pieces
- **Risk-neutral variance** can be computed from time- \( t \) asset prices
- The **covariance term** can be controlled: it is negative in theoretical models and in the data
- Formalize this key assumption as the **negative correlation condition**: 
  $$\text{cov}_t(M_T R_T, R_T) \leq 0$$
The NCC holds...  

... in lognormal models in which the market’s conditional Sharpe ratio exceeds its conditional volatility (Campbell–Cochrane 1999, Bansal–Yaron 2004, and many others).
The NCC holds . . .

1. . . in lognormal models in which the market’s conditional Sharpe ratio exceeds its conditional volatility (Campbell–Cochrane 1999, Bansal–Yaron 2004, and many others).

2. . . in a wide range of models with intertemporal investors, state variables, Epstein–Zin preferences, non-Normality, labor income.

I Proof. The given assumption implies that the SDF is proportional to\( u_0(W_t R_T) \), so we must show that \( \text{cov}(R_T u_0(W_t R_T), R_T) \leq 0 \).

This holds because \( R_T u_0(W_t R_T) \) is decreasing in \( R_T \): its derivative is \( u_0(W_t R_T) + W_t R_T u_0(W_t R_T) = u_0(W_t R_T) [W_t R_T^{-1}] \leq 0 \).
The NCC holds...

1. ...in lognormal models in which the market's conditional Sharpe ratio exceeds its conditional volatility (Campbell–Cochrane 1999, Bansal–Yaron 2004, and many others).

2. ...in a wide range of models with intertemporal investors, state variables, Epstein–Zin preferences, non-Normality, labor income.

3. ...if there is a one-period investor who maximizes expected utility, who is fully invested in the market, and whose relative risk aversion $\gamma(C) \equiv -\frac{Cu''(C)}{u'(C)} \geq 1$ (not necessarily constant).
The NCC holds...

1. ... in lognormal models in which the market’s conditional Sharpe ratio exceeds its conditional volatility (Campbell–Cochrane 1999, Bansal–Yaron 2004, and many others).

2. ... in a wide range of models with intertemporal investors, state variables, Epstein–Zin preferences, non-Normality, labor income.

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   ▶ Proof. The given assumption implies that the SDF is proportional to \( u'(W_tR_T) \), so we must show that \( \text{cov}_t (R_T u'(W_tR_T), R_T) \leq 0 \).
The NCC holds...

1. ... in lognormal models in which the market’s conditional Sharpe ratio exceeds its conditional volatility (Campbell–Cochrane 1999, Bansal–Yaron 2004, and many others).

2. ... in a wide range of models with intertemporal investors, state variables, Epstein–Zin preferences, non-Normality, labor income.

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   ▶ **Proof.** The given assumption implies that the SDF is proportional to \( u'(W_tR_T) \), so we must show that \( \text{cov}_t(R_Tu'(W_tR_T), R_T) \leq 0 \).

   ▶ This holds because \( R_Tu'(W_tR_T) \) is decreasing in \( R_T \): its derivative is \( u'(W_tR_T) + W_tR_Tu''(W_tR_T) = -u'(W_tR_T) [\gamma(W_tR_T) - 1] \leq 0 \).
Whose equity premium?

\[ \mathbb{E}_t R_T - R_{f,t} \geq \frac{1}{R_{f,t}} \varphi \] 

- Does not require that everyone holds the market
- Does not assume that all economic wealth is invested in the market
- Simply ask: What is the equity premium perceived by a rational one-period investor who holds the market and whose risk aversion is at least 1?
- This question is a sensible benchmark even in the presence of constrained and/or irrational investors
Comparison to Merton (1980)

- Merton (1980) suggested estimating the equity premium from

  \[ \text{equity premium} = \text{risk aversion} \times \text{return variance} \]

- Holds if marginal investor has power utility and the market follows a geometric Brownian motion
- No distinction between risk-neutral and real-world variance in a diffusion-based model (Girsanov’s theorem)
- The appropriate generalization relates the equity premium to \textit{risk-neutral} variance
  - \textbf{Bonus:} Risk-neutral variance is directly measurable from asset prices
Comparison to Hansen–Jagannathan (1991)

\[
\frac{1}{R_{f,t}} \text{var}^* R_T \leq \mathbb{E}_t R_T - R_{f,t} \leq R_{f,t} \cdot \sigma_t(M_T) \cdot \sigma_t(R_T)
\]

- Left-hand inequality is the **new result**
  - Good: relates unobservable equity premium to an observable quantity
  - Bad: requires the negative correlation condition
- Right-hand inequality is the Hansen–Jagannathan bound
  - Good: no assumptions
  - Bad: neither side is observable
How to measure risk-neutral variance

- We want to measure \( \frac{1}{R_{f,t}} \text{var}^* R_T = \frac{1}{R_{f,t}} \mathbb{E}_t^* R_T^2 - \frac{1}{R_{f,t}} (\mathbb{E}_t^* R_T)^2 \)
- Since \( \mathbb{E}_t^* R_T = R_{f,t} \), this boils down to calculating \( \frac{1}{R_{f,t}} \mathbb{E}_t^* S_T^2 \)
- That is: how can we price the ‘squared contract’ with payoff \( S_T^2 \)?
How to measure risk-neutral variance

- How can we price the ‘squared contract’ with payoff $S_T^2$?
- Suppose you buy:
  - 2 calls with strike $K = 0.5$
  - 2 calls with strike $K = 1.5$
  - 2 calls with strike $K = 2.5$
  - 2 calls with strike $K = 3.5$
  - etc ...
How to measure risk-neutral variance

So, \( \frac{1}{R_{f,t}} \mathbb{E}_t^* S_T^2 \approx 2 \sum_K \text{call}_{t,T}(K) \)

In fact, \( \frac{1}{R_{f,t}} \mathbb{E}_t^* S_T^2 = 2 \int_0^\infty \text{call}_{t,T}(K) \, dK \)
How to measure risk-neutral variance

Using put-call parity, we end up with a simple formula:

\[
\frac{1}{R_{f,t}} \var^*_t R_T = \frac{2}{S_t^2} \left\{ \int_0^{F_{t,T}} \text{put}_{t,T}(K) \, dK + \int_{F_{t,T}}^\infty \text{call}_{t,T}(K) \, dK \right\}
\]

- \( F_{t,T} \) is the forward price of the underlying, which is known at time \( t \)
A lower bound on the equity premium
1mo horizon, annualized, 10-day moving avg. Mid prices in black, bid prices in red
A lower bound on the equity premium

3mo horizon, annualized, 10-day moving avg. Mid prices in black, bid prices in red
A lower bound on the equity premium

1yr horizon, annualized, 10-day moving avg. Mid prices in black, bid prices in red
Robustness

- Can’t observe deep-OTM option prices
Robustness

- Even near-the-money, can’t observe a continuum of strikes

\[ \text{put}_{0,T}(K) \]
Robustness

- Both these effects mean that the true lower bound is even higher
- By ignoring deep-OTM options, we underestimate the true area under the curve
- Discretization in strike also leads to underestimating the true area, because $\text{call}_{t,T}(K)$ and $\text{put}_{t,T}(K)$ are both convex in $K$
- Maybe option markets were totally illiquid in November 2008?
- If so, we should expect to see wide bid-ask spread
- Is lower bound much lower if bid prices are used for options, rather than mid prices? No. And volume was high
The time series average of the lower bound is about 5%

It is volatile and right-skewed, particularly at short horizons
A volatility index, SVIX, gives a lower bound on the equity premium

SVIX and VIX

SVIX as a predictor variable

What is the probability of a 20% decline in the market?
SVIX and VIX

- By analogy with VIX, define

\[ SVIX_t^2 = \frac{2R_{f,t}}{(T - t) \cdot F_{t,T}^2} \left\{ \int_0^{F_{t,T}} \text{put}_{t,T}(K) \, dK + \int_{F_{t,T}}^{\infty} \text{call}_{t,T}(K) \, dK \right\} \]

- In this notation, equity premium \( \geq R_{f,t} \cdot SVIX_t^2 \)

- Compare SVIX with

\[ VIX_t^2 = \frac{2R_{f,t}}{T - t} \left\{ \int_0^{F_{t,T}} \frac{1}{K^2} \text{put}_{t,T}(K) \, dK + \int_{F_{t,T}}^{\infty} \frac{1}{K^2} \text{call}_{t,T}(K) \, dK \right\} \]

- These are definitions, not statements about pricing
SVIX and VIX

- VIX is similar to SVIX, but is more sensitive to left tail events
- SVIX measures risk-neutral variance, $SVIX^2 = \text{var}^*(R_T/R_{f,t})$
- VIX measures risk-neutral entropy,
  $VIX^2 = \log \mathbb{E}_t^*(R_T/R_{f,t}) - \mathbb{E}_t^* \log(R_T/R_{f,t})$
- What VIX does not measure: $VIX^2 \neq \frac{1}{T-t} \mathbb{E}_t^* \left[ \int_t^T \sigma^2 d\tau \right]$
Figure: VIX (dotted) and SVIX (solid). Jan 4, 1996–Jan 31, 2012
Figure shows 10-day moving average. $T = 1$ month
VIX minus SVIX

Figure: VIX minus SVIX. Jan 4, 1996–Jan 31, 2012
Figure shows 10-day moving average. $T = 1$ month
No conditionally lognormal model fits option prices

- If returns and the SDF are conditionally lognormal with return volatility $\sigma_{R,t}$ then we can calculate VIX and SVIX in closed form:

$$SVIX_t^2 = \frac{1}{T-t} \left( e^{\sigma_{R,t}^2(T-t)} - 1 \right)$$
$$VIX_t^2 = \sigma_{R,t}^2$$

- VIX would be lower than SVIX—which it never is in my sample
- No conditionally lognormal model is consistent with option prices
Outline

1. A volatility index, SVIX, gives a lower bound on the equity premium

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3. SVIX as a predictor variable

4. What is the probability of a 20% decline in the market?
Might the lower bound hold with equality?

- Time-series average of lower bound in recent data is around 5%
- Fama and French (2002) estimate unconditional equity premium of 3.83% (from dividend growth) or 4.78% (from earnings growth)
- Fama interviewed by Roll: “I always think of the number, the equity premium, as five per cent.”
- Estimates of $\text{cov}(M_T R_T, R_T)$ in linear factor models are statistically and economically close to zero
\( \text{cov}(M_T R_T, R_T) \) is negative and close to zero

<table>
<thead>
<tr>
<th></th>
<th>constant</th>
<th>( R_M - R_f )</th>
<th>( SMB )</th>
<th>( HML )</th>
<th>( MOM )</th>
<th>( \text{cov}(M_T R_T, R_T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample</td>
<td>1.072</td>
<td>-2.375</td>
<td>-0.648</td>
<td>-5.489</td>
<td>-5.572</td>
<td>-0.0018</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.746)</td>
<td>(1.011)</td>
<td>(1.131)</td>
<td>(1.033)</td>
<td>(0.0020)</td>
</tr>
<tr>
<td>Jan ’27–Dec ’62</td>
<td>1.071</td>
<td>-2.355</td>
<td>-0.587</td>
<td>-3.882</td>
<td>-5.552</td>
<td>-0.0021</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(1.034)</td>
<td>(1.747)</td>
<td>(2.163)</td>
<td>(1.565)</td>
<td>(0.0041)</td>
</tr>
<tr>
<td>Jan ’63–Dec ’13</td>
<td>1.092</td>
<td>-3.922</td>
<td>-2.400</td>
<td>-9.020</td>
<td>-5.152</td>
<td>-0.0020</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(1.272)</td>
<td>(1.475)</td>
<td>(1.795)</td>
<td>(1.427)</td>
<td>(0.0022)</td>
</tr>
<tr>
<td>Jan ’96–Dec ’13</td>
<td>1.047</td>
<td>-3.231</td>
<td>-2.327</td>
<td>-5.789</td>
<td>-2.548</td>
<td>-0.0017</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(1.981)</td>
<td>(2.224)</td>
<td>(2.491)</td>
<td>(1.637)</td>
<td>(0.0036)</td>
</tr>
</tbody>
</table>

Table: Estimates of coefficients in the 4-factor model, and of \( \text{cov}(M_T R_T, R_T) \).

- Test assets: market, riskless asset, 5 × 5 portfolios sorted on size and \( B/M \), 10 momentum portfolios; monthly data from Ken French’s website
- Estimate \( M \) and \( \text{cov}(M_T R_T, R_T) \) by GMM
Forecasting returns with risk-neutral variance

- We want to test the null hypothesis that $\mathbb{E}_t R_T - R_{f,t} = R_{f,t} \cdot SVIX_t^2$
- Run regressions
  \[ R_T - R_{f,t} = \alpha + \beta \times R_{f,t} \cdot SVIX_t^2 + \varepsilon_T \]
- Sample period: January 1996–January 2012
- Robust Hansen–Hodrick standard errors account for heteroskedasticity and overlapping observations
### Table: Coefficient estimates for the forecasting regression.

<table>
<thead>
<tr>
<th>horizon</th>
<th>$\hat{\alpha}$</th>
<th>s.e.</th>
<th>$\hat{\beta}$</th>
<th>s.e.</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mo</td>
<td>0.012</td>
<td>[0.064]</td>
<td>0.779</td>
<td>[1.386]</td>
<td>0.34%</td>
</tr>
<tr>
<td>2 mo</td>
<td>−0.002</td>
<td>[0.068]</td>
<td>0.993</td>
<td>[1.458]</td>
<td>0.86%</td>
</tr>
<tr>
<td>3 mo</td>
<td>−0.003</td>
<td>[0.075]</td>
<td>1.013</td>
<td>[1.631]</td>
<td>1.10%</td>
</tr>
<tr>
<td>6 mo</td>
<td>−0.056</td>
<td>[0.058]</td>
<td>2.104</td>
<td>[0.855]</td>
<td>5.72%</td>
</tr>
<tr>
<td>1 yr</td>
<td>−0.029</td>
<td>[0.093]</td>
<td>1.665</td>
<td>[1.263]</td>
<td>4.20%</td>
</tr>
</tbody>
</table>

- Cannot reject the null at any horizon
Forecasting returns with risk-neutral variance

<table>
<thead>
<tr>
<th>horizon</th>
<th>$\hat{\alpha}$</th>
<th>s.e.</th>
<th>$\hat{\beta}$</th>
<th>s.e.</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mo</td>
<td>−0.095</td>
<td>[0.061]</td>
<td>3.705</td>
<td>[1.258]</td>
<td>3.36%</td>
</tr>
<tr>
<td>2 mo</td>
<td>−0.081</td>
<td>[0.062]</td>
<td>3.279</td>
<td>[1.181]</td>
<td>4.83%</td>
</tr>
<tr>
<td>3 mo</td>
<td>−0.076</td>
<td>[0.067]</td>
<td>3.147</td>
<td>[1.258]</td>
<td>5.98%</td>
</tr>
<tr>
<td>6 mo</td>
<td>−0.043</td>
<td>[0.072]</td>
<td>2.319</td>
<td>[1.276]</td>
<td>4.94%</td>
</tr>
<tr>
<td>1 yr</td>
<td>0.045</td>
<td>[0.088]</td>
<td>0.473</td>
<td>[1.731]</td>
<td>0.27%</td>
</tr>
</tbody>
</table>

Table: Coefficient estimates excluding Aug ’08–Jul ’09

- Predictability is not driven by the crisis
Realized variance doesn’t predict reliably

<table>
<thead>
<tr>
<th>horizon</th>
<th>$\hat{\alpha}$</th>
<th>s.e.</th>
<th>$\hat{\beta}$</th>
<th>s.e.</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mo</td>
<td>0.049</td>
<td>[0.045]</td>
<td>-0.462</td>
<td>[0.784]</td>
<td>0.27%</td>
</tr>
<tr>
<td>2 mo</td>
<td>0.044</td>
<td>[0.043]</td>
<td>-0.341</td>
<td>[0.586]</td>
<td>0.26%</td>
</tr>
<tr>
<td>3 mo</td>
<td>0.035</td>
<td>[0.046]</td>
<td>-0.173</td>
<td>[0.722]</td>
<td>0.09%</td>
</tr>
<tr>
<td>6 mo</td>
<td>-0.025</td>
<td>[0.050]</td>
<td>1.182</td>
<td>[0.430]</td>
<td>5.45%</td>
</tr>
<tr>
<td>1 yr</td>
<td>-0.042</td>
<td>[0.068]</td>
<td>1.293</td>
<td>[0.499]</td>
<td>8.13%</td>
</tr>
</tbody>
</table>

Table: Regression $R_T - R_{f,t} = \alpha + \beta \times SVAR_t + \epsilon_T$, full sample.
Realized variance doesn’t predict reliably

<table>
<thead>
<tr>
<th>horizon</th>
<th>$\hat{\alpha}$</th>
<th>s.e.</th>
<th>$\hat{\beta}$</th>
<th>s.e.</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mo</td>
<td>$-0.007$</td>
<td>[0.049]</td>
<td>1.478</td>
<td>[1.125]</td>
<td>0.71%</td>
</tr>
<tr>
<td>2 mo</td>
<td>$-0.006$</td>
<td>[0.050]</td>
<td>1.429</td>
<td>[1.272]</td>
<td>1.13%</td>
</tr>
<tr>
<td>3 mo</td>
<td>$-0.004$</td>
<td>[0.049]</td>
<td>1.342</td>
<td>[1.265]</td>
<td>1.32%</td>
</tr>
<tr>
<td>6 mo</td>
<td>0.028</td>
<td>[0.049]</td>
<td>0.299</td>
<td>[1.424]</td>
<td>0.09%</td>
</tr>
<tr>
<td>1 yr</td>
<td>0.034</td>
<td>[0.064]</td>
<td>$-0.348$</td>
<td>[2.469]</td>
<td>0.15%</td>
</tr>
</tbody>
</table>

Table: Regression $R_T - R_{f,t} = \alpha + \beta \times SVAR_t + \varepsilon_T$, excluding Aug ’08–Jul ’09.


$$\mathbb{E}_t R_T = \frac{D}{P_t} + G$$

Important: coefficient on $D/P_t$ is not estimated but fixed \textit{a priori}

A good comparison for the risk-neutral variance approach
$R^2$ from Campbell and Thompson (2008)

<table>
<thead>
<tr>
<th>Dividend/price</th>
<th>-0.86%</th>
<th>0.21%</th>
<th>0.63%</th>
<th>0.88%</th>
<th>0.57%</th>
<th>0.67%</th>
<th>-1.30%</th>
<th>-0.21%</th>
<th>-0.54%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings/price</td>
<td>0.16</td>
<td>0.28</td>
<td>1.04</td>
<td>0.56</td>
<td>0.45</td>
<td>0.30</td>
<td>-0.53</td>
<td>-0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>Smooth earnings/price</td>
<td>0.56</td>
<td>0.53</td>
<td>1.33</td>
<td>0.80</td>
<td>0.48</td>
<td>0.51</td>
<td>-1.06</td>
<td>-0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>Dividend/price + growth</td>
<td>-0.15</td>
<td>0.18</td>
<td>0.78</td>
<td>0.18</td>
<td>0.18</td>
<td>0.59</td>
<td>0.11</td>
<td>0.11</td>
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<tr>
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<td>-0.06</td>
<td>0.12</td>
<td>0.73</td>
<td>-0.12</td>
<td>-0.12</td>
<td>0.33</td>
<td>0.05</td>
<td>0.05</td>
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<tr>
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<td>0.09</td>
<td>0.25</td>
<td>0.93</td>
<td>0.19</td>
<td>0.19</td>
<td>0.47</td>
<td>0.06</td>
<td>0.06</td>
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<tr>
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<td>-0.73</td>
<td>0.73</td>
<td>-0.12</td>
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<td>0.00</td>
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<tr>
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<td>0.30</td>
<td>0.45</td>
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<td>0.76</td>
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<td>0.41</td>
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<td>-0.28</td>
<td>0.74</td>
<td>0.04</td>
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### A: Monthly Returns

### B: Annual Returns

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<td>3.73</td>
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<td>4.65</td>
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<td>7.16</td>
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<td>0.47</td>
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<td>Smooth earnings/price + growth - real rate</td>
<td>5.34</td>
<td>5.37</td>
<td>3.19</td>
<td>-4.91</td>
<td>-4.84</td>
<td>7.32</td>
<td>2.36</td>
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<tr>
<td>Book-to-market + growth - real rate</td>
<td>-3.36</td>
<td>-4.22</td>
<td>11.85</td>
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<td>0.35</td>
<td>6.20</td>
<td>-6.20</td>
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</table>
Out-of-sample $R^2$

Fixed coefficients $\alpha = 0$, $\beta = 1$

<table>
<thead>
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<th>horizon</th>
<th>$R_{OS}^2$</th>
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<tbody>
<tr>
<td>1 mo</td>
<td>0.42%</td>
</tr>
<tr>
<td>2 mo</td>
<td>1.11%</td>
</tr>
<tr>
<td>3 mo</td>
<td>1.49%</td>
</tr>
<tr>
<td>6 mo</td>
<td>4.86%</td>
</tr>
<tr>
<td>1 yr</td>
<td>4.73%</td>
</tr>
</tbody>
</table>

Table: $R^2$ using $SVIX_t^2$ as predictor variable with $\alpha = 0$, $\beta = 1$
Are the $R^2$ too low?

No. Small $R^2 \rightarrow$ high Sharpe ratios

- We can use the predictor in a market-timing strategy
- On day $t$, invest $\alpha_t$ in the S&P 500 index and $1 - \alpha_t$ in cash
- Choose $\alpha_t$ proportional to 1-mo SVIX$_t^2$
- Earns a daily Sharpe ratio of 1.97% in sample
- For comparison, the daily Sharpe ratio of the index is 1.35%
- The point is not that Sharpe ratios are necessarily the right metric, but that apparently small $R^2$ can make a big difference
The value of a dollar invested

In cash (yellow), in the S&P 500 (red), and in the market-timing strategy (blue)


Ian Martin (LSE)

What is the Expected Return on the Market?
Risk-neutral variance vs. valuation ratios

Blue: earnings yield (Campbell and Thompson (2008)). Red: risk-neutral variance
Black Monday, 1987

- It is interesting to identify points at which my claims contrast most starkly with the conventional view based on valuation ratios.

- **In particular**: what happened to the equity premium during and immediately after Black Monday in 1987, which was by far the worst day in stock market history?

- Valuation ratios: it moved from about 5% to about 6%
  - Suppose $D/P = 2\%$ and then market halves in value. $D/P$ only increases to 4%

- Options: it exploded
  - Implied risk premium about twice as high as in the recent crisis
Risk-neutral variance exploded on Black Monday
1mo horizon, annualized and using VXO as a proxy for true measure
Risk-neutral variance vs. valuation ratios

- Campbell–Shiller: \( d_t - p_t = k + \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j (r_{t+1+j} - \Delta d_{t+1+j}) \)
- If dividend growth is unforecastable,
  \[
  d_t - p_t = k + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t r_{t+1+j}
  \]

- Dividend yield measures expected returns over the very long run
- Difference between SVIX\(_t^2\) and \( d_t - p_t \approx \) gap between short-run expected returns and long-run expected returns
  - Consider the late 1990s: 1-year expected returns (SVIX\(_t^2\)) were high, very long-run expected returns (\(D/P\)) were low
The term structure of the equity premium

- In bad times, high equity premia can mostly be attributed to very high **short-run** premia
What’s the equity premium right now?

Annualized 1-month equity premium \( \approx 20.77\%^2 = 4.3\% \)
Outline

1. A volatility index, SVIX, gives a lower bound on the equity premium
2. SVIX and VIX
3. SVIX as a predictor variable
4. What is the probability of a 20% decline in the market?
What is the probability of a 20% decline?

- Take the perspective of an investor with log utility whose portfolio is fully invested in the market.
- Expectations of such an investor obey the following relationship:

\[
\tilde{E}_t X_T = \frac{1}{R_{f,t}} \mathbb{E}_t^* [X_T R_T]
\]

- So if we can price a claim to \(X_T R_T\) then we know the log investor’s expectation of \(X_T\).
- Interpretation: “What a log investor would have to believe about \(X_T\) to make him or her happy to hold the market”
What is the probability of a 20% decline?

\[ \tilde{P}(R_T < \alpha) = \alpha \left[ \text{put}_{t,T}(\alpha S_t) - \frac{\text{put}_{t,T}(\alpha S_t)}{\alpha S_t} \right] \]
What is the probability of a 20% decline?

$T = 1 \text{ mo}$
What is the probability of a 20% decline?

$T = 2 \text{ mo}$
What is the probability of a 20% decline?

$T = 3$ mo
What is the probability of a 20% decline?

$T = 6$ mo
What is the probability of a 20% decline?

\( T = 1 \text{ yr} \)
What is the expected return on an individual stock? (joint work with Christian Wagner, Copenhagen Business School)

Our approach outperforms conventional predictors
Conclusions

- Have shown how to measure the equity premium in real time
- The results point to a new view of the equity premium
  - Extremely volatile, at faster-than-business-cycle frequency
  - Right-skewed, with occasional opportunities to earn exceptionally high expected excess returns in the short run
- Black Monday, October 19, 1987, provides the starkest illustration
  - $D/P$: annual equity premium moved from 4% to 5%
  - SVIX: equity premium was $\sim 8\%$ over the next one month