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Sustainability in a Risky World†

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How much consumption is sustainable, if "sustainability" requires that welfare should not be expected to decline over time? We impose a sustainability constraint on a standard consumption/portfolio choice problem. The constraint does not distort portfolio choice, but it imposes an upper bound on the sustainable consumption-wealth ratio, which must lie between the riskless interest rate and the expected return on wealth (and if risky capital evolves according to a geometric Brownian motion, it lies exactly halfway between the two). Sustainability requires an upward drift in wealth and consumption to compensate future generations for the increased risk they face. (*JEL* D63, D81, E21, G51, H43, Q01)

Do ethical considerations restrict the rate at which society consumes or its preference for the present over the future? Economists have answered this question in different ways.

One view is that preferences, social or individual, must be taken as given. If society discounts the future at a high rate, strongly preferring present consumption over future consumption, that preference must be respected; and if it leads to high consumption today, declining over time, that outcome must be accepted.

An alternative view, famously expressed by Ramsey (1928), is that at least for long-term discounting over the lifetimes of multiple generations, society should not discount the future at all because to do so is unethically to privilege the generation alive today over those yet unborn. Recently, this view has found powerful expression in *The Stern Review* (Stern 2006), which argues for aggressive action to combat climate change in large part on the basis of a social rate of time preference close to zero.

A third view is that social choices over consumption and saving should be subjected to an external "sustainability" constraint. Sustainability was defined by the World Commission on Environment and Development (Brundtland 1987, 8) as a consumption plan that "meets the needs of the present without compromising the ability of future generations to meet their own needs." Economists including Pezzey (1992); Solow (1993); Howarth (1995); Arrow et al. (2004); Asheim (2007); and

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Llavador, Roemer, and Silvestre (2015) have formalized this as a requirement that social value—the expected discounted value of utility from the present to the infinite future—should not decline over time. In the words of Solow (1993, 168):

> A sustainable path for the national economy is one that allows every future generation the option of being as well off as its predecessors. The duty imposed by sustainability is to bequeath to posterity not any particular thing … but rather to endow them with whatever it takes to achieve a standard of living at least as good as our own and to look after their next generation similarly.

The concept of sustainability as a constraint, rather than an objective, is consistent with the moral philosophy of Rawls (1999).[1] It can be understood as a prior principle that an ethical society should impose on itself because it would be agreed to by an individual who does not know into which of a sequence of generations they will be born. Since the time of birth is "morally arbitrary," it should not influence expected utility.

As Arrow et al. (2004) discuss, in a deterministic economy with a single form of capital that has a constant riskless rate of return, the sustainability constraint requires that the social rate of time preference does not exceed the exogenous riskless interest rate. When the constraint binds, the constrained rate of time preference equals the riskless interest rate, implying that society consumes the riskless return generated by its wealth and leaves the capital stock unchanged. Wealth, consumption, the utility and marginal utility of current consumption, and social value are then all constant over time. Sustainable consumption is only feasible when the riskless interest rate is positive. When it is, the sustainability constraint responds to the availability of an investment opportunity with a positive rate of return by allowing a greater rate of time preference and higher current consumption than would be required by Ramsey.[2]

In this paper, we extend the concept of sustainability to allow for risk. In a risky economy, with an uncertain return on capital, it is not possible to guarantee that social value remains constant over time. Instead, we impose a weaker sustainability constraint that social value—expected utility, which is itself a random variable because it is a function of current wealth—should not be *expected* to decline over time. This constraint, which has also been suggested though not formally analyzed by Howarth (1995), acknowledges the reality that social welfare is subject to random shocks, some of which cannot be controlled. In the deterministic case, our constraint reduces to the one considered by Arrow et al. (2004).

[1] Rawls (1999, 27) writes: "In justice as fairness, on the other hand, persons accept in advance a principle of equal liberty and they do this without knowledge of their more particular ends. They implicitly agree, therefore, to conform their conceptions of the good to what the principles of justice require, or at least not to press claims which directly violate them … The principles of right, and so of justice, put limits on which satisfactions have value; they impose restrictions on what are reasonable conceptions of one's good. In drawing up plans and deciding on aspirations men are to take these constraints into account."

[2] By adjusting the rate of time preference to available rates of return, the sustainability constraint responds to a critique of Ramsey made by Koopmans (1960, 1967). Koopmans (1967, 9) summarized his argument by writing: "The moral is, in my opinion, that one cannot adopt ethical principles without regard to … the anticipated technological possibilities. Any proposed optimality criterion needs to be subjected to a mathematical screening, to determine whether it does indeed bear on the problem at hand, under the circumstances assumed. More specifically, too much weight given to generations far in the future turns out to be self-defeating. It does nobody any good. How much weight is too much has to be determined in each case."

We study a continuous-time model with two forms of capital, one safe and one risky, so that society faces an asset-allocation problem as well as a consumption-savings decision. While we use the terminology of financial economics—referring, for example, to assets, wealth, consumption, and saving—we emphasize two ways in which these financial concepts should be interpreted broadly.

First, the assets we discuss are forms of capital that can be accumulated under constant returns to scale. Our model is a two-capital extension of a standard endogenous growth or *Ak* model such as Romer (1986) or more recently Barro (2023). We also consider a version of the model in which only a single form of risky capital is physically available, so risk is inescapable for society. In this case, riskless capital is in zero net supply and the riskless interest rate adjusts to ensure zero net demand for riskless investment. This version of the model is standard in the endogenous growth literature.

Second, the financial concepts we use should be interpreted to include not only the standard objects measured in national income accounts but also other conditions relevant for human well-being and productivity, including particularly environmental conditions. The risky asset, for example, could represent Earth itself, while consumption should be understood as a catchall for, among other things, the rate at which society consumes, rather than sustains, the biosphere. In these terms, the consumption-savings decision we consider is intended as a modeling metaphor that encompasses questions of resource depletion, environmental degradation, and so on. Having said that, we should be clear that our framework does not address certain important aspects of environmental sustainability, notably issues related to the economics of exhaustible resources (as studied in a deterministic setting by Dasgupta and Heal 1974; Solow 1974; and Hartwick 1977).

Returning to the financial terminology we use in the remainder of the paper, we assume that the returns on the risky asset are i.i.d., with a flexible specification that is driven both by a Brownian motion and by a Poisson jump process. The assumption of i.i.d. returns is consistent with an *Ak* model of capital accumulation. It implies that there is a unique consumption-wealth ratio at which the sustainability constraint is binding.[3] By allowing for jumps, we accommodate the literature that emphasizes the importance of rare disasters, or fat tails more generally, in the macroeconomy (Rietz 1988; Barro 2006; Weitzman 2007a; Backus, Chernov, and Martin 2011). We assume that society has a time-separable power utility function defined over aggregate consumption, and we impose the sustainability constraint on this.

Our main results are as follows.

First, when the sustainability constraint binds, the sustainable consumption-wealth ratio equals the certainty-equivalent return: the hypothetical riskless rate of return that would deliver the same expected utility to society as the actual menu of available assets. This depends on the risk aversion but not on the rate of time preference of an unconstrained representative individual in our economy.[4]

[3] In a model with diminishing returns to capital, by contrast, any constant savings rate can be sustainable although different savings rates imply different levels of steady-state consumption.

[4] This is consistent with the view of Rawls (1999, 259), who writes: "Of course, a present or near future advantage may be counted more heavily on account of its greater certainty or probability … But none of these things justifies our preferring a lesser present to a greater future good simply because of its nearer temporal position … The just savings principle for society must not, then, be affected by pure time preference, since as before the different temporal position of persons and generations does not in itself justify treating them differently."

Second, the sustainability constraint does not distort portfolio choice, which is always the same whether or not the constraint binds. In the absence of jumps, the portfolio rule is the classic one derived by Merton (1969, 1971).

Third, the sustainable consumption-wealth ratio exceeds the riskless interest rate but is less than the expected return on optimally invested wealth. In the absence of jumps, the sustainable consumption-wealth ratio lies exactly at the midpoint between these two rates of return. In the presence of jumps of deterministic size, the sustainable consumption-wealth ratio is higher than the midpoint when the jumps are downward—that is, when jumps represent bad news.

Fourth, sustainability does not require that consumption and wealth are expected to remain constant over time. In fact, consumption and wealth have positive drift in the constrained equilibrium. Intuitively, this is because risky investment causes the distribution of consumption and wealth to spread out over time, imposing more risk on later generations. To prevent risk from reducing the welfare of later generations relative to earlier ones, later generations must be compensated by higher average levels of consumption and wealth.

Fifth, the sustainable consumption-wealth ratio is higher (by a factor of risk aversion divided by risk aversion minus one) than the consumption-wealth ratio required by the Ramsey zero-time-preference rule. The difference between the two is small for very high levels of risk aversion but substantial at levels of risk aversion normally considered plausible.

The sustainable rate of time preference is not the same as the discount rate that society should apply to riskless investment projects. That discount rate is given by the riskless interest rate in the sustainable equilibrium, which is lower than the sustainable rate of time preference when the economy is exposed to risk. As a salient example, investments to mitigate climate change should be discounted at low rates if the investments are riskless and the sustainable equilibrium has a low riskless interest rate. They should be discounted at even lower rates if climate investments pay off in bad states of the world—that is, if they are analogous to insurance policies—an important point emphasized by Weitzman (2009) and Gollier (2021).

Our main analysis defines utility over aggregate consumption, in effect treating each generation equally regardless of population. This is only equivalent to treating each individual equally if population is constant over time. Population growth creates notoriously difficult issues for intertemporal ethics (Parfit 1984; Dasgupta 2001), particularly when population is itself a choice variable. However, we show in the Supplemental Appendix that if population growth is exogenous and constant, then we can modify the sustainability constraint to prevent the expected utility of an individual from declining over time. This is equivalent to subtracting the rate of population growth from all rates of return and therefore from the sustainable consumption-wealth ratio and the sustainable rate of time preference.

The literature on discounting and sustainability is enormous, and we do not attempt a complete review here. Dasgupta (2008, 2021) and Zeckhauser and Viscusi (2008) provide recent surveys. Within the literature on climate change, there has been debate between those who argue for a very low social rate of time preference, such as Cline (1992) and Stern (2006, 2016), and those who use a higher rate of time preference, such as Nordhaus (1994). Our analysis implies that a substantial rate of time preference can be consistent with the ethical criterion of sustainability in a risky world.

The organization of the paper is as follows. Section I sets up our unconstrained continuous-time model with portfolio choice over a safe and a risky asset. Section II introduces the sustainability constraint and solves the constrained model. Section III compares the sustainable consumption-wealth ratio with the consumption-wealth ratio implied by the Ramsey rule of a zero social rate of time preference. Section IV concludes. When not included in the main text, proofs of results are in the Supplemental Appendix.

I. Unconstrained Consumption and Portfolio Choice

We consider a representative agent faced with two assets or technologies, one riskless and one risky. The agent chooses society's aggregate consumption, C_t, and risky portfolio share, α, to maximize the expected discounted value of a power utility function,

$$U_0 = E_0 \int_0^{\infty} e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma} dt. \tag{1}$$

We take as given this representation of utility derived from aggregate consumption. It can be derived from individual utility of individual consumption under assumptions that permit aggregation across consumers. For example, we could assume that individuals have a constant probability of death following Blanchard (1985), that they have power utility defined over their own consumption, that they are unable to annuitize their wealth, and that the wealth of those who die is allocated to an equal number of newly born individuals. In this case, utility at each point of time is both the welfare of the generation born at that moment and the welfare of all agents alive at that time. This microfoundation for equation (1) assumes a constant population; we discuss the impact of population growth further in the Supplemental Appendix.

We assume that the rate of time preference $\rho > 0$. If individuals have a constant probability of death and do not care about their descendants, then, as Blanchard (1985) shows, ρ is the sum of the pure individual rate of time preference and the probability of death. In a more general model with intergenerational altruism, ρ will also be affected by (and declining in) the degree of altruism.

We assume that the coefficient of relative risk aversion $\gamma > 1$. In the Supplemental Appendix, we show that our main results, Results 1 and 2, extend in the expected way to the log utility case, $\gamma = 1$. It would also be easy to handle the case $0 < \gamma < 1$, but as this case requires occasional sign flips in our logic below, we rule it out to streamline the exposition.

The riskless asset has gross return R_f. It will generally be convenient to think in terms of the log riskless rate, $r_f = \log R_f$.

We require assumptions about the return on wealth that is invested rather than consumed. It will be convenient to work in continuous time for tractability. As sustainability is inherently a long-run issue, we abstract away from high-frequency variation in mean, volatility, and so on by modeling the risky return as i.i.d. over time (i.e., as a Lévy process), as in Martin (2013). We specialize slightly, within this class of processes, by specifying the risky asset's value using a combination of a Brownian motion and a Poisson process. This is a more general assumption than it may appear: As we allow for an arbitrary jump size distribution, the only cases we are ruling out, within the family of Lévy processes, are those in which infinitely many jumps can occur in a finite time interval.

We assume that the risky asset has constant expected excess return $\mu = \log(ER/R_f) > 0$, has Brownian volatility σ, and is exposed to jumps arriving according to a Poisson counting process N_t with constant arrival rate ω, where μ, σ, and ω are each constant. We write W_t for wealth at time t and $\theta = C_t/W_t$ for the consumption-wealth ratio. Under our assumptions, θ is also constant. Thus,

$$(2) \qquad \frac{dC_t}{C_t} = \frac{dW_t}{W_t} = \Big[r_f + \alpha\underbrace{(\mu + \omega EL)}_{\hat{\mu}} - \theta\Big]dt + \alpha\sigma\, dZ_t - \alpha L\, dN_t.$$

(In what follows, we will often suppress time subscripts on the random variables C_t, W_t, Z_t, and N_t to streamline the notation.)

Jumps are captured by the third term on the right-hand side of equation (2). When a jump occurs, an agent who is fully invested in the risky asset loses a fraction L of her capital. We assume that L is a random variable that is drawn in i.i.d. fashion each time a jump occurs. We also assume (with one eye on an equilibrium we study below, in which $\alpha = 1$) that L is strictly less than one, so that someone who invests fully in the risky asset is not bankrupted. We can allow L to take negative values; these represent good news for the risky asset. We write $\hat{\mu} = \mu + \omega EL$ to denote the expected excess return in the absence of jumps.

It follows that

$$(3) \qquad EC_t^{1-\gamma} = W_0^{1-\gamma}\theta^{1-\gamma}\exp\Big\{(1-\gamma)\Big(r_f + \alpha\hat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2 - \theta\Big)t + \omega E\big[(1-\alpha L)^{1-\gamma} - 1\big]t\Big\}.$$

This is a standard calculation, but we provide details in the Supplemental Appendix for convenience. Hence, the objective function (1) can be evaluated explicitly as

$$(4) \qquad U_0 = \frac{W_0^{1-\gamma}}{1-\gamma}\,\frac{\theta^{1-\gamma}}{\rho - (1-\gamma)\Big(r_f + \alpha\hat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2 - \theta\Big) - \omega E\big[(1-\alpha L)^{1-\gamma} - 1\big]}.$$

The unconstrained optimal investment and consumption choices are identified by maximizing (4) with respect to α and θ. Maximizing with respect to θ, we find that the unconstrained optimal consumption-wealth ratio, θ_{unc}, is

$$(5) \qquad \theta_{unc} = \frac{\rho + (\gamma - 1)\Big(r_f + \alpha\hat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2\Big) - \omega E\big[(1-\alpha L)^{1-\gamma} - 1\big]}{\gamma}.$$

We assume that θ_{unc} is positive. This condition implies that the denominator of (4) is positive. If it does not hold, then the integral in the definition of expected utility does not converge and expected utility is not well defined.

The optimal risky portfolio share is defined implicitly by

$$\hat{\mu} - \alpha\gamma\sigma^2 = \omega E\left[L(1-\alpha L)^{-\gamma}\right]. \tag{6}$$

In the absence of jumps, where the risky asset follows a pure Brownian motion, this simplifies to the classic Merton formula,

$$\alpha = \frac{\mu}{\gamma\sigma^2}. \tag{7}$$

The Certainty-Equivalent Return.—These equations can be simplified by introducing the concept of the certainty-equivalent return, r_{CE}, defined as the hypothetical riskless return that would give the investor the same expected utility as the actual menu of assets, conditional on the investor's choice of α and θ. Equation (4) shows that

$$r_{CE} = r_f + \alpha\hat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2 + \frac{\omega E\left[(1-\alpha L)^{1-\gamma} - 1\right]}{1-\gamma}, \tag{8}$$

which depends on risk aversion γ (both directly and indirectly through the effect of γ on the risky portfolio share α) but not on the rate of time preference ρ.[5]

Using this notation, the objective function in equation (4) can be rewritten as

$$U_0 = \frac{W_0^{1-\gamma}}{1-\gamma}\,\frac{\theta^{1-\gamma}}{\rho - (1-\gamma)(r_{CE} - \theta)} \tag{9}$$

and the solution for the unconstrained optimal consumption-wealth ratio can be simplified to

$$\theta_{unc} = \frac{1}{\gamma}\rho + \left(1 - \frac{1}{\gamma}\right) r_{CE}. \tag{10}$$

The unconstrained optimal consumption-wealth ratio is a weighted average of the rate of time preference ρ and the certainty-equivalent return r_{CE}, where the weights are risk tolerance and one minus risk tolerance.

II. A Sustainability Constraint

We formalize the notion of sustainability by imagining representatives of each generation agreeing on a time-invariant consumption-investment policy that respects a constraint that welfare should not be expected to decline over time. In doing so, we

[5] Although α, which appears in equation (8), is an equilibrium object, Result 2 shows that it does not depend on ρ.

are committing to a cardinal measure of utility, as in Harsanyi (1955), that permits welfare comparisons to be made across generations.

This implies that at time t, the representative agent will solve the consumption-investment problem studied above, subject to the extra constraint that the drift of expected utility should be nonnegative. If the representative agent is thought of as the currently living generation in an infinite dynasty, then the constraint is appropriate if she does not want her descendants to expect a lower quality of life than she does.[6]

RESULT 1: *The largest possible sustainable consumption-wealth ratio*, θ_{con}, *satisfies*

$$\theta_{con} = r_{CE}, \tag{11}$$

where r_{CE} *is defined in equation* (8). *Unlike the unconstrained consumption-wealth ratio, the sustainable consumption-wealth ratio is independent of* ρ *if the constraint binds.*

PROOF:

Equation (4) shows that expected utility at time t, U_t, is proportional to $W_t^{1-\gamma}/(1-\gamma)$. (Expected utility is itself a random variable, because it is a function of current wealth.) To understand how expected utility evolves over time, it is convenient to multiply by $1-\gamma$—which is negative under our maintained assumption that $\gamma > 1$—and work with a rescaled variable $X_t = W_t^{1-\gamma}$. By Itô's lemma, this follows the process

$$\begin{aligned} \frac{dX}{X} = {} & (1-\gamma)\left(r_f + \alpha\hat{\mu} - \theta - \frac{1}{2}\gamma\alpha^2\sigma^2\right)dt + (1-\gamma)\alpha\sigma\, dZ \\ & + \left[(1-\alpha L)^{1-\gamma} - 1\right]dN. \end{aligned} \tag{12}$$

This is a standard calculation, but we provide further detail in the Supplemental Appendix. The drift of dX/X is therefore

$$\begin{aligned} & (1-\gamma)\left(r_f + \alpha\hat{\mu} - \theta - \frac{1}{2}\gamma\alpha^2\sigma^2\right) + \omega E\left[(1-\alpha L)^{1-\gamma} - 1\right] \\ & \qquad = (1-\gamma)(r_{CE} - \theta), \end{aligned} \tag{13}$$

where we have used the fact that $E\,dN = \omega\,dt$. Sustainability requires that the drift of X is nonpositive; that is, $\theta \leq r_{CE}$. Equation (11) follows. ■

[6] One might imagine imposing other types of constraint. We could, for example, allow for a type of risk aversion over future expected utility by requiring that some concave function of future expected utility should have nondecreasing expectation. This is analytically intractable in the constant relative risk aversion (power) case, however, and indeed it is infeasible in the limit as risk aversion over future expected utility approaches infinity, as it would require expected utility—and hence wealth itself—to be nondecreasing, which is not possible unless society can entirely eliminate risk.

If the consumption-wealth ratio, θ, is larger than θ_{con}, then X has positive drift, and hence expected utility has negative drift: The optimal consumption-investment decision induces declining expected utility over time, on average.

The optimal sustainable consumption-wealth ratio, θ_{sus}, is given by whichever of θ_{con} and θ_{unc} is smaller. If the unconstrained case features a lower consumption-wealth ratio, then it certainly satisfies the constraint and delivers higher utility. If not, the unconstrained case does not satisfy the constraint, so that θ_{con} is the best we can do. Thus,

$$\theta_{sus} = \min\{\theta_{unc}, \theta_{con}\}. \tag{14}$$

Equivalently, θ_{con} is the highest possible sustainable consumption-wealth ratio.

It follows from equations (10) and (11) that

$$\theta_{unc} = \frac{1}{\gamma}\rho + \left(1 - \frac{1}{\gamma}\right)\theta_{con}. \tag{15}$$

Equations (14) and (15) have several interesting implications. First, the sustainability constraint binds if and only if $\rho > \theta_{con} = r_{CE}$ (or, equivalently, if and only if $\rho > \theta_{unc}$). Related to this, we can show that in the absence of a sustainability constraint,

$$E_0 X_t = X_0 e^{(\rho - \theta_{unc})t}. \tag{16}$$

The term $\rho - \theta_{unc}$ in equation (16) equals $(1 - 1/\gamma)(\rho - r_{CE})$. If impatience is sufficiently high that $\rho > \theta_{unc}$, or equivalently $\rho > r_{CE}$, then $X_t = W_t^{1-\gamma}$ is expected to grow without limit in an unconstrained equilibrium, so expected utility is expected to decline without limit.[7] The sustainability constraint binds in this circumstance.

Second, equation (15) shows that the moderating influence of ρ makes θ_{unc} less sensitive than θ_{con} to changes in other parameters of the model, holding ρ fixed.

Third, equation (15) implies that the behavior of an extremely risk-averse individual is little affected by the presence or absence of a sustainability constraint, as $\theta_{unc} \approx \theta_{con}$ if γ is large. This reflects the fact that highly concave utility leads an agent to choose a flat consumption path that is close to sustainable, regardless of the level of ρ.

Fourth, equations (14) and (15) show that θ_{sus} and θ_{unc} can easily be calculated from knowledge of θ_{con}, so we can focus our analysis on the determinants of θ_{con}.

Finally, we can use this analysis to analyze how and why the social discount rate used in, say, *The Stern Review* (2006) might differ from the quantity ρ that enters an individual's utility function. *The Stern Review* emphasizes the importance of the discount rate in making welfare comparisons across generations separated by long tracts of time, eventually settling on a value of 0.1 percent. Weitzman's (2007b, 707–09) review of *The Review* describes this as "a decidedly minority paternalistic view" and worries that "for most economists, a major problem … is that people are not observed to behave as if they are operating with [the discount rate] $\delta \approx 0$."

[7] Expected wealth will decline toward zero if ρ is sufficiently large, but if ρ is sufficiently close to θ_{con}, then wealth has positive drift despite the negative drift in expected utility.

In our setting, individuals unconstrained by sustainability will use the discount rate $\rho > 0$ in calculations. Might sustainability justify a lower social discount rate suitable for use in a *Stern Review*–like exercise?[8]

To answer this question, define the social discount rate $\hat{\rho}$ via the equation $\theta_{unc}(\hat{\rho}) = \theta_{sus}$. With this definition, $\hat{\rho}$ is the hypothetical discount rate that should be used by a social planner who wants to impose sustainability. Equations (14) and (15) imply that

$$\hat{\rho} = \min\{\rho, \theta_{con}\} = \min\{\rho, r_{CE}\}. \tag{17}$$

If the sustainable consumption-wealth ratio is lower than the unconstrained time discount rate ρ, this represents an alternative justification for using a social discount rate lower than an individual's discount rate, ρ; nonetheless, it suggests a higher social discount rate than does the Ramsey rule, which sets $\hat{\rho}$ equal to zero. We return to the comparison of the sustainable and Ramsey rules in Section III.

We now turn to the implications of sustainability for the portfolio choice decision. The next result shows that our sustainability constraint does not affect the optimal investment choice, so that its impact is felt purely through the consumption-savings decision, as analyzed above.

Intuitively, it is not optimal to distort portfolio choice because doing so affects expected utility in the same way in all periods. Distorting the portfolio choice decision away from the unconstrained optimum therefore does not relax the constraint, nor (by definition) does it directly benefit the objective function. By contrast, in papers such as Dybvig (1995) or Campbell and Sigalov (2022) that feature constraints on consumption as opposed to welfare, it may be optimal to distort portfolio choice relative to the unconstrained optimum in order to relax the constraint.

RESULT 2: *The optimal investment choice is unaffected by the presence of the sustainability constraint. The optimal risky asset allocation*, α, *continues to satisfy*

$$\hat{\mu} - \alpha\gamma\sigma^2 = \omega E\left[L(1 - \alpha L)^{-\gamma}\right], \tag{18}$$

as in the unconstrained case (*6*).

PROOF:

If the constraint binds, we can use it to eliminate θ from the objective function (4), giving

$$U_{con,0} = \frac{W_0^{1-\gamma}}{1-\gamma}\,\frac{\left(r_f + \alpha\hat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2 + \omega\frac{E\left[(1-\alpha L)^{1-\gamma} - 1\right]}{1-\gamma}\right)^{1-\gamma}}{\rho}. \tag{19}$$

Maximizing equation (19) with respect to α, we find the first-order condition (18). ■

As a corollary of Results 1 and 2, the sustainable strategy is Pareto efficient because it is identical to the unconstrained-optimal strategy for some choice of ρ.

[8] Weitzman (2007b) points out that model uncertainty provides another justification.

A. *Bounding the Sustainable Consumption-Wealth Ratio*

If there are no jumps, as is commonly assumed in the literature, we can use the Merton formula (equation (7)) to eliminate α from equation (8), giving

$$\theta_{con} = r_f + \frac{1}{2}\frac{\mu^2}{\gamma\sigma^2}. \tag{20}$$

Equation (20) can be rewritten, again using condition (7), as

$$\theta_{con} = \frac{1}{2} \times r_f + \frac{1}{2} \times \underbrace{\left[(1-\alpha) r_f + \alpha\left(r_f + \mu\right)\right]}_{\text{expected return on optimally invested wealth}}. \tag{21}$$

This shows that—in the absence of jumps—the certainty-equivalent return and hence the constrained consumption-wealth ratio lie halfway between the riskless return and the expected return on wealth. With plausible parameter values, the maximum sustainable consumption-wealth ratio θ_{con} is therefore considerably higher than the riskless rate. If, say, the riskless rate is $r_f = 2\%$, the expected return on risky capital is $\mu + r_f = 10\%$, its volatility is $\sigma = 20\%$, and risk aversion is 2, then the constrained consumption-wealth ratio is 6 percent.

More generally, we have the following result.

RESULT 3: *We have the bounds*

$$r_f + \frac{1}{2}\gamma\alpha^2\sigma^2 \leq \theta_{con} \leq r_f + \alpha\mu - \frac{1}{2}\gamma\alpha^2\sigma^2. \tag{22}$$

In particular, the sustainable consumption-wealth ratio lies between the riskless rate (r_f) *and the expected return on wealth* $(r_f + \alpha\mu)$.

If there are no jumps, the sustainable consumption-wealth ratio is exactly halfway between the riskless rate and the expected return on wealth.

With jumps of deterministic size L, *and assuming* $\alpha > 0$, *the sustainable consumption-wealth ratio is higher than the halfway point if jumps represent bad news*, $L > 0$, *and lower if jumps represent good news*, $L < 0$.

Result 3 shows that the constrained consumption-wealth ratio must lie between the riskless return and the expected return on wealth, with the precise location determined by the arrival rate and distribution of jump sizes. Importantly, the upper and lower bounds on the sustainable consumption-wealth ratio do not require knowledge of the frequency or size distribution of jumps, though jumps are captured indirectly via their influence on the riskless rate, expected return on risky capital, and the optimal allocation to risky capital, α. In an equilibrium in which risk is inescapable, α must equal one: In this case, only the riskless rate and average return on risky capital must be estimated. These quantities are relatively easy to estimate even in economies with jumps, as Barro (2006) points out.

Result 3 also suggests, as a rule of thumb, that if we think of rare events as representing bad news rather than good news, then the maximum sustainable consumption-wealth ratio should be *higher* than the halfway point between the riskless rate and the expected return on wealth, so that the halfway point itself is sustainable.[9]

B. *Sustainable Growth*

We have seen that sustainability places an upper limit on the rate at which wealth is consumed. Put differently, the sustainability constraint requires that there is enough saving that the expected growth rate of the economy[10] is no less than some lower limit.

RESULT 4: *Sustainability requires that consumption and wealth have positive drift in levels—that is,* $g = E\frac{dC}{C} = E\frac{dW}{W}$ *is positive.*

If $\gamma > 1$, *sustainability even requires that consumption and wealth have positive drift in logs—that is,* $g_{log} = E\,d\log C = E\,d\log W$ *is positive.*

If equilibrium requires that $\alpha = 1$*—that is, if risk is inescapable—then*

$$g = \frac{1}{2}\gamma\sigma^2 + \frac{\omega}{\gamma - 1}E\left[(1 - L)^{1-\gamma} - 1 - (\gamma - 1)L\right] \tag{23}$$

and

$$g_{log} = \frac{1}{2}(\gamma - 1)\sigma^2 + \frac{\omega}{\gamma - 1}E\left[(1 - L)^{1-\gamma} - 1 + (\gamma - 1)\log(1 - L)\right] \tag{24}$$

when the sustainability constraint binds. Lower growth rates are not sustainable. All four terms on the right-hand sides of equations (23) *and* (24) *are positive if* $\gamma > 1$.

The intuition for this result is straightforward. Growth, g, reflects the portion of the return on invested wealth, $r_f + \alpha\mu$, that is not consumed. Thus, when the sustainability constraint binds,

$$g = r_f + \alpha\mu - \theta_{con} = r_f + \alpha\mu - r_{CE}. \tag{25}$$

This difference is always positive and equals half the risk premium on invested wealth in the case where there are no jumps.

When the sustainability constraint binds, expected utility in the distant future is expected to be the same as expected utility today. But because risk cumulates over time, later generations, who are exposed to more risk, must be compensated with higher average levels of wealth and hence consumption if their expected utility is to be held constant.[11]

[9] The proof of Result 3 provides a general condition for arbitrary jump size distributions under which the sustainable consumption-wealth ratio exceeds the halfway point.

[10] As measured by the growth rate either of wealth or of consumption. The two are equivalent because the consumption-wealth ratio is constant in our setting.

[11] Result 4 illustrates the distinction between our sustainability constraint and the arithmetic and geometric constraints considered by Campbell and Sigalov (2022), which impose zero drift in wealth or in log wealth, respectively. Moreover, the Campbell and Sigalov constraints generally distort portfolio choice, whereas our sustainability constraint does not.

TABLE 1—NUMERICAL EXAMPLES

	r_f	μ	σ	ω	L	α	θ_{con}	g	g_{log}	Halfway
No disasters	0.045	0.045	0.15	0	0	1	0.0675	0.0225	0.0113	0.0675
Avoidable disasters	0.045	0.045	0.15	0.02	0.5	0.698	0.0617	0.0147	0.0076	0.0607
Unavoidable disasters	0.015	0.075	0.15	0.02	0.5	1	0.0575	0.0325	0.0174	0.0525

A counterintuitive implication of Result 4 is that as one looks into the far distant future, expected utility is overwhelmingly likely to be higher than its current value: In an economy that grows on average, expected utility at time *t* approaches its upper bound (of zero) almost surely as *t* approaches infinity. Nonetheless, expected utility is constant over time *in expectation* because (echoing the result of Martin 2012) there is a sting in the tail: There are a vanishingly small number of extreme paths in which expected utility in the future is arbitrarily low.

C. *Numerical Examples*

Table 1 presents some numerical examples to illustrate the properties we have discussed.

The first line of the table considers a case with no jumps. The riskless return and risk premium are each equal to 4.5 percent, so that the risky asset's expected return is 9 percent and the risky asset's volatility is 15 percent. With risk aversion $\gamma = 2$ (as is assumed throughout the table), Result 2 implies that it is optimal to put all wealth into the risky asset, $\alpha = 1$. The maximum sustainable consumption-wealth ratio is $\theta_{con} = 6.75\%$, by Result 1: This is exactly halfway between the riskless return and the expected return on wealth, as shown in equation (21) and reported in the final column of the table. The minimum expected growth rate that is consistent with sustainability is 2.25 percent, by Result 4. These numbers add up to the expected return on wealth, 9 percent, as required by equation (25).

The second line of Table 1 holds the riskless return and risk premium constant, but adds disasters that destroy 50 percent of wealth on arrival, and which arrive at a rate of 2 percent per year. These make the risky asset less attractive, so the optimal response is to reduce risk exposure, setting $\alpha = 0.698$. The maximum sustainable consumption-wealth ratio declines to 6.17 percent, and this is associated with the minimum sustainable growth rate $g = 1.47\%$. Again, the sustainable consumption wealth ratio and growth rate add up to the expected return on wealth, $\theta_{con} + g = r_f + \alpha\mu$.

In the economy described in the second line, the scale of two technologies—one risky, the other riskless—can be adjusted arbitrarily. But we might imagine an equilibrium in which society cannot eliminate risk no matter how much it might wish to do so. If we impose this requirement, holding the total return on the risky technology constant at 9 percent, as in the first two lines, then the riskless rate must adjust endogenously so that society is content to bear the inescapable risk of its invested wealth—that is, so that $\alpha = 1$.

The third line of Table 1 considers this case. The interest rate declines to 1.5 percent, so that the risk premium on the risky asset is 7.5 percent. At these levels, it is indeed optimal to invest fully in the risky technology. The maximum

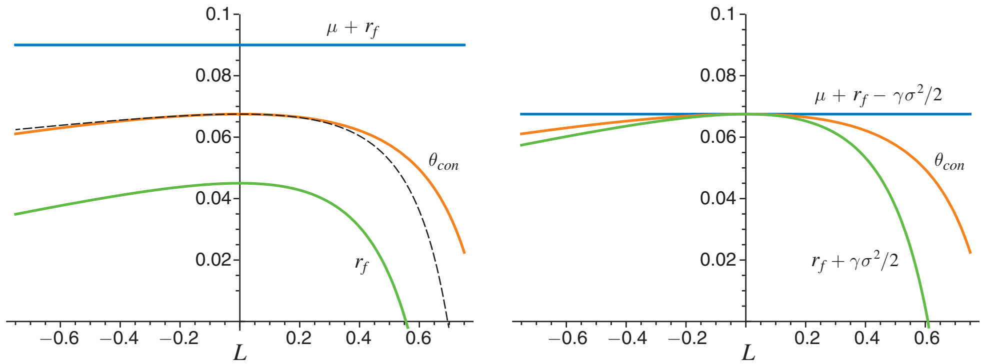

FIGURE 1. θ_{con} AND THE UPPER AND LOWER BOUNDS FOR VARIOUS DETERMINISTIC JUMP SIZES L, WITH $\gamma = 2, \sigma = 0.15, \omega = 0.02, \mu + r_f = 0.09$

Notes: Jumps are bad news if L is positive and good news if L is negative. The dashed line on the left panel indicates the midpoint of the riskless rate and expected return on the risky technology.

sustainable consumption-wealth ratio of 5.75 percent lies slightly above the halfway point between the riskless return and the expected return on the risky technology, consistent with Result 3.

Figure 1 expands on this last calibration to show how the sustainable consumption-wealth ratio varies with the severity of jumps. As in the third line of Table 1, we set risk aversion $\gamma = 2$ and assume an expected return on risky capital of 9 percent, Brownian volatility $\sigma = 15\%$, a jump probability $\omega = 2\%$, and jumps of deterministic size L. The horizontal axis shows different values for L, where positive values correspond to negative jumps (losses) in wealth and negative values correspond to positive jumps in wealth. In the left panel, the constrained consumption-wealth ratio θ_{con} is plotted along with the expected risky asset return $r_f + \mu$ (constant at 9 percent) and the endogenously determined risk-free interest rate r_f. The maximum sustainable consumption-wealth ratio is halfway between the two returns in the Brownian case; it is closer to the risky asset return in the bad jump region where $L > 0$ and closer to the risk-free interest rate in the good jump region where $L < 0$. In the right panel, the maximum sustainable consumption-wealth ratio is plotted along with the upper and lower bounds from Result 3. The bounds are tight in the Brownian case ($L = 0$) and widen out as the absolute jump size increases.

III. Sustainability and the Ramsey Rule

We have interpreted sustainability as requiring that expected utility should not be allowed to decline over time. One can imagine representatives of each generation attempting to agree (at time 0, behind the veil of ignorance) on a savings policy that gives each generation the same expected utility. Put another way, these representatives maximize the expected utility of the worst-off generation, following the "difference principle" of Rawls (1999). Equation (4) shows that this is equivalent to ensuring that $E_0 W_t^{1-\gamma}/(1-\gamma)$ (or equivalently, as the consumption-wealth ratio is constant, $E_0 C_t^{1-\gamma}/(1-\gamma)$) is constant across t.

As an alternative rule, one might also imagine considering the possibility that the representatives aim to maximize *average* utility across generations. This is equivalent to maximizing

$$\int_{t=0}^{\infty} E_0 \frac{C_t^{1-\gamma}}{1-\gamma} dt = E_0 \int_{t=0}^{\infty} \frac{C_t^{1-\gamma}}{1-\gamma} dt. \tag{26}$$

This is the problem faced by an unconstrained agent with pure time preference rate $\rho = 0$. It leads to the savings rule proposed by Ramsey (1928), who argued on ethical grounds that the rate of pure time preference should be zero. Setting $\rho = 0$ in equation (10), we arrive at the Ramsey consumption-wealth ratio

$$\theta_{Ramsey} = \left(1 - \frac{1}{\gamma}\right) r_{CE}. \tag{27}$$

The next result follows by comparing (27) with (11).

RESULT 5: *There is a simple relationship between the sustainable consumption-wealth ratio and the Ramsey consumption-wealth ratio:*

$$\theta_{Ramsey} = \left(1 - \frac{1}{\gamma}\right) \theta_{con}. \tag{28}$$

The two rules are similar at high levels of risk aversion, but the Ramsey rule is substantially more conservative at plausible values of γ. The Ramsey rule dictates 10 percent less consumption than our sustainable rule if $\gamma = 10$; 25 percent less consumption if $\gamma = 4$; and 50 percent less consumption if $\gamma = 2$. In the log utility case $\gamma = 1$, the Ramsey rule cannot be implemented at all, as it sets the consumption-wealth ratio equal to the Ramsey rate of time preference—that is, to zero.

IV. Conclusion

We have argued, in the spirit of Koopmans (1960, 1967), that the implication of an ethical criterion—sustainability—for social discounting and consumption decisions depends on the production technology available to society. Specifically, in a risky world with a binding sustainability constraint, the sustainable social rate of time preference and consumption-wealth ratio, which equal one another, are not equal to either the riskless interest rate or the risky return on invested wealth, but lie in between these two. In the special case where invested wealth has only Brownian risk and no jump risk, the sustainable social rate of time preference is the equal-weighted average of the riskless interest rate and the risky return.

We have made this point in the context of a model in which the parameters governing the distribution of returns are known. We have therefore ignored parameter uncertainty, a phenomenon emphasized by Weitzman (2007a, 2007b, 2009). We have also ignored the possibility that returns may not be i.i.d., because expected returns or risks change over time. Models with non-i.i.d. returns in general imply time-varying consumption growth and a term structure of discount rates. When consumption growth is persistent, this term structure is generally downward sloping for safe investments and upward sloping for risky ones as in the long-run risk model of Bansal and Yaron (2004). Gollier (2002) emphasizes the potential importance of a downward-sloping term structure of discount rates for social discounting. Our i.i.d. model has discount rates that are invariant to the horizon of an investment.

Although we have emphasized the sustainable social rate of time preference, we conclude by noting that this is not the same as the appropriate social discount rate that should be applied to an investment project. That discount rate depends on the project's risk. For a riskless project, the appropriate discount rate is the riskless interest rate, which is lower than the sustainable social rate of time preference in a risky world; and for a project that has the same risk as society's invested wealth, the appropriate discount rate is the expected risky return, which is higher than the sustainable social rate of time preference. Some previous discussions of social discounting have obscured these distinctions by ignoring the risk that society faces. Our analysis is deliberately simple in order to achieve clarity about these issues.

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