Decentralized Bargaining: Efficiency & the Core

Matt Elliott (Caltech) & Francesco Nava (LSE)

June 2015

Motivation

Focus: decentralized bargaining in assignment economies.

A non-cooperative bargaining model is analysed in which:

- search is directed (players choose whom to bargain with);
- players commit to their offers and cannot renegotiate.

The objectives of the presentation are:

- to highlight the efficiency properties of these models;
- to understand the structure of efficient equilibria;
- to detail inefficiencies and understand their sources;
- to micro-found classical cooperative solution concepts: the core.

The analysis focuses on Markov Perfect Equilibria (MPE).

Survey of Results

Main Results:

- a characterization of mixed strategy MPE of the game;
- necessary & sufficient conditions for efficient MPE existence;
- necessary & sufficient conditions for limiting efficient MPE existence;
- a characterization of frictions in terms of mismatch and delay.

Main Points:

- the **core refines bargaining outcomes** of non-cooperative games;
- in thin markets **frictions persist whenever the market matters**;
- a characterization of **endogenous outside options** in limiting MPE;
- a rationale for **top-down market clearing** without asymmetric info.

Even when the core match is assortative and frictions vanish, conditions for efficiency can be very demanding.

Related Literature I

1. Decentralized Non-cooperative Bargaining

   Corominas-Bosch (04), **Gale Sabourian** (06), Polanski (07), **Abreu Manea** (12a,b), Manea (15), Polanski Vega-Redondo (14)

2. Non-cooperative Bargaining in Large Markets

   Rubinstein Wolinsky (85, 90), Gale (87), Binmore Herrero (88), Gul (89), **Moreno Wooders** (02), Manea (11,13), Lauerman (2013)

3. Centralized Cooperative and Non-Cooperative Bargaining

   Shapley Shubik (71), Myerson (77), Demange et al (86), Rochford (84), Roth Vande-Vate (90), Kranton Minehart (00), **Okada** (11)

4. Search in Assignment Markets

   Shimer Smith (00), Smith (06), Jaquet Tan (07), Eeckhout Kircher (10), Lauermann Noldeke (14), Atakan (15)

Related Literature II

Compared to recent literature, the project:

- restricts attention to thin two-sided markets;
- allows for surplus heterogeneity across matches;
- analyses directed search (players to choose whom to bargain with);
- considers economies with a unique core match (generic);
- requires bargaining to take place only between pairs of players.

Assumptions invoked:

- minimize coordination problems arising with multiple core matches;
- minimize the delay frictions that arise with random matching;
- lead to a clear characterization of efficiency in such markets.

The Assignment Economy

The Assignment Economy

Consider an assignment economy:

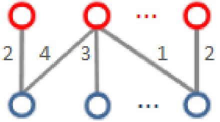
- with a set of players N;
- partitioned in two groups N_1, N_2.

A **match** (assignment) consists of a map $\mu : N \to N$ satisfying

$$\mu(i) = j \ \Leftrightarrow \mu(j) = i$$

When matched, players produce a surplus:

- $s_{ij} \geq 0$ for any $i, j \in N$;
- $s_{ij} = 0$ for any $i, j \in N_k$;
- $s_{ij} = s_{ji}$ for any $i, j \in N$.



Efficiency and the Core

An **efficient match** η satisfies

$$\sum_{i \in N} s_{i\eta(i)} = \max_{\mu} \left\{ \sum_{i \in N} s_{i\mu(i)} \right\}$$

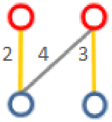
Assume that the efficient match is **unique** (generically true).

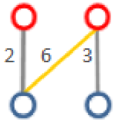
Let E denote the set of players who are **unmatched** in η.

The **core** U consists of the payoff profiles u satisfying

$$u_i + u_j \geq s_{ij} \text{ for any } i, j \in N$$

Thus if u belong to the core, $u_i + u_{\eta(i)} = s_{i\eta(i)}$ for any player i.





Bargaining Assumptions

Bargaining is **non-cooperative**.

The order of play is **random**.

Time is **countably infinite**.

Players discount time by a common factor $\delta \in (0, 1)$.

There is **complete information** about surplus and past moves.

A match between i and j generates in every period a **constant surplus**

$$(1 - \delta)s_{ij}$$

Equilibrium strategies are **Markovian**.

Decentralized Bargaining with Commitment

Decentralized Bargaining

At the beginning of the game all players are active.

A single player is selected every period to be the proposer.

Let $p_i > 0$ denote the probability of that player being i.

An active proposer i can:

- offer to another active player j a surplus split $x_{ij} \in [0, s_{ij}]$;
- delay making offers and remain active.

A player j receiving the offer decides whether:

- to reject the offer, and remain active along with the proposer;
- to accept it, become inactive along with player i, and commit to those bargaining shares for the infinite future.

Markov Perfect Equilibria and Values

The Markov state in every period consists of a set of active players A.

Given any active player set A and $i, j \in A$ let:

- $q_{ij}(A)$ be the **agreement probability** between i and j;
- $q_{ii}(A)$ be the **disagreement probability** of i;
- $V_i(A)$ be the **MPE value**;
- $v_i(A)$ be the **proposer value**.

Markov Perfection implies that player $i \in A$:

- accepts any offer above his discounted value $\delta V_i(A)$;
- has a proposer value equal to

$$v_i(A) = \max\left\{\delta V_i(A), \max_{j \in A \setminus i}\{s_{ij} - \delta V_j(A)\}\right\}.$$

MPE Existence and Characterization

For convenience define $p_A = 1 - \delta + \delta \sum_{j \in A} p_j$.

Theorem

MPE always exist.

MPE values in a subgame A satisfy

$$V_i(A) = \frac{p_i}{p_A} v_i(A) + \sum_{j \in A \setminus i} \frac{p_j}{p_A} \left[(q_{ji} + q_{jj}) \delta V_i(A) + \sum_{k \in A \setminus i,j} q_{jk} \delta V_i(A \setminus j, k) \right]$$

for some profile $(q_1, ..., q_A)$ such that for any player $i \in A$, $q_i \in \Delta(A)$ and

$$\begin{array}{lll} q_{ii} = 0 & \text{if} & v_i(A) > \delta V_i(A) \\ q_{ik} = 0 & \text{if} & v_i(A) > s_{ik} - \delta V_k(A) \text{ and } k \neq i \end{array}$$

MPE are not UNIQUE. But if (q, V) is MPE, $(q, \bar{V})$ is not for all $\bar{V} \neq V$.

Limiting Equilibria, Efficiency & Delay

Definition

An **LMPE** is the limit of a selection of the MPE correspondence as $\delta \longrightarrow 1$.

Definition

An MPE is **strongly efficient** if players immediately agree on core match.

An MPE is **weakly efficient** if players eventually agree on core match.

Efficiency defs also apply to LMPE, but require core agreement as $\delta \longrightarrow 1$.

Definition

An MPE **displays delay** if at some subgame a player with a strictly positive continuation value agrees with probability strictly smaller than 1.

Rubinstein Payoffs

The **Rubinstein Payoff** of player i is defined as

$$\sigma_i = \frac{p_i}{p_i + p_{\eta(i)}} s_{i\eta(i)}$$

AKA: limit payoffs attained by bargaining bilaterally with the core match.

The **Outside Option Payoff** of player i defined as

$$\omega_i = \max_{j \in E \cup \{i\}} s_{ij}$$

The **Shifted Rubinstein Payoff** of player i is defined as

$$\bar{\sigma}_i = \begin{cases} \omega_i & \text{if } \ \omega_i \geq \sigma_i \\ s_{i\eta(i)} - \omega_{\eta_i} & \text{if } \ \omega_{\eta(i)} \geq \sigma_{\eta(i)} \\ \sigma_i & \text{otherwise} \end{cases}$$

AKA: limit payoffs attained by bargaining bilaterally with the core match when faced with an exogenous outside option ω.

Efficient MPE and Core Spanning

Efficient MPE Values

In an efficient MPE, outside options are endogenous and vanish as the game is played. Thus, payoff reduce to bilateral bargaining payoffs.

Theorem

In any equilibrium path subgame A of a strongly efficient MPE

$$V_i(A) = \frac{p_i}{(1-\delta) + \delta(p_i + p_{\eta(i)})} s_{i\eta(i)} \quad \textit{for all } i \in A.$$

Frictions in the model can be hard (mismatch) or soft (delay). But hard frictions must affect bargaining outcomes whenever inefficiency arise.

Theorem

Any weakly efficient MPE is strongly efficient.

Thus, refer to strongly and weakly efficient MPE simply as **efficient MPE**.

In general, MPE display delay only if players match to different partners.

MPE Efficiency

The next results presents necessary and sufficient conditions for the existence of efficient MPE.

Theorem

An efficient MPE exists for all δ close to 1:

(a) if Rubinstein payoffs are in the interior of core

$$\sigma_i + \sigma_j > s_{ij} \quad \text{for all } i, j \in N \text{ such that } j \neq \eta(i);$$

(b) only if Rubinstein payoffs are in the core

$$\sigma_i + \sigma_j \geq s_{ij} \quad \text{for all } i, j \in N.$$

Efficient MPE payoffs are in the core as pairwise deviations are unilaterally profitable when players accept any offer above their discounted value.

These conditions do not imply that MPE is UNIQUE.

MPE Efficiency Intuition

The proof of the argument establishes that players:

- offer to core matches if Rubinstein payoffs are interior to the core;
- cannot offer to core matches with probability 1 when outside the core.

Outside options matter only when chosen with positive probability. But if so, frictions arise as players sometimes agree with inefficient partners.

Comments on the efficiency result:

- the proof is constructive on path;
- it relates non-cooperative bargaining outcomes to the core;
- it identifies sources of frictions when the efficient match is unique.

Efficiency Corollaries

The next observations follow directly from the previous theorem as $\delta \longrightarrow 1$.

Corollary

Any core payoff $u \in U$ is a LMPE payoff for some probabilities p.

If surplus S supports more core payoffs than S' and if an efficient MPE exists given S' and p, then an efficient MPE also exists given S and p.

If an efficient MPE exists for some probabilities p, it also exists for any probabilities p' such that

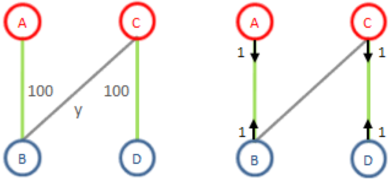
$$p_i / p_{\eta(i)} = p'_i / p'_{\eta(i)} \quad \text{for all} \quad i \in N.$$

The core of economy is spanned by LMPE of the bargaining game.

LMPE outcomes are a super-set of the core: cooperative bargaining outcomes refine non-cooperative outcomes and not viceversa.

Example: Efficiency & Mismatch

If $y \leq 100$, efficiency dictates agreement on the vertical matches.



Suppose that all players propose with equal probability and let

$$V = V_a = V_d \ \ \& \ \ W = V_b = V_c$$

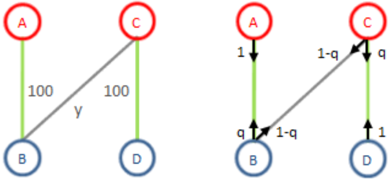
If so, an efficient MPE exists by the previous theorem since

$$2\sigma_W = 100 \geq y$$

Limiting MPE payoffs satisfy $W = 50$ and $V = 50$.

Example: Efficiency & Mismatch

If $y \in (100, 143]$, efficiency dictates agreement on the vertical matches.



If so, **no** efficient MPE exists by the previous theorem as

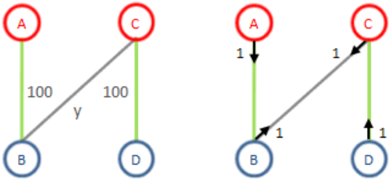
$$2\sigma_W = 100 < y$$

Strong players randomize for outside options to matter and payoffs satisfy

$$\begin{aligned} W &= \frac{y + 50 + 50q}{3 + q} \quad \text{for} \quad q = \frac{2\sqrt{2y^2 - 600y + 50000} - y}{200 - y} \\ V &= W - y + 1 \end{aligned}$$

Example: Efficiency & Mismatch

If $y \in (143, 200]$, efficiency dictates agreement on the vertical matches.



If so, **no** efficient MPE exists by the efficiency theorem as

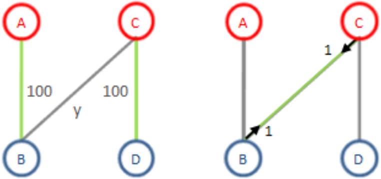
$$2\sigma_W = 100 < y$$

Strong players never offer to core matches and payoffs satisfy

$$W = \frac{50 + y}{3}$$
$$V = \frac{400 - y}{12}$$

Example: Efficiency & Mismatch

If $y \geq 200$, efficiency requires agreement on the diagonal match.



Suppose that all players propose with equal probability and let

$$V = V_a = V_d \;\; \& \;\; W = V_b = V_c$$

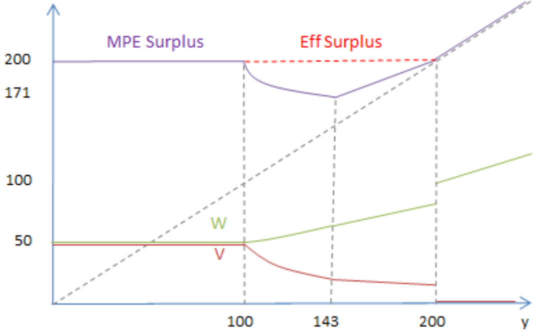
If so, an efficient MPE exists by the previous theorem since

$$\sigma_V + \sigma_W = y/2 \geq 100$$

Limiting MPE payoffs satisfy $W = \frac{y}{2}$ and $V = 0$.

Example: Efficiency & Mismatch

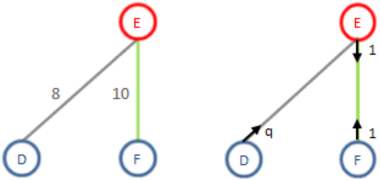
Frictions arise here as players' bargaining position evolves endogenously whenever players drop out of the market.



At $y = 200$ multiple core matches exist, and payoffs are discontinuous as there are multiple MPE (Abreu-Manea).

Example: Binding Outside Options

Only players who are single in the core can act as exogenous outside options (Sutton 86) here as they never leave the market.



The limiting payoff of the three players converge to

$$V(N) = (0, 8, 2)$$

Equilibrium is inefficient for any $\delta < 1$, as $q > 0$.

Player d acts as an exogenous outside option for e.

However d does not formally delay as his continuation value is zero.

MPE Frictions and Delay

When players do not choose whom to bargain with, MPE delay may occur as players hold out for their ideal partner.

When multiple core matches exist, MPE delay may arise because of coordination failures.

In our setting, such sources of inefficiency are ruled out, yet bargaining may remain inefficient.

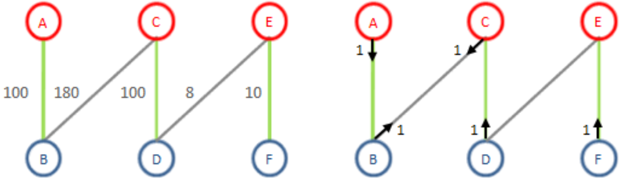
Lemma

The core match obtains with strictly positive probability in any limiting MPE without delay. However, MPE that display delay exist.

The first part exploits the acyclicity of the offer graph to show that some player always offers to his core match with positive probability.

Example: MPE Delay

Consider the market below in which players offer with the same probability.



Player e delays with probability 1 as he hopes that b and c agree.

If so, he is left in a strong position and achieve a payoff of 8, rather than bargaining with f alone to get a payoff of 5.

Unique MPE payoffs in this game converge to

$$V(N) = (55/3, 230/3, 230/3, 55/3, 13/2, 7/2)$$

Efficient LMPE and Outside Options

Strongly Efficient LMPE

Example 2 suggests that strongly efficient LMPE may exist even when no MPE is strongly efficient.

If so, only core-unmatched players can act as exogenous outside options.

Theorem

In equilibrium-path subgame A of a strongly efficient LMPE values converge to shifted Rubinstein payoffs,

$$\lim_{\delta \to 1} V_i(A) = \bar{\sigma}_i \quad \textit{for all } i \in A.$$

Moreover, a strongly efficient LMPE exists only if shifted Rubinstein payoffs are in the core

$$\bar{\sigma}_i + \bar{\sigma}_j \geq s_{ij} \quad \textit{for all } i, j \in N.$$

Only core-unmatched players can affect strongly efficient LMPE payoffs as such players are active in any equilibrium subgame of an efficient MPE.

Weakly Efficient LMPE and Sequential Agreement

Definition

A weakly efficient LMPE is a **sequential LMPE**, if at some equilibrium path subgame A such that $|A \backslash E| \geq 4$ and for some $i \in A \backslash E$

$$\lim_{\delta \to 1} \pi_{jj}(A) = 1 \quad \text{for any } j \in A \backslash \{i, \eta(i)\}.$$

In a sequential LMPE the market clears sequentially as all players delay in the limit except for a single core match.

Theorem

Any weakly efficient LMPE that is not payoff equivalent to a strongly efficient LMPE is sequential. Moreover, sequential LMPE exist.

In a sequential LMPE payoffs don't converge to shifted Rubinstein payoffs.

LMPE Efficiency Intuition

Payoff results are proven by induction on the active player set.

With sequential exit all players:

- remain in the market until a core match exits;
- thereby act as an outside option in a strongly efficient LMPE;
- affect each others' bargaining outcomes without ever agreeing.

Only those who remain active until a player's agreement affect his payoff:

- only core unmatched players play this role in strongly efficient LMPE;
- all disagreeing players play this role in sequential LMPE.

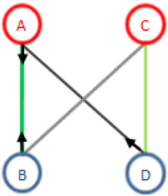
Sequential LMPE

To better understand the structure of sequential LMPE, set:

- $N = \{a, b, c, d\}$ and
- $p_i = p$ for all $i \in N$.

To avoid redundancies adopt the following labelling conventions:

- *ab* and *cd* are the core matches, $s_{ab} + s_{cd} > s_{ad} + s_{bc}$;
- *ab* is the most valuable core match, $s_{ab} \geq s_{cd}$;
- *ad* is the most valuable non-core match, $s_{ad} \geq s_{bc}$.



Weakly Efficient LMPE and Outside Option Chains

The next result characterizes 4-player sequential LMPE and shows that vertical differentiation is necessary for their existence.

Theorem

Given the labelling convention, in a sequential LMPE for all δ close to 1

$$\pi_{ab} = \pi_{ba} = \pi_{cc} = \pi_{da} + \pi_{dd} = 1, \;\; \pi_{da} > 0, \;\; \lim_{\delta\to 1} \pi_{dd} = 1.$$

Moreover, in any such LMPE

$$\begin{array}{ll} \lim_{\delta\to 1} V_a = s_{ad} - \sigma_d & \lim_{\delta\to 1} V_c = \sigma_c \\ \lim_{\delta\to 1} V_b = s_{ab} - s_{ad} + \sigma_d & \lim_{\delta\to 1} V_d = \sigma_d \end{array}$$

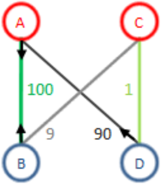
Finally, a sequential LMPE exists if and only if

$$s_{ab} > s_{ad} > \frac{s_{ab} + s_{cd}}{2} > s_{bc} > s_{cd} \;\; \text{and} \;\; \frac{s_{bc} - s_{cd}}{2(s_{ab} - s_{ad})} \geq \frac{s_{bc} + s_{cd}}{s_{ab} + s_{cd}}.$$

Weakly Efficient LMPE Comments

In a sequential LMPE:

- The market must be vertically differentiated.
- The market clears sequentially from the top down.
- The most valuable core match agrees first.
- Less valuable core matches delay & remain in the market.
- Outside option chains determine payoffs.
- The least valuable core match gets a (shifted) Rubinstein payoff.



Limit payoffs converge to $V_a = 89.5$, $V_b = 10.5$, and $V_c = V_d = .5$.

Assortative Matching and Frictions

SKIP

Assortative Matching I

Partition the set of agents into workers and firms:

- let $W = \{1, ..., w\}$ denote the set of worker types;
- let $F = \{1, ..., f\}$ denote the set of worker firm types.

The surplus of a worker-firm pair is given by a map $S : W \times F \longrightarrow \mathbb{R}_+$ st:

[C1] $S(i,j) > S(i',j)$ if $i < i'$;

[C2] $S(i,j) > S(i,j')$ if $j < j'$;

[C3] $S(i,j) + S(i',j') > S(i',j) + S(i,j')$ if $i < i'$ and $j < j'$.

C1 and C2 require workers and firms to be vertically differentiated, while C3 requires increasing differences in the surpluses of worker-firm pairs.

If so, the unique core match is the **assortative match** in which worker k is matched to firm k for all $k \leq \min\{w, f\}$.

Assortative Matching II

In any efficient MPE, the assortative match arises with probability 1.

Let p_k be the proposal probability of firm k, and q_k be that of worker k.

Lemma

If $w = f$, $p_i = q_j$ and $S(i,j) = S(j,i)$ for all $i,j \leq \min\{w,f\}$, there exists an efficient MPE for all δ sufficiently close to 1.

But, if any of these conditions fails, there exists a map S such that for all δ sufficiently close to 1 there is no efficient MPE.

The conditions for the assortative match to obtain are restrictive and require the market to be highly symmetric.

Conclusions

With commitment and decentralized bargaining:

- frictions arise if and only if the endogenous outside options bind;
- frictions arise because outside options matter only when chosen;
- the core refines non-cooperative bargaining outcomes.

Conjecture, without commitment:

- frictions vanish as players can renegotiate inefficient agreements;
- non-cooperative bargaining outcomes refine the core.

Bargaining with Separation

SKIP

Separation: Markov Bargaining

Consider the previous setup, but assume that players choose whom to bargain with as was the case in the commitment model.

Any proposer i can now:

- offer to any other player j a surplus split $x_{ij} \in [0, s_{ij}]$;
- break its current match without making offers;
- delay making offers and remain matched at its current share.

A player receiving the offer decides whether:

- to reject it and remain matched at current shares;
- to accept it, and be matched to i at the new shares.

If a match is broken, both players incur a small *separation cost* c.

Separation: MPE Efficiency

We have tried to prove the following conjecture without much success.

Conjecture:
For c sufficiently small, a weakly efficient limiting MPE always exists.

The proof of the argument should relies on three steps:

1. a proof of existence of MPE and MPE characterization (DONE);
2. a proof that LMPE always converge to steady states (DONE);
3. a proof that steady states are in the core if costs are small (HARD).

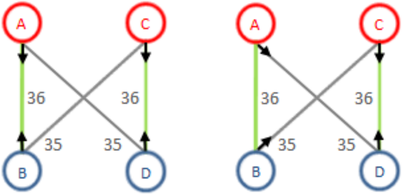
Step 2 exploits costs to constrain the extent of renegotiation.

Thank You!

MPE Multiplicity and Coordination Frictions

Inefficient MPE may exist even when conditions for efficiency hold.

Let $N = \{a, b, c, d\}$, $p_a = p_b = 0.4$, $p_c = p_d = 0.1$, and consider:



Rubinstein payoffs are in the interior of the core, but multiple MPE exist:

- an efficient one with limit payoffs $V_a = V_b = V_c = V_d = 18$;
- an inefficient one with limit payoffs $V_a = V_b = 25.6$, $V_c = V_d = 8$.

BACK1 or BACK2