Repeated Games and Networks – Literature

Francesco Nava

London School of Economics

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Local Monitoring

In many strategic environments, interaction is local and segmented:

- neighborhood stores serve different yet overlapping customers;
- informal lending/insurance is fulfilled by relatives and friends;
- the behavior of the residents affects their neighbors to a larger extent;
- a nation's policy generates externalities on neighboring nations;
- a classic case is the private provision of local public goods.

In addition to local interaction, a key feature of these environments is **local monitoring**: participants are aware of their neighbors' identities and actions, but not necessarily of the identity and actions of other players.

In these environments, it is of interest to study long run interaction when incentives can only be provided locally.

Three Strands

Three lines of research have been developed in such environments:

- The first develops **Folk Theorems**, and shows that network structure is usually irrelevant for enforcing cooperation when the frequency of interaction is high.
- The second shows how the monitoring structure affects the **maximal level of cooperation** at fixed discount rates, and broadly finds that larger and/or better connected groups are more cooperative.
- The third also fixes discount rates, and evaluates how different **communication protocols** affect the set of equilibrium payoffs and the incentives to cooperate.

Community Enforcement Classical

Community enforcement was first analyzed with random matching.

Pioneering studies by Kandori (1992) and Ellison (1994) focus on pairwise matching, and establish why collective punishments can sustain efficient equilibria when bilateral punishments fail.

Contributions include: Harrington (1995), Takahashi (2010), Deb (2014).

Although the matching and the networks literatures share methodological insights, there are significant differences:

- Whereas random matching games assume that all players potentially interact and exchange information, network games constrain interactions, monitoring and info exchange to a stable network.
- Whereas random matching games focus usually on Folk Theorems, the study of network games aims to establish a relationship between the network structure and the equilibrium payoffs.

Applications and Evidence

The use of social sanctions to deter misconduct is widely documented:

- in economics (Milgrom, North, and Weingast (1990), Greif (1993)),
- political science (Ostrom (1990), Fearon and Laitin (1996)),
- sociology (Coleman (1990), Raub and Weesie (1990)),
- law (Bernstein (1992)).

Some studies stress the importance of **cohesion** for attaining socially desirable outcomes (Coleman (1990), Greif (1993), McMillan (1995), Fearon and Laitin (1996), Uzzi (1996), Dixit (2006)).

Coleman seminal's contribution identifies a notion of **social capital**, and relates it to the underlying social architecture. Enforcement is more effective in networks with high closure and cohesion, as cohesion facilitates the implementation of social sanctions.

Others highlight the importance of **information dissemination**. Greif (2006) finds that contract enforcement between medieval Maghribi traders is effective only when a close-knit community disseminates information so to align incentives to comply with community-based sanctions.

Folk Theorems on Networks

A significant body of literature provides conditions on the interaction network for a Folk Theorem to apply. These studies establish why a **Folk theorem obtains under mild conditions** on the network structure.

A key concern here is ensuring that **players do not cooperate off-path**, as grim trigger strategies provide such strong incentives to cooperate on-path that players prefer to cooperate even after observing a deviation.

Ellison (1994) resolves this problem by introducing either a public randomization device or a milder version of grim trigger strategies that make players indifferent between cooperating and defecting on-path, and noting that cooperation is more appealing on-path than off-path.

The literature on local monitoring addresses similar concerns with related approaches. More complications arise with local monitoring as players infer the spread of defection and neighbors' beliefs about future play.

Folk Theorems: Ben-Porath and Kahneman (1996)

Most results require the network structure is common knowledge.

Ben-Porath and Kahneman (1996) considers games with public randomization in which players can make public announcements about the past behavior of other players whom they observe.

The main result shows that, as the discount factor tends to 1, the limit set of sequential equilibrium payoffs contains the set of IR payoffs, when every player is **observed by at least 2 other players**.

Two monitors are required generally to guarantee that inconsistent public announcements can be sanctioned by the community.

Results establish that if payoffs are assessed by the **liminf** of the average payoff, every IR payoff is a sequential equilibrium payoff even if players are monitored by **only 1** other player.

Folk Theorems: Renault and Tomala (1998)

Renault and Tomala (1998) analyze a model with global interactions, local monitoring, no discounting, and no explicit communication.

The main finding establishes that a **Nash Folk Theorem** applies if and only if the monitoring network is **2-connected** (i.e. if the subgraphs obtained by suppressing any one player are still connected).

The result abstracts from sequential rationality, which simplifies the problem as punishments need not be incentive compatible. Although explicit communication is ruled out, no discounting and the restriction to Nash equilibrium imply that players can use any finite number of future periods to privately communicate with neighbors at no cost.

Tomala (2010) extends the analysis to partially known networks, in which players only know their neighbors and the number of players in the network, and derives a Nash Folk Theorem.

Folk Theorems: Laclau (2012) and (2014)

Laclau (2012) considers a setup analogous to Renault and Tomala (1998), while allowing for **imperfect local monitoring** and explicit local communication (private local cheap talk).

Monitoring is imperfect, as players observe their payoff, but not the actions chosen in their neighborhood.

The main contribution shows that a Nash Folk Theorem holds when the payoff of every player is responsive to unilateral deviations (i.e. players detect local unilateral deviations, despite monitoring being imperfect).

Laclau (2014) extends results to a model in which **communication is global**, and can be either private or public.

The analysis shows that a **Sequentially Rational Folk Theorem** holds when a joint pairwise identifiability condition regarding payoff functions is satisfied. The condition requires players to detect the identity of the deviating player, when they detect a local unilateral deviation.

Folk Theorems: Stability

The classical modification of a trigger strategy devised in Ellison (1994) are **not stable** to mistakes in that defections spread over the network and cooperation is never recovered when an agent defects by mistake.

The classical solution involves **bounding** the length of the **punishments**.

Local monitoring, however, may cause **discrepancies in beliefs** about a neighbor's future actions. If there is such a discrepancy, an agent whose neighbors have different beliefs may not be able to satisfy the expectations of all them which may cause an infinite repetition of defection phases and a failure of stability.

Bounded punishment strategies may not even constitute a sequential equilibrium in a general networked setting, when discrepancies arise.

Three studies address these complications with strong assumptions (restricting attention to line graphs or allowing public randomization): Xue (2004), Cho (2010), Cho (2011).

Folk Theorems: Nava and Piccione (2014)

Nava and Piccione (2014) study local monitoring when the network structure is unknown, and when communication and public randomization are ruled out.

Their main result establishes that, for sufficiently high discount rates and any prior beliefs with full support about the network structure, **efficient sequential equilibria always exist**.

These equilibria stable whenever the network is acyclical or when discounting is ruled out. In general, despite stability failing, players will always belief that strategies revert to full cooperation.

More later...

Maximal Cooperation and Fixed Discount Factors

A major concern with Folk Theorems was disciplining behavior off-path. This part of the literature abstracts from such a concern by analyzing the **most cooperative equilibrium** in games with continuous action sets.

Such equilibria make players indifferent between cooperating and defecting on-path (as otherwise a player could be asked to cooperate more).

By essentially the same argument as in Ellison (1994), this implies that players weakly prefer to defect off-path.

Hence, a contribution of this literature is to show that grim trigger strategies provide the strongest possible incentives for cooperation on-path, not that they provide incentives for punishing off-path.

The characterization of the most cooperative equilibrium has implications for the efficiency and stability of various network configurations which are the main objectives of this literature.

Maximal Cooperation Classic

This approach was pioneered in several papers analyzing the effect of the size and structure of a group on the maximum equilibrium level of public good provision.

Classical references characterize maximum cooperation only for complete networks and public monitoring:

- Pecorino (1999) shows that public good provision is easier in large groups, as a deviation causing everyone else to defect is more costly.
- Haag and Lagunoff (2007) characterize the maximal average level of cooperation (MAC) over all stationary subgame perfect equilibria. Discount factors are heterogeneous. Rsults establish that more heterogeneous groups are less cooperative on average. The MAC also exhibits increasing returns to scale for a range of heterogeneous discount factors (i.e. larger groups are more cooperative, on average). By contrast, when the group has a common discount factor, the MAC is invariant to group size.

Haag and Lagunoff (2006)

Haag and Lagunoff (2006) examine optimal network structure in Prisoner's Dilemma with local interactions and **local monitoring**, in which individual discount factors are randomly determined. A planner chooses the network before the discount factors are realized so to maximize utilitarian welfare.

The main results restrict attention to the local trigger strategies associated to the highest utilitarian welfare. Owing to the heterogeneity of discount factors greater social conflict may arise in more connected networks.

When discount factors are known to the planner, the optimal network exhibits a **cooperative core and an uncooperative fringe**. Uncooperative players are impatient, and are connected to cooperative ones who are patient and tolerate their free riding.

By contrast, when the planner knows only the ex-ante distribution of discount factors, the optimal design either partitions individuals into **isolated cliques**, or in incomplete graphs with small overlap.

Wolitzky (2013)

Wolitzky (2013) studies cooperation in public good games with continuous actions, local monitoring, global interactions, fixed and common discount factor (players choose of a level of cooperation).

In every period, a monitoring network is realized and players receive signals about the global structure. Players perfectly observe the actions in their realized neighborhood, but observe nothing about other players' actions.

A distinguishing feature of the environment analyzed is that in every period the monitoring network must be observed by players after actions are chosen.

The study characterizes the maximum level of cooperation that can be **robustly** sustained in Perfect Bayesian equilibrium (for any information that players have about the realized monitoring network).

Wolitzky (2013)

The main contribution finds that the robust MC is always sustained in **trigger strategies**, where each player cooperates at a fixed level unless he ever observes a player failing to cooperate at his prescribed level.

Trigger strategies also maximize cooperation when players have perfect knowledge of who observed whom in the past (e.g. fixed network).

Interestingly, it is when players have less information about the monitoring structure that more complicated strategies can do better.

The actions of different players are **strategic complements** when players know who observed whom, as defecting makes every on-path history less likely when strategies are grim triggers.

This strategic complementarity breaks down, however, when players disagree about who has observed whom.

Wolitzky (2013)

The analysis compares economies in terms of robust MC in two cases:

- equal monitoring (in expectation players are monitored equally well);
- fixed monitoring (the monitoring network is fixed over time).

With equal monitoring, the effectiveness of a monitoring is determined by the **effective contagiousness**, which captures the cumulative expected present discounted number of players who learn about a deviation.

Higher levels of cooperation can be sustained if news about a deviation spreads throughout the network more quickly.

Results show that cooperation in the provision of:

- pure public goods (when the marginal benefit of cooperating independent of group size) tends to be greater in larger groups;
- divisible public goods (when the marginal benefit is inversely proportional to group size) tends to be greater in smaller groups.

Wolitzky (2013)

With a **fixed network**, a notion of network **centrality** determines both:

- which players cooperate more and
- which networks support more cooperation.

Adding links to the monitoring network necessarily increases all players' robust MAC, which formalizes the idea that individuals in better-connected groups cooperate more.

Ali and Miller (2013)

Ali and Miller (2013) compare networks in terms of MC in variable-stakes Prisoners' Dilemmas when the network is common knowledge.

The model is in continuous time and links is governed by an independent **Poisson recognition process** with a common recognition rate.

When a link is recognized a two-period game is played:

- first both players propose stakes at which to interact, and the smallest of the two proposals determines the actual stakes;
- then players engage in a Prisoners' Dilemma satisfying a strategic complementarity condition.

Players ignore both the actions chosen in interactions to which they do not belong and the time at which these interactions take place.

The analysis restricts attention to **stationary strategies** on-path. Any stationary trigger strategy prescribing stakes such that ICs bind at every on-path history is as before a PBE.

Ali and Miller (2013)

The main result establishes that any symmetric network with degree d possesses a symmetric contagion equilibrium that Pareto dominates every distinct mutual effort equilibrium (and thus identifies the optimal stakes).

The result also implies that no other stationary equilibrium has a higher value if the stage game satisfies strategic complementarity.

The argument relies on a measure of **network viscosity** (which is minimal in the clique) that captures the incentives to comply with an equilibrium.

This measure differs from the effective contagiousness in Wolitzky (2013), because in public goods games every player may punish a deviator upon receiving news of a defection, whereas in separable games only neighbors can effectively punish a deviation.

Ali and Miller (2013)

Results exploit the characterization of the optimal stakes to analyze how network structure affects aggregate welfare.

Adding links has two roles in the model:

- it helps information diffusion through contagion;
- it increases the number of interactions and thus surplus.

The main welfare implication of the model is the **optimality of cliques**.

In particular, for any network with a maximal degree smaller than d, no player attains a mutual effort equilibrium payoff that exceeds his optimal equilibrium payoff in the symmetric network of degree d.

Moreover, if the game satisfies strategic complementarity, the value in every equilibrium is less than the optimal value in the symmetric network of degree d.

Results extend to a model with separable costs of forming links.

Remarks on Fixed Discounting

These papers provide novel and interesting insights linking interaction and monitoring networks to measures of aggregate welfare.

These observations can in principle explain why community enforcement may lead to substantially different levels of cooperations across societies.

The main limitation of these studies is the restrictive class of games, as results are generally developed for Prisoners' Dilemma type games possessing a mutual minmax Nash equilibrium.

Generalizing techniques to arbitrary stage game does not seem straightforward, as the characterization of the equilibrium with the highest utilitarian welfare may become intractable.

Communication

A separate strand of the literature analyzes how equilibrium outcomes are affected by the availability of different communication technologies.

These studies include:

- Lippert and Spagolo (2011),
- Wolitzky (2014),
- Ali and Miller (2014).

The first two provide an explicit link between the communication technology and the equilibrium correspondence.

The latter analyzes evedentiary communication and cheap-talk with ostracism (i.e. when players target punishments against defectors).

Omission

There are several omission here, please see my handbook chapter for more.

A noteworthy omission is Balmaceda and Escobar (2014) who build Haag and Lagunoff (2006, 2007) to show that cohesive communities (in which players are partitioned into isolated cliques) emerge as welfare maximizing network structures.