Differentiated Durable Goods

Monopoly and Competition

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Focus: Differentiated Durable Goods

We study the incentives to differentiate products when goods are sold over time and without commitment.

The **results on the monopoly**:

- characterize limiting outcomes in terms of a **simple static problem**;
- deliver a **robust Coase conjecture** for all multi-product settings;
- develop insights on **product design**.

The **results on the competition** show why:

- **competition** can unambiguously **increase market power**;
- such instances arise when competitors choose products.

Coase Conjecture 1972

Coase's seminal conjecture first raised the time consistency problem:

- Upon selling to high value buyers, a monopolist cannot stop selling.
- If so, prices keep falling and forward looking buyers expect this.
- But if so, buyers are unwilling to pay a high price in the first place.

As the time between offers vanishes, the opening price converges to its lowest valuation and the competitive quantity is sold in a *twinkle of an eye*.

Formal proofs in Stokey 1981; Fudenberg, Levine, Tirole 1985; Gul, Sonnenschein, Wilson 1986; Asubel, Deneckere 1989 established that:

Varieties	Gap	MC	MC Time	Efficient	Competitive	Unique
1	No	Yes	Infinite	Yes	MPE	Folk
1	Yes	Yes	Finite	Yes	Yes	PBE

Monopoly: Related Literature

The more recent literature on durable goods monopoly has analysed:

1. The robustness of Coase's insight to changes in the assumptions.

   Bond-Samuelson 1984; Kahn 1986; Ausubel-Deneckere 1989; Bagnoli-Salant-Swierzbinski 1989; Sobel 1991; Fehr-Kuhn 1995; Biehl 2001; **Takeyama** 2002; **Hahn** 2006; **Inderst** 2008; McAfee-Wiseman 2008; Deb 2011; Montez 2013; Ortner 2014; **Board-Pycia** 2014.

2. Tactics that could be used to avoid the commitment problem.

   Bulow 1982; Butz 1990; Levinthaland-Purohit 1989; Waldman 1993; Choi 1994; Waldman 1996; Fudenberg-Tirole 1998; Lee-Lee 1998.

Many of these studies are cast as violations of the Coasian conclusion.

Monopoly: Contributions

In the monopoly case we establish that:

1. **Static and dynamic market-clearing** prices coincide.
2. **Optimal market-clearing profits bound PBE profits** from below.
3. **Mixing** may be required on-path **to conceal discounts**.
4. Limiting MPE profits **converge to optimal market-clearing profits**.
5. **Robustness** of these conclusions to alternative specifications.
6. **Product design** implications.

Equilibrium pricing is **neither minimal, nor competitive, nor efficient**.

But, the **Coasian logic** survives in that **optimal market-clearing and agreement govern pricing**.

Competition: Related Literature

The seminal contribution on competition by **Gul 1987**:

- considers markets in which **firms produce the same product**;
- proves a **Folk theorem** as an incumbent may benefit from entry.

These anti-competitive insights however:

- rely on **high discount factors** to sustain collusion;
- apply only when products are not differentiated;
- **do not extend to stationary equilibria**.

Competition: Contributions

Competition increases market power if the present value of profits of every seller is higher than in their respective monopoly setting.

With differentiated products, competition can increase market power:

- in all PBE **even in stationary equilibria**;
- **regardless of** the value the **discount factor**.

In the competition case we:

1. find conditions for competition to increase market power in all PBE;
2. **endogenize the choice of products** in a location choice model;
3. show that this naturally leads to increased in market power.

Model

Model: The Monopolist

The time is countably infinite, $t \in \{0, 1, ...\}$.

A **single firm** operates in the market in every period.

Two varieties, a and b, of a durable good can be produced and sold.

At each time period the monopolist sets prices for the two varieties

$$p = (p_a, p_b) .$$

The **marginal cost** of producing units of each variety is **zero**.

The monopolist discounts the future with a discount factor δ.

Its payoff amounts to the present discounted value of future profits.

Model: Buyers

All buyers have **unit-demand** for the durable good.

Buyers **exit the market** upon purchasing any one variety.

There is a unit measure of buyers.

Buyers are characterised by their values for the two products

$$v = (v_a, v_b).$$

Value profiles are **private information** of buyers.

Buyers discount the future by the common factor δ.

The payoff of consuming variety i at price p_i at date t amounts to

$$\delta^{t-1}(v_i - p_i).$$

Model: Buyers' Values

A measure $\mathcal{F}$ on the unit square $[0,1]^2$ describes the distribution of values.

Let F be the associated cumulative and V be the support.

Let F_i be the marginal cumulative of variety i and V_i be its support.

Definitions (Regularity Assumption)

The market is said to be **regular** if:

- V is convex;
- $\mathcal{F}$ is absolutely continuous on $\mathbb{R}^2$;
- its density f satisfies $f(v) \in (\underline{f}, \overline{f})$ for any $v \in V$.

The **monopolist knows** $\mathcal{F}$, but not the value of a given buyer.

Model: Information and Solution Concepts

Players observe for every previous period:

- the prices posted by the monopolist;
- (possibly) the total measure of buyers for each variety.

At any history, a strategy:

- for the monopolist specifies a profile of prices.
- for an active buyer specifies which variety to purchase, if any.

Let A^t be the set of **active buyers** at a given history h^t.

Those buyers who have yet to purchase a variety at date t.

Our results characterize the measurable **PBE** of this game.

MPE are PBE in which buyers' strategy depends **only on current prices**.

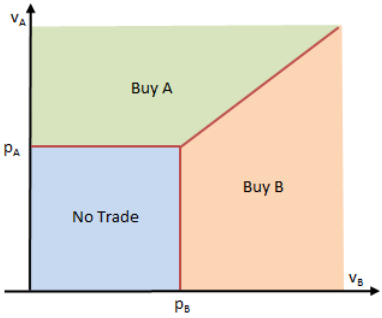
Optimal Market Clearing

Static Demand Functions

Momentarily consider the static version of the model.

Given prices, the **static demand** for product i amount to

$$d_i(p) = \mathcal{F}(v_i - p_i > \max\{v_j - p_j, 0\}).$$

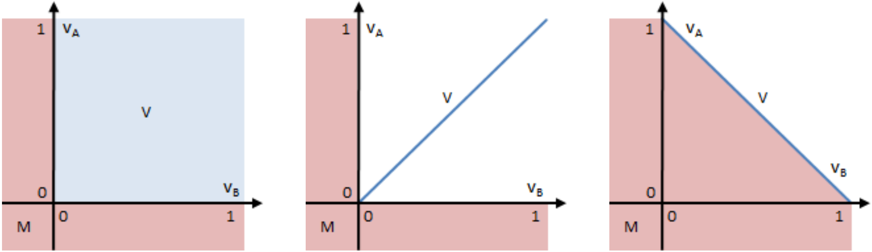


Static Market Clearing Prices

A **market clearing price** is a price profile that clears the market.

Let M be the set of market clearing prices

$$M = \left\{ p \in \mathbb{R}^2 \mid \max_i \{v_i - p_i\} \geq 0 \text{ for any } v \in V \right\}.$$



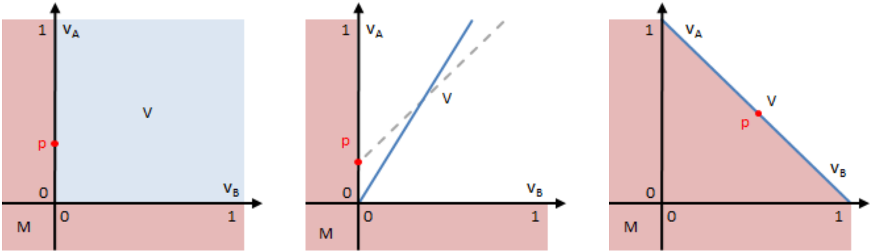
Independence   Concordance   Discordance

Static Optimal Market Clearing

An **optimal market clearing price** $\bar{p}$ solves the following static problem

$$\max_{p \in M} [d_a(p) p_a + d_b(p) p_b].$$

The value $\bar{\pi}$ of this program is the **optimal market clearing profit**.



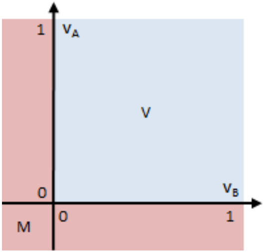
Static Market Clearing: Minimal Values

Let w_i denote the **minimal value** of variety i in the support V.

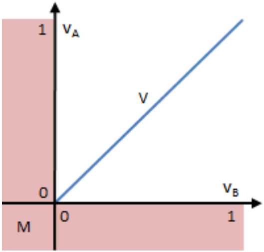
Let w_g denote the **minimal value of the durable good**

$$w_g = \min_{v \in V} \max\{v_a, v_b\}.$$

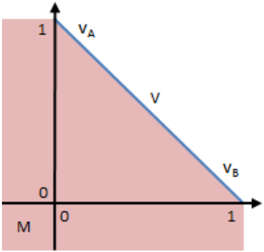
Clearly it must be that $w_g \geq \max\{w_a, w_b\}$.



$w_g = 0$



$w_g = 0$



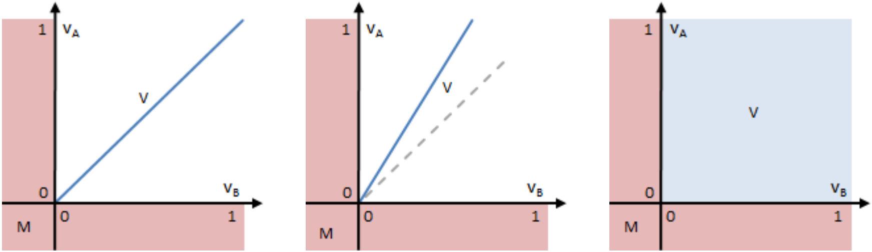
$w_g > 0$

Static Market Clearing: Special Cases

Varieties are identical if $v_a = v_b$ for all $v \in V$.

Varieties are ranked if they are not identical and $v_i \geq v_j$ for all $v \in V$.

Varieties are unranked if for any i there is $v \in V$ such that $v_i > v_j$.



Identical Ranked Unranked

Static Optimal Market Clearing: Minimal Pricing

Lemma

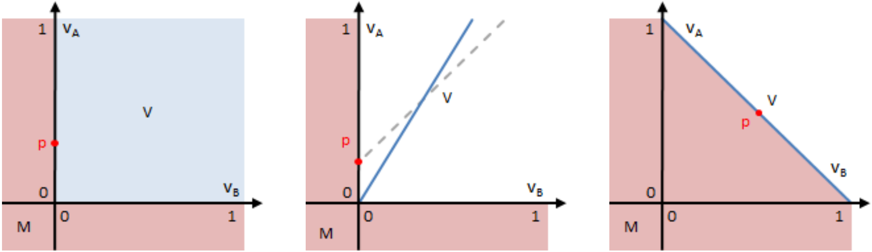
Optimal market-clearing profits:

(1) weakly exceed w_g*;*

(2) strictly exceed $\max_i w_i$ *if varieties are unranked;*

(3) equal $\min_i w_i$ *if and only if varieties are identical;*

(4) equal 0 *if and only if varieties are identical and* $(0,0) \in V$*.*



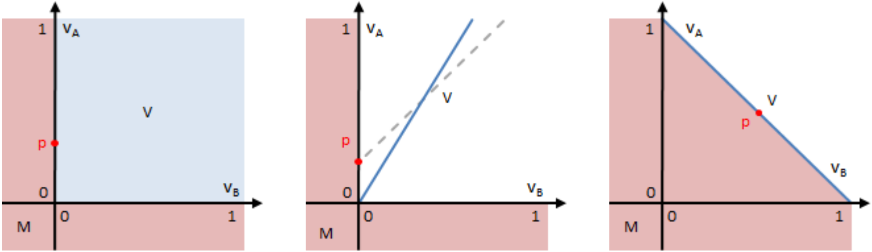
Static Optimal Market Clearing: Minimal Pricing

Lemma

Optimal market-clearing profits strictly exceed w_g*:*

(5) if varieties are ranked and $w_a = w_b$*;*

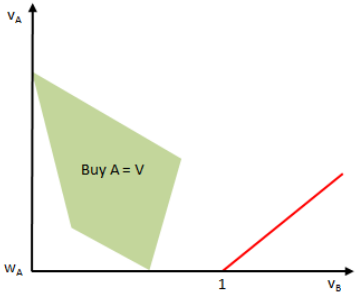
(6) if varieties are independently distributed and $w_a = w_b$*.*

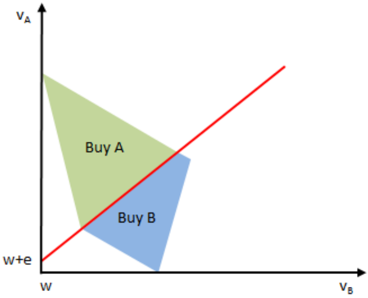


Let $w_a \geq w_b$, OMC profits:

- amount to w_a at the MC price $p = (w_a, 1)$;
- can thus amount to w_b only if $w_a = w_b = w$;
- amount to w at the MC price $(p_i, p_j) = (w + e, w)$ only if

$$d_i(w + e, w) = \mathcal{F}(v_i - v_j > e) = 0.$$





Static Optimal Market Clearing: Efficiency

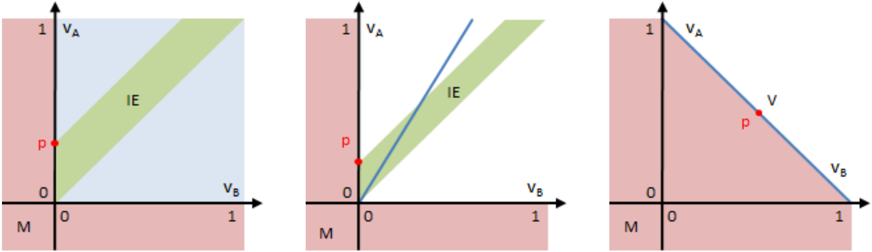
A price is **efficient** if every buyer purchases its preferred variety.

Fact

Optimal market-clearing prices are inefficient:

(1) if varieties are ranked and $w_a = w_b$;

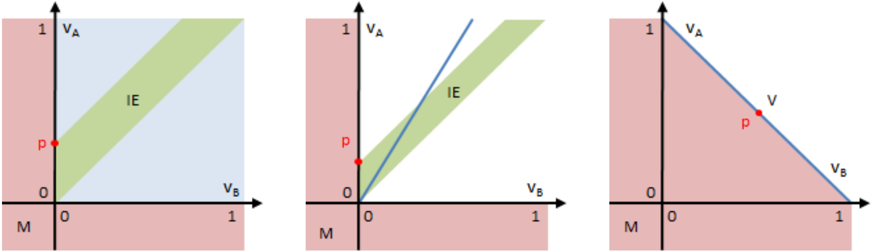
(2) if varieties are independently distributed and $w_a = w_b$.



Static Optimal Market Clearing: Comments

In contrast to the 1-variety case, market-clearing no longer implies that:

- profits amount to the minimal value of a variety or of the good;
- pricing is efficient or at the minimal value of varieties or of the good;
- profits equal 0 when there are no gaps and $(0, 0) \in V$.



Coase Conjecture

Skimming, Market Clearing & Coasian Dynamics

Next we **extend the Coase conjecture** to settings with multiple varieties.

As in seminal results **the market eventually clears** and the intuition for this coincides with the seminal Coasian insight.

As the monopolist cannot commit to prices, the market must clear or else selling more units would be profitable when prices have converged.

Coasian insights however lead to:

- static **optimal market clearing and agreement**;
- **NOT efficiency and competitive pricing**;
- and **agreement must be unimprovable**.

Skimming PBE Active Buyers

Lemma

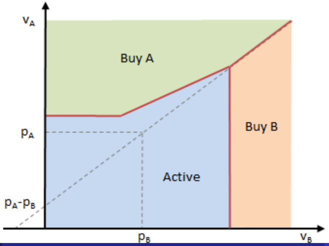
In any perfect Bayesian equilibrium, at any buyer-history:

(1) if v strictly prefers to buy variety i, so does any buyer v' such that

$$v'_i - v_i \geq \max\{0, v'_j - v_j\};$$

(2) if v prefers to buy, any buyer $v' > v$ strictly prefers to buy if

$$\delta \max_i\{v'_i - v_i\} < \min_i\{v'_i - v_i\};$$



Skimming PBE Active Buyers

$\mathcal{F}_A$ is a **truncation** of $\mathcal{F}$ on A if $\mathcal{F}_A(E) = \mathcal{F}(E \cap A)$ for $E \in \Omega$.

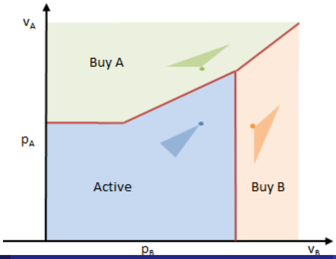
Lemma

In any perfect Bayesian equilibrium, at any buyer-history:

(3) if v prefers not to buy, any buyer $v' < v$ strictly prefers not to buy if

$$\delta \max_i \{v_i - v_i'\} < \min_i \{v_i - v_i'\};$$

(4) if the market is regular, the measure of active buyers is a truncation.



Some Intuition: Skimming Lemma

Let $\alpha_j^s(v')$ is the probability that v' purchases j at date $t+s+1$.

If buyer v is willing to purchase at date t so is buyer $v' = v + (\varepsilon, \varepsilon)$:

- As v purchases a variety

$$\max_i \{v_i - p_i\} \geq \delta U(v).$$

- As v can **mimic** buyer v' from period $t+1$ onwards

$$U(v') - U(v) \leq \sum_{s=0}^{\infty} \delta^s \left[\sum_j \alpha_j^s(v') \left(v_j' - v_j\right)\right] \leq \varepsilon.$$

- But if so, the result then follows as

$$\max_i \{v_i' - p_i\} > \max_i \{v_i - p_i\} + \delta\varepsilon \geq \delta U(v) + \delta\varepsilon \geq \delta U(v').$$

A Lower Bound on PBE Profits

The next result establishes some key properties of **any PBE**:

- Profits can equal zero only if the optimal MC profit equals zero.
- The monopolist **never undercuts on MC prices**.
- Static MC prices clear the market in the dynamic game.

Lemma

In any PBE, at any seller-history h:

(1) The monopolist never sets prices in the interior of M.

(2) All buyers purchase a variety if prices are in the interior of M.

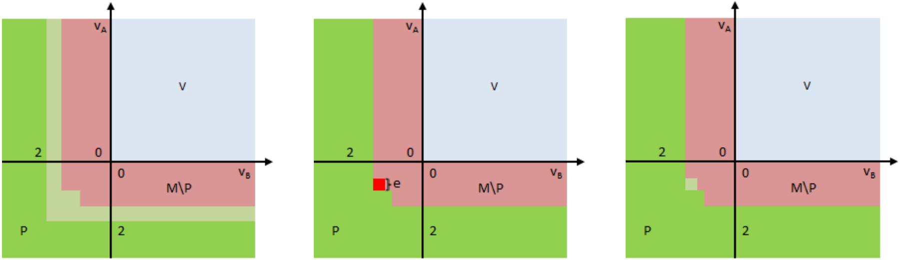
(3) The present discounted value of profits satisfies

$$\Pi(h) \geq \bar{\pi}(A)$$

Intuition: Bounding Lemma MORE

It suffices to show that all buyers accept any price in the interior of M:

- all buyers purchase when a price is below -2;
- if all buyers purchase at p, they also purchase at $p' < p$;
- all buyers purchase at any price interior to $M \backslash P$ but close to P.



Lemma

If the market is regular, in any perfect Bayesian equilibrium:
(1) every buyer purchases a variety as time diverges to infinity;
(2) if $w_g > 0$, every buyer purchases a variety in a finite time.

If $w_g > 0$, prices belong to M when few buyers are active as:

- price discrimination gains become small;
- beliefs are pinned down at any MC price;
- optimal MC profits are strictly positive in subgame.

Fact

If the market clears, the monopolist sets prices in M that maximize static MC profits given the measure of active buyers.

PBE & MPE Existence

Theorem

If the market is regular, an MPE always exists.

The theorem proves inductively the existence of a mixed strategy MPE.

Uniqueness of PBE with gaps is however **lost** with multiple varieties, partly because optimal MC prices are no longer unique.

With multiple varieties, **mixing**:

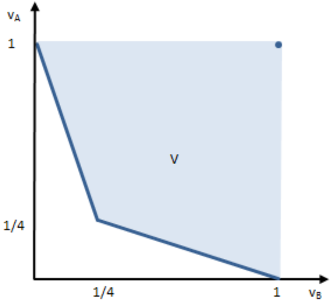
- **may take place even in the final stage**;
- not just at the initial round as with 1 variety;
- may be used **to conceal discounts** from forward-looking buyers.

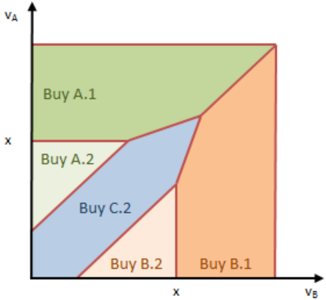
PBE Mixing: Concealing Discounts

With more varieties, **mixing** may occur even **in the final stage to conceal** from forward-looking buyers **which variety will be most discounted**.

In the following piece-wise uniform example:

- the market clears in two periods for suitable δ;
- the seller mixes on optimal MC prices at date 2.





Uniform Coase Conjecture

Theorem

If market is regular, MPE profits converge to optimal MC profits as $\delta \to 1$.

When δ is close to 1:

- consumers' **delay costs are small**;
- consumers wait a long time for a small discount;
- price discrimination gains are small (second order).

As **OMC profit is strictly positive**, deferring revenue is costly.

In low demand periods, the seller prefers to **anticipate non-negligible streams of profits** when strategies are stationary.

As in the classical case, the monopoly immediately sets prices **close to market clearing** to sell some units.

Intuition: Uniform Coase Conjecture

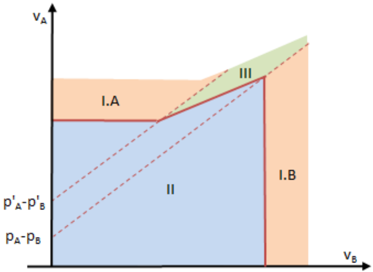
Deviating to $p^{t+1} \leq p^t$ at date t has three effects on profits:

I. It lowers the price paid by those who buy the same variety. (-)
II. It anticipates the stream of future revenue on units to be sold. (+)
III. It induces some consumers to change the product demanded. (?)

Price changes are small if a patient consumer does not delay.

Thus, if in a given period demand is small, effects I and III are small.

If so, **profits must be small** for the deviation not to be profitable.



Discussion

Costs: Market-Clearing with Uncertain Gains

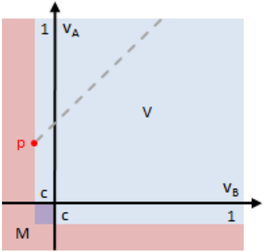
Denote the set buyers with positive gains from trade by

$$V^+ = \left\{ v \in V \mid \max_{i \in \{a,b\}} \{v_i - c_i\} \geq 0 \right\}.$$

Depletion of gains from trade or market-clearing then amounts to

$$M = \left\{ p \in \mathbb{R}^2 \mid \max_i \{v_i - p_i\} \geq 0 \text{ for all } v \in V^+ \right\}.$$

With costs, a price in the **MC prices can belong to the support.**



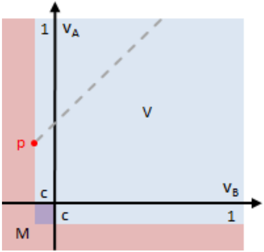
Costs: Optimal Market-Clearing

OMC profits can now **equal zero and display cross-subsidization**.

Varieties are unranked if for any i there is v such that $v_i - c_i > v_j - c_j$.

Fact

Optimal market-clearing profits are strictly positive if varieties are unranked and $w_i \geq c_i$ for some variety i.



Costs: Coasian Dynamics and Market-Clearing

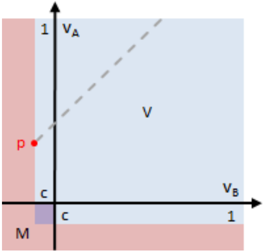
Key Coasian results however are not affected by this change.

Fact

If the market is regular, optimal market-clearing profits:

(1) are a lower bound on PBE profits;

(2) coincide with the limit of MPE profits as δ converges to 1.



Relationship to Seminal Coase

Key message: Coasian dynamics simply amount to market clearing.

Number of Varieties	Single		Multiple		
OMC Profit	0	+	0	+	
Gaps	No	Yes	No	No	Yes
Market Clearing	Yes	Yes	Yes	Yes	Yes
Bound on PBE Profit	OMC	OMC	OMC	OMC	OMC
Limit WME Profit	OMC	OMC	OMC	OMC	OMC
Time to Clear	Infinite	Finite	Infinite	Infinte	Finite
Efficiency	Yes	Yes	Possible	Rare	Rare
"Competitive" Pricing	WME	Yes	Possible	No	No
PBE Late Mixing	No	No	–	–	Yes
PBE Uniqueness	No	Yes	No	No	No

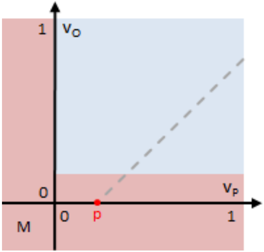
Coasian dynamics lead to OMC, not to efficiency or competitive pricing.

This is **not a failure of Coase conjecture** but a property of MC profits.

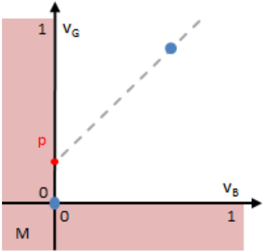
Failures of the Coase Conjecture

Related results however are often cast as **failures of Coase conjecture**:

- Outside options: Board & Pycia 2014.
- Damaged products: Hahn 2006.
- Vertically differentiated products: Inderst 2008, Takeyama 2002.



Outside Options



Vertical Differentiation

Staying in the Market or Committing to Exit

Buyers **exit the market** whenever they purchase a variety.

Such an assumption is without loss:

- if everyone purchases the preferred variety;
- if goods are consumed when purchased;
- if players commit to stay out of the market.

Otherwise buyers may:

- **remain in the market**;
- scrap their own good to buy the other;
- if it is preferred and sufficiently cheap!

This **changes** the shape of the **market clearing price** set.

Staying in the Market: Market-Clearing

Now, when a buyer v purchases variety i their values transition to

$$v_i' = 0 \text{ and } v_j' = v_j - v_i.$$

Our notion of market-clearing amounts to **depletion of gains from trade**.

Thus, for a price to clear the market:

- all buyers purchase a variety at that price;
- no buyer should switch variety at any price above its marginal cost.

The **set of market-clearing prices** then amounts to

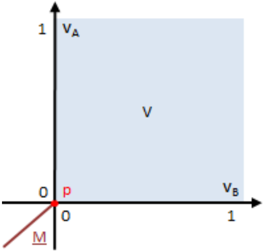
$$\underline{M} = \left\{ p \in M \mid v_i - p_i \geq v_j - p_j \;\Rightarrow\; c_j \geq v_j - v_i \text{ for all } v \in V^+ \right\}.$$

Staying in the Market: Without Costs

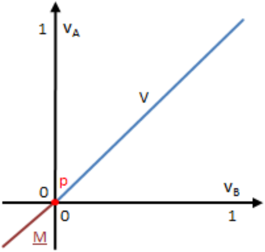
This **changes** the shape of the **market-clearing price** set

$$\underline{M} = \{p \in M \mid p_a = p_b\}.$$

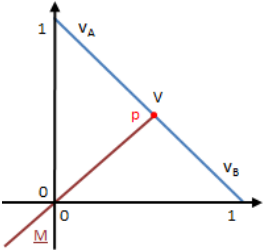
Without exit: if $\mathbf{0} \in V$ & $\mathbf{c} = \mathbf{0}$, an efficient zero-profit PBE exists.



Independence



Concordance



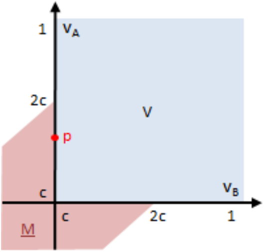
Discordance

Staying in the Market: With Costs

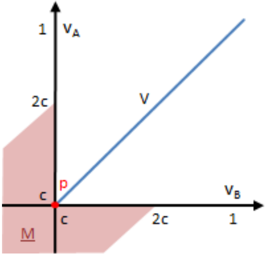
Costs **change** the shape of the **market-clearing price** set

$$\underline{M} \supseteq \{p \in M \mid c_a \geq p_a - p_b \geq -c_b\}.$$

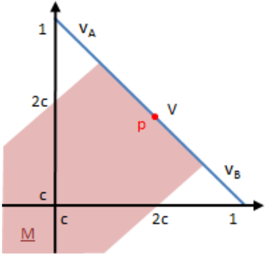
Without exit: if $\mathbf{c} >> \mathbf{0}$ though we are back to exit case.



Independence



Concordance



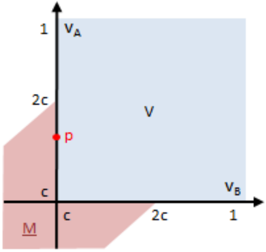
Discordance

Fact

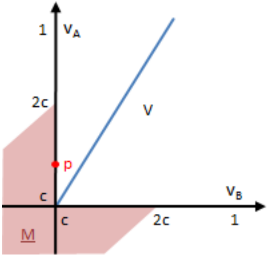
If buyers remain active and the market is regular, optimal MC profits:

(1) are a lower bound on PBE profits;

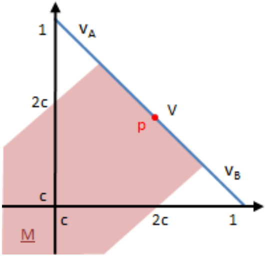
(2) coincide with the limit of MPE profits as δ converges to 1.



Independence



Concordance



Discordance

MPE vs PBE Folk Theorems

Results show that robust features of Coasian dynamics are that:

- MPE profits converge to OMC profits;
- PBE profits are bounded below by OMC profits;
- Unimprovable agreement (aka market clearing) being the essence.

The analysis did not establish whether non-stationary PBE exist that achieve higher profits than OMC as in Asubel & Deneckere 1989.

With 1 variety, a Folk Theorem arises only if OMC profits are equal to 0.

But as MPE may not be not unique with multiple varieties, **non-stationary PBE could exist** even when OMC are positive.

Competition

Differentiated Durable Goods: Competition

We consider scenarios in which:

- each firm sells a single variety;
- more than one firm can be active in the market;
- **varieties sold by competitors can be differentiated**.

We study the **effect of competition on the profit of an incumbent.**

Results establish that with competition **market power**:

- **can increase in any PBE with discordant varieties!**
- decreases **in any MPE with concordant varieties.**
- **discordance arises when competitors choose varieties**.

Similar results apply when firms can offer product lines.

A Model of Competition and Entry

Call one of the firms the **incumbent** and the other **entrants**.

First, we study how entry affects the profit of the incumbent.

When there is a single active firm and buyers are patient:

- monopoly profit must be close to w_I in any MPE;
- monopoly profit must be close to w_I in any PBE if $w_I > 0$.

Then, we **endogenize the choice of varieties**.

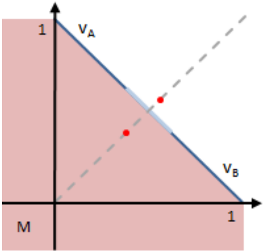
We do so, in a spatial competition model in which firms:

- **choose where to sell the product** before entering the market;
- set prices given the chosen locations after that.

Competition as Market Power I

If an entrant produces a discordant variety:

- the **worst buyers** for the incumbent are **supplied by the entrant**;
- the **incumbent is less inclined to undercut** on prices;
- the **profit** of the incumbent **can increase in any PBE.**



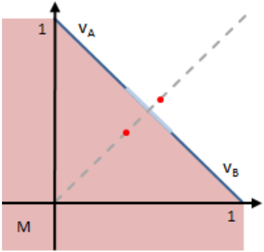
Competition as Market Power II

With discordance, the incumbent's **MPE profit** also **increases**.

Collusion is not required to raise the incumbent's profit.

In any equilibrium, the market eventually clears.

Prices can belong to the interior of the market-clearing set.

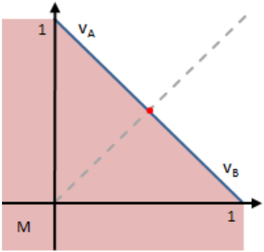


Competition and Extreme Market Power

MPE with immediate clearing can arise.

Paradoxically in such equilibria the incumbent's profit can coincide:

- **with the static monopoly profit when there is entry;**
- **with the competitive profit when there is no entry**.

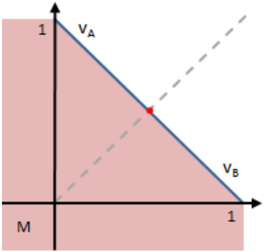


Competition: Discordance

The analysis develops **conditions on the measure of buyers for incumbent's profit to increase in any MPE** after entry.

When the incumbent suffers from the lack of commitment, **discordance**:

- is sufficient for the incumbent's MPE profit to strictly increase;
- is not necessary for the incumbent's MPE profit to increase.

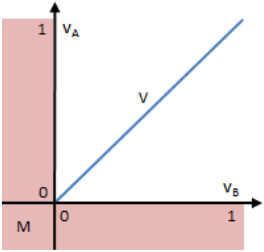


Competition: Concordance

The analysis develops **conditions on the measure of buyers for incumbent's profit to increase in any MPE** after entry.

If the entrant supplies a **concordant variety** however:

- the incumbent's profits decreases in any MPE;
- the incumbent's profits can still increase in a PBE (Gul '87).



Competition: Location Choice

Consider a **1-dimensional space and a single product**.

Varieties are identical except for the location at which they are sold.

For now, we study a setting in which:

- **at time zero firms choose where to sell the product**;
- in every other period firm set prices given their locations;
- in every other period buyers decide which variety to purchase.

Conclusions establish that with two firms:

- **never** choose **concordant** varieties;
- **always** choose varieties with **some discordance**;
- often choose to maximize product discordance.

Under weak conditions, the incumbent's profit increases with entry as competition prevents undercutting to sell to low value buyers.

Conclusions and Related Developments

We study **dynamic pricing problems without commitment** in a differentiated unit-demand setting.

Monopoly conclusions view **the Coase conjecture** as a robust result relating **limit stationary pricing** to a **simple static** pricing **problem**.

Conclusions on competition showed that with differentiated products **entry can increase market power** even when firms are unable collude.

Related projects:

We have developed **product design** results for the monopoly case.

We are studying a related **mechanism design** problem.

We are studying a related **partial commitment** problem.

Product Design

Product Design

Consider the implications of Coasian price dynamics on **product design**.

In particular we consider two product design exercises for the monopolist:

1. The first fixes marginal distributions for every variety and asks what **correlation structure** maximizes the optimal MC profit.
2. The second asks whether the monopolist prefers to reduce the **variance** of values for a given correlation structure.

The two main insights establish that:

1. **Horizontal differentiation are profit maximizing**.
2. **Niche products can be profit maximizing.**

Classical Results on Copulas

A function $C : [0, 1]^2 \to [0, 1]$ is a **copula** if:

- $C(k, 0) = C(0, k) = 0$;
- $C(k, 1) = C(1, k) = k$;
- $C(x) + C(y) \geq C(x_1, y_2) + C(y_1, x_2)$ whenever $x \geq y$.

Fact (Sklar's Theorem)

For any joint F with continuous marginals F_i there is a unique copula C st

$$F(v) = C(F_a(v_a), F_b(v_b)) \quad \textit{for all } v \in V_a \times V_b.$$

Theorem (Frechét and Hoeffding Bounds)

For any joint distribution F consistent with the two marginals,

$$L(v) \leq F(v) \leq U(v) \quad \textit{for all } v \in V_a \times V_b.$$

Copula: Examples & Bounds

- The **independent** copula exhibits **no dependence**:

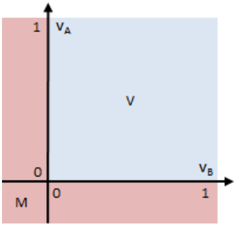
$$I(v) = F_a(v_a)F_b(v_b).$$

- The **FH upper bound** identifies the most **concordant** copula:

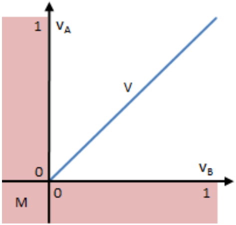
$$U(v) = \min\{F_a(v_a), F_b(v_b)\}.$$

- The **FH lower bound** identifies the most **discordant** copula:

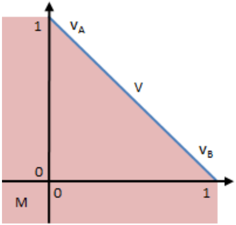
$$L(v) = \max\{F_a(v_a) + F_b(v_b) - 1, 0\}.$$



Independence



Upper Bound



Lower Bound

Profit Maximizing Copula

We identify **how dependence affects the optimal MC profits**.

Let $\bar{\pi}(F)$ be the optimal MC profit when the joint is F and solve

$$\max_C \bar{\pi}(F) \;\; \text{s.t.} \;\; F = C(F_a, F_b).$$

Theorem

For any joint F consistent with marginals F_i, it must be that

$$\bar{\pi}(F) \leq \bar{\pi}(L).$$

Full horizontal product differentiation is profit maximizing.

Intuitively, selling discordant varieties maximizes profits as segmenting the market minimizes the value is waisted on varieties that are not purchased.

Some Intuition: Profit Maximizing Copula

Let p denote the optimal MC prices given F.

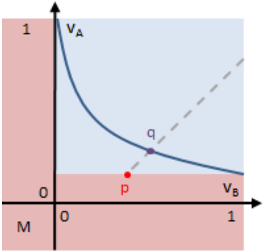
To prove this, find prices $q \in M(L)$ such that

$$q \geq p \quad \text{and} \quad d_i(p|F) = d_i(q|L).$$

As $V(L)$ is a non-increasing set, for all $\hat{p} \in V(L) \subseteq M(L)$:

$$d_i(\hat{p}|L) = \mathcal{F}\left(v_i \geq \hat{p}_i|L\right) = 1 - F_i(\hat{p}_i).$$

But if so, consider the price q satisfying $1 - F_i(q_i) = d_i(p|F)$.



Product Differentiation: Example

Consider two products with marginals uniformly distributed on

$$[0,1] \times [0,x] \quad \text{for} \quad x \in (0,1].$$

For our 3 typical copulas, optimal MC profits satisfy

C	Upper Bound	Independent	Independent	Lower Bound
B	$[0,1]$	$[0,2/3]$	$[2/3,1]$	$[0,1]$
$\bar{p}_a$	$(1-x)/2$	$(2-x)/4$	$1/3$	$1/2$
$\bar{p}_b$	0	0	0	$x/2$
$\bar{d}_a$	$1/2$	$(2-x)/4$	$2/(9x)$	$1/2$
$\bar{\pi}$	$(1-x)/4$	$(2-x)^2/16$	$2/(27x)$	$(1+x)/4$

Clearly **profits** are **maximized at the lower bound**.

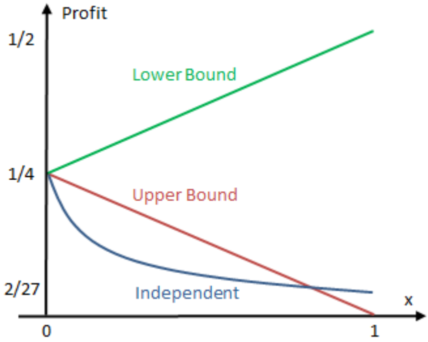
However the upper bound and the independent copula are unranked.

Product Differentiation: Example

Consider two products with marginals uniformly distributed on

$$[0,1] \times [0,x] \quad \text{for} \quad x \in (0,1).$$

For our 3 typical copulas, optimal MC profits satisfy



Designing Marginals: Mass vs Niche

With 1 variety, monopolist always **sets variance to** 0 (mass product).

A **trade-off** emerges with multiple varieties, as **variance increases**:

- buyers' **information rents** (bad for profits);
- **total surplus** as the maximal value grows (good for profits).

We ask whether the monopolist **prefers a distribution** F **to** the distribution in which all buyers value products at **the mean of** F

$$\hat{F} = [E(V|F)] .$$

We find that **variance helps only if**:

- values are discordant and
- the support is a concave map.

Designing Marginals: Liking Niche Products

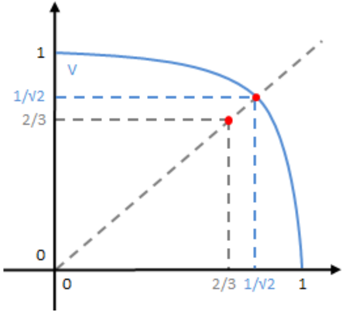
Consider the discordant distribution

$$L(v) = \max\{v_a^2 + v_b^2 - 1, 0\}.$$

The monopolist clears the market by selling all units at a price of $1/\sqrt{2}$.

The **monopolist likes the variance** as

$$\bar{\pi}(L) = 1/\sqrt{2} > 2/3 = \bar{\pi}(\hat{L}).$$



Designing Marginals: Liking Mass Products

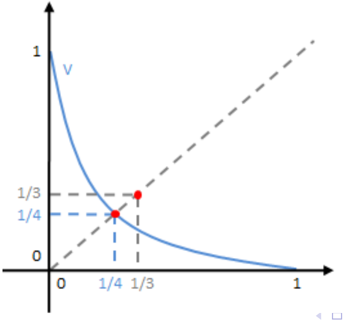
Consider the discordant distribution

$$L(v) = \max\{v_a^{1/2} + v_b^{1/2} - 1, 0\}.$$

The monopolist clears the market by selling all units at a price of 1/4.

The **monopolist dislikes the variance** as

$$\bar{\pi}(L) = 1/4 < 1/3 = \bar{\pi}(\hat{L}).$$



Omitted Materials

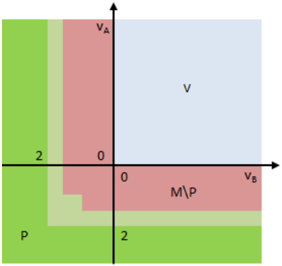
It suffices to show that all buyers buy at any price in the interior of M.

Let P be the **set of prices accepted by all buyers** "at any history".

Clearly, it must be that a price $p \in P$:

(i) if $\max_i p_i < -2$ as the payoff of all buyers exceeds 1.

(ii) if $p \leq \hat{p}$ for some $\hat{p} \in P$, as their surplus is higher than at $\hat{p}$.



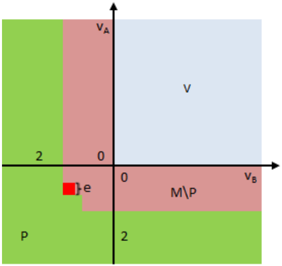
By contradiction consider a price $\hat{p} \in \bar{M} \backslash P$ such that

$$(\hat{p}_a - \varepsilon, \hat{p}_b) \in P \quad \text{and} \quad (\hat{p}_a, \hat{p}_b - \varepsilon) \in P.$$

As $\bar{M} \backslash P \neq \emptyset$, such a price $\hat{p}$ exists for any small of $\varepsilon > 0$ by (i) and (ii).

But when ε is sufficiently small:

$$\max_i \{v_i - \hat{p}_i\} > \delta \max_i \{v_i - \hat{p}_i + \varepsilon\}.$$



If so, for ε small all consumers would accept $\hat{p}$ for any PBE belief.

If a type was to reject an offer:

- she could agree no sooner than tomorrow;
- the most she could expect any one price to drop is ε;
- she would get a smaller payoff than by accepting $\hat{p}$.

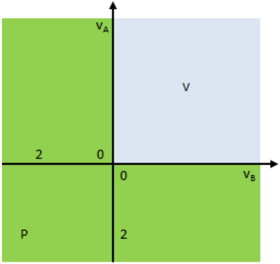
Intuition: Bounding Lemma SKIP

This contradicts the definition of P and establishes (1) and (2).

By (2), all buyers purchase a variety when prices are interior to M.

If so, the seller can secure a payoff arbitrarily close $\bar{\pi}(A) > 0$

But then (3) holds.



Some Intuition: Time to Clear Lemma SKIP

The proof establishes that **regardless of** buyers **beliefs** the monopolist:

- sets $p \in M$ if $w_g > 0$ and the measure of active buyers is small

$$\mathcal{F}(A^t) \in B_\varepsilon(0);$$

- sells to a positive measure of buyers in every period.

If the measure of those trading in next s periods were bounded above by

$$\mathcal{F}(A^t) - \mathcal{F}(A^{t+s}) < \eta,$$

PBE profits would be bounded by

$$\Pi(h^t) < \eta + \delta^s.$$

But for η small and s large, a contradiction would emerge since

$$\eta + \delta^s < \bar{\pi}(A^t).$$