Information Disclosure and Commitment: Multi-stage Quorum Rules

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Abstract. *The paper discusses how equilibrium outcomes of quorum rules are affected by different commitment and information structures. Under complete information, these rules can be manipulated by agents opposing a reform. If information is incomplete, instead, and commitments are chosen simultaneously, then the extent of the manipulation decreases. If we model the timing of their commitments to be endogenous and observable, opponents of the reform may again, by eliciting the relevant information, attain the complete information outcome, even though no information about votes was ever disclosed. Manipulation is, also, mitigated by multi-topic quorum rules, because information on timings is no longer sufficient to elicit all information. If contracts on observables are allowed, abstention-buying arises and super-majorities may be reversed. Some evidence supporting the predictions is discussed.*

Keywords and Phrases: Secrecy and Voting

1. Introduction[1]

This paper discusses how strategic voting behavior affects the outcome of a majority quorum[2] consultation when the timing of actions is endogenous and information on early commitments is potentially observable. The paper focuses on quorum rules because they provide a simple analytical example of a strategic environment in which only a subset of agents may profit from disclosure. Such rules have been often used in European plebiscites and are employed with different information structures in many parliamentary democracies.[3] For the moment we only look at the voting stage and not at the policy proposal stage. In the model the timing of actions will serve the purpose of a potentially costly communication system. Many papers document the effects of sequential voting procedures on the outcomes of preference aggregation mechanisms. But in most there is no mention about the fact that the choice on when to act ought to be considered as endogenously determined by the information structure.

We study quorum rules when information about preferences in the population is incomplete. We find that, even when agents may observe only the timing, but not the nature, of the others' commitments, the set of equilibrium payoffs of the mechanism may be, critically, affected. In fact, some of agents may, by exploiting information about timings, increase the equilibrium ex-ante probability of rejection for the policy, to a level that could have been attained in a secret quorum rule, only if all uncertainty about preferences were resolved before the voting stage. Because the model will make no assumption on the size of the committee, except for its common knowledge, all the results focus on a perfect symmetric refinement, to contain the multiplicity of equilibria.[4] We use the ex-ante probability of rejection of the policy as a measure of the extent to which agents may manipulate the mechanism, by using their timely commitments in order to communicate information. In fact, for any given quorum rule the maximal ex-ante equilibrium probability of rejection is shown to be increasing in the level of disclosure about others' commitments, because the extent of coordination among opposers may only improve. We start with a non-transferable utility model, with private, but possibly correlated types, with no cost of voting. We discuss the independent values case, but in general, we refrain from such assumption, in order to allow for uncertainty about type totals even in large populations.

We analyze multiple topic quorum rules. Such rules require agents to commit on multiple referenda at once. We find that whenever preferences among policies are perfectly correlated, none of the results is affected. We argue that if preferences are less correlated, though,

[1] The current draft consists of the preliminary analysis an application of a theoretical project still under development. An updated version of the paper will be posted regularly on *home@uchicago.edu/~franava*. In double brackets, [[·]], work that has been developed, but that does not belong to the current draft, and imminent modifications are reported. Please excuse any mistakes in this preliminary deadline-driven version.

[2] A quorum majority consultation is one in which a reform is enacted whenever a certain fraction of agents votes and the majority of that fraction votes in favor of the reform.

[3] In order for the assembly to pass a bill, it may required that a certain fraction of its members be present at the voting stage. A quorum threshold of 51 senators is required in the US senate.

[4] The symmetry refinement is justified by the assumption that agents are ex-ante identical. Hence, identities should not matter in equilibrium.

the extent of equilibrium manipulation is weakly smaller than it would be were the two consultations held separately. The result relies on the reduction of the extent of coordination among opponents of each reform. Currently, the multiple quorum analysis is limited to double quorum rules.

A section of the paper explores, through exemplary cases, the consequences of a transferability in utility assumption. When a transferable good and contracts are introduced in the game [denote TU], we argue that the extent of manipulation greatly increases, as soon as the complete secrecy assumption is dropped. In fact, having voters endogenously choose the timing of their action in a TU model, guarantees that verifiable contracts on abstention exist. Let us remark that in the TU setup much of the outcome of manipulation depends on the bargaining power of the agents, which in turn will depend on their utility from the policy. The higher the level of disclosure the bigger the extent of the manipulation, given that more incentive compatible contracts become available and markets for votes may arise. The TU analysis is at the current stage based on exemplary cases.

In most quorum rules observed preferences [voting choices] do not correspond to actual preferences [policy preferences], because of strategic voting. Hence, identification of voting preferences form vote profiles may be impossible. This follows because sincere voting may not be an equilibrium of a quorum rule and because even if it were, it would never be the unique equilibrium strategy for any possible rule and committee. Secrecy, in our model, favors the sincere voting as an equilibrium, because it reduces the extent of manipulation, but is not sufficient for it.

Some stylized facts on quorum rules we would like our model to capture are: (i) information on timing may induce reversals of relative majorities even in mechanisms with secret commitments; (ii) incomplete information and simultaneous action favor non-strategic behavior; (iii) multi-topic referenda may serve to reduce the extent of manipulation when timing are observable; (iv) in a TU setup public commitment timings may lead to reversals of absolute majorities of reformists; (v) the lack of secrecy may induce abstention levels to increase even if the cost of a vote were reduced.[5]

We report some descriptive evidence from Italian referenda which seems to support the presence of strategic abstention. But as expected strategic abstention seems to affect more single quorum or correlated multi-topic ones, than multi-topic quorums with highly heterogeneous subjects. The empirical analysis is still limited by of the relatively small sample size, but it will increase and improve with the sample.

The paper is structured as follows: in the next section, we discuss existing results and literature on the effects of sequential voting and endogenous commitment timing in a non transferable utility framework. Section three defines notation. Section four describes the, initially binary, Bayesian collective choice problems that lie in the scope of the analysis and characterizes quorum mechanisms with different levels of disclosure and commitment. For the different rules, behavior in perfect symmetric equilibria is characterized and used to

[5]This has been the case in Switzerland after the plebiscites shifted from a vote at the poll to a vote by mail.

determine the extent of strategic manipulation. In this section predictions about timing and commitments are also discussed for binary rules. In the latter section, we argue that strategic manipulation in quorum rules with public abstention may be reduced by holding multiple referenda at once, if agents can only commit to all questions at once. In the following section, we outline how results are affected by introducing a transferable good in the game. We argue that if timing of commitments is observable, a market for abstention should arise, given that some individuals strictly benefit from others abstaining and that abstention is an observable and, hence, contractable action. Finally we discuss some descriptive evidence on 50% quorum rules coming from Italian single and multi topic referenda held from 1974 to 2005. We conclude by outlining results and by describing the general project we aim at developing. Proofs are reported in the appendix.

2. Related Literature

[[To be added by the proposal date.]]

3. Notation

Let us introduce some notation and conventions used throughout. When no confusion arises, we may use a set in a superscript to denote the cardinality of the set. The following conventions are also adopted to shorten notation: $Id(\cdot)$ denotes the indicator function,[6] $\Delta(\cdot)$ denotes the simplex of a finite set,[7] for any set N and element $i \in N$, let $-i$ denote the set $N\backslash\{i\}$ and for any collection of sets $\{H_i\}_{i\in N}$ let $H \equiv \times_{i\in N} H_i$ and $H_{-i} \equiv \times_{j\in N\backslash i} H_j$, and, finally, if $H_i = H_j$ for $\forall j, i \in N$, let $H_* \equiv H_i$. Similar conventions are used to denote elements and and functions, when necessary and clear. Also, for any b-dimensional vector $h \in \times_{i=1}^{b} H_i$ let:

$$n(k|h) \equiv \textstyle\sum_{i=1}^{b} Id(h_i = k) \quad \text{for} \quad \forall k \in \cup_{i=1}^{b} H_i \quad \& \quad n(h) \equiv \{n(k|h)\}_{k\in\cup_{i=1}^{b} H_i} \tag{3.1}$$

Hence, $n(k|h)$ denotes the number of components of vector h taking value k Sometimes, we denote $(h, h') \in H \times H'$ by $h.h' \in H.H'$ to keep notation more compact. Whenever H is a product space of dimension b [i.e. $H = \times_{i=1}^{b} H_i$], for $\forall H' \subseteq H$ and $\forall B \subseteq \{1, ..., b\}$, let us denote the coordinate image of set H' on the subspace $\times_{i\in B} H_i$ by:

$$\pi_B H' = \left\{h_B \in \times_{i\in B} H_i : h_j = h'_j,\ \forall j \in B \ \ \&\text{ some } h' \in H'\right\}$$

4. Quorum Rules

In this section we discuss what are the consequences of different information and commitment structures on the equilibrium outcomes of a binary referendum quorum rule. We claim that whenever agents are allowed to commit to a vote at different stages and the tim-

[6] $Id(A) = 1$ if the predicate A is true and $Id(A) = 0$ otherwise.

[7] $\Delta(A) \equiv \left\{p \in [0,1]^{|A|} : \sum_{a\in A} p(a) = 1\right\}$ for any A such that $|A| < \infty$.

ing of commitments is observable, strategic behavior in quorum rules biases the outcome of the consultation in favor of the *status quo.* First, we describe the nature of the mechanisms and committees considered in the analysis, later equilibrium behavior. This section and the next focus on non-transferable utility models, section six discusses how one could relax this assumption and the expected consequences.

4.1. Mechanisms: Commitment and Information Structures

We start with a collective choice problem [BCC] $\Gamma = \left\{N, Y, \{S_i\}_{i\in N}, p, \{u_i\}_{i\in N}\right\}$. Here N denotes the set of agents, Y denotes the set of social outcomes, $S = \times_{i\in N} S_i$ and p denote the Cartesian product of the type spaces and the common prior on S, respectively, and $u_i : Y \times S \to \Re$ denotes the utility index of agent i. For the moment, we assume the outcome set to be binary, $Y = \{1, 0\}$, and that values are private, $u_i : Y \times S_i \to \mathbb{R}$, even though possibly correlated, $p(s_{-i}) \neq p(s_{-i}|s_i)$. For descriptive purposes, one may interpret the outcome 1 as the reform, and the outcome 0 as the *status quo.* Without loss of generality the identity of the agent may well be included as a part of the type. Hence, the utility may be written as $u : Y \times [N \times \cup_i S_i] \to \mathbb{R}$. But for analytical convenience, we, initially, assume that agents preference differ because of their type, but not because of their identity, $u : Y \times \cup_i S_i \to \mathbb{R}$. Because the labelling of types is not relevant and p is not yet specified, we assume that $S_i = S_*$ for $\forall i \in N$. Alternatively, we may enlarge, the individual type spaces so that $S'_i = S_* = \cup_{i\in N} S_i$ for $\forall i \in N$ and that the prior so that $p'(s) = Id(s \in S)p(s)$ for $\forall s \in S'$. But let us note that the latter approach, even though more general, leads to the addition of zero probability events that may affect equilibrium behavior. Given the assumptions, we write the BCC problem as $\Gamma = \{N, Y, S_*, p, u\}$. Let us remark that the BCC is symmetric, except for the prior which needs not to be.[8] We chose this approach in order embed in (S_*, p) all relevant information about agent types. For the moment, we assume utility to be non-transferable [A1]. Let us remark that we have ruled out any cost of voting by assuming that the utility index depends only on outcomes and types, but not on actions chosen. For any $s_i \in S_*$,we denote by $x(s_i) \equiv u(1|s_i) - u(0|s_i)$ the utility difference of the two policies. The set of types preferring the reform is denoted by $\overline{S} \equiv \{\overline{s} \in S_* : x(\overline{s}) > 0\}$ and the one of types opposing the reform is denoted by $\underline{S} \equiv \{\underline{s} \in S_* : x(\underline{s}) < 0\}$. Then, for any given profile of types $s \in S$ of the population, we denote the number of supporters of each option by:

$$\overline{n}(s) \equiv [\overline{n}(1|s), \overline{n}(0|s), \overline{n}(a|s)] \ \& \ \overline{n}(1|s) \equiv \textstyle\sum_{i\in N} Id(x(s_i) > 0)$$
$$\overline{n}(0|s) \equiv \textstyle\sum_{i\in N} Id(x(s_i) < 0) \ \& \ \overline{n}(a|s) \equiv \textstyle\sum_{i\in N} Id(x(s_i) = 0)$$

We refer to the first group of agents as the reformists, to the second group as the *status quo* supporters and to the others as the indifferents. We assume that there is an ex-ante positive measure of individuals that are indifferent between the two policy outcomes. That is we

[8] Ex-ante utility of any given action profile may still differ across agents.

assume that $\overline{n}(a) \equiv \sum_S \overline{n}(a|s)p(s) > 0$ [A2]. This implies that for some $i \in N$ and $s_* \in S_*$ it must be that $\overline{n}_i(a|s_*) \equiv \sum_{S_{-i}} \overline{n}(a|s)p(s_{-i}|s_*) > 0$. For a more restrictive assumption, and , $\overline{n}_i(a|s_*) > 0$ for all $i.s_* \in N.S_*$, the conclusions we draw would be strengthened. We assume that there is a positive fraction of indifferent agents to show how their presence may be exploited by supporters of the *status quo*. For any given agent type $i.s_i \in N.S_*$, we may compute the probability distribution of the number of of agents of each type in the population by:[9]

$$g_i(k|s_i) \equiv \textstyle\sum_{S_{-i}} Id(n(s) = k)p(s_{-i}|s_i) \text{ for } \forall k \in \left\{q \in \mathbb{N}^{S_*} : q'\imath = N\right\} \equiv L^{S_*}$$

A similar computation defines a measure over the number of supporters of each group $\overline{n}(s)$.[10] In our setup we want the type of an agent not only determine her preferences order on the social alternatives, but also determine her information about the distribution of types in the population at the interim stage. In fact, agents of different types may display the same preferences over social alternatives, but different information about the preferences in the population. Of course such difference in information has got to be consistent with the common prior assumption. Note that one could enlarge the type space S_* to a space $\overline{S}_*$, in such a way that $g_i(\overline{s}) = g(\overline{s}) \in \Delta(L^{\overline{S}_*})$ for any $\overline{s} \in \overline{S}_*$ by relabeling types. Types in $\overline{S}_*$ would, then, completely characterize the preferences of the agent and her interim probability distribution on type totals. The relabeling does not guarantee that the interim distribution of different agents of the same type corresponds everywhere,.but it asserts that it cannot differ in a strategically sensible way once symmetry is invoked if utility is non-transferable. Given this remark, we directly proceed by assuming that p displays sufficient symmetry to have $g_i(\overline{s}) = g(\overline{s})$ for $\overline{s} \in S_*$. It is immediate to verify that this condition holds if types are independent.

A simultaneous ε-quorum rule is an anonymous single-stage mechanism $m = \{C_*, \nu\}$. Where we denote by $C_* = C_i$ the choice set for any agent $i \in N$ and where $\nu : C \to Y$ denotes the mapping from action profiles to outcomes. Specifically, for the quorum rules studied in this section, we assume that: $C_* = \{1, 0, a\}$ and that $\nu(c) = Id(n(1|c) > n(0|c))\, Id(N - n(a|c) > N\varepsilon)$ for $\forall c \in C \equiv C_*^N$. Hence, $\varepsilon \in [0, 1]$ is a parameter characterizing the fraction of voters necessary to enact the reform.[11] The mechanism and the collective choice problem induce a Bayesian game of incomplete information defined by: $\Gamma^B = \{N, C_*, S_*, p, \overline{u}\}$, for $\overline{u}(\cdot|s_i) \equiv u(\nu(\cdot)|s_i)$. If the prior is degenerate, $p(s) = 1$, we denote the corresponding complete information game as $\Gamma^C(s) = \left\{N, C_*, \overline{\overline{u}}\right\}$, for $\overline{\overline{u}}(\cdot) \equiv u(\nu(\cdot)|s_i)$. If the interim distributions are degenerate for any given type $i.s_i \in N.S_*$, $p(s_{-i}|s_i) = 1$, agents know all the information at any interim information state and play according to the complete information game $\Gamma^C(s)$.

Simultaneous mechanism hold as an assumption that all agents have to commit to their

[9] Where $n(s)$ is defined by [3.1] and ι denotes the unit vector of dimension S_*.

[10] In that case, $\overline{g}_i(k|s_i) \equiv \sum_{S_{-i}} Id(\overline{n}(s) = k)p(s_{-i}|s_i)$, for $\forall k \in \left\{q \in \mathbb{N}^3 : q'\imath = N\right\} = L^3$.

[11] If $\varepsilon = 0$ the quorum rule corresponds to majority. If $\varepsilon = 1$ it is equivalent to unanimity.

actions at the same stage. Or, alternatively, they assume that agents may commit sequentially, but that by doing so they cannot disclose any information before the final commitment stage.[12] In many cases, though, quorum rules are implemented without such stringent requirements. In fact, an alternative rule may allow agents to choose when to commit to any given action and allow others to observe the timing of such commitments or some of the predetermined actions. Alternatively, it could allow for pre-play communication, mediated or not. Let us note whenever the timing of early commitments may be observed at latter stages it may serve as a channel for coordinating actions. Additionally, note that if one assumes the actions at a given stage to be non-binding at latter stages[13] then it could be argued that the sole scope of that stage is cheap talk.

For instance, consider the following multi-stage anonymous history independent mechanism with timing of commitments and disclosure: $M = \{T, C_*, Z, Y, \alpha, \sigma, \nu_M\}$. Where M is such that:

$$\begin{gathered}
C_* \equiv \{1, 0, a\} \quad \text{for } \forall i \in N \\
\alpha(c_{i.t}) \equiv \left\{ \begin{array}{ll} C_* & c_{i.t} = \{a, \emptyset\} \\ c_{i.t} & c_{i.t} \in \{1, 0\} \end{array} \right\} \text{ for } \forall i.t \in N.T \\
C^T(\alpha) \equiv \left\{ c^T \in C^T : c_{i.t} \in \alpha(c_{i.t-1}),\ \forall i.t \leq N.T \right\} \\
\nu_M\left(c^T\right) \equiv \nu\left(\pi_T c^T\right) = \nu\left(c_T\right) \quad \text{for } \forall c^T \in C^T(\alpha) \\
\sigma : T \times C^T(\alpha) \to \Delta(Z)
\end{gathered} \tag{4.1}$$

Here, T denotes the discrete [finite or countable] collection of time periods in which agents may take actions. A countable T, may be used to describe a strategic environment in which agents have an arbitrary large number of stages in which to commit before some a given time period.[14] By C_* and α we denote the initial choice set and a commitment mapping, identifying which deviations are available at time $t+1$ to an agent after any given stream of commitments leading to $c_{i.t} \in C_*$ at time t. $C^T(\alpha)$ denotes the set of possible action profiles for the commitment map α. If T is infinite we consider $c_T \equiv \lim_{t \uparrow \infty} \pi_t c^t \in C$ for any $c^T \in C^T(\alpha)$.[15] The outcome map $\nu_M : C^T(\alpha) \to Y$ is assumed to depend only on the limiting commitments and to be defined by the simultaneous quorum outcome map. Finally, the mapping σ defines the distribution of the publicly observed signal, known to agents before the choice instance, as a function of predetermined commitments. Specifically, we assume that for any $t > 1$ and $c^T \in C^T(\alpha)$ we have that $\sigma(t, c^T) = \sigma_t(c^{t-1}) \in \Delta(Z)$. This implies that at any stage agents' the signal depends solely on predetermined commitments through its distribution. By Z we denote the appropriately chosen space of realizations of the

[12] This intuitive claim will be proven in what follows.

[13] We refer to action that bind behavior at latter stages as commitments, and to non-binding actions, simply as actions.

[14] Since one may define a one-to-one map form $\mathbb{N}$ to any countable subset of any interval $[\underline{t}, \bar{t})$.

[15] The limit on the discrete set are well defined given that only sequences in $C^T(\alpha)$ are considered. For all such sequences a limit may be defined with respect to a discrete metric. Similar notation is used for other variables.

signal and by Z_t the signal as random variable. Throughout the analysis of the multi-period mechanism we assume that agents display perfect recall. Hence, at any given commitment stage the information available to an agent consists of her type, her stream of pervious commitments and of the stream of past signals. That is, $s_{i.t} \equiv (s_i, c_{i.1}, ..., c_{i.t-1}, z_2, ..., z_t) \in \Im_{i.t}$.[16]

We look at several information structures with different levels of disclosure. One that is interim-secret, so that for $\forall c^{t-1}, \overline{c}^{t-1} \in C^{t-1}(\alpha) \equiv \pi_{\{1,...,t\}} C^T(\alpha)$:

$$\sigma_t(c^{t-1}) = \sigma_t(\overline{c}^{t-1}) \tag{4.2}$$

One that is deterministic $\sigma_t(c^{t-1}) \in Z$ and displays public abstention totals at any interim stage for $\forall c^{t-1}, \overline{c}^{t-1} \in C^{t-1}(\alpha)$:

$$\sigma_t(c^{t-1}) = \sigma_t(\overline{c}^{t-1}) \Leftrightarrow n(a|c_{t-1}) = n(a|\overline{c}_{t-1}) \tag{4.3}$$

In which case, it is without loss to assume that: $\sigma_t(c^{t-1}) = n(a|c_{t-1})$, or the more empirically relevant $\sigma_t(c^{t-1}) = N - n(a|c_{t-1})$. An information structure with individual abstention totals may, similarly, be defined and has different implications whenever utility is transferable. An information structure with random public abstention totals in which at any interim stage only a random variable Z_t, distributed according to $\sigma_t(c^{t-1}) \in \Delta(Z)$ may be observed. And such that $\forall c^{t-1} \in C^{t-1}(\alpha)$ and $Z \supseteq N$:

$$E(Z_t|c^{t-1}) = \textstyle\sum_{z \in Z} z\sigma_t(z|c^{t-1}) = n(a|c_{t-1}) \tag{4.4}$$

Finally, a deterministic one, $\sigma_t(c^{t-1}) \in Z$, that displays full disclosure about past actions at all stages for $\forall c^{t-1}, \overline{c}^{t-1} \in C^{t-1}(\alpha)$:

$$\sigma_t(c^{t-1}) \neq \sigma_t(\overline{c}^{t-1}) \Leftrightarrow c^{t-1} \neq \overline{c}^{t-1} \tag{4.5}$$

Let us remark that all of the information structures described for the multistage rule only involve a public signaling. We denote the multistage game of incomplete information induced by the multistage mechanism by: $\Gamma^M(\sigma) = \left\{N, T, (C_*, \alpha), (S_*, p), (Z, \sigma), \{\overline{u}_i\}_{i \in N}\right\}$.

Before we turn to equilibrium analysis, let us, briefly, discuss properties of the outcome mapping of an ε-quorum rule. Note that in any quorum rule whenever agents are assumed to vote non-strategically,[17] there are profiles of types, $s \in S$, in which the reform passes, $\nu(\overline{n}(s)) = 1$, even though the share of its supporters in the committee is smaller than the quorum level, $\overline{n}(1|s) < N\varepsilon$. But for such a profiles of types the *status quo* supporters, $\overline{n}(0|s)$, may prevent the reform from passing by collectively abstaining. In fact, for $\widehat{n}(1|c) = \overline{n}(1|s)$ and $\widehat{n}(a|c) = N - \overline{n}(1|s)$ we have that $\nu(\widehat{n}(c)) = 0$. The quorum outcome map never allows

[16] For $\Im_{i.t} \equiv S_* \times C_*^{t-1}(\alpha) \times Z^{t-1}$ and $C_*^{t-1}(\alpha) \equiv \left\{c^{t-1} \in C_*^{t-1} : c_r \in \alpha(c_{r-1}),\ \forall r < t\right\}$.

[17] By non-strategic voting we mean sincere voting. That is: only indifferents abstain and supporters of each side vote according to their preferences. Technically, for $\forall s \in S$ we have $n(c(s)) = \overline{n}(s)$ for $c_i(s_i) = Id(u_i(1|s_i) > u_i(0|s_i)) + aId(u_i(1|s_i) = u_i(0|s_i))$.

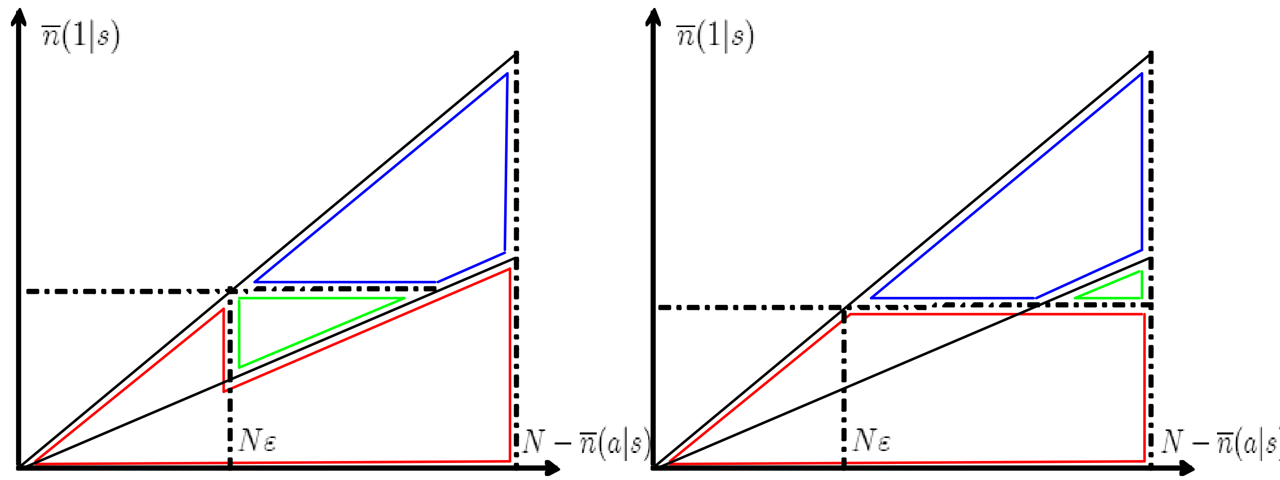


Figure 1: Sincere Voting (left) and Strategic Abstention (right) Acceptance (green-blue) & Rejection (red) Regions

reformists to benefit form a vote that does not reflect the preferences, but gives incentives to *status quo* supporters to coordinate their actions on either abstention or a vote in against the reform depending on the preferences in the population and the quorum level. In fact, for *status quo* supporters, whenever $\varepsilon \geq .5$, collective abstention always leads to a weakly preferred outcome. But, whenever $\varepsilon = 0$, a vote against is their only weakly dominant action. Voting for the reform is always weakly dominated if one opposes it. The left plot of figure 1 depicts the reform acceptance [blue and green] and rejection [red] region in the space of type totals in the case that agents vote sincerely for some $\varepsilon \in (0, .5)$. The right plot, instead, depicts the acceptance [blue and green] and the rejection [red] region whenever *status quo* supporters collectively abstain. Clearly, whenever there is uncertainty about the space of types, agents opposing the reform, if considered as a group, face a trade-off between voting sincerely, certainly loosing if the population profile of preferences is in the green region of the left plot, and strategically abstaining, loosing whenever the profile is in the green region of the right plot.

4.2. Equilibria and Disclosure

Throughout the analysis, the symbol γ will be used denote profiles of behavioral strategies. In a simultaneous quorum rule a profile γ, consists of a collection of maps $\gamma_i : S_* \to \Delta(C_*)$, one for $\forall i \in N$.[18] In a multi-stage rule, instead, a profile of behavioral strategy, γ^T, consists of a collection of maps $\gamma_{i.t} : \Im_{i.t} \to \Delta(\alpha(c_{i.t-1}))$, one for $\forall i.t \in N.T$. Hence, γ^T defines, for each agent at each stage, a map from information states, $s_{i.t} \in \Im_{i.t}$, to probability distributions over the set available commitments, $\alpha(c_{i.t-1})$, which is known by perfect recall, $c_{i.t-1} \in s_{i.t}$.

In this subsection, when analyzing the simultaneous rule, we assume that indifferents

[18]These are maps from the type space to the set of probability distributions over the action set.

always abstain $x(\overline{s}) = 0 \Rightarrow \gamma_*(a|\overline{s}) = 1$ [A3]. We restrict attention to this type of behavior because, even though voting costs are ruled out of the model, we want to entertain the possibility that a fraction of indifferents prefers to abstain.[19] An equivalent assumption will be made when analyzing multi-stage rules: $x(\overline{s}) = 0 \Rightarrow \gamma_{i.t}(a|\overline{s}) = 1$ for $\forall i.t \in N.T$ [A3′].

Given the assumptions, let us start discussing the equilibria and their properties for the simultaneous move quorum majority game. Suppose that uncertainty were completely disclosed at the interim stage, after types are revealed, but before actions are taken.[20] If this were the case, opponents of the reform may, by acting cohesively, maximize the ex-ante probability of rejection for the policy. In fact, there exists a symmetric perfect Bayes Nash equilibrium which leads to the rejection of the reform for any profile of types belonging to the union of the rejection regions discussed in figure 1. For instance, such rejection region, depicted in figure 2, is attained by the following perfect equilibrium strategies for $\forall s \in S$:

$$\gamma_{FI}(s) = \left\{ \begin{array}{lc} \gamma_{FI}(1|s) = 1 & x(s_i) > 0 \\ \gamma_{FI}(0|s) = 1 & x(s_i) < 0 \cap \overline{n}(1|s) > \varepsilon N \\ \gamma_{FI}(a|s) = 1 & \text{otw.} \end{array} \right\} \tag{4.6}$$

Let us remark that there is no other perfect equilibrium profile of strategies yielding a bigger rejection region for the reform in the simultaneous full disclosure game. In fact, if we denote by:

$$P^{FI}(0) \equiv 1 - \textstyle\sum_{s \in S} Id(\overline{n}(1|s) > N\varepsilon \vee \overline{n}(0|s))p(s)$$

the ex-ante probability of rejection of the full disclosure game when equilibrium [4.6] is played, we claim that:

Claim 1. *For any game Γ^M induced by a mechanism M satisfying assumptions [A1] and conditions [4.1], there exists no perfect equilibrium leading to a higher ex-ante probability of rejection for the reform than $P^{FI}(0)$.*

We will look with particular interest to equilibria maximizing the rejection region for different informational assumptions. We do so because the extent of such region may be used to measure the advantage that the mechanism gives to *status quo* supporters.

When information is incomplete, we assume that types profiles with zero prior or interim probability are treated as impossible by agents. We refine our search to symmetric [behaviorally equivalent types behave the same] perfect Bayes Nash equilibria [denote SPBE]. Because no assumption is made on the size of the population, but for its finiteness, and multiplicity problems may arise, perfection and symmetry enable us to focus on a sensible set of equilibria. We wish to compare mechanisms by the maximal value that the ex-ante probability of rejection for the reform may attains in some perfect symmetric equilibrium

[19]This is equivalently achieved through costs that are infinitesimal with respect to the probability of being pivotal.

[20]Full disclosure may result from the interim distribution being degenerate for any agent type. That is, $p(s_{-i}|s_i)$ for some $s_{-i} \in S_*^{N-1}$ and $\forall i.s_i \in N.S_*$. Or it may result from an informative signal about others' preference types after the own preference type was picked, which should also be modeled in p.

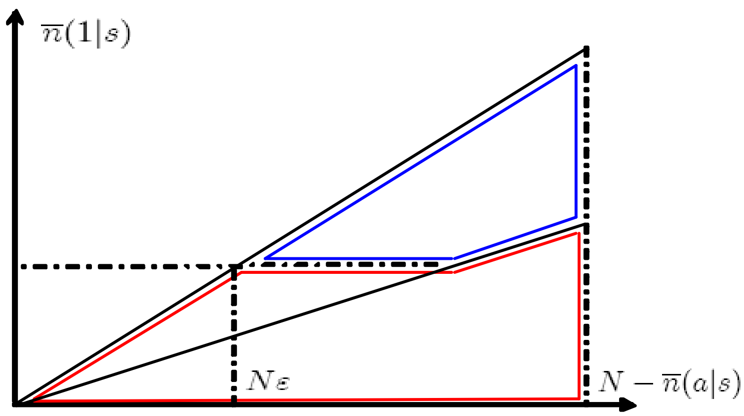


Figure 2: Full Disclosure Acceptance & Rejection Regions

of the induced game. We believe the comparison to be a sensible because it suggestive of the extent to which outcomes may be manipulated in equilibrium through the resolution of uncertainty. For any behavioral strategy profile γ, such that $\gamma_i : S_* \to \Delta(C_*)$ for $\forall i \in N$, we may define the ex-ante probability of rejection of the simultaneous game, Γ^B, by:

$$P^B(0|\gamma) = 1 - \sum_{s\in S} \left[\sum_{c\in C} \nu(c) \prod_{i\in N} \gamma_i(c_i|s_i)\right] p(s)$$

The agent expected utility at the interim stage,[21] is given by:

$$U_i(\gamma|s_i) \equiv \sum_{s_{-i}\in S_{-i}} \left[\sum_{c\in C} u(\nu(c)|s_i) \prod_{j\in N} \gamma_j(c_j|s_j)\right] p(s_{-i}|s_i) = u(0|s_i) + x(s_i)\Pr_i(1|\gamma, s_i)$$

$$\Pr_i(1|\gamma, s_i) = \sum_{s_{-i}\in S_{-i}} \left[\sum_{c\in C} \nu(c) \prod_{j\in N} \gamma_j(c_j|s_j)\right] p(s_{-i}|s_i)$$

Hence, all that matters to agents is how their actions affect the interim probability of the reform.

Since we focus on symmetric equilibria, we assume that whenever $s_i = s_j$ and, consequently, $[x(s_i) = x(s_j) \cap g(s_i) = g(s_j)]$, we have that $\gamma_j(s_j) = \gamma_i(s_i) = \gamma_*(s_i)$. By symmetry we have that for any $\overline{s} \in S_*$:

$$\Pr_i(1|\gamma, s_i = \overline{s}) = \Pr(1|\gamma, \overline{s}) = \sum_{l\in L(S_*)} \left[\sum_{c\in C} \nu(c) \prod_{s'\in S_*} \gamma_*(c_j|s')^{l(s')}\right] g(l|\overline{s})$$

Let us denote a probability distribution in the interior of the simplex by $\gamma_i^o : S_* \to \Delta^o(C_*)$. When agent act simultaneously, perfection requires that if $\overline{s} \in \overline{S}$, then:[22]

$$[\gamma_*(1|\overline{s}) = 1] \in \arg\max_{\overline{\gamma}_i \in \Delta(C_*)} U(\overline{\gamma}_i, \gamma_{-i}^o|\overline{s}) \text{ for } \forall \gamma_j^o : S_* \to \Delta^o(C_*)$$

$$\text{since } \Pr(1|\gamma_*, \gamma_{-i}^o, \overline{s}) \geq \Pr(1|\overline{\gamma}_i, \gamma_{-i}^o, \overline{s}) \text{ for } \forall \overline{\gamma}_i \in \Delta(C_*)$$

[21] After information about types is revealed.

[22] For $\Pr(1|\gamma_*, \gamma_{-i}^o, \overline{s}) = \sum_{l\in L(S_*)} \left[\sum_{c_{-i}\in C_{-i}} \nu(1, c_{-i}) \gamma(c_j|\overline{s})^{l(\overline{s})-1} \prod_{s'\in S_*\setminus\overline{s}} \gamma_*(c_j|s')^{l(s')}\right] g(l|\overline{s})$ and $\nu(1, c_{-i}) = Id(n(1|c_{-i}) + 1 > n(0|c_{-i}))\, Id(N - n(a|c_{-i})) > \varepsilon N)$.

That is: because a reformist can be pivotal in favor of the reform only by voting for it, on both the quorum margin $N - n(a|c_{-i}) = \varepsilon N$ and on the majority margin $n(1|c_{-i}) = n(0|c_{-i})$, she better do so in any perfect Bayesian equilibrium of the simultaneous game. For the simultaneous rule such behavior guarantees that the ex-ante probability of rejection cannot exceed $P^{FI}(0)$. A similar perfection argument shows that if $\underline{s} \in \underline{S}$, then:

$$\gamma_*(\underline{s}) \in \arg\max_{\overline{\gamma}_i \in \Delta(C_*)} U(\overline{\gamma}_i, \gamma^o_{-i}|\underline{s}) \text{ for } \forall \gamma^o_j : S_* \to \Delta^o(C_*) \Leftrightarrow [\gamma_*(1|\underline{s}) = 0]$$
$$\text{since } \Pr(1|\gamma_*, \gamma^o_{-i}, \underline{s}) \geq \Pr(1|\overline{\gamma}_i, \gamma^o_{-i}, \underline{s}) \text{ for } \forall \overline{\gamma}_i \in \Delta(C_*) \text{ s.t. } \overline{\gamma}_i(1|\underline{s}) > 0$$

Hence, the only dimension characterizing a symmetric perfect equilibrium of the simultaneous quorum rule is the probability of abstention for all types $\underline{s} \in \underline{S}$. Such probability depends on the strategy of the other agents with types in $\underline{S}$ and on the interim beliefs about the distribution of types in the population specifically. For any prior distribution on the type space there is a symmetric perfect Bayesian equilibrium in which all agents types $\underline{s} \in \underline{S}$ strategically abstain:

$$\gamma_{SA} = \left\{ \begin{array}{ll} \gamma_{SA}(a|s_i) = 1 & \text{if } s_i \notin \overline{S} \\ \gamma_{SA}(1|s_i) = 1 & \text{if } s_i \in \overline{S} \end{array} \right\} \in \mathcal{E}^{SP}(\Gamma^B)$$

This follows because if only reformist were to vote a non-reformist would never have an incentive to vote against the reform since the probability of being pivotal on the abstention margin would by far exceed the one of being pivotal on the majority margin. In fact, $\forall \underline{s} \in \underline{S}$ there $\exists \delta > 0$:[23]

$$[\gamma_*(a|\underline{s}) = 1] = \arg\max_{\overline{\gamma}_i \in \Delta(C_*)} U(\overline{\gamma}_i, \gamma^o_{-i}|\overline{s})$$
$$\forall \gamma^o_j \in B_\delta(\gamma_*) \equiv \left\{\gamma^o \in \Delta^o(C_*)^{S_*} : \|\gamma^o - \gamma_{SA}\|_\infty \leq \delta\right\} \text{ for } \forall j \in N \backslash i$$

Because $\forall \gamma^o_{-i} \in B_\delta(\gamma_*)^{N-1}$ it must be the case that:

$$\begin{gathered} \Pr(N - n(a|c_{-i}) = N\varepsilon \cap n(1|c_{-i}) > n(0|c_{-i})|\gamma^o_{-i}, \underline{s}) > \\ \Pr(n(1|c_{-i}) = n(0|c_{-i}) - 1 \cap N - n(a|c_{-i}) > N\varepsilon|\gamma^o_{-i}, \underline{s}) \end{gathered} \tag{4.7}$$

[[The complete argument will be reported in appendix.]] Let us remark that whether sincere voting is a perfect symmetric equilibrium of the simultaneous game with incomplete information depends on the prior and the type space. The sincere voting profile defined by:

$$\gamma_{SV} = \left\{ \begin{array}{ll} \gamma_{SV}(1|s_i) = 1 & \text{if } s_i \in \overline{S} \\ \gamma_{SV}(0|s_i) = 1 & \text{if } s_i \in \underline{S} \\ \gamma_{SV}(a|s_i) = 1 & \text{otw.} \end{array} \right\}$$

[23] The norm used is $\left\|\gamma^o_j - \gamma_*\right\|_\infty = \sup_{s \in S_*} \sup_{c \in C_*} \left|\gamma^o_j(c|s) - \gamma_*(c|s)\right|$.

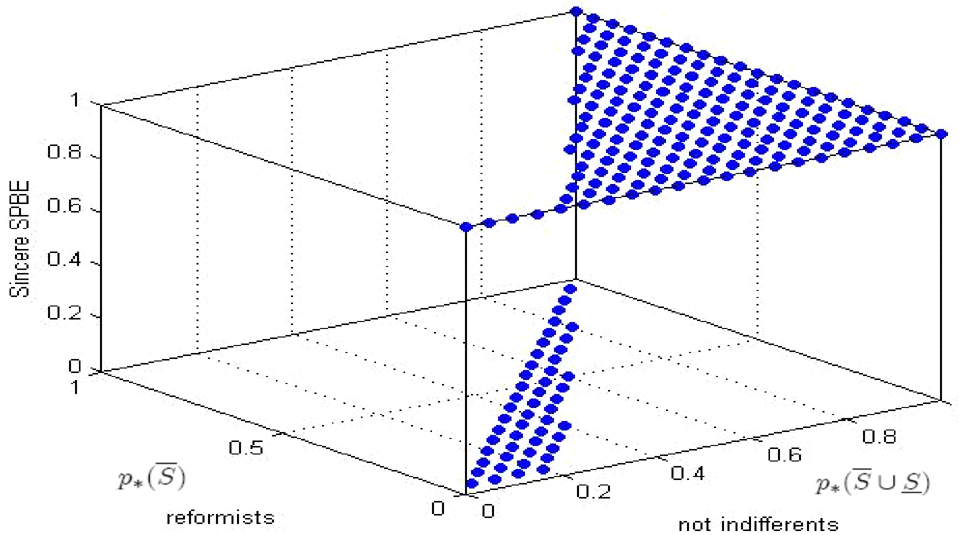


Figure 3: Sincere Voting Equilibria on the Elevated Plane

gives rise to symmetric perfect equilibrium $\gamma_{SV} \in \mathcal{E}^{PS}(\Gamma^B)$ if and only if condition $\forall\delta > 0$, $\exists\gamma^o_{-i} \in B_\delta(\widehat{\gamma}_*)^{N-1}$ such that condition [4.7] holds with the reversed and possibly weak inequality.[24] In fact, if types were independently distributed, according to $p_* \in \Delta(S_*)$, the sufficient condition for sincere voting to be a perfect symmetric equilibrium would be:

$$\sum_{n=(N\varepsilon+1)/2}^{N/2}\left[\frac{N-1!}{n!(n-1)!(N-2n)!}q_1^n q_0^{n-1}(1-q_1-q_0)^{N-2n}\right] \geq \tag{4.8}$$
$$\geq \sum_{n=(N\varepsilon+1)/2}^{N\varepsilon}\left[\frac{N-1!}{n!(N\varepsilon-n)!(N(1-\varepsilon)-1)!}q_1^n q_0^{N\varepsilon-n}(1-q_1-q_0)^{N(1-\varepsilon)-1}\right]$$

for $q_c \equiv \sum_{s_i \in S_*}\gamma_{SV}(c|s_i)p_*(s_i)$ for $\forall c \in C_*$. Intuitively, the condition requires a status quo supporter to be more likely to be pivotal on the reform margin than on the quorum margin given the equilibrium behavior of others. Figure 3 depicts on the elevated plane, for a population of 171 agents, the values of p_* for which sincere voting is an equilibrium of the independent simultaneous move game for $\varepsilon = 1/3$.[25] For boundary quorum values, as discussed in the previous section, the full information rejection region can be obtained even under incomplete information. Namely:

Remark 1. *Under assumption [A3]: whenever $\varepsilon \in [.5, 1]$, the perfect equilibrium maximizing the ex-ante probability of rejection of the simultaneous rule is strategic abstention, γ_{SA}.*[26] *Hence, $P^B(0|\gamma_{SA}) = P^{FI}(0)$. Whenever $\varepsilon = 0$, sincere voting, γ_{SV}, is the only perfect equilibrium of the simultaneous rule. Again, $P^B(0|\gamma_{SV}) = P^{FI}(0)$.*

[24] If $n(a|s) < N - 2N\varepsilon$ is known and $N - 2N\varepsilon \in (0, N]$, then sincere voting is always an equilibrium.

[25] The population size is, in the example, limited by 16 digit precision, but the result extends.

[26] Recall: $\gamma_{SA} = \left\{\begin{array}{ll} \gamma_{SA}(1|s_i) = 1 & \text{if } s_i \in \overline{S} \\ \gamma_{SA}(a|s_i) = 1 & \text{otw.} \end{array}\right\}$.

Let us remark that when types are independent, the extent of the uncertainty about type totals is limited in a large population by the law of large numbers.[27] Hence, if one assumes that types are independent, as the population increases, the following strategy converges to the full information rejection region and is an equilibrium of the simultaneous quorum game for any value of $\varepsilon > 0$ and $N > 1$:

$$\gamma_{AF} = \left\{ \begin{array}{ll} \gamma_{AF}(1|s_i) = 1 & x(s_i) > 0 \\ \gamma_{AF}(0|s_i) = 1 & x(s_i) < 0 \cap p_*(\overline{S}) \in (N\varepsilon/N-1, p_*(\underline{S})] \\ \gamma_{AF}(a|s_i) = 1 & \text{otw.} \end{array} \right\}$$

So that $\lim_{N\uparrow\infty} P_N^B(0|\gamma_{AF}) = P^{FI}(0)_-$, given that all the relevant incomplete information is revealed in the limit. Independence guarantees that such strategy is, always, an equilibrium, since status quo supporters always pool their actions given the common prior and since they vote against only when the chance of hitting the majority margin is grater than the one of hitting the quorum margin.[28] Additionally, note that in the simultaneous game with independent types, whenever and $p_*(\overline{S}) + p_*(\underline{S}) < 1$,[29] the upper-bound on the ex-ante probability of rejection, $P^{FI}(0)$, can never be exactly attained, because there is always a positive probability of coordinating on the bad action given that agents cannot exactly condition their actions on the preferences in the population. Hence:

REMARK 2. *Under assumptions [A2-3]: whenever $\varepsilon \in (0, .5)$, $p_*(\underline{S}) > 0$, $p_*(\overline{S}) > 0$ and types are independent, there exists no perfect Bayesian equilibrium leading to the upper-bound on the ex-ante probability of rejection, $P^B(0) < P^{FI}(0)$.*[30]

A similar strategy may be constructed for correlated types by replacing the second line with: $\gamma_{AF}(0|s_i) = 1$ if $x(s_i) < 0$ and $E(\overline{n}(a|s)|s_i) \leq N - 2N\varepsilon$.[31] This strategy is for almost any type space an equilibrium of the simultaneous rule. Note that this strategy does not require opponents to pool their action. But opponents will be lead to some coordination by the common prior assumption and by exchangeability.

Note that mixed perfect symmetric equilibria in which opponents of the reform pool their actions, may exist only if all of them share the same interim distribution on preferences. For instance, if all of them are of the same type. Additionally, remark that if the interim probability distribution on types were sufficiently correlated to the actual profile of types, so that revelation were almost complete for all types, opponents may still be able to attain or closely approach the complete information rejection region in some equilibrium by not pooling their actions. Hence, let us impose conditions to guarantee that there is some relevant uncertainty about the realization of the type profile in the committee.

[27] By independence, the fraction of agents of any given type converges to the prior probality of that type.

[28] Since condition [4.8] always holds, for $q_c \equiv \sum_{s_i \in S_*} \gamma_{AF}(c|s_i)p_*(s_i)$ for $\forall c \in C_*$ whenever $p_*(\overline{S}) \in (N\varepsilon/N-1, p_*(\underline{S})]$.

[29] The third inequality always holds by [A2] whenever the first two hold.

[30] For $P^B(0) \equiv \max_{\gamma \in \mathcal{E}^{SP}(\Gamma^B)} P^B(0|\gamma)$. And where $\mathcal{E}^{SP}(\cdot)$ denotes the SPB equilibrium strategies.

[31] If $N > N - 2N\varepsilon > 0$. Otherwise remark 1 defines the appropriate strategy.

[[The set of sufficient conditions for $P^B(0) < P^{FI}(0)$ is currently being relaxed in the correlated case and will be added as soon as possible. Here and in a Remark. Sorry for the delay.]]

Let us, now, switch to multi-period rules, again assuming that indifferents never commit [A3′]. If we focus on a multi-period secret quorum rule rather that on a simultaneous one the results are not affected. In fact, because of the secrecy in such mechanism, an agent may condition her commitment at any stage only on information about their previous choices. Hence, profiles of strategies leading to the same distribution of terminal commitments yield the same expected utility at any stage. But because no information can be shared or obtained by others, it follows that all equilibria of the multi-period secret game may be represented as equilibria of the one stage simultaneous game by looking at the distribution of final commitments induced by the multi-period strategy. More, specifically, let us note that for the interim secret information structure for any $s_{i.t} = (c_i^{t-1}, s_i)$ utility and beliefs are defined by:[32]

$$U_{i.t}(\gamma^T|c_i^{t-1}, s_i) = u(0|s_i) + x(s_i)\Pr_{i.t}(1|\gamma^T, s_{i.t})$$
$$\Pr_{i.t}(1|\gamma^T, s_{i.t}) \equiv \textstyle\sum_{s_{-i}\in S_{-i}} \sum_{\overline{c}^T\in C^T(\alpha)} \nu(\overline{c}_T) \prod_{r\geq t}\prod_{j\in N} \gamma_{j.r}(\overline{c}_{j.r}|s_{j.r})\beta_{i.t}(\overline{c}^{t-1}, s_{-i}|s_{i.t})$$
$$\beta_{i.t}(\overline{c}^{t-1}, s_{-i}|s_{i.t}) \equiv Id(\overline{c}_i^{t-1} = c_i^{t-1}) \textstyle\prod_{r<t}\prod_{j\in N\setminus i} \gamma_{j.r}(\overline{c}_{j.r}|s_{j.r})p(s_{-i}|s_i)$$

and that the probability of a reform is independent of the timing and information $\forall i.t \in N.T$:

$$\Pr_{i.1}(1|\gamma^T, s_i) = \Pr_{i.t}(1|\gamma^T, s_{i.t}) = \Pr_i(1|\overline{\gamma}, s_i) \text{ for } \forall s_{i.t} = (c_i^{t-1}, s_i) \in C_*^{t-1}(\alpha)\times S_*$$
$$\overline{\gamma}_i(k_*|s_*) \equiv \textstyle\sum_{c_i^T\in C_*^T(\alpha)} Id(c_T = k_*)\prod_{t\in T}\gamma_{i.t}(c_{i.t}|c_i^{t-1}, s_*) \text{ for } \forall k_*, s_* \in C_*\times S_*$$

Consequently, the distribution on final commitments of any agent is independent of others' strategies and only strategies γ^T such that the corresponding $\overline{\gamma}\in\mathcal{E}^{SP}(\Gamma^B)$ can belong to the equilibrium set of strategies of the secret game, $\mathcal{E}^{SP}(\Gamma^M)$. This guarantees that if one cares just about secret mechanisms assuming that agents take actions at the same stage poses no restriction, as one would expect. The timing of the commitments will not be determined in equilibrium, in the secret mechanism, but the nature of the commitments will be by perfection.

For any multi-period quorum mechanism, it is easily verified that abstention at the last [limiting] stage and a commitment to a vote against the reform at any stage are weakly dominated actions for the any reformist. Hence, in all perfect equilibria all pro-reform agents will commit to a vote in favor of the reform at some stages with probability one. Similarly, whoever opposes the reform will never commit to a vote in favor of the reform by perfection. Hence, in all symmetric perfect equilibria the only variables to be determined are the timing of commitment for the reformist and the timing and probability of commitment for non reformist. Let us remark that if the set of time periods is countably infinite, independently of the prior distribution there exists an equilibrium of the public abstention totals quorum

[32] In appendix, equivalent definitions for the non secret cases are reported.

rule in which opponents induce complete disclosure of all relevant information. In fact, such outcome may be obtained by the following strategy, for $s_{i.t} \in \Im_{i.t}$ and $z_t = N - n(a|c_{t-1})$:

$$\widehat{\gamma}^T = \left\{ \begin{array}{lc} \widehat{\gamma}_t(1|s_{i.t}) = 1 & \text{if } s_i \in \overline{S} \cap t \in (\widehat{t}(s_i), \infty) \\ \widehat{\gamma}_t(0|s_{i.t}) = 1 & \text{if } s_i \in \underline{S} \cap z_t > N\varepsilon \\ \widehat{\gamma}_t(a|s_{i.t}) = 1 & \text{otw.} \end{array} \right\} \tag{4.9}$$

Where $\widehat{t}(s_i)$ denotes a finite number of stages after which agent type $s_i \in \overline{S}$ commits. Such strategy profile completely reveals $\overline{n}(1|s)$ in a finite number of periods with probability one. And, consequently, leads to the same policy rejection region of the simultaneous full information mechanism. Hence, for $P^M(0) \equiv \max_{\gamma^T \in \mathcal{E}^{PS}(\Gamma^M)} P^M(0|\gamma^T)$ and:[33]

$$P^M(0|\gamma^T) = 1 - \sum_{s\in S} \sum_{c^T \in C^T(\alpha)} \nu\left(c_T\right) \prod_{t\in T} \prod_{j\in N} \gamma_{j.t}(c_{j.t}|s_{j.t}(s, c^{t-1}))p(s)$$

the following claim holds.

CLAIM 2. *A countable time period quorum rule, M, with public abstention rates, $z_t = n(a|c_{t-1})$, always possesses an equilibrium in which the ex-ante probability of rejection of the policy is maximized by opposers of the reform alone. $P^M(0) = P^{FI}(0)$.*

A similar result holds if one assumes that the game lasts a stochastic number of periods and ends in a finite number of time periods with probability one. Namely, because reformists fear not to be able to commit at their next stage, they may be led to an early vote and, hence, to reveal all relevant information.

The full disclosure rule with infinite time periods described by [4.5] may not be exploited from opposers of the reform to attain any better rejection region in any symmetric perfect equilibrium. In fact, $\widehat{\gamma}^T$ is still one of equilibria leading to the maximal ex-ante rejection region. Note that the two rules may display drastically different equilibrium sets, when utility is transferable, because the set of contractable actions increases from $\{a\}$ to C_*.

[[The section will proceed with the analysis of the random public abstention rule and of finite time period rules. In finite rules often backward induction effects and the existence of simultaneous terminal stage, may prevent or reduce the extent of strategic manipulation in absence of voting costs and if utility is NTU.]]

5. Multiple Quorum Rules

This section discusses the extent to which strategic manipulation may be reduced by holding several referenda at once. Two generalizations of the single-topic rule will be introduced and analyzed in the context of multi-topic quorums.

In single-topic quorum rules with infinite time periods and public abstention totals, opponents manage to exploit the mechanism because the communication channel they are

[33] Since the information structure is deterministic, otherwise one would replace $\gamma_{j.t}(c_{j.t}|s_{j.t})$ with $\sum_{z\in Z} \gamma_{j.t}(c_{j.t}|s_{j.t-1}, c_{j.t-1}, z)\sigma_t(z|c^{t-1})$.

given can be used to extract all relevant information. But if agents were to vote on several propositions at once and only the timing of the first commitment, rather than the timing of each commitment, were observable, we claim that the extent of the manipulation could be greatly reduced. Indeed, whether such expedient solves the manipulation problem, depends on the correlation among preferences for different policies. Intuitively, if preferences on different policies are not perfectly and positively correlated across agents we would expect holding the referenda together increase turnout and reduce the extent of strategic abstention.

The analysis in this section is, currently, limited to double-topic binary quorum rules.[34] For simplicity, we assume that preferences on different policies to be separable [A4], but possibly correlated. Hence, $u'(y|s_i) = w(y'|s_i) + w'(y''|s_i)$ for any $y = (y', y'') \in Y' = \{0,1\}^2$. Again, we may define the utility differences for each policy, given a type $s_i \in S_*$ by $x(s_i) = w(1|s_i) - w(0|s_i)$ and by $x'(s_i) = w'(1|s_i) - w'(0|s_i)$. For notational convenience, let us define the preference type sets by $S_{11} \equiv (\overline{S} \cap \overline{S}')$, $S_{00} \equiv (\underline{S} \cap \underline{S}')$, $S_{aa} \equiv (S_a \cap S'_a)$, $S_{1a} \equiv (S_1 \cap S'_a)$ [and so on], for $\overline{S} \equiv \{\overline{s} \in S_* : x(\overline{s}) > 0\}$, $\overline{S}' \equiv \{\overline{s} \in S_* : x'(\overline{s}) > 0\}$, $\underline{S}$, $\underline{S}'$ respectively defined and $S_a \equiv S_* \backslash (\overline{S} \cup \underline{S})$ and $S'_a \equiv S_* \backslash (\overline{S}' \cup \underline{S}')$.

For this setup, a possible generalization of the single-topic simultaneous rule consists of:

$$\begin{gathered} c = (c', c'') \in C_*^2 \equiv C'_* \\ y = \nu'(c) = (\nu(c'), \nu(c'')) \end{gathered}$$

The rule assumes that both reforms are faced with the same outcome map ν , defined in the previous section, and quorum level ε. It, also, assumes that agents can affect the outcome of both topics separately by choosing an action from C_* for both of them. Note that if the two referenda are held simultaneously the separability assumption guarantees that voting on each topic according to some equilibrium of the single-topic rule given the interim distribution on types for that topic is an equilibrium. Hence, implications are not affected under the simultaneity assumption.[35] Again, if the full disclosure at the interim stage is assumed in the simultaneous rule, it is possible to construct upper-bounds for the ex-ante probabilities of rejection of the two policies, $P^{FI}(0)$ and $P'^{FI}(0)$.[36] Indeed, such upper-bounds may be obtained by playing separately according to [4.6] on each topic separately.[37]

The corresponding multi-stage rule is defined by:

$$\alpha'(c_{i.t}) \equiv \left\{ \begin{array}{cc} C_*^2 & c_{i.t} \in \{(a,a), \emptyset\} \\ c_{i.t} & \text{otw.} \end{array} \right\} \tag{5.1}$$
$$\nu'_M\left(c^T\right) = \nu'\left(c_T\right) \quad \text{for } \forall c^T \in C^T(\alpha')$$

[34] The generalization to multi-topic rules should be straightforward, once analysis will be complete.

[35] If the double quorum is played simultaneously all combinations equilibrium strategies of the two single simultaneous single quorums are equilibria of the double quorum.

[36] For $P^{FI}(0) = 1 - \sum_{s \in S} Id(\overline{n}(1|s) > N\varepsilon \vee \overline{n}(0|s))p(s)$ and $P'^{FI}(0) = 1 - \sum_{s \in S} Id(\overline{n}'(1|s) > N\varepsilon \vee \overline{n}'(0|s))p(s)$ when $\overline{n}(1|s) \equiv \sum_{i \in N} Id(x(s_i) > 0)$ and $\overline{n}'(1|s) \equiv \sum_{i \in N} Id(x'(s_i) > 0)$ and accordingly.

[37] By playing according to a $\gamma_{*2} : S_* \to \Delta(C'_*)$, defined in appendix, consistently with the full disclosure strategy of the single quorum, opponents of each policy may strategically enlarge its rejection region.

The mapping α', that requires agents to either to abstain or to commit on all topics at once, is central for our notion of the multi-topic multi-stage rule. Currently, we focus on a rule with public timing of commitments and a countable number of time periods:

$$\sigma'_t(c^{t-1}) = N - n((a,a)|c_{t-1}) \tag{5.2}$$

The information structure, σ', displays the publicly number, but not the nature of previous commitments. Hence, voters at time $t+1$ know how many of them chose to abstain on both topics up to time t. Note that, for information structure σ', even if only reformists vote on any given topic and $n((a,a)|c_{t-1}) < N\varepsilon$, it can be the case that no reform passes the quorum. This was impossible in the single quorum rule. But as in single-topic quorums it must be that, if $n((a,a)|c_{t-1}) \geq N\varepsilon$, agents know that none of the reforms has yet passed. Again, it must be the case that there exists no perfect symmetric equilibrium strategy of that can lead to a bigger rejection region for both reforms in an endogenous timing mechanism. [[We are working on a claim that guarantees that both probabilities of rejection for the reforms do not exceed the upper-bounds in any SPBE of a mechanism satisfying conditions [5.1] & [5.2].]]

Before we turn to general result for the multi-stage let us characterize two extreme cases. Again let us assume that voters that are indifferent on any given policy abstain. In the first, suppose that preferences are perfectly and positively correlated across policies. For instance, suppose $w(y|s_i) = w'(y|s_i)$ for any $(y, s_i) \in Y \times S_*$. Given the assumption, preferences at any stage are:[38]

$$U_{i.t}(\gamma|s_{i.t}) = x(s_i)\left[\Pr_{i.t}((1,1)|\gamma, s_{i.t}) - \Pr_{i.t}((0,0)|\gamma, s_{i.t})\right] + con(s_i)$$

If this were the case, the multi-topic rule could be reduced to and played as a single-topic one, given the correlation among preferences. In fact, the following symmetric perfect Bayesian equilibrium strategy would, trivially, attain the full disclosure rejection region, for $s_{i.t} \in \Im_{i.t}$ and $z_t = N - n((a,a)|c_{t-1})$:

$$\gamma^{+T} = \left\{ \begin{array}{lc} \gamma_t^+((1,1)|s_{i.t}) = 1 & \text{if } s_i \in S_{11} \cap t \in (t^+(s_i), \infty) \\ \gamma_t^+((0,0)|s_{i.t}) = 1 & \text{if } s_i \in S_{00} \cap z_t > N\varepsilon \\ \gamma_t^+((a,a)|s_{i.t}) = 1 & \text{otw.} \end{array} \right\}$$

Hence, if preferences are positively and perfectly correlated, the maximal extent of manipulation cannot be reduced by holding the two rules jointly.

In the second example, preferences are still assumed to be perfectly correlated, but negatively. For instance, $w(y|s_i) = -w'(y|s_i)$ for any $(y, s_i) \in Y \times S_*$. In this scenario, all agents, except for the indifferents, care for one and only one reform to pass and preferences

[38] For $con(s_i) \equiv w(1|s_i) + w(0|s_i)$ for any $s_i \in S_*$.

at any stage are given by:

$$U_{i.t}(\gamma|s_{i.t}) = x(s_i)\left[\Pr_{i.t}((1,0)|\gamma, s_{i.t}) - \Pr_{i.t}((0,1)|\gamma, s_{i.t})\right]$$

In this committee, for all reformists not committing to an action profile with probability one before the game ends is a weakly dominated strategy. The multi-stage quorum rule for this committee possesses the following symmetric perfect equilibrium independently of the prior, for $s_{i.t} \in \Im_{i.t}$:[39]

$$\gamma^{-T} = \left\{ \begin{array}{ll} \gamma_t^-((1,a)|s_{i.t}) = 1 & \text{if } s_i \in S_{10} \cap t \in (t^-(s_i), \infty) \\ \gamma_t^-((a,1)|s_{i.t}) = 1 & \text{if } s_i \in S_{01} \cap t \in (t^-(s_i), \infty) \\ \gamma_t^-((a,a)|s_{i.t}) = 1 & \qquad\qquad \text{otw.} \end{array} \right\}$$

Additionally, the above described strategic abstention strategy may be the unique equilibrium. In fact, remark that, in this example, both factions of reformists would benefit from the knowledge of the number of indifferents, $n((a,a)|s)$, because members could, by cohesively conditioning their actions upon that information, attain the maximal rejection region for the undesired policy. But if agents were to act upon the disclosed information by voting against depending on the number of voters, delaying commitments in order to receive better information may be optimal for all non-indifferent agents and prevent any other equilibrium from arising. In fact, because for all agents that are not indifferent, the equilibrium probability of not committing in a finite number of time periods is zero by perfection, any reformist would benefit from a deviation that delays his commitment.[40] Also, note that sincere voting without conditioning actions on information may not be equilibrium, in part because of the reasons discussed in the single topic section. Hence, for preferences that are perfectly negatively correlated it may be the case that the double topic multi-stage rule with observable commitment totals never attains the maximal ex-ante probability of rejection for the policy. But this would not the case if $n((a,a)|s) = n((a,a)|s_i)$ for any $i.s_i \in N.S_*$, because all reformist may, by only being aware of the number of indifferents, attain the maximal rejection probability for the respective policies. In general we have that:

Remark 3. *If $p(s \in [S_{11} \cup S_{00} \cup S_{aa}]^N) = 1$, the upper-bounds, $P^{FI}(0)$ and $P'^{FI}(0)$, may always attained in a SPB equilibrium of a countable time period public commitment totals quorum rule. And similarly if $p(s \in [S_{1a} \cup S_{0a} \cup S_{aa}]^N) = 1$ or $p(s \in [S_{10} \cup S_{00} \cup S_{a0}]^N) = 1$.*[41]

In fact, for any prior satisfying these conditions the problem may be mapped to a single-topic quorum rule. Hence, the results on the upper-bounds do still hold.

Let us, now, modify the outcome map, slightly, in order to draw results that are more related to the evidence discussed in the section eight. Let:

$$\overline{\nu}\left(c', c\right) = Id\left(n(1|c') > n(0|c')\right) Id(N - n((a,a)|c) > N\varepsilon) \text{ for } \forall c = (c', c'') \in C_*'^N$$

[39] The equilibrium is unstable to joint deviations.

[40] More details about the argument will be added in appendix, as soon as possible.

[41] Clearly, the same result holds for the reversed policy order.

The map differs from the pervious one, only to the extent that the quorum is determined for both policies from the number of commitments rather from the number of votes on the given topic. Consequently, we redefine the double-topic simultaneous and multi-stage maps as:

$$\overline{\nu}'(c) = (\overline{\nu}(c', c), \overline{\nu}(c'', c))$$
$$\overline{\nu}'_M\left(c^T\right) = \overline{\nu}'\left(c_T\right) \quad \text{for } \forall c^T \in C^T(\alpha')$$

For this outcome map in all perfect equilibria whenever agents choose to commit they will do so sincerely on both topics, since once the quorum margin is affected, the majority margin is all that matters to them. This phenomenon depends on the specific nature of the mechanism that prevents any policy from not passing the quorum margin if the other one does. Again this rule is equivalent to a single topic rule if conditions in Remark 2 are met. Let us assume as we did for the other rules that $s_i \in S_{aa} \Rightarrow \gamma_t((a, a)|s_{i.t}) = 1$ and that $s_i \in S_{xa} \Rightarrow \gamma_t((y, a)|s_{i.t}) = 1$ for some $x, y \in \{1, 0, a\}$. Perfection of all equilibria considered in the multi-stage rule requires that in any SPB equilibrium:

$$\begin{array}{rc}
\gamma_t((1,1)|s_{i.t}) = 1 & \text{if } s_i \in S_{11} \cap t \in (t_*(s_i), \infty) \\
\gamma_t((1,a)|s_{i.t}) = 1 & \text{if } s_i \in S_{1a} \cap t \in (t_*(s_i), \infty) \\
\gamma_t((a,a)|s_{i.t}) = 1 & \text{if } s_i \in (S_{00} \cup S_{a0} \cup S_{0a}) \cap z_t \leq N\varepsilon \\
\gamma_t((0,0)|s_{i.t}) = 1 & \text{if } s_i \in S_{00} \cap z_t > N\varepsilon \\
\gamma_t((0,a)|s_{i.t}) = 1 & \text{if } s_i \in S_{0a} \cap z_t > N\varepsilon \\
\gamma_t((1,0)|s_{i.t}) = 1 & \text{if } s_i \in S_{10} \cap z_t > N\varepsilon
\end{array}$$

Lines two, five and six of the above matrix hold respectively for reversed preference types. Hence, the only part of the equilibrium strategy not determined by these restriction is the behavioral strategy for types in S_{10} and S_{01} whenever the quorum is not met. The behavior of these preference types when the quorum is not met depends on the specific nature of the committee But remark, that this mechanism, *per se*, tends to increase the correlation among the number of voters on the two policies, with respect to the previous rule, because strategic abstention cannot be exercised independently on both reforms.

[[For the moment, the analysis of double-topic rules proceeds exemplary cases, but we plan on finding more conditions & results before extending the discussion.]]

6. Comments on Transferable Utility

If one allows for utility to be transferable results become more dramatic. Suppose that a good is introduced in the strategic environment and that this good enters linearly and separably the utility function. So that $\forall i \in N$:

$$u(y, b, s) = u(y|s_i) + b_i$$

Additionally, assume that good b_* is transferable across agents and that all agents are endowed with a finite and non-negative amount of the transferable good. We denote by $\overline{b} \in B_*^N \subset \mathbb{R}_+^N$ the profile of endowments for the committee and we let it be a part of the type space. If agents can allowed write enforceable contracts on observable variables, it may be profitable for some to do so if the multi-stage rule displays information disclosure. For the moment, we rule out contracts on the outcomes of the mechanism, but we plan on relaxing this assumption as research proceeds.[42] Hence, if the mechanism were secret or simultaneous no incentive compatible contract would exist because there would be no variable to be contracted upon. But in a multi-stage mechanism with observable individual abstention,[43] satisfying [4.1], agents deriving benefits from the *status quo* may write contracts with other agents, possibly with reformists, in order to get them to abstain. In fact, such contracts are incentive compatible, given that they prevent any moral hazard on the side of the recipient, to the extent that abstention increases the interim rejection probability. This phenomenon for specific configurations of the model may lead to reversals of absolute majorities of reformists. In fact, as displayed in the top-left corner of figure 4, if information is fully disclosed at the interim stage there are profiles of endowments for which the reform passes only nobody opposes it and more than the quorum favor it. In such, occasions all agents either abstain strategically or are bought to abstain. Now:

$$P_T^{FI}(0) = 1 - \sum_{s \in S} Id(\overline{n}(1|s) > N\varepsilon \cap \overline{n}(0|s) = 0)p(s)$$

And for specific endowment type spaces it may be the case that even in a finite number of voting stages.$P^M(0) = P_T^{FI}(0)$.[44] This means that most of the absolute majorites of reformists can be *a priori* overturned by vote buying in a mechanism with individually observable abstention.

Also, reformists have incentives to write contract, but less than opponents. In fact, in a rule with individual public abstention, reformists may only profit from writing contracts that require voting. Hence, there will be room for moral hazard, because contracts cannot condition payments on the nature of the vote given that it is unobservable. Let us look upper-bounds on the ex-ante probability of acceptance by assuming full disclosure at the interim stage, as we did for the rejection probabilities. Before we do so, let us separate out three cases, which in a NTU environment had identical behavioral implications. Namely: (I) the vote of indifferents cannot be bought, (II) the vote of indifferents can be bought and (III) agents are allowed to abstain even in the poll and, hence, not even voting is contractable. Even though, assumption (I) may sound unreasonable, let us remark that we only need there to be a group of agent that would not want to vote for the biggest offer they could get, for it to be satisfied. The biggest acceptance region for the policy given (I) under

[42] A justification for this assumption may be that contract on outcomes cannot time consistent in one shot games.

[43] The information structure displays individual public abstention if $\forall c^{t-1}, \overline{c}^{t-1} \in C^{t-1}(\alpha)$ we have that: $\sigma_t(c^{t-1}) = \sigma_t(\overline{c}^{t-1}) \Leftrightarrow [c_{i.t-1} = a \Leftrightarrow \overline{c}_{i.t-1} = a, \forall i \in N\ ]$.

[44] For instance if only non-reformist possess money and in excess of the cumulative utility difference for the reform.

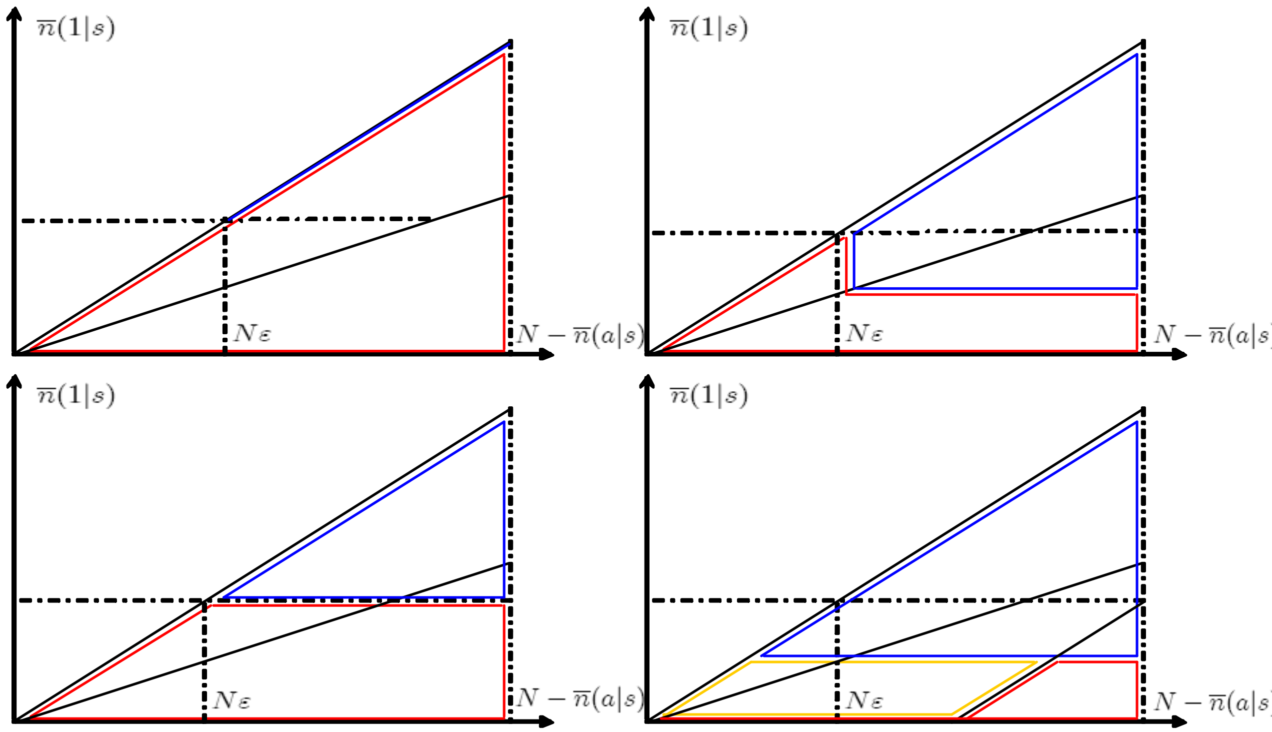


Figure 4: Maximal Rejection Region (up left) & Maximal Acceptance Region: with Unbuyable Indifferents (up right), with Secret Abstention (down left), and with Buyable Indifferents (down right)

full disclosure is depicted in the top-right corner of figure 4. This outcome can be obtained, for specific endowment type spaces, by buying abstention and voting from the appropriate number of opponents. Hence, there exists endowment spaces for which:

$$P_I^{FI}(1) \equiv \sum_{s\in S} Id(\overline{n}(1|s) > N\varepsilon/2 \cap \overline{n}(a|s) < N - N\varepsilon)p(s)$$

If, instead, (II) is assumed, the biggest acceptance region for the policy under full disclosure depends on how indifferents commit when their vote is bought. If indifferents commit in favor of the policy the rejection region is depicted in red in the bottom-right corner of figure 4. If, instead, they commit against the rejection region is the union of the orange and red regions.[45] These outcomes can be obtained, for specific endowment type spaces, by buying abstention and voting from the appropriate number of opponents and indifferents. Hence, for the two cases, respectively, there are endowment profiles for which:

$$P_{II}^{FI}(1) \equiv \sum_{s\in S} Id(\overline{n}(1|s) > N\varepsilon/2)p(s)$$
$$P_{II'}^{FI}(1) \equiv \sum_{s\in S} Id(\overline{n}(1|s) > 0 \vee (N\varepsilon/2 \wedge N\varepsilon - \overline{n}(a|s)))p(s)$$

Finally, assumption (III) modifies C_* to $\overline{C}_* \equiv \{1, 0, a, \overline{a}\}$, where $\overline{a}$ denotes abstention at

[45] For intermediate cases yield intermediate results.

the poll, and the outcome and action set maps to:

$$\nu\left(c\right)=Id\left(n(1|c)>n(0|c)\right)Id(N-n(a|c)-n(\overline{a}|c)>N\varepsilon)$$
$$\alpha(c_{i.t})\equiv\left\{\begin{array}{ll}C_{*} & c_{i.t}=\{a,\emptyset\}\\ c_{i.t} & c_{i.t}\in\{1,0,\overline{a}\}\end{array}\right\}\ \forall i.t\in N.T$$

Let us remark that even though this assumption is inconsequential in a NTU setup, it reduces the maximal ex-ante acceptance region for the reform, by preventing votes to become contractable. This assumption is quite common in referendum implementations. If (III) holds, hence, the maximal attainable ex-ante acceptance region is:

$$P_{III}^{FI}(1)\equiv\sum_{s\in S}Id(\overline{n}(1|s)>N\varepsilon)p(s)$$

And it can be obtained by buying some of the opponents' abstention, for specific endowment type spaces.[46] Note that: $P_{II}^{FI}(1)=1-P^{B}(0|\gamma_{SA})$. Clearly, for type spaces such that all three upper-bound may be attained, we have that: $P_{III}^{FI}(1)\leq P_{I}^{FI}(1)\leq P_{II}^{FI}(1)$. Again, non-reformists seem to be advantaged by the mechanism, given that there are profile of preference over policies in the committee that cannot be overturned independently of the monetary endowments in the population even when population is fully disclosed.

[[More details on the TU case [also regarding the multi-stage rule] and some analytical examples have been developed, and will be added as soon as possible.]]

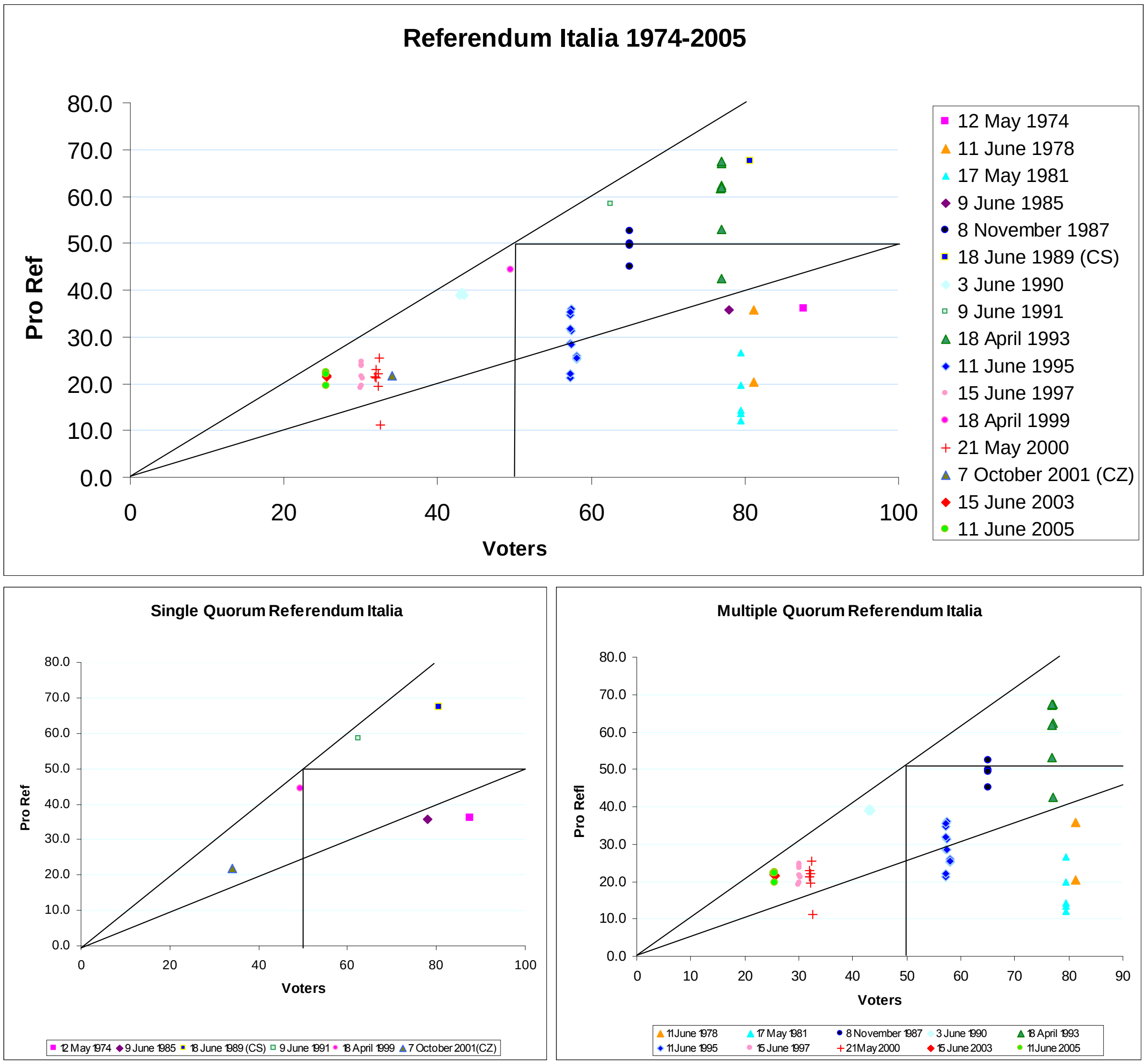
7. Descriptive Evidence in Italian Referenda

The empirical evidence consists, at the current stage, of a descriptive analysis of plebiscites with a 50% quorum performed in Italy between 1974 and 2005. In that period 61 referenda with quorum were voted on 16 occasions. In six dates single-topic referendum were held. In the remainder multi-topic plebiscites were held. In some of the occasions though the topics of the were highly related, in others the questions regarded highly heterogeneous problems. The Italian data on referenda consists of the polling totals collected by the ministry of interior and is publicly available. Referendum in Italy are usually held on one or two days. The number of voters is publicly disclosed and recorded at several hours during the voting dates by the ministry of interior. Additionally, let us remark that abstention is potentially observable at an individual level, because no law prohibits the observation of entry at polls. Hence, in a transferable utility setup it seems plausible to study abstention as a contractable action. And though law prohibit the acquisition of votes, we would expect the choice of casting a vote to be influenced by outside factors[47] on sensitive policy issues. Some evidence in support of this effect can be found by looking at opinion polls. In fact, from the preliminary analysis it appears that when reformists are close to minority, even if in relative majority in the opinion polls, the percentage change in the fraction of reformists

[46] Here, we did assume [A3] and that if a vote were bought and $x(\overline{s})=0\Rightarrow\gamma_{*}(\overline{a}|\overline{s})=1$.

[47] Because a vote may reveal one's preferences, casting it may make the agent worse off if she wants to go unrecognized and she expects to have only a small chance at being pivotal.

from the opinion poll to the poll appears to be negatively correlated to the abstention rate. This of course could, also, be explained by the probability of being pivotal of reformists decreasing. But if one believes in the infinitesimal cost assumption, the former explanation seems more compelling than the latter. Let us comment on the regularities our small sam-



ple. For single-topic quorums, we find that no reform was ever enacted with votes in the manipulable rejection region.[48] And that whenever a reform was enacted the fraction of pro reform votes was significantly above an absolute majority. This seems to suggest that opponents manage to effectively use abstention to strike down reforms. A similar pattern seems to emerge for multi-topic rules whenever all questions are closely related to one another. The picture is quite different when multi-topic rules are used for subjects that which have different support bases. In fact, we observe that in such cases turnout tends to be higher

[48] The triangle with no observations on the left bottom of figure 4.

and some reforms may be passed just by a relative majority, if some of the other reforms are either passed with a absolute majority or rejected by votes against. Let us remark that in all quorum rules the number of voters on each subject seem to be highly correlated among referenda held on the same dates. A possible explanation for this phenomenon hinges on the equilibrium behavior described for the second type of multi-topic rule. In fact, the Italian law requires the quorum to be set on the number of agents entering the poll, rather than on the number of agents voting on the single policies. Hence, we would expect whoever votes to vote sincerely on all topics, as discussed at the end of section 5. Certainly, if one were to add non infinitesimal costs, the high correlation among the different topic vote totals would have an additional justification. An final regularity found in the data on multi-topic referendum, is that the spread of votes among policies of a multi-topic rule seems to be proportional to the number of votes on the average topic.

More data is being gathered about other European Union countries in which referenda with smaller quorum levels were held. In such cases, in fact, our model would predict a higher level of strategic sophistication. Let us remark that abundant Swiss data about referenda does not apply to our analysis because requirements for approval of the reform are not quite a quorum.[49] Additionally, Switzerland has recently reformed its voting scheme for plebiscites from one with vote at the polls to one with vote by mail. Let us remark that because of the big change in the information structure, we would expect outcomes to be drastically affected. Especially, when utility is transferable.

[[Preliminary result will be added in a data appendix, time permitting. Further analysis will be performed after the proposal date.]]

8. Conclusions and Further Projects

This paper consists of a preliminary analysis of the effects of endogenous sequential commitment on equilibrium outcomes of quorum rules with disclosure. We find that in single topic quorum rules whenever the timing of commitments is observable all the relevant incomplete information may be disclosed in equilibrium and that the probability of rejection may be increased to its maximal complete information value. In fact, the observation of the timing of commitment may serve as a coordination device for agents opposing the reform. In simultaneous voting rules there is no means of coordination, but for the common knowledge of the equilibrium strategy and, hence, there is no possibility of tieing one's actions to others'. In fact, our commitment structure may be interpreted as a costly communication system, because commitments are potentially revealing actions.

We have argued through specific examples that these effects may be mitigated by multiple quorum rules at once, whenever preferences on topics are not strongly correlated and only the total number of voters is observable. In fact, a single communication channel may not be sufficient to disclose all relevant incomplete information in a multiple-topic rule. The section on multi-topic referenda is still preliminary and results will be extended and refined.

[49] Magiority of population and majority of cantonal votes are the requirements.

We discuss how results may be affected when a transferable good is included in the system, given that the sequential commitment rules give rise to incentive compatible contracts. We claim, that under this circumstance, the extent of strategic manipulation of quorum rule increases. In fact, for given profiles of resources, both super-majorities of reformists and opponents may be overturned by abstention-buying. The rule still favors opponents though, given that there are profiles of preferences for which the reform does not pass independently of the profile of endowments in the committee and that this is never the case for reformists. In fact, TU exacerbates the extent of equilibrium manipulation and may lead to full disclosure even in finite stage games, because of the incentive compatible contacts available to agents. Some general result of the section are still under development.

The evidence gathered from Italian referenda, even though analyzed at a very descriptive level seems to support the theoretical conclusions regarding strategic abstention in both single and multi-topic 50% quorum rules. We plan on gathering additional evidence on referenda with smaller quorums, to verify if the more complex implications of such rules hold. Comparisons between polls and opinion polls will also be reported in latter drafts and will be used to measure the extent of strategic voting.

This project was initially supposed to belong, as an application, to a theoretical project on information disclosure and commitment that is still being constructed. In such project our interest lies in the characterization of the set of payoffs, of a Bayesian game, that can be sustained as equilibria by changing the commitment structure and level of disclosure of the original game. Let us remark that if there is no-commitment and actions at early stages serve as mere communication, the multi-period expansion of the game may be mapped to a long cheap talk game. Consequently, there exists a convex set of payoffs of the initial game that may be implemented through communication, as equilibria of the multi-stage no commitment game. Also, remark that if one, instead, is free to choose any commitment structure, all points in the feasible set can be implemented as equilibria. Such boundary results are, already, present in different forms in the literature. Hence, our objective for this project would be the characterization of the set of equilibrium payoffs for intermediate commitment structures.

Our proposed research for the next academic year encompasses the completion of both the referendum project and the commitment and disclosure project. The referendum paper requires further development on the following points: robustness to different information structures [finite time]; generalization form double to multi quorum rules: a more formal exposition and characterization of TU results and some further data analysis. The other project is currently based on two player games with incomplete information and includes, at the current stage, many exemplary cases and boundary results, but has still no general characterization for the intermediate cases. [[Upon request, a more detailed description of the other project will be added by the proposal date.]]

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9. Appendix

Claim 3. *For any game Γ^M induced by a mechanism M satisfying conditions [4.1], there exists no perfect equilibrium leading to a greater ex-ante probability of rejection for the reform than $P^{FI}(0)$.*

Proof. This follows because in all perfect equilibria of mechanisms satisfying conditions [4.1] $\gamma_T(s_{i.T})$ must be such that $\lim_{t\to T}\gamma_t(1|s_{i.t}) = 1$ for $\forall s_i^T \in \Im_i^T$ for which $s_i \in \overline{S}$. In fact, if $s_i \in \overline{S}$, $\exists t(s_{i.t}) < |T|$:

$$U_{i.t}(1, \gamma^T_{-i.s_{i.t}}|s_{i.t}) \geq U_{i.T}(\overline{\gamma}_{i.t}(s_{i.t}), \gamma^T_{-i.s_{i.t}}|s_{i.t}) \text{ for } \forall\overline{\gamma}_{i.t}(s_{i.t}) \in \Delta(C_*) \tag{9.1}$$

In fact, not committing in the limit is a weakly dominated action, because the reformist would forego any chance of being pivotal. Since, she can be pivotal in favor of the reform just by voting pro reform, reformists always should do so before or at the final stage with probability one. Perfection prevents the possibility of a reformist never committing, because even though disclosure may reduce the pivot probability, it cannot alter the fact that yes votes induce the pivot. Hence, any profile of types $s \in S$ such that $\overline{n}(1|s) > \{N\varepsilon \vee \overline{n}(0|s)\}$ must to lead to a reform in any perfect equilibrium of a game Γ^M satisfying [4.1]. And this proves that for $s_{i.t} \equiv \left(s_i, c_{i.1}, ..., c_{i.t-1}, z_2(c^1), ..., z_t(c^{t-1})\right) \in \Im_{i.t}$:

$$\begin{aligned} & P^M(0) = \max_{\gamma^T\in\mathcal{E}^{PS}(\Gamma^M)} 1 - \textstyle\sum_{s\in S}\sum_{c^T\in C^T}\nu\left(c_T\right)\prod_{t\in T}\prod_{i\in N}\gamma_t(c_{i.t}|s_{i.t})p(s) \leq \\ \leq\ & P^{FI}(0) = 1 - \textstyle\sum_{s\in S} Id(\overline{n}(1|s) > N\varepsilon \vee \overline{n}(0|s))p(s) \end{aligned}$$

Which is our initial claim. [Details may be added.] ■

Claim 4. *The countable time period quorum rule, M, with public abstention rates, $z_t = N - n(a|c_{t-1})$, always possesses an equilibrium in which the ex-ante probability of rejection of the policy is maximized by opposers of the reform alone. $P^M(0) = P^{FI}(0)$.*

Proof. We only need to show that $\widetilde{\gamma}^T$, defined [4.9],in belongs to $\mathcal{E}^{PS}(\Gamma^M)$. By [A4] we do not need to check behavior of indifferents. And by [9.1] we know that the strategy is optimal for reformists given the others' behavior. Hence, we only need to check that the opponents actions are optimal. Clearly, deviation committing to a vote in favor is never optimal, as discussed above. Also, note that committing to a no vote if the quorum were not met may only lead to worst outcomes, since whenever $z_t \leq N\varepsilon$:

$$U_{i.t}(a, \gamma^T_{-i.s_{i.t}}|s_{i.t}) - U_{i.t}(0, \gamma^T_{-i.s_{i.t}}|s_{i.t}) = -\Pr(\overline{n}(1|s) = N\varepsilon \cap \overline{n}(0|s) < N\varepsilon|s_{i.t})x(s_i) \geq 0$$

But, whenever $z_t > N\varepsilon$, never committing is suboptimal:

$$U_{i.t}(0, \gamma^T_{-i.s_{i.t}}|s_{i.t}) - U_{i.t}(\{a\}_t^\infty, \gamma^T_{-i.s_{i.t}}|s_{i.t}) = -\Pr(\overline{n}(1|s) > N\varepsilon \cap \overline{n}(1|s) = \overline{n}(0|s)+1|s_{i.t})x(s_i) \geq 0$$

Additionally, the timing of commitments, given that the quorum has been passed, does not reveal any more relevant information and does not affect the outcome. ■

- Stage probabilities of rejection and belief distribution for non-secret infinite time period mechanisms are:

$$\Pr_{i.t}(1|\gamma^T, s_{i.t}) \equiv \sum_{s_{-i}\in S_{-i}} \sum_{c^T\in C^T(\alpha)} \nu\left(c_T\right) \prod_{r\geq t} \prod_{j\in N} \gamma_{j.r}(c_{j.r}|s_{j.r}(s, c^{r-1}))\beta_{i.t}(c^{t-1}, s|s_{i.t})$$

$$\beta_{i.t+1}(c^t, s|s_{i.t}) \equiv \frac{Id((s_i, c_i^t, \sigma^t(c^t)) = s_{i.t}) \prod_{r\leq t} \prod_{j\in N\backslash i} \gamma_{j.r}(c_{j.r}|s_{j.r}(s, c^{r-1}))p(s)}{\sum_{s_{-i}\in S_{-i}} \sum_{c_{-i}^t\in C_{-i}^t(\alpha)} \prod_{r\leq t} Id(z_r \in \sigma_r(c^r)) \prod_{j\in N\backslash i} \gamma_{j.r}(c_{j.s}|s_{j.r}(s, c^{r-1}))p(s)}$$

- Let us report as promised in the multiple quorum section, the equilibrium strategy of the double quorum full disclosure simultaneous mechanism that maximizes the ex-ante probability of rejection for the two policies. Let $D \equiv [-1, 0; 0, 1]$, $X(s_i) \equiv [x(s_i), x'(s_i)]$ and $Q(s) \equiv [\overline{n}(1|s) - \varepsilon N, \overline{n}'(1|s) - \varepsilon N]$. Then for any $s \in S$ the strategy is defined by:

$$\gamma_{*2}(s) = \left\{ \begin{array}{cc} \gamma_{*}((1,1)|s) = 1 & X(s_i) \gg 0 \\ \gamma_{*2}((1,0)|s) = 1 & DX(s_i) \ll 0 \cap \overline{n}'(1|s) > \varepsilon N \\ \gamma_{*2}((0,1)|s) = 1 & DX(s_i) \gg 0 \cap \overline{n}(1|s) > \varepsilon N \\ \gamma_{*2}((0,0)|s) = 1 & X(s_i) \ll 0 \cap Q(s) \gg 0 \\ \gamma_{*2}((1,a)|s) = 1 & DX(s_i) \ll 0 \cap \overline{n}'(1|s) < \varepsilon N \\ \gamma_{*2}((a,1)|s) = 1 & DX(s_i) \gg 0 \cap \overline{n}(1|s) < \varepsilon N \\ \gamma_{*2}((0,a)|s) = 1 & X(s_i) \ll 0 \cap DQ(s) \ll 0 \\ \gamma_{*2}((a,0)|s) = 1 & X(s_i) \ll 0 \cap DQ(s) \gg 0 \\ \gamma_{*2}((a,a)|s) = 1 & \text{otw.} \end{array} \right\}$$