Sales and Collusion in a Market with Storage
Web-Appendix

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Abstract

Sales are a widespread and well-known phenomenon documented in several product markets. This paper presents a novel rationale for sales that does not rely on consumer heterogeneity, or on any form of randomness to explain such periodic price fluctuations. The analysis is carried out in the context of a simple repeated price competition model, and establishes that firms must periodically reduce prices in order to sustain collusion when goods are storable and the market is large. The largest equilibrium profits are characterized at any market size. A trade-off between the size of the industry and its profits arises. Sales foster collusion, by magnifying the intertemporal links in consumers' decisions.

Keywords: Storage, sales, collusion, cartel size, repeated games.

JEL classification: L11, L12, L13, L41.

The first section discusses some useful derivations, the second section covers comparative statics omitted from the main text, while the third extends the analysis to asynchronous sales.

1 Derivatives and Signs

Recall that for any strategy $\gamma \in \mathcal{C}$ equilibrium and deviation profits in the two critical periods respectively satisfy:

$$\begin{aligned}
\Pi_0(\gamma) &= \left[\alpha_0 + \frac{1-\delta}{1-\delta^x}\left[\sigma(\varkappa\alpha_S + \alpha_0) - \alpha_0\right]\right]\mu c \\
\Pi_1(\gamma) &= \left[\alpha_0 + \frac{\delta^{\varkappa-1}-\delta^{\varkappa}}{1-\delta^{\varkappa}}\left[\sigma(\varkappa\alpha_S + \alpha_0) - \alpha_0\right]\right]\mu c \\
\Delta_0(\gamma) &= (1-\delta)\sigma\mu c \\
\Delta_1(\gamma) &= (1-\delta)\alpha_0\mu c
\end{aligned}$$

Derivatives at $t = 0$:

$$\begin{aligned}
\frac{d\Pi_0(\gamma)}{d\mu} &= \left[\alpha_0 + \frac{1-\delta}{1-\delta^x}\left[\sigma(\varkappa\alpha_S + \alpha_0) - \alpha_0\right]\right] c > 0 \\
\frac{d\Pi_0(\gamma)}{d\sigma} &= \frac{1-\delta}{1-\delta^{\varkappa}}(\varkappa\alpha_S + \alpha_0)\mu c > 0 \\
\frac{d\Pi_0(\gamma)}{d\varkappa} &= \left[\frac{1-\delta}{1-\delta^{\varkappa}}\sigma\alpha_S + \log\delta\frac{\delta^{\varkappa}-\delta^{\varkappa+1}}{(1-\delta^{\varkappa})^2}\left[\sigma(\varkappa\alpha_S + \alpha_0) - \alpha_0\right]\right]\mu c > 0 \\
\frac{d\Delta_0(\gamma)}{d\mu} &= (1-\delta)\sigma c \geq 0 \ \ \& \ \ \frac{d\Delta_0(\gamma)}{d\sigma} = (1-\delta)\mu c > 0 \ \ \& \ \ \frac{d\Delta_0(\gamma)}{d\varkappa} = 0
\end{aligned}$$

To sign $d\Pi_0(\gamma)/d\varkappa$ consider harder case, namely $\alpha_S = 1$. If so:

$$\frac{d\Pi_0(\gamma)}{d\varkappa} = \frac{1-\delta}{1-\delta^{\varkappa}}\left[1 + \varkappa\log\delta\frac{\delta^{\varkappa}}{1-\delta^{\varkappa}}\right]\sigma\mu c \geq 0$$

which is positive, since:

$$x\log\delta = \log\delta^x \geq 1 - \frac{1}{\delta^{\varkappa}}$$

Similarly, derivatives at $t = 1$, satisfy:

$$
\begin{aligned}
\frac{d\Pi_1(\gamma)}{d\mu} &= \left[\alpha_0 + \frac{\delta^{\varkappa-1}-\delta^{\varkappa}}{1-\delta^{\varkappa}}\left[\sigma(\varkappa\alpha_S+\alpha_0)-\alpha_0\right]\right] c > 0 \\
\frac{d\Pi_1(\gamma)}{d\sigma} &= \frac{\delta^{\varkappa-1}-\delta^{\varkappa}}{1-\delta^{\varkappa}}(\varkappa\alpha_S+\alpha_0)\mu c > 0 \\
\frac{d\Pi_1(\gamma)}{d\varkappa} &= \left[\frac{\delta^{\varkappa-1}-\delta^{\varkappa}}{1-\delta^{\varkappa}}\sigma\alpha_S + \log\delta\frac{\delta^{\varkappa-1}-\delta^{\varkappa}}{(1-\delta^{\varkappa})^2}\left[\sigma(\varkappa\alpha_S+\alpha_0)-\alpha_0\right]\right]\mu c \\
\frac{d\Delta_1(\gamma)}{d\mu} &= (1-\delta)\alpha_0 c \;\;\&\;\; \frac{d\Delta_1(\gamma)}{d\sigma} = \frac{d\Delta_1(\gamma)}{d\varkappa} = 0
\end{aligned}
$$

Again, to sign $d\Pi_1(\gamma)/d\varkappa$ consider the harder case, namely $\alpha_S = 1$. If so:

$$
\frac{d\Pi_1(\gamma)}{dx} = \frac{\delta^{\varkappa-1}-\delta^{\varkappa}}{1-\delta^{\varkappa}}\left[1+\varkappa\log\delta\frac{1}{1-\delta^{\varkappa}}\right]\sigma\mu c \leq 0
$$

which is negative, since:

$$
\log\delta^{\varkappa} \leq \delta^{\varkappa} - 1
$$

Moreover $d\Pi_1(\gamma)/d\varkappa > 0$, when $\alpha_0 = 1$. Thus, the sign of $d\Pi_1(\gamma)/d\varkappa$ depends on the fraction of consumers with storage in the economy. Notice that the resulting critical ratios are independent of μ:

$$
\begin{aligned}
R_0(\gamma) &= \frac{1}{1-\delta}\frac{\alpha_0}{\sigma} + \frac{1}{1-\delta^{x}}\left[(\varkappa\alpha_S+\alpha_0)-\frac{\alpha_0}{\sigma}\right] \\
R_1(\gamma) &= \frac{1}{1-\delta} + \frac{\delta^{\varkappa-1}}{1-\delta^{\varkappa}}\left[(\varkappa\alpha_S+\alpha_0)\frac{\sigma}{\alpha_0}-1\right]
\end{aligned}
$$

Derivatives at $t = 0$:

$$
\begin{aligned}
\frac{dR_0(\gamma)}{d\sigma} &= -\left[\frac{\delta-\delta^{x}}{(1-\delta^{x})(1-\delta)}\right]\frac{\alpha_0}{\sigma^2} < 0 \\
\frac{dR_1(\gamma)}{d\sigma} &= \left[\frac{\delta^{\varkappa-1}}{1-\delta^{\varkappa}}\right]\left[\frac{\varkappa\alpha_S+\alpha_0}{\alpha_0}\right] > 0 \\
\frac{dR_0(\gamma)}{d\varkappa} &= \frac{1}{1-\delta^{\varkappa}}\alpha_S + \log\delta\frac{\delta^{\varkappa}}{(1-\delta^{\varkappa})^2}\left[\varkappa\alpha_S+\alpha_0-\frac{\alpha_0}{\sigma}\right] \geq 0 \\
\frac{dR_1(\gamma)}{d\varkappa} &= \frac{\delta^{\varkappa-1}}{1-\delta^{\varkappa}}\frac{\alpha_S}{\alpha_0}\sigma + \log\delta\frac{\delta^{\varkappa-1}}{(1-\delta^{\varkappa})^2}\left[\frac{\varkappa\alpha_S+\alpha_0}{\alpha_0}\sigma-1\right]
\end{aligned}
$$

where the sign of $dR_1(\gamma)/d\varkappa$ coincides with that of $d\Pi_1(\gamma)/d\varkappa$.

Notice that $R_1(\gamma) - R_0(\gamma) = 0$ requires:

$$\sigma^2 \left[\delta^{x-1} \frac{\alpha_S \varkappa + \alpha_0}{\alpha_0} \right] + \sigma \left[\frac{1 - \delta^{\varkappa - 1}}{1 - \delta} - [\alpha_S \varkappa + \alpha_0] \right] - \alpha_0 \left[\frac{\delta - \delta^{\varkappa}}{1 - \delta} \right] = 0$$

Such condition always has unique positive solution which satisfies $\sigma^\circ \in [\alpha_0, \alpha_0/\delta^{\varkappa-1}]$, since it is negative both at $\sigma = 0$ and at $\sigma = \alpha_0$, and positive at $\sigma = \alpha_0/\delta^{\varkappa-1}$. However, the solution could in principle require $\sigma^\circ > 1$. If so, the solution to the general program $\max_{\sigma\in[0,1]} \min_t R_t(\gamma) = R_1(\gamma)$ will satisfy $\sigma(\varkappa) = \min\{1, \sigma^\circ\}$.

2 Comparative Statics

The comparative static results on the size of the set $\mathcal{E}$ developed in the previous section also, hold without further modifications. As expected, more sale strategies will be stable: when consumers are more patient; when more consumers have access to storage; or when the market becomes more profitable.

Proposition 13 *The size of the set $\mathcal{E}$ decreases with c and α, and increases with v and δ.*

The result is proven by studying how the bounds characterizing the set of stable strategy $\mathcal{E}$ vary with the free parameters. A larger fraction of consumers with storage increases the size of $\mathcal{E}$, since more sale discounts are stable at any frequency $\varkappa$. Similarly patience δ, and profitability $v - c$, increase the size of $\mathcal{E}$, since the storage constraint $\kappa(\varkappa) \geq \sigma$ is easier to satisfy when such variables grow.

3 Multiple Markets and Asynchronized Sales

From the previous discussion, it may appear that coordination in sales is necessary to achieve any stability gain. In contrast, we provide a simple example to argue that sale strategies do not need to synchronized. In particular, we show that when firms operate in multiple markets, sales do not need to be simultaneous and symmetric both within and across markets. Consider a variation on the previously described economy in which there are two identical markets A and B, each with a mass $1/2$ of consumers, and an even number $n \geq 4$ of firms operating in both markets.[1] Corollary 9 provides

[1]Notice that there are no stability gains due to the multi-market setup, since markets are symmetric, since firms' objective functions are not strictly concave, and since returns to scale are constant (Bernheim and Whinston 1990, and Spagnolo 1999).

sufficient conditions for the existence of a stable sale strategy $\gamma_k \in \mathcal{E}$ in each market $k \in \{A, B\}$. Each of these strategies still requires firms to charge the fixed markup, μ_k, in almost every period, and to periodically hold sales by reducing the markup to $\mu_k \sigma_k$ every $\varkappa_k$ periods. The most stable sale strategy in such an environment still prescribes to set $\gamma_k = \gamma^*$ in each market $k \in \{A, B\}$. Such a sale strategy sustains collusion if in any period $t \in \{0, 1\}$

$$n \leq \frac{\Pi_t(\gamma_A) + \Pi_t(\gamma_B)}{\Delta_t(\gamma_A) + \Delta_t(\gamma_B)} = R_t(\gamma^*), \tag{1}$$

since $\Pi_t(\gamma_k) = \Pi_t(\gamma^*)$ and $\Delta_t(\gamma_k) = \Delta_t(\gamma^*)$ for any $k \in \{A, B\}$.

Now consider a strategy $\hat{\gamma}$ in which the markup in each market is fixed to $\hat{\mu}_k = \mu^*$, but different firms hold sales in different markets every $\varkappa^*$ periods. In particular, consider a strategy in which sales occurring along the equilibrium path satisfy in every period t:

(1) if $\mathrm{mod}(t, 2\varkappa^*) = 0$, firms $\{1, 2, ..., n/2\}$ set a discount $\hat{\sigma}_A = \sigma^*$ in market A and $\hat{\sigma}_B > \sigma^*$ in market B, while all the remaining firms set $\hat{\sigma}_B = \sigma^*$ in market B and $\hat{\sigma}_A > \sigma^*$ in market A;

(2) if $\mathrm{mod}(t, 2\varkappa^*) = \varkappa^*$, firms $\{n/2 + 1, ..., n\}$ set a discount $\hat{\sigma}_A = \sigma^*$ in market A and $\hat{\sigma}_B > \sigma^*$ in market B, while all the remaining firms set $\hat{\sigma}_B = \sigma^*$ in market B and $\hat{\sigma}_A > \sigma^*$ in market A;

(3) if $\mathrm{mod}(t, 2\varkappa^*) \notin \{0, \varkappa^*\}$, $\hat{\sigma}_A = \hat{\sigma}_B = 1$ for every firm in every market.

Note that any firm charging $\hat{\sigma}_k > \sigma^*$ in market k does not collect profits in that market during a sales period. Also, observe that the total profit across markets is constant for each firm and equal to the total profit achieved in case of simultaneous sales. Without loss consider period 0 and a firm $i \in \{1, 2, ..., n/2\}$ and note that

$$\begin{aligned}
\Pi_{0i}(\hat{\gamma}_A) &= \frac{1}{2}\left[\frac{\delta - \delta^x}{1 - \delta^x}\alpha_0 + \frac{1 - \delta}{1 - \delta^{2x}}\left[2\left(\varkappa \alpha_S + \alpha_0\right)\sigma\right]\right]\mu c, \\
\Pi_{0i}(\hat{\gamma}_B) &= \frac{1}{2}\left[\frac{\delta - \delta^x}{1 - \delta^x}\alpha_0 + \frac{\delta^x - \delta^{x+1}}{1 - \delta^{2x}}\left[2\left(\varkappa \alpha_S + \alpha_0\right)\sigma\right]\right]\mu c, \\
&\Rightarrow \Pi_{0i}(\hat{\gamma}_A) + \Pi_{0i}(\hat{\gamma}_B) = \Pi_0(\gamma_A) + \Pi_0(\gamma_B).
\end{aligned}$$

Moreover, note that in any period t the deviation profits of each player coincide in each market k with those of the most stable sale strategy since $\Delta_t(\hat{\gamma}_k) = \Delta_t(\gamma_k)$. The few last observations in turn imply that strategy $\hat{\gamma}$ is as stable as most stable strategy γ^*. Without loss of generality consider the incentives to deviate in period 0 of a for a firm $i \in \{1, 2, ..., n/2\}$ holding a sales in market A:

$$n \leq \frac{\Pi_0(\hat{\gamma}_A) + \Pi_0(\hat{\gamma}_B)}{\Delta_0(\hat{\gamma}_A) + \Delta_0(\hat{\gamma}_B)} = R_0(\gamma^*)$$

which is equivalent to condition (1). Similarly, incentives to comply with the equilibrium strategy remain unaffected in periods without sales. Thus, maximal cartel size under which a sale strategy sustains collusion remains unaffected even with asynchronized sales. Hence, an asynchronized sale strategy would strictly dominate a simultaneous sale strategy for any arbitrarily small menu cost incurred by firms while changing prices.

The previous argument required the number of firms operating in each market to exceed four. This was necessary, since the deviation payoff $\Delta_0(\hat{\gamma}_k)$ would increase, if a single firm held sales in market k, as $\hat{\sigma}_k > \sigma^*$ for any firm not holding sales. If so, the largest sustainable cartel with asynchronized sales would be smaller than with synchronized sales, as stability is inversely related to the lowest price charged by a competing firm. Note that the straightforward extension of the multi-market model to asymmetric markets would generate sale strategies which are not synchronized across markets as well as within markets.

Let $\check{\gamma}$ denote the variant of strategy $\hat{\gamma}$ in which a single firm has sales in market A in periods $\text{mod}(t, 2\varkappa^*) = 0$ and in market B in periods $\text{mod}(t, 2\varkappa^*) = \varkappa^*$. Again consider an economy in which $S = 1$, $\delta = 0.95$, $\alpha = 0.15$, $v = 10$, and $c = 1$. Fix the threat markdown of all the firms not selling in a market k during a sales period to $\hat{\sigma}_k = 0.2$. As expected, the stability of strategy $\check{\gamma}$ is smaller compared to γ^*, as the maximal equilibrium cartel size declines, whenever deviation profits grow

	n	Π	σ	μ	$\varkappa$
γ^m	20.0	9.00	1.00	9	$\forall$
γ^+	20.5	8.72	0.94	9	2
γ^*	28.6	1.96	0.15	9	2
$\hat{\gamma}$	28.6	1.96	0.15	9	2
$\check{\gamma}$	24.7	1.96	0.15	9	2

4 Proofs

Proof Proposition 13. First note that by proposition 5, a sale strategy with period $\varkappa$ belongs to $\mathcal{E}$ if and only if:

$$\delta^{\varkappa-1} \geq \frac{1}{1+1/\bar{\mu}} \frac{\alpha}{\varkappa-\alpha(\varkappa-1)} + \frac{1/\bar{\mu}}{1+1/\bar{\mu}} = h(\alpha, 1/\bar{\mu}) \tag{2}$$

Further notice that such condition is harder to satisfy when either $1/\bar{\mu}$ or α increase since:

$$\begin{aligned} \frac{dh(\alpha,1/\bar{\mu})}{d\alpha} &= \frac{1}{1+1/\bar{\mu}} \frac{\varkappa}{(\varkappa-\alpha(\varkappa-1))^2} > 0 \\ \frac{dh(\alpha,1/\bar{\mu})}{d1/\bar{\mu}} &= \frac{1}{(1+1/\bar{\mu})^2} \left[\frac{(1-\alpha)\varkappa}{\varkappa-\alpha(\varkappa-1)}\right] > 0 \end{aligned}$$

Thus, the size of the set $\mathcal{E}$ decreases with both $1/\bar{\mu}$ and α. To establish the comparative statics on c and v, simply note that $d1/\bar{\mu}/dv < 0$ and that $d1/\bar{\mu}/dc > 0$. The final observation on δ is trivial the left hand side of (2) increases in δ. ■