

Proofs of Propositions 6 and 7

Proof of Proposition 6. Recall first that $\Pi_{\Delta F}$ increases with E and decreases with b , whereas $W_{\Delta F}$ increases with both E and b . Thus, starting at $\{b_N, E_N\}$, whenever $b < b_N$, \tilde{E} has to increase to keep $W_{\Delta F} = 0$ whereas \hat{E} has to decrease to maintain $\Pi_{\Delta F} = 0$. The opposite is true when $b > b_N$. It then follows from the definition of \bar{E} that, in both cases, it must be strictly between \tilde{E} and \hat{E} whenever $b \neq \{0, b_N\}$. When $b = 0$, it is immediate that $\bar{E} = \tilde{E}$. When $b = b_N$, it is also clear that $\tilde{E} = \bar{E} = \hat{E}$.

To find out the slope of $\bar{E}(b)$, we first totally differentiate the identity that defines $\bar{E}(b)$:

$$\frac{\partial W_{\Delta F}(\bar{E}(b), b)}{\partial b} + \frac{\partial W_{\Delta F}(\bar{E}(b), b)}{\partial E} \frac{d\bar{E}(b)}{db} + \Pi_{\Delta F}(\bar{E}(b), b) + b \left[\frac{\partial \Pi_{\Delta F}(\bar{E}(b), b)}{\partial b} + \frac{\partial \Pi_{\Delta F}(\bar{E}(b), b)}{\partial E} \frac{d\bar{E}(b)}{db} \right] = 0.$$

After rearrangement, this expression becomes

$$(A9) \quad \frac{d\bar{E}(b)}{db} = - \frac{\partial [W_{\Delta F}(\bar{E}(b), b) + b \Pi_{\Delta F}(\bar{E}(b), b)] / \partial b}{[\partial W_{\Delta F}(\bar{E}(b), b) / \partial E] + b [\partial \Pi_{\Delta F}(\bar{E}(b), b) / \partial E]}.$$

The denominator of (A9) is strictly positive, since an increase in the number of export sectors unambiguously increases the impact of the FTA on both aggregate welfare and aggregate profits. Now, using the analog of (A8) for aggregate welfare and profits, we have that the numerator of (A9) is equal to $\Pi_{\Delta F}(\bar{E}(b), b)$. It follows that $\text{sgn}[d\bar{E}(b)/db] = -\text{sgn}[\Pi_{\Delta F}(\bar{E}(b), b)]$. For $b < b_N$, then, $d\bar{E}(b)/db < 0$ because $\bar{E}(b) > \hat{E}(b)$, which implies that $\Pi_{\Delta F}(\bar{E}(b), b) > 0$. For $b > b_N$, $d\bar{E}(b)/db > 0$ because $\bar{E}(b) < \hat{E}(b)$, which implies that $\Pi_{\Delta F}(\bar{E}(b), b) < 0$. At $b = b_N$, $d\bar{E}(b)/db = 0$, since $\bar{E}(b) = \hat{E}(b)$. QED

Proof of Proposition 7. To prepare for the proofs of each lemma and proposition under the oligopolistic structure, denote by q^j the sales of a firm from country j in the home market, $j = H, Y, Z$. The first order necessary condition that defines q^j is represented by

$$(A10) \quad P(t, t_i) - c - q^j(t, t_i) - t_j = 0.$$

When firm j represents a firm from country Y , $t_j = t_i$; if it represents a firm from country Z , $t_j = t$; if it represents a domestic firm, $t_j = 0$. Summing this condition over all firms, we obtain

$$n[P(t, t_i) - c] - D(t, t_i) - n_y t_i - n_z t = 0.$$

Using $D = A - P$ and rearranging this expression, we find the market's equilibrium price as a function of tariffs and the other parameters of the model:

$$(A11) \quad P(t, t_i) = [A + cn + n_y t_i + n_z t] / (1 + n).$$

Substituting (A11) in the first order condition of a domestic firm, whose local sales I denote henceforth by $q(t, t_i)$, we find

$$(A12) \quad q(t, t_i) = [A - c + n_y t_i + n_z t] / (1 + n).$$

To show that Lemma 1 holds under the oligopolistic structure, note that now price is greater than marginal costs. Hence, equation (6) becomes

$$(A13) \quad \frac{d\pi^m(t)}{dt} = q(t) \frac{dP(t)}{dt} + [(P(t) - c)] \frac{dq(t)}{dt},$$

where $x(t) \equiv x(t, t)$ for any variable x . Using (A13) and its analog under an FTA, the effect of a marginal increase in the external tariff on the profits of the domestic firm changes with an FTA as follows:

$$\frac{d\pi_F^m(t)}{dt} - \frac{d\pi^m(t)}{dt} = q_F(t) \frac{dP_F(t)}{dt} + [(P_F(t) - c)] \frac{dq_F(t)}{dt} - q(t) \frac{dP(t)}{dt} - [(P(t) - c)] \frac{dq(t)}{dt}.$$

However, it follows from (A11) and (A12) that the *changes* in both the price level and the sales of a domestic firm due to a higher external tariff are independent of the internal tariff; hence, they are unaffected by the constraint setting $t_i = 0$ under the FTA. The expression above can then be rewritten as

$$\frac{d\pi_F^m(t)}{dt} - \frac{d\pi^m(t)}{dt} = [q_F(t) - q(t)] \frac{dP(t)}{dt} + [(P_F(t) - P(t))] \frac{dq(t)}{dt}.$$

The constraint $t_i = 0$ does affect, however, the *level* of those two variables. As is clear from (A11) and (A12), $P_F(t) \equiv P(t, 0) < P(t, t) \equiv P(t)$ and $q_F(t) \equiv q(t, 0) < q(t, t) \equiv q(t)$ at any given external tariff. Since $dq(t)/dt > 0$ and $dP(t)/dt > 0$, we obtain $d\pi_F^m(t)/dt < d\pi^m(t)/dt$, as Lemma 1 indicates.

To show that Lemma 2 holds under the oligopolistic structure, note first that (A10) implies that $q^j = q^b - t_j, j = Y, Z$. Thus, the equilibrium sales of a firm from country Y and those of a firm from country Z can be represented, respectively, as $q(t, t_i) - t_i$ and $q(t, t_i) - t$. Accounting for that, one can rewrite the objective function in (21) as

$$\begin{aligned}
(A14) \quad & W(t, t_i) + b\Pi^M(t, t_i) = CS(t, t_i) + TR(t, t_i) + (1+b)n_b\pi(t, t_i) = \\
& = \frac{1}{2}[A - P(t, t_i)]^2 + n_z[q(t, t_i) - t] + n_y[q(t, t_i) - t_i]t_i + (1+b)n_b[P(t, t_i) - c]q(t, t_i).
\end{aligned}$$

Now, it follows from (A11) and (A12) that $dq(t, t_i)/dt$ and $dP(t, t_i)/dt$ are both constant. Thus,

$$\frac{d^2[W(t, t_i) + b\Pi^M(t, t_i)]}{dt^2} = \left[\frac{dP(t, t_i)}{dt} \right]^2 + 2n_z \left[\frac{dq(t, t_i)}{dt} - 1 \right] + 2n_b(1+b) \frac{dP(t, t_i)}{dt} \frac{dq(t, t_i)}{dt}$$

is constant as well. It is also negative. To see that, notice that, since $dP(t, t_i)/dt = dq(t, t_i)/dt = n_z/(1+n)$, the expression is negative even when $b = b'_{max} = 1/2n_b$. This proves that Lemma 2 hold under the oligopolistic structure as well.

To see that the same applies to Lemma 3, it suffices to show that $dt(t_i)/dt_i$ in this case as well. Using the notation defined in the original proof and recalling that $dq(t, t_i)/dt$ and $dP(t, t_i)/dt$ are constant, Lemma 2 implies that

$$\begin{aligned}
\frac{dt}{dt_i} &= - \frac{\partial \text{FONC}(t, t_i, b) / \partial t_i}{\partial \text{FONC}(t, t_i, b) / \partial t} = \\
&= \frac{1}{k} \left\{ \frac{dP(t, t_i)}{dt} \frac{dP(t, t_i)}{dt_i} + n_z \frac{dq(t, t_i)}{dt_i} + n_y \frac{dq(t, t_i)}{dt} + (1+b)n_b \left[\frac{dP(t, t_i)}{dt} \frac{dq(t, t_i)}{dt_i} + \frac{dq(t, t_i)}{dt} \frac{dP(t, t_i)}{dt_i} \right] \right\}.
\end{aligned}$$

Since $k > 0$ (Lemma 3), $dq(t, t_i)/dt_j > 0$ and $dP(t, t_i)/dt_j > 0$ for $j = i, e$, $dt(t_i)/dt_i > 0$, confirming that Lemma 3 is also robust to the change in market structure.

Given lemmas 1-3, the proofs of propositions 1 and 2 are entirely analogous to the case under perfect competition considered in Section II. Regardless of market structure, Proposition 3 follows from Proposition 2 and the definition of political rents, whereas Proposition 4 follows from the definition of political viability and propositions 2 and 3. Proposition 5 is similarly independent of the market structure.

It remains to be shown, then, only that lobbying for and against FTAs does not produce any qualitative change in Proposition 4 under the oligopolistic structure. Central to this result is the fact that the FTA increases the sum of consumers' surplus and tariff revenue under this structure: $CS_{\Delta F} + TR_{\Delta F} > 0$. To see this, first use (A11) and (A12) to write $CS + TR$ as a function of t_i and t :

$$\begin{aligned}
& CS(t, t_i) + TR(t, t_i) = \\
& = \frac{1}{1+n} \left\{ \frac{1}{2(1+n)} [(A-c)n - n_y t_i - n_\zeta t]^2 + [A-c + n_y t_i - (1+n_b + n_y)t] n_\zeta t + [A-c + n_\zeta t - (1+n_b + n_\zeta)t_i] n_y t_i \right\},
\end{aligned}$$

which is equivalent to

$$(A15) \quad CS + TR = \frac{1}{1+n} \left\{ \frac{1}{2(1+n)} [(A-c)n - (n_y + n_\zeta)t^p]^2 + [A-c - (1+n_b)t^p](n_y + n_\zeta)t^p \right\}$$

in the absence of the FTA and to

$$(A16) \quad CS_F + TR_F = \frac{1}{1+n} \left\{ \frac{1}{2(1+n)} [(A-c)n - n_\zeta t_F^p]^2 + [A-c - (1+n_b + n_y)t_F^p] n_\zeta t_F^p \right\}$$

under the FTA. When there is no political economy, we can find the political tariff with and without the FTA by solving $FONC_F(b=0) = 0$ and $FONC(b=0) = 0$, respectively. Calculations reveal that

$$t^* = \frac{(A-c)(1+2n_b)}{2(1+n_b)^2 + n_y + n_\zeta} \quad \text{and} \quad t_F^* = \frac{(A-c)(1+2n_b)}{2(1+n_b + n_y)^2 + 2n_y n_\zeta + n_\zeta}.$$

Substituting these expressions back in (A15) and (A16), we obtain

$$CS(b=0) + TR(b=0) = \frac{(A-c)^2 \left\{ \frac{1}{2} n^2 + (1+2n_b)[(n_b^2 + n_y + n_\zeta)(1+n_b) + \frac{1}{2} n_b^2] \right\}}{[2(1+n_b)^2 + n_y + n_\zeta]^2}$$

and

$$CS_F(b=0) + TR_F(b=0) = \frac{(A-c)^2 \left\{ \frac{1}{2} n^2 (1+2n_y)^2 + (1+2n_b)[(n_b + n_y)^2 (\frac{3}{2} + n_b + 2n_y) + (1+n_b + n_y)(1+2n_y)n_\zeta] \right\}}{[2(1+n_b + n_y)^2 + 2n_y n_\zeta + n_\zeta]^2}.$$

After some manipulation, one finds that $CS_{\Delta F}(b=0) + TR_{\Delta F}(b=0) > 0$ (see Ornelas 2003). Now, by Proposition 2, $dW_{\Delta F}/db = d(CS_{\Delta F} + TR_{\Delta F} + \Pi_{\Delta F}^M)/db > 0$ and $d\Pi_{\Delta F}^M/db < 0$. Therefore, $d(CS_{\Delta F} + TR_{\Delta F})/db > 0$. Since $CS_{\Delta F}(b=0) + TR_{\Delta F}(b=0) > 0$, it then must be the case that $CS_{\Delta F} + TR_{\Delta F} > 0$ for any b . Hence, $W_{\Delta F} < 0$ is possible only if $\Pi_{\Delta F} < 0$, so $W_{\Delta F} < 0$ implies $W_{\Delta F} + b\Pi_{\Delta F} < 0$ and welfare-reducing FTAs cannot be politically feasible (from criterion (19)). Conversely, if $\Pi_{\Delta F} < 0$ and $W_{\Delta F} \in (0, -b\Pi_{\Delta F})$, the FTA is welfare-improving but $W_{\Delta F} + b\Pi_{\Delta F} < 0$, so welfare-improving FTAs may not be politically feasible either. QED