Job and wage mobility
with minimum wages and imperfect compliance∗

Professor Zvi Eckstein
Tel Aviv University
and Bank of Israel

Doctor Suqin Ge
Virginia Tech

Doctor Barbara Petrongolo
London School of Economics
and CEP (LSE)

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Abstract

We propose a job search model with minimum wage regulations and imperfect compliance to explain the doubling of the mean and variance of hourly earnings of white males during the first eighteen years of labor market experience. The model encompasses job mobility and on-the-job wage growth as sources of wage dynamics, and is estimated by simulated GMM using data from the NLSY79. Our estimates provide a good fit for the observed levels and trends of the main job and wage mobility data, and for the increase in the mean and variance of wages over the life cycle, as well as for the fall in the fraction of workers paid below the minimum wage. Job mobility explains 40% to 50% of the observed wage growth. Increases in the minimum wage and/or compliance deliver small effects on the wage distribution and the nonemployment rate.

Keywords: job search; wage growth; minimum wages; compliance.

JEL: J42, J63, J64

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1 Introduction

Data on labor market histories give evidence of rich employment and earnings dynamics in the years following labor market entry. Most notably, according to the National Longitudinal Survey of Youths 1979, real hourly earnings of white, male high school graduates double during the first eighteen years in the labor market, and so does their variance, while nonemployment rates and job mobility decline. Another interesting piece of evidence is that upon labor market entry an important fraction of high school graduates are paid below the US federal minimum wage: among white males this figure is about 20% initially, then falls to 10% in the first three years in the labor market, and settles around 2-3% from year ten onwards.¹ In this paper we intend to assess the potential of a simple job search model to jointly explain the observed employment and earnings dynamics, and in particular the three main stylized facts concerning mean earnings, their variance, and the extent of noncompliance to minimum wage regulations along the careers of young males.

We will embody minimum wage regulations and imperfect compliance in an otherwise standard job search model that encompasses both job mobility and on-the-job wage growth as potential sources of wage dynamics. Both employed and nonemployed workers search for jobs, and wage offers are random draws from a known, exogenous wage distribution. Wage offers reflect firm specific productivity, as in the Lucas and Prescott (1974) islands’ model. There is a minimum wage in this economy, and due to imperfect compliance only a fraction of firms whose productivity falls below the minimum wage leaves the market.

The model is estimated by simulated method of moments using data from the National Longitudinal Survey of Youths 1979. In the empirical analysis we use two separate sets of moments, namely the sequence of monthly moments of labor market states, transitions and wages; and moments computed on employment cycles that are delimited by subsequent nonemployment spells. Unobserved heterogeneity is controlled for non-parametrically by allowing for either a two-type or a four-type mixture distribution. We allow for measurement error in observed wages, which in turn implies that wage observations below the minimum wage, paid below the minimum wage.

¹Despite the findings of Ashenfelter and Smith (1979), pointing at a 25-35% rate of noncompliance to minimum wage regulations in the US, most of the recent literature on labor market effects of minimum wages has typically overlooked compliance issues. More recent work by Weil (2005), based on data from the US apparel industry, detects a 54% noncompliance rate among employers, with about 27 minimum wage violations per 100 workers.
wage may stem from both noncompliance and error. However, while measurement error affects the variance of the observed wage distribution without affecting job-to-job transitions, which are instead based on true wage realizations, true wages below the minimum wage affect job mobility, and this will be the basis of our identification strategy.

The two sets of moments used for identification deliver very similar sets of parameter estimates, which have plausible magnitudes and are in line with existing estimates of search model with similar ingredients (see the extensive survey by Eckstein and van den Berg, 2007). The arrival rate of job offers is higher for nonemployed than for employed workers, and these rates differ between two (or four) unobserved types of individuals, delivering a decreasing hazard of leaving nonemployment. With an estimated annual rate of wage growth on-the-job around 2.2-2.5%, our estimates predict that job search and mobility explain between 40% and 50% of total wage growth during the first eighteen years of labor market experience. The rest is accounted for by wage growth on-the-job.

When fitting a two-type mixture distribution, we find that the unobserved types have roughly equal size. One type is less employable, with relatively low arrival rates of job offers and mean wage offers, and a reservation wage below the federal minimum wage, which implies that this type is affected by minimum wage regulations. The other type has much higher arrival rates of job offers and mean wage offers, and a reservation wage above the minimum wage, and thus is not affected by minimum wage regulations. The estimated noncompliance rate is about 25%. That is, a quarter of firms whose productivity falls below the minimum wage do not leave the market and offer instead wages below the minimum. This figure in turn translates into a steady state proportion of jobs paying less than the minimum wage equal to roughly 11%. As the US legislation on the minimum wage caters for a limited number of exemptions to minimum wage regulations that we may not fully identify in our data, this figure may also include such exempt cases.

Our estimated model fits reasonably well the sets of monthly and cycle moments obtained on the data. In particular, it does a very good job at predicting the initial eighteen years of wage growth from 8 to 16 dollars per hour, the level and the upward trend in their wage variance, and the decline in the fraction of workers paid below the minimum wage. To our knowledge, this work is the first to use a simple search model
to replicate both first and second moments of the wage distribution over the life cycle, as well as the extent of noncompliance to minimum wage regulations.

The comparative statics analysis of changes in the minimum wage and/or in the extent of compliance delivers very limited effects of either variable on the nonemployment rate, while significantly affecting the proportion of workers paid below the minimum wage. For example, a 10% increase in the minimum wage increases nonemployment by about 0.3-0.4 percentage points, depending on labor market experience, and this is a sort of upper bound for the employment effects of minimum wages, as (complying) firms whose productivity falls below the minimum wage have no option but to leave the market.2

The literature on labor market effects of minimum wage regulations is very large and we do not attempt to cover it all here.3 Work that is most closely related to ours includes a number of papers that analyze minimum wage policies in estimable search equilibrium models. In Eckstein and Wolpin (1990), who extend and estimate the equilibrium search model of Albrecht and Axell (1984), a binding minimum wage drives less profitable firms out of business, and thus shifts the wage distribution to the right and reduces the effective arrival rate of job offers to unemployed workers. van den Berg and Ridder (1998) introduce minimum wages in an equilibrium wage posting model with employed job search à la Burdett and Mortensen (1998), and obtain a continuous equilibrium wage distribution, truncated at the minimum wage. A similar equilibrium model is later used by van den Berg (2003) to study the potential of minimum wages to rule out a Pareto dominated equilibrium in an environment with multiple equilibria. Finally, Flinn (2006) estimates a matching model with Nash wage bargaining, where both the contact rate between workers and firms and the acceptance rate respond to the imposition of a minimum wage.

Similarly as in these papers, we let the wage offer distribution and the arrival rate of job offers respond to the introduction of a legally binding minimum wage. In doing this we adopt a simple framework with

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2If firms have instead monopsony power and may offer wages below productivity, the minimum wage impact on nonemployment could be smaller or even positive, see Manning (2003, Chapter 12).

3See Brown (1999) for a review of empirical work on the labor market effects of the minimum wage. The consensus formed in the 1990s concerning the negative employment effects of minimum wages for youths has been challenged by Card and Krueger (1995), who, based on a number of policy studies, detect no detrimental effects of minimum wages on employment (see Brown, 1999, Table 3). See also Keenan (1995), Newmark and Wascher (2000) and Card and Krueger (2000) for more recent contributions to this debate.
exogenous compliance to minimum wage regulations. The rest of the paper is organized as follows. Section 2 presents our job search model. Section 3 describes our sample of white, male high school graduates, obtained from the NLSY79. Section 4 gives details of the estimation method, based on efficient simulated GMM. Section 5 presents our estimation results and simulates the effect of changes in the minimum wage and noncompliance on both nonemployment and the wage distribution. Section 6 finally concludes.

2 The job search model

We construct a continuous time search model in a stationary labor market environment with three main ingredients: (i) on-the-job search; (ii) minimum wages with imperfect compliance by firms; and (iii) exogenous on-the-job wage growth.

The economy is populated by infinitely lived, identical workers, whose mass is normalized to 1. At each moment in time a worker can be either nonemployed (a state denoted by \( n \)) or employed (a state denoted by \( e \)). When nonemployed, workers enjoy some real return \( b \) (typically including the value of leisure and unemployment insurance benefits, net of search effort costs), and receive job offers at a Poisson rate \( \lambda_n \). When employed, they enjoy a real wage \( w \), which grows at an exogenous rate \( g \), and receive job offers at a Poisson rate \( \lambda_e \). Existing jobs are hit by idiosyncratic shocks, which occur at a Poisson rate \( \delta \). The instantaneous discount rate is \( r \). New wage offers for the employed and the nonemployed are randomly drawn from some known, exogenous distribution \( F(w) \). Once an individual accepts a wage \( w \), his wage on the same job grows with tenure, \( \tau \), such that \( w_{\tau} = we^{g\tau} \).

A simple way to motivate our choice of wage distribution follows Lucas and Prescott (1974) island model. In particular, we assume that there exists a mass 1 of small firms, with one job each, which can be either occupied by a worker or vacant. Firms differ in their productivity, and the wage rate at each firm is equal to productivity, with resulting zero profits. Thus the wage offer distribution \( F(w) \) mimics the exogenous productivity distribution across firms. In Lucas and Prescott (1974) model, productivity in each island is

\[^{4}\text{Endogenizing firms' compliance strategies would ideally require longitudinal, linked employer-employee data. When using worker level data, as we do in this paper, this task would require strong identifying assumptions on firm behavior, which are probably not superior to our assumption of exogenous compliance, while making estimated model much less tractable.}\]

\[^{5}\text{We do not explicitly model job search behavior, and this leaves the arrival rates of job offers exogenous. Endogenizing search effort and the associated arrival rates (as in a previous version of this paper) would not alter our main results.}\]
subject to idiosyncratic shocks, and workers spend effort in order to locate better matching opportunities and eventually relocate across islands in pursuit of wage gains. In our model, the productivity on each job grows at a deterministic and homogeneous rate $g$, but better matching opportunities arise to workers through search on-the-job.

Workers in this set-up are ex-ante homogeneous, as they face the same arrival rates of job offers and the same wage offer distribution, and are ex-post heterogeneous due to luck, which determines whether a worker receives a job offer and the associated initial wage, and seniority, which determines wage growth on-the-job. In the empirical implementation we introduce heterogeneity in the fundamental parameters of the model by allowing for two (or four) unobserved workers type. For simplicity of exposition, the model works with the assumption of ex-ante homogeneity.

There is a minimum wage in the economy, denoted by $w_M$, which would drive $F(w_M)$ firms out of business if they were all forced to comply with minimum wage regulations. However, under imperfect compliance, only a proportion $1 - \alpha$ of these firms leave the market, and workers still face some positive probability to receive a wage offer below the minimum wage. The number of operating firms is thus reduced to $1 - (1 - \alpha)F(w_M)$.

As the number of active firms falls with the minimum wage, arrival rates of job offers both off- and on-the-job needs to be adjusted accordingly. Following the matching function literature, we assume that the arrival rate is increasing and concave in the number of firms, with constant elasticity $\eta$, $0 < \eta < 1$.\(^6\) Thus, the new arrival rates are $\tilde{\lambda}_n(\alpha, w_M) = \lambda_n(1 - (1 - \alpha)F(w_M))^\eta$ and $\tilde{\lambda}_e(\alpha, w_M) = \lambda_e(1 - (1 - \alpha)F(w_M))^\eta$. The resulting wage offer density is $\tilde{f}(w; \alpha, w_M) = \frac{f(w)}{1 - (1 - \alpha)F(w_M)}$ for all $w \geq w_M$ and $\alpha \tilde{f}(w; \alpha, w_M)$ for all $w < w_M$. The limiting case $\alpha = 0$ represents full compliance, with zero wage density for all $w < w_M$ and $\tilde{f}(w; 0, w_M) = \frac{f(w)}{1 - F(w_M)}$ for all $w \geq w_M$. When $\alpha = 1$ there is no effective minimum wage regulation in the economy and the wage density is simply $f(w)$.\(^7\)

Search strategies of individuals can be characterized using standard lifetime value functions. The lifetime

\(^6\)Typically matching rates would depend on both the number of firms and the number of jobseekers, but the number of jobseekers is here normalized to 1.

\(^7\)Introducing an exogenous noncompliance parameter $\alpha$ is a simple way to keep the wage offer distribution continuous and differentiable. Modelling explicitly compliance decisions, while going beyond the scope of this paper, generates a spike at the minimum wage and a discontinuity in the wage distribution (see Ashenfelter and Smith, 1979 and Lott and Roberts 1995).
value of employment in a job paying \( w_t \) is \( V_e(w_t) \), where \( \tau \) denotes tenure on the current job. The lifetime value of nonemployment is denoted by \( V_n \). As \( V_e(w_t) \) increases with \( w_t \) while \( V_n \) is independent of \( w_t \), there exists a unique reservation wage \( w^* \) such that \( V_n = V_e(w^*) \), describing the wage acceptance policy of the nonemployed. By the same logic, the employed optimally accept the first wage offer at or above their current wage.

When the minimum is binding (\( w_M > w^* \)), the flow value of unemployment can be written as:

\[
rV_n = b + \tilde{\lambda}_n(\alpha, w_M) \left[ \alpha \int_{w^*}^{w_M} [V_e(w) - V_n] \tilde{f}(w; \alpha, w_M)dw + \int_{w_M}^{w} [V_e(w) - V_n] \tilde{f}(w; \alpha, w_M)dw \right].
\]  
(1)

Similarly, the flow value of employment is given by:

\[
rV_e(w_t) = w_t + gw_tV'_e(w_t) + \delta [V_n - V_e(w_t)] + \tilde{\lambda}_e(\alpha, w_M) \left[ \alpha \int_{w_t}^{w_M} [V_e(w) - V_e(w_t)] \tilde{f}(w; \alpha, w_M)dw \right. \\
+ \int_{w_M}^{w} [V_e(w) - V_e(w_t)] \tilde{f}(w; \alpha, w_M)dw \]  
for \( w_t \geq w_M \),  
(2)

\[
rV_e(w_t) = w_t + gw_tV'_e(w_t) + \delta [V_n - V_e(w_t)] + \tilde{\lambda}_e(\alpha, w_M) \left[ \alpha \int_{w_t}^{w_M} [V_e(w) - V_e(w_t)] \tilde{f}(w; \alpha, w_M)dw \right. \\
+ \int_{w_M}^{w} [V_e(w) - V_e(w_t)] \tilde{f}(w; \alpha, w_M)dw \]  
for \( w_t < w_M \),  
(3)

where terms in \( V'_e(w_t) \) represent the appreciation in the value of employment due to returns to tenure.

The value of the reservation wage can be solved for by setting equation (1) equal to equation (3), evaluated at \( w = w^* \) and \( \tau = 0 \), exploiting the continuity of \( V_e(w_t) \) at \( w_M \):

\[
w^* = b - gw^*V'_e(w^*) + \left( \tilde{\lambda}_e(\alpha, w_M) - \tilde{\lambda}_e(\alpha, w_M') \right) \left[ \alpha \int_{w^*}^{w_M} [V_e(w) - V_n] \tilde{f}(w; \alpha, w_M)dw + \int_{w_M}^{w} [V_e(w) - V_n] \tilde{f}(w; \alpha, w_M)dw \right]
\]

\[
= b - gw^*V'_e(w^*) + \left( \tilde{\lambda}_e(\alpha, w_M) - \tilde{\lambda}_e(\alpha, w_M') \right) \left[ \int_{w^*}^{w_M} [1 - \tilde{F}(\alpha, w_M)]V'_e(w)dw - \alpha \int_{w^*}^{w_M} \tilde{F}(w; \alpha, w_M)V'_e(w)dw \right],
\]  
(4)

where \( \tilde{F}(\cdot; \alpha, w_M) = \frac{\tilde{F}(\cdot)}{1 - \lambda(\alpha, w_M)} \).

To solve the model one still needs to obtain an expression for \( V'_e(w_t) \). In Appendix A we derive the explicit solution for \( V'_e(w_t) \) for \( w_t \geq w_M \) (equation 13) and \( w_t < w_M \) (equation 14). Substituting (13) and
(14) into (4) gives a unique solution for the reservation wage, which enables us to fully characterize search decisions and simulate labor market dynamics.

In the rest of the paper this framework will be used to simulate and estimate transitions from the first nonemployment spell after education into employment, the subsequent job-to-job mobility and associated wage growth, and transitions back into nonemployment following involuntary job loss. Based on these, we will estimate the probability of nonemployment and that of employment, distinguishing between below and above the minimum wage.

This framework is also useful to describe the impact of changes in the minimum wage on nonemployment. An increase in the minimum wage lowers the arrival rate of job offers for both the employed and the nonemployed in the same proportion. If the nonemployed have higher arrival rates than the employed, as typically the case, the absolute fall in $\tilde{\lambda}_n(\alpha, w_M)$ is going to be greater than the absolute fall in $\tilde{\lambda}_e(\alpha, w_M)$, and this tends to lower the reservation wage. At the same time it raises the expected value of all acceptable wage offers (first term in the bottom row of equation (4)) and removes some mass of firms paying below $w_M$ (second term in the bottom row of equation (4)). The impact of higher minimum wages on the reservation wage is thus ambiguous. A fall in $\alpha$, meaning better compliance of firms to minimum wage regulations, has the same qualitative impact of an increase in $w_M$, that is, it lowers arrival rates of job offers and has an ambiguous impact on the reservation wage.

The impact of the minimum wage and compliance on nonemployment is solely determined by the unemployment exit hazard, given that the job destruction rate $\delta$ is exogenous. Such hazard is given by

$$h_n(\alpha, w_M) = \tilde{\lambda}_n(\alpha, w_M) \left[ 1 - (1 - \alpha) \tilde{F}(w_M; \alpha, w_M) - \alpha \tilde{F}(w^*; \alpha, w_M) \right].$$

While the arrival rate $\tilde{\lambda}_n(\alpha, w_M)$ falls with $w_M$ and $1 - \alpha$, the impact of these two variables on $w^*$ and therefore on the acceptance rate $[1 - (1 - \alpha) \tilde{F}(w_M; \alpha, w_M) - \alpha \tilde{F}(w^*; \alpha, w_M)]$ is ambiguous. Thus their impact on the reemployment hazard is also theoretically ambiguous, and will only be assessed empirically. In the practice, it will turn out that the minimum wage and compliance parameters have a negligible impact on the reservation wage, and thus most of the action of minimum wage regulations on nonemployment works
through a reduction in the arrival rate of job offers.

3 Data

We use data drawn from the National Longitudinal Survey of Youths, which contains information on a sample of 12,686 respondents who were between 14 and 22 years of age as of January 1979 (NLSY79). We focus our analysis on a fairly homogenous population, who is relatively likely to both participate in the labor force and receive wage offers below the minimum wage. For this purpose we include in our sample white, high school graduate males. Specifically, we select white males who have completed at most 12 years of schooling and declare to hold a high school degree, having excluded those who (i) ever went to the army; (ii) ever declared to be in college; (iii) ever declared to have a college or professional degree. We further restrict our sample to those who completed high school between age 17 and 19. These restrictions leave us with a sample of 577 individuals, with almost 12(months)x18(years) work history observations per individual. Information on selected respondents is available since January 1978. We assume that labor market entry coincides with the month of high school graduation, and construct later monthly work histories using retrospective information contained in the NLSY work history files.

Labor Market States We first construct individual monthly employment status between January 1978 and December 1998. We define an individual as employed in a given month if he works at least 10 hours per week and at least three weeks in the month, or during the last two weeks in the month. Otherwise, an individual is classified as nonemployed, and we do not further distinguish between unemployed and out of the labor force.

The work history information is employer-based, thus a “job” should be understood as an uninterrupted employment spell with a given employer. Multiple jobs held contemporaneously are treated as new jobs altogether, with an associated wage equal to the average of hourly wages, and working hours equal to the sum of working hours on the different jobs. Tenure in a job is completed when a new job is recorded or when an individual is back in nonemployment. Figure 1 shows the monthly employment rate by potential experience,
measured as time since high school graduation. As expected, the data show a trend increase in employment rates, starting off around 70% upon high-school graduation and reaching 90% ten years later. Clear patterns of seasonality can also be detected. In particular, employment rates are highest in March/April, and lowest in November/December.

We find that 55% of individuals are employed in the month they complete high school. This may happen because job search starts while in school or, more likely, because students may take up temporary and part-time jobs while in school. The latter explanation also seems supported by the clear seasonal pattern of employment rates during the last year before graduation. We assume that individuals employed before graduation enter the “official” labor market upon graduation, but we treat the proportion of individuals employed at labor market entry as an initial condition in our analysis.

Figure 2 plots transitions across labor market states by labor experience. All transitions display a downward trend during the first ten years, especially the employment to nonemployment hazard, and then they stay roughly constant for the additional eight years. Again, there is strong evidence of seasonality, with large monthly fluctuations. Survival rates in a given state are plotted in Figure 3. The probability of staying on the same job increases from 65% to 90% over the sample period, while that of staying out of work decreases from 20% to about 8%. An important goal of the search model described above is to fit the levels and trends of these transition rates, although the model is not primarily suited to fit the monthly seasonal fluctuations, which are mostly driven by aggregate, as opposed to life-cycle, factors.

Wages and Employment Cycles In order to describe labor market dynamics, it will be useful to organize the longitudinal data into cycles that begin and end in the nonemployment state, as also done by Wolpin (1992). Cycle \( c \) in a given labor market career is denoted by

\[ \text{cycle}^c(\text{ne}^c, e_1^c, e_2^c, \ldots), \]

where \( \text{ne}^c \) denotes the duration of the \( c \)th nonemployment spell, and \( e_1^c, e_2^c, \ldots \) denote uninterrupted job spells within cycle \( c \), with associated wages \( w_1^c, w_2^c, \ldots \). For individuals who start working before graduation, the first cycle starts with their first job instead of nonemployment.
Table 1: Employment and wage cycles

<table>
<thead>
<tr>
<th>Cycle 1</th>
<th>Cycle 2</th>
<th>Cycle 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>NE</td>
<td>Job 1</td>
<td>Job 2</td>
</tr>
<tr>
<td>(no. of obs.)</td>
<td>190</td>
<td>574</td>
</tr>
<tr>
<td>Duration</td>
<td>8.07</td>
<td>33.96</td>
</tr>
<tr>
<td>Mean wage</td>
<td>8.22</td>
<td>10.37</td>
</tr>
<tr>
<td>Std. dev. wage</td>
<td>3.43</td>
<td>4.28</td>
</tr>
<tr>
<td>(no. of obs.)*</td>
<td>306</td>
<td>192</td>
</tr>
<tr>
<td>Wage below ( w_M )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportion (%)</td>
<td>21.90</td>
<td>6.77</td>
</tr>
<tr>
<td>Mean wage</td>
<td>4.89</td>
<td>5.17</td>
</tr>
<tr>
<td>Std. dev. wage</td>
<td>1.38</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Notes. Durations are expressed in months and wages are expressed in 2000 dollars. * denotes nonmissing values on wages.

The wage concept that we use is the hourly pay rate on each job, provided in the work history file. This is derived by combining information on respondents’ usual earnings (inclusive of tips, overtime and bonuses, before deductions) with each employer for whom the respondent worked since the last interview date, with information on the applicable unit of time (per day, per hour, etc.). We next obtain real hourly wages in 2000 dollars, by deflating nominal wages using the monthly CPI published by the BLS. We finally top and bottom code hourly wages at $150 and $1.0, respectively, up until 1990; and at $200 and $1.5 afterwards. The mean hourly wage in our sample increases from about $8 upon high school graduation to $16 eighteen years later (see Figure 4, left-hand scale). The standard deviation of wages also doubles during this time period, from about $4 to $8 (see Figure 4, right-hand scale).

Table 1 provides descriptive information on employment and wages over the first three jobs in the first three cycles (we are not analyzing further cycles or further jobs within the first three cycles because of very small cell size). Both nonemployment and job durations fall between the first and the second cycle. As expected, mean wages increase with job moves within cycles. However, job changes with intervening nonemployment (across cycles) are typically associated with wage losses.

The minimum wage The federal hourly minimum wage for covered nonexempt employees has increased during our sample period from $2.65 in 1978 to $5.15 in 1998. However, the real value of the minimum wage,
deflated using the monthly CPI, has fallen during the same period by about 16% (see Figure 5). Several states also have state-level minimum wage laws, and where an employee is subject to both state and federal minimum wage laws, he or she is entitled to the higher of the two. As of March 2006, 19 states had minimum wage rates above the federal level, with a maximum of $7.63 per hour in Washington State.

Some minimum wage exemptions apply under specific circumstances. For example, as of 1998, a minimum wage of $4.25 per hour applied to workers under the age of 20 during their first 90 consecutive calendar days of employment with an employer. After 90 days or when the employee reaches age 20, he or she must receive a minimum wage of $5.15. Full-time students can be paid not less than 85% of the minimum wage before they graduate or permanently leave school. Student learners aged 16 or older can be paid not less than 75% of the minimum wage for as long as they are enrolled in a vocational education program. Workers normally receiving tips are also exempt from minimum wage regulations. Most important exemptions should not feature in our data, as we exclude students from our sample, and although we include tipped workers, their wage data are inclusive of tips. Having said this, other exemptions would feature as part of noncompliance with minimum wage regulations.

In our NLYS sample, about 20% of individuals work below the federal minimum wage upon high-school graduation, as shown in Figure 6. This proportion halves to 10% during the next three years, and then gradually falls further to around 2%. In the same Figure we also report the corresponding series obtained on the CPS, with the same sample restrictions as those used for the NLSY, and this happens to be even higher than the series based on the NLSY. Part of the difference between these two series is attributable to the fact that the CPS does not allow to distinguish between high school graduates and GED, who are thus included in the CPS sample used for Figure 6, while they are excluded from the NLSY sample.

Conditionally on ever working below the minimum wage, the average number of months worked below the minimum wage is 13.5, and the average number of jobs held below the minimum wage is 1.5. The mean job duration below the minimum wage is 8.9 months. These pieces of evidence indicate that, if wages are

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*Exemptions are documented by the US Department of Labor, Employment Standards Administration Wage and Hour Division at www.dol.gov/esa/.
reported without error, noncompliance with the minimum wage law is quite important among young high school graduates.

Table 2 gives the number of individuals making transitions from nonemployment to jobs, and between jobs, conditional on wages above or below the minimum wage. Most transitions to jobs paying less than the minimum wage originate in nonemployment, and most workers earn wages above the minimum wage once they switch job. Very few workers are observed to move from a job paying more than the minimum wage to one paying less. As direct transitions from higher-paying to lower-paying jobs can only be reconciled with our model if wages are measured with error, such transitions help us separately identify measurement error in wages and noncompliance.

4 Estimation

Specification. We estimate the model using simulated moments. In particular, we use efficient simulated GMM, in which the weighting matrix is computed as the inverse of the covariance of the sample moment conditions.

In the empirical implementation we allow for unobserved heterogeneity in arrival rates, job destruction rates and in the parameters of the wage offer distribution by estimating a mixture model with two types of individuals, with \( \pi \) denoting the proportion of type one. As robustness checks, we also reestimated the model using a four-point mixture (see Table 7 in Appendix C), and allow the heterogeneity distribution to depend on individual covariates (see Table 8 in Appendix C). The wage density function is assumed to be log normal, with \( \ln w \sim N(\mu, \sigma_w^2) \). We allow for measurement error in observed wages, such that \( \ln w^o = \ln w + u \), where \( w^o \) is the observed wage, \( w \) is the true wage and the error term is normally distributed, \( u \sim N(0, \sigma_u^2) \), and independent of the true wage. The time preference parameter \( r \) is set to 4% annually, which corresponds to about 0.3% monthly. We cannot estimate the matching function parameter \( \eta \) directly, as in order to identify it one should need data on firms or vacancies. As in much of the related literature, we set \( \eta = 0.5 \) (see Petrongolo and Pissarides, 2001, for supporting evidence), but we perform some sensitivity analysis on \( \eta \) within the range of variation found in the empirical matching function literature.
<table>
<thead>
<tr>
<th>Nonemployed</th>
<th>Cycle 1</th>
<th>Cycle 2</th>
<th>Cycle 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>NE to 1st job above $w_M$</td>
<td>87</td>
<td>264</td>
<td>217</td>
</tr>
<tr>
<td>NE to 1st job below $w_M$</td>
<td>29</td>
<td>47</td>
<td>25</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>116</td>
<td>311</td>
<td>242</td>
</tr>
<tr>
<td>1st job above $w_M$</td>
<td>93</td>
<td>123</td>
<td>88</td>
</tr>
<tr>
<td>Move to 2nd job above $w_M$</td>
<td>4</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>239</td>
<td>264</td>
<td>217</td>
</tr>
<tr>
<td>1st job below $w_M$</td>
<td>22</td>
<td>14</td>
<td>11</td>
</tr>
<tr>
<td>Move to 2nd job below $w_M$</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>67</td>
<td>47</td>
<td>25</td>
</tr>
<tr>
<td>2nd job above $w_M$</td>
<td>86</td>
<td>67</td>
<td>50</td>
</tr>
<tr>
<td>Move to 3rd job above $w_M$</td>
<td>2</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>179</td>
<td>165</td>
<td>115</td>
</tr>
<tr>
<td>2nd job below $w_M$</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Move to 3rd job below $w_M$</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>13</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>3rd job above $w_M$</td>
<td>59</td>
<td>48</td>
<td>33</td>
</tr>
<tr>
<td>Move to 4th job above $w_M$</td>
<td>6</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>130</td>
<td>77</td>
<td>67</td>
</tr>
<tr>
<td>3rd job below $w_M$</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Move to 4th job below $w_M$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>2</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>4th job above $w_M$</td>
<td>31</td>
<td>28</td>
<td>18</td>
</tr>
<tr>
<td>Move to 5th job above $w_M$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>70</td>
<td>55</td>
<td>38</td>
</tr>
<tr>
<td>4th job below $w_M$</td>
<td>6</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Move to 5th job below $w_M$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>7</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
We estimate the reservation wage directly using equation (4). In particular, measurement error in reported wages means that the reservation wage does not need to coincide with the lowest accepted wage. As 55% of individuals in our sample worked before graduation, we assume that a separate labor market exists before graduation, and we estimate an initial (period 0) reservation wage \( w^*_0 \), which characterizes the pre-graduation job search. The parameters of the model to be estimated are summarized in the vector

\[
\theta = [\lambda_{n1}, \lambda_{e1}, \lambda_{n2}, \lambda_{e2}, w^*_1, w^*_2, \mu_1, \mu_2, \sigma_{w1}, \sigma_{w2}, \delta_1, \delta_2, w_{01}, w_{02}, \pi, \sigma^2_w, \alpha, g]' \]

The number of parameters to be estimated is modest relatively to empirical models that attempt to provide a fit of the same data (see for example Keane and Wolpin, 1997).

**Data and simulations.** As described above, we have a sample of 577 white, male high school graduates. Let \( d_{it} = 1 \) if individual \( i \) is working and \( d_{it} = 0 \) if individual \( i \) is not working \( t \) months since graduation. We observe \( d_{it} \) for \( i = 1, \ldots, 577 \) and \( t = 1, \ldots, T_i \); and \( w_{it} \) for all \( i = 1, \ldots, 577 \) and \( t = 1, \ldots, T_i \) in which \( d_{it} = 1 \).

For the simulated sample, we simulate unconditional moments, that is predicted values of a given outcome in a given month, based on the simulated values for the previous month. In the practice, if individual \( i \) is employed in month \( t \), with probability \( \lambda_n \) he receives a wage offer \( w \) drawn from a distribution

\[
\begin{align*}
\frac{\phi(w)}{1 - (1 - \alpha)\Phi(w_M)} & \quad \text{for } w \geq w_M \\
\frac{\alpha \phi(w)}{1 - (1 - \alpha)\Phi(w_M)} & \quad \text{for } w < w_M
\end{align*}
\]

where \( \phi(\cdot) \) is the log normal wage density function with mean \( \mu \) and variance \( \sigma^2_w \) and \( \Phi(\cdot) \) is the corresponding c.d.f. If \( w > w^* \), he moves from nonemployment to employment. If at \( t \) he is employed at a wage \( w_{it} \), with probability \( \lambda_e \) he receives a random wage draw \( w' \) from the distribution (5)-(6). If \( w' > w_{it} \), he moves on to the new job, otherwise he stays on the current job and his wage increases to \( w_{it+1} = w_{it} (1 + g) \).\(^9\) In any period he may go back to nonemployment with probability \( \delta \). Finally, a small adjustment should cater for the

\(^9\)While the theoretical model of Section 2 is provided in continuous time, the estimated model is clearly specified in discrete time.
fact that in the initial month, we have both employed and nonemployed individuals. Thus if an individual is employed at $t = 0$, we simulate a wage $w_{i0}$ such that $w_{i0} \geq w^*_{o0}$.

For all $i$ and $t$, we run 25 simulations, and then take their average for constructing simulated moments. In each simulation $s$, the model predicts $d_{st}$ and $w_{st}$, conditional on $d_{st-1}$ and $w_{st-1}$, where $w_{st}$ denotes the “true” simulated wage. The model also predicts the observed simulated wage $w^*_{os}$ according to $w^*_{os} = w_{st} + u_{st}$, where $u_{st}$ is drawn from $N(0, \sigma^2_u)$. When an individual is not employed, the wage is not simulated. For each individual we thus generate a sequence of simulated outcomes $[d_{st}, w_{st}]$, which follows the observed sequence $[d_{it}, w_{it}]$ for $t = 1, ..., T_i$.

**Monthly moments and identification** We use two sets of moments: the first set is computed by months in the labor market, and the second set is computed by employment cycle. Monthly moments include nonemployment rates, the proportion of individuals who move from nonemployment to employment, the proportion of individuals who move from job to job, the proportion of individuals who move from employment to nonemployment, the mean wage, its standard deviation, the mean wage below the minimum wage, the standard deviation of the wage below the minimum wage, and the proportion of individuals that work below the minimum wage (see Appendix B for the exact definitions of moments).

Transition moments from nonemployment to employment are used to identify the offer arrival rate when nonemployed. Similarly, job-to-job transitions identify the offer arrival rate when employed and transitions from employment to nonemployment identify the job destruction rate. The reservation wage is identified by the nonemployment rate. Wage moments can identify the parameters of the wage offer distribution. In particular, the initial wage identifies the initial reservation wage. The mean and the variance of the wage offer distribution are identified from the observed monthly mean and variance as well as from job-to-job transitions.

A key aspect of the paper is the identification of the noncompliance parameter $\alpha$. The proportion of workers who earn below the minimum wage identifies $\alpha$, but it should be noted that this moment is also affected by measurement error in observed wages. However, conditional on the true wage offer distribution, measurement
error affects the variance of the observed wage distribution, without affecting job-to-job transitions, and this will be the basis of our identification strategy. In particular, if the true wage offer distribution is correctly specified as $\ln w^o = \ln w + u$, the variance of the observed wage offer distribution, job-to-job transitions and the proportion of workers earning less than the minimum wage allow us to separately identify $\alpha$, $\sigma_u^2$ and $\sigma_w^2$.

**Cycle moments and identification** The second set of moments is based on the first three employment cycles. We first use duration moments, namely mean nonemployment and employment duration on the first three jobs in the first three employment cycles. Second, we use wage moments, namely mean and standard deviation of wages (either overall or below the minimum wage) on this sequence of jobs. Third, we use transition moments, including the proportion of individuals who start the first three cycles from nonemployment, the proportion of individuals who move from the first to the second job and from the second to the third job in the first three cycles. Last, we also use the proportion of individuals who work below the minimum wage on the first three jobs in the first three cycles.

As with monthly moments, nonemployment duration identifies the offer arrival rate when nonemployed. Job-to-job transitions identify the offer arrival rate when employed. The reservation wage is identified by the wage on the first job. The first and second moments of the wage offer distribution are identified by the mean wage and its variance. The job destruction rate is identified by the nonemployment rate when new cycles start. The initial reservation wage is identified by the initial nonemployment rate and the mean wage on the first job in the first cycle (being significantly lower than the wage on the first job in the second and third cycles). As discussed above, the variance of the observed wage offer distribution, job-to-job transitions and the proportion of workers earning less than the minimum wage allow us to separately identify $\alpha$, $\sigma_u^2$ and $\sigma_w^2$.

**Implementation.** Let $m_k$ be moment $k$ in the data, computed on the observed sequence $[d_{it}, w_{it}]$. The corresponding simulated moment is denoted by $m_k^S(\theta)$, and it is obtained as an average across 25 simulations, $m_k^S(\theta) = \frac{1}{25} \sum_{s=1}^{25} m_k^s(\theta)$, where $\theta$ denotes the parameter vector to be estimated. The $m_k^s(\theta)$ elements are in turn computed on the simulated sequence $[d_{it}^s, w_{it}^s]$, as illustrated in Appendix B. The vector of moment
conditions is

\[ g(\theta)' = [m_1 - m_1^S(\theta), \ldots, m_k - m_k^S(\theta), \ldots, m_K - m_K^S(\theta)], \]

where \( K \) is the number of moments used. The objective function to be minimized with respect to \( \theta \) is

\[ J(\theta) = g(\theta)' W g(\theta), \]

where the weighting matrix \( W \) is the inverse of the covariance matrix of the moment conditions.\(^{10}\) When using monthly moments, the very large set of moment conditions (9 moments * 216 months) delivers a 1944*1944 covariance matrix to be inverted. Our estimation programme always failed to invert such matrix, and thus we reduced the number of moments to be matched to two monthly moments per year (the first and the sixth months for each year), delivering only 324 moment conditions, and an invertible covariance matrix. With cycle moments, we only had 66 moment conditions and the associated covariance matrix would invert without problems.

5 Results

5.1 Estimates

Our main results are presented in Table 3, where the two sets of estimates are based on monthly and cycle moments, respectively. The two (unobserved) types of individuals are allowed to differ in all parameters but in the level of compliance \( \alpha \), the measurement error variance \( \sigma_u \) and the rate of wage growth on-the-job, \( g \).

Starting from estimates on monthly moments, our parameter values have plausible magnitudes and are in line with previous estimates of the parameters of a search model with search on-the-job. Specifically, arrival rates of job offers are higher for the nonemployed than for the employed, and these rates differ across types of individuals, delivering an overall decreasing hazard rate. Type 1 individuals, representing almost exactly half of our sample, have lower job offer arrival rates while either employed or nonemployed, a higher job destruction rate, a lower mean wage offer and a lower reservation wage. This is the only group whose reservation wage falls below the minimum wage, and thus the only one for which minimum wage regulations

\(^{10}\)Note that while \( d_{1i} \) is a step function of the parameters, the simulated moment that we then match to the data is the proportion of individuals with \( d_{1i} = 1 \) for each level of labor market experience, which is in principle “nearly” continuous. We then minimize our objective function combining both a random search algorithm (the simplex) and a gradient search algorithm.
Table 3: Parameter estimates of a search model - two-type mixture distribution

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates based on monthly moments</th>
<th>Estimates based on cycle moments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coef. (s.e.)</td>
<td>coef. (s.e.)</td>
</tr>
<tr>
<td>$\lambda_{n1}$</td>
<td>0.428 (0.003)</td>
<td>0.479 (0.054)</td>
</tr>
<tr>
<td>$\lambda_{n2}$</td>
<td>0.658 (0.035)</td>
<td>0.767 (0.097)</td>
</tr>
<tr>
<td>$\lambda_{e1}$</td>
<td>0.116 (0.001)</td>
<td>0.125 (0.009)</td>
</tr>
<tr>
<td>$\lambda_{e2}$</td>
<td>0.237 (0.013)</td>
<td>0.237 (0.015)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.499 (0.004)</td>
<td>0.499 (0.017)</td>
</tr>
<tr>
<td>$w_{11}$</td>
<td>2.496 (0.025)</td>
<td>2.496 (0.307)</td>
</tr>
<tr>
<td>$w_{21}$</td>
<td>11.507 (0.238)</td>
<td>10.771 (0.248)</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>1.636 (0.002)</td>
<td>1.635 (0.035)</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>1.669 (0.011)</td>
<td>1.654 (0.025)</td>
</tr>
<tr>
<td>$\sigma_{w1}$</td>
<td>0.511 (0.003)</td>
<td>0.502 (0.015)</td>
</tr>
<tr>
<td>$\sigma_{w2}$</td>
<td>0.522 (0.005)</td>
<td>0.522 (0.013)</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.064 (0.00051)</td>
<td>0.045 (0.003)</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.00412 (0.00027)</td>
<td>0.013 (0.00071)</td>
</tr>
<tr>
<td>$w_{01}$</td>
<td>2.958 (0.046)</td>
<td>2.903 (0.112)</td>
</tr>
<tr>
<td>$w_{02}$</td>
<td>6.791 (0.064)</td>
<td>7.487 (0.135)</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.012 (0.00060)</td>
<td>0.008 (1.246)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.262 (0.002)</td>
<td>0.279 (0.027)</td>
</tr>
<tr>
<td>$g$</td>
<td>0.00209 (0.00031)</td>
<td>0.00184 (0.00021)</td>
</tr>
</tbody>
</table>

Notes. The sample includes white, male high school graduates from the NLSY. Number of observations: 577. Estimation methods: Simulated GMM.

are binding. One would expect reservation wages during the final year of high school to be lower than after graduation, and this is indeed the case for type-2 individuals, while not for type-1 individuals, although the two estimated values are not significantly different from each other when using cycle moments.

The unobserved types are identified from the panel dimension of the data. One way to illustrate identification is to look at the posterior probability of being type 1, conditional on a particular event. For example, one can compute the posterior probability of being type 1, conditional on observing a transition from employment to nonemployment. Given the model estimates, this is

$$
\Pr(type 1|e \to ue) = \frac{\Pr(e \to ue|type 1) \Pr(type 1)}{\Pr(e \to ue)} = \frac{\delta_1 \pi_1}{\delta_1 \pi_1 + \delta_2 \pi_2} = 0.939,
$$

implying that type-1 individuals represent about 94% of employment to nonemployment transitions.

The novelty of our results consists in providing an estimate for the extent of noncompliance of firms’ wage offers to minimum wage regulations, represented by the parameter $\alpha$. We find that the arrival rate of job
offers below the minimum wage is about a quarter of that above the minimum wage. Having estimated $\alpha$, the steady state proportion of jobs that pay less than the minimum wage is given by

$$\frac{\pi_1 \alpha [F_1 (w_M) - F_1 (w_1^*)]}{\pi_1 [1 - F_1 (w_1^*)] + \pi_2 [1 - F_2 (w_2^*)]} = 0.112.$$  

This number is clearly lower than $\alpha$ as only type 1 individuals are affected by the compliance level. Furthermore, it is only the density of offers between the reservation wage of type-1 individuals and $w_M$ that matters for the job count below $w_M$.

Finally, wage growth on-the-job is 0.209% per month, corresponding to 2.5% per year. This estimate is close to the upper bound for wage growth on-the-job suggested by Bowlus and Neumann (2006) on the NLSY. One can use this estimate to infer the proportion of wage growth that is explained by job search and mobility, as opposed to wage growth on-the-job. Over the entire sample period, the mean hourly wage grows by $8.10, from $8.32 in the first month to $16.42 in the last month. An individual who is spending this eighteen years span on the same job would see his wage increase by $4.71, from $8.32 to 8.32 * (1.00209)^{215} = 13.03. Thus 58.2% of wage growth is due to on-the-job growth and the remaining 41.8% is due to job mobility. This last figure is slightly higher than the estimates obtained by Topel and Ward (1992) on the Longitudinal Employer-Employee Data.

The estimates obtained on cycle moments are quite similar to those obtained on monthly moments, including the ranking of values for type 1 and type 2 individuals. Two differences may be worthwhile mentioning. First, when using cycle moments we obtain a lower (and non-significant) estimate of the measurement error variance, but the estimate for $\alpha$ stays very similar to that obtained on monthly moments. Second, wage growth on-the-job is slightly lower, predicting a virtually equal role of job mobility and wage growth on-the-job in explaining wage growth during the first eighteen years of labor market experience.

We also carry an overidentification test of our econometric model on both sets of estimates. When $\theta$ is estimated by efficient GMM, Hansen (1982) shows that the overidentifying restrictions ($OIR$) test statistic

$$OIR = \left[ \frac{1}{N} \sum_{i=1}^{N} g (w_i, \theta) \right]' W \left[ \frac{1}{N} \sum_{i=1}^{N} g (w_i, \theta) \right]$$

is asymptotically distributed as $\chi^2 (K - q)$ under $H_0 : E[g(w_i, \theta_0)] = 0$, where $K$ denotes the number of
moment conditions and $q$ is the number of parameters to be estimated. Note that $OIR$ equals the GMM objective function evaluated at the estimated value of $\theta$. On our estimates we obtain $OIR = 3614$ on monthly moments and $OIR = 578$ on cycle moments. Both these realizations are higher than the corresponding 5% critical values. In particular, when using monthly moments $K = 324$, $q = 18$, and the 5% critical value of $\chi^2(306)$ is 347.8. When using cycle moments, $K = 66$, $q = 18$, and the 5% critical value of $\chi^2(48)$ is 65.2. Thus the overidentification test fails in both cases. This is a recurrent feature of models with these characteristics (see for example Hansen and Singleton, 1982), and in particular in our case such failure is not really surprising given that we fail to match seasonal fluctuations in relevant moments, and thus our model is bound to miss an important source of variation in the data. In the next section we provide an alternative assessment of our model, based on its capacity to fit the data.

5.2 Model Fit

Figures 7-15 show the fit of all the monthly moments that are used for estimation of the parameters of interest. In general the estimates obtained do quite a good job at reproducing levels and trends in the data moments. In particular, the model fits well the life cycle decrease in nonemployment (Figure 7), and the slight increase in the transition rate from nonemployment to employment (Figure 8) during the first 10 years in the labor market, but fails to fit the corresponding seasonal fluctuations, which seem to have a purely aggregate component. Also, the model fails to fit the decreasing trend in transitions from employment to nonemployment, but it does fit its average level (Figure 9). This failure is easily explained by the fact that transitions into nonemployment are driven by an exogenous job destruction rate, which would not respond to tenure, wages or seasonal factors. Indeed, the only potential source of dynamics in the nonemployment inflow here is the unobserved heterogeneity in the job destruction rate, and the data show that a two-type heterogeneity may not be sufficient to fit its decreasing trend. Next, the model fits well the fall in job-to-job transitions (Figure 10).

A model with a constant wage offer distribution for the entire sample period and constant wage growth on-the-job fits well the eighteen years of mean hourly wage growth from about $8 to $16 (Figure 11), with 11 An extension that endogenizes $\delta$ would be beyond the scope of this paper and would not alter the substance of our model.
job mobility explaining about 40\% of such growth. The upward trend in the hourly wage variance is also well predicted by the model (Figure 12). Furthermore, simulated moments replicate well the mean of wages below the minimum wage (Figure 13), although they tend to underpredict their variance (Figure 14).

Figure 15 finally presents model and data moments for the proportion of individuals working below the minimum wage, and suggests that a model with an exogenous, constant noncompliance parameter provides a very close fit of the level and trend of the proportion of workers paid below the minimum wage.\(^\text{12}\) As the estimated model replicates well the proportion of workers earning less than the minimum wage, it can provide a sound basis to illustrate the implications of changes in the compliance rate on labor market outcomes.

The fit of cycle moments is shown in Table 4. The duration of both nonemployment and jobs in the first three cycles is reproduced fairly accurately by our estimates, as well as the associated mean wages and variances. When looking at the proportion of individuals paid below the minimum wage, the model does a good job at replicating data moments in the first cycle, but not during the second and third cycles. Such failure is at least partly due to smaller cell sizes for these cycles. A similar caveat applies to the model fit of job-to-job transitions.

### 5.3 Policy experiments

We finally use our estimated model to get quantitative implications of changing the level of the minimum wage and the rate of noncompliance with the minimum wage.

Table 5 reports the estimated impact of a 10\% increase in the minimum wage, from $5.15 to $5.67. A comparison of columns 3 and 4 shows that such policy raises the nonemployment rate in this sample by 0.3-0.4 percentage points, depending on labor market experience. This estimated effect is lower than most estimates obtained in the literature for young adults, exploiting minimum wage variations across US states (see Brown, 1999, Table 3). The estimated elasticity of the nonemployment rate to the minimum wage, computed in correspondence of this increase, is around 0.24. This is an average of the estimated values of such elasticities.

\(^{12}\)If anything, one should note that such proportion is slightly overstated by our estimates towards the end of the sample period, and thus our model fit may be possibly improved by expressing \(\alpha\) as an increasing function of the ratio of the minimum wage to the average wage in the economy. This ratio is clearly falling over time in our sample, and thus may deliver a lower value of \(\alpha\) for the later years.
Table 4: Employment and wage cycles

<table>
<thead>
<tr>
<th></th>
<th>Cycle 1</th>
<th>Cycle 2</th>
<th>Cycle 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NE</td>
<td>Job 1</td>
<td>Job 2</td>
</tr>
<tr>
<td>Duration</td>
<td>data</td>
<td>8.07</td>
<td>33.96</td>
</tr>
<tr>
<td></td>
<td>model</td>
<td>7.96</td>
<td>23.84</td>
</tr>
<tr>
<td>Mean wage</td>
<td>data</td>
<td>8.22</td>
<td>10.37</td>
</tr>
<tr>
<td>Std. dev. wage</td>
<td>data</td>
<td>3.43</td>
<td>4.28</td>
</tr>
<tr>
<td></td>
<td>model</td>
<td>4.56</td>
<td>4.86</td>
</tr>
<tr>
<td>Wage below $w_M$</td>
<td>data</td>
<td>21.90</td>
<td>6.77</td>
</tr>
<tr>
<td></td>
<td>model</td>
<td>22.30</td>
<td>4.90</td>
</tr>
<tr>
<td>Mean wage</td>
<td>data</td>
<td>4.89</td>
<td>5.17</td>
</tr>
<tr>
<td></td>
<td>model</td>
<td>4.48</td>
<td>4.98</td>
</tr>
<tr>
<td>Std. dev. wage</td>
<td>data</td>
<td>1.38</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>model</td>
<td>1.02</td>
<td>0.81</td>
</tr>
<tr>
<td>Moving from</td>
<td>data</td>
<td>51.74</td>
<td>52.07</td>
</tr>
<tr>
<td></td>
<td>model</td>
<td>59.08</td>
<td>39.20</td>
</tr>
<tr>
<td>Moving from</td>
<td>data</td>
<td>54.55</td>
<td>48.76</td>
</tr>
</tbody>
</table>

Notes. Durations are expressed in months and wages are expressed in 2000 dollars. Transitions are expressed in %.

Across all years of experience. A similar comparison between column 3 and column 5 illustrates the effects of a 10% fall in the minimum wage, delivering a fall in nonemployment of 0.3-0.5 percentage points, depending on labor market experience, with an associated elasticity of 0.27. Although the model used to obtain employment effects of the minimum wage is not linear, we do not find evidence of strong nonlinearities in our estimated effects around a 10% change in the minimum wage.

According to our model, the employment effect of the minimum wage comes from a combination of lower job offer arrival rates and (ambiguous) changes in the reservation wage. However, empirically we find that $w_M$ has a negligible impact on $w^*$. Thus the increase in nonemployment driven by the increase in the minimum wage is almost entirely driven by the fact that $(1 - \alpha) [F(w'_M) - F(w_M)]$ firms leave the market, where $w'_M$ is the new value of the minimum wage.

It should be finally noted that our estimates are obtained in correspondence of an elasticity of the meeting technology between workers and firms with respect to job seekers equal to 0.5, for which there is large support.

We also performed some sensitivity analysis with respect to alternative values of $\eta$, looking at the employment...
effects of the same changes in the minimum wage as those studied in Table 5. Re-estimating the model with 
\( \eta = 0.3 \) yields changes in the nonemployment rate between 0.1 and 0.2 percentage points, following variations 
of plus or minus 10% in the minimum wage. When \( \eta = 0.7 \), the implied changes in nonemployment are in 
the range of 0.4-0.6. Our estimated results are thus not too sensitive to the adopted parameter value of \( \eta \).

Table 5 also reports the estimated impact of changes in the minimum wage on a number of moments of 
the wage distribution. As expected, the increase in the minimum wage raises mean wages, as shown in panel 
2 of the Table, but the corresponding elasticity is modest, in the range 0.12-0.16. Inequality, as measured 
by the 90 to 10 percentile wage ratio, is hardly affected, except perhaps for higher experience cells (panel 
3). Finally, panel 4 shows that an increase in the minimum wage substantially increases the proportion of 
workers paid below it. This model thus suggests that plausible changes in the minimum wage generate large 
changes in noncompliance, with modest changes in both the nonemployment rate and the wage distribution 
above the minimum wage.

The labor market effects of changing the noncompliance parameter \( \alpha \) are sort of symmetric to the effects 
of changing the minimum wage. These are reported in Table 6 for the estimated value of \( \alpha = 0.26 \) and 
alternative values \( \alpha = 0, 0.5, 1 \). Eliminating noncompliance, i.e. going from \( \alpha = 0.26 \) to \( \alpha = 0 \) raises the 
nonemployment rate by half of a percentage point on average, with a slight increase in the mean wage and a 
slight fall in wage dispersion. The proportion of workers paid below the minimum wage obviously drops to 
zero after 2 years in the labor market. The reason why this proportion may not be zero upon labor market 
entry lies in the existence of a pre-graduation labor market, which is not subject to the minimum wage, and 
to measurement error. The other extreme case with \( \alpha = 1 \) corresponds to the elimination of minimum wage 
regulations, with an average decrease in the nonemployment rate of 1.2 percentage points and a fall in the 
mean wage of about 6.7%.

6 Conclusions

This paper aims to explain monthly work trajectories of white, male high school graduates using a continuous 
time search model with both nonemployed and employed job search and minimum wages. We extended an
Table 5: The effect of a 10% change in the minimum wage

<table>
<thead>
<tr>
<th>Years in the labor market</th>
<th>Model $w_M = 5.15$</th>
<th>Counterfactuals $w_M = 5.67$</th>
<th>Counterfactuals $w_M = 4.64$</th>
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<tr>
<td>Non-employment rate</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>26.2</td>
<td>26.6</td>
<td>25.7</td>
</tr>
<tr>
<td>2-4</td>
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<td>5-9</td>
<td>11.7</td>
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<tr>
<td>10-18</td>
<td>11.4</td>
<td>11.7</td>
<td>11.1</td>
</tr>
<tr>
<td>Mean wage (in 2000 dollars)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>9.5</td>
<td>9.6</td>
<td>9.4</td>
</tr>
<tr>
<td>2-4</td>
<td>11.8</td>
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<td>13.5</td>
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<td>13.4</td>
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<tr>
<td>10-18</td>
<td>15.2</td>
<td>15.5</td>
<td>15.0</td>
</tr>
<tr>
<td>90/10 percentile wage ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3.1</td>
<td>3.1</td>
<td>3.1</td>
</tr>
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<td>3.1</td>
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<td>3.2</td>
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<td>5-9</td>
<td>3.6</td>
<td>3.5</td>
<td>3.7</td>
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<tr>
<td>10-18</td>
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<td>4.4</td>
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<tr>
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<td></td>
</tr>
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<td>11.6</td>
<td>14.7</td>
<td>8.5</td>
</tr>
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<td>4.4</td>
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<td>3.0</td>
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<td>2.8</td>
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<tr>
<td>10-18</td>
<td>3.8</td>
<td>5.2</td>
<td>2.7</td>
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Table 6: The effect of changes in compliance to the minimum wage

<table>
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<th>Years in the labor market</th>
<th>Model α = 0.26</th>
<th>Counterfactuals α = 0</th>
<th>Counterfactuals α = 0.5</th>
<th>Counterfactuals α = 1.0</th>
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<td>10.9</td>
<td>10.2</td>
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<tr>
<td>Mean wage (in 2000 dollars)</td>
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<td></td>
<td></td>
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<td>9.5</td>
<td>9.8</td>
<td>9.2</td>
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<td>15.8</td>
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<tr>
<td>90/10 percentile wage ratio</td>
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<td></td>
<td></td>
</tr>
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<td>3.1</td>
<td>2.8</td>
<td>3.3</td>
<td>3.6</td>
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<tr>
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<td>3.8</td>
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<td>4.7</td>
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<tr>
<td>10-18</td>
<td>4.2</td>
<td>3.9</td>
<td>4.4</td>
<td>5.3</td>
</tr>
<tr>
<td>% below the minimum wage</td>
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<td></td>
<td></td>
<td></td>
</tr>
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<td>3.8</td>
<td>0.0</td>
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<td>11.7</td>
</tr>
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</table>
otherwise conventional job search model by considering imperfect compliance to minimum wage regulations, and this is supposed to cater for the fact that early in their careers a large fraction of individuals in our sample are observed to earn less than the federal minimum wage. We interpret such wage observations as the result of either noncompliance (or exemptions) to the minimum wage, or measurement error, and separately identify the two factors by exploiting their different impact on labor market transitions.

We have assumed that firms offer wages that reflect their idiosyncratic productivity, and thus firms whose productivity falls below the minimum wage either exit the market or choose not to comply. In this scenario a binding minimum wage reduces the arrival rate of job offers and has an ambiguous, though small, impact on both the reservation wage and the acceptance rate. Employment effects of the minimum wage (and compliance parameters) thus mostly stem from their impact on the arrival rate of job offers.

We estimated the proposed model by simulated GMM, and the model fit revealed that a simple search model with heterogeneous firms can fit fairly well individual transitions from school to work, job-to-job transitions, and wage growth. Finally, policy experiments show that both changes in the minimum wage and the extent of noncompliance have a very modest impact on both nonemployment rates of young adults and various moments of the wage distribution.
References


Appendix A: Computation of $V'_{e'}$

Let $x(\alpha, w_M) = [1 - (1 - \alpha) F (w_M)]^{-(1-\eta)}$. Using integration by parts, (2) and (3) can be rewritten as:

$$rV_e(w_r) = w_r + gw_rV'_e(w_r) + \delta [V_n - V_e(w_r)]$$

$$+ x(\alpha, w_M) \lambda_c \int_{w_r}^{w_M} [1 - F(w)]V'_e(w) \, dw \text{ for } w_r \geq w_M$$

(7)

and

$$rV_e(w_r) = w_r + gw_rV'_e(w_r) + \delta [V_n - V_e(w_r)] + x(\alpha, w_M) \int_{w_r}^{w_M} [1 - F(w)]V'_e(w) \, dw$$

$$+ \int_{w_r}^{w_M} [1 - \alpha F(w) - (1 - \alpha) F(w_M)]V'_e(w) \, dw \text{ for } w_r < w_M,$$

respectively. Differentiating (7) and (8) with respect to $w_r$ yields:

$$gw_rV''_e(w_r) = \{r + \delta + x(\alpha, w_M) \lambda_c [1 - F(w_r)] - g\} V'(w_r) - 1, \text{ for } w_r \geq w_M$$

(9)

$$gw_rV''_e(w_r) = \{r + \delta + x(\alpha, w_M) \lambda_c [1 - \alpha F(w_r) - (1 - \alpha) F(w_M)] - g\} V'(w_r) - 1,$$

(10)

for $w_r < w_M$.

Consider the case $w_r \geq w_M$ first, and let $u(w_r) = -\frac{1}{gw_r} \{r + \delta - g + x(\alpha, w_M) \lambda_c [1 - F(w_r)]\}$ and $q(w_r) = -\frac{1}{gw_r}$. Equation (9) implies

$$V'_e(w_r) = e^{-\int u(w_r) \, dw_r} \left[ A + \int q(w_r) e^\int u(w_r) \, dw_r \, dw_r \right]$$

$$= e^{-\int_{w_0}^{w_r} u(s) \, ds} \left[ A + \int_{w_0}^{w_r} q(s) e^\int u(s) \, dz \, ds \right],$$

(11)

where $A$ is an arbitrary constant. Now let

$$R(w_r; w_0) = -\int_{w_0}^{w_r} u(s) \, ds$$

$$= \int_{w_0}^{w_r} \frac{r + \delta - g + x(\alpha, w_M) \lambda_c [1 - F(s)]}{gs} \, ds.$$

(12)

This in turn implies

$$\int_{w_0}^{w_r} q(s) e^{\int u(s) \, dz} \, ds = \int_{w_0}^{w_r} q(s) e^{-R(s)} \, ds = -\int_{w_0}^{w_r} \frac{1}{gs} e^{-R(s)} \, ds.$$
Having set $\tau = 0$ in (11), one obtains $V'_{e}(w_0) = A = \int_{w_0}^{\infty} \frac{1}{gs}e^{-R(s)}ds$ and thus

$$V'_{e}(w_\tau) = e^{R(w_\tau)} \left[ \int_{w_0}^{\infty} \frac{1}{gs}e^{-R(s)}ds - \int_{w_0}^{w_\tau} \frac{1}{gs}e^{-R(s)}ds \right]$$

$$= e^{R(w_\tau)} \int_{w_\tau}^{\infty} \frac{1}{gs}e^{-R(s)}ds$$

$$= \int_{w_\tau}^{\infty} \frac{1}{gs}e^{R(w_\tau)-R(s)}ds,$$

where

$$R(w_\tau) - R(s) = \int_{s}^{w_\tau} r + \delta - g + x(\alpha,w_M)\lambda_e[1 - F(z)]dz.$$

Therefore:

$$V'_{e}(w_\tau) = \int_{w_\tau}^{\infty} \frac{1}{gs}e^{\left( \int_{s}^{w_\tau} r + \delta - g + x(\alpha,w_M)\lambda_e[1 - F(z)]dz \right)}ds, \quad (13)$$

is the solution for $w_\tau > w_M$.

Similarly, when $w_\tau < w_M$

$$V'_{e}(w_\tau) = \int_{w_\tau}^{\infty} \frac{1}{gs}e^{\left( \int_{s}^{w_\tau} r + \delta - g + x(\alpha,w_M)\lambda_e[1 - \alpha F(z) - (1-\alpha)F(w_M)]dz \right)}ds. \quad (14)$$

**Appendix B: Moments**

**Monthly Moments** We use following formulas to compute monthly moments ($\tau = 1, 2, \cdots, 216$) in the data. Let $mne$ be the vector of monthly non-employment rates. Each of its element, $mne(\tau)$, is determined by

$$mne(\tau) = \frac{\sum_i I(d_{i\tau} = 0)}{\sum_i I(d_{i\tau} = 0) + \sum_i I(d_{i\tau} = 1)},$$

where $I(\cdot)$ is an indicator function. Similarly transition rates from non-employment to employment are given by

$$mtr(\tau) = \frac{\sum_i I(d_{i\tau} = 0, d_{i\tau+1} = 1)}{\sum_i I(d_{i\tau} = 0)}, \tau = 1, 2, \cdots, 215.$$

Other transition rates are defined accordingly. Means and standard deviations of monthly wage are calculated according to

$$mw(\tau) = \frac{\sum_i (w_{i\tau}>0) I(d_{i\tau} = 1 | w_{i\tau}>0)}{\sum_i I(d_{i\tau} = 1 | w_{i\tau}>0)}; \quad stdw(\tau) = \sqrt{\frac{\sum_i ((w_{i\tau} - mw(\tau))^2 | w_{i\tau}>0)}{\sum_i I(d_{i\tau} = 1 | w_{i\tau}>0) - 1}}.$$
The wage moments below the minimum wage use the formulas above, having conditioned \( w_{i\tau} > w_M \). Finally, the proportion of individuals working below the minimum wage is

\[
mp(\tau) = \frac{\sum_i I(w_{i\tau} < w_M | w_{i\tau} > 0)}{\sum_i I(d_{i\tau} = 1 | w_{i\tau} > 0)}.
\]

Simulated moments are defined similarly, based on \( d_{i\tau}^s \) and \( w_{i\tau}^s \). For example simulated monthly nonemployment rates are defined as

\[
mne^S(\tau) = \frac{1}{25} \sum_{s=1}^{25} \frac{\sum_i I(d_{i\tau}^s = 0)}{\sum_i I(d_{i\tau}^s = 0) + \sum_i I(d_{i\tau}^s = 1)}.
\]

**Cycle Moments**  To calculate the empirical moments, we follow each individual \( i \) for the first three cycle and the first three jobs in each cycle, i.e. \( cycle^c(e^c_j) \) where \( c = 1, 2, 3 \) denotes cycles and \( j = 0, 1, 2, 3 \) denotes spells within cycles (specifically \( j = 0 \) indicates the initial nonemployment spell and \( j = 1, 2, 3 \) indicates the first three jobs). We convert monthly data into \([D_{ij}^c, w_{ij}^c]\) where \( i \) denotes individuals, \( w \) denotes wages and \( D \) denotes duration in a given state. For example \( D_{12}^1 = 10 \) means that individual \( i \) has worked for 10 months on the second job of his first employment cycle and \( D_{00}^2 = 5 \) means that individual \( i \) has been nonemployed for 5 months at the beginning of his second cycle. \( w_{ij}^c \) represents the accepted wage in job \( j \) in cycle \( c \), which is the first wage observation on that particular job.

The average duration of job \( j \) in cycle \( c \) is computed as

\[
mdur^c_j = \frac{\sum_i D_{ij}^c}{\sum_i I(D_{ij}^c \geq 1)}, \quad c = 1, 2, 3, j = 0, 1, 2, 3.
\]

Means and standard deviations of accepted wage are computed as

\[
mw^c_j = \frac{\sum_i (w_{ij}^c | w_{ij}^c > 0)}{\sum_i I(D_{ij}^c \geq 1 | w_{ij}^c > 0)}; \quad stdw^c_j = \sqrt{\frac{\sum_i ((w_{ij}^c - mw^c_j)^2 | w_{ij}^c > 0)}{\sum_i I(D_{ij}^c \geq 1 | w_{ij}^c > 0) - 1}}, \quad c, j = 1, 2, 3.
\]

Wage moments below the minimum wage are defined in a similar fashion. The proportion of workers paid below the minimum wage on job \( j \) in cycle \( c \) is

\[
mp^c_{j \tau} = \frac{\sum_i I(w_{ij}^c < w_M | w_{ij}^c > 0)}{\sum_i I(D_{ij}^c \geq 1 | w_{ij}^c > 0)}, \quad c, j = 1, 2, 3.
\]
The proportions of workers who move from job 1 to job 2, and from job 2 to job 3 in cycle $c$ are

\[
tr_c^1 = \frac{\sum_i I(D_{c1}^i \geq 1, D_{c2}^i \geq 1)}{\sum_i I(D_{c1}^i \geq 1)}; \quad tr_c^2 = \frac{\sum_i I(D_{c2}^i \geq 1, D_{c3}^i \geq 1)}{\sum_i I(D_{c2}^i \geq 1)}, \quad c = 1, 2, 3.
\]

Like monthly moments, simulated cycle moments are defined for each simulation $s$ and we then take averages across 25 simulations.

**Appendix C: Robustness tests**

Table 7 below reports estimates based on monthly moments allowing for four, as opposed to two, types in the mixture model for unobserved heterogeneity.

The fit of the two- and four-type models can be compared based on a $\chi^2$ test. The four-type model is unrestricted, and the corresponding efficient GMM unrestricted estimator $\theta_u$ minimizes

\[
J_N(\theta) = \left[ \frac{1}{N} \sum_{i=1}^{N} g(w_i, \theta) \right]' \Omega_N^{-1} \left[ \frac{1}{N} \sum_{i=1}^{N} g(w_i, \theta) \right].
\]

The two-type model imposes $h = 16$ restrictions, and the corresponding restricted estimator $\theta_r$ minimizes $J_N(\theta)$ subject to the $h$ restrictions. The difference in maximized values of the objective function can be compared using the difference test statistic (Newey and West, 1987)

\[
D = N[J_N(\theta_r) - J_N(\theta_u)],
\]

which is asymptotically $\chi^2(h)$ distributed under $H_0$. The 5% threshold for $\chi^2(16)$ is equal to 26.296. With our estimates $D = 577 \times (3614 - 3517) = 55969 > \chi^2(16)$, thus the two-type restrictions are rejected.

Despite the better fit of the four-type model, the general picture emerging from these estimates is very close to that provided by Table 3. In particular, arrival rates of job offer are very similar for types 1 and 2 (those with relatively poorer labor market prospects) and for types 3 and 4 (with better labor market prospects). Also, half of the sample is not affected by the minimum wage, because their reservation wage exceeds the minimum wage. Finally, the noncompliance estimate is virtually identical to that obtained in Table 3. The only noteworthy difference between the two sets of results is the estimate of wage growth on-the-job, which in Table 7 is about four times smaller than in Table 3.
Table 7: Parameter estimates of a search model - four-type mixture distribution

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<th>coef.</th>
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<tr>
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<td>(0.108)</td>
</tr>
<tr>
<td>$w^0_{04}$</td>
<td>6.797</td>
<td>(0.080)</td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.030</td>
<td>(0.002)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.258</td>
<td>(0.003)</td>
</tr>
<tr>
<td>$g$</td>
<td>0.00049</td>
<td>(0.00003)</td>
</tr>
</tbody>
</table>

Notes. The sample includes white, male high school graduates from the NLSY. Number of observations: 577. Estimation methods: Simulated GMM.
In a further robustness test we allow for dependence on observed characteristics in the heterogeneity distribution. Our sample is homogeneous along gender, age, race and education dimensions, but it makes sense to ask whether individual probabilities of belonging to a given sub-population respond to other covariates such as ability, family background, etc. We implement this test by modelling the probability of belonging to type 1, \( \pi_i \), as a logit function of such characteristics: 

\[
\pi_i = \frac{\exp(X_i\beta)}{1 + \exp(X_i\beta)},
\]

where the \( X_i \) vector includes controls for mother’s schooling, father’s schooling, AFQT score\(^{13}\), age at high school graduation, living with both parents at age 14, and number of siblings, and \( \beta \) is the associated parameter vector.

The results are reported in Table 8. The probability of belonging to type 1, which is the one with poorer labor market prospects, falls with all these controls except mother education. Having said this, allowing for these interactions does not alter in any meaningful way the other estimated parameters with respect to the baseline estimates of Table 3.

\(^{13}\)This is normalized by year of birth, having subtracted the mean score for an individual’s birth year cohort from his raw score.
Table 8: Parameter estimates of a search model - allowing for covariates in $\pi$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Coef. (s.e.)</th>
<th>Coef. (s.e.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>0.428 (0.006)</td>
<td></td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.658 (0.048)</td>
<td></td>
</tr>
<tr>
<td>$\lambda_c1$</td>
<td>0.122 (0.002)</td>
<td></td>
</tr>
<tr>
<td>$\lambda_c2$</td>
<td>0.245 (0.027)</td>
<td></td>
</tr>
<tr>
<td>$w^*_1$</td>
<td>2.498 (0.068)</td>
<td></td>
</tr>
<tr>
<td>$w^*_2$</td>
<td>13.149 (0.372)</td>
<td></td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>1.637 (0.002)</td>
<td></td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>1.687 (0.029)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{w1}$</td>
<td>0.498 (0.006)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{w2}$</td>
<td>0.522 (0.009)</td>
<td></td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.065 (0.0009)</td>
<td></td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.0061 (0.0004)</td>
<td></td>
</tr>
<tr>
<td>$w^*_{01}$</td>
<td>2.990 (0.122)</td>
<td></td>
</tr>
<tr>
<td>$w^*_{02}$</td>
<td>6.934 (0.101)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.014 (0.002)</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.260 (0.005)</td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>0.00285 (0.00009)</td>
<td></td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>2.250 (0.037)</td>
<td></td>
</tr>
</tbody>
</table>

Covariates in $\pi_i$:

- constant: 0.037 (0.001)
- mother’s schooling: -0.172 (0.003)
- father’s schooling: -0.003 (0.0002)
- AFQT: -0.024 (0.0006)
- age at graduation: -0.260 (0.007)
- living with both parents at 14: -0.057 (0.002)
- number of siblings: -0.003 (0.0003)

Notes. The sample includes white, male high school graduates from the NLSY. Number of observations: 577. Estimation methods: Simulated GMM.