A dynamic measure of inflation

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Abstract

This paper shows that conventional measures of cost-of-living inflation, based on static models of consumption, suffer from two problems. The first is an intertemporal substitution bias, as these measures neglect the ability of consumers to borrow and lend in response to price changes. The second problem is the omission of intertemporal prices, which capture relevant relative prices for a consumer who lives for many periods. I propose a dynamic price index (DPI) that solves these problems. Theoretically, I show that the DPI: is forward-looking, responds by more to persistent shocks, includes assets prices, and distinguishes between durable and non-durable goods' prices. Dynamic inflation in the United States from 1970 to 2008 differs markedly from the CPI, it is close to serially uncorrelated, it is mostly driven by the prices of houses and bonds, and it is twice as high as the CPI in 2008.

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Three questions arise if prices are uncertain and change over time:

1. If you have two children, one year apart, and wish to give each a bequest at a certain age, how much more should you give the younger one relative to what you gave the older one, so that they are equally well-off, in spite of the different prices they face?

2. If you are managing the endowment of a long-lived institution (e.g., a university), what must be the minimum return on the endowment this year given current prices, so that the institution is able to serve future generations at least as well as it did last year with the past prices?[1]

3. If you are about to retire and live off the savings in your private retirement account, how much must the account have, given today's prices, so that you are as well off as you would have been had you retired last year?

The answer to all of these questions involves adjusting a nominal amount by a scalar between two successive years in response to changing prices. This scalar is a *price index*, a function of all the prices measuring the change in the (broadly understood) cost of living.

One feature of the three questions is that they are explicitly dynamic. Not only they involve a comparison between two points in time, but also, and more importantly, they involve entities that exist for many periods and face uncertainty. However, conventional measures of the cost of living are static, derived from models of agents that live for only one period. This paper provides a *dynamic measure of inflation* that answers the three questions. For short, I refer to it as the dynamic price index, or DPI.[2]

The paper is organized as follows. Section 1 further motivates the need for dynamic measures of inflation and surveys the related literature. Section 2 works through a simple two-period model that highlights the main properties of dynamic inflation. Section 3 describes a more general model of intertemporal behavior and defines the DPI. Section 4 studies its theoretical properties, and section 5 takes a first stab at constructing a benchmark DPI for the United States. Section 6 concludes.

[1] This goal is sometimes called intergenerational equity (Association of American Universities, 2008).

[2] A more appropriate, though more lengthy, nomenclature might be DS-COL-CPI, for **D**ynamic **S**tochastic **C**ost-**o**f-**L**iving **C**onsumer **P**rice **I**ndex.

1 Motivation and relation to the literature

The three questions stated in the introduction involve large sums of money: intended bequests account for a large fraction of aggregate wealth; in 2007, U.S. university endowments alone totalled at least \$411 billion; and in the United States in 2001, 52% of the population had a retirement account, these held 28% of the value of household's financial assets, and pension funds held \$4.5 trillion in assets in 2004.[3] Adjusting these accounts in response to price changes to satisfy equity concerns is only one of the many issues involved in managing them. But the sums involved are large enough that a price index that is useful for decisions on bequests, endowments, and retirement should be in high demand.[4]

A common practice to measure the cost of living is to measure the change in expenditure required to buy a fixed basket of goods when facing a new set of prices, as Laspeyres or Paasche suggested. Economists have long criticized this fixed-basket approach since, in general, consumers will substitute across goods in response to changes in prices, so keeping baskets fixed will lead to a substitution bias.[5] Rather than keeping *baskets* fixed, instead one should keep *utility* fixed. Konus (1924) first defined a cost-of-living price index as the welfare measure of compensating variation in response to price changes. If $V(W, \mathbf{p}^t)$ is the indirect utility function of a consumer mapping the wealth she has, W, and the prices she faces, $\mathbf{p}^t$, to the standard of living she can achieve by acting optimally, then the cost-of-living price index π_{t+1} is the solution to:[6]

$$V(\pi_{t+1}W, \mathbf{p}^{t+1}) = V(W, \mathbf{p}^t). \tag{1}$$

The model of behavior behind the indirect utility function has so far been almost ex-

[3] Sources: NACUBO 2007 Endowments Study, Survey of Consumer Finances, 2001, Tables 5.E and 4; Flow of Funds, Tables L.119.b and L.119.c.

[4] How much can indexing matter? A curious incident provides an answer. When social security and disability benefits were first indexed to CPI-inflation in 1972, a mistake was made in the indexation formula. Among other factors, this contributed to make the social security system so generous that it quickly fell into disrepute. This led by the early 1980s to widespread reforms that scaled down the system, including an adjustment of the indexing formula (Bound and Burkhauser, 1999, pp. 3454-3456). Boskin and Jorgenson (1997) calculate that if an estimated 1.1% upward-bias of the CPI due to quality change was corrected, government savings on social programs indexed to the CPI would in a decade lower federal debt by \$1,066 billion dollars.

[5] On quantifying this substitution bias, see Shapiro and Wilcox (1996), Boskin et al (1997) and the more recent discussion in the National Research Council (2002).

[6] The price index is sometimes defined using instead the expenditure function. Duality implies that it is equivalent to define it as in (1).

clusively the classical model of a consumer who lives for one period and maximizes a utility function subject to a static budget constraint and no uncertainty.[7] Using these static models with consumers that live for many periods creates at least two problems. First, by ignoring time, static models suffer from an intertemporal substitution bias. Consumers that live for more than one period react to higher prices today relative to the future by substituting away from present into future consumption, therefore partially attenuating the welfare impact of the price change. Second, static models suffer from an intertemporal price omission. For the consumer's welfare the relative prices of apples today versus in the future are as relevant as the prices of apples versus other goods today, yet the former are ignored.

This paper addresses these problems by using the modern theory of consumption (e.g., Deaton, 1992) modeling people that maximize utility over many periods and are subject to shocks to measure inflation. The cost-of-living price index is still defined by (1), as all cost-of-living price indices are, but the underlying model of behavior is now dynamic and stochastic, explicitly taking into account intertemporal substitution and intertemporal prices.

Dynamic measures of inflation have two further appeals relative to their static counterparts. The first is parsimony: economists now routinely use stochastic dynamic models of consumption to study consumption, business cycles or growth. It is natural to use these models also to measure the cost of living. Second, the DPI treats time in a theoretically coherent way, using a model of people that live for at least two periods when comparing prices at two dates, and recognizing that their consumption basket includes both consumption in the present as well as in all future dates.

The consideration of intertemporal trade-offs in the context of price indices was, to my knowledge, first articulated by Alchian and Klein (1973). They proposed a definition of a dynamic price index with complete markets, noted that it would include futures prices, and proceeded to use asset prices in the estimation of money demand equations. In this paper, I define the DPI in the more realistic case of incomplete insurance markets, I characterize the influence of different prices on the price index, and I actually construct a dynamic measure

[7] Fisher and Shell (1972), Diewert and Montmarquette (1983), Pollak (1989) and Diewert (2001) give thorough presentations of this approach, and Jorgenson and Slesnick (1999) provide an econometric application of the static approach to indexing retirement accounts.

of U.S. inflation to answer indexation questions.

A few other authors have expanded on Alchian and Klein (1973). Jorgenson and Yun (2001) discuss how to deal with dynamics but no uncertainty, Boskin (2005) defends the merits of taking a dynamic approach, and Diewert (2002) criticizes the assumption of complete Arrow-Debreu markets. All of them highlight in different ways the importance of solving the problem in this paper. Pollak (1989, chapter 3) studies whether it is possible to form period sub-indices in an intertemporal context. Some of his results can be applied to the DPI; however, this paper focuses on a different set of questions. Shibuya (1992) and Wynne (1994) are the only studies that I am aware of that tried to build a price index taking dynamics and uncertainty into account. They used very restrictive assumptions though and can be seen as special cases of the more general results in this paper.

Other authors have informally defended the inclusion of asset prices in price indices. Goodhart (2001) persuasively argues that house prices should receive a special treatment. This paper provides a theoretical foundation to many of his comments. Another literature assumes the CPI is the correct measure of inflation, but asset prices are useful in forecasting static inflation (Cecchetti et al, 2000, Stock and Watson, 2003). This paper instead notes that asset prices enter directly into a dynamic measure of inflation.

Finally, this paper studies price indices that measure the cost of living. There are many other measures of welfare in response to shocks that are not price indices in public finance (e.g. Auerbach and Kotlikoff, 1987), and many other measures of inflation for other purposes for which dynamic considerations are important (e.g., Basu and Fernald, 2002, Mankiw and Reis, 2003, Geanakoplos, 2005, Reis and Watson, 2008).

2 Dynamic inflation in a simple economy

Consider an economy populated by generations that live for two periods, young and old. The problem of a young person is:

$$V(w, p_a, p_b, q) = \max_{c_a, c_b, c'_a, c'_b, e} \left\{ \frac{u(c_a, c_b)^{1-1/\gamma}}{1-1/\gamma} + \beta \mathbb{E}\left[\frac{u(c'_a, c'_b)^{1-1/\gamma}}{1-1/\gamma} \right] \right\} \text{ s.t.} \tag{2}$$

$$p_a c_a + p_b c_b + qe \leq w \tag{3}$$

$$p'_a c'_a + p'_b c'_b \leq (q' + 1)e \tag{4}$$

There are two goods, indexed by a and b, that sell for prices (p_a, p_b) and (p'_a, p'_b) in the two periods, respectively. The consumer's static utility function is Cobb-Douglas, $u(c_a, c_b) = c_a^{\alpha} c_b^{1-\alpha}$, he discounts the future by the factor β, and has an intertemporal elasticity of substitution γ. He lives off his wealth in period 1, w, and can save by buying e units of a consol for price q that next period earns a coupon of 1 and sells for price q'. The log of each price follows an AR(1) with independent, zero-mean, normal innovations: $\ln(p'_a) = \eta_a \ln(p_a) + \varepsilon_a$, $\ln(p'_b) = \eta_b \ln(p_b) + \varepsilon_b$, and $\ln(q' + 1) = \eta_q \ln(q) + \varepsilon_q$.[8]

A philanthropist in this economy can make unrestricted gifts to each of the generations at the start of their life, effectively controlling their starting wealth. The philanthropist cares about the members of each generation equally, and has a Rawlsian social welfare function, so she wants to make sure that they are all equally well off. The question she asks is: how much more wealth should the t generation have relative to the $t-1$ generation? The three questions posed in the introduction are conceptually the same as this one, as they all involve computing a measure of inflation to index an account that will be used by a long-lived agent that can shift funds across time.

The standard answer to this question would be to adjust wealth by the static cost-of-living price index. Letting the indirect static utility function be

$$v(w - qe, p_a, p_b) = \max\{u(c_a, c_b) : p_a c_a + p_b c_b \leq w - qe\}, \tag{5}$$

static (gross) inflation, s_t, is defined as $v(s_t(w_{t-1} - e_{t-1}q_{t-1}), p_{a,t}, p_{b,t}) = v((w_{t-1} - e_{t-1}q_{t-1}), p_{a,t}, p_{b,t})$. Using the Cobb-Douglas preferences,

$$\ln(s_t) = \alpha \Delta \ln(p_{a,t}) + (1 - \alpha) \Delta \ln(p_{b,t}). \tag{6}$$

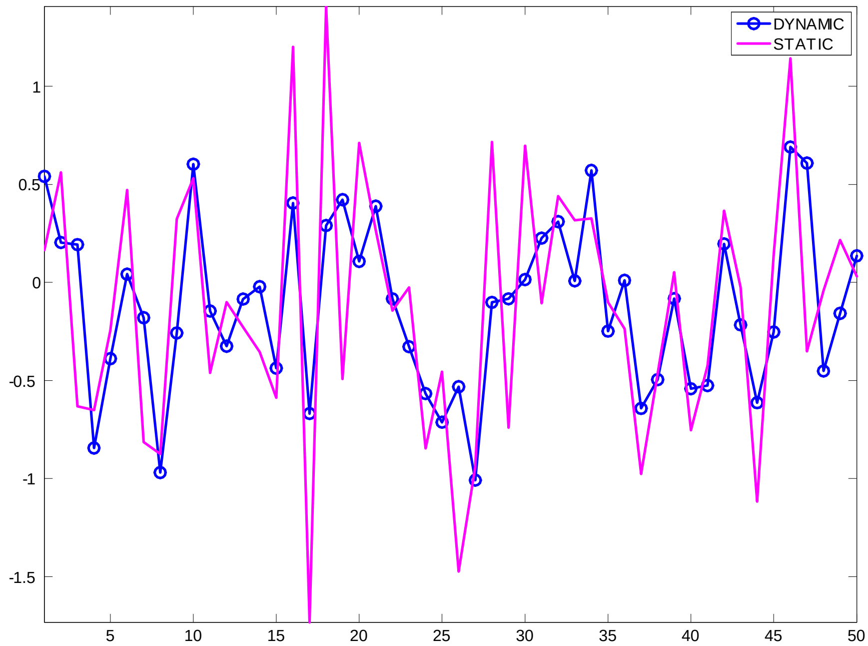
However, the generation's welfare is given by $V(.)$, not $v(.)$, so the right answer to the philanthropist's question is instead a dynamic measure of inflation π_t such that $w_t = \pi_t w_{t-1}$ so:

$$V(\pi_t w_{t-1}, p_{a,t}, p_{b,t}, q_t) = V(w_{t-1}, p_{a,t-1}, p_{b,t-1}, q_{t-1}). \tag{7}$$

This dynamic measure of inflation takes into account the fact that the consumer will live

[8] The "twist" to the process for asset prices helps to make the problem analytical but plays no other important role in the results.

Figure 1: Simulated dynamic and static inflation



for more than one period, and will optimally allocate consumption across life in response to different prices at different ages. Standard calculations (details in the appendix) give the indirect utility function, and using this to solve for π_t in equation (7) gives dynamic inflation as a weighted average between static inflation and another term, where $\theta(p_a, p_b, q)$ is a function of the three prices:

$$\ln(\pi_t) = \ln(s_t) + \left(\frac{1}{\gamma - 1}\right) \ln\left(\frac{\theta(p_{a,t}, p_{b,t}, q_t)}{\theta(p_{a,t-1}, p_{b,t-1}, q_{t-1})}\right). \tag{8}$$

The first conclusion is that, in general, *dynamic inflation is not equal static inflation.* Figure 1 illustrates that the difference is quantitatively significant in a 50-period simulation using formula (8), where the correlation between the two measures is 0.67.[9]

To understand what drives these differences, I use an approximation around the non-

[9] The figure is drawn using $\gamma = 0.5$, $\alpha = 1/3$, $\beta = 0.5$, $\eta_a = \eta_b = 0$, $\eta_q = 0.5$, $\sigma_a = \sigma_b = \sigma_q = 0.05$.

stochastic steady state. Letting g denote gross consumption growth in this steady state:

$$\begin{aligned}\ln(\pi_t) \quad &\approx \quad \alpha\left(\frac{1+\beta g^{1-1/\gamma}\eta_a}{1+\beta g^{1-1/\gamma}}\right)\Delta\ln(p_{a,t})+(1-\alpha)\left(\frac{1+\beta g^{1-1/\gamma}\eta_b}{1+\beta g^{1-1/\gamma}}\right)\Delta\ln(p_{b,t}) \\ &\quad + \left[\frac{\beta g^{1-1/\gamma}(1-\eta_q)}{1+\beta g^{1-1/\gamma}}\right]\Delta\ln(q_t) \qquad (9)\end{aligned}$$

Imagine that every good's price is i.i.d. and increases by 1%. For $\alpha = \beta = 0.5$, static inflation is $s_t = 1\%$, but dynamic inflation is $\pi_t = 0.67\%$. Misguided by the static measure, the philanthropist would be overly generous to the new generation. The reason is that she would ignore the response of the member of this generation to the temporarily higher prices by saving less for the future, partially buffering the negative impact of the higher prices on welfare. The second conclusion is that the ability to transfer funds over time attenuates the impact of price changes on the cost of living. *There is an intertemporal substitution bias that leads static inflation to overstate dynamic inflation.* This bias is larger if the elasticity of intertemporal substitution is larger or if the shocks are more transitory, since in both cases the consumer is more willing and more able to buffer the shocks via intertemporal trade. Only in the limits, when either $\gamma \to 0$, or the price shocks are random walks, does dynamic inflation equal static inflation, as in these cases the consumer is either unwilling or unable to engage in any intertemporal substitution.[10]

Finally, *asset prices affect dynamic inflation but not static inflation.* If the asset price rises temporarily, static inflation is unchanged, but dynamic inflation is higher. The philanthropist should compensate the person because expected returns are now lower, so the costs of transferring funds from the present to the future has gone up, and the effective price of future consumption is higher. Only if returns are serially uncorrelated is it correct to ignore equity prices as the static measure does.

The correct dynamic measure of inflation for a long-lived agent is therefore different from static inflation and takes into account substitution over time, the persistence of shocks, and the price of financial assets. The next section shows that these properties hold more generally.

[10] The reader may wonder whether it is intertemporal subsitution or risk aversion driving the results. Using preferences that distinguish betwen the two, one can show that exactly the same expression (9) defines the DPI with the elasticity of intertemporal substitution appearing and not the coefficient of relative risk aversion (details available from the author).

3 The theoretical framework

The person receiving the bequest in the first question, the institution using the endowment, or the person contemplating retirement, can all be thought of as solving an intertemporal consumption problem. In the first case, the utility function refers to the well-being of each child, in the second case, it refers to an objective function of an institution, and in the third case, to the well-being of the person close to retirement.[11]

3.1 The model of behavior

Compactly written, the problem at date t consists of choosing $\{C_{j,t+i}, S_{j,t+i}, B_{j,t+i}\}_{i=0}^{\infty}$ for all j to maximize:

$$V(.) = \mathbb{E}_t \left[\sum_{i=0}^{\infty} \beta^i U(C_{1,t+i}, ..., C_{N,t+i}, S_{1,t+i}, ..., S_{D,t+i}) \right] \tag{10}$$

subject to:

$$\sum_{j=1}^{N} P_{j,t+i} C_{j,t+i} + \sum_{j=1}^{D} R_{j,t+i} S_{j,t+i} + \sum_{j \in \{B,E\}} Q_{j,t+i} B_{j,t+i} \leq W_{t+i}, \tag{11}$$

$$W_{t+1+i} = \sum_{j=1}^{A} D_{j,t+i} B_{j,t+i} + \sum_{j=1}^{D} R_{j,t+1+i}(1-\delta_j) S_{j,t+i}, \tag{12}$$

$$W_{t+1+i} \geq 0, \quad C_{j,t+i} \geq 0, \quad S_{j,t+i} \geq 0, \tag{13}$$

$$\text{for } i = 0, 1, 2, ..., \quad \text{and } W_t = A_t. \tag{14}$$

In words, the consumer maximizes total welfare, which equals the expected discounted sum of period utilities. Period utility is strictly monotonic in each of its arguments, overall concave, unbounded above and below, with $\lim_{x \to 0} \partial U(.)/\partial x = +\infty$ for each of its arguments x. The agent faces a constant probability of dying, which combined with impatience, leads to a discount factor $\beta < 1$, and obtains utility from consuming non-durable goods, each denoted by $C_{j,t+i}$, and from a flow of durable goods that is proportional to the stock, $S_{j,t+i}$.

The consumer allocates his wealth each period W_{t+i} to the uses in (11). He can acquire

[11] The interpretation of an institution managing an endowment as an infinitely-lived consumption problem has a long history, e.g. Merton (1993, chapter 21).

non-durables at prices $P_{j,t+i}$, invest in the durables at prices $R_{j,t+i}$, or buy and sell one-period financial assets in amount $B_{j,t+i}$ that trade at the price vector $Q_{j,t+i}$. Many of the results in this paper apply to an arbitrary set of assets, but for concreteness, I will specialize to the case where there are two financial assets: bonds, which pay a certain amount next period, and equity, which pays a dividend and can be re-sold.

The consumer starts date t with wealth A_t and from then on has two sources of wealth.[12] The first is the payoff from the financial asset, $D_{j,t+i}$, so the return on holding an asset is $D_{j,t+i+1}/Q_{j,t+i}$. The second is the market value of the stock of durables after depreciation, where δ_j is the depreciation rate of durable j. Finally, the constraints in (13) impose that consumption cannot be negative and that the consumer cannot run Ponzi schemes, which in this case reduces to always having non-negative wealth.

The last component of the model to specify is the stochastic processes. Because the aim of this paper is to calculate the measure of inflation to index an account against price changes, I assume that prices are the only source of uncertainty: $\mathbf{p}_{t+i}$ is a random vector containing the prices for non-durables, durables and assets, while tastes and the quality of goods are known. This assumption implies that the resulting index will be a *price* index, that responds to prices but nothing else. I further assume that the consumer perceives $\mathbf{p}_{t+i}$ as following a Markov process so that $\mathbf{p}^t = (\mathbf{p}_t, \mathbf{p}_{t-1}, \mathbf{p}_{t-2}, ...)$ constitutes a sufficient statistic for forming his expectations. This implies that, at any date, the consumer's wealth and this vector of past prices are the state variables affecting plans, so the indirect utility function (or value function) is $V(W_{t+i}, \mathbf{p}^{t+i})$, measuring the standard of living.[13]

3.2 Defining the dynamic price index

The answer to the three questions is given by a measure of inflation:

[12] I abstract from other sources of income. For two of the questions, the funding of a charity and the retirement problem, this is a roughly accurate assumption. For the bequests question, labor income may be important, but if it is either deterministic or perfectly spanned by the financial assets, we can think of the present value of labor income as being part of A_t (Merton, 1993). I abstract from other forms of labor income in order to avoid the controversial question of whether changes in wages are a change in a price (of leisure) that the price index should include, or a non-price shock that it should control for. In the first case, then interpreting one of the $C_{j,t}$ as leisure, the analysis is unchanged. In the second case, see footnote 14.

[13] For there to exist an optimal solution to the consumer problem leading to a finite value function requires some constraint on the stochastic process for prices. This is to ensure that, following a shock, prices do not go to zero too quickly driving consumption to infinity and so leading to unbounded utility. It is difficult to state these conditions for a general Markov process for all of the prices. Later, when I specialize to low-order Markov processes, I verify these conditions case by case.

Definition: *The dynamic price index π_t is the scalar that solves:*

$$V(\pi_t A_{t-1}, \mathbf{p}^t) = V(A_{t-1}, \mathbf{p}^{t-1}). \tag{15}$$

For the question on bequests, the intertemporal consumers are the children. Then, the left-hand side of (15) is the welfare of the younger child, while the right-hand side is the welfare of the older, and the parent wishes to equate them. For the question on donations, the consumer is the institution and the utility function is the goal that it tries to pursue by spending its endowment. The left-hand side of (15) now is a measure of how it attains its goals at date t, while the right-hand side is the same measure at $t-1$, so that π_t is the necessary adjustment to the endowment in order to maintain its ability to sustain its goals given the different prices. Finally, for the worker contemplating retirement, if he is offered an adjustment to his retirement account of π_t conditional on the prices $\mathbf{p}^t$, he will be indifferent between retiring at either date t or at date $t-1$.

A few remarks clarify this definition. First, note that since the DPI is defined taking as base the previous period, it corresponds to a measure of inflation. This is not essential: taking the base in the right-hand side of (15) with respect to a fixed date in time would lead to a measure of the price level.

Second, the DPI is an once-and-for-all adjustment to the wealth in the account. In principle, one could use the standard theory of annuities to convert this amount into an equivalent stream of payments over time.

Third, the DPI measures the total adjustment in wealth required to leave the agent equally well-off. If the agent has sources of income other than his parent / philanthropist / prospective retiree, then the required contribution would take these into account in hitting the desired target wealth.[14] Moreover, if changes in financial prices lead to current capital gains and losses or if the payoff of some assets includes insurance payments against price changes, these may partially provide for the adjustments measured by the DPI. Because the same A_{t-1} appears on both sides of (15), the DPI will measure the total adjustment,

[14] In the case of no-price shocks, like changes in wages, tastes, or quality of goods, we may want the price index to not respond to them. Letting z^t be a sufficient statistic for the no-price shocks, indirect utility would be $V(W_t, p^t, z^t)$, and following Pollak (1989), a conditional price index could be either (i) $V(\pi_{t+1} W_t, p^{t+1}, z^t) = V(W_t, p^t, z^t)$, or (ii) $V(\pi_{t+1} W_t, p^{t+1}, z^{t+1}) = V(W_t, p^t, z^{t+1})$, or (iii) perhaps $\mathbb{E}_z[V(\pi_{t+1} W_t, p^{t+1}, z^{t+1})] = V(W_t, p^t, z^t)$, where $\mathbb{E}_z[.]$ integrates over the density for z^{t+1} as of date t. In the first two cases, one keeps non-price shocks fixed at a base period, and in the third, one uses the consumer's expectations of the shocks.

regardless of its source.

Fourth, note that to answer the questions posed in the introduction only requires a model of consumer behavior. The effect that providing this index may have in the general equilibrium of an economy is an interesting question that is not addressed here.[15]

4 Theoretical properties of dynamic inflation

After stating a few general properties that establish a common ground between the DPI and static price indices, this section will start with a simpler version of the consumer problem and progressively build in ingredients towards the general problem. Sometimes, to be more concrete, I focus on log-preferences:

$$U(.) = \sum_{j=1}^{N} \alpha_{N,j} \ln\left(C_{j,t+i}\right) + \sum_{j=1}^{D} \alpha_{D,j} \ln\left(S_{j,t+i}\right), \tag{16}$$

4.1 Basic properties

A check that the questions posed have an answer is provided by:[16]

Proposition 1 *If prices and wealth are positive and finite, the DPI exists and is unique.*

Samuelson and Swamy (1974) showed that a static cost-of-living index is independent of wealth if preferences are homothetic. With time-separable and homothetic preferences, the value function is still homothetic, and the same result applies to the DPI:

Proposition 2 *The DPI is independent of wealth A_t as long as $U(.)$ is homothetic.*

Another useful property of conventional price indices is that they move one-to-one with pure inflation. That is, if all nominal prices and payoffs increase proportionally by the same amount M, so no relative prices change but there is only a change in the unit of account, then the price index increases by M as well. The DPI has this property:

Proposition 3 *The DPI is proportional to M.*

[15] Who would provide these indices is another interesting question. There is also no impediment for private financial institutions to supply accounts indexed to the DPI, especially since there is already a competitive market in retirement accounts and endowment management. These could be supported by holding portfolios that replicate its portfolio, and the aggregate risk could be diversified interntionally, as suggested by Shiller (2003).

[16] The proofs of all the propositions are in the appendix.

Generally, static and dynamic measures of inflation will be different as intertemporal measures of welfare and equivalent variation are generically different from static measures, as illustrated by Blackorby, Donaldson and Maloney (1984). It is instructive though to investigate the special case when they are the same. This will be true if there is no scope for any intertemporal substitution, which happens if all price shocks are permanent:[17]

Proposition 4 *With log-utility, if goods prices all follow random walks and financial asset returns are all i.i.d., up to a first-order approximation, the DPI equals the static cost-of-living price index.*

4.2 Long lives and looking forward

To start off, consider a simpler version of the consumer problem in (10)-(13) in which there is no uncertainty nor any trade of resources over time, as there is no access to durable goods or financial assets. In order to have non-zero consumption after the first period, assume there is an annuity contract converting the initial assets into a fixed stream of nominal income every period. The only dynamic element in this case comes from the consumer caring about total, rather than period, utility.

Proposition 5 *If there is no uncertainty, no durable goods, no assets, and a constant annuity payment of* $(1-\beta)A_t$ *every period:*
a) If $P_{j,t+1+i} \geq P_{j,t+i}$ *for all* i *and* j*, then* $\ln(\pi_{t+i+1}) \geq 0$.
b) In two situations A *and* B*, such that:* $(P_{t+1+i}/P_{t+i})^A \geq (P_{t+1+i}/P_{t+i})^B$ *for all* i*,* $\ln(\pi^A_{t+i+1}) \geq \ln(\pi^B_{t+i+1})$.
c) If the price sequence is: $P_{j,t+i} = P$ *for* $i \neq h$*,* $P_{j,t+h} > P$ *for all* j*, then* $\ln(\pi_{t+1}) > 0$.

The first two results are sensible properties of any price index. The first result states that if prices are rising, the DPI is above one, and the second result adds that if prices rise faster, the DPI is higher. However, even in this bare case where the only link between dates comes from the consumer living for many periods, the DPI is not identical to a conventional static price index. The third result considers a case where prices are always the same with one exception, in h periods, when prices will be higher. The consumer today, realizing prices will rise, requires an increase in her nominal wealth to be able to afford these higher

[17] This result would hold exactly, without any approximation, if there were no durables.

prices at the future date. The DPI is above one already today even though prices will only increase in the future. Because consumers are forward looking, so is the price index, reacting to news on future prices.

4.3 Non-durables goods prices and intertemporal trade

Next, I allow for intertemporal trade and uncertainty on non-durable goods prices.

Proposition 6 *Assume that there are no durable goods and, without loss of generality, separate each price into the product of a common stochastic component and an idiosyncratic shock: $P_{j,t} = P_t \hat{P}_{j,t}$. Then, up to first-order approximations around a steady-state:*
a) If P_t is i.i.d. then $\ln(\pi_{t+1}) \approx (1-\beta)\ln(P_{t+1}/P_t)$
b) If $\Delta\ln(P_{t+1}) = \eta\Delta\ln(P_t) + \varepsilon^P_{t+1}$, then $\ln(\pi_{t+1}) \approx \ln(P_{t+1}/P_t)/(1-\beta\eta)$
c) If $\hat{P}_{j,t}$ and $\hat{P}_{k,t}$ are independent and i.i.d. over time $(\partial\ln(\pi_{t+1})/\partial\ln(\hat{P}_{j,t+1}))/(\partial\ln(\pi_{t+1})/\partial\ln(\hat{P}_{k,t+1})) = P_{j,t+1}C_{j,t+1}/P_{k,t+1}C_{k,t+1}$.

Since P_t is the static price index, the first result shows that a 1% increase in static inflation raises dynamic inflation by less than 1%. If periods are years, so $\beta \approx 0.96$, the static price index is biased up by a factor of 25. The reason for the smaller impact of price shocks on the DPI is that consumers can use financial assets to borrow against temporarily high prices, attenuating their impact on welfare by smoothing consumption.

Empirically, goods' prices are closer to a first-order autoregression in log differences. The second result shows that if η is positive, a 1% increase in static inflation raises dynamic inflation by more than 1%. Higher prices today now imply that the consumer should expect even higher prices in the future, so he requires a larger increase in wealth today in order not to be worse off. The larger is the persistence of shocks, the larger their impact on dynamic inflation.

Turning to the third result, the relative marginal impact of shocks to two non-durable prices equals their ratio of relative expenditures. Intuitively, if the consumer cares more about a good and allocates a larger amount of spending to this good, then an increase in its price affects her cost of living by more.

4.4 Asset prices

One crucial difference between the DPI and a static price index is the role of asset prices. For a consumer that lives for many periods, the relevant consumption basket includes not only consumption of different goods today, but also future consumption. If today's relative price between two goods affects the price index, then so must the relative price between today and the future. Asset prices measure precisely these relative prices, so in general, they affect dynamic inflation. Moreover, note that what matters to the consumer are expected relative prices, or expected capital gains and losses. An increase in asset prices may induce current capital gains and losses but dynamic inflation does not respond to these since, by the definition in (15), wealth is kept fixed. Because dynamic inflation measures the total change in wealth to leave the consumer equally well-off, it is expected relative prices going forward that matter.

The next proposition assesses the quantitative significance of asset prices in the DPI:

Proposition 7 *Assume that there are no durable goods and, without loss of generality, separate each asset price into the product of a common and an idiosyncratic component:* $Q_{B,t+i} = Q_{t+i}\hat{Q}_{B,t+i}$ *and* $Q_{E,t+i} = Q_{t+i}\hat{Q}_{E,t+i}$. *Assume that* Q_{t+i} *is i.i.d. Then, up to a first-order approximation:*

a) In response to the common shock to asset prices: $\ln(\pi_{t+1}) \approx \beta \ln(Q_{t+1}/Q_t)$

b) If equity returns, $D_{E,t+1}/Q_{E,t}$*, are i.i.d., then* $\partial \ln(\pi_t)/\partial \ln(\hat{Q}_{E,t}) = 0$.

c) If $\hat{Q}_{B,t}$ *and* $\hat{Q}_{E,t}$ *are independent and i.i.d. over time, then* $(\partial \ln(\pi_{t+1})/\partial \ln(\hat{Q}_{E,t+1}))/(\partial \ln(\pi_{t+1})/\partial \ln(\hat{Q}_{B,t+1})) = \hat{Q}_{E,t+1}B_{E,t+1}/\hat{Q}_{B,t+1}B_{B,t+1}$.

Higher asset prices raise the DPI because they make it more costly to transfer funds for future consumption. The first result shows that asset prices can matter a lot. Again if $\beta \approx 0.96$, they have an impact on dynamic inflation 24 times greater than that of goods' prices. The second result show that again the persistence of the shocks matters. If equity returns are i.i.d. (a rough description of the actual data), then they do not affect the DPI because, in this case, higher equity prices today have no implication for expected future returns. Thus, no relative prices change and neither does welfare or the cost of living. The third result shows that the price of each asset receives a weight in the DPI that is proportional to its portfolio share.

We can combine the insights so far in a special case for which there is an analytical solution for the DPI:

Proposition 8 *If there are log-preferences, only common price shocks P_t, and only one asset with a 1-period return I^M_{t+i+1}, then:*

$$\ln(\pi_{t+1}) = \ln\left(P_{t+1}/P_t\right) + (1-\beta)\sum_{i=1}^{\infty}\beta^i \mathbb{E}_{t+1}\left[\ln\left(\frac{P_{t+1+i}I^M_{t+1}}{P_{t+i}I^M_{t+i+1}}\right) - \ln\left(P_{t+1}/P_t\right)\right] \quad (17)$$

$$+(1-\beta)\sum_{i=1}^{\infty}\beta^i\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\ln\left(P_{t+i}/\prod_{j=1}^{i} I^M_{t+j}\right) \quad (18)$$

This simple formula separates movements in the DPI into the sum of three components. The first term is the static price index, while the other two terms involve expected future goods' price inflation discounted by interest rates. The second term measures the difference between expected future prices and current prices, the intertemporal substitution effect of Proposition 6. The third term captures the revision in expectations about future prices, the price news effect of Proposition 5. Asset prices matter because they affect the effective interest rates used by households to discount the future. Higher asset prices (or lower interest rates) imply that the consumer's retirement account will be worth less in the future in terms of consumption goods and thus raises the cost of living.

4.5 Durable goods' prices

Finally, I introduce durable goods. They are particularly interesting because they combine features of both goods and assets: they yield utility, and they also transfer wealth across time. These two sides of a durable j are well captured by $u_{j,t+1}$, the ex post user cost of holding it between t and $t+1$:

$$u_{j,t+1} = R_{j,t} - (1-\delta)R_{j,t+1}/I^M_{t+1}. \quad (19)$$

Holding the durable for one period requires paying $R_{j,t}$ for it at date t and then selling the remainder after depreciation for $R_{j,t+1}$ at date $t+1$, noting that the opportunity cost of investing a $t+1$ dollar in durables is $1/I^M_{t+1}$ dollars at date t, where I^M_{t+1} is the return on the portfolio of financial assets.

Proposition 9 *If the log price of durable j and non-durable i follow first-order autoregres-*

sions with coefficients η_j and η_i, then:

$$\frac{\partial \ln(\pi_t)/\partial \ln(R_{j,t})}{\partial \ln(\pi_t)/\partial \ln(P_{i,t})} \approx \frac{S_j \partial U(.)/\partial S_j}{C_i \partial U(.)/\partial C_i} \times \left(\frac{1-\beta\eta_i}{1-\beta\eta_j}\right) \times \mathbb{E}_t\left(\frac{\partial u_{j,t+1}}{\partial R_{j,t}} \times \frac{R_{j,t}}{u_{j,t+1}}\right). \tag{20}$$

Moreover:

$$\frac{\partial u_{j,t+1}}{\partial R_{j,t}} \times \frac{R_{j,t}}{u_{j,t+1}} = \frac{1-\eta_j(1-\delta)\left(R_{j,t+1}/R_{j,t}I^M_{t+1}\right)}{1-(1-\delta)\left(R_{j,t+1}/R_{j,t}I^M_{t+1}\right)}. \tag{21}$$

The proposition gives the relative marginal impact of a change in durable j's price relative to a change in the price of non-durable i. The first fraction in the expression captures the effect of expenditure shares, and the second the role of persistence, both of which were discussed in section 4.2. What is special about durable prices is the third fraction. If user costs are always proportional to prices, it equals one. This is the case if the log of the price of the durable follows a random walk. In this case, durability is irrelevant: whether the good is durable or not, it has the same weight on the DPI.

If instead shocks are transitory, then user costs rise by more than 1% in response to a 1% rise in prices. A higher durable good's price then hurts the consumer in two ways: first because it raises the current price paid for the good, and second because the consumer expects a capital loss on holding this asset since the price is expected to fall. In this case, durable goods have a larger weight on the DPI, and one that increases with $1-\delta$, the durability of the good. In the extreme case in which the price of the durable is i.i.d and the return on bonds and durables are approximately the same, the last fraction in (20) equals $1/\delta$. For a very durable good such as housing, for which $\delta \approx 0.02$, a temporary increase in its price raises the DPI by about 50 times more than a comparable increase in a non-durable price.

Alternatively, if shocks to durable prices are very permanent, in the sense that a 1% increase in $R_{j,t}$ comes with an expected increase in $R_{j,t+1}$ of more than 1%, then even though the consumer is hurt by paying more for the good, she benefits from the expected capital gains on it. The user cost of the durable falls, and so the change in its price has a smaller impact on the cost of living.[18]

[18] One of the most important consumer durable goods is housing. Transaction costs and changes in quality are two features of housing that the model ignores but which future research should explore. In related work, Bajari, Benkard and Krainer (2005) study the impact of a change in house prices on welfare and provide some related results to the ones in this section. They did not consider uncertainty or the persistence of shocks however. Diewert (2003) provides a thorough review of the current state of knowledge on how to include house prices in price indices.

4.6 Summary of theoretical results

To sum up, dynamic inflation shares some of the common features of static measures: it is independent of retirement wealth as long as utility is homothetic, it increases one-to-one with an increase in all prices, it responds more to goods that have a larger expenditure share, and it is positive if prices trend upwards and higher the steeper this trend. However, considering dynamics leads to several new features that make dynamic inflation very different from a conventional static price index. It is forward-looking, because consumers are forward-looking. Consumers engage in intertemporal substitution, and ignoring this can lead to a large bias. The more persistent are shocks, the larger their impact on the DPI. Asset prices generally affect the price index, and may do so by a large amount, although in the special case where equity prices follow a random walk, they do not affect the DPI. Finally, durable goods are special in that changes in their price have a double impact on the DPI: through the change in expenditure and through expected capital gains and losses.

5 Dynamic inflation in the United States

This section takes a first stab at building a dynamic measure of U.S. inflation. A full-fledged DPI would have to include hundreds of prices and deal with many measurement issues involving durables and taxes.[19] The more modest aim of the calculations that follow is to show the steps involved in building a DPI and to identify its broad movements since 1970.

To take the general model to the data, I introduce a few modifications. First, I consider a small number of broad consumption and asset categories. In particular, there are four non-durable goods (food, energy, services, others), two durable goods (shelter, others), and two assets (equity and bonds). The goods span the whole of the consumer price index (CPI), which I will take as the comparison.

Second, I assume log-preferences as in (16), so the intertemporal elasticity of substitution is one. This reduces the number of parameters to calibrate and is a useful benchmark for richer models of consumer demand.

Third, I assume that adjusting durables is costly. Otherwise, because user costs are very volatile in the data, there are counterfactually wild swings on the stock of durables.

[19] Triplett (1983) provides a lucid discussion of many practical issues.

The adjustment cost function is quadratic and preserves the homotheticity of the indirect utility function.

Fourth, I assume that agents perceive a stochastic process for asset prices that is the sum of a Markov process plus a rare event that wipes out equity's value and occurs with some probability every period. As Barro (2006) shows, this allows the simple model to be roughly consistent with the asset market facts and to match the share of equity in the portfolios that we observe in the data.

5.1 Data and calibration

The time period is one year and the sample goes from 1970 to 2004. The data are the (log) price series for the six goods from the CPI database, with the exception of housing, for which I use the Conventional Mortgage House Price Index produced by Freddie Mac.[20] Asset returns come from the Center for Research in Security Prices, with equity referring to the value-weighted index of stocks in the NYSE, AMEX and NASDAQ, and bond returns to the average yield on 3-month Treasury bills.

Turning to the calibration of the parameters, the discount factor β is $1/1.04$ to match a steady-state 4% annual real return. The taste parameters α_j match the relative shares in household expenditures in the CPI. They are allowed to vary deterministically over time to match the weight changes in each revision of the CPI. The depreciation rates, δ_j, for shelter and durables are 1.6% and 21.1%, respectively, from the Fixed Assets Table of the Bureau of Economic Analysis. The degree of adjustment costs for each durable is set to match the variance of expenditures in that durable.

Finally, for the price dynamics, the perceived probability of a rare event is set to match the 27% average equity portfolio share from Ameriks and Zeldes (2004). Bar a rare event (which in the sample never takes place), the price dynamics are governed by a first-order vector autoregression in first differences, estimated by least squares.

[20] The BLS shelter series suffers from well-known deficiencies: (i) it includes both rents and house prices, but the two are close to uncorrelated (Verbrugge, 2005); (ii) there is a break in 1983, when the BLS went from using reported sales prices to computing "rental-equivalent prices" for non-sold houses by matching them with similar rented ones, so the series before and after 1983 are, strictly speaking, non-comparable; (iii) the series seems to be biased downwards at least over long horizons (Gordon and van Goethem, 2007). The Freddie Mac index gives a weighted indexed of house prices measured by repeat sales.

5.2 Dynamic U.S. inflation

Figure 1 plots annual U.S. inflation using the DPI, the CPI, and a static measure that uses the same series and parameters values as the DPI. The DPI is strikingly different from the CPI; their correlation is merely 0.34.[21] While the two series are equally volatile, with standard deviations of 2.8% and 3.1%, the DPI is noticeably less persistent than the CPI. The serial correlation of dynamic inflation is 0.10, while that of the CPI is 0.81. Figure 2 shows the trends in each series, by plotting at each date the annualized accumulated inflation over the past decade ($\sum_{i=0}^{9} \ln(\pi_{t-i})/10$). The series are much closer (correlation of 0.85 with the CPI and 0.92 with the SPI), but even at these lower frequencies there are important differences, the most striking during the 2000s. At the start of the decade high house prices push static inflation over the CPI, as a gap arose between the house prices I use and the rental-equivalent series of the BLS. Dynamic inflation was even higher, as house prices have a large effect on the DPI. In 2008, the last year of the sample, the DPI shot up as bond prices rose dramatically, so that by then annual dynamic inflation was 7.3% compared with 3.7% in the CPI.

Table 2 tries to get behind what drives the differences between the DPI and the CPI. The first row shows the weight on the DPI of changes in prices in the static price index, that is when each price follows an independent random walk (recall Proposition 4). The second row shows the standard deviation of changes in the log of each price, with equity prices standing out as being much more volatile than any other price. However, the third row reports the coefficient on an AR(1) regression for each price. Note that equity returns are close to being serially uncorrelated. Consequently, despite being volatile, equity returns have a small impact on the DPI. Unlike equity, bond returns are quite persistent and so are house prices.

Row four shows the marginal impact of shocks to each price using the independent AR(1)'s as the forecasting model.[22] The assumption of independence allows one to associate changes in the price with structural shocks to that price. Noticeably, house prices receive a large weight—they are very durable and very persistent, two of the features that lead to a large impact on the DPI. Likewise, bond prices get a large weight because bond returns

[21] The correlation between CPI and the static price index (SPI) is 0.84, while the correlation between the SPI and the DPI is 0.29.

[22] In contrast, with the VAR, shocks are not identified, and it seems hopeless to try to come up with enough identifying restrictions.

Figure 2: Annual measures of inflation: dynamic, static and CPI

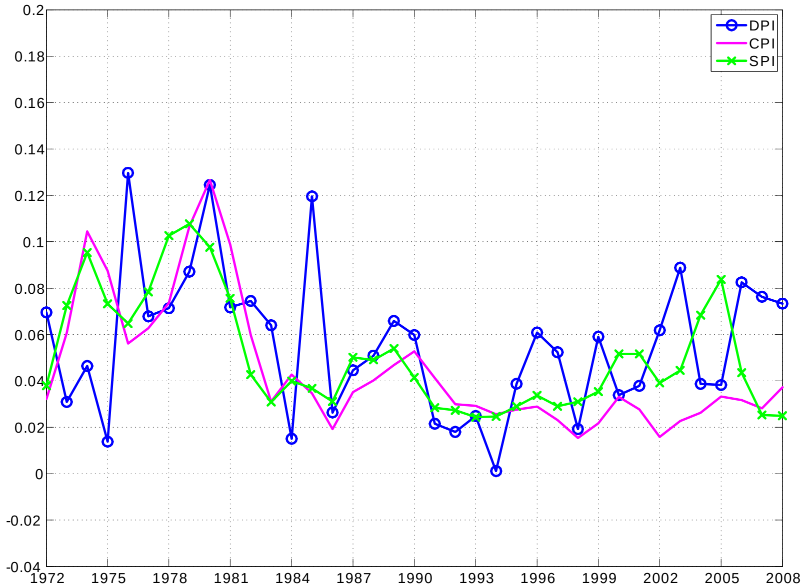
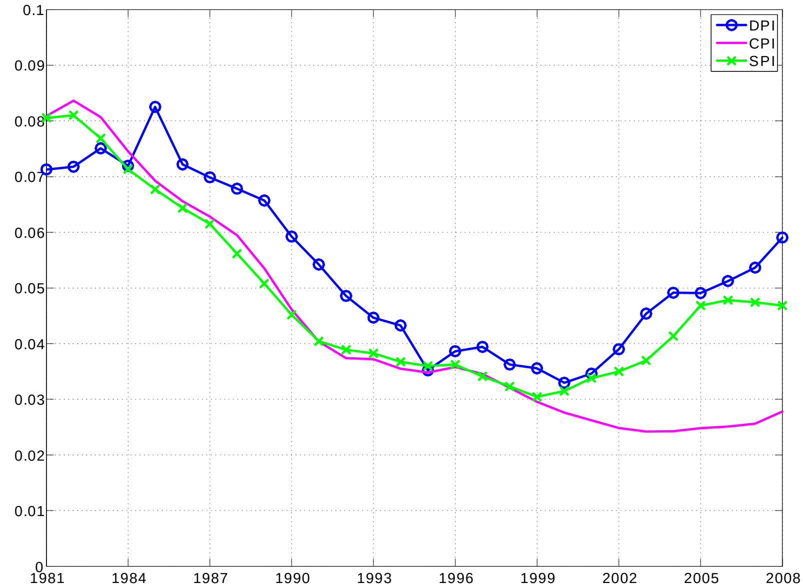


Figure 3: Decade measure of inflation: dynamic, static and CPI



are very persistent.

Table 2. The volatility of the components of the DPI and their average impact

	Food	Energy	Services	Other non-durables
Static weights	0.12	0.05	0.14	0.11
Standard deviation	0.03	0.08	0.03	0.02
Serial correlation	0.69	0.45	0.73	0.72
Dynamic weights with AR(1)	0.34	0.09	0.45	0.37
	Housing	**Other durables**	**Equity**	**Bonds**
Static weights	0.48	0.10	0	0
Standard deviation	0.03	0.04	0.19	0.03
Serial correlation	0.72	0.87	0.00	0.83
Dynamic weights with AR(1)	0.60	0.58	0.00	1.99

These different pieces of evidence paint the following picture of dynamic inflation. First, even though it includes equity prices, dynamic inflation is not more volatile than static inflation, because returns are close to being i.i.d.[23] Second, the DPI is less serially correlated because it responds to news, rather than to the actual changes in prices. Third, housing plays an important role, because it is very durable and its price changes are quite persistent. Fourth, bond prices are the other main driving force in the DPI for two reasons: first, because changes in bond prices tend to be very persistent, and second because they affect the DPI both directly and also indirectly through the user cost of housing. Fifth, the high dynamic inflation of the last decade is partly due to the sharp increase in house prices and partly due to the more recent rise in bond prices.

5.3 Alternatives and robustness

Three inputs went into the DPI: the price series, the agent's forecasting model, and the consumer's parameters.

[23] Goodhart (2001) and Bryan, Cecchetti and O'Sullivan (2001) worried that any price index that included equity prices would be very volatile. The DPI dispels this worry.

As an alternative to the house price series, I considered the House Price Index produced by the Office of Federal Housing Enterprise Oversight. For the other goods' price series, I considered the personal consumption expenditures series from the Bureau of Economic Analysis, that are revised backwards with GDP revisions and update expenditure shares more frequently than the BLS. These alternative price series led to almost identical results for the DPI.

Turning to the forecasting model, I also estimated vector autoregressions of order 2 and 3, but these had little impact on the DPI. In the baseline case, I took first differences of the series. Standard unit root tests strongly rejected the hypothesis that the price series are stationarity, while there is mixed support for stationarity in first-differences. I consider alternatives in two directions. In one direction, I estimated a Bayesian VAR on the levels of the prices (rather than their first differences) using a Minnesota prior. In the other direction, I supposed that the first-differences in prices are cointegrated and used the CPI as the common trend.[24] For both cases, the conclusions on what is driving the DPI and how dynamic inflation has evolved over time were unchanged.

On the consumer parameters, I considered setting the depreciation rates so that the model matches two moments from the Survey of Consumer Finances: the share of wealth held in durables relative to financial assets, and the ratio of wealth in real estate relative to wealth in other consumer durables. These led to annual depreciation rates of 2.4% and 10.8%. This higher depreciation rate for housing lowers the role of house prices in the DPI, but the main conclusions again remain.

Finally, I set parameters focusing on each of the three questions in the introduction. For the first question, the baseline DPI using facts for the average U.S. household seemed appropriate. For the third question, I instead calibrated the taste parameters and the probability of the rare event to match the expenditure shares and the equity share (20%) of the population aged 55 to 64, again using the data from the BLS and from Ameriks and Zeldes (2004). The second question is more challenging since 60% of universities expenditures are on wages, and data on their expenditure shares is harder to obtain. I introduce two new expenditure categories, for academic and other employees, and measure

[24] Johansen's trace test of the number of cointegrating relations lends some support to this hypothesis (at the 5% significance level), by finding 7 cointegrating relations between the 8 price series (Stock and Watson, 1988).

their price using wages for professors and for the aggregate economy, respectively. The sample now starts in 1975. The parameters α_j are re-calibrated to hit expenditure shares obtained from several sources, and the perceived probability of the rare event is re-calibrated to match a 59% share of equity in financial investments. The appendix contains more details on the data sources.

The three series are plotted in figures 4 and 5 for the annual and decade measures. They are all quite close, with correlation coefficients above 0.9. The main noticeable difference is that, towards the end of the sample, the DPI for universities is lower. The two reasons behind this difference again point to the key role of housing and bonds in the DPI. First, salaries have a large weight on university expenditure and they have grown at a slower pace than housing costs in recent years. Second, universities hold a smaller share of bonds in their portfolios, so they are less affected by the recent increases in bond prices.

6 Conclusion

Some of the properties of the dynamic measure of inflation derived in this paper may at first be a little startling. On second thought, perhaps they should not be so surprising. Hall's (1978) seminal work on the role of dynamics and uncertainty on consumption led to a radically new view of the properties of consumption. Contrary to previous knowledge, economists learned that: (i) it is news, not changes in income or prices that matter, so consumption growth is little serially correlated (ii) the persistence of income shocks is a key determinant of the marginal propensity to consume, (iii) asset prices, or asset yields, alone determine expected consumption growth, and (iv) durables have different dynamics from non-durables.

This paper brought this modern model of consumption, with dynamics and uncertainty, into the study of cost-of-living price indices. It found results that mirror those in the consumption literature: (i) news matter so inflation is little serially correlated, (ii) persistence of price-shocks determine their impact on inflation, (iii) asset prices matter, and (iv) durables are different from non-durables. A cost-of-living price index is the dual of a model of consumption, so the properties of dynamic inflation mirror those of Hall's study of consumption. Since economists find dynamic models of consumption appealing, they should be attracted to dynamic measures of the cost of living.

Figure 4: Annual dynamic inflation for bequests, university endowments, and prospective retirees

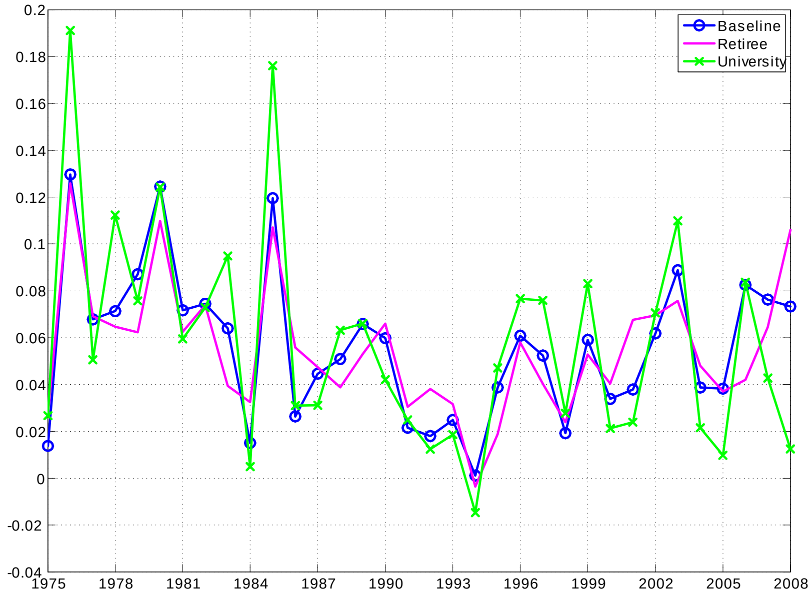
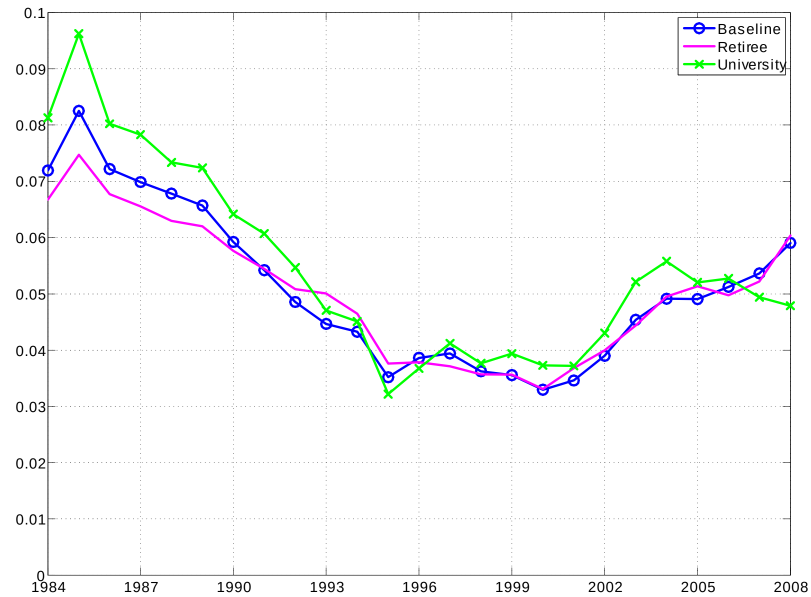


Figure 5: Decade dynamic inflation for bequests, university endowments, and prospective retirees



Computing the DPI does not pose any insurmountable data problems. Relative to the static approach, one needs new information on consumer's preferences for trading over time, which is already routinely collected for portfolio and retirement advice, and on expectations of future price changes, for which we have massive amounts of data from markets and in the BLS records. The static approach does not need this information solely because it ignores the future.

Much remains to be done in future work. Empirically, one could consider more disaggregated categories of goods and more involved utility functions that better fit the cross-sectional patterns of demand. Better measures of price expectations would also be useful, whether using direct surveys, or using statistical and economic models. Theoretically, this paper used the standard model of consumption over time under uncertainty in order to isolate the conceptual difference between dynamic and static inflation. Research on consumption has found that borrowing constraints, household production, non-convex adjustment costs, habits, temptations, and inattentiveness, can all substantially improve the empirical performance of the model. Incorporating these features in dynamic measures of inflation is left for future work.

Appendix

A.1. Solution of the simple model in section 2

Letting λ and λ' be the Lagrange multipliers on the two periods' budget constraints, the necessary and sufficient optimality conditions for $(c_a, c_b, c_a' c_b', e, \lambda, \lambda')$ are:

$$\alpha u(c_a, c_b)^{1-1/\gamma} = p_a c_a \lambda \tag{22}$$

$$(1-\alpha) u(c_a, c_b)^{1-1/\gamma} = p_b c_b \lambda \tag{23}$$

$$\alpha u(c_a', c_b')^{1-1/\gamma} = p_a' c_a' \lambda' \tag{24}$$

$$(1-\alpha) u(c_a', c_b')^{1-1/\gamma} = p_b' c_b' \lambda' \tag{25}$$

$$\beta E\left[\left(q'+1\right)\lambda'\right] = q\lambda, \tag{26}$$

together with (3) and (4) holding as equalities. You can guess and verify the solution:

$$p_a c_a = \alpha \theta(p_a, p_b, q) w, \tag{27}$$

$$p_b c_b = (1-\alpha)\theta(p_a, p_b, q) w, \tag{28}$$

$$p_a' c_a' = \alpha \left[1 - \theta(p_a, p_b, q)\right] \left(\frac{q'+1}{q}\right) w \tag{29}$$

$$p_b' c_b' = (1-\alpha)\left[1 - \theta(p_a, p_b, q)\right] \left(\frac{q'+1}{q}\right) w \tag{30}$$

where the function $\theta(p_a, p_b, q)$ is defined by:

$$\theta(p_a, p_b, q)^{-1} - 1 = \left\{\beta E\left\{\left[\frac{q}{q'+1} \times \left(\frac{p_a'}{p_a}\right)^{\alpha} \times \left(\frac{p_b'}{p_b}\right)^{1-\alpha}\right]^{(1-\gamma)/\gamma}\right\}\right\}^{\gamma}. \tag{31}$$

The log-normality of the prices can be used to further simplify the term on the right-hand side. Plugging into the utility function gives the value function:

$$V(w, p_a, p_b, q) = \left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right]^{1-1/\gamma} \left(\frac{w^{1-1/\gamma}}{1-1/\gamma}\right) \frac{\theta(p_a, p_b, q)^{-1/\gamma}}{\left(p_a^{\alpha} p_b^{1-\alpha}\right)^{1-1/\gamma}}. \tag{32}$$

For the approximation, note that in the non-stochastic steady-state, the right-hand side of (31) equals $\beta g^{1-1/\gamma}$. A log-linearization of (8) and (31) around this non-stochastic steady state then delivers:

$$\ln(\pi_t) = \ln(s_t) + \left(\frac{\beta g^{1-1/\gamma}}{1+\beta g^{1-1/\gamma}}\right)\left\{(1-\eta_q)\Delta\ln(q_t) + \alpha(\eta_a - 1)\Delta\ln(p_{at}) + (1-\alpha)(\eta_b - 1)\Delta\ln(p_{bt})\right\},$$

where I used the stochastic processes for each price. Rearranging gives the solution in (9).

A.2. Proof of Proposition 1: It is a standard result (e.g. Carroll, 2004) that this consumption problem has a solution with a continuous value function that increases with W_t. As π_t varies from 0 to infinity, the left-hand side of (15) therefore increases continuously from $-\infty$ to $+\infty$. Since for positive and finite wealth and prices, the right-hand side of (15) is a finite number, a unique solution to the equation exists.

A.3. Proof of Proposition 2: Consider the following transformation of the problem: $\{C_{j,t+i}, S_{j,t+i}, B_{j,t+i}, W_t\}_{i=0}^{\infty} \rightarrow \{\lambda C_{j,t+i}, \lambda S_{j,t+i}, \lambda B_{j,t+i}, \lambda W_t\}_{i=0}^{\infty}$ for all j for a non-zero scalar λ. The feasibility set of the maximization problem is unchanged, while the objective function goes through the transformation $\sum \beta^t U(C,S) \rightarrow \sum \beta^t \lambda U(C,S)$ if the utility function is linear homogenous. Therefore, $V(\lambda W_t, \mathbf{p}^t) = \lambda V(W_t, \mathbf{p}^t)$, so the value function is also linear homogenous. Letting $\lambda = \pi_t / W_{t-1}$ and $\lambda = 1/W_{t-1}$ on the two sides of (15), respectively, it follows that: $\ln(\pi_t) = \ln(V(1, \mathbf{p}^{t-1})) - \ln(V(1, \mathbf{p}^t))$, which does not depend on wealth. The extension to the homothetic case follows immediately.

A.4. Proof of Proposition 3: Consider the following transformation of the problem: $\{W_t, p^t, D_t\} \rightarrow \{MW_t, Mp^t, MD_t\}$ for a non-zero scalar M. The feasibility set of the maximization problem is unchanged and so is the objective function. Thus, the transformation leaves the value function unchanged. From the definition of the DPI, $\pi_t = M$.

A.5. Proof of Proposition 4: The dynamic program is:

$$V(W_t, \mathbf{p}^t) = \max_{C_{j,t}, S_{j,t}, b_t} \left\{ \sum_{j=1}^{N} \alpha_j \ln(C_{j,t}) + \sum_{j=1}^{D} \alpha_j \ln(S_{j,t}) + \beta \mathbb{E}_t \left[V\left(W_{t+1}, \mathbf{p}^{t+1}\right)\right] \right\}, \quad (33)$$

$$W_{t+1} = I_{t+1}^M \left(W_t - \sum_{j=1}^{N} P_{j,t} C_{j,t} - \sum_{j=1}^{D} u_{j,t+1} S_{j,t} \right), \quad (34)$$

$$I_{t+1}^M = b_t D_{E,t+1}/Q_{E,t} + (1-b_t) D_{B,t+1}/Q_{B,t}. \quad (35)$$

where b_t is the portfolio share in equities such that $b_t/(1-b_t) = Q_{E,t}B_{E,t}/Q_{B,t}B_{B,t}$, and $u_{j,t+1}$ is the user cost of durable j defined in (19). First, note that if asset returns are i.i.d.

then I^M_{t+1} is i.i.d. But then, since I^M_{t+1} only affect wealth next period, and asset prices only affect the problem via I^M_{t+1}, it follows that the value function is independent of asset prices.

Next, transform the variables $\tilde{C}_{j,t} = P_{j,t}C_{j,t}/W_t$, $\tilde{S}_{j,t} = R_{j,t}S_{j,t}/W_t$, $\tilde{u}_{j,t+1} = u_{j,t+1}/R_{j,t}$ and re-write the problem as

$$V(W_t, \ln(P_{j,t}), \ln(R_{j,t})) = \max_{C_{j,t}, S_{j,t}, b_t} \left\{ \begin{array}{c} \sum_{j=1}^N \alpha_j \ln(\tilde{C}_{j,t}) + \sum_{j=1}^D \alpha_j \ln(\tilde{S}_{j,t}) + \ln(W_t) \\ -\sum_{j=1}^N \alpha_j \ln(P_{j,t}) - \sum_{j=1}^D \alpha_j \ln(R_{j,t}) \\ +\beta \mathbb{E}_t \left[V\left(W_{t+1}, \ln(P_{j,t+1}), \ln(R_{j,t+1})\right)\right] \end{array} \right\} \tag{36}$$

$$W_{t+1} = I^M_{t+1} W_t \left(1 - \sum_{j=1}^N \tilde{C}_{j,t} - \sum_{j=1}^D \tilde{u}_{j,t+1} \tilde{S}_{j,t} \right), \tag{37}$$

The envelope theorem condition with respect to wealth is:

$$V_W(W_t, .) = \frac{1}{W_t} + \beta \mathbb{E}_t \left[\frac{V_W(W_{t+1}, .) W_{t+1}}{W_t} \right], \tag{38}$$

where $V_x \equiv \partial V(.)/\partial x$. This implies that $W_t V_W(W_t, .) = 1/(1-\beta)$, which after integrating implies that $V(W_t, .) = \ln(W_t)/(1-\beta) + T(\ln(P_{j,t}), \ln(R_{j,t}))$, where $T(.)$ is an unknown function. From the definition of the DPI, it then follows that:

$$\ln(\pi_t) = (1-\beta) \left[T(\ln(P_{j,t-1}), \ln(R_{j,t-1})) - T(\ln(P_{j,t}), \ln(R_{j,t})) \right]. \tag{39}$$

The optimality conditions are:

$$\frac{\alpha_j}{\tilde{C}_{j,t}} = \beta W_t \mathbb{E}_t \left[I^M_{t+1} V_W(W_{t+1}, .) \right] = \left(\frac{\beta}{1-\beta} \right) \mathbb{E}_t \left[\frac{1}{1 - \sum \tilde{C}_{i,t} - \sum \tilde{u}_{i,t+1} \tilde{S}_{i,t}} \right] \tag{40}$$

$$\frac{\alpha_j}{\tilde{S}_{j,t}} = \beta W_t \mathbb{E}_t \left[\tilde{u}_{j,t+1} I^M_{t+1} V_W(W_{t+1}, .) \right] = \left(\frac{\beta}{1-\beta} \right) \mathbb{E}_t \left[\frac{\tilde{u}_{j,t+1}}{1 - \sum \tilde{C}_{i,t} - \sum \tilde{u}_{i,t+1} \tilde{S}_{i,t}} \right] \tag{41}$$

where the equalities follow from the solution for the marginal utility of wealth, and using the budget constraint to rearrange.

Evaluating these optimality conditions at the non-stochastic steady-state, we get the steady-state solutions: $\beta I^M = 1$, $\bar{C}_j = \alpha_j(1-\beta)$, and $\bar{u}_j \bar{S}_j = \alpha_j(1-\beta)$. Log-linearizing the first-order conditions around this steady-state and letting hats denote the log-deviations:

$$\hat{C}_{j,t} + \left(\frac{1-\beta}{\beta}\right)\mathbb{E}_t\left[\sum \alpha_i \hat{C}_{i,t} + \sum \alpha_i \left(\hat{S}_{i,t} + \hat{u}_{i,t+1}\right)\right] = 0 \tag{42}$$

$$\hat{S}_{j,t} + \mathbb{E}_t\left(\hat{u}_{j,t+1}\right) + \left(\frac{1-\beta}{\beta}\right)\mathbb{E}_t\left[\sum \alpha_i \hat{C}_{i,t} + \sum \alpha_i \left(\hat{S}_{i,t} + \hat{u}_{i,t+1}\right)\right] = 0. \tag{43}$$

The solution to these equations is:

$$\hat{C}_{j,t} = 0 \text{ and } \hat{S}_{j,t} = -\mathbb{E}_t\left(\hat{u}_{j,t+1}\right), \tag{44}$$

which going back to the budget constraint implies that

$$\hat{W}_{t+1} = \hat{W}_t + \hat{I}^M_{t+1} - \left(\frac{1-\beta}{\beta}\right)\sum_{j=1}^{D}\alpha_j\left[\hat{u}_{j,t+1} - \mathbb{E}_t\left(\hat{u}_{j,t+1}\right)\right] \tag{45}$$

Returning to the Bellman equation in (36) and using the known solution for $V(.)$:

$$\begin{aligned}\frac{\ln(W_t)}{1-\beta} + T(\ln(P_{j,t}), \ln(R_{j,t})) &= \sum_{j=1}^{N}\alpha_j \ln(\tilde{C}_{j,t}) + \sum_{j=1}^{D}\alpha_j \ln(\tilde{S}_{j,t}) + \ln(W_t) \\ &\quad -\sum_{j=1}^{N}\alpha_j \ln(P_{j,t}) - \sum_{j=1}^{D}\alpha_j \ln(R_{j,t}) \\ &\quad +\beta\mathbb{E}_t\left[\frac{\ln(W_{t+1})}{1-\beta} + T(\ln(P_{j,t+1}), \ln(R_{j,+1t}))\right]\end{aligned} \tag{46}$$

Using the log-linearized solutions in (44) and (45), and cancelling terms:

$$\begin{aligned}T(\ln(P_{j,t}), \ln(R_{j,t})) &= -\sum_{j=1}^{D}\alpha_j\mathbb{E}_t\left(\hat{u}_{j,t+1}\right) - \sum_{j=1}^{N}\alpha_j \ln(P_{j,t}) - \sum_{j=1}^{D}\alpha_j \ln(R_{j,t}) \\ &\quad +\beta\mathbb{E}_t\left[\hat{I}^M_{t+1}/(1-\beta) + T(\ln(P_{j,t+1}), \ln(R_{j,+1t}))\right].\end{aligned} \tag{47}$$

But, with i.i.d. financial returns $\mathbb{E}_t\left(\hat{I}^M_{t+1}\right) = 0$ and with random-walk durables prices $\mathbb{E}_t\left(\hat{u}_{j,t+1}\right) = 0$. The solution to the difference equation above, using the fact that all prices follow a random-walk, then is:

$$T(\ln(P_{j,t}), \ln(R_{j,t})) = -\frac{\sum_{j=1}^{N}\alpha_j \ln(P_{j,t}) + \sum_{j=1}^{D}\alpha_j \ln(R_{j,t})}{1-\beta}. \tag{48}$$

Going back to the definition of the DPI in (39), therefore:

$$\ln(\pi_t) = \sum_{j=1}^{N} \alpha_j \Delta \ln(P_{j,t}) + \sum_{j=1}^{D} \alpha_j \Delta \ln(R_{j,t}), \tag{49}$$

which is, of course, just the static price index.

A.6. Proof of Proposition 5: The assumptions reduce the problem to:

$$V(A_t, ...) = \max_{\{C_{t+i}\}_{i=0}^{\infty}} \left\{ \sum_{i=0}^{\infty} \beta^i U(C_{1,t+i}, ..., C_{N,t+i}) : \sum_{j=1}^{N} P_{j,t+i} C_{j,t+i} \leq (1-\beta) A_t \text{ for } i = 0, 1, ... \right\} \tag{50}$$

With the assumptions on the utility function, it is clear that indirect utility increases with wealth, and the budget constraints always bind. Then, because all three scenarios involve a tightening of the budget set, they imply a lower value. This must be offset by higher wealth, which implies higher dynamic inflation to solve equation (15). With log-preferences, it is easy to derive:

$$\ln(\pi_t) = (1-\beta) \sum_{i=0}^{\infty} \beta^i \sum_{j=1}^{N} \alpha_j \ln(P_{j,t+i}/P_{j,t-1+i}), \tag{51}$$

confirming the general result.

A.7. Proof of Proposition 6: The consumer problem is:

$$\begin{aligned} V(W_t, \mathbf{p}^t) &= \max_{C_{j,t}, b_t} \left\{ U(C_{1,t}, ..., C_{N,t}) + \beta \mathbb{E}_t \left[V\left(W_{t+1}, \mathbf{p}^{t+1} \right) \right] \right\}, & (52) \\ W_{t+1} &= I_{t+1}^{M} \left(W_t - P_t \sum_{j=1}^{N} \hat{P}_{j,t} C_{j,t} \right), & (53) \end{aligned}$$

where I_{t+1}^{M} is defined in (35).

Starting with result a), since P_t is i.i.d., the value function includes assets and only the contemporaneous common component in prices: $V(A_t, P_t, .)$. Recall the definition of dynamic inflation, $V(\pi_t A_{t-1}, P_t, .) = V(A_{t-1}, P_{t-1}, .)$, and note that if $P_t = P_{t-1}$, then $\pi_t = 1$. A first-order Taylor approximation of the left hand-side around this point gives:

$$V(A_{t-1}, P_{t-1}) + V_W W_{t-1} \ln(\pi_t) + V_P P_{t-1} \Delta \ln(P_t) \approx V(A_{t-1}, P_{t-1}). \tag{54}$$

Re–arranging gives the result:

$$\ln(\pi_t) \approx -\left(\frac{PV_P}{WV_W}\right)\Delta\ln(P_t). \tag{55}$$

To compute the needed derivatives, use the envelope theorem conditions:

$$\begin{aligned} V_W(W_t,.) &= \beta\mathbb{E}_t\left[I^M_{t+1}V_W(W_{t+1},.)\right], & (56)\\ V_P(W_t,.) &= -\beta\left(\sum\hat{P}_{j,t}C_{j,t}\right)\mathbb{E}_t\left[I^M_{t+1}V_W(W_{t+1},.)\right]. & (57)\end{aligned}$$

Replacing this into the approximate expression for dynamic inflation in (55) gives:

$$\ln(\pi_t) \approx \left(\frac{\sum_{j=1}^N P_jC_j}{W}\right)\Delta\ln(P_t). \tag{58}$$

Finally, note that from the budget constraint:

$$\frac{W_{t+1}}{W_t} = I^M_{t+1}\left(1 - \frac{\sum_{j=1}^N P_{j,t}C_{j,t}}{W_t}\right). \tag{59}$$

At a steady-state, $W_{t+1} = W_t$ and $\beta I^M = 1$, which implies that $\sum_{j=1}^N P_jC_j/W = 1-\beta$. This gives result a). Result c) follows by almost identical steps.

For result b), note that the relevant state variables in the value function now are: $V(A_t, \Delta\ln(P_t), .)$. A similar first-order approximation then shows that:

$$\ln(\pi_t) \approx -\left(\frac{V_{\Delta\ln(P)}}{WV_W}\right)\Delta\ln(P_t). \tag{60}$$

The envelope theorem condition for $\Delta\ln(P_t)$ now is:

$$\begin{aligned} V_{\Delta\ln(P)}(W_t,.) &= -\beta\left(\sum P_{j,t}C_{j,t}\right)\mathbb{E}_t\left[I^M_{t+1}V_W(W_{t+1},.)\right]\\ &\quad -\beta\mathbb{E}_t\left[\eta V_{\Delta\ln(P)}(W_{t+1},.)\right]. & (61)\end{aligned}$$

In steady-state, this implies that

$$V_{\Delta\ln(P)} = -(1-\beta)WV_W/(1-\eta\beta), \tag{62}$$

which once plugged in the approximation gives the result.

A.8. Proof of Proposition 7: The dynamic problem is:

$$V(W_t, \mathbf{p}^t) = \max_{C_{j,t}, b_t} \left\{ U(C_{1,t}, ..., C_{N,t}) + \beta \mathbb{E}_t \left[V\left(W_{t+1}, \mathbf{p}^{t+1}\right)\right]\right\}, \tag{63}$$

$$W_{t+1} = I_{t+1}^M \left(W_t - \sum_{j=1}^{N} P_{j,t} C_{j,t} \right), \tag{64}$$

$$I_{t+1}^M = \frac{b_t D_{E,t+1}/\hat{Q}_{E,t} + (1-b_t) D_{B,t+1}/\hat{Q}_{B,t}}{Q_t}. \tag{65}$$

Starting with result a), similar steps to those in proposition 6 show that an approximation of dynamic inflation is:

$$\ln(\pi_t) \approx -\left(\frac{QV_Q}{WV_W}\right) \Delta \ln(Q_t). \tag{66}$$

The envelope theorem conditions with respect to wealth and asset prices are:

$$V_W(W_t, .) = \beta \mathbb{E}_t \left[I_{t+1}^M V_W(W_{t+1}, .)\right], \tag{67}$$

$$V_Q(W_t, .) = -(\beta/Q_t)\left(W_t - \sum P_{j,t} C_{j,t}\right) \mathbb{E}_t \left[I_{t+1}^M V_W(W_{t+1}, .)\right]. \tag{68}$$

Using these in the approximation gives:

$$\ln(\pi_t) \approx \left(1 - \sum P_j C_j / W_t\right) \Delta \ln(Q_t). \tag{69}$$

As shown in the proof of proposition 6, at the steady state $\sum_{j=1}^{N} P_j C_j / W_t = 1 - \beta$, from where result a) follows.

Turning to result b), note that in the statement of the problem above, $Q_{E,t}$ only affects the problem via the return on equity and I_{t+1}^M. If the return on equity is i.i.d, then the forecast of I_{t+1}^M does not depend on $Q_{E,t}$. Thus, the value function $V(.)$ does not depend on $Q_{E,t}$ and so neither does dynamic inflation.

Now to result c). Identical steps to those used to prove result a) show that by the implicit function theorem:

$$\frac{\partial \ln(\pi_t) / \ln(\hat{Q}_{E,t})}{\partial \ln(\pi_t) / \ln(\hat{Q}_{B,t})} = \frac{\hat{Q}_E V_{Q_E}}{\hat{Q}_B V_{Q_B}}. \tag{70}$$

The envelope theorem conditions with respect to the two separate asset prices are:

$$V_{Q_E}(W_t, .) = -\frac{b_t\beta}{Q_{E,t}}\left(W_t - \sum_{j=1}^{N} P_{j,t}C_{j,t}\right)\mathbb{E}_t\left[\left(\frac{D_{E,t+1}}{Q_{E,t}}\right) \times V_W(W_{t+1}, .)\right], \tag{71}$$

$$V_{Q_B}(W_t, .) = -\frac{(1-b_t)\beta}{Q_{B,t}}\left(W_t - \sum_{j=1}^{N} P_{j,t}C_{j,t}\right)\mathbb{E}_t\left[\left(\frac{D_{B,t+1}}{Q_{B,t}}\right) \times V_W(W_{t+1}, .)\right]. \tag{72}$$

In turn, the optimality condition with respect to b_t is:

$$\left(\frac{\beta}{Q_t}\right) E_t\left[V_W(W_{t+1}, .) \times \left(W_t - \sum P_{j,t}C_{j,t}\right) \times \left(\frac{D_{E,t+1}}{\hat{Q}_{E,t}} - \frac{D_{B,t+1}}{\hat{Q}_{B,t}}\right)\right] = 0 \tag{73}$$

Using this to combine the previous two envelope theorem conditions delivers:

$$\frac{\hat{Q}_E V_{Q_E}}{\hat{Q}_B V_{Q_B}} = \frac{b}{1-b}, \tag{74}$$

Using the definition of b, result c) follows.

A.9. Proof of Proposition 8: The dynamic program is

$$V(W_t, \mathbf{p}^t) = \max_{C_{j,t}}\left\{\sum_{j=1}^{N}\alpha_j \ln(C_{j,t}) + \beta\mathbb{E}_t\left[V\left(W_{t+1}, .\right)\right]\right\}, \tag{75}$$

$$W_{t+1} = I_{t+1}^M\left(W_t - \sum P_{j,t}C_{j,t}\right). \tag{76}$$

The Euler equation for the consumption of each good j is:

$$1 = \beta\mathbb{E}_t\left[I_{t+1}^M\left(\frac{P_{j,t}C_{j,t}}{P_{j,t+1}C_{j,t+1}}\right)\right], \tag{77}$$

It is easy to see that the solution $P_{j,t}C_{j,t} = \alpha_j(1-\beta)W_t$ satisfies the Euler equations as well as the budget constraint, implying a law of motion for wealth: $W_{t+1} = \beta I_{t+1}^M W_t$.

The value function at date t equals the expected sum of discounted utility obtained by behaving optimally. Using the optimal consumption choices and the evolution of wealth and summing over time, gives the value function:

$$V(.) = const. + \frac{\ln(W_t)}{1-\beta} - \sum_{i=0}^{\infty}\beta^i\sum_{j=1}^{N}\alpha_j\mathbb{E}_t\left[\ln(P_{j,t+i})\right] + \sum_{i=0}^{\infty}\beta^i\mathbb{E}_t\left[\sum_{j=0}^{i-1}\ln\left(I_{t+j}^M\right)\right]$$

Evaluating expectations and using the definition of the DPI gives the result.

A.10. Proof of Proposition 9: The dynamic program now is:

$$V(W_t, \ln(P_{i,t}), \ln(R_{j,t})) = \max_{C_{j,t}, S_{j,t}} \{U(C_{1,t}, ..., S_{D,t}) + \beta \mathbb{E}_t \left[V\left(W_{t+1}, \ln(P_{i,t+1}), \ln(R_{j,t+1})\right)\right]\}, \tag{78}$$

$$W_{t+1} = I^M_{t+1} \left(W_t - \sum_{j=1}^{N} P_{j,t} C_{j,t} - \sum_{j=1}^{D} u_{j,t+1} S_{j,t} \right), \tag{79}$$

where I^M_{t+1} is defined in (35), and user costs are defined in equation (19). Using the definition of the DPI and the implicit function theorem:

$$\partial \ln(\pi_t)/\partial \ln(P_{i,t}) = -V_{\ln(P_i)}/V_W W, \tag{80}$$

$$\partial \ln(\pi_t)/\partial \ln(R_{j,t}) = -V_{\ln(R_j)}/V_W W, \tag{81}$$

so the goal is to find $V_{\ln(R_j)}/V_{\ln(P_i)}$. The envelope theorem for durable j and non-durable i implies:

$$V_{\ln(R_j)}(W_t, .) = -\beta S_{j,t} \mathbb{E}_t \left[V_W(W_{t+1}, .) I^M_{t+1} u_{j,t+1} \left(\frac{R_{j,t} \partial u_{j,t+1}}{u_{j,t+1} \partial R_{j,t}} \right) \right] + \beta \eta_j \mathbb{E}_t[V_{\ln(R_j)}(W_{t+1}, .)] \tag{82}$$

$$V_{\ln(P_i)}(W_t, .) = -\beta P_{i,t} C_{i,t} \mathbb{E}_t[V_W(W_{t+1}, .) I^M_{t+1}] + \beta \eta_i \mathbb{E}_t[V_{\ln(P_i)}(W_{t+1}, .)]. \tag{83}$$

The first order conditions for these two goods in turn are:

$$\frac{\partial U(.)}{\partial S_j} = \beta \mathbb{E}_t \left[V_W(W_{t+1}, .) I^M_{t+1} u_{j,t+1} \right] \tag{84}$$

$$\frac{\partial U(.)}{\partial C_i} = \beta P_{i,t} \mathbb{E}_t \left[V_W(W_{t+1}, .) I^M_{t+1} \right] \tag{85}$$

Using the approximations: $\mathbb{E}_t[V_{\ln(R_j)}] \approx V_{\ln(R_j)}$ and $\mathbb{E}_t \left[V_W I^M_{t+1} u_{j,t+1} \left(\frac{R_{j,t} \partial u_{j,t+1}}{u_{j,t+1} \partial R_{j,t}} \right) \right] \approx \mathbb{E}_t \left[V_W I^M_{t+1} u_{j,t+1} \right] \mathbb{E}_t \left(\frac{R_{j,t} \partial u_{j,t+1}}{u_{j,t+1} \partial R_{j,t}} \right)$, the result in the proposition follows. The second result follows from the definition of user costs.

A.11. Empirical model of the DPI: The consumer problem is just like in (10)-(13), with preferences given by (16), with $N = 4$ and $D = 2$, and with a change to (11) to include adjustment costs:

$$\sum_{j=1}^{4} P_{j,t+i}C_{j,t+i} + \sum_{j=1}^{2} R_{j,t+i}S_{j,t+i} + \sum_{j\in\{B,E\}} Q_{j,t+i}B_{j,t+i}$$
$$\leq \quad W_{t+i} + \sum_{j=1}^{2} \frac{\phi_j}{2}\left(\frac{S_{j,t+i}R_{j,t+i}/W_t}{S_{j,t-1+i}R_{j,t-1+i}/W_{t-1}} - 1\right)^2 \left(\frac{W_{t+i}}{W_{t-1+i}}\right) R_{j,t-1+i}S_{j,t-1+i}$$

Let z_t be the 8x1 vector with all first-differences of prices and returns: $z_t = (\Delta \ln(P_{1,t})$, ..., $\Delta \ln(P_{4,t})$, $\Delta \ln(R_{1,t})$, $\Delta \ln(R_{2,t})$, $\ln(D_{B,t}/Q_{B,t-1})$, $\ln(\varepsilon_t D_{E,t}/Q_{E,t-1}))$. I assume that:

$$z_t = \Phi z_{t-1} + v_t,$$

where v_t follows a multivariate normal distribution with zero mean, $\mathbb{E}_t[v_t v_t'] = \Psi$, and $\mathbb{E}_t[v_t v_{t-k}'] = \mathbf{0}$ for $k \neq 0$. As for the rare event ε_t, each period with probability ς it equals 0 and otherwise equals 1.

The calibrated parameters are (β, $\boldsymbol{\alpha}$, δ_1, δ_2, ϕ_1, ϕ_2, ς), following the details in the text, while Φ and Ψ are estimated by fitting a VAR to the data. To deal with the large number of state variables, I use perturbation methods to solve the dynamic program.

A.12. DPI construction for a university: I introduce two new non-durable goods, faculty and other (administrative, clerical and service) employees, so $N = 6$ now. The salaries of faculty are measured using the data since 1975 from the American Association of University Professors; those of other employees using compensation per hour in the nonfarm business sector from the Bureau of Labor Statistics.

Turning to the preference weights, $\boldsymbol{\alpha}$, I use data from the Commonfund institute for their Higher Education Price Institute on the non-durable expenditure of universities, to obtain the relative weights of faculty salaries (30%), other salaries (30%), and non-durable goods (10%), apportioning the non-durables goods to the four categories using the same proportion as in the CPI. The remaining 30% are expenditure on durables, which I again apportion to housing (20%) and others (10%) using the same weights as in the CPI. I was unable to find a systematic analysis of the relative share of spending on durables (or capital projects) by universities. The 30% number used for durables comes form a particular university (Yale) that makes their budget publicly available.

Finally, for the portfolio shares, I use data from NACUBO from 1989 to 2001, aggregating into equity investments in five categories, U.S. equity, non U.S. equity, hedge funds, private equity and venture capital, and aggregating three categories into bonds, U.S. bonds, non-U.S. bonds, and cash. Equity accounts for 59% of the portfolio and bonds for 37%, with the residual 2% invested in natural resources and real estate.

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