Algorithm for the transfers model with fixed capital

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1 Final goods firm

There is a continuum of atomic firms with the following identical production function:

$$Y_t = A_t \overline{K}^{\alpha} X_t^{1-\alpha} \quad \text{and the aggregator} \quad X_t = \left(\int x_t(j)^{1/\mu} dj \right)^{\mu}$$

The firm chooses $\{x_t(j)\}_{j=0}^{1}$ taking as given $\{p_t(j)\}_{j=0}^{1}$. Hence the objective is:

$$\max_{\{x_t(j)\}_{j=0}^{1}} A_t \overline{K}^{\alpha} X_t^{1-\alpha} - \int p_t(j) x_t(j) dj$$

Cost minimization yields:

$$p_t = \left(\int p_t(j)^{1/(1-\mu)} dj \right)^{1-\mu}, \qquad p_t X_t = \int p_t(j) x_t(j) dj, \qquad x_t(j) = X_t \left(\frac{p_t(j)}{p_t} \right)^{-\mu/(\mu-1)}$$

and profit maximization yields:

$$p_t = (1-\alpha) A_t \left(\frac{\overline{K}}{X_t} \right)^{\alpha}$$

Therefore, any atomic firm obtains the following real profit, denoted by R_t^k:

$$R_t^k = Y_t - p_t X_t = \alpha Y_t$$

Since all firms are identical, we abstract from index notations of each final good firm. The total profit from the final goods firms are hence R_t^k as well.

The ownership of final goods firms can be assumed in different manners. We assume that each household can trade the shares.

1.1 Lucas tree ownership of final goods profit

There is a continuum of households, indexed by i. The first case to consider is when households trade shares of the total real profit. Denote the share of each household as $s^k_{i,t}$ and the price of the share as p^k_t. Shares must add up to 1:

$$\int_0^1 s^k_{i,t} di = 1$$

In each period, households trade the amount of shares at a given market price p^k_t. For the previously held shares, they obtain the profit R^k_t as dividends, as well as the current market price p^k_t. This implies that the gross real return rate of holding shares from period $t-1$ to t is $\frac{p^k_t + R^k_t}{p^k_{t-1}}$. For consistency of notation, write $k_{i,t+1} = p^k_t s^k_{i,t+1}$, the total expenditure of buying shares in consumption units. The budget constraint (without social transfers) can be written as follows:

$$c_{i,t} + k_{i,t+1} = \left(\frac{p^k_t + R^k_t}{p^k_{t-1}} \right) k_{i,t} + s_{i,t} w_t n_{i,t} + d_t$$

As far as we are concerned about a nonstochastic steady state ($p^k_t = p^k \quad \forall t$), the budget constraint can be written as follows:

$$c + k' = \left(1 + \frac{R^k}{p^k} \right) k + swn + d$$

Again for notational consistency, write $r = \frac{R^k}{p^k}$. The algorithm of obtaining a steady state is as follows:

1. Guess X_0 and r_0.

2. Obtain w_0, L_0, and d_0 (intermediate good firms).

$$w_0 = \frac{(1-\alpha)A_0}{\mu} \left(\frac{\overline{K}}{X_0} \right)^{\alpha}, \qquad L_0 = X_0, \qquad d_0 = (\mu - 1) w_0 L_0$$

3. Obtain R^k_0 (final good firms) and p^k_0.

$$R^k_0 = \alpha Y_0 = \alpha A_0 \overline{K}^{\alpha} X_0^{1-\alpha}, \qquad p^k_0 = \frac{R^k_0}{r_0}$$

4. Solve the household problem.

$$V(k,s,h) = \max_{c,k',n}\{u(c) + \beta E V(k',s',h')\}$$
$$c + k' = (1+r_0)k + s w_0 n + d_0$$
$$k' \geq 0, \quad c \geq 0, \quad n \in \{0,1\}$$

5. Solve the decision problem of the households to obtain $k'^*(k,s,h)$ and $n^*(k,s,h)$.

6. Use this to obtain the stationary distribution $F(k,s,h)$.

7. Update new guesses: X_1 and r_1

$$X_1 = \left(\int s^{1/\mu} n^*(k,s,h) dF(k,s,h)\right)^{\mu}$$
$$Y_1 = A\overline{K}^{\alpha} X_1^{1-\alpha}, \qquad R_1^k = \alpha Y_1, \qquad p_1^k = \int k'^*(k,s,h) dF(k,s,h)$$
$$r_1 = \frac{R_1^k}{p_1^k}$$

For the transition dynamics, guess $\{X_t\}_{t=1}^T$ and $\{r_t\}_{t=1}^T$. Also guess $\{p_t^k\}_{t=0}^{T-1}$. Starting from $t = T$ to $t = 1$, obtain w_t (p_t), L_t, and d_t as usual. Conduct value function iteration to get $c_t^*(k,s,h)$, $n_t^*(k,s,h)$, and $k_t'^*(k,s,h)$. From this, obtain the distribution $\{F_t(k,s,h)\}_{t=1}^T$. Starting from $t = 1$, for each $F_t(k,s,h)$, p_{t-1}^k, X_t, Y_t, R_t^k and r_{t-1} are obtained by the following formula:

$$X_t = \left(\int s^{1/\mu} n_t^*(k,s,h) dF_t(k,s,h)\right)^{\mu}$$
$$Y_t = A_t\overline{K}^{\alpha} X_t^{1-\alpha}, \quad R_t^k = \alpha Y_t, \quad p_{t-1}^k = \int k dF_t(k,s,h)$$
$$r_{t-1} = \frac{p_{t-1}^k - p_{t-2}^k}{p_{t-2}^k} + \frac{R_{t-1}^k}{p_{t-2}^k}$$

Update X_t and r_t and repeat until convergence.