Maintaining Central-Bank Financial Stability under New-Style Central Banking[*]

Robert E. Hall
Hoover Institution and Department of Economics, Stanford, and NBER
rehall@stanford.edu; website: stanford.edu/∼rehall
Ricardo Reis
Department of Economics, Columbia, NBER, and CEPR
rreis@columbia.edu; website: columbia.edu/∼rr2572

July 16, 2015

Abstract

Since 2008, the central banks of advanced countries have borrowed trillions of dollars from their commercial banks in the form of interest-paying reserves and invested the proceeds in portfolios of risky assets. We investigate how this new style of central banking affects central banks' solvency. A central bank is insolvent if its requirement to pay dividends to its government exceeds its income by enough to cause an unending upward drift in its debts to commercial banks. We consider three sources of risk to central banks: interest-rate risk (the Federal Reserve), default risk (the European Central Bank), and exchange-rate risk (central banks of small open economies). We find that a central bank that pays dividends equal to a standard concept of net income will always be solvent—its reserve obligations will not explode. In some circumstances, the dividend will be negative, meaning that the government is making a payment to the bank. If the charter does not provide for payments in that direction, then reserves will tend to grow more in crises than they shrink in normal times. To prevent this buildup, the charter needs to provide for makeup reductions in payments from the bank to the government. We compute measures of the financial strength of central banks, and discuss how different institutions interact with quantitative easing policies to put these banks in less or more danger of instability. We conclude that the risks to financial stability are real in theory, but remote in practice today.

---

[*] JEL E42 E58. Hall's research was supported by the Hoover Institution and Reis's by a grant from the Institute for New Economic Thinking. This research is part of the National Bureau of Economic Research's Economic Fluctuations and Growth and Monetary Economics Programs. We are grateful to David Archer, Marco Bassetto, Marco Del Negro, Jeff Huther, John Leahy, Dirk Niepelt, Monika Piazzesi, Chris Sims, Carl Walsh, and Michael Woodford for helpful comments. This is a significantly revised version of a paper with a similar title that circulated in February of 2013. All data and programs for this paper are available from Reis's website.

The Bank of Japan in the 1990s, followed in the 2000s by the Federal Reserve, the European Central Bank, the Bank of England, and the Swiss National Bank, made dramatic changes in the conduct of monetary policy in response to financial crises. They borrowed trillions of dollars from commercial banks by expanding reserves and paying positive or receiving negative interest on them, while using the proceeds to enlarge their holdings of government bonds and other risky investments. Critics of the new style of central-bank policy worry that, either once market interest rates begin to rise as the crisis recedes, or if the bonds in the central bank's portfolio default, or if the exchange rate rises suddenly, these central banks will come under stress. Might the double burden of capital losses on central banks' portfolios and the rising cost of paying market interest on trillions of dollars of reserves threaten the banks' financial stability?

In Appendix A, we survey the literature on central bank financial stability. Studies have so far fallen into two strands. The first investigates how central-bank capital may affect monetary policy and inflation. This research takes as given that the central bank remains solvent, and explores the effect of different policies on inflation, under that constraint. The second strand characterizes circumstances when the central bank capital can be negative, focussing on the peculiar accounting rules of central banks and on the items in their balance sheet. Because of this focus, that literature's lessons apply only to the central bank under study, usually the Federal Reserve, but do not extend to other institutional setups.

Our approach is different. We take as given the commitment to a target for inflation, and explore whether the central bank is financially stable. We provide an economic analysis of how the central bank's charter determines whether the bank faces the danger of exploding reserves. Our theory is simple but flexible enough to accommodate most central banks in the world. Our formal model of central bank risk, founded in modern finance theory, describes the evolution of a central bank's financial position, and shows when can it become unsustainable. We spell out why a central bank's reserves may be on an exploding path and which institutions and policies protect against exploding reserves and guarantee financial stability.

We start by laying out the resource constraint faced by the central bank, focusing on the path of reserves. Central banks are unique among financial institutions in their power to compel banks to make loans to the central bank in the form of reserves. We introduce three forms of risk to the assets of the central bank: interest-rate risk that comes with the central bank holding long-term bonds that fall in value when interest rates rise; default risk that comes with holding bonds that may become impaired in some states of the world; and exchange-rate risk that arises when central banks hold foreign currency and gold in their assets.

Section 2 spells out the meaning of central-bank financial stability. A central bank is an independent arm of government whose relation to the government is contractual. In particular, the bank has a dividend rule in its charter governing the cash flows from the bank to the government and—potentially—the cash flows from the government to the bank. There is a bias toward cash flowing from the bank to the government because the charter generally specifies that the bank can issue growing volumes of currency and thus earn seignorage. A central bank can always meet its obligation to deliver the mandated dividend to the government because it has the unlimited power to borrow from commercial banks by issuing reserves. But a central bank cannot continue as a functioning financial institution independent of the government if it appears to be on a path of issuing an exploding volume of reserves. In that case, a central bank would be engaging in a Ponzi scheme. A principal objective of the paper is to derive conditions under which a central bank is financially stable, in the precise sense that its volume of reserves is a stationary function of the state of the economy. To put it differently, we ask what dividend rules prevent a central-bank Ponzi scheme.

We apply the theory in section 3 to appraise the current financial strength of the Federal Reserve. We show that the new obligation to pay rising interest rates as the economy returns to normal barely has an effect on outstanding reserves and does not threaten an explosion of reserves. However, quantitative easing in response to a financial crisis, by expanding the Fed's portfolio of government bonds and significantly lengthening its maturity, exposes the Fed to capital losses during the recovery from the crisis, when interest rates rise. We find that in almost all conditions the Fed would be able to withstand this interest-rate risk and would resume paying a substantial positive dividend to the Treasury after some years of zero dividend. At that point, reserves would be back to their normal level, so there is no threat that the crisis would trigger a permanent increase in reserves. The answer to the question—is the Fed at risk for losing the ability to stabilize prices because it becomes financially unstable?—is an unambiguous no, given the historical volatility of interest rates and bond prices.

We discuss how different policies for managing the exit from quantitative easing affect the stability of the Fed in the face of interest-rate risk. We show that if the only concern was to minimize the risk of an explosion of reserves, the Fed would unwind the QE bulge in its bond portfolio before raising interest rates. Our theory also allows us to study the institutional practice of not marking losses to market. We show that this does not affect the present value of losses, but only their time path. Marking to market leads to large reported losses at the time of exit from the crisis, followed by reported gains. The accounting conventions of the Fed smooth reported income, reducing the negative reported income when the economy returns to normal after a crisis, but lowering reported income for several years later.

Section 5 focuses on default risk by applying the model to the ECB's finances. One result is that if default risk is priced into the bonds that the central bank buys, then it has a small effect on financial stability. Otherwise, the ECB could suffer significant losses. This section also studies the role that repurchase agreements, which are short-term debt obligations of banks collateralized by bonds and other securities, have on the solvency of the central bank. The ECB has traditionally held a substantial fraction of its assets as repos. We find that, as long as banks meet their obligations to repay the loans even when the collateral value declines, the ECB's repo position significantly limits the risk of instability because the ECB suffers capital losses only if the banks fail to repay the debt *and* the collateral loses value by more than the repo haircut. Only if there is a comprehensive financial meltdown such that repos suffer default would the ECB's stability be compromised.

In 2015, the ECB announced it would significantly increase the size of its balance sheet, and move towards direct holdings of bonds and away from repos. We conclude that this policy increases the potential losses the ECB might incur and so increases the risk to the ECB's stability.

Section 6 turns to exchange-rate risk. We show that foreign reserves lead to volatile net income for a central bank often leading to losses. We conjecture that this is the reason why central banks in many developing countries have considerable power to retain earnings and run large deferred accounts. These institutions allow these central banks to deal with their heightened risk of negative income.

Exchange rate pegs are a practice that can lead to central bank reserve explosion. We consider both the case of a central bank trying to defend a peg, as well as the more recent experience of the Swiss National Bank. We show how the decision of the SNB to abandon its peg can be justified as a response to a serious risk to its stability. Finally, we find that a central bank that undertakes a successful inflation stabilization program will often see its stability compromised, and illustrate this finding with the example of emerging markets.

New-style central banking, with large balance sheets and payment of interest on reserves, makes the topic of central bank stability relevant and important. We conclude that central banks with inappropriate dividend rules may face the risk of reserve explosion, and that this may happen under a variety of scenarios. But we also conclude that the risks of this happening to the Fed and the ECB are remote and that losses can be managed by a temporary buildup of reserves that is reversed well before the next major adverse shock is likely to occur. We show how several institutions, including central bank deferred accounts, mark-to-market accounting, repurchase agreements, and exchange rate pegs can have a significant effect on the stability of the central bank.

1 The Resource Constraint of the Central Bank

A central bank issues reserves, which are one-period debt claims on the central bank held by commercial banks. We denote their real value by V in units of output. Reserves are the economy's unit of value. One unit of output costs p units of reserves; p is the price level. The real value of a unit of reserves is $1/p$. The central bank has the power to issue reserves to commercial banks and to use the proceeds to buy securities, usually government debt. Modern central banks pay interest, positive or negative, on reserves.

1.1 The financial environment

We use the scalar s to denote the state of the economy, which obeys a Markov process in the set $\{1, \cdots, S\}$, and has a transition probability $\omega_{s,s'}$, where a prime ($'$) denotes next period's value. We use the terminology in the rest of the paper that a variable is *state-dependent* if it is a function of only s and possibly s'. We let $m_{s,s'}$ denote the stochastic discount factor for a real payoff. Then the real present value of a random real payoff one period in the future, $y_{s'}$, is

$$\mathbb{V}(y_{s'}) = \sum_{s'} \omega_{s,s'} m_{s,s'} y_{s'}. \tag{1}$$

We assume that policy sets the state-dependent nominal interest rate on reserves to i_s. Let the rate of inflation be $\pi_s = p'/p - 1$, where p is the price level, noting that we have assumed that inflation is pre-determined one period ahead, a useful simplification that simplifies the algebra but plays no other role in our analysis. Then, the real value of one unit of reserves as of the current period is

$$\mathbb{V}\left(\frac{1+i_s}{p'}\right)=\frac{1+i_s}{p'(1+r_s)}, \tag{2}$$

where r_s is the safe one-period real interest rate. Because the real value of \$1 of reserves this period must be $1/p$, this equation leads to the Fisher equation that pins down the rate of inflation as

$$\pi_s=\frac{1+i_s}{1+r_s}-1. \tag{3}$$

If the central bank wants to achieve some rate of inflation, it must choose a nominal interest rate consistent with that aim and the equation above. This derivation rests on the assumption of saturation in reserves, so that reserves are valued solely for their cash payoff and not for any service yield.

In an economy with monetary non-neutrality, the application of this relationship to determine the appropriate monetary policy is a challenge, because there is feedback from the nominal interest rate to the real interest rate. In this paper, we consider an economy with state-dependent rates of inflation π_s, real interest rates r_s, and nominal rates i_s, satisfying the Fisher relationship. We need not, and do not, enter the thicket of modern monetary economics to find the optimal policy or to determine the strength of the feedback from the nominal interest rate to the real interest rate. We do make the assumption that monetary policy, as embedded in i_s, is invariant to the choice of dividend rule in the central bank's charter.

An alternative central bank policy would be a state-dependent price-level policy, so that we could write the price level as a function of the state of the economy, p_s. We do not pursue this approach because a state-dependent price level implies essentially zero inflation—every time the economy returns to a given state, the change in the price level since the first time it was in that state is exactly zero. No central bank has an inflation target of zero. The announced targets of central banks in advanced countries cluster around two percent per year. It is therefore more realistic to assume a state-dependent rate of inflation with all of the π_ss non-negative and presumably near two percent.

Finally, no theory of inflation is complete without a statement of how fiscal policy interacts with monetary policy. To focus on central-bank stability, we must take a stand on the solvency of the overall government. We portray the central bank as an arm of a government that is committed to a fiscal policy that satisfies the intertemporal budget constraint by adjusting taxes and spending under all possible realizations of random events, without relying on inflationary finance from the central bank. See Hall (2014) for evidence that the U.S. satisfied that hypothesis prior to 2000 but appears to have departed seriously as a result of the financial crisis of 2008.

1.2 The central bank's assets and their risks

Central banks typically hold debt instruments of different maturities as well as gold and foreign currencies. Before the financial crisis in 2008, the Federal Reserve held almost entirely Treasury bills and bonds, while the ECB held a significant amount of gold together with repo claims on commercial banks, which were collateralized by short-term government debt. The Swiss National Bank, as is the case with most small open economies, held a significant amount of foreign currency reserves. With the crisis, all three central banks borrowed extensively from commercial banks to increase their holdings of debt instruments to include longer maturities, higher default risks, and more foreign currency.

We lump all central-bank assets into a single category called bonds, and denote by $B_s \geq 0$ the total number of bonds held by the central bank. In the model, a bond is an instrument that pays a real coupon $\theta c_s + (1-\theta)e_{s'}$ next period, where $e_{s'}$ is the relative price change of the non-domestic assets held by the central bank. Because these assets in most central banks consist largely of foreign currency, $e_{s'}$ is approximately the rate of depreciation of the domestic real exchange rate and θ is the fraction of domestic assets in the central bank's portfolio. In the second period, the bond will pay a fraction $(1-\delta)$ of this payoff, and then $(1-\delta)^2$ in its third period, and so on. Lower values of δ correspond to portfolios with longer maturities. We add up bonds in terms of the amount of output they will pay in the current period so, each period, the bonds inherited from the previous period shrink by the factor $1-\delta$, and $1/\delta$ is the average maturity of the portfolio holdings. The bonds, which sell for price q_s, then give a payoff equal to the difference between the coupon payments and the repayment of the principal by a factor δ. Appendix B shows how to obtain this composite bond from mixing investments in short-run bonds, long-run bonds, gold and foreign currency, as we observe in central banks' balance sheets.

The asset holdings of the central bank pose three distinct risks, captured by our bond. These risks are especially acute during financial crises. First is the risk that the bonds will not pay the full coupon. We assume that the coupon on the bonds is one in all states apart from the financial crisis state S, when bonds may suffer an impairment captured by $c_S \leq 1$. This possibility is pressing in Europe, where the ECB holds a significant amount of Greek debt with market values that imply substantial default probabilities.

The second risk comes from changes in bond prices resulting from changes in discounts. The price of the bonds q_s satisfies the no-arbitrage asset-pricing recursion:

$$q_s = \mathbb{V}(\theta c_s + (1-\theta)e_{s'} + (1-\delta)q_{s'}). \tag{4}$$

Domestic short-term bonds are free from this risk, but if $\delta < 1$, changes in discounts and thus in real interest rates will cause differences in q_s across states, especially as the economy enters and exits the crisis state. This is a particularly pressing issue for the United States today, as interest rates are expected to rise soon. Because we want to remain in the simple environment of a recursive model, we cannot deal with term-structure changes except within the geometric model of bond payoffs.

The third risk comes from changes in the price of gold and foreign assets and is captured by the exchange-rate change $e_{s'}$. In the central bank of most small open economies, where θ is close to zero, this is the main source of risk.

The three parameters, δ, c_S, and θ, describe the sources of risk: interest-rate risk, default risk and exchange rate. If all three parameters are equal to one, the bond is riskless, and

$$q_s = \frac{c_s}{1+r_s}. \tag{5}$$

1.3 Seignorage and inflation

The only source of income to a central bank besides its portfolio of assets is seignorage income from printing currency. All central banks keep the market values of reserves and currency at exact parity by standing ready to exchange one for the other. Therefore, the policy variable is high-powered money, the sum of reserves and currency. Market forces determine the split between the two components, and because seignorage is only earned from currency, but not from interest-paying reserves, the central bank does not directly control the seignorage it earns. Rather, central banks expand their portfolios to high levels almost entirely by borrowing from commercial banks at market interest rates, and some of these new reserves may be converted to currency.

Because the volume of currency is stable in advanced countries, we take the public's real holdings of currency to be a function N_s of the state s. The central bank's real seignorage revenue is:

$$n_{s,s'} = \frac{p' N_{s'} - p N_s}{p'} = N_{s'} - \frac{N_s}{\pi_s}. \tag{6}$$

Although a central bank does not control seignorage directly, by using its policy rule to keep prices on target, it affects the demand for currency and thus the amount of seignorage. Real seignorage increases with inflation until it reaches a maximum and then declines. Seignorage is a form of taxation on the real currency held by the public, where inflation acts as the implicit tax, and there is a point beyond which higher taxes reduce revenue. Cagan (1956) is the classic derivation of this function starting from the properties of the desire for real money balances, and Hilscher, Raviv and Reis (2015) provide recent estimates of the seignorage function. We assume that seignorage is state-dependent, and estimate the dependence from the relation between the inflation target and seignorage.

1.4 The resulting resource constraint

The central bank's real reserves follow the law of motion:

$$V' - V = r_s V + q_{s'}[B_{s'} - (1-\delta)B_s] + d_{s'} - [\theta c_s + (1-\theta)e_{s'}]B_s - n_{s,s'}. \tag{7}$$

When the economy is in state s' and was previously in state s, the central bank issues additional reserves, $V' - V$, to fund the sum of: (1) real interest on the previous level of reserves, $r_s V$, (2) net bond purchases at real cost $q_{s'}[B_{s'} - (1-\delta)B_s]$, and (3) payment of dividends to the government, $d_{s'}$. Funding needs are reduced by (4) receipts of interest this period on last period's bond holdings, $[\theta c_s + (1-\theta)e_{s'}]B_s$, and (5) receipts of seignorage, $n_{s,s'}$.

Equation (7) ignores the operating expenses of the central bank because these are usually small relative to its assets. The same applies to the revenue from direct loans to banks and to the mandatory statutory dividends that central banks are often committed to pay to the providers of their paid-in capital. We also ignore the use of resources for quasi-fiscal operations such as bailing out the banking system.

The simplest version of equation (7) is for a central bank that issues no reserves ($V = V' = 0$) and holds no bonds ($B_s = B_{s'} = 0$). In this case, it boils down to:

$$d_{s'} = n_{s,s'}. \tag{8}$$

In this case, the central bank transfers its seignorage revenue to the Treasury every period.

With outstanding reserves that pay interest, and a portfolio of bonds that fluctuate in value, the resource constraint of the central bank becomes more interesting. In principle, either reserves or bond holdings could adjust to make the constraint hold in every state of the world. During the recent financial crisis, both the Federal Reserve and the ECB altered their holdings of bonds to intervene in selected financial markets, so in the next section we will take the choices of B_s as given and take V as the endogenous variable. In the following sections we will study how different policies for B_s affect the risks of instability.

2 The Financial Stability of a Central Bank

The traditional measure of a corporation's financial standing is the accounting measure variously termed capital, equity, or net worth. There are typically three reasons to measure it: to calculate the residual winding up value of the corporation, to assess its market value, and to ascertain the weight of equity versus credit in the firm's funding.

None of these reasons applies to a central bank. Central banks cannot be liquidated because their creditors cannot demand to convert their credit into anything but what they already hold, currency and reserves. Therefore, there is no meaning to the residual value of a central bank because the central bank cannot be wound up. Central banks also do not have a meaningful market value, because their goal is not profits, and shares in the bank are typically not traded. Finally, governments own the central bank, they often deposit funds with the central bank, and most of the assets of the central bank are government liabilities. The traditional distinction between equity-holders and credit-holders is blurry and confusing for a central bank.

Nonetheless, central banks may run into financial difficulties. There is no bankruptcy court for central banks but if private agents refuse to hold its liabilities, the central bank can no longer operate and is effectively insolvent. Theoretically, this implies a hyperinflation (literally, that $p = \infty$) because reserves become worthless, but in practice it comes with a currency reform such that the old central bank effectively closes its doors and a new central bank with new liabilities starts over.

Insolvency is an off-equilibrium outcome of a central bank that has a payout requirement d_s that is sufficiently in excess of its income from seignorage and bond interest so that its reserves rise without limit. Specifically, with explosive dividend payouts, for any $\epsilon > 0$ and stated reserve level, there is a finite time horizon such that the probability that the reserve level will exceed the stated level is $1 - \epsilon$. In a well-run country, any perception that existing policies regarding the central bank will lead to a reserve explosion results in an immediate correction through reduced dividend payout requirements. We will discuss dividend rules that guarantee the absence of reserve explosion.

2.1 Dividends and central bank independence

Central banks pay dividends d_s to their governments. If the central bank and the Treasury were perfectly integrated, the resource constraint in equation (7) would have little bite. The dividends d_s would just record the transfers between a particular branch and the overall government, but these would all be subsumed within the operations of the government as a whole. It would make little sense to talk of the solvency of the central bank instead of the solvency of the government as a whole. The individual accounts of most government agencies do not merit any special attention.

Central banks are different from government agencies. In most advanced countries, the central bank is a separately organized entity, with independence to pursue its mandate. There are many definitions and measures of how independent a central bank is, but a minimal one is that it need not ask the government for resources—see Cukierman's (2008) survey. That is, *a central bank is independent as long as it adheres to its dividend rule and the rule does not imply explosive reserve growth.* The success of the fundamental political strategy of an independent central bank hangs on the presence in its charter of a non-explosive dividend rule. Goodfriend (1994) discusses the independence and solvency of the Fed from this perspective.

The rest of the paper explores alternative dividend rules with respect to their capacity to deliver stable independent central banking free from the danger of reserve explosion. Although some of the rules we consider have roots in accounting concepts, we focus entirely on the economic criterion of reserve stability.

2.2 Net income and the meaning of negative dividends

Most central banks have dividend rules based on a concept of net income. The bank pays its net income to the Treasury as a dividend. Because net income can be negative, a dividend rule calling for payments based on net income may call for payments from the government to the bank. Historically, this issue has not arisen for the Federal Reserve, because it enjoyed high seignorage, high income from holding short-term bonds funded almost entirely with currency, low levels of reserves, and zero or low interest paid on reserves. But with the expansion of reserves and the resulting necessity to pay near-market rates on reserves, negative income may occur.

We assume that the government and the central bank have committed to a stated dividend rule, and that the public understands the commitment. Absent a well-understood commitment, payments to the central bank may be mischaracterized as *ad hoc* recapitalizations of a weak central bank, a potential source of failure of the principle of central-bank independence. Governments and the public should understand the implications of tying dividends directly to a measure of net income. Either they should accept that negative dividends are appropriate under the central bank's charter, or adopt a charter that modifies dividends by permitting zero dividend payments during periods when net income is negative, together with makeup increases in dividends once net income turns positive. We will discuss both of these types of rules.

2.3 The real mark-to-market dividend rule

If the central bank's net income is calculated the way an economist would, in real terms with its portfolio marked to market, the dividend would obey the *real mark-to-market dividend rule*:

$$d_{s'} = [\theta c_s + (1-\theta)e_{s'} + q_{s'} - q_s - \delta q_{s'}])B_s + n_{s,s'} - r_s V. \tag{9}$$

The dividend is (1) the return on the bond portfolio, the coupon payment $\theta c_s + (1-\theta)e_{s'}$ plus the capital gain $q_{s'} - q_s$ less the depreciation $\delta q_{s'}$, all applied to the number of bonds, plus (2) the seignorage revenue from issuance of currency, $n_{s,s'}$, less (3) the deduction of the interest paid on reserves $r_s V$.

A simple result follows:

Proposition 1 *Under the real mark-to-market dividend rule in equation* (9)*, the law of motion of reserves is:*

$$V_s = q_s B_s + V_0 - q_0 B_0. \tag{10}$$

Here $V_0 - q_0 B_0$ *is an initial condition. Reserves are a function of state-dependent bond values* $q_s B_s$ *and the initial condition, so reserves cannot explode.*

Proof: Substitute equation (9) for $d_{s'}$ in equation (7). The result simplifies to

$$V' - q_{s'} B_{s'} = V - q_s B_s. \tag{11}$$

Therefore, $V - q_s B_s$ is constant across periods. Letting the initial condition be $V_0 - q_0 B_0$ gives the result. □

During a crisis in state S, a central bank practicing quantitative easing will expand its bond holdings and fund the expansion with added reserves. When the crisis is over, and the bond-holding rule B_s assigns a normal, smaller level of bond holdings, reserves will contract as the bank sells bonds and uses the proceeds to pay off its debt to commercial banks. The proposition shows that reserves are state dependent and the central bank is always able to honor its debts whatever is the payment on reserves that it chooses.

2.4 The nominal mark-to-market dividend rule

In practice, central bank dividend rules use nominal rather than real accounting. Net income is defined as the change in nominal net worth that would occur if dividends were zero. Nominal net worth is

$$pW = p(q_s B_s - V - N_s), \tag{12}$$

with currency N_s treated as a liability. Then, by definition, nominal net worth pW is constant under a rule that sets dividends equal to that concept of net income. This dividend rule also meshes well with the notion that central banks engage in open-market operations, where the sum of reserves and currency—the monetary base—moves one-to-one with the purchase or sale of bonds.

Under a state-dependent price-level policy, we could derive results parallel to the ones for the real mark-to-market dividend rule—real reserves would be state dependent and have a constant additive term interpreted as an initial condition. But a state-dependent inflation policy, where all of the π_ss are positive, and the price level rises continually in a non-state-dependent way, is more realistic. Combining the nominal payment-on-reserves rule, with the law of motion for reserves in equation (7) and the condition that $p'W' = pW$ with equation (12) gives the *nominal mark-to-market dividend rule*:

$$d_{s'} = \left[\theta c_s + (1-\theta)e_{s'} + q_{s'} - \frac{q_s}{1+\pi_s} - \delta q_{s'}\right] B_s - \frac{i_s V}{1+\pi_s}. \tag{13}$$

The first distinction from equation (9) is that net income is calculated in nominal terms, so that higher inflation raises measured capital gains $q_{s'} - q_s/(1+\pi_s)$. The second distinction is that central-bank accounting treats the growth of currency and resulting increase in bond holdings as exactly offsetting, because currency is treated as a liability, even though the present value of its future cash burden is zero. Given the accounting convention that the income statement is the first difference of the balance sheet, nothing makes its way to the income statement to reveal the income from seignorage.

The constancy of nominal net worth implies

Proposition 2 *Under the nominal mark-to-market dividend rule with* $W_0 = 0$*, reserves follow the state-dependent law of motion,*

$$V_s = q_s B_s - N_s. \tag{14}$$

If $W_0 > 0$*, beyond this state-dependent component, there is a time-dependent component which dies away if inflation is always positive and is a constant if inflation is always zero.*

Proof: The constancy of nominal net worth implies that, in time-dependent notation:

$$q_t B_t - V_t - N_t = \frac{p_0}{p_t} W_0, \tag{15}$$

so

$$V_t = q_t B_t - N_t - \prod_{\tau=0}^{t-1} \left(\frac{1}{1+\pi_\tau}\right) W_0. \tag{16}$$

With $W_0 = 0$ we obtain equation (14), the state-dependent component. The last term on the right-hand side is: zero if $W_0 = 0$, depends on time and dies away if $W_0 > 0$ and $\pi_t > 0$, and is constant if $W_0 > 0$ and $\pi_t = 0$ for all t. $\square$

In general, from an arbitrary starting point of real net worth W_0, reserves are not state dependent. Rather, they are the sum of a state-dependent component $q_t B_t - N_t$ and a negative component associated with the initial level of real net worth that dies away with time. The dying away would be at a constant rate if the rate of inflation were constant across all states.

Under this dividend rule, reserves fluctuate not only because of changes in bond holdings but also as the public varies its demand for currency. As before, reserves during a crisis, V_S, may be high, but they are non-explosive.

2.5 When is net income negative?

Before the crisis, the Federal Reserve's portfolio consisted almost exclusively of short-term government bonds. Old-style central banking was characterized by $\delta = \theta = c_S = 1$. In this case, under the real mark-to-market dividend rule:

$$d_{s'} = n_{s,s'} + r_s(q_0 B_0 - V_0). \tag{17}$$

Historically, seignorage has almost always been positive, as have been real interest rates. Therefore, in the 100-year history of the Federal Reserve under old-style central banking, the Federal Reserve always paid a positive dividend to the Treasury.

Paying interest on reserves and holding investments with interest-rate, default and exchange-rate risk opens the possibility of negative dividends. Substituting the law of motion for reserves, equation (25), into the real mark-to-market dividend rule in equation (9) describes dividends in terms of exogenous variables and the initial condition:

$$d_{s'} = n_{s,s'} + r_s(q_0 B_0 - V_0) + [\theta c_s + (1-\theta)e_{s'} - \delta q_{s'} - r_s q_s]B_s + (q_{s'} - q_s)B_s. \tag{18}$$

New-style central banking comes with a new term beyond seignorage and the return on capital. It is the sum of (1) $[\theta c_s + (1-\theta)e_{s'} - \delta q_{s'} - r_s q_s]B_s$, the difference between the coupon payment adjusted for bond depreciation less the real rate on reserves, and (2) $(q_{s'} - q_s)B_s$, the capital gain on the bond portfolio. If $\delta = \theta = 1$ this second term is exactly zero. But for a central bank that holds long-term bonds or foreign assets, net income is more likely to be negative when the bond repayment is impaired, when the real exchange rate appreciates, or when there is a capital loss in the bond portfolio.

Using instead the nominal dividend rule in equation (13), we find, with the same substitutions,

$$d_{s'} = \frac{i_s}{1+\pi_s}(N_s + W_0) + \left[\theta c_s + (1-\theta)e_{s'} + (1-\delta)q_{s'} - \frac{1+i_s}{1+\pi_s}q_s\right]B_s. \tag{19}$$

The first term is the inflation tax on holders of currency, who pay the tax by holding banknotes instead of interest-bearing assets. The quantity within the square brackets is the difference between the actual and the expected payment on the bonds. It is zero on average, but it may well be negative if coupons or future prices fall below average. Impaired coupons, real exchange rate appreciation, falls in bond prices, and rises in real interest rates are potential sources of negative net income.

Equation (19) holds whether interest is paid on reserves or not. Paying interest on reserves requires resources, but because it also lowers net income in the dividend rule, it has no effect on the evolution of reserves or on the equilibrium dividends paid. What distinguishes old-style from new-style central banking is instead the holding of risky assets, foreign and domestic, increasing the risk from changes in real interest rates, coupons, and exchange rates. If $\delta = \theta = c_s = 1$ and only one-period domestic safe bonds were held, the term within square brackets would be zero, and the central bank would always have positive dividends equal to the inflation tax.

2.6 Implications of a dividend rule that excludes negative dividends

In the case of the Fed, which has used the net income rule to pay its dividends, there is no formal financial arrangement between it and the rest of the government. Nothing obligates the Treasury to make a payment to the Fed if its net income is negative. The appropriation of government funds would have to be approved by fiscal authorities, subject to the political process. Negative income opens the door to political interference in monetary policy decisions.

One solution is to set the dividend to zero in times when net income is negative

$$d_s = \max(y_s, 0), \tag{20}$$

where y_s is the measure of net income otherwise used to set dividends. In this case, the central bank issues additional reserves to compensate for the foregone payment from the government, in the amount of the shortfall:

$$z_s = d_s - y_s. \tag{21}$$

Consider first a central bank subject to the real mark-to-market dividend rule. An increase in reserves resulting from a positive shortfall z has the same permanent effect that we analyzed in connection with the initial condition in proposition 1. We let Z denote the cumulative sum of the zs, so $Z' = Z + z_{s'}$. It is not a function of the current state alone.

Rather, since equation (20) and equation (21) imply that z_s is always non-negative, as long as there is a positive probability of negative net income, Z is a weakly increasing sequence over time that will tend to infinity. Retracing the steps that led to proposition 1 gives:

Proposition 3 *If the central bank pays real mark-to-market dividends according to the rule in equation* (20)*, with net income given by equation* (9)*, reserves are:*

$$V_t = q_t B_t + (V_0 - q_0 B_0) + Z_t, \tag{22}$$

so they drift upward without limit and are explosive.

The rule in equation (20) is unsustainable as it violates the no-Ponzi-scheme condition on central bank reserves.

The situation is somewhat different in the more realistic case of a central bank subject to a nominal mark-to-market rule and positive state-dependent inflation. Combining the rule in equation (13) with the constraint that dividends are non-negative in equation (21) and the law of motion for reserves in equation (7), we obtain:

$$W' = \frac{W}{1+\pi_s} - z'. \tag{23}$$

Whenever dividends are higher than net income, z is positive, and net worth falls. In nominal terms, $p'z'$ gives the the decline in nominal net worth this period.

Let pZ now denote the shortfall of nominal net worth from its initial level $p_0 W_0$. With zero inflation, this is still the cumulative sum of the zs, but with positive inflation,

$$Z' = \left(\frac{1}{1+\pi_s}\right) Z + z_{s'}. \tag{24}$$

Inflation makes the real shortfall in net worth fall over time if the π_ss are positive. In this case, Z obeys a stationary autoregressive time-series process.

Given the law of motion for net worth, the same steps that led to proposition 2 yield the following result:

Proposition 4 *If the central bank pays nominal mark-to-market dividends according to the rule in equation* (20)*, with net income given by equation* (13)*, reserves are:*

$$V_t = q_t B_t - N_t - \prod_{\tau=0}^{t-1} \left(\frac{1}{1+\pi_\tau}\right) W_0 + Z_t. \tag{25}$$

The term $q_t B_t - N_t$ is state-dependent. If inflation is positive, the term in W_0 dies away and Z_t is stationary, so reserves V are stationary. If inflation is always zero, the term in W_0 is constant, and Z_t drifts upwards without limit, so reserves V are non-stationary.

A corollary of the proposition is that nominal reserves pV track pZ. Inflation plays an important role with a nominal mark-to-market dividends rule. If inflation is zero or, similarly, if the price-level is state-dependent, then reserves again explode as a result of the Treasury not paying into the central bank. With positive inflation, real reserves are stationary. A policy that tries to keep nominal net worth constant leads to falling real net worth when inflation is positive. As a result, when the central bank fails to receive funds from the Treasury, inflation erodes this real shortfall over time and keeps real reserves stationary. Higher inflation increases the financial stability of the central bank both by raising seignorage and by eroding the real shortfall in Treasury backing.

2.7 Deferred assets and retaining earnings

The dividend rules of many central banks cause the bank to recover from the issuance of reserves in lieu of reimbursement of negative income by setting subsequent dividends that are less than net income. The rules do not allow the central bank to refuse to hand over its net income to the fiscal authorities for arbitrary prolonged periods of time—see Amtenbrink (2010).

We model the relationship by hypothesizing an account balance D that measures the backlog of negative net income realizations that the Treasury did not cover by making payments to the central bank. D is a deferred asset capturing the claim that the central bank has on its own future remittances to the Treasury. Many central banks do not record this claim, but simply act on it to recover their accounting equity after losses, while the accounting rules of the Fed (Federal Reserve Act, section 7, 12 USC 290) record the claim as a negative liability. When D is positive, the central bank can draw it down by paying a dividend d' less than net income y'. On the other hand, when y' is negative, the balance D rises by $-y'$. The dividend rule is

$$d' = \max(y' - D, 0). \tag{26}$$

We also assume that an upper limit $\bar{D}$ applies to the balance. Central-bank charters are vague about this limit but ultimately the central bank is part of the government and fiscal authorities can, at their discretion, reclaim the surplus accumulated by the central bank. It is quite plausible that if the balance in D was high the Treasury would perceive the payments associated with this debt to the central bank as an effective recapitalization. A balance above $\bar{D}$ would put in question the independence of the central bank in the same way that we argued before when we set this limit to zero. At the limit, $\bar{D}$ may be the present value of seignorage, so that the central bank does not need to receive a positive transfer from the fiscal authority in present value, as shown in Reis (2013b).

Balances in the D account decline with inflation, matching the decline that occurs in the extra component of reserves, Z. Putting all of these elements together, we get the law of motion,

$$D' = \min\left(\bar{D}, \frac{1}{1+\pi_s}(D - \max(y' - d', 0) + \max(-y', 0))\right). \tag{27}$$

As before, the law of motion of the bulge in reserves caused by missed Treasury payments, Z, is

$$Z' = \frac{1}{1+\pi_s} Z + d' - y'. \tag{28}$$

This provision, with a reasonably generous value of $\bar{D}$, will cancel a bulge of reserve issuance following an episode of negative net income by cutting subsequent dividends and using the funds to pay off the bulge of reserves. The degree of protection depends on $\bar{D}$ and the frequency and magnitude of negative incomes compared to offsetting positive incomes in normal times. In an infinite lifetime, the buildup of reserves Z will rise above any finite level with positive probability at some time, but allowing the central bank to pay low dividends to recover lowers this probability.

The message from these results is that, whatever the dividend rule or the inflation policy, negative net income poses a challenge. If the Treasury does not make payments to the central bank in times of negative net income, reserves will have to rise.

2.8 A quantitative measure of the financial strength of a central bank

A central bank is financially sound if its charter specifies a dividend rule that excludes exploding reserves. The balance in the deferred account D is a useful metric for judging the bank's stability. It is almost identical to Z, the excess of its reserve obligations. If the dividend rule has the prospect for payments from the Treasury that will pay off its obligation, reserves will revert to their state-dependent levels in propositions 1 and 2. D becomes a second state variable for the central bank.

For the Fed and possibly other central banks, the Treasury has discretion over the repayment of its deferred obligation—the charter does not deal explicitly with the issue. A central bank relying on a large discretionary intervention of the Treasury is financially weak, and the larger the amount, the weaker the bank. Therefore, while we started this section by arguing that the usual case for the usefulness of accounting capital or equity does not apply to the central bank, under the nominal mark-to-market rule, Z measures precisely the shortfall in the net worth of the central bank. Our analysis provides an economically motivated measure of the financial strength of a central bank. Rather than formulate a concept of capital for central banks and raise red flags if capital is depleted, a better approach is to monitor how close the central bank's deferred claim on the Treasury is to its upper limit $\bar{D}$, or how long it will take the Treasury to pay off this deferred obligation.

3 The Federal Reserve and Interest-Rate Risk

The main lesson from the previous section is that the stability of a central bank depends crucially on what happens to its dividends when net income is negative. Prior to 2008, the Fed held mostly safe short-term Treasury debt with small potential capital losses and it paid no interest on reserves. As a result, net income was always positive. In contrast, during and after the crisis, the Fed functioned as a highly profitable hedge fund, borrowing at low rates in the short-term market to fund holdings of higher-yield longer-term debt, thereby generating substantial positive net income.

The stability of the Fed is at stake when interest rates increase, as the increase raises the payment on reserves and lowers the value of the Fed's portfolio of longer-term bonds. In this section, we simulate what would happen to reserves and dividends under different scenarios for the recovery. Any answer to the question of how the federal government would deal with a situation calling for the Treasury to pay off its deferred obligation to the Fed is conjectural. Our approach is to take real interest rates as given, and to study how different inflation targets, different institutions for calculating dividends, and different management of the exit from the crisis would affect the Fed's volume of reserves. We begin by describing how our stylized model approximates the Fed and how we measure the relevant variables and calibrate the model.

3.1 The Fed and the model

A central bank's institutions in our model are captured by the parameters δ, c_S, and θ and by its dividend rule. The Fed barely has any foreign investments and it almost exclusively holds securities issued by the Treasury or agency securities guaranteed by it. For the Fed, $c_s = \theta = 1$.

Of course the Fed does not hold delta bonds, but rather Treasury bills, notes, and bonds of various maturities. All actual Treasury instruments pay off face value at maturity, rather than melting away like a delta bond, and notes and bonds make periodic coupon payments as well. There is no computational obstacle to building a model that keeps track of the historical path of purchases and sales and records gains and losses only upon sale. But the complexity of the model would stand in the way of the points we want to make in this paper. Instead, we use the data in the annual report of the Federal Reserve on the value and maturity of the Treasury securities it holds, to calculate the value-weighted average maturity of the Fed's financial assets. The parameter δ is the reciprocal of the average maturity. Between 2009 and 2013, δ averaged 0.128. We keep δ constant throughout, but historically δ was higher, and in the future the Fed will likely reduce the maturity of its portfolio as it exits the crisis. Variations in δ by state of the economy, though feasible, would much complicate the model. Our procedure is reasonably accurate, because the central risk to the Fed occurs upon exiting the crisis state, when the portfolio is large. Prior to the crisis, the portfolio is smaller, so our exaggeration of the capital gain at the beginning of the crisis is not large.

We describe the Fed's financial stability under the hypothesis that it pays dividends according to the nominal mark-to-market rule discussed in the previous section. In fact, the Federal Reserve does not mark its bond portfolio to market, so unrealized capital gains do not enter into its calculations of net income and are not paid out as dividends. We discuss this practice and how it affects the results in section 3.9.

3.2 The states of the economy

Our annual data cover the period from 1954 through 2013. To define the states of the U.S. economy, we use the realized real T-bill rates, measured as the nominal rate on one-year bills minus the increase in the Consumer Price Index. We allow for 5 states, where the first four correspond to the behavior of the interest rate until 2008, and the real rates for these states are the medians within the quartiles of the distribution of the rate over that period. Since then, the economy has been in state 5, the crisis state, with a real rate essentially minus the inflation rate, because the nominal rate has been close to zero. The transition into state 5 from 2007 to 2008 occurred out of state 1.

Exogenous model inputs (in percents)					*Endogenous bond prices q*
State number	*Safe rate, r*	*Bond holdings, qB*	*Currency, N*	*Inflation, p'/p-1*	
1	4.33	4.78	4.86	3.00	6.71
2	1.92	5.44	5.45	2.70	6.83
3	1.32	5.29	5.31	3.33	6.96
4	-0.61	5.60	5.37	6.60	7.22
5	-1.47	14.48	6.48	1.87	7.64

Table 1: Inputs Derived from U.S. Data

Our estimates imply that the economy has a financial crisis with a probability of 1.7 percent per year. Based on recent experience, we take the exit rate to be 20 percent per year as a rough estimate. We also assume the transition will be into state 3, the median state. We therefore assume that a crisis has an expected life of 5 years. The estimated transition matrix appears in Table 4 in Appendix C, together with the implied stationary distribution across states.

3.3 The inputs of the model

Table 1 shows the values of the variables in the model at each state of the economy, based on data from the annual financial statements of the Federal Reserve System. We measure the holdings of bonds $q_s B_s$ as the total U.S. Treasury and agency securities held by the Federal Reserve system. To calculate currency holdings N_s, we use currency in circulation. The difference between the two is close to the reserves outstanding, as the Fed's accounting net worth is relatively small. We divide all nominal variables by GDP and use the GDP deflator as our measure of inflation π_s. Note that in the crisis state, the bond portfolio of the Fed is more than twice as large as in the other four states.

The last column of the table shows asset prices for the approximating delta-bond. Recall that the price of delta-bonds is given by

$$q_s = \mathbb{V}(c_s + (1 - \delta) q_{s'}).$$

From state number	*To state number*				
	1	*2*	*3*	*4*	*5*
1	0.245	0.322	0.399	0.564	0.824
2	0.157	0.243	0.329	-	-
3	0.090	0.172	0.254	0.431	-
4	0.044	0.128	0.212	0.392	-
5	-	-	-0.855	-	0.276

Table 2: Net Income in the Ergodic Distribution, as a Percent of GDP

We use this delta-bond price to account for capital gains and losses on the portfolio of the Fed. To apply the $\mathbb{V}$ operator, we need to have the stochastic discount factor $m_{s,s'}$. A reasonable class of SDFs satisfies

$$m_{s,s'} = \beta \frac{\mu_{s'}}{\mu_s}. \tag{29}$$

For example, in the representative-agent consumption CAPM with time-separable expected utility, μ_s is marginal utility. Under this assumption, the definition for the real interest rate gives a condition that the SDF must satisfy:

$$\beta(1 + r_s) \sum_{s'} \omega_{s,s'} \mu_{s'} - \mu_s = 0. \tag{30}$$

With the normalization $\mu_1 = 1$, this is a system of N equations in N unknowns, $\beta, \mu_2, \ldots, \mu_N$ that takes as inputs the data on r_s. Though nonlinear, the system solves easily by standard methods. Bond prices then solve the linear pricing recursion. Appendix C shows the auxiliary parameters that come out of applying the formula. This approach to asset pricing was also taken by Hall (2014), and is closely related to the approach taken by Ross (2015).

3.4 The Fed's stability in normal times

We begin with the case where the Treasury pays the Fed immediately if the Fed's net income is negative, so that proposition 2 holds and there are no concerns about stability. In this case, reserves are state-dependent, and we assume that any initial effect from W_0 has long since melted away on account of positive chronic inflation. Table 2 shows the net income in the ergodic distribution, with blank entries for states with zero probability.

The Fed earns a positive net income in every pair of originating and succeeding states but one—the Fed has negative net income only when the economy exits from the crisis, from state 5 to state 3. Negative income is a substantial 0.85 percent of GDP, but this event only happens once every 59 years. Moreover, when the Fed enters a crisis, its dividends are abnormally high at 0.82 percent of GDP, and while the crisis lasts, net income is 0.28 percent of GDP. The Fed is financially stable under new-style central banking.

3.5 The Fed in crisis and recovery

Only because of crises is the Fed's financial stability potentially at stake. We track the Fed's finances during an extreme but still plausible worst-case scenario, where the economy moves over a 15-year period from state 1 to a crisis in state 5 for five years and a recovery in state 3 for the remaining 9 years. Throughout, we assume 2-percent inflation. We set the initial conditions to match the level of reserves in 2007.

At the outset of the crisis, the Fed borrows \$1.26 trillion from the banking system to buy bonds. High levels of bond holdings financed by high levels of reserves continue until the end of the crisis, when the economy shifts to state 3. The Fed then sells \$1.31 trillion in bonds and uses the funds to retire a similar volume of reserves, which decline to almost their initial level. Reserves are elevated during the crisis state, but are otherwise low and stable during the other states. The nominal interest rate starts at a fairly high level prior to the crisis, plunges to zero for the crisis years, and then resumes a moderate positive level for the remaining time in state 3. The bond price follows the price level on a trend, but rises at the beginning of the crisis and falls when the crisis ends. Therefore, the Fed faces significant reserve enlargement at the recovery, when the fall in bond prices causes capital losses in its large portfolio, and the rising interest rate raises payments on reserves.

Figure 1 shows the Fed's dividend to the Treasury and the flows that determine it. The least important determinant is the payment on reserves, which is a deduction from the dividend. During the crisis the interest rate paid is zero, and outside of the crisis reserves are small so the interest paid has a barely visible negative effect. The coupon earnings from the bond portfolio less depreciation are likewise of little importance. Deducting depreciation is the equivalent of not counting the return of principal as part of the earnings from a bond. The most important determinant of dividends is the capital gain on the bonds. The Fed receives a capital gain upon entering the crisis state, with negative real interest, and pays a

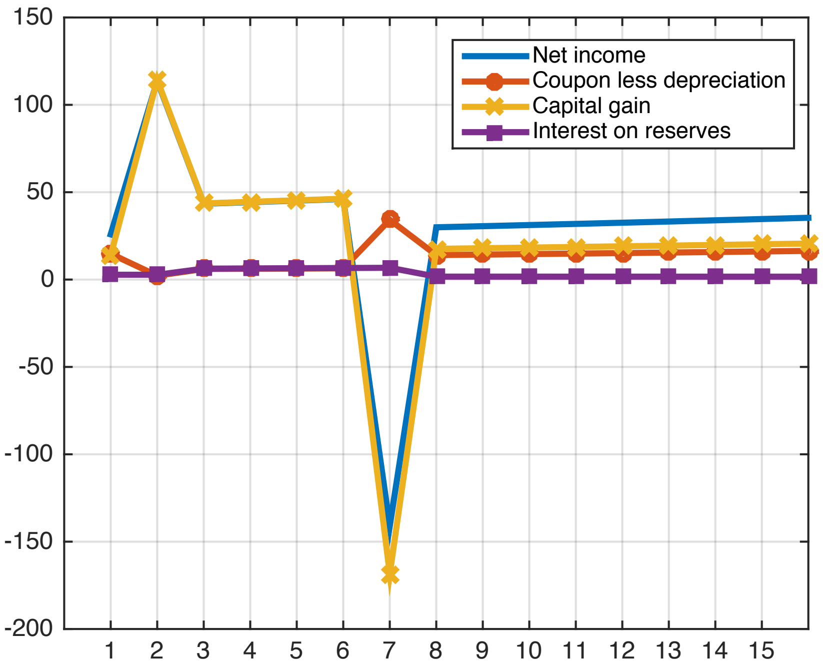


Figure 1: Components of the Fed's Dividend to the Treasury

corresponding dividend. Upon exiting the crisis state, the Fed suffers a capital loss. The loss is larger than the gain because it applies to the large portfolio acquired during the crisis.

Figure 2 shows the flows that raise or reduce reserves. Interest on reserves is a minor factor contributing to growth and seignorage is a minor factor contributing to shrinkage. The big factor is purchasing and selling bonds. When the crisis strikes, the Fed expands reserves to buy bonds; when it ends, the Fed sells a large volume of bonds and pays down reserves. Dividend payments, which add to reserves, also have noticeable roles at the beginning of the crisis—when capital gains from the lower interest rate accrue and are paid to the Treasury—and at the end of the crisis—when capital losses from the higher interest rate accrue, which the Treasury funds.

3.6 Financial stability when inflation is the only tool for dealing with negative net income

Our next calculations show how reserves evolve if the Treasury keeps all net income when it is positive but makes no provision for compensating the central bank for negative net income. This was the case described in proposition 4.

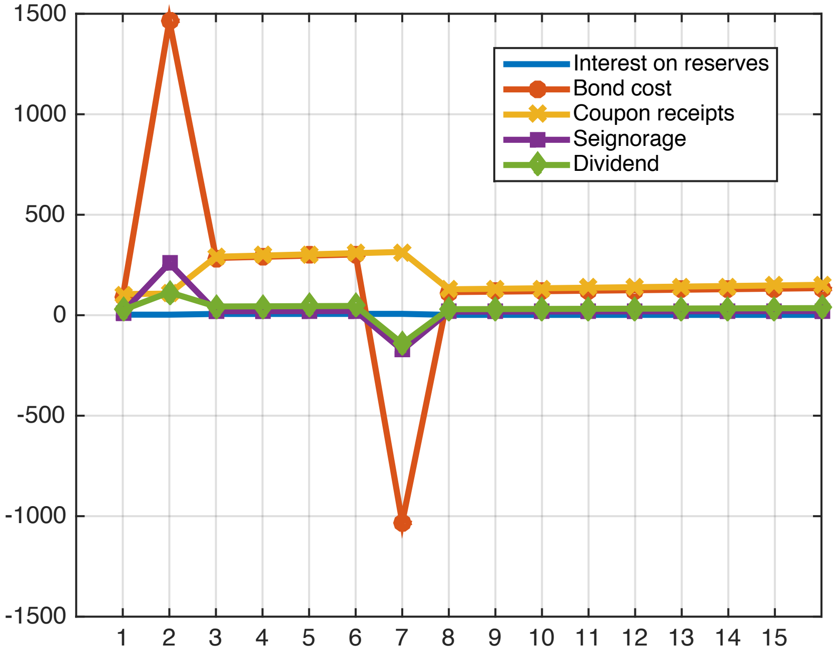


Figure 2: Flows into and out of Reserves

In this case, real reserves grow each time the economy recovers from a crisis but shrink every year from inflation. At the exit of the crisis, the balance in the Z account rises to 0.7 percent of GDP. With a 2 percent inflation target it would then take 35 years for it to decline by half. An interesting question is how much inflation would be needed to speed this transition. Providing an accurate estimate would require having reliable estimates of the seignorage function. Instead, we provide a lower bound by assuming that the higher inflation would have no effect on the real holdings of currency thus maximizing the seignorage that the central bank could generate.

If the central bank raises its inflation target to 4 percent after exiting the crisis, then the balance in the Z account is 0.5 percent of GDP 10 years after the crisis is over, which is slightly above two years of net income. In order to be one year of net income away within a decade, the inflation target would have to increase from 2 percent to 6.2 percent. If instead, the central bank raised the inflation target right away at the exit of the crisis to generate enough seignorage income to compensate for the whole of the capital losses, then inflation would have to rise to 18.8 percent on the year of exit from the crisis. These large amounts reflect the limited ability to raise seignorage via higher inflation.

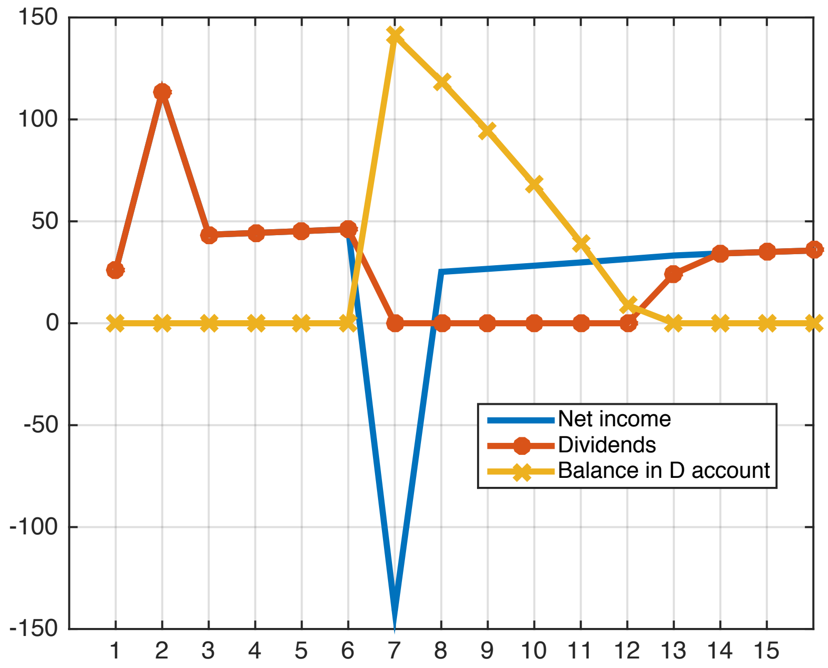


Figure 3: The D Account after a Period of Negative Income

3.7 Financial stability with deferred Treasury reimbursement of negative net income

Our next calculations show how reserves evolve if the Treasury does not pay the Fed on a current basis for negative net income, but allows the Fed to retire the extra reserves through the D account we described earlier. We take the upper limit to be $\bar{D} = 0.02$ or 2 percent of GDP, a limit that is not binding in our scenario. The result is that the balance D is the same as the extra reserves Z, which is our dollar measure of the Fed's financial condition.

Figure 3 shows the operation of the D account under the assumptions in Figure 2. Until the crisis ends, dividends equal net income, resulting in a large payout to the Treasury when the Fed's bond portfolio appreciates at the onset of the crisis. When the economy recovers, net income is negative for a year. The Fed issues extra reserves to cover the loss. The balance in the D account rises by $141 billion, along with a bulge in reserves of the same amount. For the next 5 years, the Fed pays the Treasury zero dividends and gradually works off the balance in D and the extra reserves that had been issued when the crisis ended. In the following year, the Fed pays a positive dividend but less than net income. In the succeeding years, $D = Z = 0$ and reserves are back to their state-dependent normal values.

3.8 Managing the balance sheet: speed and timing of the exit

We conclude that there is a theoretical danger to the Fed's financial stability from capital losses when exiting the crisis. Anything that lowers the fall in bond prices reduces the danger of instability. To investigate this conclusion, first we make the exit from the crisis occur to state 1, rather than to state 3. The fall in bond prices when exiting the crisis is now higher and so are the capital losses. Figure 4 shows that the amount accumulated in the Fed's deferred claim on the Treasury account is now much larger. Our conclusion that the Federal Reserve is still at little risk is also approximately unchanged. Now, the maximum deferred compensation to the Fed would be \$209 billion. It takes now 10 years, instead of 6, to reclaim dividends from the Treasury to bring this amount to zero.

How could the Fed's dividend rule be changed to lower this risk? One possibility is to manage the exit from the crisis differently. In particular, we assumed that the Fed shrinks its balance sheet back to pre-crisis levels right as the recession ends. Because it sells a large share of its portfolio at the same time as real interest rates rise and bond prices fall, the Fed realizes a large loss. The Fed has suggested instead selling its crisis portfolio slowly. Figure 4 shows what would happen to the Fed's D account—the extra reserves it issues to make up for the Treasury's non-payment against negative net income—if it takes 5 years to sell the bonds bought during the crisis. This policy of slow winding down makes almost no difference to realized net income and thus to the D balance.

If slow shrinkage of the balance sheet makes little difference, perhaps doing the opposite helps. The opposite, in this case, consists in reversing quantitative easing before interest rates rise. We add a sixth state and repeat our simulations to calculate the net income and reserve borrowing of the Fed, if in that sixth state, the balance sheet falls to its pre-crisis size, but the nominal interest rate stays at zero. The following period, the crisis is over as we transition back to state 3 as before. Note that this experiment makes the generous assumption that the Fed is able to sell its assets still at their high crisis-level prices, as agents only learn about the end of the crisis the following year. Figure 4 shows that selling before would lower the risks to the Fed's stability. Still, the deferred account stays positive for two years.

An early exit from quantitative easing lowers the risk of instability. But managing the balance sheet does not eliminate the risk. As long as the Fed marks its portfolio to market,

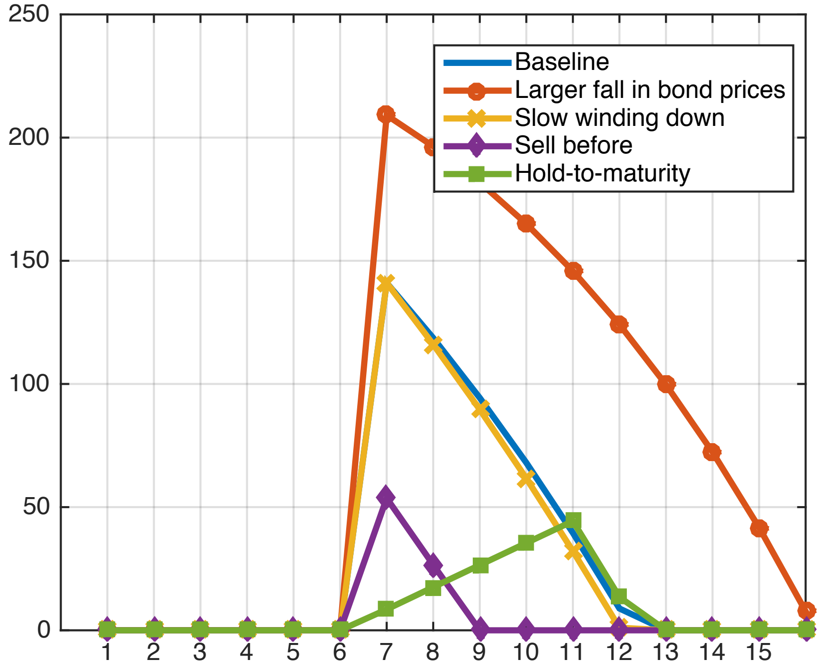


Figure 4: The D Account under Alternative Policies and Conditions

its income for the purposes of its dividend to the Treasury will be sharply negative at the end of the crisis, whether it sells the bonds faster or slower.

3.9 Institutional remedies: marking the Fed's portfolio

All the policy counterfactuals so far assumed that the Fed's bond holdings were marked to market each year in determining net income y. The Fed is more likely to have trouble under this dividend rule than under its actual dividend rule, because it earns capital gains on its portfolio upon entering the crisis state, which it would pay out immediately to the Treasury. In actuality, the Fed uses hold-to-maturity accounting. It computes net income as if its bonds were worth the nominal principal returned at maturity.

If the Fed sells all of its bonds when interest rates rises, as in our baseline case, hold-to-maturity accounting would make little difference to measured net income. When the crisis ends, the fall in bond prices would coincide with the Fed bringing the size of its assets back to its pre-crisis level. Therefore, the Fed would sell most of its bonds and realize the capital losses, just as in our mark-to-market benchmark.

Instead, if the Fed slowly winds down its portfolio, there would be a noticeable difference, as shown in Figure 4 in the line labeled hold-to-maturity. The basic problem with the dividend rules we have been studying is that they generate a burst of negative income in a recovery. Any accounting system that smooths measured net income will avoid or ameliorate the danger of instability that arises from occasional bursts of negative income that do not result in payments from the Treasury to the Fed.

A slow winding down of the portfolio without marking to market accomplishes the smoothing. Because most of the bonds the Fed bought during the crisis would mature in the post-crisis years, their losses would take the form, not of the immediate capital loss that we record, but rather of a stream of coupon payments at rates lower than the current market rate during the crisis years. By recording the bonds at an artificially high price on its accounts, the Fed would record lower returns on them than the market interest rate that it pays on reserves. The interest paid on reserves now shows up as a leading source of losses, for several years after the crisis, instead of the immediate capital loss at the time of the exit. The present value of dividends is the same: accounting rules do not create free lunches.

The hold-to-maturity line in Figure 4 shows that the path of the D account, with a slow winding down, spreads the recorded capital losses over the 5 years of the winding down. Now, when the economy exits the crisis state, the Fed's dividend is based on a small recorded loss associated with the selling of only one-fifth of the Fed's bonds. In the following years though, the payment of interest on reserves leads to persistent recorded losses for 5 years. When the crisis ends and the Fed's income is low, both because of low coupons on its holdings of bonds purchased during the crisis years and because of paying now-higher rates on reserves, the Fed goes through a long period of losses. The amount in the D account actually rises over the first four years of the recovery. During this time, the Fed pays no dividend to the Treasury. For the next two years, the Fed also pays no dividend, even though its recorded net income is positive, as the Fed draws down the D account.

The policy of slow winding down of the large bond portfolio acquired in the crisis, together with hold-to-maturity accounting, has the effect of smoothing the Fed's dividend relative to our assumption of marking to market. The policy avoids a big dividend payment at the beginning of the crisis, but instead involves many periods of smaller but negative net income. If it is the size of the balance in the deferred account at any one time that matters, then this accounting convention is effective at increasing the Fed's financial strength. If instead what would put the independence of the Fed in question would be an increase in the number of years with negative recorded income, then actual policy makes the Fed more fragile relative to marking to market.

4 The European Central Bank and Default Risk

The European Central Bank is the coordinating agency of the Eurosystem, comprising the ECB and the national central banks of the euro countries. Throughout our discussion, we use the name ECB, though our law of motion for reserves describes the Eurosystem, while our analysis of stability relates more closely to the ECB. As long as no major country leaves the euro area, this distinction is unimportant.

4.1 The ECB and the model

Since late 2008, the ECB has pursued policies similar to those of the Fed. Its assets have risen substantially funded by borrowing from banks by issuing reserves, which pay close-to-market interest rates. The ECB only reports the maturity of a subset of the securities that it holds, those purchased under the Securities Market Program. Using the reported maturities at the end of 2012, we set $\delta = 0.233$.

Like most central banks, but unlike the Fed, the ECB has traditionally held a large share of its assets as gold and foreign reserves. While the expansion of the balance sheet after 2008 involved almost only European securities, therefore reducing the share of gold and foreign reserves, the share of domestic assets was still on average 0.461 of the direct holdings of assets by the ECB after 2008. We set θ to this value.

More interesting, the ECB faces significant default risk on its holdings of sovereign bonds from countries in the periphery. In the model, the ECB suffers a default loss of $1 - c_S$ of its coupon expectation every period that it remains in the financial crisis state. Note that the bond-pricing equation assumes that all bonds resume paying full coupon rates once the economy exits state S—the bank does not lose a fraction $1 - c_S$ of the value of its bond holdings. As an extreme illustrative scenario, we consider the possibility that the ECB's bond holdings of Greek, Irish, Portuguese, and Spanish bonds in 2012 all went into default, and that the resulting drop in value matched the 65 percent haircut that was applied to private holders of Greek debt in 2012. This extreme scenario corresponds to $c_S = 0.858$.

4.2 Data and calibration for the euro area

Our data for Europe cover the period from 2000 through 2013. We take the exogenous real interest rates to be the 1-year Euribor rate minus inflation measured by the euro-area Harmonized Index of Consumer Prices. We define two states, one between 2000 and 2008, and the crisis covering 2009-13. With only one observation of each, it is impossible to estimate a transition matrix. Instead, we take the probabilities associated with a crisis to be the same as for the United States—the euro-area economy enters a financial crisis with probability 0.016 and the stationary probability of a crisis is 0.081. Our source for the ECB's finances is its last weekly financial statement published each year. For the depreciation in the holdings of foreign assets, we take a weighted average of the change in the price of gold and in the real effective exchange rate, with weights given by the shares of gold and foreign securities in the ECB's portfolio.

Table 3 shows the key exogenous variables for the ECB. As in the United States, the crisis corresponds to the nominal interest rate falling to zero, and the size of asset holding by the ECB more than doubling. The table shows the change in asset prices under two circumstances: if markets perceive the possibility of the default that we study, or if they believe instead that $c_2 = 1$ as happened in the data, since no default occurred. In a crisis, the lower real interest rates raise asset prices, and the impaired coupon lowers them. Therefore, when there is default, the change in asset prices getting into and out of the crisis is smaller than it would be if all assets were safe, and so the capital gains and losses will be smaller as well.

4.3 Default and the ECB's financial stability with direct holdings

Figures for the ECB corresponding to those we presented for the Fed are available in appendix D. As in the United States, a crisis comes with an increase in bond prices and a fall in nominal interest rates, and an expansion in bond holdings together with reserves. Capital gains and losses dominate the movements of the dividend paid, while the outlays for bond purchases and receipts from bond sales, along with coupon payments from bond held, dominate the movements of reserves. Default though now also plays a role.

Figure 5 shows the predicted path for net income and for the balance in the D account after a 5-year crisis as in the last section. For these calculations, we assume the ECB receives no payments from the euro governments when the bank's net income is negative, and that the

Exogenous model inputs (in percents)

State number	*Safe rate, r*	*Direct bond holdings, qBd*	*Repos, qBr*	*Currency, N*	*Inflation, p'/p-1*	*Foreign asset returns e-1*
1	1.28	6.07	4.87	6.29	2.10	4.79
2	-0.23	12.21	8.39	9.37	1.72	6.14

Coupons and endogenous bond prices, q

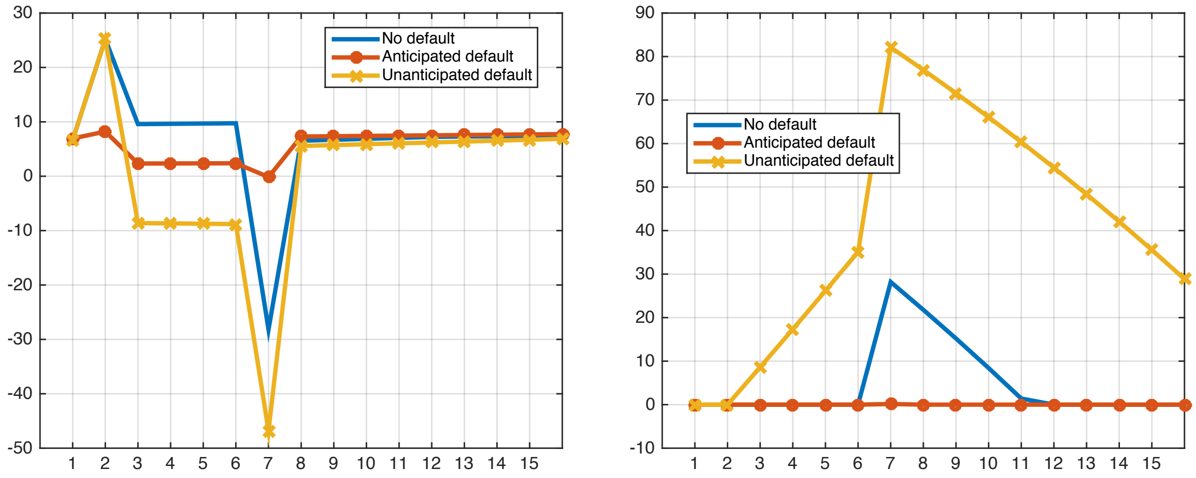
State number	*Coupon, c*	*Anticipated default, q*	*Unanticipated default, q*
1	1.00	4.18	4.19
2	0.86	4.19	4.37

Table 3: Inputs for the ECB

ECB pays no dividend after an episode of negative income until its D balance is exhausted. The figure shows three cases. In the first case, there is no default ($c_S = 1$) in the crisis, just as we observed in the data, whereas in the other two, coupons are impaired. The difference between them is that, in one, which we call *unanticipated default*, the prices of the bonds do not reflect the possibility of a default. In the second case, *anticipated default*, the prices of the bonds in both periods reflect the rational expectation that the economy may enter state 2 and default occurs. The first case tries to capture the possibility that the default comes as a surprise to the markets, that bond prices did not anticipate, whereas in the second case the central bank would have bought the bonds that fueled the expansion in its balance sheet at low prices, given their imminent default.

When there is no default, the source of risk to the financial stability of the ECB is similar to that for the Fed. When the economy exits the crisis, assets prices fall, and the ECB's sales of €592 billion of its assets comes with a capital loss of €28 billion. The deferred account has a positive balance for 5 years.

With an anticipated default, on the one hand, the flow of coupon income is lower than before, and no longer covers the depreciation of the bond as before, lowering net income. On the other hand, the lower coupon implies that bond prices during the crisis are lower, so the capital losses when exiting the crisis are smaller. The two effects close to cancel out so that, both during the crisis as well as upon exit, the net income of the ECB is close to zero. This



(a) Net income

(b) Balance in D account

Figure 5: The ECB's Finances under Three Default Scenarios

confirms our previous conclusion that capital gains and losses in the central bank's portfolio from asset price movements is the main source of stability risk.

With an unanticipated default though, the central bank now both suffers the losses from the impaired coupons during the crisis, and the capital losses at the time of exit from the crisis. In this case, the balance in the deferred account is considerable, peaking at €82 billion and it takes many years to pay it off.

4.4 Institutional remedies: repo holdings

The baseline scenario considered only the direct holding of securities by the ECB. The ECB has traditionally conducted its monetary policy using repurchase agreements with banks. These repos are effectively term-lending to commercial banks collateralized by government bonds or government-guaranteed securities. The ECB requires collateral in excess of the value of the loan, by as much as 15 percent (the haircut), and frequently revalues its collateral using market prices and requires extra collateral to be posted to lower its credit risk. A credit loss from a repo involves both default by the borrowing bank and an impairment of the value of the collateral in excess of the haircut. The ECB would suffer credit losses on its repos only in the event of a general financial meltdown in the euro countries serious enough to extinguish the haircut and cause the borrowing bank to default. Default would result in reorganization of the bank and its exclusion from future borrowing from the ECB.

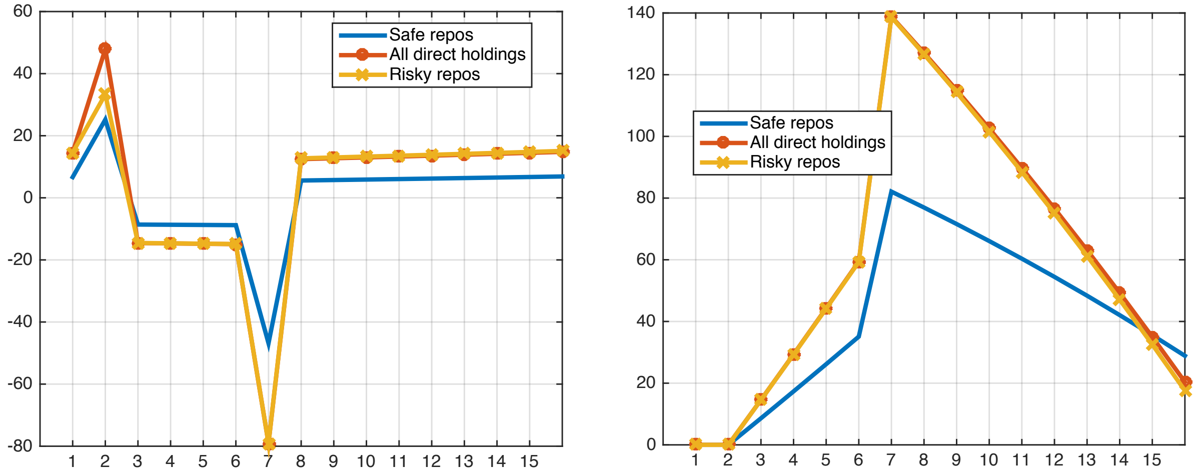
Repos have floating rates indexed to the one-week repo rate, so the relevant interest rate is still the short-term safe rate r_s. We let $q_s B_s^d$ denote the real value of the securities held directly and $q_s B_s^r$ denote the real value of repos, so the total assets of the ECB are $B_s = B_s^d + B_s^r$. If a meltdown has low probability, we can treat repos as safe loans, with no risk of capital gains or losses to the ECB. In that case, repos have the same financial character as negative reserves. Issuing reserves and investing the proceeds in repos has a neutral effect on stability because this asset and liability earns exactly the same interest rate. Likewise, any coupon payments or capital gains and losses stay with the bank, not the ECB. Therefore, all the formulas from section 2 still apply as long as we replace B_s with B_s^d and V with $V - q_s B_s^r$.

At the other extreme, with a significant probability of meltdown, the ECB faces the possibility of capital losses, when euro banks become insolvent and the ECB's collateral loses value by more than the haircuts. In this case, we need to take into account the asymmetry inherent in ownership of any debt claim, that the owner will never receive more than face value but may receive less. Letting $\mathbb{I}$ be an indicator for the bank's failure to pay off, net income under the nominal mark-to-market rule becomes:

$$\begin{aligned} d_{s'} &= \left(\theta c_s + (1-\theta)e_{s'} + (1-\delta)q_{s'} - \frac{q_s}{1+\pi_s}\right) B_s^d - \frac{i_s(V - B_s^r)}{1+\pi_s} \\ &\quad + \mathbb{I}\left(\theta c_s + (1-\theta)e_{s'} + (1-\delta)q_{s'} - \frac{(1+i_s)q_s}{1+\pi_s}\right) B_s^r \end{aligned} \tag{31}$$

The term preceded by $\mathbb{I}$ reflects the ECB's capital loss if the bank fails to pay off the repo, but also no longer pays the interest rate due on the repo contract. Our earlier analysis of the conditions when reserves are stationary continues to apply. But in terms of our simulations that follow, the change when banks fail to pay off on repos on impaired assets can be substantial.

Before 2008, the duration of the repos was mostly one week, with the longest being 3 months, so it would take extremely wide price movements in a short period of time for a bank to become insolvent and not honor its repos with the ECB. Since 2008, 6-month and 12-month repos became dominant, and in December 2011 and February 2012, the ECB lent €1 trillion through 3-year long-term refunding operations, effectively three-year repos. Taking a worst case scenario, we assume that $\mathbb{I} = 1$ if the return from holding the bonds is negative. That is, banks do not pay off the repo when the economy exits the crisis.



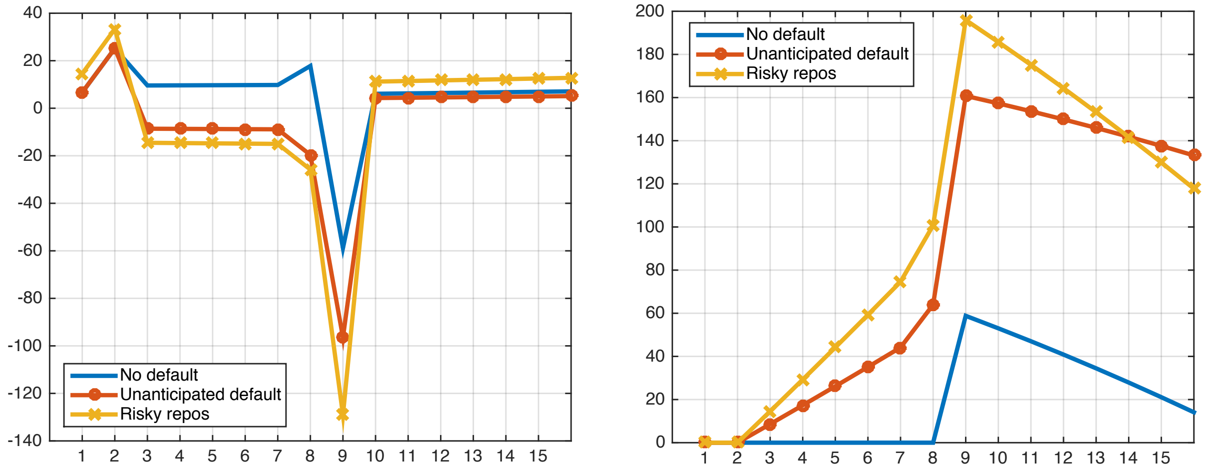
(a) Net income

(b) Balance in the D account

Figure 6: The ECB's Finances under Different Types of Bond Holdings

Figure 6 shows the net income and reserve borrowings of the ECB under different perspectives on the repos. In the first case, "safe repos", we take the view that only bonds held directly lead to risk, as in the previous figure. This scenario corresponds to $\mathbb{I} = 0$ in the law of motion. In the second case, we take the other extreme view that all of the bonds held by the ECB are direct holdings, whether they are collateral for repos or not, in the same way we did for the Federal Reserve. Finally, in the third scenario, "risky repos", we take the more realistic asymmetric case with $\mathbb{I} = 1$. The ECB bears capital losses in excess of haircuts, but not gains, on the bonds that it holds as collateral. For all cases, we assume the case of unanticipated default, since this was the one for which losses were larger in our previous experiment.

If banks always perform on their repurchase agreements, the ECB 's financial stability suffers little risk, as we discussed above. If, instead, the banks fail on their repos when asset prices fall, the ECB receives the collateral. The losses are €76 billion. Compared to the case were the ECB held all securities directly, with the repos, the ECB earns less while entering and staying in the crisis, because the positive capital gains on the bonds stay with the banks. But, when there are large capital losses, then the possibility that repos are honored significantly lowers the risk to the ECB's stability.



(a) Net income

(b) Balance in D account

Figure 7: The ECB's Finances after Quantitative Easing

4.5 Quantitative easing

Initially, in response to the crisis, the ECB expanded its repo program by both extending its maturity and increasing its size. These long-term repurchase obligations (LTRO) extended the usual procedures of operation of the ECB although, by continuing to accept government securities that most credit agencies rated as junk and doing so for longer periods of time, some of these new repos were farther from the safe repo case and closer to risky repos. At the same time, the ECB also bought securities directly through the Outright Monetary Transactions program (OMT), accounting for the increase in direct holdings in Table 6.

At the start of 2015, the ECB announced a new policy, the Public Sector Purchase Programme, to buy €1.08 trillion in government securities. This policy of quantitative easing will roughly double the value of securities held directly by the ECB. To investigate the effect of this policy on the financial stability of the ECB, we add a third state to the model. It is the same as the crisis state, except that the ECB's direct bond holdings double relative to the crisis state. This state lasts for two years, before the economy finally exists the crisis, back into state 1. Figure 7 describes the effect of this policy on the financial position of the ECB.

If there is no default, so that the securities that the ECB plans to buy turn out to be safe, the larger balance sheet implies that capital losses at the time of the exit are now significantly larger. As a result, the ECB accumulates a balance in its D account that only disappears 9 years after the exit. If coupons turn out to be unexpectedly impaired, the losses are much larger, and the deferred account peaks at €196 billion, taking a very long time to recover. Engaging in quantitative easing raises the risk of instability relative to enlarging the repo program. Of course, this may be more than offset by potential benefits of this policy in pursuing the ECB's policy goals.

5 A Small Open Economy Central Bank and Exchange-Rate Risk

The assets of most central banks of small open economies include neither repo contracts nor holdings of long-term bonds. Rather, gold and short-term foreign government debt are the dominant assets, and currency and reserves are the main liabilities. For these banks, $\delta = c_s = 1$, but the share of foreign-denominated assets is large, so θ is close to zero.

5.1 Exchange-rate risk and institutions

Relative to the Fed and the ECB, the accounts of the central bank of a typical small open economy are easy to read. Because foreign short-term goverment debt is easy to value, the way in which net income is assessed is close to our mark-to-market rule. And because monetary operations involve exchanging foreign debt for domestic reserves, their assets are directly held, without the complications of repo contracts. Therefore, the net income for a typical, small open-economy central bank is:

$$d_{s'} = n_{s,s'} + r_s(\mathbb{V}(e_{s'})B_s - V_s) + [e_{s'} - (1 + r_s)\mathbb{V}(e_{s'})])B_s. \tag{32}$$

As well as seignorage and the return on its holdings of domestic bonds, the central bank earns a return on its portfolio of foreign assets. It is positive if the exchange rate depreciation ($e_{s'}$) is larger than was expected.

When the foreign exchange holdings (B_s) are small, net income is close to seignorage, which is typically positive. Starting with the Asian financial crisis of the 1990s, many central banks started accumulating large amounts of foreign assets. Combined with the large volatility of exchange rate, this exposure leads these economies to experience negative net income. Klüh and Stella (2002) and more recently Filardo and Grenville (2012) discuss many of these cases in the past thirty years.

These central banks sometimes experience negative net income, so we might expect that they are more likely to take precautions to prevent unstable growth of reserves. This proposition turns out to be true. For instance, the central bank of Mexico has the power to limit dividends if there is a need to retain earnings against possible future losses due to movements in exchange rates, or to bring down reserves after past unexpected losses and restore accounting capital to positive levels. Between 2009 and 2014, the Banco de Mexico paid no dividend in spite of positive net income.

By contrast, the current charter of the Banco do Brasil provides that if net income is negative, the fiscal authority must pay the bank the amount of the negative income. If this commitment is truly unlimited, the Banco de Brasil would always be solvent as long as the overall government is solvent—an application of our proposition 1.

Relative to the Fed and the ECB, the dividend rules of the central banks of most small open countries have more power to limit dividends when net income is positive, following earlier periods of negative net income, just as in our model of the deferred account. Accordingly, in spite of frequent losses, these banks' reserves will be stable.

5.2 Exchange-rate pegs and financial stability

The combination of pegged exchange rates and large balance sheets in the form of foreign reserves constitutes the new style of central banking for most small open economies. In the typical case, the central bank tries to prevent the exchange rate from depreciating, by selling foreign reserves. Insofar as this move is unexpected, the exchange rate depreciates by less than what was expected by markets. Real interest rates rise as the central bank defends the peg, and these higher expenses in paying the return on reserves are not offset by gains in the foreign currency portfolio. Therefore, $e_{s'} - (1 + r_s)\mathbb{V}(e_{s'}) < 0$ and the central bank realizes losses. These central banks hold high B_s to defend the peg.

Central banks are at risk of accumulating losses in the process of defending unrealistically high pegs. Eventually, the peg is abandoned. When this happens, the large depreciation of the currency would lead to large capital gains. But, by then, typically the central bank is left with foreign bond holdings close to zero, so the gain ends up being minimal. The capital gain does not offset the previous losses. A pattern of successive defense of unrealistic pegs may lead to unstable growth of reserves.

A more unusual case arose out of the Swiss National Bank's actions in 2015, when the SNB unexpectedly dropped an earlier ceiling on its exchange rate with the euro. The Swiss franc appreciated quickly, and the SNB realized a large loss. Our framework helps to understand this decision from three perspectives. First, the announcement of QE by the ECB led to the anticipation that the SNB's holdings of euro-denominated debt would increase even further to defend the peg. Second, the exchange rate of the euro-pegged Swiss franc relative to the dollar had been depreciating steadily, leading to the expectation that a future abandonment of the peg would come with a very large appreciation relative to the euro. Third, the reserve debt of the SNB was already close to the size of the GDP of Switzerland, so that there was limited fiscal room for either the Treasury to transfer funds or even for future net income to provide for present losses. Therefore, in early 2015, the SNB, given its intention of sooner or later abandoning the peg to the euro, could anticipate that if it waited just a few more months to do so, it would face a more negative $e_{s'} - (1 + r_s)\mathbb{V}(e_{s'})$, multiplying a much larger B_s, generating larger losses $d_{s'}$, which neither the Treasury not future seignorage might be able to offset. The SNB had to abandon the peg immediately to prevent a large expansion of its outstanding reserves and the risk of insolvency.

A final interesting case connected to pegs arises when a small open economy in a developing country successfully engages in an inflation stabilization program. If this was partly unexpected, it leads to an appreciation of the exchange rate. Central-bank losses would ensue. Central bank reserve expansion may be a perverse consequence of central bank success at controlling inflation.

6 Concluding Remarks

We have provided a systematic analyses of central bank financial health. We have applied a reasonable definition of central bank financial stability as the achievement of stationary levels of the bank's reserve borrowings from commercial banks. We went on to study what affects stability, and when is it in danger. After describing the resource constraint of the central bank, we showed that if that rule is to pay a dividend equal to net income, the central bank is always solvent. However, under new-style central banking, that is, with central banks that pay interest on reserves, have large balance sheets, and have exposure to interest-rate, default, and exchange-rate risk, net income will sometimes be negative. This rule requires periodic payments from the government to the central bank. Without this direct fiscal backing or its equivalent, in the form of programmed reductions in future dividends following an episode of negative income, the central bank could be on an unstable path of endlessly borrowing in the form of reserves.

We studied the workings of a deferred account to keep reserves under control. We found that its balance provides an operational measure of the financial strength of the central bank. We then showed how different institutions, such as not marking assets to market, holding them as repos, or being able to build provisions against future losses may work to increase the financial strength of the central bank. We showed how to make our theory applicable to most central bank finances, and how to use it to inform policy choices, such as when to manage the exit from U.S. QE, to measure the effects of European QE, and to abandon the peg of the Swiss franc to the euro.

Our analysis keeps real interest rates (r_s), central bank asset holdings (B_s), and inflation (π_s) at given values, as background for our study of the evolution of the central bank's net income (y), reserves (V), and financial strength (D) for a rule on dividends (d). In our applications, we studied how differences in each of the real economy, financial policy or inflation policy affected the financial stability of the central bank. Future work can build on our results to jointly study financial stability and each of these three policies. Our results already provide initial guidance on all three fronts.

Incorporating the determination of r_s would involve models of monetary non-neutrality—the fact that central-bank policies that set nominal interest rates have effects on real activity. There is already some research in the past two years that uses our framework to do this; we describe it in appendix A.

This paper provides some first steps in designing financial policy, as captured in our bond-holding variable, B_s. We showed that capital gains and losses on these bond holdings are the main issue in central-bank stability. Larger asset holding, longer duration of bonds, and more direct holdings instead of repos during the crisis increase the risks. Reducing the size of the portfolio before interest rates rise implies lower losses when the rise occurs. These effects of different bond-holding policies on financial stability may or may not be offset by benefits that bond purchases by the central bank may have on financial markets. Research has been active recently in quantifying those benefits

Finally, with respect to inflation policy π_s, we found that higher inflation improves financial stability by both increasing seignorage as well as by raising the rate at which the extra reserves from previous shortfalls fall in their real value. Still, we found that even small losses for the Fed would require very large increases in inflation.

References

Amtenbrink, Fabian, "Securing Financial Independence in the Legal Basis of a Central Bank." In Milton and Sinclair (2010) pp. 83–95.

Archer, David and Paul Moser-Boehm, "Central Bank Finances," BIS Papers no 71 April 2013.

Bank for International Settlements, *Are Central Bank Balance Sheets in Asia Too Large?* BIS Papers no 66, Bank for International Settlements, October 2012.

Bassetto, Marco and Todd Messer, "Fiscal Consequences of Paying Interest on Reserves," *Fiscal Studies*, December 2013, *34*, 413–436.

Benigno, Pierpaolo and Salvatore Nistico, "Central Bank Balance Sheet, Liquidity Trap, and Quantitative Easing," 2015. LUISS Guido Carli and Sapienza University of Rome.

Berriel, Tiago and Saroj Bhattarai, "Monetary Policy and Central Bank Balance Sheet Concerns," *Contributions to Macroeconomics*, 2009, *9* (1), 1.

Berriel, Tiago C. and Arthur Mendes, "Central Bank Balance Sheet, Liquidity Trap, and Quantitative Easing," 2015. PUC-Rio.

Bhattarai, Saroj, Gauti Eggertsson, and Bulat Gafarov, "Time Consistency and the Duration of Government Debt: A Signalling Theory of Quantitative Easing," 2014. UT Austin and Brown University.

Buiter, Willem H., "Seignorage," NBER Working Paper 12919 February 2007.

Cagan, Phillip, "The Monetary Dynamics of Hyperinflations," in Milton Friedman, ed., *Studies in the Quantity Theory of Money*, The University of Chicago Press, 1956, pp. 25–117.

Carpenter, Seth B., Jane E. Ihrig, Elizabeth C. Klee, Daniel W. Quinn, and Alexander H. Boote, "The Federal Reserve's Balance Sheet and Earnings: A Primer and Projections," 2013. Finance and Economics Discussion Series 2013-03.

Christensen, Jens, Jose Lopez, and Glenn Rudebusch, "A Probability-Based Stress Test of Federal Reserve Assets and Income," *Journal of Monetary Economics*, 2015, *forthcoming.*

Cukierman, Alex, "Central Bank Independence and Monetary Policymaking Institutions: Past, Present and Future," *European Journal of Political Economy*, 2008, *24* (4), 722 – 736.

Dalton, John and Claudia Dziobek, "Central Bank Losses and Experiences in Selected Countries," April 2005. IMF Working Paper 05-72.

Del Negro, Marco and Christopher A. Sims, "When Does a Central Bank's Balance Sheet Require Fiscal Support?," *Journal of Monetary Economics*, 2015, *forthcoming.*

Filardo, Andrew and Stephen Grenville, "Central Bank Balance Sheets and Foreign Exchange Rate Regimes: Understanding the Nexus in Asia," in Bank for International Settlements, ed., *Are Central Bank Balance Sheets in Asia Too Large?*, BIS Papers No 66, Bank for International Settlements, October 2012, pp. 76–110.

Goodfriend, Marvin, "Why We Need an 'Accord' for Federal Reserve Credit Policy: A Note," *Journal of Money, Credit and Banking*, August 1994, *26* (3), 572–80.

——, "Monetary Policy as a Carry Trade," *Monetary and Economic Studies*, November 2014, *32*, 29–44.

Greenlaw, David, James D. Hamilton, Peter Hooper, and Frederic S. Mishkin, "Crunch Times: Fiscal Crises and the Role of Monetary Policy," NBER working paper 19297 August 2013.

Hall, Robert E., "The Long Slump," *American Economic Review*, April 2011, *101* (2), 431–69.

——, "Fiscal Stability of High-Debt Nations under Volatile Economic Conditions," *German Economic Review*, February 2014, *15* (1), 4–22.

Hilscher, Jens, Alon Raviv, and Ricardo Reis, "Measuring the Market Value of Central Bank Capital," 2015. Brandeis University and Columbia University.

Ize, Alain, “Spending Seigniorage: Do Central Banks Have a Governance Problem,” *IMF Staff Papers*, December 2007, *54* (3), 563–589.

Klüh, Ulrich and Peter Stella, “Central Bank Financial Strength and Policy Performance: An Econometric Evaluation,” International Monetary Fund Working Paper 08-176 2002.

Leone, Alfredo, “Institutional and Operational Aspects of Central Bank Losses,” in “Frameworks for Monetary Stability: Policy Issues and Country Experiences,” International Monetary Fund, 1994.

Milton, Sue and Peter Sinclair, *The Capital Needs of Central Banks*, Routledge, 2010.

Quinn, Stephen F. and William Roberds, “Death of a Reserve Currency,” FRB Atlanta Working Paper 2014-17 September 2014.

Reis, Ricardo, “Central Bank Design,” *Journal of Economic Perspectives*, Fall 2013, *27* (4), 17–44.

____ , “The Mystique Surrounding the Central Bank’s Balance Sheet, Applied to the European Crisis,” *American Economic Review*, May 2013, *103* (3), 135–140.

____ , “Different Types of Central Bank Solvency and the Central Role of Seignorage,” *Journal of Monetary Economics*, 2015, *forthcoming.*

Ross, Steve, “The Recovery Theorem,” *The Journal of Finance*, 2015, *70* (2), 615–648.

Sims, Christopher A., “Fiscal Aspects of Central Bank Independence,” February 2003. Princeton University.

Stella, Peter, “Do Central Banks Need Capital?,” International Monetary Fund Working Paper 97-83 1997.

____ , “Central Bank Financial Strength, Transparency and Policy Credibility,” *IMF Staff Papers*, 2007, *52* (2), 335–365.

Vergote, Olivier, Werner Studener, Ioannis Efthymiadis, and Niall Merriman, “Main Drivers of the ECB Financial Accounts and ECB Financial Strength over the First 11 years,” ECB Occasional Paper Series no 111 May 2010.

Appendices

A Survey of the Literature

One strand of work has studied the accounting principles and the history of central banks around the world to document when the accounting capital of the central bank is negative. Milton and Sinclair (2010) is a useful starting point, and Dalton and Dziobek (2005), Ize (2007) and Leone (1994) are earlier examples. This literature struggles to explain conceptually why central bank capital matters and why central banks operate with negative capital so often. Our theoretical framework explains why balance-sheet capital *per se* does not determine the stability of a central bank, but that we can use it as an imperfect measure of the amount Z by which the central bank would have to be funded to return reserves to their stationary path. Moreover, our study of different institutions and our formal treatment of the determinants of central bank stability complement this literature's collection of case studies.

Stella (1997), Stella (2007), and Klüh and Stella (2002) discuss whether measures of central bank accounting capital constrain monetary policy actions. Berriel and Bhattarai (2009) formalize the effects that a concern for its capital will have on the interest rates chosen by a central bank, while Bhattarai, Eggertsson and Gafarov (2014), Berriel and Mendes (2015), and Benigno and Nistico (2015) discuss how the maturity composition of the balance sheet affects the ability of the central bank to commit to future monetary policies and to keep inflation on target. Our approach is the opposite, and complementary to these papers. They take as given the stability of the central bank and study its effects on interest rates and inflation. We take interest rates and inflation as given and study the implications for stability.

We focus on the solvency of the central bank from the perspective of central bank independence and define it as generating a state-dependent, non-explosive path for reserves. Buiter (2007) and Sims (2003) also discuss the separation between the Treasury and the central banks' resource constraints in terms of central bank independence. We take the next step by showing that the dividend rule connecting these two branches of the government defines the financial stability of the central bank and by studying the effect of different rules. Reis (2015) proceeds by discussing different perspectives on central bank solvency in terms of different constraints on dividends.

A complementary approach sprang up after we circulated the first draft of this paper. It assumes that the dividend rule is that the present value of dividends must be non-negative. In this case, the central bank will remain independent of the fiscal authorities and solvent as long as it satisfies an intertemporal budget constraint, as emphasized by Reis (2013b). Bassetto and Messer (2013) discuss how the payment of interest on reserves affects this intertemporal constraint. Del Negro and Sims (2015) study how inflation may have to change to generate the seignorage revenue that will keep the central bank solvent. Hilscher et al. (2015) measure the present value of the central bank dividends and estimate by how much it changes with inflation. Quinn and Roberds (2014) present the fascinating case of the Bank of Amsterdam in 1781-1792, which became intertemporally insolvent after significant losses, leading to the disappearance of the Dutch bank florin, following the process that we described in section 2.

A series of papers studied the particular risk for the net income of the Federal Reserve of its recent balance sheet. Carpenter, Ihrig, Klee, Quinn and Boote (2013) and Greenlaw, Hamilton, Hooper and Mishkin (2013) present statistical projections for components of the balance sheet of the Federal Reserve, while Christensen, Lopez and Rudebusch (2015) use a dynamic term structure model to obtain precise estimates of the interest-rate risk facing the Fed. Our two conclusions from section 3 agree with theirs: the main risk facing the Fed is interest-rate risk, and it materializes into lower net income when exiting the crisis. While our calculations are more imprecise and take a worst case scenario, these other authors carefully calculate plausible estimates. At the same time, they do not answer why negative net income would matter, while we provide a theoretical analysis. Also, while their applied study is relevant only for the Fed, we provide a general theory that applies to any central bank.

Outside of the United States, a considerable literature had documented how the exposure to exchange-rate risk often led central banks to record negative net income. Vergote, Studener, Efthymiadis and Merriman (2010) discusses the first eleven years of the ECB and the Bank for International Settlements (2012) the experience of Asian countries pegging exchange rates. We provide a theoretical framework to understand these episodes.

Some papers have studied the institutions that may affect the solvency of the central bank. Archer and Moser-Boehm (2013), like us, emphasize the central role of the rule for dividends in determining the financial stability of the central bank, and provide a rich discussion of the institutions in place at many central banks. Goodfriend (1994) discusses an exceptional transfer from the Fed to the Treasury in the early 1990s and how it compromised central bank independence. Goodfriend (2014) starts from our warning that the Fed will likely have negative net income, and suggests that it should provision its earnings—that is, pay down some reserves so as to create a buffer of borrowing power to deal with future situations where net income is negative. Reis (2013a) makes the more radical proposal to sever completely the resource link between the central bank and the Treasury by having the central bank dividends paid to a public trust fund that supports a public good, like basic research in the social sciences.

B Multiple Central Bank Investments

Consider a central bank that can split its investment at date t between long-term infinitely-lived bonds B_t^L, short-term one-period bonds B_t^S, and short-term foreign assets B_t^F. Their respective prices are q_t^L, q_t^S and ϵ_t, where the latter is the real exchange rate for the foreign bond. The total value of the portfolio today then is:

$$q_t^L B_t^L + q_t^S B_t^S + \epsilon_t B_t^F \equiv q_t B_t,$$

where $q_t B_t$ is the defined total value of the portfolio.

The payoff of the portfolio one period later is:

$$(\tilde{c}_t + q_{t+1}^L) B_t^L + B_t^S + \epsilon_{t+1} B_t^F.$$

since $\tilde{c}_t$ is the coupon on the long-term bond, while the short-term bond and the foreign bond have no coupon. This will match our formulation in equation (7) if it equals:

$$q_{t+1}[\theta c_t + (1-\theta) e_{t+1} - (1-\delta)] B_t.$$

One can verify that this is true as long as:

$$\begin{aligned}
\theta_t &= 1 - \frac{\epsilon_t B_t^F}{q_t B_t}, \\
\delta_t &= \frac{q_t^S B_t^S}{\theta_t q_t B_t}, \\
q_t &= q_t^L, \\
e_{t+1} &= \frac{\epsilon_{t+1} q_t}{\epsilon_t}, \\
c_t &= \tilde{c}_t (1-\delta_t) + \frac{\delta_t q_t}{q_t^S}.
\end{aligned}$$

From state number	*Transition probabilities to state*					*Stationary proba-bilities*
	1	*2*	*3*	*4*	*5*	
1	0.429	0.214	0.214	0.071	0.071	0.237
2	0.308	0.308	0.385	0.000	0.000	0.220
3	0.231	0.308	0.154	0.308	0.000	0.220
4	0.071	0.143	0.143	0.643	0.000	0.237
5	0.000	0.000	0.200	0.000	0.800	0.085

Table 4: Transition Matrix and Stationary Distribution of States of the U.S. Economy

Therefore, the relevant assumption is that θ_t and δ_t are constant over time, or in other words, that the central bank keeps the portfolio shares of short-term, long-term and foreign bonds constant. If this condition holds, the artificial delta composite bond in the main text will accurately capture the payoffs of this more complicated and realistic portfolio.

C The US Data and Simulations

Table 4 shows the estimated transition matrix among the states of the model and the stationary distribution across the states. The subjective discount factor β in the model is 0.985.

Applying our procedure to determine bond prices gives the results in Table 5. Notice that marginal utility is quite low in the crisis state, 5, corresponding to a high level of consumption in that state. To rationalize the low real interest rate in the crisis state in terms of the consumption Euler equation, it must be the case that consumption is expected to fall when in the crisis. Accordingly, consumption in the crisis must be unusually high. Obviously aggregate consumption does not behave this way across our states. Hall (2011) discusses how low real interest rates in the crisis might be consistent with the consumption paths of a limited group of wealthier consumers who participate in securities markets. Most consumers, in that view, are at a corner in their intertemporal consumption problems, so the slope of their consumption profiles does not contribute to asset pricing. He points to data from the Survey of Consumer Finances to support this hypothesis.

Figure 8 shows the nominal interest rate and the nominal bond price over our simulation period.

State	*Safe rate,* r	*Coupon,* c	*Marginal utility,* μ	*Bond price,* q	*Nominal rate,* i
1	0.043	1.000	1.000	6.711	0.075
2	0.019	1.000	0.987	6.834	0.047
3	0.013	1.000	0.966	6.958	0.047
4	-0.006	1.000	0.926	7.225	0.059
5	-0.015	1.000	0.839	7.643	0.004

Table 5: Prices and Returns in the U.S. Data

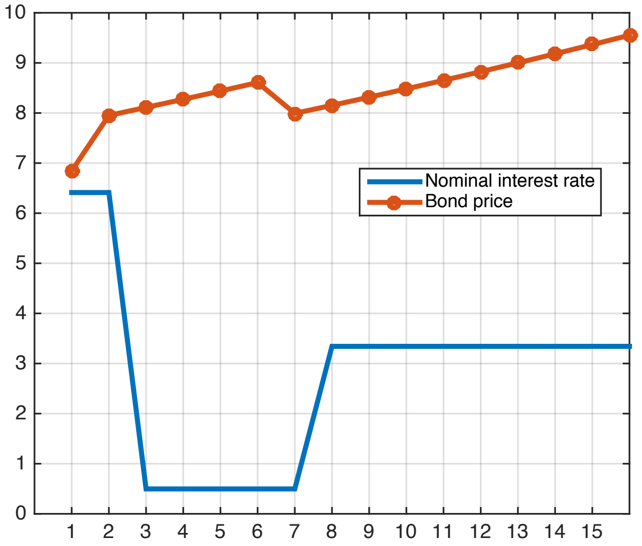


Figure 8: U.S. Interest Rate and Bond Price in Simulation

From state number	To state: 1	To state: 2	Stationary proba-bilities
1	0.983	0.017	0.919
2	0.198	0.802	0.081

Table 6: Transition Matrix and Stationary Distribution for the Euro-Area

State	Safe rate, r	Coupon, c	Foreign returns, e	Marginal utility, μ	Bond price, q	Nominal rate, i
1	0.013	1.000	1.048	1.000	4.182	0.034
2	-0.002	0.858	1.061	0.934	4.194	0.015

Table 7: Prices and returns for the Euro-Area

D The Euro-Area Data and Simulations

Table 6 shows the transition matrix and the stationary distribution for the Euro-Area data. We derived the matrix by assuming a stationary probability of 0.081 for the crisis state, and a conditional probability of entering a crisis of 0.016, the numbers for the United States.

Table 7 shows the financial environment that results from our pricing exercise at the ergodic distribution. This is in the case where default happens and is anticipated by agents, so it is reflected in bond prices. In turn, Table 8 shows the corresponding ergodic distribution of dividends, which is positive in all states.

Figure 9, Figure 10, and Figure 11 show the simulation results for the ECB under our crisis scenario and with unanticipated defaults. We set the inflation target to 0.5 percent so that the nominal interest rate is close to zero in the crisis state. The results are similar to the ones for the United States.

From state number	To state number: 1	To state number: 2
1	0.209	0.222
2	0.116	0.142

Table 8: Dividends in the Ergodic Distribution for the Euro-Area

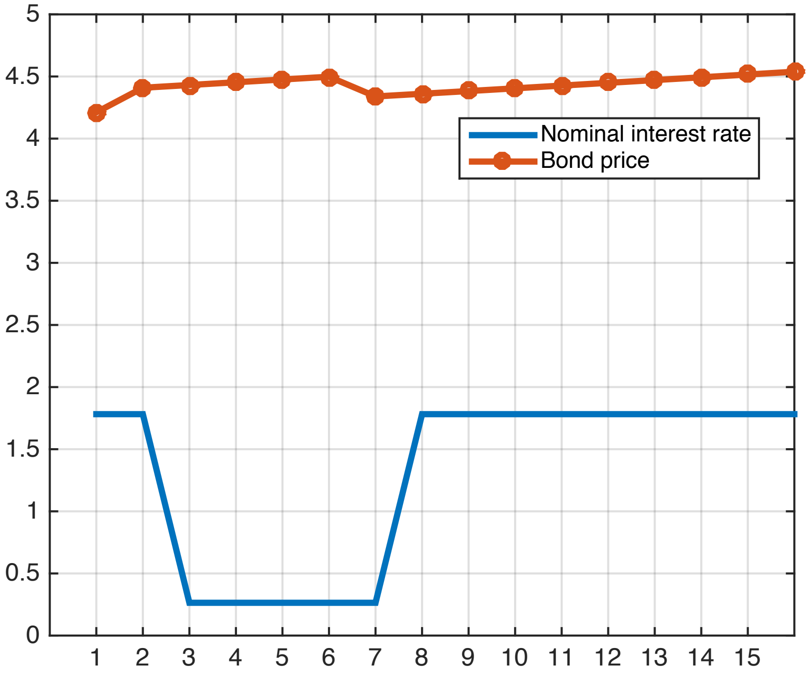


Figure 9: Interest Rate and Bond Prices in the Euro-Area

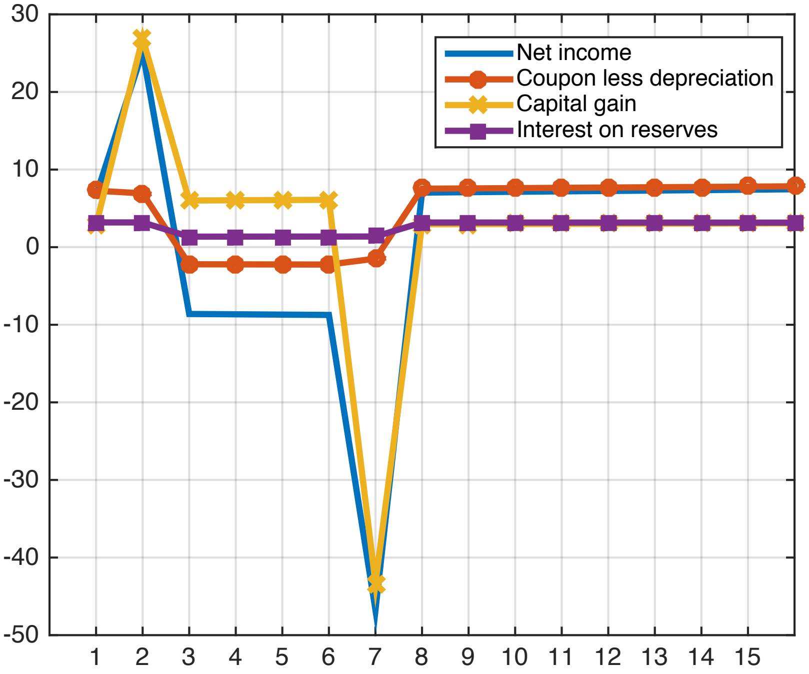


Figure 10: Components of the ECB's Dividend

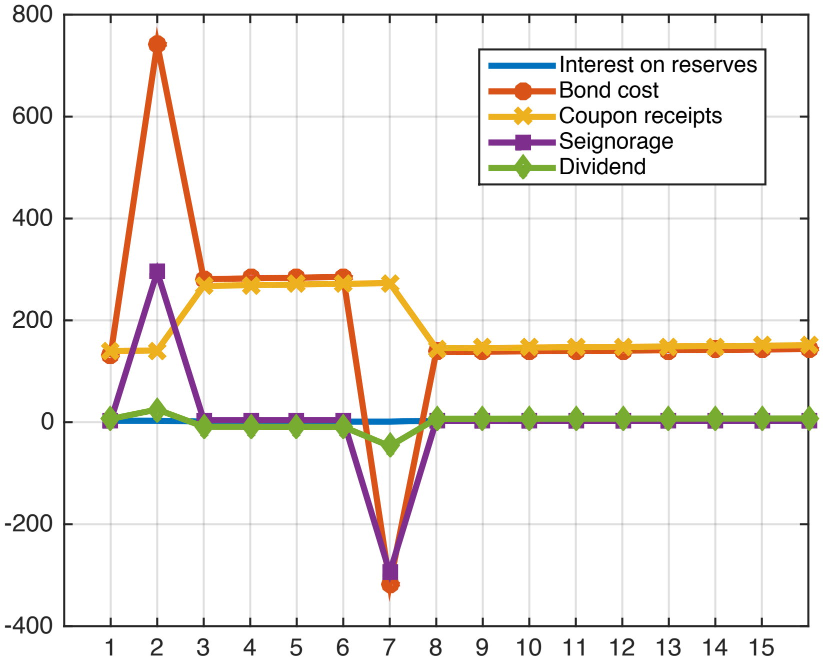


Figure 11: Flows into and out of ECB Reserves