The Sovereign-Bank Diabolic Loop and ESBies*

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Abstract

We propose a simple model of the sovereign-bank diabolic loop, and establish four results. First, the diabolic loop can be avoided by restricting banks domestic sovereign exposures relative to their equity. Second, equity requirements can be lowered if banks only hold senior domestic sovereign debt. Third, such requirements shrink even further if banks only hold the senior tranche of an internationally diversified sovereign portfolio known as ESBies in the euro-area context. Finally, ESBies generate more safe assets than domestic debt tranching alone; and, insofar as the diabolic loop is defused, the junior tranche generated by the securitization is itself risk-free.

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The "diabolic loop" or nexus between sovereign and bank credit risk was the hallmark of the 2009-12 sovereign debt crisis in the periphery of the euro area. In Greece, Ireland, Italy, Portugal, and Spain, the deterioration of sovereign creditworthiness reduced the market value of banks' holdings of domestic sovereign debt. This reduced the perceived solvency of domestic banks and curtailed their lending activity. The resulting bank distress increased the chances that banks would have to be bailed out by their (domestic) government, which increased sovereign distress even further, engendering a "bailout loop". Moreover, the recessionary impact of the credit crunch led to a reduction in tax revenue, which also contributed to weakening government solvency in these countries, triggering a "real-economy loop". These two concomitant feedback loops are illustrated in Figure 1.

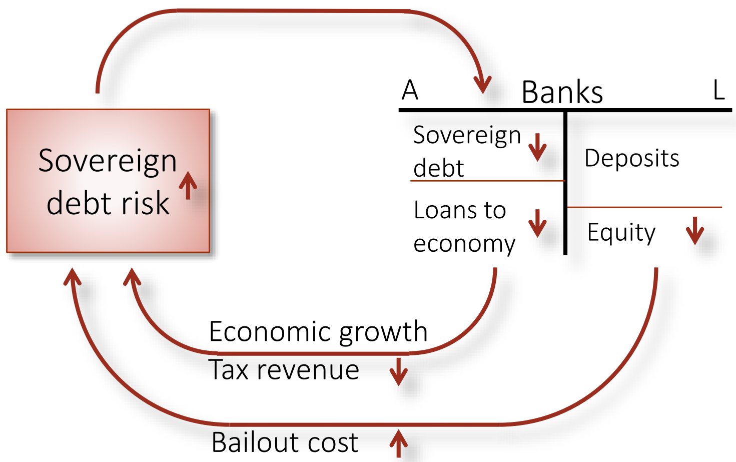

Figure 1: Two Diabolic Loops. Source: Brunnermeier et al. (2011)

There are three ingredients to these feedback loops. First, the home bias of banks' sovereign debt portfolios, which makes their equity value and solvency dependent on swings in the perceived solvency and market value of their own government's debt (Carlo Altavilla, Saverio Simonelli and Marco Pagano, 2015). Second, the inability of governments to commit ex-ante not to bailout domestic banks, since bailout is optimal once banks are distressed. Third, free capital mobility, which ensures that international investors' perceptions of future government solvency – whether warranted by fiscal fundamentals or not – are incorporated in the market value of domestic government debt. To break these loops, policy must remove at least one of these three ingredients. So far, capital controls are the only policy remedy adopted in response to the diabolic loop, in Cyprus and Greece.

In this paper we analyze the proposal by Brunnermeier et al. (2011), which aims to eliminate the diabolic loop by reducing the sensitivity of banks' sovereign debt portfolios to domestic sovereign risk. The proposal envisions that banks' sovereign bond holdings would consist mainly of the senior tranche of a well-diversified portfolio. This seniority structure could be achieved via a simple securitization, whereby financial intermediaries use a well-diversified portfolio of euro-area sovereign bonds to back the issuance of a senior tranche, labeled "European Safe Bonds" (or ESBies), and a junior tranche, named "European Junior Bonds" (or EJBies). ESBies would have very little exposure to sovereign risk, owing to the "double protection" of diversification and seniority: relative to a simple diversified portfolio of sovereign debt, ESBies would enjoy the additional protection provided by seniority. The impact of a sovereign default would be absorbed in the first instance by the junior tranche, which would not be held by banks. The creation of such a safe asset would be important both for financial stability and for the conduct of monetary policy (Brunnermeier and Yuliy Sannikov, 2015).

This paper shows that restricting euro-area banks to hold ESBies would effectively isolate banks from domestic sovereign risk, and thereby defuse the "diabolic loop" between sovereign and bank credit risk. Interestingly, both features of ESBies – diversification and seniority – are needed. On the one hand, the price of a diversified but not tranched sovereign debt portfolio would still depend on swings in the perceived creditworthiness of euro area governments, especially if they are correlated across countries due to a generalized "flight to quality". On the other hand, tranching the sovereign debt of an individual country does not produce enough safe domestic securities in countries with weaker fiscal positions or limited sovereign debt issuance. In contrast, performing the tranching on a large pool of imperfectly correlated sovereign bonds would generate a large stock of an essentially risk-free euro-area sovereign asset, the liquidity and safety of which would be attractive for both banks and non-banks.

Last but not least, the issuance of such a security would not require any form of "fiscal solidarity" among euro area governments: each government would remain entirely responsible for its own solvency, and the market price of its debt would remain a signal of its perceived solvency. This absence of joint liability stands in contrast to euro-bond proposals, such as the blue-red bond proposal by Jakob Von Weizsäcker and Jacques Delpla (2011).

1 One-Country Model

Consider a single country with stochastic tax revenue, resulting in a high or low primary surplus. We show that a "sunspot-driven" repricing of the country's sovereign risk can result in bailouts of banks or other systemic financial institutions, which can lead to sovereign default when the primary surplus turns out to be low. In the absence of such repricing, the government never defaults. Effectively, the sunspot acts as a selection device among two equilibria – one with bailout and possible default, and another with no bailout and no default. A key condition for the first equilibrium to exist – and hence for the diabolic loop to arise – is that banks hold a sufficiently large fraction of the stock of domestic sovereign debt.

There are four domestic agents. First, the government, which prefers higher to lower output, as this is associated with greater tax revenue. Second, dispersed depositors, which run on insolvent banks if the government does not bail them out, and also pay taxes. Third, bank equity holders, which use all of their capital for the initial equity, so they cannot recapitalize banks subsequently. Finally, investors in government bonds, whose beliefs determine the price of sovereign debt subject to a sunspot that may lead to repricing of sovereign risk. For simplicity, all agents are risk neutral and there is no discounting, so that the risk-free interest rate is zero. Short-term deposits yield extra utility compared to long-term government debt due to their convenience value in performing transactions.[1]

The model has four dates: 0, 1, 2, 3. All consumption takes place at the final date 3. At $t = 0$, the government issues a unit of a zero coupon bond at price B_0 with face value $\underline{S} > 0$, which is repaid probabilistically in the last period. The government primary surplus S (absent the diabolic loop) is low $\underline{S}$ with probability π and high $\overline{S} > \underline{S}$ with probability $1 - \pi$. We denote by B_t the price of the bond at each date t. Next, we denote by α the share of debt owned by banks in the original period, the remaining fraction $1 - \alpha$ being held by other risk-neutral investors. Hence, at time $t = 0$, banks hold αB_0 in sovereign debt on the asset side of their balance sheet, as well as an amount L_0 of loans to the real economy. On the liability side of their balance sheets are deposits D_0 and equity E_0.

At date $t = 1$ a sunspot occurs with probability p.[2] When a sunspot is observed, investors become pessimistic: they expect partial government default in the last period, which in equilibrium will be a true belief. Hence, the price of the government bond drops from B_0 to

[1] This is necessary to justify the demand for bank deposits backed by sovereign debt. Otherwise, banks would not need to hold sovereign debt.

[2] The sunspot carries no fundamental information about the primary surplus revealed in $t = 3$.

B_1 and banks suffer marked-to-market capital losses of $-\alpha(B_1 - B_0)$.[3] If this leads banks' equity to drop below zero, banks are insolvent. We assume that insolvent banks cannot roll-over maturing loans of size ψL_0. This is assumed to lead to an output loss, which lowers the government's tax revenue by $\tau\psi L_0 \geq 0$ at $t = 3$. At date $t = 2$ the government must decide whether to bail out banks, before discovering its actual tax revenue at $t = 3$. A bailout involves the issuance of additional government bonds, which are given to the banks as extra assets. If the government chooses not to bailout, a further ψL_0 of loans are not rolled-over, resulting in even lower tax revenues at $t = 3$.

Finally, at date $t = 3$, the government's fiscal surplus is realized. If no sunspot occurred, the surplus is just the stochastic variable S, while if the sunspot occurred at $t = 1$ *and* a bailout at $t = 2$, the surplus is $S - \tau\psi L_0 + \alpha(B_1 - B_0) + E_0 =: S - C$, where C is the implied (endogenous) bailout cost, which includes the tax loss due to credit crunch in $t = 1$.

We make three parametric assumptions. First, the government's primary surplus before bailout costs remains positive:

$$\underline{S} - \tau\psi L_0 \geq 0. \tag{A1}$$

Second, banks' aggregate equity is sufficiently small that the diabolic loop occurs at least if exposure is maximal ($\alpha = 1$):

$$E_0 < (1 - p)\,\pi\tau\psi L_0. \tag{A2}$$

Third, if the surplus is high, the government can still fully repay its debt even after a bailout at $t = 2$ for any α and any p (even for $\alpha = 1$ and $p = 0$):[4]

$$\overline{S} - \underline{S} \geq \frac{\tau\psi L_0 - E_0}{1 - \pi}. \tag{A3}$$

The Diabolic Loop The diabolic loop occurs if the fraction of sovereign debt held by banks exceeds a threshold or equivalently if banks' equity is below a critical level. When investors become pessimistic due to the sunspot, the price of sovereign debt drops, making banks insolvent. This prompts the government to bail them out, which precipitates default and justifies investors' pessimism.

When the primary surplus at $t = 3$ is $\underline{S}$, after a bailout the government can only pay $\underline{S} - C$. Therefore, the price of debt at $t = 1$ is $B_1 = \underline{S} - \pi C$, so $\pi C \equiv \Delta_1$ is the price discount relative to its face value $\underline{S}$. The price of the debt in period 0 is the probability-

[3]Note that even if banks' asset were not marked to market, the diabolic loop would still arise if depositors or other creditors panic as result of depreciation of banks' assets.

[4]This assumption is only used to simplify calculations, but can easily be relaxed.

weighted average of sunspot and no-sunspot prices: $B_0 = \underline{S} - \pi p C$, with a price discount $\pi p C \equiv \Delta_0 = p\Delta_1$. Recalling the definition of the bailout cost C and of prices B_0 and B_1, and noticing that $B_1 - B_0 = -(1-p)\Delta_1$, the discount at $t = 1$ is

$$\Delta_1 = \pi\left[\tau\psi L_0 - \alpha(B_1 - B_0) - E_0\right] \tag{1.1}$$

$$= \frac{\pi(\tau\psi L_0 - E_0)}{1 - \alpha\pi(1-p)}.$$

Hence, banks are left with negative equity if

$$\alpha(B_1 - B_0) + E_0 < 0 \tag{1.2}$$

$$\Leftrightarrow E_0 < \alpha(1-p)\pi\tau\psi L_0,$$

where the equivalence follows from

$$B_1 - B_0 = -\frac{(1-p)\pi}{1 - \alpha(1-p)\pi}(\tau\psi L_0 - E_0). \tag{1.3}$$

When banks are left with negative equity, the government bails them out if the capital shortfall is smaller than the cost $\tau\psi L_0$ of not bailing them out, i.e.,

$$\alpha(B_1 - B_0) + E_0 + \tau\psi L_0 > 0 \tag{1.4}$$

$$\Leftrightarrow E_0 > \left[2\alpha(1-p)\pi - 1\right]\tau\psi L_0.$$

where the equivalence follows from (1.3).

If banks' equity is below the threshold in (1.2), and the bailout condition (1.4) holds, then the sunspot leads to the diabolic-loop equilibrium. In this equilibrium, the price drop (1.3) is higher in absolute value (i) the smaller bank equity E_0, (ii) the larger the fraction α of sovereign debt held by banks, (iii) the higher the probability π of low fiscal surplus, and (iv) the smaller the sunspot probability p (as a very unlikely sunspot is less priced in B_0).[5]

Conversely, if banks' equity is above the threshold in (1.2), then the sunspot does not lead to the diabolic-loop equilibrium, *given the price* B_0. The price B_0, however, is computed under the assumption that the diabolic loop occurs with probability p. For the diabolic loop to occur with probability zero, banks' equity must be above the threshold in (1.2) for any p,

[5] Equation (1.2) can hold for some parameter values because of (A2). Moreover, (1.2) and (1.4) can hold simultaneously for a subset of these values because $\alpha(1-p)\pi < 1$.

including $p = 0$, i.e.,

$$\Leftrightarrow E_0 \geq \alpha\pi\tau\psi L_0 := \underline{E}_0. \tag{1.5}$$

This defines the minimum equity threshold $\underline{E}_0$. Equivalently, one can impose on banks an aggregate position limit on government bonds α^*, given their initial equity E_0. The total supply of safe (diabolic-loop-free) assets to the banks is $\alpha^*\underline{S}$, since bonds are risk-free. This effectively limits the amount of safe deposits that the banking system can generate.[6]

Proposition 1 summarizes these results.

Proposition 1. *(i) To avoid the diabolic loop, the ratio of bank equity to sovereign exposure must be at least* $\pi\frac{\tau\psi L_0}{\underline{S}}$.
(ii) The maximum amount of safe assets available to banks is $\alpha^* B_0 = \frac{E_0}{\pi\tau\psi L_0}\underline{S}$. *Equivalently,* $\frac{\underline{E}_0}{\alpha\underline{S}}$ *is the minimum ratio of aggregate bank equity to sovereign exposure.*

Sovereign Debt Tranching We consider an alternative to an upper bound on bank holdings of debt. Sovereign debt could be split into a senior and a junior tranche, with banks permitted to hold only the senior tranche. We will show that the diabolic loop is ruled out if the face value, F^s, of the senior tranche (the tranching point) or the bank's senior tranche holdings, α^s, is sufficiently low (for a given equity level E_0) or equivalently, $E_0 > \underline{E}_0^s := \alpha^s\pi[\tau\psi L_0 - (\underline{S} - F^s)]$). In other words, the diabolic loop equilibrium can be ruled out by picking appropriate pairs (α^s, F^s). Tranching shrinks the region in which the diabolic loop can occur: intuitively, this is because it shifts risk arising from sovereign debt from banks to holders of the junior tranche. The analysis is the same as in the case of no tranching except that C is replaced by $C^s - (\underline{S} - F^s)$. Now, the bailout cost C^s reflects the price drop in the senior bond and the additional term $-(\underline{S} - F^s)$ reflects the reduction in bailout costs due to the additional protection provided by the junior tranche.

Insofar as tranching eliminates the risk of bailouts, it also makes the junior tranche risk free as in this model the government may default only if it bails out the banks.

Tranching increases the total supply of safe assets, $\alpha^s F^s$ to the banking sector. To see this, suppose banks increase their senior bond holdings, α^s. This may expose them to the diabolic loop. But by picking a lower face value F^s one can still rule out the diabolic loop. We show that the required decline in F^s is small enough that $\alpha^s F^s$, i.e. the total value of safe assets, increases.

Stating these results formally:[7]

[6]Note that if (1.5) holds, then the bailout condition (1.4) also holds for any p.
[7]The proof of this and the next proposition is relegated to an on-line appendix.

Proposition 2. *(i) For a given security structure F^s, to avoid the diabolic loop, the ratio of banks' aggregate equity to sovereign exposure must be at least* $\pi\frac{\tau\psi L_0-(\underline{S}-F^s)}{F^s}$*, where the term* $(\underline{S}-F^s)$ *reflects the protection afforded by the junior tranche.*
(ii) If $E_0 > \underline{E}_0^s$*, the junior bond is also safe.*
(iii) If F^s *is chosen so as to maximize the amount of safe assets for the banking sector, tranching generates larger amounts of safe assets than no tranching. Equivalently, tranching lowers the equity to be held by banks per unit of sovereign exposure.*

2 Two-Country Model

Now consider two symmetric countries. The realizations of their primary surpluses absent bailout interventions is independently distributed. Both governments issue zero coupon bonds with face value $\underline{S}$. If banks held only their own government sovereign bond, we would effectively be in the single country case: sovereign default is only correlated to the extent that sunspots are correlated. Suppose instead that an intermediary securitizes a symmetric pool made of government bonds issued by the two countries. If banks rebalance their portfolios slightly towards this pooled asset, they will be less exposed to a drop in the price of domestic debt. So, they need less equity to avoid the diabolic loop. This is the benefit of pooling.

But, if banks in both countries replace their entire domestic sovereign holdings with the pooled asset, all banks end up with identical portfolios. Now, repricing of sovereign debt cannot occur in one country without occurring in the other. For bailout to occur in one of the two countries, the repricing of its domestic debt should be large enough that the implied price drop of the pooled asset would trigger insolvency of its domestic banks. But then, by symmetry the banks of the other country are also insolvent, and require a bailout. Hence, complete pooling leads to perfect contagion. This is the curse of pooling.

This illustrates an important insight: simply requiring banks to hold a pooled asset – or an equivalently diversified portfolio of sovereign bonds – might actually lead to contagion across countries, if it makes their sovereign debt portfolios very similar.

But contagion is contained if banks hold only the senior tranche, $\alpha^{\mathcal{E}}$, of such a pooled asset, i.e. ESBies. Pooling and tranching interact positively, since repricing of ESBies after a sunspot is smaller than that of a senior bond of a single country. Intuitively, tranching the pooled asset allows senior bond holders to push losses onto the junior bond holders in a greater number of states than tranching the debt of a single country. Hence, banks' equity requirements can be reduced. Still, the junior bond would be itself isolated from repricing risk due to a sunspot: insofar as the diabolic loop is avoided, banks' losses are an off equilibrium phenomenon so that even junior bonds are risk-free. Of course, in a more general model, in which default does not only arise from the diabolic loop, junior bonds would not be entirely risk-free.

Pooling and tranching enables a maximal supply of safe assets to banks. The logic is the same as tranching in a single country but, when applied to pooled sovereign debt, the (off-equilibrium) risk can be shifted more effectively to the junior bond holders. As a result, tranching combined with pooling increases the supply of safe assets further. Proposition 3 states this formally.

Proposition 3. *(i) Given the tranching point* $F^{\mathcal{E}}$*, ESBies lower the required ratio of equity to sovereign exposure compared to single country tranching (for* $\alpha^{\mathcal{E}} = \alpha^s$*).*
(ii) If this ratio is upheld, the junior bond is also safe.
(iii) If $F^{\mathcal{E}}$ *and* $\alpha^{\mathcal{E}}$ *are chosen so as to maximize the amount of safe assets for the banking sector, ESBies generate a larger amount of safe assets than single country tranching.*

3 Conclusion

This paper adds to a recent literature on the feedback loop between sovereign and bank solvency risk (Brunnermeier et al., 2011, Obstfeld, 2013, Acharya, Drechsler and Schnabl, 2014; Cooper and Nikolov, 2013, Farhi and Tirole, 2015; Leonello, 2014), by providing a simple model and using it to explore whether and how the loop can be defused by restricting banks' portfolio of sovereign holdings.

First, we find that what matters is the ratio of banks' equity to their domestic sovereign exposures: the diabolic loop can equivalently be defused by raising banks' equity requirements or by restricting their holdings of domestic sovereign debt.

Second, requiring banks to hold only a senior tranche of domestic sovereign debt is more effective than requiring them to diversify their sovereign portfolios across countries.

Third, tranching is most effective if applied to a diversified sovereign debt portfolio: requiring banks to hold only the senior tranche of such a portfolio – i.e. ESBies – reduces their equity requirement per dollar of sovereign holdings, relative to a regime where they are required to hold senior domestic sovereign debt only. Intuitively, the reason is that using *both* pooling and tranching allows more of the sovereign risk generated by each government to be shifted onto the junior bond-holders, hence away from the banks of the corresponding country – thus eliminating the need for the government to intervene with a bailout. Accordingly, we show that ESBies generate a larger amount of safe assets than domestic debt tranching alone, if the split between senior and junior bonds is optimally designed.

Finally, insofar as ESBies succeed in defusing the diabolic loop and associate endogenous risk, in equilibrium even the junior bond is risk-free.

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A Mathematical Appendix

A.1 Proof of Proposition 2

To prove claim (i), note that if the space (α^s, F^s) is split into a subset in which there exists a sunspot observed with some probability p that gives rise to the diabolic loop and a subset, $\mathcal{N}$, in which the diabolic loop never occurs, identifying the boundary of $\mathcal{N}$ will enable us to characterize the diabolic-loop region. To do so, we compute senior bond prices under the diabolic-loop equilibrium and require that the losses associated with the diabolic loop reduce bank equity exactly to zero in the limit when the sunspot probability is $p = 0$.

If the sunspot is not observed, debt trades at its no default-value $\underline{S}$, and the same holds for the senior tranche, which trades at F^s. If the sunspot is observed and banks require a recapitalization, the cost to the government is $C^s \equiv \tau\psi L_0 - \alpha(B_1^s - B_0^s) - E_0$, where B_t^s denotes the price of the senior tranche. If the surplus at $t = 3$ is $\overline{S}$, the government can repay its debt in full after incurring the cost C^s because of A3, so that the senior tranche pays its face value F^s; if instead the surplus is $\underline{S}$, the government can only pay $\underline{S} - C^s$ and the senior tranche yields $F^s - [C^s - (\underline{S} - F^s)]$, where $\underline{S} - F^s$ is the loss absorbed by the junior tranche. Hence, the price of the senior tranche at $t = 1$ is $B_1^s = F^s - \pi[C^s - (\underline{S} - F^s)]$. So the analysis is the same as in the case of no tranching except that C is replaced by $C^s - (\underline{S} - F^s)$. This amounts to replacing $\tau\psi L_0$ in Equation (1.5) by $\tau\psi L_0 - (\underline{S} - F^s)$. In other words, the bailout is avoided if

$$E_0 \geq \alpha^s \pi \left[\tau\psi L_0 - (\underline{S} - F^s)\right] =: \underline{E}_0^s. \tag{4}$$

This proves claim (i).

Claim (ii) follows by noticing that a diabolic loop cannot occur if banks' equity is $E_0 > \underline{E}_0^s$, so that the junior bond is also risk-free.

To prove claim (iii) note that for pairs (α^s, F^s) on the boundary of the no-diabolic-loop subset $\mathcal{N}$, the inequality (4) holds with equality. The right-hand side of (4) is increasing in both α^s and F^s, which means that at the boundary if banks hold a larger fraction of the senior tranche α^s, this tranche must have a lower face value F^s, and vice versa. We want to find the pair $(\alpha^{s*}, F^{s*}) \in \mathcal{N}$ that maximizes the total value of safe assets available to the banking system:

$$\max_{(\alpha^s, F^s) \in \mathcal{N}} \alpha^s F^s = \max_{(\alpha^s, F^s) \in \mathcal{N}} \frac{E_0 F^s}{\pi \left[\tau\psi L_0 - (\underline{S} - F^s)\right]}.$$

The maximand is decreasing in F^s, because $\underline{S} > \tau\psi L_0$. Therefore, the maximization requires setting the optimal face value F^{s*} at the lowest possible value that meets (4) with equality.

In turn, this requires setting α^s at its upper bound $\alpha^{s*} = 1$, so that

$$F^{s*} = \underline{S} + \frac{E_0}{\pi} - \tau\psi L_0 < \underline{S}, \tag{5}$$

where the inequality follows from A2. Since the solution for $\max_{(\alpha^s, F^s)\in\mathcal{N}} \alpha^s F^s$ differs from the no-tranching solution, tranching allows the economy to generate a larger amount of safe assets for the banking system. QED

A.2 Proof of Proposition 3

As in the case where tranching occurs in a single country, we wish to characterize the set $\mathcal{N}$ of pairs $(\alpha^{\mathcal{E}}, F^{\mathcal{E}})$ that rule out the diabolic-loop equilibrium. To do so, we initially compute prices of ESBies for a given $(\alpha^{\mathcal{E}}, F^{\mathcal{E}})$ under the diabolic-loop equilibrium and require that bank equity drops exactly to zero in the limit when the sunspot probability is $p = 0$. Consider the parameter region in which the senior tranche incurs losses when the (union-wide) sunspot is observed. There are two scenarios to be considered:

First, suppose equity E_0 is large enough that ESBies incur losses only in the worse-case outcome at $t = 3$, in which both countries have primary surplus $\underline{S}$ realization. In this scenario, which occurs with probability π^2, the pooled asset pays $\underline{S} - C^{\mathcal{E}}$, and the senior tranche pays $F^{\mathcal{E}} - [C^{\mathcal{E}} - (\underline{S} - F^{\mathcal{E}})]$. Hence, junior bond holders are wiped out. Clearly, ESBies are better protected than a single country senior bond, where the low surplus realization occurs with probability π.

Second, for lower equity levels E_0 the diabolic loop might be so large that ESBies might incur losses if only one of the two countries has a low primary surplus realization. In this case the pooled asset pays $\underline{S} - \frac{1}{2}C^{\mathcal{E}}$ and the junior bond holder will be wiped out in three of the four possible surplus realizations. This case occurs with probability $2\pi(1-\pi)$.

In the first scenario, in which ESBies only default in the state where surplus realization is $\underline{S}$ for both governments, the following inequality must hold

$$\underline{S} - \frac{1}{2}C^{\mathcal{E}} \geq F^{\mathcal{E}}. \tag{6}$$

If (6) holds, then the price of the senior tranche in period 1 is $B_1^{\mathcal{E}} = F^{\mathcal{E}} - \pi^2[C^{\mathcal{E}} - (\underline{S} - F^{\mathcal{E}})]$. The analysis is the same as in the one-country case with tranching except that π is replaced by π^2. A recapitalization is not needed if

$$E_0 \geq \alpha^{\mathcal{E}} \pi^2 \left[\tau\psi L_0 - (\underline{S} - F^{\mathcal{E}})\right]. \tag{7}$$

In the second scenario, where (6) is violated, if one country has surplus $\overline{S}$ and the other $\underline{S}$, the senior tranche receives $F^{\mathcal{E}} - [\frac{1}{2}C^{\mathcal{E}} - (\underline{S} - F^{\mathcal{E}})]$ and its price at $t = 1$ is

$$\begin{aligned} B_1^{\mathcal{E}} &= F^{\mathcal{E}} - \left[\frac{1}{2} 2\pi(1-\pi) + \pi^2\right] C^{\mathcal{E}} + \left[2\pi(1-\pi) + \pi^2\right] (\underline{S} - F^{\mathcal{E}}) \\ &= F^{\mathcal{E}} - \pi \left[C^{\mathcal{E}} - (2-\pi)(\underline{S} - F^{\mathcal{E}})\right]. \end{aligned}$$

The analysis is the same as in the one-country case with tranching except that we must replace $\underline{S} - F^{\mathcal{E}}$ by $(2-\pi)(\underline{S} - F^{\mathcal{E}})$. A recapitalization is not needed if

$$E_0 \geq \alpha^{\mathcal{E}} \pi \left[\tau\psi L_0 - (2-\pi)(\underline{S} - F^{\mathcal{E}})\right]. \tag{8}$$

Setting $\alpha^{\mathcal{E}} = \alpha^s$ and $F^{\mathcal{E}} = F^s$ in (7) and (8) and comparing them with (4), it follows that the lower bound on equity to sovereign exposure ratio is less stringent with ESBies than with single country tranching. This completes part (i) of the proof.

The claim in part (ii) follows directly from the Equations (7) and (8) which rule out the diabolic loop equilibrium.

To prove the claim in part (iii), note that in the first scenario the pair $(\alpha^{\mathcal{E}*}, F^{\mathcal{E}*})$ that maximizes the value of the safe asset available to the banks satisfies (7) with equality, and $\alpha^{\mathcal{E}*} = 1$ by the same argument as in the one-country case. The resulting value of the senior tranche is analogous to (5) in the one-country case with tranching:

$$F^{\mathcal{E}*} = \underline{S} + \frac{E_0}{\pi^2} - \tau\psi L_0. \tag{9}$$

Since π is now replaced by π^2, we have $F^{\mathcal{E}*} > F^{s*}$: pooling and tranching generates a larger supply of the safe asset than tranching in each country separately.

We must finally check that ESBies suffer no losses even in the next to worst-case scenario, i.e. (6) is satisfied. Noting that

$$C^{\mathcal{E}*} = \tau\psi L_0 - \alpha(B_1^{\mathcal{E}*} - B_0^{\mathcal{E}*}) - E_0 = \tau\psi L_0 + \alpha^{\mathcal{E}*}\Delta_1^{\mathcal{E}*} - E_0,$$

and that in the two-country case with tranching $\Delta_1^{\mathcal{E}}$ is given by an equation analogous to

(1.1) where $\tau\psi L_0$ is replaced by $\tau\psi L_0 - (\underline{S} - F^{\mathcal{E}})$ and π by π^2, the no-loss condition (6) can be rewritten as

$$\underline{S} - F^{\mathcal{E}*} - \frac{1}{2}\left[\tau\psi L_0 + \alpha^{\mathcal{E}*}\frac{\pi^2(\tau\psi L_0 - E_0 - (\underline{S} - F^{\mathcal{E}*}))}{1 - \alpha^{\mathcal{E}*}\pi^2} - E_0\right] \geq 0. \tag{10}$$

We next set $\alpha^{\mathcal{E}*} = 1$ and $F^{\mathcal{E}*}$ equal to its value in (9). Because these values satisfy (7) with equality, the sum of the second and third term in the square bracket of (10) is zero, so using (9), (10) becomes

$$E_0 \leq \frac{1}{2}\pi^2\tau\psi L_0, \tag{11}$$

which is part of the parameter space for E_0 we consider under A2.

For the second scenario, in which (6) is violated, going through the same steps as for the first scenario, we find that the pair $(\alpha^{\mathcal{E}*}, F^{\mathcal{E}*})$ that maximizes the value of safe investment available to the banks satisfies $\alpha^{\mathcal{E}*} = 1$ and

$$F^{\mathcal{E}*} = \underline{S} - \frac{1}{(2-\pi)}\left[\tau\psi L_0 - \frac{E_0}{\pi}\right]. \tag{12}$$

The face value $F^{\mathcal{E}*}$ is larger than in the one-country case because by A2 the term in square brackets is positive. We are in the second scenario, i.e. (6) is violated, if equity is in the region

$$\frac{1}{2}\pi^2\tau\psi L_0 < E_0 < \pi\tau\psi L_0.$$

QED