Central Banks Going Long

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Long-term interest rates have for long played an ambiguous role in the operation of monetary policy. The Federal Reserve Act of 1913 that created the Federal Reserve set the monetary policy objective to be: "… to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates." But, after the Treasury Fed accord of 1951, the Fed dropped the third of these objectives, and has since referred to itself as having a "dual mandate." More recently, when policymakers discuss the effect of new monetary policies, from forward guidance to quantitative easing, they commonly state their impact on longer-term interest rates as a proof of success. As short-term interest rates stay close to zero, policies that directly target long-term rates can be considered to control inflation, together with macroprudential policies that affect the risk premium in long-term bonds.

In principle, a central bank could issue reserves and make loans at one arbitrary maturity, and use its lending and deposit facilities together with open-market operations to target the market interest rate at this maturity. Almost all central banks choose very short maturities, from the traditional focus on the overnight Federal Funds rate by the Federal Reserve, to the one-week main refinancing operations of the European Central Bank.[1] In September 2016, the Bank of Japan announced a new policy of "yield-curve control," which targets a rate of 0% for the 10-year government yield. If inflation stays away from target for long, as it happened in Japan, other central banks may consider going long as well.

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1. An exception is the Swiss National Bank, whose target interest rate is a 3-month money-market rate.

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To evaluate these potential policies in the future, this paper looks at the past. During the late 1940s, the long-term interest rate played a crucial role in U.S. monetary policy, both in its operations and in its goals. The Federal Reserve was not unique in this regard, as the Bank of England focused policy in part on long-term interest rates, following the recommendations of the 1959 Radcliffe Report. This paper describes the context behind these two historical experiments, and analyzes their role in determining inflation through the lenses of a model of inflation with interest-rate rules. Each of these experiments was different, but each went well beyond just using the long-term interest rate as one of many indicators of the state of economy. Central banks went long, significantly changing the composition of their balance sheets and adapting their procedures to focus monetary policy on long-term interest rates. In the context of interest-rate rules, the long-term interest rate was not just one more variable on the right-hand side, but crossed to the left-hand side of the policy rule.

Motivated by these historical episodes, this paper discusses different ways in which the familiar model of monetary policy can integrate long-term interest rates as a policy tool with the dynamics of inflation. While each case is different, the results brought together suggest that focusing on long-term interest rates leads to more volatile and less anchored inflation.

The economic analysis requires linking the dynamics of inflation, short-term nominal interest rates, and long-term yields. There is an extensive literature on the yield curve and inflation, including several tractable models and extensions that have successfully fit the data.[2] A barrier to merging them with the study of inflation is that they are mostly set in continuous time with shocks that follow diffusions, while most work in monetary economics uses linearized models in discrete time.[3] To overcome this barrier, this paper presents the classic inflation control problem in continuous model. This may prove to be useful in other contexts. Methodologically, it pushes forward a research agenda promoted by Brunnermeier and Sannikov (2017), who argue that bringing continuous-time tools to macroeconomics will allow models to better incorporate endogenous risk premia and financial frictions.

2. See Piazzesi (2010) for a survey and Smith and Taylor (2009) for an estimated model closer to the one in this paper.

3. On the study of inflation, Jones and Kulish (2013) and McGough, Rudebusch and Williams (2005) are the closest papers in the literature in their treatment of long rates, but they work with linearized discrete-time new Keynesian models. Gallmeyer, Hollifield and Zin (2005) are closer from the perspective of the yield curve.

Sections 1 to 3 of the paper set up the model, solve for the dynamics of inflation, and characterize the dynamics of the yield curve, respectively. Sections 4 and 5 discuss the two case studies of central banks going long, and applying the model to understand them. Section 6 discusses the recent Japanese experience. Section 7 concludes with lessons for central banks that consider going long, and discusses future research to integrate the study of monetary policy with long-term interest rates.

1. Controlling Inflation in Continuous Time

I first describe the choices facing the private sector, then the central bank's policies, and finally define the equilibrium interaction between the two. Subsection 1.4 provides general-equilibrium microfoundations.

1.1 The Private Sector

A representative household chooses how much to save in a real riskless bond that, in exchange for one unit of consumption today, returns for sure $R_t^{(s)}$ units of consumption s periods from now. Letting m_t denote the marginal utility of consumption at date t, then the optimal holdings of this bond must satisfy the Euler equation:

$$m_t = \mathbb{E}_t \left(m_{t+s} R_t^{(s)} \right). \tag{1}$$

Buying an extra unit of the bond lowers utility by the left-hand side of this equation, but is expected to raise it by the right-hand side. At the optimum, the net effect must be zero.

Taking the limit as s becomes a time interval dt that is infinitesimally close to zero, and since $R_t^{(s)}$ is known at date t, gives the continuous-time version of this equation:

$$\mathbb{E}_t \left(\frac{dm_t}{m_t} \right) = -r_t dt, \tag{2}$$

where r_t is the return on an instantaneous bond. Using the language of the Ramsey model, this equation states that marginal utility must decline at the same rate as the safe return to savings at an intertemporal optimum.

I assume that the utility function of the households is time-separable and has constant relative risk aversion. Therefore: $m_t = \beta^t c_t^{-\gamma}$, where

$\beta \in (0.1)$ is the discount factor, c_t is consumption, and $\gamma > 0$ is the coefficient of relative risk aversion. As in baseline new Keynesian economies, there is no capital or investment; therefore markets clear when consumption equals output y_t.

The key assumption in this economy is that prices are flexible, so the classical dichotomy holds. As illustrated in Woodford (2003), Cochrane (2011), or more recently in Hall and Reis (2016), the economic problem of pinning down the price level by using interest-rate rules is conceptually unchanged if there are nominal rigidities and a Phillips curve. Adding nominal rigidities complicates the expressions and may require linearizing the equilibrium conditions, but the qualitative conclusions on when inflation is pinned down remain unchanged.

Given this assumption, it is then a mere simplification to further assume that output is exogenous, as in an endowment economy. (For the unconvinced readers, subsection 1.4 endogenizes the evolution of output as a function of technology shocks.) In particular, I assume that output follows a random walk, that has normally distributed innovations with standard deviation σ_y, and a stochastic mean growth rate g_t.[4] This trend, in turn follows a stationary autoregressive process, with long-run mean $\bar{g}$, speed of mean reversion κ_g, and normal shocks with standard deviation σ_g. In continuous-time notation, this is compactly written as:

$$\frac{dy_t}{y_t} = g_t dt + \sigma_y dz_t^y, \tag{3}$$

$$dg_t = -\kappa_g (g_t - \bar{g})dt + \sigma_g dz_t^g. \tag{4}$$

The shocks are independent Wiener processes, so they are normally distributed with mean zero and variance $\mathbb{E}_0[(z_t^y)^2] = \mathbb{E}_0[(z_t^g)^2] = t$.

To solve for the real interest rate, note that market clearing in the goods market implies that: $m_t = \beta^t y_t^{-\gamma}$. Using Ito's lemma to take time derivatives of this expression:

$$\frac{dm_t}{m_t} = [\ln\beta - \gamma g_t + 0.5\gamma(\gamma+1)\sigma_y^2]dt - \gamma\sigma_y dz_t^y. \tag{5}$$

4. To be more precise, time is continuous, y_t is a stochastic variable defined on a filtered probability space, and Z_t^y is an adapted Brownian motion in this space. The same applies to all other stochastic variables in this paper.

Then, the Euler equation gives the solution for the real interest rate:

$$r_t = \gamma g_t - \ln\beta - 0.5\gamma(\gamma+1)\sigma_y^2. \tag{6}$$

The first two terms on the right-hand side are the standard ones from the Ramsey model: higher growth rates or more patient households increase the equilibrium real interest rate. The third term captures the precautionary savings effect that more uncertainty on output induces the consumer to save more and this lowers the real interest rate in equilibrium. A virtue of working in continuous time is that this precautionary savings term is present and analytic; in discrete-time linearized setups it is zero, and in numerical solutions it appears only as higher-order terms.

Collecting all the results gives the real equilibrium:

Lemma 1. *Real variables do not depend on monetary policy and the marginal utility of consumption and the real interest rate are given by:*

$$\frac{dm_t}{m_t} = -r_t dt - \gamma\sigma_y dz_t^y, \tag{7}$$

$$dr_t = -\kappa_g (r_t - \bar{r})dt + \gamma\sigma_g dz_t^g. \tag{8}$$

where $\bar{r} = \gamma\bar{g} - \ln\beta - 0.5\gamma(\gamma+1)\sigma_y^2$.

1.2 The Central Bank

Central banks take deposits from banks, commonly called reserves. This liability is crucial in the modern monetary system, because people make electronic payments by using cards and other means of payments issued by banks. These give rise to large gross cross-bank liabilities every day. Reserves are the settlement currency used by the banks to clear these transactions among themselves.

If the deposits at the central bank have maturity s, then the usual central-bank policy is to promise a safe nominal return of $I_t^{(s)}$ per unit of currency held as reserves. I assume that the demand for liquidity is satiated (Reis, 2016*a*), so that the central bank can perfectly choose this return and the private agents in the economy, represented by the representative consumer, choose to hold these deposits according to the optimality condition:

$$\mathbb{E}_t\left(\frac{m_{t+s} I_t^{(s)}}{p_{t+s}}\right) = \frac{m_t}{p_t}. \tag{9}$$

The price level p_t appears because reserves are the unit of account in the economy. In the extreme case where reserves are instantaneous deposits, then the differential version of this condition is:

$$\mathbb{E}_t\left(\frac{d(m_t / p_t)}{(m_t / p_t)}\right) = -i_t dt, \tag{10}$$

where i_t is the nominal interest rate on an instantaneous deposit at the central bank.

The central bank is independent, and its dividend rule is to rebate net profits every instant to the fiscal authority. By the result in Hall and Reis (2015), the central bank is therefore always solvent, as its reserves satisfy a no-Ponzi scheme condition. Fiscal considerations then play no role in the determination of inflation.[5]

Following a long line of work, I assume that the central bank adopts a feedback rule for the choice of the interest rate. The first component of this rule is a constant inflation target π^*.[6] A strict reading of the mandate of most central banks sets π^* to a constant equal to 2% at an annual rate.[7]

The central bank then responds to any deviation of actual inflation dp_t/p_t from this target by raising interest rates by an amount $\phi \geq 0$ in the next instant of time. The assumption that this is positive corresponds to the famous Taylor principle (since it corresponds to $e^{\phi} \geq 1$.)

Most central banks, however, do not engage in such strict inflation targeting, but rather adopt a policy of flexible inflation targeting. In any given period, they target an inflation rate different from π^* depending on the state of the economy. This is optimal in many models of nominal rigidities.[8] As a result, interest rates rise and fall to push inflation above or below the strict inflation target temporarily in order to stabilize real activity.

Moreover, when inflation is on target, then the nominal interest rate must mimic changes in real interest rates. Yet, most central banks find it difficult to measure the right real interest and respond to it instantly, or more generally to track the state of the business cycle. Errors in measurement lead to changes in interest rates.

5. For a discussion of the multiple fiscal channels between central banks and Treasuries, see Reis (2018).

6. Letting the target vary over time deterministically would make no difference to the results.

7. For instance, if a one-unit period in the model corresponds to one week, then $\pi^* = 0.02/52$.

8. See Woodford (2010) or Ball, Mankiw and Reis (2005).

Finally, almost no central bank follows a rule, but rather chooses a path for monetary policy from the aggregation of the opinions of different committee members. As opinions of the individuals in charge of decision, or the composition of the committee changes, this will lead to changes in interest rates.

Whether it is in response to desires to stabilize real fluctuations, due to mis-measurement of the actual state of the business cycle, or because of monetary policy shocks, then even if inflation is at π^*, nominal interest rates may vary. I capture the combination of all these factors through a random nominal interest rate target, or intercept x_t, that also follows a Markov process with long-run mean $\bar{x}$ and dz_t^x shocks .

Finally, central banks smooth interest rates at a rate $\rho > 0$.

Combining all these ingredients, and assuming for now that the central bank sets policy in terms of the instantaneous interest rate on reserves, gives the monetary policy rule:

$$d(i_t - x_t) = -\rho(i_t - x_t)dt + \phi\left(\frac{dp_t}{p_t} - \pi^* dt \right). \tag{11}$$

1.3 The Equilibrium

Because the classical dichotomy holds in this economy, all the real variables are already pinned down. What remains to determine is the price level. A rational expectations equilibrium is a path for the price level $\{p_t \in \Re^+ : t \geq 0\}$ given the real equilibrium in lemma 1 and the monetary policy rule in equation (11). Following a long literature on new Keynesian dynamic stochastic general equilibrium (DSGE) models of monetary policy, I focus on a narrower definition of equilibrium:

Definition 1. *A bounded homoskedastic Markov perfect equilibrium is a function for expected inflation* $\pi(r,x) : \Re^2 \rightarrow \Re$ *and three constants,* α_y, α_g, α_x *such that:*

$$\frac{dp_t}{p_t} = \pi(r_t, x_t)dt + \alpha_y \sigma_y dz_t^y + \alpha_g \sigma_g dz_t^g + \alpha_x \sigma_x dz_t^x, \tag{12}$$

where equations (7), (8) and (11) hold, and expected inflation satisfies:

$$\lim_{T\rightarrow\infty} \mathbb{E}_t \left(e^{-\varepsilon (T-t)} \pi(r_T, x_T) \right) = 0 \tag{13}$$

for any $\in > 0$.

There are restrictions imposed on this definition relative to the rational-expectations equilibrium. First, since the state of the economy is captured by the real interest rate and the nominal interest rate, and (r_t, x_t) follows a Markov process, the restriction to look only at a Markov equilibrium is natural. This rules out the possibility that sunspots drive inflation. Second, since all variances are independent of time, the definition imposes that the variance of inflation also do not depend on time. Therefore, the responses to shocks, stacked in the column vector $Z_t = (z_t^y, z_t^g, z_t^x)$ are given by a column vector of constants $\alpha = (\alpha_y\sigma_y, \alpha_g\sigma_g, \alpha_x\sigma_x)$ rather than by three functions of the state vector. I conjecture that allowing for sunspot shocks by letting inflation depend also on some other $\alpha_\epsilon dz_t^\epsilon$, or allowing the responses of inflation to shocks to depend on (r_t, x_t) would actually make no difference: in equilibrium, $\alpha_\epsilon = 0$ and the other αs would not depend on the state of the economy.

More important is the assumption of boundedness. Cochrane (2011) provides a scathing critique of this assumption as an equilibrium selection device. It is not micro-founded since it does not follow from optimal-behavior or market-clearing conditions. Moreover, it plays an important role, since variations of it can dramatically change the results. The long literature on interest-rate rules has proposed other related boundary conditions, Obstfeld and Rogoff (1983) being a famous example, and there is also an extensive literature using other monetary policies to control the price level (Reis, 2016*b*). I follow Woodford (2003) and the extensive literature after it in maintaining this assumption because there is little in the analysis that brings any new light to the issues involved.

Given the stochastic process for marginal utility in equation (7), and for prices in equation (12), Ito's lemma gives the expected rate of change in m_t/p_t. By the Euler equation (10), this is equal to the instantaneous nominal interest rate. This gives a modified Fisher equation as a no-arbitrage condition between nominal and real bonds:

$$i_t = r_t + \pi_t - \alpha'\alpha - \gamma\sigma_y^2\alpha_y. \tag{14}$$

As usual, the nominal interest rate is equal to the sum of the real interest rate and expected inflation, the two first terms on the right-hand side, respectively. However, shocks to inflation introduce two extra terms. First, because of the convexity of returns, more variable inflation subtract from the realized real returns on nominal bonds. Second, there is an inflation risk premium. If positive shocks to inflation come at times when the marginal utility of consumption is high, then nominal bonds will have a realized return that is lower when returns are more valuable. Thus, holding a nominal bond comes with risk, and so it must pay a higher nominal interest rate to compensate for this risk. The focus on a homoskedastic equilibrium makes this risk premium constant, which is counterfactual. Allowing for heteroskedasticity in the growth rate of output or in the shocks to monetary policy would easily lead to a time-varying risk premium, and future work should explore its role.

1.4 Where does the Price Level Come From?

Because reserves are the unit of account in the economy, their real value is, by definition, $1/p_t$. It is the absence of arbitrage between private bonds and reserves at the central bank that pins down the price level. Outside of equilibrium, if the price level were too high, then reserves would cost less, which would make banks want to sell private bonds and deposit more reserves at the central bank. As the supply of reserves is fixed by the central bank this "excess demand" for reserves would make their value fall, which comes through the price level rising back to equilibrium.

This description of equilibrium may strike some readers as odd in two ways. First, output was taken as exogenous, as in an endowment economy. Second, there was no mention of goods' prices. Both of these features resulted from not having any mention of firms selling goods and setting prices. This section shows how introducing these makes no difference.

Assume that the representative agent solves the following problem:

$$\max_{\{\{c_{t,j}\},l_t\}} \mathbb{E}_0 \int_0^\infty \beta^t \left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{l_t^{1+\psi}}{1+\psi} \right) dt \tag{15}$$

$$\text{s.t. } c_t = \left(\int_0^1 c_{t,j}^{1/\mu} dj \right)^{\mu}, \tag{16}$$

$$d(b_t + v_t) = \left(i_t - \frac{dp_t}{p_t} \right) v_t + r_t b_t + (w_t l_t + k_t - c_t) dt. \tag{17}$$

The representative household chooses its consumption of a continuum of varieties ($c_{t,j}$) and hours worked (l_t) for a real wage (w_t) to maximize expected discounted utility, subject to its preference for different varieties and to a flow budget constraint where labor and investment income is complemented with dividends from firms (k_t). For simplicity, this assumes only instantaneous bonds (b_t) and reserves (v_t) are held, but allowing for higher maturities would not change the argument.

The optimal behavior of consumers is then characterized by the two Euler equations already presented in equations (2) and (10), the flow of resources combined with a transversality condition, and finally the optimality condition for labor supply:

$$c_t^{\gamma} l_t^{\psi} = w_t. \tag{18}$$

The real wage is equal to the marginal rate of substitution between labor and consumption.

A continuum of monopolistic firms operate a technology $y_{t,j} = a_t l_{t,j}$ to produce each variety of good subject to the common productivity a_t. Using their monopoly power, the optimal price they charge is a markup over costs:

$$\frac{p_{t,j}}{p_t} = \frac{\mu w_t}{a_t}. \tag{19}$$

A general equilibrium of this economy is a situation where households and firms behave optimally, and all market clear. There is a market for labor, so that $l_t = \int l_{t,j} dj$. In the goods market, $c_{t,j} = y_{t,j}$, which leads to $c_t = y_t$. Finally, the supply of real bonds and nominal reserves are both zero on net: $b_t = v_t = 0$.

This economy maps exactly into the price determination problem defined before. To see this, note that because prices are flexible, the symmetry of the problem leads to $p_{t,j} = p_t$. It then follows from combining equations (18) and (19) that:

$$\mu y_t^{\gamma+\psi} = a_t^{1+\psi}. \tag{20}$$

Therefore, given an exogenous stochastic process for technology such that a_t is a random walk in logs with a stochastic stationary trend, this maps exactly into the assumption on y_t. The model is fully microfounded with firms that choose prices.

The process by which an equilibrium price level is attained in the economy can be explained differently. If the price level is hypothetically too high, the private agents realize that the return on savings in reserves at the central bank is high. They therefore cut consumption to save more. But, as they cut consumption, this lowers the demand for goods, which in turn leads firms to want to cut their prices, thus making the price level fall back to equilibrium.

In Walrasian general equilibrium economies, either this story or the one at the start of this subsection are equally valid. All markets, for savings, for bonds, for reserves, for goods, and for labor must jointly clear, so excess demand or supply in any one of them comes with excess demand or supply in all others. Firms are choosing prices, and households are responding to them by consuming more or less, by saving more or less, and by depositing more funds at the central bank or not, all together and at once.

2. The Dynamics of Inflation

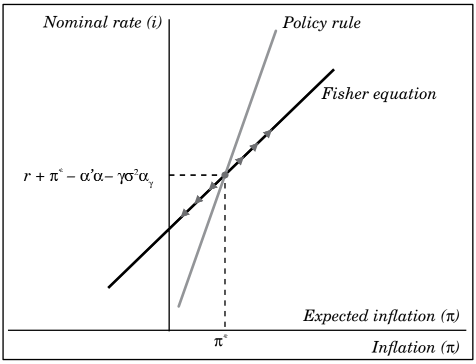
This section solves the mathematical problem set out in the previous section: to solve for the dynamics of inflation in equation (12), subject to the equilibrium Fisher equation (14), the policy Taylor rule in equation (11), and the boundary condition in equation (13).

2.1 A Phase Diagram for Expected Inflation

The Fisher equation is a linear relation between the nominal interest rate and expected inflation with slope 1 and vertical intercept $r_t - \alpha'\alpha - \gamma\sigma_y^{\,2}\alpha_y$. At a steady state with no shocks, the policy rule is also a line, with slope ϕ/ρ, so that if expected inflation is equal to π^*, then the nominal rate is x_t. Figure 1 shows the phase diagram for the dynamics of expected inflation. The equilibria are movements along the Fisher line, such that if the economy is above the policy rule, then the interest rate will fall, and rise conversely.

The dynamic system is clearly unstable as long as ϕ/ρ. Therefore, inflation must always stay at the intersection of the two lines. Otherwise, it would diverge to infinity violating the boundedness condition. This is the famous Taylor condition, adapted to account for interest-rate smoothing. Intuitively, as long as the central bank commits to raising interest rates when expected inflation increases from target, then, from the Fisher equation, this will raise expected inflation. But because this further raises inflation the next instant, it leads to a new rise in interest rates, and a further rise in expected inflation. If private agents in the economy rule out from their expectations these infinite forward-looking possibilities where inflation explodes at an accelerating pace, as captured in equation (13), then this disciplines their initial inflation expectations to not deviate from target.

Figure 1. Phase Diagram for Expected Inflation



Source: Author's elaboration.

The level of real interest rates r_t or monetary policy x_t together with shocks to both of them determines where expected and actual inflation are at a point in time. Understanding these responses requires moving beyond the phase diagram, fully solving the model.

2.2 Analytical Solution for Expected Inflation

Taking time differences of the Fisher equation gives:

$$di_t = dr_t + d\pi_t. \tag{21}$$

In turn, using the Fisher equation to replace i_t in the Taylor rule, and the dynamics of inflation to replace for dp_t / p_t gives:

$$di_t = dx_t - \rho(r_t + \pi_t - \alpha'\alpha + \gamma\sigma_y^2\alpha_y - x_t)dt + \phi\left(\pi_t dt + \alpha' dZ_t - \pi^* dt\right). \tag{22}$$

Equating the right-hand sides of the previous two equations and rearranging gives the law of motion for expected inflation:

$$d\pi_t = (\phi - \rho)(\pi_t - \pi^*)dt - d\varepsilon_t - \rho\varepsilon_t dt + \phi\alpha' dZ_t. \tag{23}$$

This expression defined a new variable: $\varepsilon_t = r_t - x_t + \pi^* - \alpha'\alpha - \gamma\sigma_y^2\alpha_y$. For now, take this as being just a convenient way to collect terms in what would otherwise be a long and messy expression.

Take expectations of the differential equation at date t, and let hatted variables denote the expected value of actual variables, e.g., $d\hat{\varepsilon}_t = \mathbb{E}_t(d\varepsilon_t)$. Expected inflation π_t then evolves according to:

$$d(\hat{\pi}_t + \hat{\varepsilon}_t - \pi^*) + (\rho - \phi)(\hat{\pi}_t + \hat{\varepsilon}_t - \pi^*)dt = -\phi\hat{\varepsilon}_t dt. \tag{24}$$

This is a standard ordinary differential equation that has the solution

$$\pi_t = \pi^* - \hat{\varepsilon}_t + e^{-(\phi-\rho)(T-t)}(\hat{\pi}_T + \hat{\varepsilon}_T - \pi^*) + \int_t^T \phi e^{-(\phi-\rho)(s-t)}\hat{\varepsilon}_s ds. \tag{25}$$

Taking the limits as T goes to infinity and imposing the boundary condition gives the solution for expected inflation:

$$\pi_t = \pi^* + \left(\frac{\rho}{\phi - \rho}\right)\varepsilon_t + \int_0^\infty \phi e^{-(\phi-\rho)s}\mathbb{E}_t(\varepsilon_{t+s} - \varepsilon_t)ds, \tag{26}$$

as long as $\phi > \rho$. Mathematically, equation (24) shows why the Taylor condition is necessary: it makes expected inflation an explosive process since positive deviations from target lead to further increases in the gap between expected inflation and target.

2.3 The Deviations of Expected Inflation from Target

Inflation deviates from target due to the terms on the right-hand side of equation (26). Recall that:

$$\varepsilon_t = (r_t + \pi^* - \alpha'\alpha - \gamma\sigma_y^2\alpha_y) - x_t. \tag{27}$$

If $\varepsilon_t = 0$ at all dates, then expected inflation will always be on target. An omniscient, long-lived, and inflation-nutter central bank would perfectly control inflation by choosing x_t to mimic one-to-one movements in real interest rates. Since x_t would be perfectly negatively correlated with r_t, the only state variable in the economy would be the real interest rate and monetary policy would introduce no extra source of uncertainty to any nominal variable.

But a central bank that has trouble tracking and measuring r_t in real time, that wants to use interest rates to have inflation deviate from the target in order to stimulate economic activity, or that in its deliberative process changes its views on the appropriate policy, will not be able or willing to set x_t to keep inflation at π^* at all dates. Extending the model to have a time-varying risk premium would make it even more unlikely for the central bank to measure in real time changes in $\sigma_t^2\alpha_y$ and adjust the interest rate in response to them.

The opposite case is one in which the setting of nominal interest rates by monetary policy is independent of the real interest rate. In this case, we can take x_t to follow an exogenous process:

$$dx_t = -\kappa_x (x_t - \bar{x}) d_t + \sigma_x dz_t^x, \tag{28}$$

where the shocks dz_t^x are independent from the shocks to output, dz_t^y and dz_t^g. The appendix covers the intermediate case where x_t only partially adjusts to changes in r_t.

Using the stochastic processes for real interest rates and policy interest rates in equations (8) and (28), one can evaluate the expectations and the integral in equation (26).[9] The final solution for expected inflation is:

$$\begin{aligned}\pi(r_t, x_t) &= \pi^* + \left(\frac{\rho}{\phi - \rho}\right)\left(\bar{r} + \pi^* - \alpha'\alpha - \gamma\sigma_y^2\alpha_y - \bar{x}\right) \\ &+ \left(\frac{\rho - \kappa_g}{\kappa_g + \phi - \rho}\right)(r_t - \bar{r}) - \left(\frac{\rho - \kappa_x}{\kappa_x + \phi - \rho}\right)(x_t - \bar{x}).\text{C.}\end{aligned} \tag{29}$$

Expected inflation is a linear function of the two state variables.

The first line of this equation has the intercept for inflation. A central bank that cannot fully keep track of movements in real interest rates or in inflation risk premia still has to figure out what these are on average and then set its average interest rate appropriately. In times when secular changes in productivity may have led to changes in safe real rates, or when the long-run inflation risk premium may be changing due to financial crises, this normal interest rate to which monetary policy should converge is not easy to assess, but it plays a crucial role in keeping inflation on target.

The second line of the expression above shows the sensitivity of expected inflation to the state of the economy. Depending on the persistence of interest-rate changes, shocks to monetary policy can raise or lower expected inflation. This is to be expected because, of course, forever-higher nominal interest rates unambiguously raise inflation, since they correspond to an effective increase in the inflation

9. Simply recall that $\mathbb{E}_t(r_{t+s} - r_t) = (\bar{r} - r_t)(1 - e^{-\kappa_r s})$ and likewise for $\mathbb{E}_t(x_{t+s} - x_t)$, and that $\int_0^\infty e^{-\phi t} dt = 1/\phi$.

target. A different question is whether actual inflation rises or falls with positive shocks to nominal interest rates. I turn to this question next.

2.4 Shocks to Inflation

The final step is to solve for inflation's response to shocks in the vector α. Subtracting equation (23) from equation (24):

$$d\pi_t - d\hat{\pi}_t = d\hat{\varepsilon}_t - d\varepsilon_t + \phi\alpha' dZ_t \tag{30}$$

Using the solution for expected inflation in equation (29) and the definition of ε_t in equation (27), this equation becomes:

$$\begin{aligned}&\left(\frac{\rho-\kappa_g}{\kappa_g+\phi-\rho}\right)\gamma\sigma_g dz_t^g - \left(\frac{\rho-\kappa_x}{\kappa_x+\phi-\rho}\right)\sigma_x dz_t^x \\ &= -\gamma\sigma_g dz_t^g + \sigma_x dz_t^x + \phi\alpha_y\sigma_y dz_t^y + \phi\alpha_g\sigma_g dz_t^g + \phi\alpha_x\sigma_x dz_t^x .\end{aligned} \tag{31}$$

This equation must hold for all realizations of the shocks. Therefore, the solution is:

$$\alpha_x = -\frac{1}{\kappa_x+\phi-\rho} \tag{32}$$

$$\alpha_g = \frac{\gamma}{\kappa_g+\phi-\rho} \tag{33}$$

$$\alpha_y = 0. \tag{34}$$

The first interesting result is that a positive shock to the nominal interest rate lowers actual inflation. The effect is smaller the more aggressive the Taylor rule coefficient response in future periods is, the less persistent the shock is, and the more interest rates are smoothed. The higher the variance of these monetary policy shocks, the higher the variance of inflation deviations from target.

The second result is that permanent shocks to output that do not move real interest rates have no effect on inflation. Similarly, sunspot nominal shocks that do not move real interest rates would have no effect on inflation. Moreover, since all the responses to shocks depend on parameters that are time-invariant, the equilibrium has a constant variances of shocks to inflation. This justifies the conjecture that restricting attention to Markov homoskedastic equilibrium is not limiting.

A summary of the analytical solution of the model is in the next proposition:

Proposition 1. *The bounded homoskedastic Markov equilibrium has expected inflation* $\pi(x_t, r_t)$ *given by equation (29) and the response to shocks* α *given by equations (32)-(34).*

3. Equilibrium Interest Rates

Combining the solution for expected inflation in equation (29) with the Fisher equation in equation (14) gives the equilibrium dynamics of the short-term interest rate. The next lemma states it formally.

Lemma 2. *In equilibrium, the instantaneous nominal interest rate is:*

$$i_t = \theta_0 + \theta_x x_t + \theta_r r_t \tag{35}$$

where $\theta_0 = [\phi/(\phi-\rho)](\pi^* - \alpha'\alpha) + [\phi\kappa_x/(\phi-\rho)(\kappa_x + \phi - \rho)]\bar{x} - [\phi\kappa_g/(\phi-\rho)(\kappa_g + \phi - \rho)]\bar{r}$ *and* $\theta_x = (\kappa_x - \rho)/(\kappa_x + \phi - \rho)$ *and* $\theta_r = \phi/(\kappa_g + \phi - \rho)$.

In this simple model, the nominal interest rate is an affine function of the two state variables, the state of the real economy and the stance of monetary policy. Therefore, the model fits into the general family of affine models of the term structure (Piazzesi, 2010).

The key result from this literature then follows (and is proven in the appendix):

Lemma 3. *Define the yield on the bond as* $i_t^{(s)} = \log(I_t^{(s)})$*. In equilibrium, it is:*

$$i_t^{(s)} = \delta_0(s) + \delta_i(s) i_t + \delta_x(s) x_t \tag{36}$$

where $\delta_i(s) = (1 - e^{-\kappa_g s})/(\kappa_g s)$.

3.1 Two Limitations to Going Long

The relation between long and short rates in the lemma results from the absence of arbitrage along the yield curve. A central bank that follows a Taylor rule for the overnight rate cannot separately set an exogenous target for the long rate that disrespects the equation in this lemma. Otherwise, if $i_t^{(s)}$ were larger than the expression in the lemma, private banks and investors would all want to deposit long-dated reserves at the central bank and want to hold zero instantaneous reserves. If the inequality flipped, so would the balance sheet of the central bank suddenly, from long to short reserves. The central bank, pushed from one corner to the next, would have to adjust its assets correspondingly and quickly, otherwise it would be exposed to losses that could endanger its solvency.

Moreover, for long maturities, s is large, so $\delta_t^{(s)}$ is expected to be quite small as long as the shocks to real interest rates are not very persistent (so κ_g is not too small). This says that temporary changes in short-term interest rates move long rates less than one-to-one. Stated backwards, it means that if the central bank targets the long rate, then any policy decision to change it will have a large impact on short-term interest rates. Today, central banks change their policy rate infrequently in a lumpy way, say every so many weeks by 25 basis points. If they did the same while going long, then the days before any policy meeting would come with intense speculation on short bonds in the days before, as the short rate would be expected to move by several percentage points at the time of the policy announcement. Going long requires a large change in operating procedures, with more frequent meetings of policy committees that would make single-digit basis point decisions.

3.2 From the Model to the Data

Another implication of lemma 3 is that long-term interest rates are linear functions of the instantaneous interest rate.[10] This affine property of the model is very convenient on many accounts. First, this class of models has been extensively taken to the data on interest rates of different maturities. Second, it has been extended in different directions. One could, for instance, consider shocks to the long-run growth rate of the economy akin to news shocks, or stochastic volatility in the growth rate mapping into uncertainty shocks, and so incorporate these two recent popular business-cycle literatures into the determination of inflation and the study of long-term interest rate policies.

Third, we can easily incorporate other state variables. For instance, Greenwood, Hanson and Vayanos (2016) introduce limits to

10. Smith and Taylor (2009) impose this linearity and obtain a related result to lemma 3 to focus on how changes in affect $\delta_x^{(s)}$.

arbitrage in the bond market so that there are two extra linear factors corresponding to the actual bond holdings by the central bank and their expected mean at different maturities. In their model, when central banks go long in the sense of buying government bonds of different maturities, they affect long-term interest rates. In this paper instead, central banks go long directly by choosing the value of the long-term interest rate. Merging the two models would provide a rich theory for how quantitative easing policies can affect inflation.

4. The United States Pre-Accord: 1942-1951

The behavior of the Federal Reserve during the Great Depression is one of the most studied in monetary history. In turn, modern analyses of U.S. monetary policy almost exclusively focus on the behavior of the Fed after the Treasury-Fed Accord of 4 March 1951, described by Friedman and Schwartz (1963) as: "Few episodes in American monetary history have attracted so much attention in the halls of Congress and in academic quarters alike." Considerably less attention has been spent on the period that goes from World War II to the Accord. This was a period when the Federal Reserve went long.[11]

4.1 Pegging Interest Rates

The United States entered World War II on 8 December 1941. As almost always happens when a country enters a major war, the primary goal of economic policy became the financing of large war expenditures, and the Treasury was its leading executor. The Federal Reserve was a subordinate, as monetary policy's role was to ensure that the banks that it regulated and the financial markets in which it intervened would provide a steady demand for the government bonds. While the Treasury officially managed the public debt, the Federal Reserve was supposed to ensure that the government bonds were sold at a favorable price.

The particular approach implemented by the United States during this time was announced in April 1942. One part of this policy was that the Fed stood ready to buy and sell 90-day Treasury bills at a fixed rate of 3/8%. The T-bill rate then became the effective policy rate. Certificates of deposit could be discounted at rates that still changed

11. Standard references for the history of the Federal Reserve are Friedman and Schwartz (1963) and Meltzer (2010).

from time to time to respond to demands in the banking sector, but the peg on the T-bill rate was the focus of the policy. Correspondingly, Treasury bills, not reserves, became the major liquid asset in the balance sheet of banks. Knowing that these could be bought and sold from the Fed at a fixed price at any time, banks did not need reserves, for T-bills were just as liquid.

While much has been made of the policy of pegging interest rates, it actually lasted for a relatively short period of time. The Federal Reserve continuously clashed with the Treasury about raising the T-bill rate, especially at the end of the War when inflation accelerated. Eventually, in July 1947, the Fed raised the T-bill rate after striking a bargain with the Treasury that involved the payment to the Treasury of a significant share of the net income it had accumulated. Further increases immediately followed, so that by December the bill rate was 1%, and one year later, by the end of 1948, it was set at 1 1/8%.

Between 1949 and 1951, there was an intense political struggle between the Treasury, partly backed by the president, and the Fed. At times, it seems worthy of a political drama TV series (Hetzel and Leach, 2001). It started with the FOMC statement in June 1949 that it intended to change the interpretation of the mandate to keep a peg on interest rates. A crucial shock arrived in 1950 with the intensification of the Korean war. Real interest rates rose as a response, and large government deficits were expected. Moreover, the anticipation of price controls led to a sharp increase in inflation, mostly for durable goods. On one side, the Treasury became nervous about keeping the peg on the price of its debt, especially given the prospect of another long conflict. On the other side, the Fed worried that to keep its interest rates low, it would have to issue reserves to buy more government bonds, and that this would fuel credit and inflation. In 1951, in testimony to Congress, the chairman of the Federal Reserve system unequivocally stated that: *"As long as the Federal Reserve is required to buy government securities at the will of the market for the purpose of defending a fixed pattern of interest rates established by the Treasury, it must stand ready to create new bank reserves in unlimited amount. This policy makes the entire banking system, through the action of the Federal Reserve System, an engine of inflation."*[12]

The Treasury-Fed Accord of March 1951 declared a truce between the Treasury and the Fed. In spite of having little legal force, and in itself stating little of substance, Fed Chairman Martin masterfully

12. U.S. Congress 1951, p. 158.

interpreted it in a way that affirmed the independence of the Fed from the Treasury from then onwards. One fundamental implication of the Accord for the conduct of U.S. monetary policy was that supporting the national debt was no longer an objective for monetary policy, which became concentrated on macroeconomic and price stability. Another implication was that the peg on the bill rate was lifted and the Fed gained full autonomy over the setting of interest rates.

There was a third implication of the Treasury-Fed Accord. Since then and all the way until the adoption of quantitative easing in 2008, the Federal Reserve focused its attention on short-term interest rates and conducted the bulk of its open-market operations by using Treasury bills. This was not the case before the Accord.

4.2 Ceiling Policy

While the peg for the T-bill rate gets much of the attention, it only lasted for five years. More persistent, and arguably more significant, was a different part of the March 1942 policy, which remained in force until March 1953: an explicit ceiling of 2.5% for the 10-year yield. Friedman and Schwartz (1963) argue that, unlike the peg on the bill rate, the Fed was in general favorable to this policy. The bond support program, as it was called, had originated intellectually within the Fed.

While the War lasted, the yield on Treasury bills was low relative to the yield on longer-dated Treasury bonds. As a result, banks were happy to hold bonds earning higher returns, exchanging them for Treasury bills at the Fed whenever they needed liquidity. The Fed rarely needed to intervene, and its assets mostly consisted of Treasury bills.

This changed between 1945 and 1948. The Treasury started issuing many more long-term bonds with the goal of delaying the payment of the wartime debt. Yields rose, reaching 2.37% in November 1947 and forcing the Fed to step in with a large-scale purchase of bonds to keep the ceiling unbroken. In 24 December of that same year, the Fed released a mere suggestion that it might allow for small deviations from the ceiling, and the long-term yield immediately jumped to 2.45%, thus demonstrating the active role the Fed was playing in the bond market. As the slope of the yield curve shrank, the private sector shifted the composition of its portfolio towards Treasury bills. Correspondingly, the maturity of the Fed's bond portfolio expanded.

Noticeably, while between 1947 and 1950 the Fed raised the bill rate several times and wanted to raise it more and more often, it always stayed committed to the ceiling on the bond rate. In fact, on 16 October 1947, the Board of Governors wrote a letter to the Treasury Secretary where, in the process of defending the change in the bill rate, it stated: *"We can assure you that these actions will not affect the maintenance of the 2 1/2 percent rate for the outstanding long-term government bonds."*

This changed with the Korean War. The flattening of the yield curve intensified the pressure for the Fed's balance sheet to grow and become longer. In 1950, Chairman Eccles advocated a relaxation of the ceiling on bond yields, but was strongly opposed by President Truman who, having imposed wage controls in 1951, was adamant that long-term mortgage rates would not increase. Moreover, the Treasury warned of a large financial crisis in bond markets if the ceiling was dropped. Following the Accord, the Fed did not explicitly abandon its interest rate ceiling; it did so only a full two years later, in March 1953. Only then did the Fed start selling bonds at a fast pace. Intellectual and policy support for a "bills-only" policy with regards to the Fed's balance sheet arose, and remained for many years to come, as the Fed moved completely away from its going-long policy.

4.3 Turning to the Model: Pegs

Focusing on the peg of short-term rates that lasted between 1942 and 1947, if the peg was expected to last forever, then inflation becomes indeterminate. This policy corresponds to $\phi = \rho = 0$ and to a constant x_t in the model. In this case, combining the Fisher equation in (14) with the policy rule, now gives:

$$\pi_t = \bar{x} - r_t + \alpha'\alpha + \gamma\sigma_y^2\alpha_y. \tag{37}$$

Since r_t is a stationary process, this satisfies the boundedness condition. Therefore, this equation is the sole condition with which to pin down the evolution of inflation. This is one equation in several variables: expected inflation and the response of inflation to each of the shocks. The result is indeterminacy.

This is not the classic indeterminacy result of Sargent and Wallace (1975). In a deterministic model, $\alpha = 0$, so the equation above uniquely pins down inflation. Sargent and Wallace (1975) instead emphasized that the initial price level is indeterminate, not inflation. With uncertainty, there is another form of indeterminacy (Nakajima and Polemarchakis, 2005). Monetary policy can at best pin down a risk-adjusted measure of expected inflation, the breakdown between expected inflation and the actual inflation response to shocks is indeterminate.

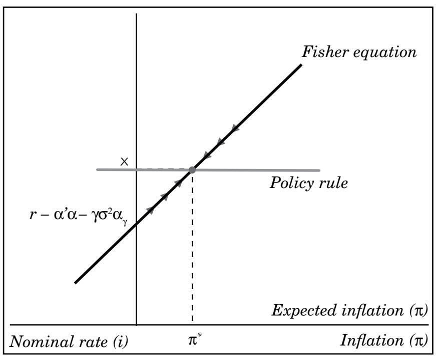
Figure 2 plots the phase diagram for this case, where the policy rule is now a horizontal line. Clearly, the system is globally stable: after a shock to the real economy (which shifts the Fisher relation) or a shock to monetary policy (which shifts the policy rule), inflation will converge back to the new intersection of the two lines. The boundedness condition puts no restriction on equilibrium. Yet, with risk, the change in inflation in response to the shocks (α) affects the location of the Fisher relation. Therefore, there are multiple possible combinations of inflation and its responsiveness to shocks that are consistent with equilibrium.

The Fed instead set policy in terms of the 90-day rate. A peg on a long rate implies that $i_t^{(s)}$ equals a constant ι. Using lemma 3, this implies that:

$$\iota = \delta_0(s) + (\delta_i(s) + \delta_x(s)). \tag{38}$$

Given the one-to-one correspondence between $\bar{x}$ and ι, the peg on a long rate can be analyzed by using the same phase diagram and the same mathematics as a peg on the short rate: it simply corresponds to a different choice of $\bar{x}$. Indeterminacy of inflation remains, and going long is immaterial.

Figure 2. A hard peg



Source: Author's elaboration.

4.4 Ceilings and Inflation

The supposed hard peg only lasted for a little over 5 years; by comparison, the policy rate was unchanged in the United States for 7 years, between December 2009 and December 2016. An alternative interpretation of the policy at the time is that the Fed followed a feedback rule for interest rates, as in equation (11), but with a very high extent of interest rate smoothing (low ρ) and a relatively low sensitivity of interest rates to inflation (low ϕ). However, the analysis of section 2 does not apply. For more than a decade, the Fed had a ceiling on long rates. That is, there was an exogenous ι for the 10-year rate such that monetary policy followed the feedback rule unless it implied a violation of the constraint $i_t^{(s)} \leq \iota$. If so, then the interest rate was unchanged.

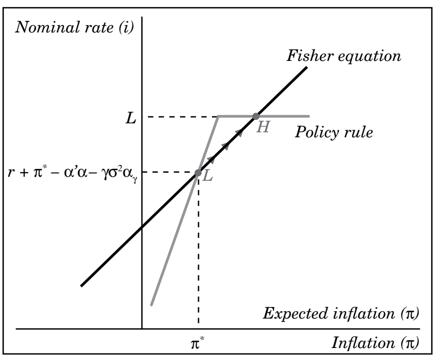
Figure 3 plots the phase diagram matching this case. For simplicity of the graph, consider the case where all shocks are zero, so that $i_t^{(s)} = \int_t^s i_j dj$ and take the intercept in the policy rule to be consistent with the inflation target: $x_t = \overline{x} = r + \pi^*$. Then, starting from a point where the short-term rate equals the long-term rate, the policy resembles a peg. Therefore, the policy rule at point H is horizontal and stays so up to the point where it intersects the feedback rule. Given the monotonicity of the interest rate in the dynamic system, it then follows that the interest rate will be at the bound for all levels of inflation between point H and the point where the ceiling intersects the unbounded policy rule.

There are now two equilibria: the previous unstable one, with inflation on target at point L, and a new globally stable equilibrium at point H, with persistently high inflation. This model allows one to make sense of the conflict between the Fed and the Treasury in the late 40s and of the dynamics of inflation at the time. At first, the economy was close to the L equilibrium. Given small shocks to the real interest rate that shifted the Fisher equation, the Fed would make small adjustments to the bill rate (changes in x_t) to shift the policy rule, and make sure that the interaction L still implied inflation at π^*. These adjustments had to be negotiated with the Treasury, but they were essential since, if the shocks pushed for higher inflation and the Fed was not quick to raise x_t and shift the policy rule, it risked being to the right of the L point and entering escape dynamics towards the H equilibrium. Post 1945, when these positive inflation shocks happened, the political tension between the Fed and the Treasury was therefore large, and concentrated on the level of the T-bill rate.

In 1950, the intensification of the Korea War implied a large increase in r_t shifting the Fisher equation upwards. Controlling inflation would require a sharp increase in the bill rate x_t to shift the policy rule upwards as well and keep inflation on target. The tension intensified and the Accord had to follow. The ceiling played a crucial role because as the two upward-sloping line segments shift upwards, any further positive shocks to inflation would quickly set in dynamics that ultimately lead to point H. Translating this into economics, as real and nominal rates rise, the yield curve flattens, and this reduces the room for further shocks to not make the ceiling put a binding constraint on short rates.

By 1953, it was clear that the Fed must let go of the ceiling. Even with control of short rates and potentially a more aggressive policy in the form of a higher ϕ, still there was a real danger that a future shock would start dynamics towards the H point. The statements by the Fed at the time, of fearing that the policy of pegging the long rate would lead to inflation getting out of hand, are justified by this simple model. The ceiling put a strain on monetary policy because any mistake in setting x_t too low, would lead the economy to enter a stable path where inflation monotonically rises and converges to the high-inflation equilibrium in point H. Abandoning the ceiling was the way to prevent the high-inflation stable equilibrium from becoming the dominant reality in the United States.

Figure 3. Long Ceiling Policy



Source: Author's elaboration.

According to the model, the way in which the Fed went long at the behest of the Treasury was ultimately unsustainable. It created a high-inflation equilibrium that might have been reached and set its stable roots in the U.S. were it not for the strong intervention of the Fed to break its ties with the Treasury.

5. The Radcliffe Commission and U.K. Monetary Policy in the 1960s

On 3 May 1957, the chancellor of the exchequer set up a "Committee on the Working of the Monetary System," headed by Lord Radcliffe. Its official goal was ambitious and wide-ranging: "to inquire into Britain's monetary and credit mechanism, and to make recommendations." It deliberated for more than two years, questioning more than two hundred witnesses, and receiving more than one hundred special memoranda, until the final report was presented in August 1959.

The Radcliffe Report's purported to explain how monetary policy worked and how it should work. Unsurprisingly, it attracted both strong support as well as violent disagreement across the globe. In the academic world, in 1960 alone, there were special articles in the *American Economic Review*, the *Journal of Finance*, and the *Review of Economics and Statistics* devoted to the Report. The prominent monetary economist Anna Schwartz for many years argued that the Report was misguided (Schwartz, 1987). Its policy principles explicitly guided the Bank of England's monetary policy during the 1960s, and were arguably influential for longer, so that the Report plays a central role in any history of the Bank of England in the XXth century.[13]

5.1 Prelude: Criticisms of Monetary Policy in the 1950s

Throughout the 1950s, the U.K. economy was still recovering from the devastating effects of World War II. There were many direct economic controls in place and a large stock of public debt outstanding. The maturity of that debt was low relative to what had been typical, which led to constant pressure to refinance bonds that would come due.

13. Good references for monetary policy in the period before and after the Report, drawing links to long-term interest rates, are Dimsdale (1991), Goodhart (1999), Batini and Nelson (2005), Capie (2010), and Allen (2014).

The Bank of England was not independent, since it operated under the control of the Treasury. Reducing unemployment was the dominant goal of economic policy, and following the prevalence of Keynesian thought, fiscal policy directed to controlling aggregate demand was perceived as the best way to achieve it. Monetary policy was mostly devoted to managing international reserves and preventing fluctuations in the value of the exchange rate. Therefore, almost all of the changes in the main policy rate—the rate at which the Bank of England lends to banks—came in response to international shocks that affected the exchange rate. This led to frequent accusations that the Bank was too short-sighted, since it focused on short rates as opposed to keeping long rates low, a policy of "cheap money" that was popular in Keynesian circles.

As in the United States, right after the war there was an explicit target for the 10-year rate on government bonds of 2.5%. However, it was implemented quite differently. If investors required higher returns to buy the bonds, the Treasury simply refused to sell them. As a result, when during the 1950s the Bank would increase short-term interest rates in response to foreign shocks while keeping long rates fixed, the market for long-term gilts would dry up, and the Treasury would issue mostly Treasury bills. This led to further criticism of the Bank for undermining the national goal of extending the maturity of the stock of government debt.

Academics were likewise critical of the Bank, as this was a time of fervent debate on the role of monetary policy. Students of the gold standard thought that the central bank should be solely in charge of setting an interest rate to affect currency markets. In turn, Treasury officials saw macroeconomic policy through the lenses of a tradeoff between unemployment and inflation, in the spirit of the Phillips curve. More dominant was the view that credit policies were the main tool for a central bank to affect financial markets, while only a minority argued that monetary aggregates were important.

Following large sudden increases in the bank rate in 1955 and 1957 partly to stop an outflow of international reserves and the 1957 rise to power of prime minister Macmillan, the Radcliffe Committee was formed to clarify the role of monetary policy and the functions of the Bank of England. The Radcliffe Committee's hearings became a public arena where competing views of monetary policy were debated.

5.2 The Report's View of Monetary Policy

While the Report was unanimously approved by its members, it did not offer a clear list of conclusions and recommendations. Still, most contemporary readers summarized its contribution in a list of five points. The first four of these have attracted much academic attention already. These are: First, the recommendation that monetary policy has many different goals, sprayed throughout the Report without any clear discussion of policy tradeoffs, and no clear connection between them beyond the fact that central bank actions could in principle be relevant to each of them. Second, the downplay of monetary aggregates or, more generally, of the role of money in affecting macroeconomic outcomes due to the combination of a view that velocity is infinitely elastic and a preference for a broader and looser concept of "liquidity" as the relevant influence on aggregate demand and inflation. Third, the preference for explicit credit policies and controls as the tool that the Bank of England should use to complement the role of fiscal policy in steering aggregate demand. Fourth, a conventional and unremarkable discussion of the role of international reserves and exchange-rate volatility.

The fifth conclusion concerned the role of interest rates, especially at longer maturities, in monetary policy. This is the part of the Radcliffe Report relevant for this paper. It is the most grounded and clearly argued of the five main points, because it builds and expands on the 1945 National Debt Enquiry. Unlike the targets for liquidity, which were never concretely implemented, the advice on interest rates was influential in the setting of Bank of England policy in the 1960s.

The Radcliffe Report saw the management of the public debt as a fundamental goal of monetary policy. This was to be done by setting interest rates at many different maturities since policymakers *"[…] must have and must consciously exercise a positive policy about interest rates, long as well as short, and about the relationship between them."*[14] The quantities of government bonds held at different maturities would then be decided in markets according to investors' demand. The Radcliffe Report implicitly rejected the no-arbitrage view of the term structure, and was closer instead to a clientele perspective, where in each maturity separately, the central bank could choose a price, and market forces would determine the finite quantity that cleared the market.

14. Radcliffe Report, page 337.

The Report went further and dismissed the idea that in setting interest rates, the central bank would have a significant effect on aggregate demand. It likewise dismissed a connection between money and interest rates. Finally, it was critical of the Bank of England for focusing on short-term interest rates, and blamed the failings of monetary policy in the previous decade on its neglect of active management of long-term interest rates.

Throughout the 1960s, U.K. monetary policy devoted itself first to stabilizing the exchange rate and capital flows through the setting of short-term interest rates, and second to managing the yield curve and the cost of government financing through the setting of long-term interest rates. The Radcliffe Report urged the central bank to estimate the "right level" for interest rates. While the Bank never explicitly embraced focusing on one particular long-term interest rate, it continuously estimated a perceived "trend" in yields, which throughout the 1960s kept on rising. Managing the issuance of bonds of different maturities, using credit controls, regulating banks, and adjusting the bank rate were all tools used to ensure that a steady demand for government bonds materialized at the desired target.

5.3 The Bank of England Going Long

One way to interpret U.K. monetary policy is as pegging around an exogenous ι_t. Inflation was not a target for monetary policy, and changes in ι_t either followed some statistically estimated trend, or occurred infrequently as a result of political compromises with the Treasury and changing views on the need to stimulate investment. The going-long policy consisted of focusing monetary policy operations on a long interest rate and choosing this somewhat independently of inflation or aggregate demand. The central bank focused instead on devising a target for the long rate, ι_t, which following Radcliffe, was exogenous to inflation.

It is not a big stretch though to instead model the Bank of England's policy as a feedback rule for the long rate:

$$d(i_t^{(s)} - \iota_t) = -\rho(i_t^{(s)} - \iota_t)dt + \phi_\iota\left(\frac{dp_t}{p_t} - \pi^* dt\right), \tag{39}$$

with a small ϕ_t and a large extent of smoothing ρ. The history of policy decisions at the time has some episodes where an increase in inflation expectations is followed by a discussion of whether to adjust the target for long-term interest rates.

Lemma 3 mapped long-term interest rates into the instantaneous rate. From this map follows the result:

Lemma 4. *The policy rule for long-term interest rates in equation (39) leads to inflation dynamics as in proposition 1 with* $\phi = \phi_t / \delta_i(s)$ *and*

$$x_t = \frac{\iota_t - \delta_0(s)}{\delta_i(s) + \delta_x(s)}. \tag{40}$$

The proof is as follows. Conjecture that the policy rule leads to a rule for instantaneous rates as in equation (11) for some ϕ and some x_t. Then, from lemma 3, we know that it $i_t^{(s)} - \iota_t = \delta_0(s) + \delta_i(s) i_t + \delta_x(s) x_t - \iota_t = \delta_i(s)[i_t - (\iota_t - \delta_0(s) - \delta_x(s)x_t)/\delta_i(s)]$. The conjecture will be verified if $\phi = \phi_\iota / \delta_i(s)$ and if

$$\delta_i(s) x_t = \iota_t - \delta_0(s) - \delta_x(s) x_t. \tag{41}$$

Rearranging gives the expression in the lemma.

In a sense, all central banks follow a rule of this type, as few set a truly instantaneous interest rate, but instead set overnight or one week interest rates. For these short maturities, $\delta_0(s)$ and $\delta_x(s)$ are close to zero, while $\delta_i(s)$ is close to 1. The result in lemma 4 shows that the properties of inflation derived in section 2 then applies with no modifications to these actual policies.

When the central bank goes long, instead, s is large and so $\delta_i(s)$ is small. Section 2 discussed three main determinants of inflation, which using lemma 4 we can now apply to the policy of targeting long-term interest rates.

First, it must be that $\iota > \delta_i(s)\rho$ for inflation to be determinate. Since $\delta_i < 1$, the condition for determinacy is therefore less stringent than it was for shorter rates.

Second, section 2 noted that it takes a precise setting of $\bar{x}$ to make inflation equal its target on average. This translates into an average target for the long-term rate $\bar{\iota}$ that follows the formula in the lemma above. To calculate accurately the real interest rate and the inflation risk premium, the policymaker must also now understand all the determinants of long-term yields, from their long-run average to their sensitivity to each shock. The problem is harder.

The third result in section 2 was that the variance of inflation depended on the variance of the interest rate. Given uncertainty on the parameters that determine the yield curve, setting the interest rate exactly to keep inflation on target (a choice of ι_t to hit $\varepsilon_t = 0$)

appears to be harder. Moreover, the insistence on lowering as much as possible the burden of paying for the national debt and the reluctance in linking interest rates to the evolution of inflation suggests that ι_t was not chosen to attempt to keep ε_t close to zero. Finally, exogenous shocks to ι_t may lead to a larger impact on inflation if $\delta_i(s)+\delta_x(s)<1$.

In conclusion, using long-term rates as the policy tool is consistent with controlling inflation and involves similar considerations as using short-term rates. In fact, the one-to-one map between long and short rates in lemma 3, implies that the set of equilibria that a going-long policy can achieve is the same as the set of equilibria that an equivalent policy for the short rate can achieve, in the sense of lemma 4. However, uncertainty on the shape of the yield curve suggest that this strategy likely comes with higher level and variability of inflation and nominal interest rates.

5.4 Spreads as Targets

At the same time, the Bank of England had multiple targets for different rates. As discussed in section 3, setting more than one interest rate independently would potentially create arbitrage opportunities across the yield curve. Instead, one can think of monetary policy as moving more than one interest rate in tandem to satisfy no-arbitrage. A simple way to model this is as a policy rule for the slope of the yield curve:

$$d(i_t^{(s)} - i_t - \iota_t) = -\rho(i_t^{(s)} - i_t - \iota_t) + \phi_\iota\left(\frac{dp_t}{p_t} - \pi^* dt\right). \tag{42}$$

Similar steps to those in lemma 4 show that

Lemma 5. *The policy rule for the slope of the yield curve in equation (42) leads to inflation dynamics as in proposition 1 with* $\phi=\phi_\iota/(\delta_i(s)-1)$ *and:*

$$x_t = \frac{\iota_t - \delta_0(s)}{\delta_i(s) + \delta_x(s) - 1}. \tag{43}$$

Taking again the relevant case where s is large so $\delta_i(s)$ is small, the outcomes under a slope policy differ significantly from those under a long policy in one aspect. Pinning down inflation requires that $\phi>\rho$ and the higher is ϕ, the lower the variability of inflation in response to shocks. Using the result in the lemma, this requires ϕ_ι to be negative, and significantly so. That is, the model suggests that the central bank should commit to increasing its 10-year yield target by less than its overnight interest rate when inflation increases.

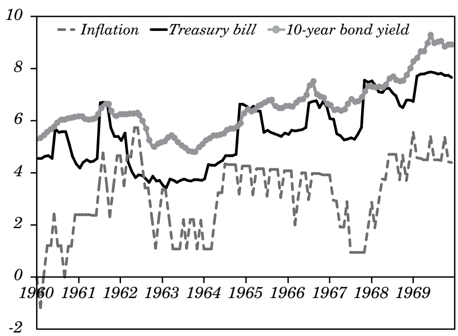
Controlling inflation requires flattening the yield curve in order to lower inflation. Conversely, stimulating inflation requires low overnight rates today that are expected to rise in the future.

To be clear, this result follows in this model because only monetary effects are at play. With nominal rigidities, lowering long rates by flattening the yield curve may stimulate investment which, through the Phillips curve, may raise inflation. Moreover, quantitative easing policies may instead lower term premium, which could affect inflation. Still, any model that has a Fisher equation and a feedback interest-rate rule will have the channel described above, according to which the slope of the yield curve should respond negatively to inflation.

5.5 Inflation Outcomes

To conclude, using long-term interest rates as the tools in feedback rules is consistent with keeping inflation under control. The conditions and economic logic are similar to those in the more familiar case where the policy rate is a short-term rate. However, the analysis suggested that without a precise understanding of the yield curve, its slope, and how it responds to shocks, keeping inflation under control will be hard.

Figure 4. U.K. Interest Rates and Inflation, 1960-70



Source: Author's elaboration.

Figure 4 shows the path of interest rates and inflation during the 1960s in the U.K. Interest rates crept up from 1965 onwards, thus revealing the failure to pin them down at a natural rate. While, as the figure showed, the slope of the yield curve was small, the fiscal problems of the government persisted and intensified and, eventually, ended in a request for an IMF loan a few years later. Towards the end of the decade, inflation started accelerating, and by the early 1970s the Bank of England stopped going long, with the strategy deemed a failure.

6. The Bank of Japan Going Long

Since 1985, annual core CPI inflation in Japan only exceeded 2% in two years and inflation expectations were equally low. In response to fears of deflation, in 1997 the Bank of Japan (BoJ) gradually went long, making this policy explicit at the end of 2016.

Between July 1996 and March 1999, the BoJ expanded the size of its balance sheet by saturating the market for reserves. Starting from March 2001, the BoJ gradually introduced quantitative easing by committing to buy government bonds and to lend to banks in horizons that gradually rose all the way to 3 months. The interest-rate policy was clearly laid out in the Directive of 12 February 1999, which stated that: *"The Bank of Japan will provide more ample funds and encourage the uncollateralized overnight call rate to move as low as possible. To avoid excessive volatility in the short-term financial markets, the Bank of Japan will, by paying due consideration to maintaining market function, initially aim to guide the above call rate to move around 0.15%, and subsequently induce further decline in view of the market developments."* The BoJ repeatedly used forward guidance to state its intention to keep the overnight rate low until inflation expectations rose.

The first stage of this policy was unsuccessful insofar as the price level barely moved between 1997 and 2010, and inflation expectations stayed tightly anchored at 0. In a second stage, between 2010 and 2016, the BoJ rolled out a new policy, the qualitative and quantitative easing (QQE), committing to buy many other assets beyond government bonds. The balance sheet grew rapidly but, more importantly, it changed its composition to become more varied.

The second stage produced an increase in the rate of core inflation, from close to –2% in 2010 to slightly above 1% in 2015. Yet, after an initial jump in inflation in 2013, rising by 1.5% in a little over one year, inflation fell again throughout the second half of 2014 and 2015, so that by the middle of 2016, inflation was back to –0.5%. Consensus inflation expectations started falling since mid to end of 2015, far from their intended 2% inflation target. This led to a third stage in policy in September 2016: the yield-curve control.

The BoJ announced a target not just for the overnight central bank rate, but also for an intended yield on the 10-year government bond rate and, in the future, potentially other maturities as well. The BoJ announced a desired target of 0% for the 10-year government bond rate, while the target for the overnight rate was –0.1%. This was implemented by adjusting the purchase programs of bonds at the 10-year maturity to stay near the target.

It is too early to know how this policy will be pursued in the future. The analysis in this paper suggested that depending on whether the BoJ going long is formulated as: (i) a peg, (ii) a ceiling, (iii) a feedback rule for long rates, (iv) a rule for the term spread, or (v) something else, this has very different implications for how to stimulate inflation and for the dangers that may arise. Alternatively, perhaps the policy of the BoJ consists of separate pegs for the overnight and 10-year rate, as in the model of Reis (2017). Whichever way, if central banks follow the lead of the BoJ and go long, both history and theory should try to inform their policy choices.

7. Conclusion

In the past decade, it became common among policymakers to discuss monetary policies in terms of their impact on long-term interest rates. For instance, in her survey of the conduct of monetary policy and the role of quantitative easing by the Federal Reserve during the crisis, Chair Janet Yellen (2017) wrote: *"For this reason, the Committee turned to asset purchases to help make up for the shortfall by putting additional downward pressure on longer-term interest rates."* The Bank of Japan has gone further by announcing an explicit 0% target for the 10-year rate. Central banks have been going long by increasingly focusing on longer-term interest rates.

This paper went back in history to discuss the experience of the Federal Reserve in the 1940s and the Bank of England in the 1960s. They were different in interesting ways, and mapping them to explicit policies in a model is subject to interpretation. The analysis in this paper suggested several caveats to going long. First, unless it is implemented carefully, it can put the solvency of the central bank at risk or lead to much volatility in interest rate markets. Second, a ceiling on long-term rates creates a stable equilibrium with high inflation to which the economy can easily escape if there are positive shocks to inflation. Third, a feedback rule for long rates requires very precise knowledge of the yield curve and how it changes with separate shocks. Fourth, making the slope of the yield curve the policy tool requires steepening the yield curve, raising long rates relative to short rates, in order to raise inflation.

The analysis required linking long- and short-term interest rates, and inflation in a tractable way that keeps the effects of uncertainty and risk premiums. This suggested setting the problem of inflation control in an economy where shocks follow continuous-time diffusions. This opens the door for future work to introduce frictions, such as nominal rigidities and financial imperfections, in order to improve the model of the endogenous determination of inflation and term premia in the yield curve.

APPENDIX

A. Partial Adjustment to Real Interest Rates

Imagine now that nominal interest rates adjust only partially to real interest rates. This is achieved by having the policy choice of nominal interest rates, x_t, follow instead:

$$x_t - \overline{x} = \zeta(r_t - \overline{r}) + (1-\zeta)\tilde{x}_t \tag{A1}$$

where, with a slight abuse of notation, now it is $\tilde{x}_t$ that follows an exogenous stationary process:

$$d\tilde{x}_t = -\kappa_x \tilde{x}_t d_t + \sigma_x dz_t^x. \tag{A2}$$

Finally, set $\overline{x} = \overline{r} + \pi^* - \alpha'\alpha - \gamma\sigma_y^2\alpha_y$, so that on average inflation is on target.

Now, if $\zeta = 1$, we are back in the first case covered in the text, in which $\varepsilon_t = 0$ at all dates (equation (27)). If $\zeta = 0$, we are in the second case, and the solution for inflation is the one given by equation (29).

Under this new rule:

$$\varepsilon_t = (r_t - \overline{r}) - (x_t - \overline{x}) = (1-\zeta)(r_t - \overline{r} - \tilde{x}_t). \tag{A3}$$

If therefore follows that:

$$\mathbb{E}_t(\varepsilon_{t+s} - \varepsilon_t) = (1-\zeta)(r_t - \overline{r})(e^{-\kappa_g s} - 1) - (1-\zeta)\tilde{x}_t(e^{-\kappa_x s} - 1). \tag{A4}$$

Plugging this into equation (26) and rearranging gives the new solution for expected inflation:

$$\begin{aligned}\pi(r_t, x_t) = \pi^* &+ \left(\frac{\rho - \kappa_g}{\kappa_g + \phi - \rho}\right)(1-\zeta)(r_t - \overline{r}) \\ &- \left(\frac{\rho - \kappa_x}{\kappa_x + \phi - \rho}\right)(1-\zeta)\tilde{x}_t.\end{aligned} \tag{A5}$$

Clearly, if $\zeta = 1$, then inflation is on target, while if $\zeta = 0$, this equation is equivalent to equation (29), thus nesting the two cases in the text.

Finally, turning to the shocks on inflation, now equation (30) leads to:

$$\left(\frac{\rho-\kappa_g}{\kappa_g+\phi-\rho}\right)(1-\zeta)[dr-d\hat{r}]-\left(\frac{\rho-\kappa_x}{\kappa_x+\phi-\rho}\right)(1-\zeta)[d\tilde{x}_t-d\hat{x}_t]$$
$$=-\gamma\sigma_g dz_t^g+\sigma_x dz_t^x+\phi\alpha_y\sigma_y dz_t^y+\phi\alpha_g\sigma_g dz_t^g+\phi\alpha_x\sigma_x dz_t^x. \tag{A6}$$

Collecting terms this becomes:

$$\alpha_x=-\frac{1-\zeta}{\kappa_x+\phi-\rho} \tag{A7}$$

$$\alpha_g=\frac{\gamma(1-\zeta)}{\kappa_g+\phi-\rho} \tag{A8}$$

$$\alpha_y=0. \tag{A9}$$

This again matches the solution in the main text for the two polar cases.

B. Proof of Lemma 2

Combine equations (14) and (29) and simplify by grouping terms to get the solution.

C. Proof of Lemma 3

Start with the Euler equation:

$$\mathbb{E}_t\left(\frac{m_{t+s}p_t}{m_t p_{t+s}Q_t^{(s)}}\right)=1 \tag{A10}$$

where I have used the notation $Q_t^{(s)}\equiv\frac{1}{I_t^{(s)}}$, to denote the price (the inverse of the yield) of the s-long bond. The differential version of this equation is:

$$\mathbb{E}_t\left(\frac{dQ_t^{(s)}}{Q_t^{(s)}}-\frac{\partial Q_t^{(s)}/\partial s}{Q_t^{(s)}}dt\right)+\mathbb{E}_t(m_t/p_t)$$
$$+\mathbb{E}_t\left(\frac{d(m_t/p_t)}{(m_t/p_t)}\frac{dQ_t^{(s)}}{Q_t^{(s)}}\right)=0, \tag{A11}$$

where the second term inside the first parentheses takes into account the fact that an instant later, the bond's maturity is shorter.

Guess that $\log Q_t^{(s)} = a(s) - b(s)r_t - c(s)x_t$, with undetermined coefficients $a(s), b(s), c(s)$.

Then, using Ito's lemma, it follows that:

$$\frac{dQ_t^{(s)}}{Q_t^{(s)}} = -b(s)dr_t - c(s)dx_t + \frac{1}{2}\left(b(s)^2\gamma^2\sigma_g^2 dt + c(s)^2\sigma_x^2 dt \right). \tag{A12}$$

Using this to replace into the pricing condition, and evaluating the expectations gives a long expression, where each of the four lines matches each of the four terms in the pricing equation:

$$\begin{aligned} & b(s)\kappa_g(r_t - \bar{r})dt + c(s)\kappa_x(x_t - \bar{x})dt \\ & + \left(\frac{b(s)^2}{2} \right)\gamma^2\sigma_y^2 + \left(\frac{c(s)^2}{2} \right)\sigma_x^2 \\ & - (a'(s) - b'(s)r_t - c'(s)x_t) \\ & -(\theta_0 + \theta_r r_t + \theta_x x_t) \\ & -b(s)\gamma\sigma_g^2(\alpha_g + \gamma) - c(s)\alpha_x\sigma_x^2 = 0. \end{aligned} \tag{A13}$$

Since this equation must hold for each and every realization of the state variables, one can match the coefficients in x_t to get an ordinary differential equation:

$$b(s)\kappa_g + b'(s) - \theta_r = 0. \tag{A14}$$

Together with the boundary condition that $b(0) = 0$, this has the simple solution:

$$b(s) = \theta_r(1 - e^{-\kappa_g s}) / \kappa_g \tag{A15}$$

Similarly, one can easily solve for $a(s)$ and $c(s)$.

Finally, by the definition of the long rate:

$$i_t^{(s)} = \frac{\log(I_t^{(s)})}{s} = \frac{-\log(Q_t^{(s)})}{s} = \frac{-a(s) + b(s)r_t + c(s)x_t}{s} \tag{A16}$$

Using lemma 2 to replace out r_t, this delivers the expression in lemma 3, where $\delta_i(s) = b(s)/(s\theta_r)$.

References

Allen, W.A. 2014. "Monetary Policy and Financial Repression in Britain, 1951-59." Palgrave MacMillan.

Ball, L., N.G. Mankiw and R. Reis. 2005. "Monetary Policy for Inattentive Economies." *Journal of Monetary Economics* 52(4): 703–25.

Batini, N. and E. Nelson. 2005. "The U.K.'s Rocky Road to Stability." Federal Reserve Bank of St. Louis Working Paper No. 2005–020.

Brunnermeier, M.K. and Y. Sannikov. 2017. "Macro, Money and Finance: A Continuous-Time Approach." In *Handbook of Macroeconomics*, vol. 2B. The Netherlands: North-Holland.

Capie, F. 2010. *The Bank of England: 1950s to 1979*. Cambridge, UK: Cambridge University Press.

Cochrane, J.H. 2011. "Determinacy and Identification with Taylor Rules." *Journal of Political Economy* 119(3): 565–615.

Dimsdale, N.H. 1991. "British Monetary Policy since 1945." In *The British Economy since 1945*, edited by N.F.R. Crufts and N.W.C. Woodwood. Oxford, UK: Clarendon Press.

Friedman, M. and A.J. Schwartz. 1963. "A Monetary History of the United States, 1867–1960." Princeton University Press.

Gallmeyer, M.F., B. Hollifield, and S.E. Zin. 2005. "Taylor Rules, McCallum Rules and the Term Structure of Interest Rates." *Journal of Monetary Economics* 52(5): 921–50.

Goodhart, C.A.E. 1999. "Monetary Policy and Debt Management in the United Kingdom: Some Historical Viewpoints." In *Government Debt Structure and Monetary Conditions*, edited by K. Alec Chrystal. London, UK; Bank of England.

Greenwood, R., S.G. Hanson, and D. Vayanos. 2016. "Forward Guidance in the Yield Curve: Short Rates versus Bond Supply." In *Monetary Policy through Asset Markets: Lessons from Unconventional Measures and Implications for an Integrated World*. Vol. 24, edited by Elías Albagli, Diego Saravia, and Michael Woodford. Santiago: Central Bank of Chile.

Hall, R.E. and R. Reis. 2015. "Maintaining Central-Bank Solvency under New-Style Central Banking." NBER Working Paper 21173.

————. 2016. "Achieving Price Stability by Manipulating the Central Bank's Payment on Reserves." NBER Working Paper 22761.

Hetzel, R.L. and R. Leach. 2001. "The Treasury-Fed Accord: A New Narrative Account." *FRB Richmond Economic Quarterly* 87(1): 33–56.

Jones, C. and M. Kulish. 2013. "Long-term Interest Rates, Risk Premia and Unconventional Monetary Policy." *Journal of Economic Dynamics and Control* 37(12): 2547–61.

McGough, B., G.D. Rudebusch, and J.C. Williams. 2005. "Using a Long-Term Interest Rate as the Monetary Policy Instrument." *Journal of Monetary Economics* 52(5): 855–79.

Meltzer, A. 2010. *A History of the Federal Reserve, Volume 1: 1913–1951.* Chicago, IL: University of Chicago Press.

Nakajima, T. and H. Polemarchakis. 2005. "Money and Prices Under Uncertainty." *Review of Economic Studies* 72(1): 223–46.

Obstfeld, M. and K. Rogoff. 1983. "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" *Journal of Political Economy* 91(4): 675–87.

Piazzesi, M. 2010. "Affine Term Structure Models." In *Handbook of Financial Econometrics*. 691–766. The Netherlands: North-Holland.

Reis, R. 2016*a*. "Funding Quantitative Easing to Target Inflation." In *Designing Resilient Monetary Policy Frameworks for the Future*. Jackson Hole Symposium: Federal Reserve Bank of Kansas City.

———. 2016*b*. "How do Central Banks Control Inflation? A Guide to the Perplexed." Detailed lecture notes.

———. 2017. "Qualitative and Quantitative Easing with Yield Curve Control." LSE manuscript.

———. 2018. "Can the Central Bank Alleviate Fiscal Burdens?" In *The Oxford Handbook of the Economics of Central Banking*, edited by D. Mayes, P.L. Siklos, and J.-E. Sturm. Oxford, UK: Oxford University Press.

Sargent, T.J. and N. Wallace. 1975. "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule." *Journal of Political Economy* 83(2): 241–54.

Schwartz, A.J. 1987. *Money in Historical Perspective.* University of Chicago Press.

Smith, J.M. and J.B. Taylor. 2009. "The Term Structure of Policy Rules." *Journal of Monetary Economics* 56(7): 907–17.

Woodford, M. 2003. "Interest and Prices: Foundations of a Theory of Monetary Policy." Princeton, NJ: Princeton University Press.

———. 2010. "Optimal Monetary Stabilization Policy." In *Handbook of Monetary Economics*, vol. 3, edited by B.M. Friedman and M. Woodford. Elsevier: North Holland.