The Market for Inflation Risk

Saleem Bahaj[1,2] Robert Czech[2] Sitong Ding[3] Ricardo Reis[3]

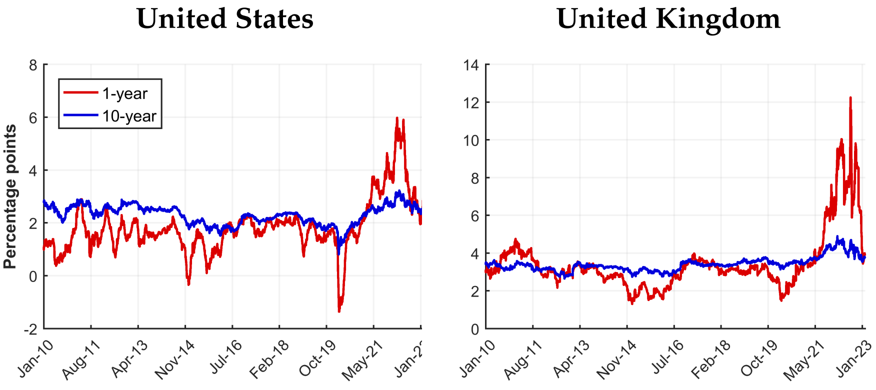
[1]UCL

[2]Bank of England

[3]LSE

April 2025

Breakeven prices of inflation swaps



How reliable are movements in inflation swap prices as indicators of expected inflation? How quickly do they incorporate information, and how do they compare with survey data? Who are the relevant market participants, and how and why do they transfer inflation risk between them? What are the supply and demand curves for protection against inflation?

What the paper does

1) **Quantities behind the prices:** universal data on transactions in UK market.
   → Facts: segmentation across maturities, banks net bearers of inflation risk.

2) **Identification strategies:** for segmented markets' models
   → Heteroskedasticity (time series), granular instrumental variable (cross-section), sign/timing restrictions (high frequency).

3) **Estimates of expected inflation:** cleaned of frictions.
   → Versus swap prices, at short and long horizons

4) **Estimates of this market:** the drivers of a hedging market
   → What shocks drive prices, and what are the slopes of supply and demand

5) **From beliefs to trading:** do banks put their money where their mouth is?
   → When one expects more inflation does it purchase more inflation protection?

2. The facts about this market

Data

- Swap contract: bilateral agreement where the floating leg payer pays realized inflation over the length of the contract, while the fixed leg payer pays fixed rate, called the *breakeven inflation rate*. The *price*

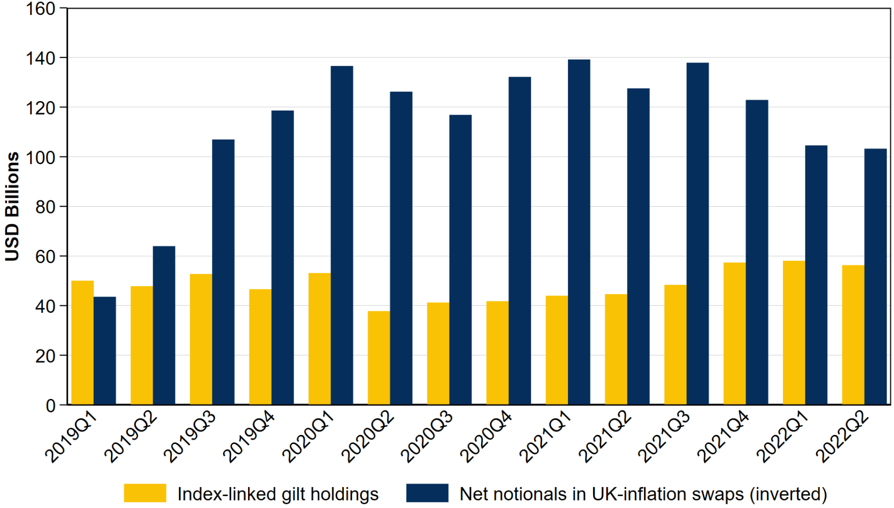
- Payoff: the two parties continuously exchange payments to ensure the contract remains at zero net present value: variation margin. Payments tied to changes in expected inflation. RPI is the measure

- Gross notional: amount swapped that links the size of cash flows to inflation and the fixed rate (netted upon settlement). 110-130% of GDP in UK (compared to 20% of indexed bonds). Net notional is whether an institution is overall a net buyer (if positive) or seller (if negative) of protection against high inflation.

- Source of data: EMIR trade repository data, 2019-2023, 25 million trades.

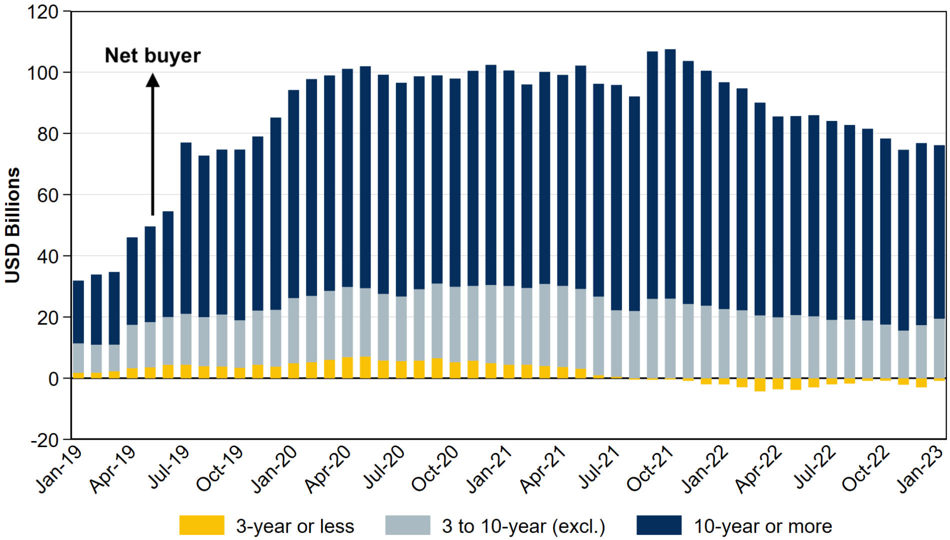
- Dealer client segment: $1.1 trillion in gross notional terms.

FACT 1: DEALERS ARE NOT NEUTRAL MARKET MAKERS

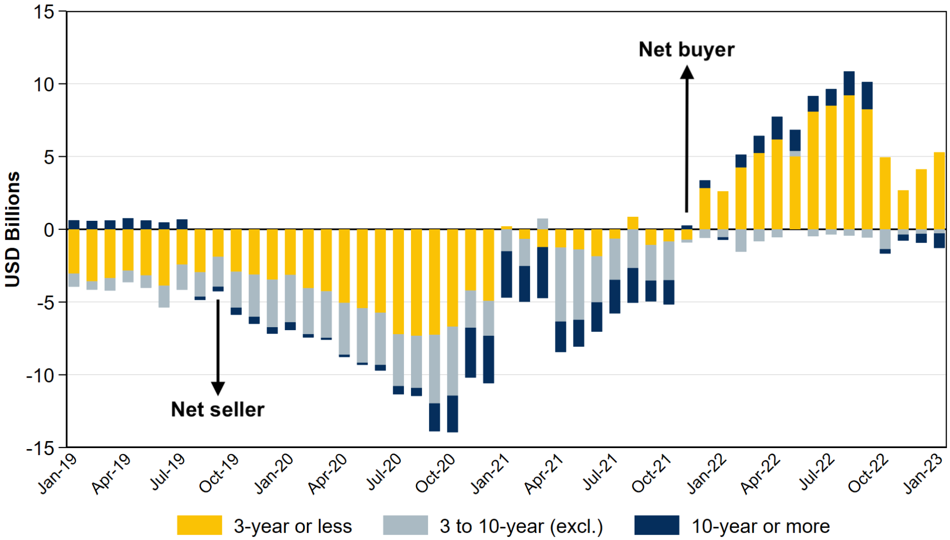


The 15% overshoot in UK inflation over 2021-2023 cost UK banks \$15bn $\approx$ 3% of their capital in cash flow terms.

Fact 2: pension funds buy protection at long maturities

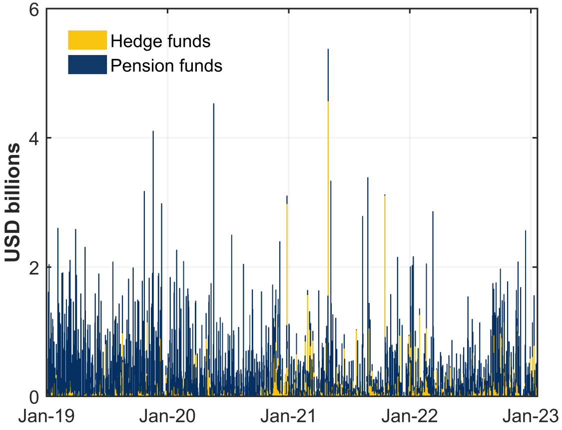


Fact 3: hedge funds trade risk at short maturities

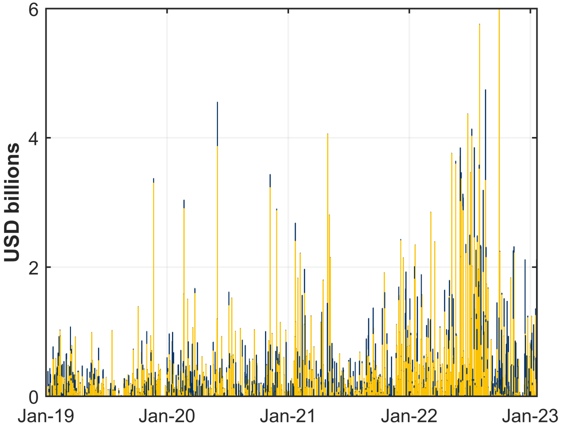


SEGMENTATION CLEARER IN TRADING ACTIVITY

Volume: long maturity



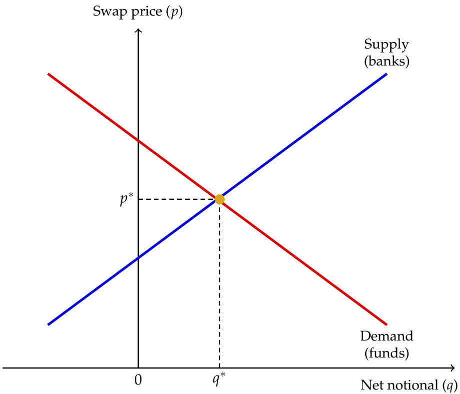
Volume: short maturity



3. A model of the market and identification problem

The market

Figure 2 A stylized description of the market for inflation swaps

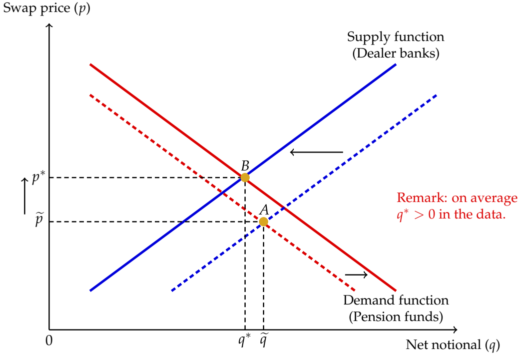


Banks supply insurance and pension and hedge funds demand it

- $q \neq 0$ reflects gains from trade from disagreement about expected inflation
- p is a risk-aversion-wealth-weighted average of risk-adjusted expected inflation

Frictional component

(a) The long maturity segment and the effect of shocks to supply (ε_b) and demand (ε_f)

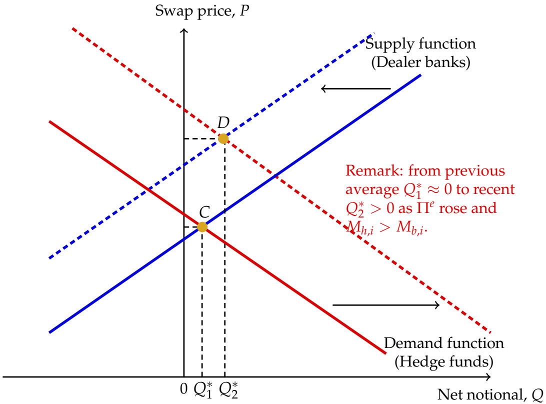


Pension fund mandates and bank constraints shift demand and supply:

- Background risk and operational limits, e.g., indexed linked liabilities, balance sheet space
- Regulatory and trading constraints, e.g., margin limits, market making
- Changes in p due to frictions.

Identification problem

(b) The short maturity segment and the response to an inflation shock (ε_π)



Increase in expected inflation

- Shifts both demand and supply upwards
- Change in P reflects this fundamental
- But cannot distinguish by simultaneous shock to frictions on supply and demand.

More formally: problem of agents

- Pension funds (f), hedge funds (h), dealer banks (b). Bank i with assets $a_{b,i}$ goal:

$$\mathbb{E}_{b,i}\left[U(a'_{b,i})\right]$$

- Swap costs $p(P)$ pays off $\pi(\Pi)$. Other assets $e_{b,i}$ and background risk $y_{b,i}$

$$a'_{b,i} = a_{b,i} + (\pi - p)q_{b,i} + (\Pi - P)Q_{b,i} + (d - s)e_{b,i} + y_{b,i}$$

- Expectations unrestricted about inflation

$$\mathbb{E}_{b,i}(\pi) = \mu_{b,i}\pi^e \quad \text{with} \quad \Big(\sum_{i\in\Theta_f}\mu_{f,i} + \sum_{i\in\Theta_b}\mu_{b,i}\Big) / \left(|\Theta_f| + |\Theta_b|\right) = 1$$

- Trading constraint (with LM $\lambda^{L,*}_{b,i}, \lambda^{S,*}_{b,i}$):

$$G^L_b(q_{b,i}, Q_{b,i}, z_{b,i}) \geq 0 \quad \text{and} \quad G^S_b(Q_{b,i}, q_{b,i}, z_{b,i}) \geq 0$$

- *(Segmented markets.)* Pension funds do not participate in the short maturity segment, $Q_{f,i} = 0$; hedge funds do not participate in the long maturity segment, $q_{h,i} = 0$.

Functional form assumptions

1) CARA preferences with $\widetilde{\gamma}_{d,i} = \gamma_{d,i}/a_{d,i}$

$$U(.) = -\exp\left(-\widetilde{\gamma}_{b,i} a'_{b,i}\right)$$

2) Normal beliefs with $\mathbb{E}_{f,i}[d] = \theta_d$ and $\mathbb{E}_{f,i}[y_{f,i}] = 0$, and with $\sigma_{d,y_{f,i}} = 0$ to focus on inflation.

3) Inflation at different horizons covaries with market returns following a one-factor structure, so that $\rho_{\pi,\Pi} = \rho_{\pi,d}\rho_{\Pi,d}$.

Long maturity demand and market

Demand from dealer bank

$$\frac{q^*_{b,i}}{a_{b,i}} = \underbrace{\frac{\mu_{b,i}\pi^e - p^*}{\gamma_{b,i}\sigma^2_\pi(1-\rho^2_{\pi,d})}}_{\textit{price and beliefs}} - \underbrace{\left(\frac{\sigma_d}{\sigma_\pi}\right)\left[\frac{\theta_d - s}{\gamma_{b,i}\sigma^2_d(1-\rho^2_{\pi,d})}\right]\rho_{\pi,d}}_{\textit{hedging demand}} - \underbrace{\left[\frac{1}{(1-\rho^2_{\pi,d})\sigma^2_\pi}\right]\left(\frac{\sigma_{\pi,y_{b,i}}}{a_{f,i}} + \frac{\lambda^{L,*}_{b,i} g^*_{f,i}}{\gamma_{b,i}}\right)}_{\textit{frictions}}$$

Dealers on other side, similar problem. Market clearing:

$$q^* \equiv \sum_{i\in\Theta_f} q^*_{f,i} = -\sum_{i\in\Theta_b} q^*_{b,i}$$

Frictionless swap prices

$\widetilde{p}$ is the fundamental price of a long horizon inflation swap if there are complete markets to fully insure institution-specific income risk, so $\sigma_{\pi,y_{b,i}} = \sigma_{\pi,y_{f,i}} = \sigma_{\pi,y_{h,i}} = 0$, and the trading constraints do not bind for any agent in either market segment, $\lambda_{b,i}^{L,*} = \lambda_{f,i}^{*} = \lambda_{b,i}^{S,*} = \lambda_{h,i}^{*} = 0$. In equilibrium it is:

$$\widetilde{p} = \underbrace{\left[\frac{\sum_{i\in\Theta_f} \widetilde{\gamma}_{f,i}^{-1}\mu_{f,i} + \sum_{i\in\Theta_b} \widetilde{\gamma}_{b,i}^{-1}\mu_{b,i}}{\sum_{i\in\Theta_f} \widetilde{\gamma}_{f,i}^{-1} + \sum_{i\in\Theta_b} \widetilde{\gamma}_{b,i}^{-1}}\right]}_{\equiv\Lambda,\ \textit{risk-adjusted size-weighted dispersion of beliefs}} \underbrace{\pi^e}_{\textit{expected inflation}} - \underbrace{\left(\frac{\theta_d - s}{\sigma_d\sigma_\pi}\right)\rho_{\pi,d}}_{\textit{risk premium}}.$$

Shocks ε_π are innovations to π^e or to $\rho_{\pi,d}$.

Frictional premium

The price of a long-horizon swap is:

$$p^* = \widetilde{p} - \underbrace{\frac{\sum_{i\in\Theta_f}\left\{\sigma_{\pi,y_{f,i}} + \frac{\lambda^*_{f,i} g^*_{f,i}}{\widetilde{\gamma}_{f,i}}\right\}}{\sum_{i\in\Theta_f}\widetilde{\gamma}^{-1}_{f,i} + \sum_{i\in\Theta_b}\widetilde{\gamma}^{-1}_{b,i}}}_{\textit{frictional demand from pension funds}} - \underbrace{\frac{\sum_{i\in\Theta_b}\left\{\sigma_{\pi,y_{b,i}} + \frac{\lambda^{L,*}_{b,i} g^{L,*}_{b,i}}{\widetilde{\gamma}_{b,i}}\right\}}{\sum_{i\in\Theta_f}\widetilde{\gamma}^{-1}_{f,i} + \sum_{i\in\Theta_b}\widetilde{\gamma}^{-1}_{b,i}}}_{\textit{frictional supply from dealer banks}}$$

Changes in $z_{f,i}$ shifting $\lambda^*_{f,i}$ and changes in $\sigma_{\pi,y_{f,i}}$ shift aggregate demand, shocks ε_f.

Changes in $z_{b,i}$ shifting $\lambda^{L,*}_{b,i}$ and $\sigma_{\pi,y_{b,i}}$ shift the supply curve, shocks ε_b.

Spillover from short-horizon market: shock ε_h affects Q^*, which shifts $G^L_b(q_{b,i}, Q_{b,i}, z_{b,i})$, which shifts $g^{L,*}_{b,i}$ and so p^*

Identification problem

Data: $\mathbf{Y} = (Q, P, q, p)'$ on prices and quantities 2 Jan 19 to 10 Feb 23, 1,078 daily observations, covering 210 pension funds, 30 hedge funds, and 16 dealer banks (13 in the short market). Long maturity is $>=$ 10 years and short is $\leq$ 3-year maturity.

Shocks that drive it: $\boldsymbol{\varepsilon} = (\varepsilon_h, \varepsilon_f, \varepsilon_b, \varepsilon_\pi)$

Identification problem: Need to learn about the 4x4 matrix $\boldsymbol{\Psi}$.

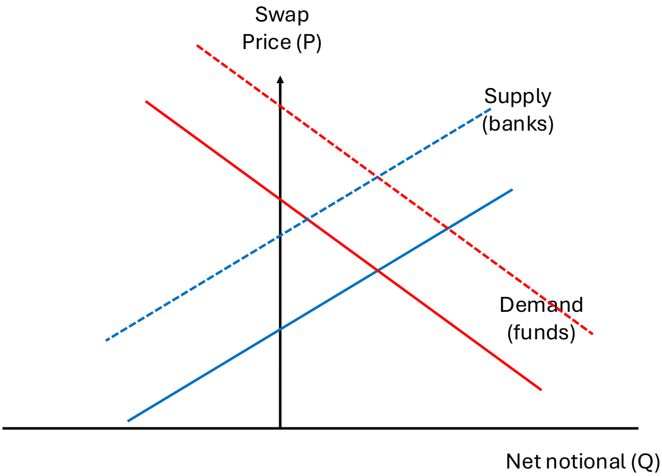
$$\mathbf{Y} = \boldsymbol{\Psi}\boldsymbol{\varepsilon}$$

Estimation: add dynamics, Bayesian VAR with 3 lags and a deterministic constant

$$\mathbf{Y}_t = \mathbf{c} + \sum_{\ell=1}^{L} \boldsymbol{\Phi}_\ell \mathbf{Y}_{t-\ell} + \mathbf{u}_t \quad \text{and} \quad \mathbf{u}_t = \boldsymbol{\Psi}\boldsymbol{\varepsilon}_t.$$

4. Three identification strategies

1. Heteroskedasticity across time



Fundamental had a higher relative variance on announcement days

- Inflation fundamental news is lumpy
- Data: 49 monthly dates when the data on UK RPI inflation was released, and 2 Truss moments
- **Exploit the time-series length of our data**

Formally

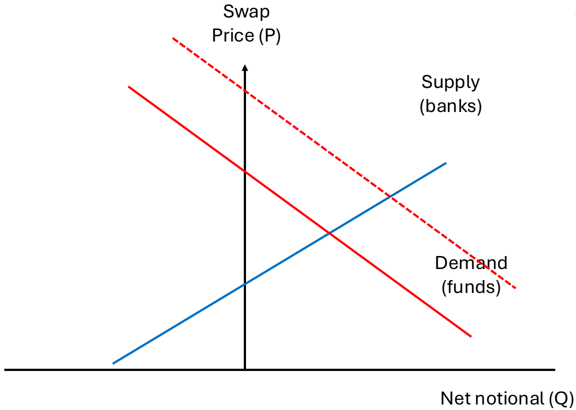
Assumption. *(Heteroskedascity in the fundamental shock at known dates)* Let $\mathbf{\Sigma}_H$ denote the diagonal variance-covariance matrix of the shocks $\boldsymbol{\varepsilon}$ at data release dates, and $\mathbf{\Sigma}_L$ the one at other dates. We assume:

1) *The largest diagonal of* $\Sigma_H \Sigma_L^{-1}$ *is greater than one, unique and corresponds to the ratio of variances of the fundamental shock between release and other dates.*

2) Ψ *does not change between release and non-release dates.*

In data: the (median) estimate of the largest eigenvalue is 1.43.

2. Granularity across institutions



Size-weighted sum of idiosyncratic shocks non-zero in expectation

- Demand system is a panel factor model

$$\frac{q_{f,i,t}}{a_{f,i,t}} = \boldsymbol{\omega}'_{f,i}\mathbf{F}_t + \widetilde{\varepsilon}_{f,i,t}$$

- Build granular IV: $GIV_{f,t} = \sum_{i \in \Theta_f} a_{f,i,t}\tilde{\varepsilon}_{f,i,t}$. Valid instrument for ε_f as orthogonal by construction, relevant if LLN fails.
- With $GIV_{h,t}$ and $GIV_{b,t}$, have instruments for three liquidity shocks. Fundamental follows
- **Exploit the cross-section trading behavior variation of our data**

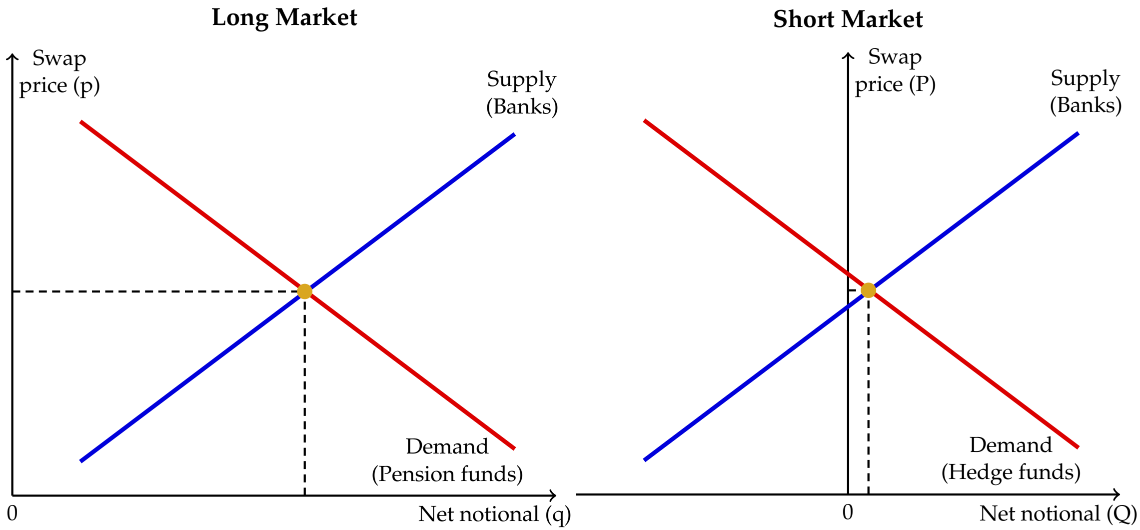
Formally

Assumption: *(Granularity of the institutions.)* The data on asset positions $a_{f,i,t}$, $a_{h,i,t}$ and $a_{b,i,t}$ is granular in that:

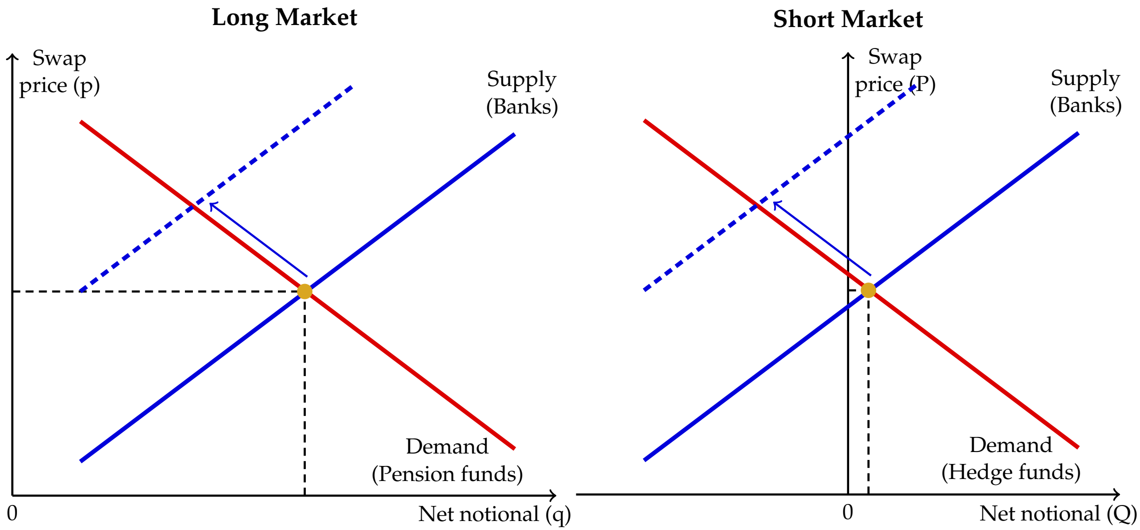
$$\mathbb{E}(GIV_{f,t}\varepsilon_{f,t}) \neq 0 \text{ and } \mathbb{E}(GIV_{b,t}\varepsilon_{b,t}) \neq 0 \text{ and } \mathbb{E}(GIV_{h,t}\varepsilon_{h,t}) \neq 0.$$

- The size of pension funds' gross positions in the long horizon market can be well described by Zipf's law (estimated power coefficient is -0.9).
- Interactive fixed effects model with 21 (as opposed to two) factors.
- The instruments are relevant: the first-stage F-statistics are: 72.4 for $GIV_{f,t}$, 22.3 for $GIV_{h,t}$ and 43.5 for $GIV_{b,t}$.

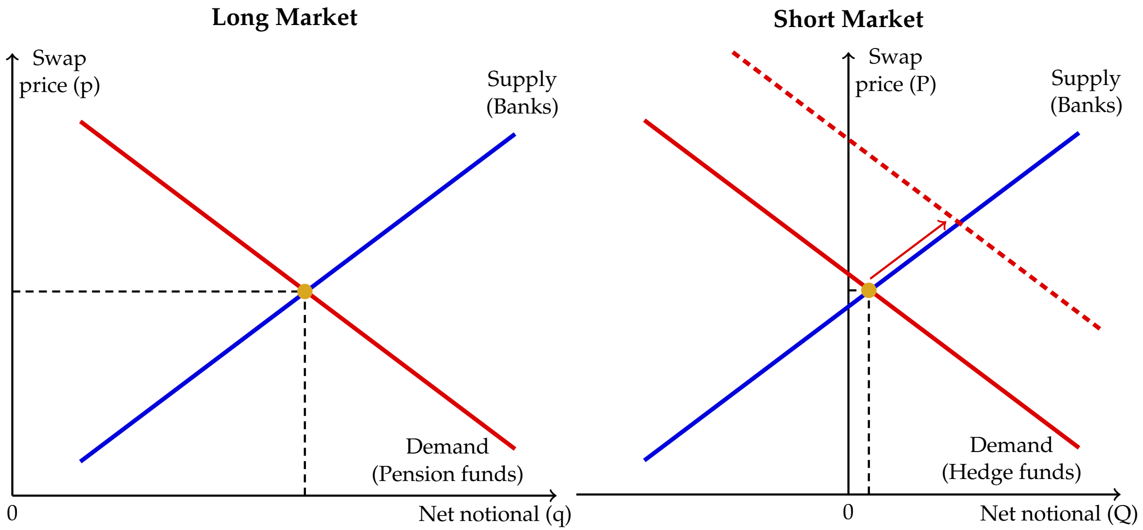
3. Exploit segmented markets



3. Exploit segmented markets



3. Exploit segmented markets



FORMALLY

Assumption: *(Desk separation within the day.)* The dealers' capacity constraints are independent of each other: $\partial G_b^S(\cdot,\cdot)/\partial q_{b,i} = 0$ and $\partial G_b^L(\cdot,\cdot)/\partial Q_{b,i} = 0$ so that they are:

$$G_b^S(Q_{b,i}, z_{b,i}) \geq 0 \quad \text{and} \quad G_b^L(q_{b,i}, z_{b,i}) \geq 0\,. \tag{1}$$

Assumption: *(Differential reactiveness to fundamental news about inflation.)* Dealer banks respond more to long maturity expected inflation than pension funds but less to short maturity expected inflation than hedge funds:

$$\frac{\sum_{i\in\Theta_b} \widetilde{\gamma}_{b,i}^{-1}\mu_{b,i}}{\sum_{i\in\Theta_f} \widetilde{\gamma}_{f,i}^{-1} + \sum_{i\in\Theta_b} \widetilde{\gamma}_{b,i}^{-1}} > \frac{\sum_{i\in\Theta_f} \widetilde{\gamma}_{f,i}^{-1}\mu_{f,i}}{\sum_{i\in\Theta_f} \widetilde{\gamma}_{f,i}^{-1} + \sum_{i\in\Theta_b} \widetilde{\gamma}_{b,i}^{-1}}\,,$$

$$\frac{\sum_{i\in\Theta_h} \widetilde{\gamma}_{h,i}^{-1}M_{h,i}}{\sum_{i\in\Theta_h} \widetilde{\gamma}_{h,i}^{-1} + \sum_{i\in\Theta_b} \widetilde{\gamma}_{b,i}^{-1}} > \frac{\sum_{i\in\Theta_b} \widetilde{\gamma}_{b,i}^{-1}M_{b,i}}{\sum_{i\in\Theta_f} \widetilde{\gamma}_{h,i}^{-1} + \sum_{i\in\Theta_b} \widetilde{\gamma}_{b,i}^{-1}}\,.$$

IMPLICATIONS

- **Exploiting high frequency of the data**
- Sign restrictions, the zeros in blue come from the desk separation assumption, and the signs in red come from the differential reactiveness assumption.

$$\begin{pmatrix} \text{short qty} \\ \text{short price} \\ \text{long qty} \\ \text{long price} \end{pmatrix} = \underbrace{\begin{pmatrix} + & 0 & - & + \\ + & 0 & + & + \\ 0 & + & - & - \\ 0 & + & + & + \end{pmatrix}}_{\boldsymbol{\Psi}} \begin{pmatrix} \text{hedge fund demand} \\ \text{pension fund demand} \\ \text{dealer-bank supply} \\ \text{fundamental} \end{pmatrix}$$

5. Estimates of expected inflation

The shocks across identification strategies

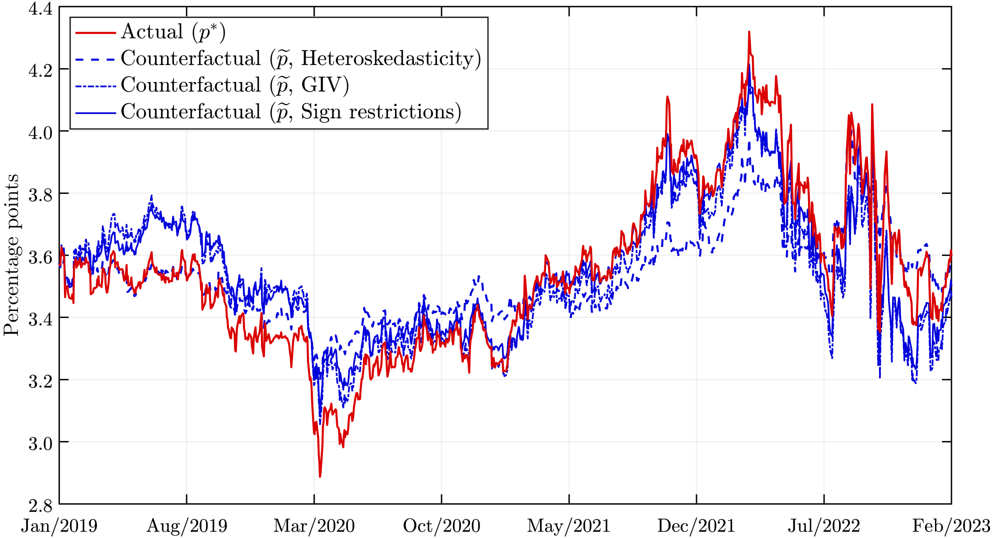
Correlations of median fundamental shock from all three strategies (SR, GIV, Hetero):

$$\begin{bmatrix} 1 & 0.98 & 0.84 \\ \cdot & 1 & 0.89 \\ \cdot & \cdot & 1 \end{bmatrix}$$

These correlations can also be interpreted as overidentification tests:

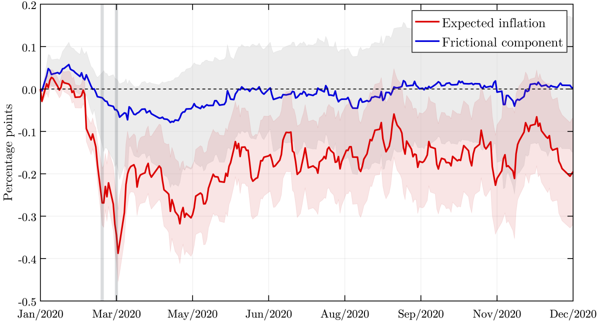
- ε_π from strategy 2 have higher relative variance in the release dates used in strategy 3 in 99% of draws.
- $\frac{1}{T}\sum_{t=1}^{T} GIV_{\nu,t}\hat{\varepsilon}_{\pi,t}$ for $\nu \in \{f, h, b\}$ from strategies 1 & 3 are $(-0.0068, 0.0089, 0.031)$ and $(-0.061, 0.012, 0.056$, respectively, supporting strategy 2
- IRFs from strategies 1 & 2 confirm the sign restrictions in strategy 3. Differential reactiveness & desk separation assumptions hold in the microdata.

The evolution of risk-neutral expected inflation

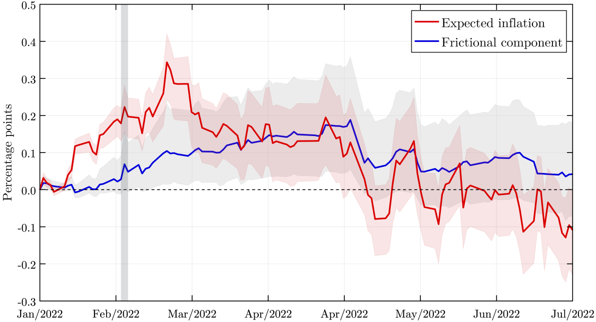


Two episodes: Pandemic and Ukraine

(a) COVID-19 pandemic period

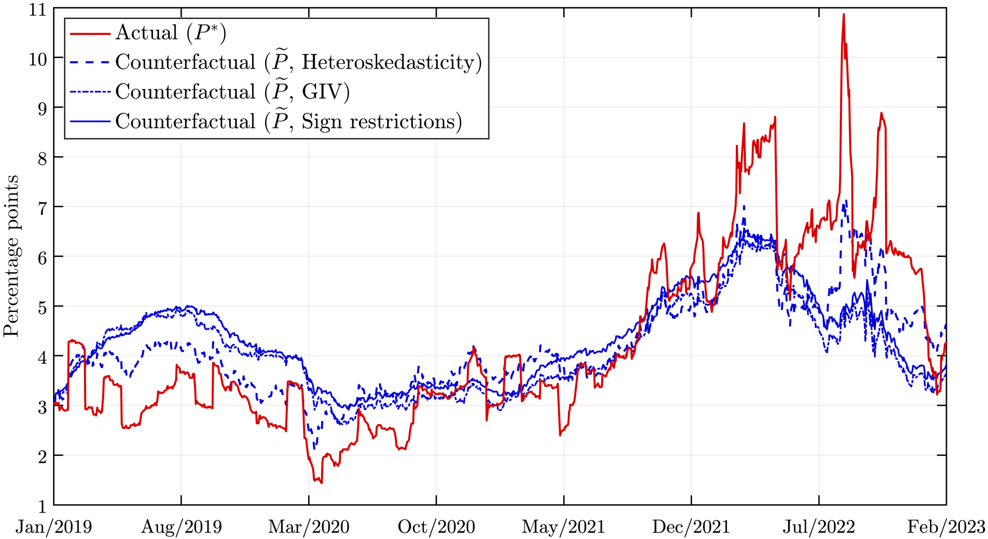


(b) Ukraine invasion



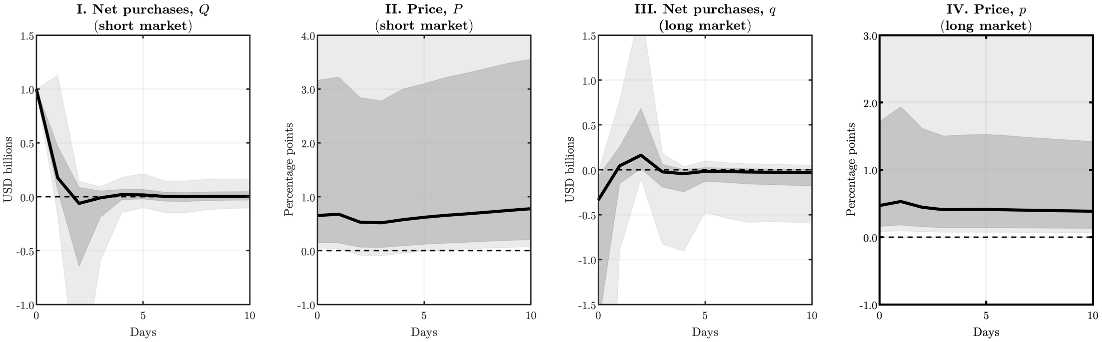
Conventional measures have overstated the fluctuations in long horizon inflation expectations.
A rule of thumb: 0.87 per each 1.

ST prices noisier but have lower-frequency information

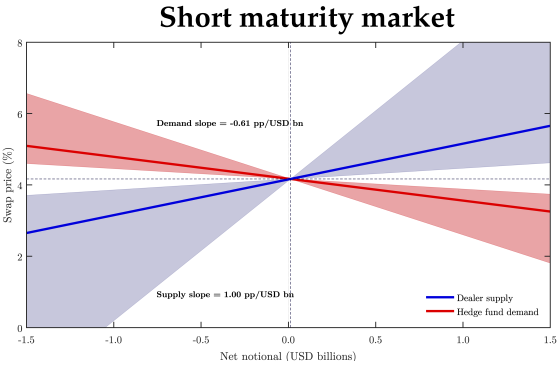


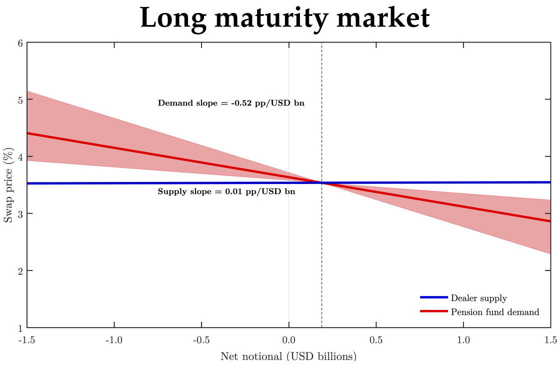
6. Estimates of how the market shifts inflation risk

The speed of price adjustment to shocks



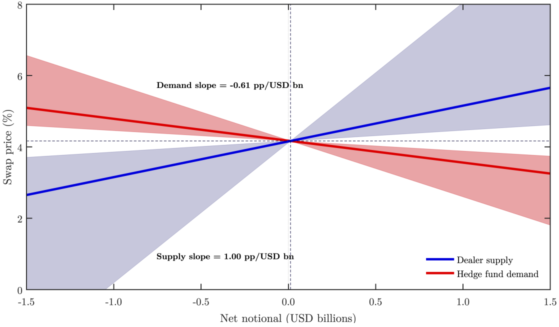
DEMAND CURVES: SIMILAR SLOPE BUT 3-TIMES VOLATILITY



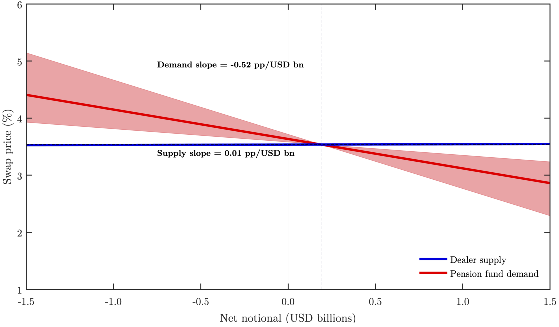


Slope of supply function horizontal in long market

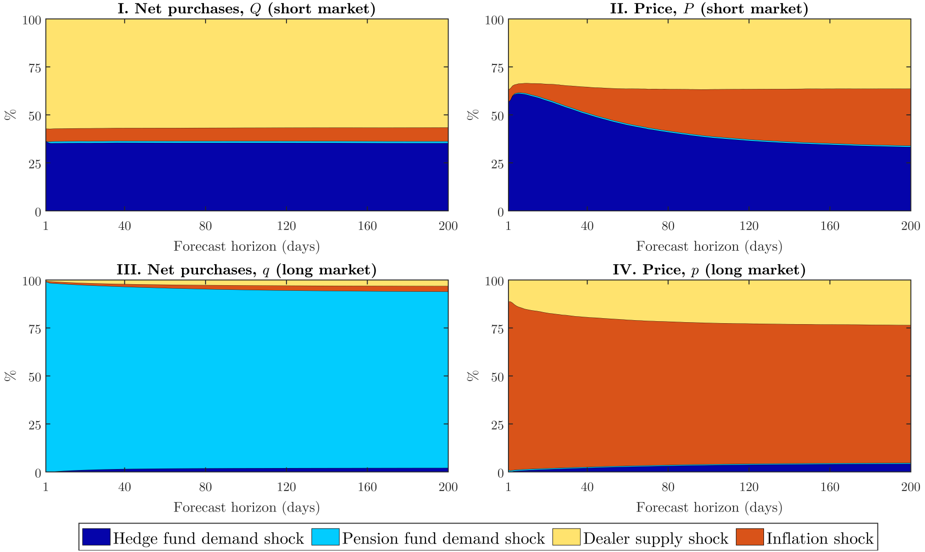
Short maturity market



Long maturity market



The drivers of inflation swap prices



7. Links to survey expectations and liquidity

Markets versus surveys

- The sensitivity of a dealer banks's trading to shocks to expected inflation:

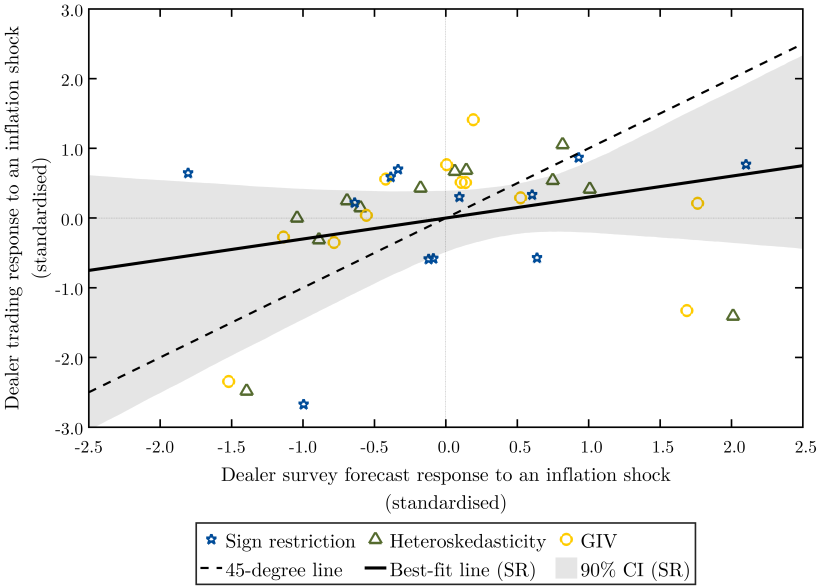
$$\frac{Q_{b,i,t}}{a_{b,i,t}} = constant_b + \beta_{b,i}\varepsilon_t^{\pi} + v_{b,i,t} \quad \text{in model} \quad \beta_{b,i} = \frac{M_{b,i} - \Lambda}{\gamma_{b,i}\sigma_{\pi}^2(1-\rho_{\pi,d}^2)}$$

- Sensitivity of survey answers about expected inflation to expected inflation:

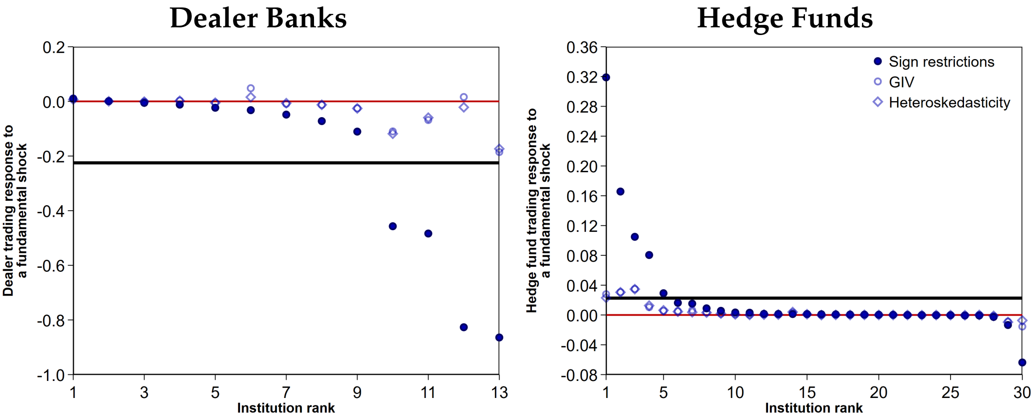
$$\Delta\hat{\Pi}_{b,i,t}^e = constant_b + \phi_{b,i}\varepsilon_t^{\pi} + u_{b,i,t} \quad \text{in model} \quad E_{b,i}(\Pi) = M_{b,i}\Pi^e$$

- *Do banks put their money where their mouth is: those that update their beliefs of inflation upwards after a shock also buy more inflation protection that same day.*

Match between markets and surveys

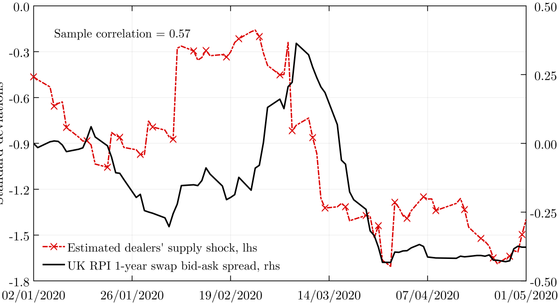


Relative price impact disperse and driven by few (short maturity)

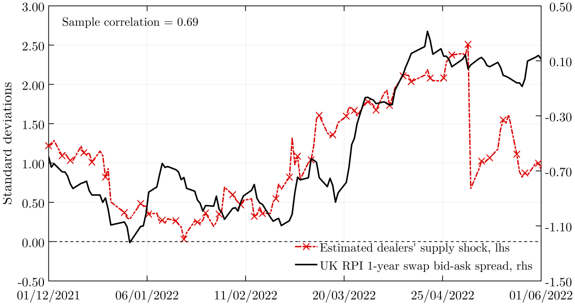


Comparisons with market bid-ask spreads

(a) COVID-19 period



(b) Ukraine war period



8. Conclusions

Conclusions

1) Facts: at short horizons, hedge funds and dealers alternate between -/+ positions. At long horizons, dealers provide inflation protection to pension funds.

2) Identification strategies for segmented markets: exploit information/variability in daily frequency, concentration across institutions, and time series.

3) Expected inflation: swap prices overstate extent of unanchoring at long horizons, are unreliable at short horizons.

4) Financial market: weak informationally efficient. At long horizons, supply curve is flat, fluctuations in quantities reflect shocks to trading frictions while expectations account for 3/4 of movements in prices. At short horizons, frictions affecting hedge funds are almost as significant as those affecting dealer banks.

5) There is a significant correlation between the beliefs of banks in answering surveys about expected inflation and their trading activity in the inflation swap market.