Optimal Automatic Stabilizers*

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Abstract

Should the generosity of unemployment benefits and the progressivity of income taxes depend on the presence of business cycles? This paper proposes a tractable model where there is a role for social insurance against uninsurable shocks to income and unemployment, as well as inefficient business cycles driven by aggregate shocks through matching frictions and nominal rigidities. We derive an augmented Baily-Chetty formula showing that the optimal generosity and progressivity depend on a macroeconomic stabilization term. This term pushes for an increase in generosity and progressivity when the level of economic activity is more responsive to social programs in recessions than in booms. A calibration to the U.S. economy shows that taking concerns for macroeconomic stabilization into account raises the optimal unemployment insurance replacement rate by 18 percentage points but has a negligible impact on the optimal progressivity of the income tax. More generally, the role of social insurance programs as automatic stabilizers affects their optimal design.


Keywords: Counter-cyclical fiscal policy; Redistribution; Distortionary taxes.

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1 Introduction

The usual motivation behind large social welfare programs, like unemployment insurance or progressive income taxation, is to provide social insurance and engage in redistribution. An extensive literature therefore studies the optimal progressivity of income taxes typically by weighing the disincentive effect on individual labor supply and savings against concerns for redistribution and for insurance against idiosyncratic income shocks.\(^1\) In turn, the optimal generosity of unemployment benefits is often stated in terms of a Baily-Chetty formula, which weighs the moral hazard effect of unemployment insurance on job search and creation against the social insurance benefits that it provides.\(^2\)

For the most part, this literature abstracts from aggregate shocks, so that the optimal generosity and progressivity do not take into account business cycles. Yet, from their inception, an auxiliary justification for these social programs was that they were also supposed to automatically stabilize the business cycle.\(^3\) The classic work that focussed on the automatic stabilizers relied on a Keynesian tradition that ignored the social insurance that these programs provide and their disincentive effects on employment. Some recent work brings these two orthogonal literatures together, but so far it has focused on the positive effects of the automatic stabilizers, falling short of computing optimal policies.\(^4\)

The goal of this paper is to answer two classic questions—How generous should unemployment benefits be? How progressive should income taxes be?—but taking into account their automatic stabilizer benefits as well as their social insurance benefits. We present a model in which there is both a role for social insurance as well as aggregate shocks and inefficient business cycles. We introduce unemployment insurance and progressive income taxes as automatic stabilizers, that is, programs that do not directly depend on the aggregate state of the economy even if the aggregate size of the programs changes with the composition of income in the economy. We then solve for the ex ante socially optimal replacement rate of unemployment benefits and progressivity of personal income taxes in the presence of uninsured income risks, precautionary savings motives, labor market

\(^1\)Mirrlees (1971) and Varian (1980) are classic references, and more recently see Benabou (2002), Conesa and Krueger (2006), Heathcote et al. (2014), Krueger and Ludwig (2013), and Golosov et al. (2016).

\(^2\)See the classic work by Baily (1978) and Chetty (2006).

\(^3\)Musgrave and Miller (1948) and Auerbach and Feenberg (2000) are classic references, while Blanchard et al. (2010) is a recent call for more modern work in this topic.

\(^4\)See McKay and Reis (2016) for a recent model, DiMaggio and Kermani (2016) for recent empirical work, and IMF (2015) for the shortcomings of the older literature.
frictions, and nominal rigidities.

Our first main contribution is to provide a new, theoretical definition of an automatic stabilizer. We show that a business-cycle variant of the Baily-Chetty formula for unemployment insurance and a similar formula for the optimal choice of progressivity of the tax system are both augmented by a new macroeconomic stabilization term. This term equals the expectation of the product of the welfare gain from eliminating economic slack with the elasticity of slack with respect to the replacement rate or tax progressivity. Even if the economy is efficient on average, economic fluctuations may lead to more generous unemployment insurance or more progressive income taxes, relative to standard analyses that ignore the automatic stabilizer properties of these programs. This term captures the automatic stabilizer nature of social insurance programs.

The second contribution is to characterize this macroeconomic stabilization term analytically to understand the different economic mechanisms behind it. Fluctuations in aggregate economic slack, measured by the unemployment rate, the output gap, or the job finding rate, can lead to welfare losses through four separate channels. First, they may create a wedge between the marginal disutility of hours worked and the social benefit of work. This inefficiency appears in standard models of inefficient business cycles, and is sometimes described as a result of time-varying markups (Chari et al., 2007; Galí et al., 2007). Second, when labor markets are tight, more workers are employed raising production but the cost of recruiting and hiring workers rises. The equilibrium level of unemployment need not be efficient as hiring and search decisions do not necessarily internalize these tradeoffs. This is the source of inefficiency common to search models (e.g. Hosios, 1990). Third, the state of the business cycle alters the extent of uninsurable risk that households face both unemployment and income risk. This is the source of welfare costs of business cycles that has been studied by Storesletten et al. (2001), Krebs (2003, 2007), and De Santis (2007). Finally, with nominal rigidities, slack affects inflation and the dispersion of relative prices, as emphasized by the new Keynesian business cycle literature (Woodford, 2010; Gali, 2011). Our measure isolates these four effects cleanly in terms of separate additive terms in the condition determining the optimal extent of the social insurance programs.

As for the elasticity of slack with respect to social programs, unemployment benefits and progressive taxes can stabilize the economy even if these policies are themselves not responsive to the business cycle. For one, these policies redistribute across groups who may have different marginal
propensities to consume. In the case of unemployment insurance, the magnitude of this redistribution increases when more people become unemployed in a recession. Moreover, these policies mitigate precautionary savings motives by providing social insurance. Because the risk in pre-tax incomes rises in a recession, the effect of this social insurance on aggregate demand rises as well, so these policies stabilize aggregate demand. We further show that if prices are flexible so aggregate demand matters little, or if monetary policy aggressively stabilizes the business cycle, then little role is left for the social programs to work as stabilizers.

Our third contribution is to investigate the magnitude of the macroeconomic stabilization term and the key mechanisms behind it. We calculate the optimal unemployment replacement rate and tax progressivity, and we compare these values to what one would find in the absence of aggregate risk. We find a large effect on unemployment insurance: with business cycles, the optimal unemployment replacement rate rises from 35 to 53 percent. However, the level of tax progressivity has little stabilizing effect on the business cycle so the presence of aggregate shocks has almost no effect on the optimal degree of progressivity.

Our analytical results allow us to interpret these numerical results by allowing us to quantify the tradeoffs between incentives, social insurance, and macroeconomic stabilization as well as the constituent mechanisms of the macroeconomic stabilization term. This highlights the usefulness of the propositions for isolating the key forces at hand. Quantitatively, the automatic stabilizer term is large in the case of unemployment benefits because of the interaction between two forces. First, unemployment benefits stabilize the business cycle by dampening the destabilizing feedback loop between unemployment fears, precautionary savings and aggregate demand. Second, stabilizing the business cycle is important in welfare terms because recessions lead to concentrated and long-lasting losses for individuals through the cyclicality of uninsurable idiosyncratic risk.

Finally, we use numerical analysis as a laboratory to relax some assumptions that we make for analytical tractability. Namely, our baseline results rely on a degenerate wealth distribution for tractability and we numerically evaluate how heterogeneity in wealth alters the relative strengths of the tradeoffs in our analysis.

There are large literatures on the three topics that we touch on: business cycle models with incomplete markets and nominal rigidities, social insurance and public programs, and automatic stabilizers. Our model of aggregate demand has some of the key features of new Keynesian models.
with labor markets frictions (Gali, 2011) but that literature focuses on optimal monetary policy, whereas we study the optimal design of the social insurance system. Our model of incomplete markets builds on McKay and Reis (2016), Ravn and Sterk (2017), and Heathcote et al. (2014) to generate a tractable model of incomplete markets and automatic stabilizers. This simplicity allows us to analytically express optimality conditions for generosity and progressivity, and to, even in a more general case, easily solve the model numerically and so be able to search for the optimal policies. Finally, our paper is part of a surge of work on the interplay of nominal rigidities and precautionary savings, but this literature has mostly been positive whereas this paper’s focus is on optimal policy.  

On the generosity of unemployment insurance, our work is closest to Landais et al. (2018) and Kekre (2018). They also couch their analysis in terms of the standard Baily-Chetty formula by considering the general equilibrium effects of unemployment insurance. The main difference is that they study benefits as discretionary policy instruments that vary over the business cycle, while we study how the presence of business cycles affects the ex ante fixed level of benefits. Our focus is on automatic stabilizers, an ex ante passive policy, while they consider active stabilization policy. This focus leads us to investigate channels through which even constant policies shape the dynamics of the business cycle. Moreover, our model includes aggregate uncertainty, and we also study income tax progressivity.

On income taxes, our work is closest to Benabou (2002) and Bhandari et al. (2018). Our dynamic heterogeneous-agent model with progressive income taxes is similar to the one in Benabou (2002), but our focus is on business cycles, so we complement it with aggregate shocks and nominal rigidities. Bhandari et al. (2018) is one of the very few studies of optimal income taxes with aggregate shocks. Like us, those authors emphasize the interaction between business cycles and the desire for redistribution. However, they solve for the Ramsey optimal fiscal policy, which adjusts the tax instruments every period in response to shocks, while we choose the ex ante optimal rules for generosity and progressivity. This is consistent with our focus on automatic stabilizers, which

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5See Oh and Reis (2012); Guerrieri and Lorenzoni (2017); Auclert (2019); McKay et al. (2016); Kaplan et al. (2018); Werning (2015).
7Unlike the United States, where the duration of unemployment benefits is often increased during recessions, in most OECD countries the terms of unemployment insurance programs do not change over the business cycle, as described in http://www.oecd.org/els/soc/.
8Werning (2007) also studies optimal income taxes with aggregate shocks and social insurance.
are ex ante fiscal systems, rather than counter-cyclical policies.

Finally, this paper is related to the modern study of automatic stabilizers and especially our earlier work in McKay and Reis (2016). There, we considered the positive question of how the actual automatic stabilizers implemented in the US alter the dynamics of the business cycle. Here we are concerned with the optimal fiscal system as opposed to the observed one.

The paper is structured as follows. Section 2 presents the model, and section 3 discusses its equilibrium properties. Section 4 derives the macroeconomic stabilization term in the optimality conditions for the two social programs. Section 5 discusses its qualitative properties, the economic mechanisms that it depends on, and its likely sign. Section 6 calibrates the model, quantifies the macro stabilization term and its effects on the optimal automatic stabilizers. Section 7 concludes.

2 The Model

The main ingredients in the model are: uninsurable income and employment risks, social insurance programs, and nominal rigidities so that aggregate demand matters for equilibrium allocations. Time is discrete and indexed by $t$.

2.1 Agents and commodities

There are two groups of private agents in the economy: households and firms. Households are indexed by $i$ in the unit interval, and their type is given by their productivity $\alpha_{i,t} \in \mathbb{R}_0^+$ and employment status $n_{i,t} \in \{0, 1\}$. Every period, an independently drawn share $\delta$ dies, and is replaced by newborn households with no assets and productivity normalized to $\alpha_{i,t} = 1$. Households derive utility from consumption, $c_{i,t}$, and publicly provided goods, $G_t$, and derive disutility from working for pay, $h_{i,t}$, searching for work, $q_{i,t}$, and being unemployed according to the utility function:

$$E_0 \sum_t \beta^t \left[ \log(c_{i,t}) - \frac{h_{i,t}^{1+\gamma}}{1+\gamma} - \frac{q_{i,t}^{1+\kappa}}{1+\kappa} + \chi \log(G_t) - \xi (1 - n_{i,t}) \right].$$  \hfill (1)

The parameter $\beta$ captures the joint discounting effect from time preference and mortality risk, while $\xi$ is a non-pecuniary cost of being unemployed.\textsuperscript{9}

\textsuperscript{9}If $\hat{\beta}$ is pure time discounting, then $\beta \equiv \hat{\beta}(1 - \delta)$.
The final consumption good is provided by a competitive final goods sector in the amount $Y_t$ that sells for price $p_t$. It is produced by combining varieties of goods in a Dixit-Stiglitz aggregator with elasticity of substitution $\mu/(\mu - 1)$. Each variety $j \in [0, 1]$ is monopolistically provided by a firm with output $y_{j,t} = \eta_t^A l_{j,t}$, where $l_{j,t}$ is the effective units of labor hired by the firm at wage $w_t$ and $\eta_t^A$ is an exogenous productivity shock.

### 2.2 Asset markets and social programs

Households can insure against mortality risk by buying an annuity, but they cannot insure against risks to their individual skill or employment status. The simplest way to capture this market incompleteness is by assuming that households can only hold a single risk-free annuity, $a_{i,t}$, that has a gross real return $R_t$.\(^\text{10}\)

The net supply of inside assets is zero, while there is a stock of government bonds $B$. Following Krusell et al. (2011), Ravn and Sterk (2017), and Werning (2015), we use a strong assumption that will make the distribution of wealth tractable: households cannot borrow, $a_{i,t} \geq 0$, and $B = 0$ so bonds are in zero supply. This assumption facilitates our theoretical analysis. We relax this assumption in an extension of our quantitative analysis.

The government provides two social insurance programs. The first is a progressive income tax such that if $z_{i,t}$ is pre-tax income, the after-tax income is $\lambda_t z_{i,t}^{1-\tau}$. The overall level of taxes determined by $1 - \lambda_t \in [0, 1]$, together with the size of government purchases $G_t$, pin down the size of the government. The object of our study is instead the automatic stabilizer role of the government, so our focus is on $\tau \in [0, 1]$. This determines the progressivity of the tax system. If $\tau = 0$, there is a flat tax at rate $1 - \lambda_t$, while if $\tau = 1$ everyone ends up with the same after-tax income. In between, a higher $\tau$ implies a more convex tax function, or a more progressive income tax system.

The second social program is unemployment insurance. A household qualifies as long as it is unemployed ($n_{i,t} = 0$) and collects benefits that are paid in proportion to what the unemployed worker would earn if she were employed. Suppose the worker’s productivity is such that she would

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\(^{10}\)A standard formulation for asset markets that gives rise to these annuities is the following: A financial intermediary sells claims that pay one unit if the household survives and zero units if the household dies, and supports these claims by trading a risk-less bond with return $\bar{R}$. If $a_i$ are the annuity holdings of household $i$, the law of large numbers implies the intermediary pays out in total $(1 - \delta) \int a_t \text{d}t$, which is known in advance, and the cost of the bond position to support it is $(1 - \delta) \int a_t \text{d}t/\bar{R}$. Because the risk-less bond is in zero net supply, then the net supply of annuities is zero $\int a_t \text{d}t = 0$, and for the intermediary to make zero profits, $R_t = \bar{R}_t/(1 - \delta)$. 

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earn pre-tax income $z_{i,t}$ if she were employed, then her after-tax unemployment benefit is $b\lambda_t z_{i,t}^{1-\tau}$.\textsuperscript{11} Our focus is on the replacement rate $b \in [0, 1]$, with a more generous program understood as having a higher $b$.\textsuperscript{12}

Our goal is to characterize the optimal fixed levels of $b$ and $\tau$ set ex ante as automatic stabilizers. These are programs that can automatically stabilize the business cycle without policy intervention, so $b$ and $\tau$ do not depend on time or on the state of the business cycle. In this design problem, we are following the tradition in the literature on automatic stabilizers that makes a sharp distinction between built-in properties of programs as opposed to feedback rules or discretionary choices that adjust these programs in response to current or past information.\textsuperscript{13}

2.3 Key frictions

There are three key frictions in the economy that create the policy trade-offs that we analyze.

2.3.1 Individual productivity risk

Labor income for an employed household is $\alpha_{i,t} w_t h_{i,t}$, where $\alpha_{i,t}$ is idiosyncratic productivity or skill and $w_t$ is the wage per effective unit of labor. The productivity of households evolves as

$$\alpha_{i,t+1} = \alpha_{i,t} \epsilon_{i,t+1}$$

with $\epsilon_{i,t+1} \sim F(\epsilon; x_t)$, (2)

and where $\int \epsilon dF(\epsilon; x_t) = 1$ for all $t$, which implies that the average idiosyncratic productivity in the population is constant and equal to one.\textsuperscript{14} The distribution of shocks varies over time so that the model generates cyclical changes in the distribution of earnings risks. We capture this dependence through the aggregate slack in the economy, $x_t$. As we explain below, all the labor market variables are driven by a common factor in equilibrium and this is captured by $x_t$. A higher $x_t$ implies that the economy is tighter, or that the economy is using resources more intensively. The structure of

\textsuperscript{11} It would be more realistic, but less tractable, to assume that benefits are a proportion of the income the agent earned when she lost her job. But, given the persistence in earnings, both in the data and in our model, our formulation will not be quantitatively too different from this case. Also, in our notation, it may appear that unemployment benefits are not subject to the income tax, but this is just the result of a normalization: if they were taxed and the replacement rate was $\tilde{b}$, then the model would be unchanged and $b \equiv \tilde{b}^{1-\tau}$.

\textsuperscript{12} In our model, focusing on the duration of unemployment benefits instead of the replacement rate would lead to similar trade offs, so we refer to $b$ more generally as the generosity of the program.

\textsuperscript{13} Perotti (2005) among many others.

\textsuperscript{14} Since newborn households have productivity 1, the assumption is that they have average productivity.
our model and our optimal policy problem guide our choice of $x_t$ and we define it formally below.

When we get to our quantitative analysis, cyclical idiosyncratic productivity risk will be important because it leads business cycles to have substantial welfare costs. Our function $F(\epsilon, x_t)$ stands in for the concentrated and long-lasting costs of recessions (see Storesletten et al., 2004; Guvenen et al., 2014; Davis and von Wachter, 2011). Davis and von Wachter (2011) find that workers laid off as part of a mass layoff have twice as large long-term earnings losses when the layoff occurs in a recession rather than an expansion. This evidence supports the perspective that aggregate business cycle conditions have long-term consequences for individual workers. The loss of skills during non-employment spells is one potential economic mechanism that could explain these earnings dynamics. A premise of our work is that these costs can be reduced by stabilizing the business cycle. To date the literature has not settled on a theoretical understanding of these cyclical dynamics of income changes, which leads us to adopt a reduced-form approach instead.

2.3.2 Employment risk

The second source of risk is employment. At the start of the period, a fraction $v \in [0, 1]$ of households loses employment and must search to regain employment. Search effort $q_{i,t}$ leads to employment with probability $M_t q_{i,t}$, where $M_t$ is the job-finding rate per unit of search effort. In what follows we lay out conditions under which all households choose a common search effort $q_t$ in which case the unemployment rate evolves as

$$u_t = [u_{t-1} + v (1 - u_{t-1})] (1 - q_t M_t).$$  \hfill (3)

Firms must post vacancies at a cost to hire workers. As in Blanchard and Galí (2010), the cost per hire is increasing in aggregate labor market tightness, which is just equal to the ratio of hires to searchers, or the job-finding rate $M_t$. The hiring cost per hire is $\psi_1 M_t^{\psi_2}$, denominated in units of final goods where $\psi_1$ and $\psi_2$ are parameters that govern the level and elasticity of the hiring costs. Since aggregate hires are the difference between the beginning of period employment rate $(1 - v)(1 - u_{t-1})$ and the realized employment rate $1 - u_t$, aggregate hiring costs are:

$$J_t \equiv \psi_1 M_t^{\psi_2} [(1 - u_t) - (1 - v)(1 - u_{t-1})].$$  \hfill (4)
We assume a law of large numbers within the firm so the average productivity of hires is 1.

In this model of the labor market, there is a surplus in the employment relationship since, on one side, firms would have to pay hiring costs to replace the worker and, on the other side, a worker who rejects a job must continue searching for a job thereby foregoing wages. This surplus creates a bargaining set for wages, and there are many alternative models of how wages are chosen within this set, from Nash bargaining to wage stickiness, as emphasized by Hall (2005). We assume a convenient wage rule for the analytical results:

\[
    w_t = \bar{w} A_t (1 - J_t / Y_t) x_t^\zeta.
\]

The assumption is that the real wage per effective unit of labor depends on three variables, aside from a constant \( \bar{w} \). First, it increases proportionately with aggregate effective productivity \( A_t \), as it would in a frictionless model of the labor market and product markets. \( A_t \equiv \eta^A_t / S_t \) includes the exogenous component, \( \eta^A_t \), and an endogenous component, \( S_t \), coming from price dispersion.\(^{15}\) Second, it falls when aggregate hiring costs are higher, so that some of these costs are passed from firms to workers. The justification is that when hiring costs rise, the economy is poorer and this raises labor supply, which the fall in wages exactly offsets. Since these costs are quantitatively small in our calibration this assumption has little effect on the predictions of the model but allows us to avoid carrying this uninteresting wealth effect on labor supply throughout the analysis. Third, wages rise with an elasticity of \( \zeta \) when the labor market is tighter captured by the common factor of labor market variables, \( x_t \).

Qualitatively, the wage rule does not play a large role in our analysis, but it is useful in simplifying the choice of labor supply on the intensive margin. If labor supply were fixed on the intensive margin, as in most search models of the labor market, then we would not need this assumption. Still, to justify it, Appendix A provides a Nash bargaining protocol that gives rise to a wage rule of this form.\(^{16}\) A notable feature of this wage rule is that the policy parameters do not directly affect wages, although they indirectly affect them through \( x_t \) for example. Appendix A writes a more general wage rule that allows policy parameters to directly affect wages and shows that it would

\(^{15}\)Most of the variance in \( A_t \) comes from the exogenous component so quantitatively it does not matter if we include the price-dispersion term in the wage rule or not. Theoretically, it is convenient to include it.

\(^{16}\)An implication of Appendix A is that the wage given by (5) always lies within the bargaining set implied by this bargaining protocol.
lead to similar, but somewhat more complicated, theoretical results. As for quantitative results, we consider this case in section 6.5.

2.3.3 Intermediate goods production and nominal rigidities

Firms cannot set their price equal to their desired price every period because of nominal rigidities. Firm $j$ chooses its reset price and will hire workers as necessary to meet demand at that price. The firm’s stock of workers declines with the job separation rate $\nu$. Employed workers set their own hours taking the hourly wage as given. We show below they all make the same choice $h_t$. The aggregate profits of these firms are distributed among employed workers in proportion to their skill, which can be thought of as representing bonus payments in a sharing economy.

For our theoretical analysis, we assume a simple, canonical model of nominal rigidities that captures most of the qualitative insights from New Keynesian economics (Mankiw and Reis, 2010). Every period an i.i.d. fraction $\theta$ of firms can set their prices $p_{j,t} = p^*_t$ to the desired markup over marginal cost, while the remaining set their price to equal what they expected their optimal price would be: $p_{j,t} = \mathbb{E}_{t-1} p^*_t$. The analytical benefit of this assumption is that the degree of price dispersion in the economy reflects only current conditions and is not itself a state variable. For our quantitative analysis, we use Calvo-style nominal rigidities. Appendix B.4 states the firm’s decision problem under both formulations of nominal rigidities.

2.4 Other government policy

Aside from the two social programs that are the focus of our study, the government also chooses policies for nominal interest rates, government purchases, and the public debt. Starting with the first, we assume a standard interest rate rule for nominal interest rates $I_t$:

$$I_t = \bar{I} \pi^{\omega_x} \pi_t^{\omega_x} \eta_t^I,$$

where $\omega_\pi > 1$ and $\omega_x \geq 0$. The exogenous $\eta_t^I$ represent shocks to monetary policy.\(^{17}\)

Turning to the second, government purchases follow:

$$G_t = \chi C_t \eta_t^G,$$

\(^{17}\)As usual, the real and nominal interest rates are linked by the Fisher equation $R_t = I_t / \mathbb{E}_t [\pi_{t+1}]$. 

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where $\eta^G_t$ are random shocks. Absent these shocks, this rule states that the marginal utility benefit of public goods offsets the marginal utility loss from diverting goods from private consumption.

The government runs a balanced budget by adjusting $\lambda_t$ to satisfy:

$$G_t = \int n_{i,t} \left( z_{i,t} - \lambda_t z_{1,t}^{1-\tau} \right) - \left( 1 - n_{i,t} \right) b \lambda_t z_{1,t}^{1-\tau} di,$$

where $z_{i,t}$ denotes the income of household $i$ should they be employed. It is well known, at least since Aiyagari and McGrattan (1998), that in an incomplete markets economy like ours, changes in the supply of safe assets will affect the ability to accumulate precautionary savings. Deficits or surpluses may stabilize the business cycle by changing the cost of self-insurance. In the same way that we abstracted above from the stabilizing properties of changes in government purchases, this lets us likewise abstract from the stabilizing property of public debt, in order to focus on our two social programs.\textsuperscript{18}

### 2.5 Measuring slack

The state of the business cycle is measured by the tightness of the labor market: $x_t = M_t$. As the only input to production is labor, it is natural to use a measure of resource utilization with a labor market variable and the job-finding rate is a convenient choice. Conditional on the policy parameters, $b$ and $\tau$, the labor market variables all move together in response to fluctuations in labor demand (we return to this point in Section 3.4). Given these close relationships, any of the labor market variables could serve as a measure of resource utilization. However, as we vary the social insurance system we would like to disentangle changes in the labor market that arise from the incentive effects of the social insurance system from those that affect fluctuations in labor demand. The unemployment rate and measures of hours reflect incentives as well as labor demand because they depend on household search effort and intensive-margin labor supply decisions that are affected by UI replacement rates and marginal tax rates. We define $x_t$ as the job-finding rate because it is a measure of labor market tightness that is not directly affected by these incentive considerations.

\textsuperscript{18} In previous work (McKay and Reis, 2016), we found that allowing for deficits and public debt had little effect on the effectiveness of stabilizers. This is because, in order to match the concentration of wealth in the data, almost all of the public debt is held by richer households who are already close to fully self insured. The same turns out to be the case in this economy.
3 Equilibrium and the role of policy

Our model combines idiosyncratic risk, incomplete markets, and nominal rigidities, and yet it is structured so as to be tractable enough to analytically investigate optimal policy. An aggregate equilibrium is a solution for 17 endogenous variables using a system of equations summarized in Appendix B.4, together with the exogenous processes $\eta^A_t$, $\eta^G_t$, and $\eta^I_t$. To further simplify the analytical investigation we make use of three parameter restrictions. First, agents never die, $\delta = 0$, which allows us to easily compute the welfare effects of skill risk. Second, the job separation rate, $\upsilon$, is one, which means that the unemployment rate is not a state variable. Third, the volatility of fiscal shocks is zero so $\eta^G_t = 1 \forall t$, which means public spending is at the efficient level. In addition to these parameter assumptions, we make use of the sticky-information form of nominal rigidities. These assumptions are in effect until we arrive at our quantitative analysis in Section 6.

3.1 Inequality and heterogeneity

The following result plays a crucial role in simplifying the analysis:

**Lemma 1.** All households choose the same asset holdings, hours worked, and search effort, so $a_{i,t} = 0$, $h_{i,t} = h_t$, and $q_{i,t} = q_t$ for all $i$.

To prove this result, note that the decision problem of a household searching for a job at the start of the period is:

$$V^s(a, \alpha, S) = \max_q \left\{ MqV(a, \alpha, n = 1, S) + (1 - Mq)V(a, \alpha, n = 0, S) - \frac{q^{1+\kappa}}{1+\kappa} \right\}, \quad (9)$$

where we used $S$ to denote the collection of aggregate states. The decision problem of the household at the end of the period is:

$$V(a, \alpha, n = 1, S) = \max_{c, h, a'} \left\{ \log c - \frac{h^{1+\gamma}}{1+\gamma} + \chi \log(G) + \beta \mathbb{E} \left[ (1 - \upsilon) V(a', \alpha', 1, S') + \upsilon V^s(a', \alpha', S') \right] \right\},$$

$$V(a, \alpha, n = 0, S) = \max_{c, a'} \left\{ \log c + \chi \log(G) - \xi + \beta \mathbb{E} \left[ V^s(a', \alpha', S') \right] \right\},$$

both subject to: $a' + c = Ra + \lambda (n + (1 - n)b) [\alpha(wh + d)]^{1-\tau}$.

In the decision problem of the unemployed household, the UI payment is exogenous and the $h$ that
appears in the budget constraint should be understood as a parameter equal to the equilibrium choice of their employed counterpart, \( h(a, \alpha, 1, S) \).

Starting with asset holdings, since no agent can borrow and bonds are in zero net supply, then it must be that \( a_{i,t} = 0 \) for all \( i \) in equilibrium because there is no gross supply of bonds for savers to own. Turning to hours worked, the intra-temporal labor supply condition for an employed household is:

\[
c_i h_i^\gamma = (1 - \tau)\lambda_t z_i^\gamma w_t \alpha_{i,t},
\]  

(10)

where the left-hand side is the marginal rate of substitution between consumption and leisure, and the right-hand side is the after-tax return to working an extra hour to raise income \( z_{i,t} \). More productive agents want to work more. However, they are also richer and consume more. The combination of our preferences and the budget constraint imply that these two effects exactly cancel out so that in equilibrium all employed households work the same hours:

\[
h^\gamma_t = \frac{(1 - \tau)w_t}{w_t h_t + d_t},
\]  

(11)

where \( d_t \) is aggregate dividends per employed worker.\(^{19}\)

Finally, the optimality condition for search effort is:

\[
q_{i,t}^\kappa = M_t [V(a_{i,t}, \alpha_{i,t}, 1, S) - V(a_{i,t}, \alpha_{i,t}, 0, S)].
\]  

(12)

Intuitively, the household equates the marginal disutility of searching on the left-hand side to the expected benefit of finding a job on the right-hand side, which is the product of the job-finding probability \( M_t \) and the increase in value of becoming employed. Appendix B.1 shows that this increase in value is independent of \( \alpha_{i,t} \). The key assumption that ensures this is that unemployment benefits are indexed to income \( z_{i,t} \) so the after-tax income with and without a job scales with idiosyncratic productivity in the same way. This then implies that \( q_{i,t} \) is the same for all households.

The lemma clearly limits the scope of our study. We cannot speak to the effect of policy on asset holdings, and differences in labor supply are reduced to having a job or not, which ignores

\(^{19}\)To derive this, substitute \( z_{i,t} = \alpha_{i,t}(w_t h_{i,t} + d_t) \) and \( c_{i,t} = \lambda_t z_{i,t}^{1-\tau} \) into (10).
diversity in part-time jobs and overtime. At the same time, it has the substantial payoff that we do not need to keep track of the cross-sectional distribution of wealth to characterize an equilibrium. Thus, our model can be studied analytically and global, non-linear numerical solutions are easy to compute. Moreover, the social programs that we study are arguably more concerned with income, rather than wealth inequality, and the vast majority of studies of the automatic stabilizers also ignores any direct effects of wealth inequality (as opposed to income inequality) on the business cycle.

Even though there is no wealth inequality, there is a rich distribution of income and consumption driven by heterogeneity in employment status \(n_{i,t}\) and skill \(\alpha_{i,t}\) in our model. In Section 6, we are able to fit the more prominent features of income inequality in the United States by parameterizing the distribution \(F(\epsilon, x)\). Moreover, in our model, there is a rich distribution of individual prices and output across firms, \((p_{j,t}, y_{j,t})\), driven by nominal rigidities. And finally, the exogenous aggregate shocks to productivity, monetary policy, and government purchases, \((\eta^A_t, \eta^I_t, \eta^G_t)\), affect all of these distributions, which therefore vary over time and over the business cycle. In spite of the simplifications and their limitations, our model still admits a rich amount of inequality and heterogeneity.

### 3.2 Quasi-aggregation and consumption

Define \(\tilde{c}_t\) as the consumption of the average-skilled \((\alpha_{i,t} = 1)\), employed agent. The consumption of individual \(i\) is given by:

\[
c_{i,t} = \alpha_{i,t}^{1-\tau}(n_{i,t} + (1 - n_{i,t})b)\tilde{c}_t.
\]

Integrating across \(i\) gives aggregate consumption:

\[
C_t = \mathbb{E}_t \left[ \alpha_{i,t}^{1-\tau} \right] (1 - u_t + u_t b)\tilde{c}_t.
\]

The next property that simplifies our model is proven in Appendix B.2.
Lemma 2. Consumption dynamics follows a modified Euler equation:

\[
\frac{1}{c_t} = \beta R_t E_t \left\{ \frac{1}{c_{t+1}Q_{t+1}} \right\},
\]

(15)

with: \[Q_{t+1} \equiv \left[ 1 + \nu (1 - q_{t+1}M_{t+1}) (b^{-1} - 1) \right] E \left[ \epsilon_{t,t+1}^{-1} \left( 1 - \tau \right) \right].\]

(16)

and equation (14) gives \( c_{i,t} \).

The variable \( Q_{t+1} \) captures how uninsurable risk affects aggregate consumption dynamics through precautionary savings motives. The more uncertain is income, the larger is \( Q_{t+1} \) and so the larger are savings motives leading to steeper consumption growth. This Euler equation is the key equation through which precautionary savings motives affect fluctuations in output. Specifically, the term \( \nu (1 - q_{t+1}M_{t+1}) (b^{-1} - 1) \) is central to our analysis. With probability \( \nu (1 - q_{t+1}M_{t+1}) \) the employed worker becomes unemployed next period in which case the marginal utility is increased by a factor \( b^{-1} - 1 \). A more generous unemployment insurance system dampens the precautionary savings motive stemming from employment risk. Similarly, a more progressive income tax lower the dispersion of after-tax income growth due to skill shocks.

3.3 Policy distortions and redistribution over the business cycle

Social policies not only affect aggregate consumption, but also all individual choices in the economy, introducing both distortions and redistribution. Combining the optimality condition for hours with the wage rule gives (see Appendix B.3):

\[
h_t = \left[ \bar{w} (1 - \tau) \right]^{\frac{1}{1+\gamma}} x_t^\frac{\xi}{1+\gamma}.
\]

(17)

A more progressive income tax lowers hours worked by increasing the ratio of the marginal tax rate to the average tax rate.

Moving to search effort, Appendix B.3 shows that with \( \nu = 1 \):

\[
q_t^c = M_t \left[ \xi - \frac{h_t^{1+\gamma}}{1+\gamma} - \log(b) \right].
\]

(18)

This states that the marginal disutility of searching for a job is equal to the probability of finding a job times the increase in utility of having a job. This utility gain is equal to the difference between
the non-pecuniary cost of unemployment and the disutility of working, minus the loss in utility units of reducing consumption by a factor $b$. More generous benefits therefore lower search effort. Intuitively, they lower the value of finding a job, so less effort is expended looking for one.

Equation (13) shows that more productive and employed households consume more, as expected. Social policies redistribute income and equalize consumption. A higher $b$ requires larger contributions from all households, lowering $\bar{c}_t$, but raises the consumption of the unemployed relative to the employed. In turn, a higher $\tau$ lowers the cross-sectional dispersion of consumption because it reduces the income of the rich more than that of the poor. The state of the business cycle affects the extent of the redistribution by driving both unemployment and the cross-sectional distribution of productivity risk.

Finally, nominal rigidities lead otherwise identical firms to charge different prices, and this relative-price dispersion lowers efficiency. The social insurance system will alter the dynamics of aggregate demand leading to different dynamics for nominal marginal costs, inflation, and price dispersion. The degree of price dispersion is given by:

$$S_t \equiv \int \frac{(p_t(j)/p_t)^{\mu/(1-\mu)}}{\mu/(1-\mu)} \frac{df}{d_j} = \left[ \frac{\theta + (1-\theta) \left( \frac{E_{t-1}p_t^*}{p_t^*} \right)^{\mu/(1-\mu)} }{\mu/(1-\mu)} \right]. \quad (19)$$

Integrating over the intermediate good production functions and using the demand for each variety it follows that $Y_t = A_t h_t (1 - u_t)$ where $A_t \equiv \eta_t^A / S_t$.

### 3.4 The structure of the labor market and the role of $x_t$

Let us reinforce the point that the job-finding rate is a useful summary measure of labor market tightness. To keep this discussion simple, we will abstract from the efficiency loss from price dispersion although the argument can be extended to allow for that. Equations (3), (17), (18), and $Y_t = A_t h_t (1 - u_t)$ form a system of four equations in the five variables $Y_t$, $h_t$, $u_t$, $M_t$, and $q_t$. Suppose the demand side of the economy pins down $Y_t$, then the supply side of the economy adjusts to meet this demand. The solution to these four equations will determine the equilibrium values of $h_t$, $u_t$, $M_t$, and $q_t$. As labor utilization fluctuates all of these variables fluctuate together and any one of them could be used as a measure of labor market tightness for a fixed social insurance
The measure of slack $x_t$ plays two roles. First, it appears in several reduced form equations: the interest rate rule, the wage rule, and the distribution of idiosyncratic skill risk. Based on the argument above, we view this as an innocuous choice as we could have substituted with, say $u_t$, with no change in the dynamics of the economy and we will in fact do that in our quantitative analysis. $x_t$ is also a measure of business cycle conditions that appears in our theoretical analysis of changes in the social insurance channel. As discussed in Section 2.5, $M_t$ is our preferred measure because it is not directly affected by labor supply incentives.

4 Optimal policy and insurance versus incentives

All agents in our economy are identical ex ante, making it natural to take as the target of policy the utilitarian social welfare function. Using equations (13) and (14) and integrating the utility function in equation (1) gives the objective function for policy $E_0 \sum_{t=0}^{\infty} \beta^t W_t$, where period-welfare is:

$$W_t = E_t \log \left( \alpha_{i,t}^{1-\tau} \right) - \log \left( E_t \left[ \alpha_{i,t}^{1-\tau} \right] \right) + u_t \log b - \log (1 - u_t + u_t b) + \log(C_t) - (1 - u_t) Q^{1+\kappa} \left( 1 + \gamma \right) + \chi \log(G_t) - \xi u_t. \quad (20)$$

The first line shows how inequality affects social welfare. Productivity differences and unemployment introduce costly idiosyncratic risk, which is attenuated by the social insurance policies. The second line captures the usual effect of aggregates on welfare. While these would be the terms that would survive if there were complete insurance markets, recall that the incompleteness of markets also affects the evolution of aggregates, as we explained in the previous section.

The policy problem is then to pick $b$ and $\tau$ once and for all to maximize equation (20) subject to the equilibrium conditions.

4.1 Optimal unemployment insurance

Appendix C derives the following optimality condition for $b$:

$^{20}$If the separation rate is less than 1, then the solution to this system of equations will also depend on the lagged unemployment rate.
Proposition 1. The optimal choice of the generosity of unemployment insurance $b$ satisfies:

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{b} \left( 1 - \frac{1}{b} \right) \frac{\partial \log (b\tilde{c}_t)}{\partial \log b} \right|_{x,q} + \frac{\partial \log \tilde{c}_t}{\partial \log u_t} \left|_x \right. \frac{\partial \log u_t}{\partial b} \left. \right|_x + dW_t \right\} = 0. \quad (21)
$$

Equation (21) is closely related to the Baily-Chetty formula for optimal unemployment insurance. The first term captures the social insurance value of changing the replacement rate. It is equal to the percentage difference between the marginal utility of unemployed and employed agents times the elasticity of the consumption of the unemployed with respect to the benefit. If unemployment came with no differences in consumption, this term would be zero, and likewise if giving higher benefits to the unemployed had no effect on their consumption. But as long as employed agents consume more, and raising benefits closes some of the consumption gap, then this term will be positive and call for higher unemployment benefits.

The second term gives the moral hazard cost of unemployment insurance. It is equal to the product of the elasticity of the consumption of the employed with respect to the unemployment rate, which is negative, and the elasticity of the unemployment rate with respect to the benefit, which arises out of reduced search effort. Higher replacement rates induce agents to search less, which raises equilibrium unemployment, and leads to higher taxes to finance benefits.

In the absence of general equilibrium effects, these would be the only two terms as they are derivatives keeping the state of the business cycle $x$ fixed. They capture the standard trade-off between insurance and incentives in the literature but now averaged across time. With business cycles and general equilibrium effects, there is an extra macroeconomic stabilization term. The larger this term is, the more generous optimal unemployment benefits should be. We explain this shortly, but first, we turn to the income tax.

4.2 Optimal progressivity of the income tax

Appendix C shows the following:
Proposition 2. The optimal progressivity of the tax system $\tau$ satisfies:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\text{Cov}(\alpha_{t,0}^{1-\tau}, \log \alpha_{t,0})}{\mathbb{E}_t[\alpha_{t,0}]} + \frac{\beta}{1-\beta} \frac{\text{Cov}(\epsilon_{t,t+1}^{1-\tau}, \log \epsilon_{t,t+1})}{\mathbb{E}_t[\epsilon_{t,t+1}]} \right\} + \left( \frac{A_t}{C_t} - h_t^{\gamma} \right) (1 - u_t) \left. \frac{\partial h_t}{\partial \tau} \right|_x + \left. \frac{\partial \log \tilde{c}_t}{\partial \log \tilde{u}_t} \right|_x \left. \frac{\partial \log u_t}{\partial \tau} \right|_x + \frac{dW_t}{dx_t} \frac{dx_t}{d\tau} = 0. \tag{22}$$

The three rows again capture the trade-offs between insurance, incentives, and macroeconomic stabilization, respectively. Starting with the first row, the first term gives the welfare benefit of redistributing already existing differences in income, as captured by the initial dispersion of skills. The second term gives the welfare benefits of reducing the dispersion in after-tax incomes due to skill shocks that the household is exposed to in the future. Both terms in the first row have a similar structure and are both positive.\(^{21}\)

The second row gives the incentive costs of raising progressivity, which are the welfare costs of the response of aggregate hours, $(1 - u_t) h_t$, to tax progressivity holding $x_t$ fixed. The first term on the row corresponds to the changing $h_t$ and the second term corresponds to changing $u_t$, which arises through search effort choices. A more progressive tax system raises marginal tax rates and reduces the incentive to supply labor on the intensive margin. Similarly, the tax system affects the relative rewards to being employed and therefore alters household search effort and the unemployment rate.

Finally, the third row captures the concern for macroeconomic stabilization in a very similar way to the term for unemployment benefits. A larger stabilization term in (22) justifies a more progressive tax.

4.3 The macroeconomic stabilization term

The two previous propositions clearly isolate the automatic-stabilizing role of the social insurance programs in a single term. It equals the product of the welfare benefit of changing slack and the response of slack to policy. If business cycles are efficient, the macroeconomic stabilization term is zero. That is, if the economy is always at an efficient level of slack, so that $dW_t/dx_t = 0$, then there is no reason to take macroeconomic stabilization into account when designing the stabilizers.

\(^{21}\)Each of the terms involves the covariance of two increasing functions of a single random variable, which is positive if the underlying random variable has positive variance. The denominators are positive because $\alpha_t$ and $\epsilon_t$ take positive values.
Intuitively, the business cycle is of no concern for policymakers in this case.

Even if business cycles are efficient on average or the stabilizers have no effect on the average level of slack, the stabilizers can still have stabilization benefits. This is because:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t \left\{ dW_t \frac{dx_t}{db} \right\} = \sum_{t=0}^{\infty} \beta_t \left\{ \mathbb{E}_0 \left[ dW_t \right] \mathbb{E}_0 \left[ \frac{dx_t}{db} \right] + \text{Cov} \left[ dW_t, \frac{dx_t}{db} \right] \right\},$$

(23)

so that even if $\mathbb{E}_0 \left[ dW_t \frac{dx_t}{db} \right] = 0$, a positive covariance term would still imply a positive aggregate stabilization term and an increase in benefits (or more progressive taxes). Our model therefore provides a definition of a social policy that serves as an automatic stabilizer: it stimulates the economy more in recessions, when slack is inefficiently high. The stronger this effect, the larger the program should be. In the next section, we discuss the sign of this covariance and what affects it.

5  Inspecting the macroeconomic stabilization term

Understanding the automatic stabilizer nature of social programs requires understanding separately the effect of slack on welfare, $dW_t/dx_t$, and the effect of the social policies on slack, $dx_t/db$ and $dx_t/d\tau$. Instead of trying to measure the covariance between these two unobservables in the data, a daunting task, we proceed to characterize their structural determinants in terms of familiar economic channels that have been measured elsewhere.

5.1 Slack and welfare

There are five separate channels through which the business cycle may be inefficient in our model, characterized in the following result:

**Proposition 3.** The effect of macroeconomic slack on welfare can be decomposed into:

$$\frac{dW_t}{dx_t} = \left( 1 - u_t \right) \left[ \frac{A_t}{C_t} \left( \frac{S_t}{C_t} \right) \frac{dt}{dx_t} \right] \frac{dh_t}{dx_t} - \frac{Y_t}{C_t} \frac{d\phi}{dx_t} + \frac{1}{C_t} \frac{\partial C_t}{\partial x_t} \left| x \right\| \frac{du_t}{dx_t} - \frac{1}{C_t} \frac{\partial J_t}{\partial x_t} \left| u \right\|$$

(24)

$$- \left( \xi - \log b - \frac{h_t^{1+\gamma}}{1+\gamma} \right) \frac{\partial u_t}{\partial x_t} q + \frac{1-b}{1-u_t+u_t b} \frac{du_t}{dx_t} + \beta \frac{d}{1-\beta} \int \log \left( \frac{\epsilon^{1-\gamma}}{1-\gamma} dF(\epsilon, x_t) \right) dF(\epsilon, x_t)$$

$$\text{unemployment-risk} \quad \text{income-risk}$$
The first term captures the effect of the labor wedge or markups. In the economy, \( A_t/C_t \) is the marginal product of an extra hour worked in utility units, while \( h_t^{\gamma} \) is the marginal disutility of working. If the first exceeds the second, the economy is underproducing, and increasing hours worked would raise welfare.

The second term captures the effect of slack on price dispersion. Because of nominal rigidities, aggregate shocks will lead to price dispersion. In that case, changes in aggregate slack will affect inflation, via the Phillips curve, and so price dispersion. This is the conventional welfare cost of inflation in new Keynesian models.

The third and fourth terms capture the standard extensive margin trade-off in models with costly matching. On the one hand, tightening the labor market lowers unemployment and raises consumption. On the other hand, it increases hiring costs. If \( \partial C_t/\partial u_t|_x du_t/dx_t > \partial J/\partial x_t \), welfare rises as the labor market gets tighter.\(^{22}\)

The terms in the second line of equation (24) focus on inequality and its effect on welfare. If the extent of income risk is cyclical, which the literature has demonstrated starting with Storesletten et al. (2004), then raising economic activity reduces income risk and so raises welfare. In our model, there is both unemployment and income risk, so this works through two channels.

The fourth and fifth term capture the effect of slack on unemployment risk. For a given aggregate consumption, more unemployment has two effects on welfare. First, there are more unemployed who consume a lower amount. The term \( \xi - \log b - h_t^{1+\gamma}/(1 + \gamma) \) is the utility loss from becoming unemployed. Second, those who are employed consume a larger share (dividing the pie among fewer employed people).

The sixth and final term captures the effect of slack on the distribution of skill shocks. Slack affects welfare by changing the distribution \( F(\epsilon, x_t) \) and we will emphasize pro-cyclical skewness of the distribution. By the concavity of the log function, a more negatively skewed \( F(.) \) results in more welfare losses.

\(^{22}\)The partial derivative of \( C_t \) with respect to \( u_t \) given \( x_t \) is defined mathematically in Appendix C. It is the gain in consumption from putting more people to work but without changing wages, hours on the intensive margin, price dispersion, or the other consequences of changing \( x_t \).
5.2 Three special cases

To better understand these different channels of welfare effects, and link them to the literature before us, we consider three special cases that correspond to familiar models of fluctuations.

5.2.1 Frictional unemployment

Consider the special case where prices are flexible ($\theta = 1$), there is no productivity risk ($\text{Var}(\epsilon) = 0$), and labor supply does not vary on the intensive margin because hours worked are constant ($\gamma = \infty$). The only source of inequality is then unemployment, due to the costly process of search and matching. Therefore, equation (24) becomes:

$$\frac{dW_t}{dx_t} = \frac{1}{C_t} \frac{dC_t}{dx_t} - \frac{1}{C_t} \frac{\partial J_t}{\partial x_t} \bigg|_u \left( \xi - \log b - \frac{h^{1+\gamma}_t}{1+\gamma} \right) \frac{\partial u_t}{\partial x_t} \bigg|_q + \frac{1}{1-u_t + u_t b} \frac{du_t}{dx_t}. \quad (25)$$

as only the extensive margin effect and the unemployment risk are now present.

In this special case, our model is close to the one in Landais et al. (2018). They discuss the macroeconomic effects of unemployment benefits from the perspective of their effect on labor market tightness by changing the worker’s bargaining position and wages on the one hand and, on the other hand, their impact on dissuading search effort.

5.2.2 Real business cycle effects

Next, we consider the case of flexible prices ($\theta = 1$), constant search effort ($\kappa = \infty$), an exogenous job finding rate ($M_t$ exogenous), and a log-normal $F(\epsilon, x_t)$ with variance of log $\epsilon$ given by $\sigma^2(x_t)$ and mean $-0.5 \times \sigma^2(x_t)$. With nominal rigidities and search removed, what is left is the labor wedge and the effect of cyclical income risk on welfare, so equation (24) simplifies to

$$\frac{dW_t}{dx_t} = (1-u_t) \left[ \frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dx_t} - \frac{\beta}{1-\beta} (1-\tau)^2 \frac{\sigma^2(x_t)}{2} \frac{d}{dx_t}. \quad (26)$$

In this case, our paper fits into the standard analysis of business cycles in Chari et al. (2007) through the first term, and into the study the costs of business cycles due to income inequality.

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23When $M_t$ is constant we need to define slack differently from $x_t = M_t$. In this case, the role of $x_t$ is to change the wage and change labor supply on the intensive margin. The wage will need to adjust to clear the labor market as in the three-equation New Keynesian model and then the wage rule, equation (5), becomes the definition of $x_t$.\footnote{When $M_t$ is constant we need to define slack differently from $x_t = M_t$. In this case, the role of $x_t$ is to change the wage and change labor supply on the intensive margin. The wage will need to adjust to clear the labor market as in the three-equation New Keynesian model and then the wage rule, equation (5), becomes the definition of $x_t$.}
emphasized by Krebs (2003) through the second term.

5.2.3 Aggregate demand effects

Traditionally, the literature on automatic stabilizers has focussed on aggregate demand effects following a Keynesian tradition. When there is no productivity risk (Var($\epsilon$) = 0), job search effort is constant ($\kappa = \infty$) and the labor market’s matching frictions are constant ($M_t$ is constant), equation (24) simplifies to:

$$\frac{dW_t}{dx_t} = (1 - u_t) \left[ \frac{A_t}{C_t} - h^*_t \right] \frac{dh_t}{dx_t} - \frac{Y_t}{C_t S_t} \frac{dS_t}{dx_t},$$

so only the markup effects are present, both through the labor wedge and through price dispersion.

Appendix D.2 shows that a second-order approximation of $W_t$ around the flexible-price, socially-efficient level of aggregate output $Y^*_t$ and consumption $C^*_t$ transforms this expression into

$$\frac{dW_t}{dx_t} = \left( \frac{Y^*_t}{C^*_t} \right) \left[ \left( 1 - \frac{\gamma}{Y^*_t} \right) \frac{dY_t}{dx_t} + \left( \frac{1 - \theta}{\theta} \frac{\mu}{\mu - 1} \right) \left( \frac{E_{t-1} p_t - p_t}{E_t - 1} \right) \frac{dp_t}{dx_t} \right],$$

In this case, our model fits into the baseline new Keynesian framework developed in Woodford (2003). Raising slack affects the output gap and the price level, through the Phillips curve, and this affects welfare through the two conventional terms in the expression. The first is the effect on the output gap, and the second the effect on surprise inflation. These are the two sources of welfare costs in this economy.

5.3 Social programs and slack

We now turn attention to the second component of the macroeconomic stabilization term, either $dx_t/db$ in the case of unemployment benefits, or $dx_t/d\tau$ in the case of tax progressivity. Fully characterizing these relationships is difficult so instead we use the equations of the model to explain the key channels through which constant social insurance policies can stabilize aggregate demand and the business cycle. When we turn to the quantitative analysis we will demonstrate how this logic plays out in the numerical solution.

To build intuition, use the logic that the Euler equation (15) determines $\tilde{c}_t$ and then equation (14) gives $C_t$. The Euler equation depends on the precautionary savings motive $Q_{t+1}$ given by
equation (16). $Q_{t+1}$ depends on the after-government idiosyncratic risk, which depends on the social insurance system. Specifically, the household faces both unemployment risk and skill risk. The unemployment risk is dampened by the UI system and the skill risk is dampened by progressive taxes. Crucially, both of these pre-government risks become more severe in a recession: with a lower job-finding rate, there is a greater chance of an EU transition and the distribution of skill shocks becomes less favorable in a recession through the distribution $F(\epsilon, x_t)$. When the pre-government risks increase, the level of social insurance becomes more important. For this reason, aggregate demand becomes more sensitive to the level of $b$ and $\tau$ in recessions.

Equation (14) shows an additional consideration of UI benefits. Taking $\tilde{c}_t$ as given from the Euler equation, the aggregate consumption of employed households is fixed, but the consumption of unemployed households depends on the transfers they receive, which are a ratio $b$ of their employed counterparts income. With $b < 1$, aggregate consumption $C_t$ is less than the total consumption of employed households $\tilde{c}_t \mathbb{E}_t \left[ \alpha_{t,t}^{1-\tau} \right]$ by a factor $1 - u_t + u_t b$. As the unemployment rate rises, the consumption of the unemployed become more important to the aggregate and so aggregate consumption depends more strongly on the level of transfers.

These arguments motivate why $dx_t/db$ and $dx_t/d\tau$ can be counter-cyclical even if $b$ and $\tau$ are constant. So far, this discussion has taken a partial-equilibrium perspective holding real interest rates and job-finding rates constant. These variables are also determined in equilibrium and their responses will dampen or amplify the response of $x_t$ to social insurance. Real interest rates will typically attenuate the fluctuations in aggregate demand caused by social insurance. On the other hand, the dynamics of the job-finding rate can amplify fluctuations in aggregate demand because low aggregate demand leads to a slack labor market, which in turn increases the risk of becoming unemployed further reducing aggregate demand.\(^{24}\)

If business cycles are inefficient in the sense that tightness is inefficiently low in a recession, then we expect a positive covariance between $dW_t/dx_t$ and the elasticities of slack with respect to policy. This positive covariance implies a positive aggregate stabilization term and more generous unemployment benefits and a more progressive tax system even if the business cycle is efficient on average.\(^{24}\)

\(^{24}\)Similar reinforcing dynamics arise out of unemployment risk in Ravn and Sterk (2017), Den Haan et al. (2015), and Heathcote and Perri (2018).
6 Quantitative analysis

We have shown that the presence of business cycles leads to a macroeconomic stabilization term in the determination of the optimal generosity of unemployment insurance and the progressivity of income taxes, and that this term likely makes these programs more generous and progressive, respectively. We now turn to numerical solutions to evaluate whether the macroeconomic stabilization term is quantitatively significant. We use the analytical formulas presented above to understand the mechanisms driving the numerical results.

6.1 Calibration and solution of the general model

We solve the model using global methods, as described in Appendix E.2, so that we can accurately compute social welfare, assuming that the economy starts at date 0 at the deterministic steady state. We then numerically search for the values of $b$ and $\tau$ that maximize the social welfare function, and compare these with the maximal values in a counterfactual economy without aggregate shocks, but otherwise identical.

In Section 3, we introduced several assumptions for tractability that we now relax. Specifically, we allow for mortality, persistent unemployment, government spending shocks, and Calvo-style pricing. When we estimate our interest rate rule and cyclical idiosyncratic risk and in our specification of the wage rule we use $x_t = (1 - u_t)/(1 - \bar{u})$. This point is not substantively important because $M_t$ and $u_t$ move together over the business cycle as explained in Section 3.4, but we make this change because $u_t$ is more easily measured for calibration purposes than $M_t$.

Table 1 shows the calibration of the model, dividing the parameters into different groups. The first group has parameters set ex ante to standard choices in the literature. Only the last one deserves some explanation. $\psi_2$ is the elasticity of hiring costs with respect to labor market tightness, and we set it at 1 as in Blanchard and Gali (2010), in order to be consistent with an elasticity of the matching function with respect to unemployment of 0.5 as suggested by Petrongolo and Pissarides (2001).

Panel B contains parameters individually calibrated to match time-series moments. For the preference for public goods, we target the observed average ratio of government purchases to GDP

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25 In applying our analytical propositions to our numerical results we continue to use $M_t$ as our measure of the business cycle.
<table>
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</tr>
<tr>
<td>$\psi_2$</td>
<td>1</td>
<td>Elasticity of hiring cost</td>
<td>Blanchard and Galí (2010)</td>
</tr>
<tr>
<td>Panel B. Parameters individually calibrated</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.262</td>
<td>Preference for public goods</td>
<td>$G/Y = 0.207$</td>
</tr>
<tr>
<td>$\omega_{\pi}$</td>
<td>1.66</td>
<td>Mon. pol. response to $\pi$</td>
<td>Estimated interest rate rule</td>
</tr>
<tr>
<td>$\omega_u$</td>
<td>0.133</td>
<td>Mon. pol. response to $u$</td>
<td>Estimated interest rate rule</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.094</td>
<td>Job separation rate</td>
<td>3.8% EU trans. prob.</td>
</tr>
<tr>
<td>$F(\epsilon, \cdot)$</td>
<td>mix-normals</td>
<td>Productivity-risk process</td>
<td>See Appendix E.1</td>
</tr>
<tr>
<td>Panel C. Parameters jointly calibrated to steady-state moments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.982</td>
<td>Discount factor</td>
<td>3% annual real interest rate</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>0.831</td>
<td>Average wage</td>
<td>Unemployment rate = 6.1%</td>
</tr>
<tr>
<td>$\psi_1$</td>
<td>0.0309</td>
<td>Scale of hiring cost</td>
<td>Recruiting costs of 3% of pay</td>
</tr>
<tr>
<td>$1/\kappa$</td>
<td>0.0476</td>
<td>Search effort elasticity</td>
<td>$d \log u/d \log b</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.236</td>
<td>Pain from unemployment</td>
<td>Leisure benefit of unemployment</td>
</tr>
<tr>
<td>Panel D. Parameters jointly calibrated to volatilities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.9</td>
<td>Autocorrelation of shocks</td>
<td>See fn. 28</td>
</tr>
<tr>
<td>StDev($\eta^A$)</td>
<td>0.46%</td>
<td>TFP innovation</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>StDev($\eta^I$)</td>
<td>0.44%</td>
<td>Monetary policy innovation</td>
<td>StDev($u_t$) = 1.59%</td>
</tr>
<tr>
<td>StDev($\eta^G$)</td>
<td>4.63%</td>
<td>Government purchases innovation</td>
<td>StDev($G_t/Y_t$) = 1.75%</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>1.68</td>
<td>Elasticity of wage w.r.t. $x$</td>
<td>StDev($h_t$)/StDev($1 - u_t$) = 0.568</td>
</tr>
<tr>
<td>Panel E: Automatic stabilizers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>0.810</td>
<td>UI replacement rate</td>
<td>See text</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.151</td>
<td>Progressivity of tax system</td>
<td>Heathcote et al. (2014)</td>
</tr>
</tbody>
</table>

Table 1: Calibrated parameter values and targets
in the US in 1984-2007. For the monetary policy rule, we use OLS estimates of equation (6). We calibrate the job separation rate, \( \nu \), so that we match the EU transition rate from Table 2 of Krusell et al. (2017).\(^{26}\) Finally, we estimate a version of the income innovation process specified in equation (2) using a mixture of normals as a flexible parameterization of the distribution \( F(\epsilon'; \cdot) \). Two of the mixture components shift with the unemployment rate to match the observed pro-cyclical skewness of earnings growth rates documented by Guvenen et al. (2014). This parametric income process is similar to the one in McKay (2017) and Appendix E.1 provides additional details.\(^{27}\) As a check on our calibration, the model implies a cross-sectional variance of log consumption of 0.40, while in 2005 CEX data the variance of log consumption of non-durables was around 0.35 (Heathcote et al., 2010).

Panel C has parameters chosen jointly to target a set of moments. We target the average unemployment rate between 1960 to 2014 and recruiting costs of 3 percent of quarterly pay, consistent with Barron et al. (1997). The parameter \( \kappa \) controls the marginal disutility of effort searching for a job, and we set it to target a micro-elasticity of unemployment with respect to benefits of 0.5 as reported by Landais et al. (2018). Last in the panel is \( \xi \), the non-pecuniary costs of unemployment. In the model, the utility loss from unemployment is \( \log(1/b) - h^{1+\gamma}/(1 + \gamma) + \xi \), reflecting the loss in consumption, the gain in leisure, and other non-pecuniary costs of unemployment. We set \( \xi = h^{1+\gamma}/(1 + \gamma) \) in the steady state of our baseline calibration so that the benefit of increased leisure in unemployment is dissipated by the non-pecuniary costs.

Panel D calibrates the three aggregate shocks in our model that perturb productivity, monetary policy, and public expenditures. We have found that monetary shocks are particularly important in that they account for the majority of the variance of the unemployment rate. In each case, the exogenous process is an AR(1) in logs with common autocorrelation.\(^{28}\) We set the variance of the TFP shock to the posterior mean of the estimates in Smets and Wouters (2007). The other

\(^{26}\)We use the Abowd-Zellner panel of the table converted to a quarterly frequency. As our model abstracts from non-participation we measure the EU transition rate as \( f_{EU}/(1 - f_{EN}) \) where \( f_{EU} \) and \( f_{EN} \) represents the entries in the transformed table.

\(^{27}\)We include unemployment fluctuations in the income process we simulate to match the empirical moments so the contribution of unemployment to observed changes in income distributions is accounted for.

\(^{28}\)In unreported results we have found that our results are insensitive to the persistence of the fiscal and technology shocks, but are sensitive to the persistence of the unemployment rate. In particular, the aggregate demand consequences of the precautionary savings motive accumulate in the forward-looking Euler equation so more persistent shocks to unemployment risk have larger effects. Our choice of persistence of 0.9 generates a similarly persistent unemployment rate, which aligns well with the data. The quarterly persistence of the unemployment rate is between 0.92 and 0.98 depending on how one accounts for low-frequency components.
two variances are set to match the standard deviation of the unemployment rate and the standard deviation of $G_t/Y_t$. We set $\zeta = 1.68$ to match the standard deviation of hours per worker relative to the standard deviation of the employment-population ratio.

Finally, panel E has the baseline values for the automatic stabilizers. For $\tau$ we adopt the estimate of 0.151 from Heathcote et al. (2014). In calibrating $b$ we target the observed degree of insurance that households have against unemployment shocks, as measured by the change in consumption upon unemployment. We set $b = 0.81$, consistent with a 19% decline in consumption when unemployed since the literature has found consumption changes between 16% and 21%.\textsuperscript{29} These calibrated values for $b$ and $\tau$ do not directly enter our analysis of optimal policy but are used to jointly calibrate the other structural parameters of the model.

As a check on the model’s performance, the standard deviation of hours, output, and inflation in the model are 0.75%, 2.53%, and 0.81%. The equivalent moments in the US data 1960-2014 are 0.84%, 1.32%, and 0.55%.

### 6.2 Optimal automatic stabilizers

Our first main quantitative result is that aggregate shocks increase the optimal $b$ to 0.811 from 0.746 in the absence of aggregate shocks. Our interpretation of the magnitude of $b$ is the level of insurance against unemployment shocks at the level of a household. Specifically, in an unemployment spell, consumption is a fraction $b$ of what it would have been had the household been employed. $b$ is large relative to typical UI replacement rates because many households have other sources of insurance with multiple earners being especially important. So to express $b$ in terms of a replacement rate, we conduct a simple calculation in which the hypothetical unemployment spell only affects one of the two workers in the household and the UI system partially replaces that worker’s income while the other worker continues working as normal. We also adjust for the fact that $b$ is an after-tax measure while UI replacement rates are typically pre-tax. We then have the relationship:

$$\left( \text{replacement rate} \times \frac{1}{2} + \frac{1}{2} \right)^{1-\tau} = b.$$

\textsuperscript{29}See Stephens Jr (2004), Aguiar and Hurst (2005), Saporta-Eksten (2014), and Chodorow-Reich and Karabarbounis (2016).
Using this conversion of $b$ and $\tau$ we find an optimal replacement rate of 53% with aggregate shocks as compared to 35% percent without.

Figure 1 shows the positive effects of changing the unemployment benefit. Raising the generosity of unemployment benefits hurts the incentives for working, so unemployment rises somewhat. However, it has a substantial macroeconomic stabilizing effect.

Turning to taxes, we find that optimal tax progressivity actually falls to 0.221 relative to 0.248 without aggregate shocks. To see that this result implies a small change in progressivity, consider the following. The 80-20 ratio of pre-tax wage income among those 25 to 60 years old in the 2001 SCF is 4.76. $\tau = 0.248$ implies an 80-20 ratio for after-tax wage income of 3.24 while $\tau = 0.221$ leads to 3.37. Our finding of a lower $\tau$ in the presence of business cycle risk results from our joint optimization over $b$ and $\tau$. If we hold $b$ fixed and optimize only over $\tau$, then $\tau$ is hardly affected by business cycles. Macroeconomic stabilization does not factor strongly into the choice of tax progressivity because the trade-off between stabilizing the economy and distorting the economy is much less favorable for $\tau$ than it is for $b$. We return to this point below.

How much is gained from using the optimal policy? Keeping $\tau$ at its optimum, but using $b = 0.746$ leads to a consumption-equivalent welfare loss of 0.3% relative to the optimal policy. Keeping $b$ at its optimum, but using $\tau = 0.246$ leads to a welfare loss of 0.1%.
6.3 Using the analytical propositions to understand the numerical results

What drives the large automatic stabilization role for unemployment benefits, but not for income tax progressivity? Our analytical results provide guidance on the key economic channels at play.

Figure 2 uses Proposition 1 and Proposition 3 to understand why the optimal $b$ rises in the presence of business cycle risk. Those propositions were derived under a few parameter restrictions and sticky-information and Appendix E.3 explains how the propositions can be extended to the richer setting considered here. The curves labelled “Insurance”, “Incentives”, and “Macro stabilization” plot the marginal welfare gains and losses from increasing $b$ that are due to the three policy trade-offs.

Figure 2 shows that, as $b$ rises, the incentive costs worsen, because the value of working gets closer to the value of unemployment, and the insurance gain diminishes, as the consumption (and therefore marginal utilities) of employed and unemployed become closer. In the absence of the macro stabilization term, the optimal $b$ would be the one at which these two terms balance, which is near $b = 0.75$. Yet, the macro stabilization term is positive and large, so the actual optimal $b$ is much larger. This role of the program, which has been neglected so far, is as important as the incentives and redistribution roles that the literature has emphasized instead. A concern for automatic stabilizers makes the unemployment insurance system significantly more generous.

The third and last striking result from Figure 2 is that the macroeconomic stabilization term is driven by the covariance term in equation (23), not by the steady state inefficiency in the economy. The benefits from stabilization do not come from closing the average gap between the level of activity and its optimal level, but rather from attenuating the amplitude of the business cycle.

The right panel of Figure 2 unpacks the sources of the business-cycle stabilization benefits in terms of the different sources of inefficient fluctuations that we characterized in Proposition 3. Each curve in the figure corresponds to a component of the marginal welfare gain or loss from reducing slack displayed in Proposition 3. The dominant component is clearly the reduction in idiosyncratic risk that results from a higher $b$. By stabilizing the economy, more generous unemployment benefits reduce the risk that households face in their pre-government incomes. This channel is distinct from the insurance benefit, which is the smoothing of post-government income for a given risk to pre-government income. There are two sources of idiosyncratic risk, skills and unemployment, but the welfare effects are driven almost entirely by the skills component. The inefficient utilization of labor
Figure 2: Marginal welfare gain from changing $b$ for $\tau = 0.221$. The quantities in the left panel correspond to the terms in Proposition 1. The covariance term shows that the macroeconomic stabilization term is driven by the covariance term in equation (23). The terms in the right panel correspond to $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{dW_t}{dx_t} \frac{dx_t}{db_t}$ with $\frac{dW_t}{dx_t}$ broken into components as in Proposition 3. Both figures are scaled to units of consumption equivalent welfare per unit change in $b$.

...on the extensive margin is in fact negative, as raising $b$ raises the unemployment rate on average. The other components that are plotted in the figure, are small in contrast.

Turning to taxes, Figure 3 shows the policy tradeoffs for the choice of $\tau$. The key feature of this figure is how large the insurance and incentive tradeoffs are in the left figure. These considerations dominate the choice of optimal $\tau$. To illustrate this point, we can observe that $\text{Cov} \left( \frac{dW}{dx}, \frac{dx}{db} \right)$ is about half as big as $\text{Cov} \left( \frac{dW}{dx}, \frac{dx}{db} \right)$. To compare these two covariances, we must ask how a unit increase in $\tau$ compares to a unit increase in $b$? One way to proceed is to express these changes in terms of the welfare loss from distorting the economy by changing $\tau$ or $b$. To do so we divide by the incentives term from our proposition. This gives a ratio of 0.05 for $\tau$ and a ratio of 1.3 for $b$. Thus, the stabilization benefit per unit of incentive distortion is much higher for $b$ than for $\tau$.

6.4 When do automatic stabilizers matter?

We now consider several alternative specifications of the economy in order to show the features of the economy that create a role for automatic stabilization in the choice of the unemployment insurance benefit. In particular we evaluate how $\text{Cov} \left( \frac{dW}{dx}, \frac{dx}{db} \right)$ changes as we modify the model. This covariance can be split into the correlation and the two standard deviations. The correlation is high in the model so we focus on the standard deviations. A large value for $\text{StDev} \left( \frac{dW}{dx} \right)$ indicates...
Figure 3: Marginal welfare gain from changing $\tau$ for $b = 0.811$. The quantities in the left panel correspond to the terms in Proposition 2. The terms in the right panel correspond to $E_0 \sum_{t=0}^{\infty} \beta^t \frac{dW_t}{dx_t}$ with $\frac{dW_t}{dx_t}$ broken into components as in Proposition 3. Both figures are scaled to units of consumption equivalent welfare per unit change in $\tau$.

<table>
<thead>
<tr>
<th></th>
<th>Optimal $b$</th>
<th>StDev $\left( \frac{dW}{dx} \right)$</th>
<th>StDev $\left( \frac{dx}{db} \right)$</th>
<th>Cov $\left( \frac{dW}{dx}, \frac{dx}{db} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) No aggregate shocks</td>
<td>0.746</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>(ii) Baseline</td>
<td>0.811</td>
<td>0.264</td>
<td>0.105</td>
<td>0.027</td>
</tr>
<tr>
<td>(iii) Flexible prices</td>
<td>0.748</td>
<td>--</td>
<td>See note.</td>
<td>--</td>
</tr>
<tr>
<td>(iv) Aggressive monetary policy</td>
<td>0.748</td>
<td>0.152</td>
<td>0.036</td>
<td>0.005</td>
</tr>
<tr>
<td>(v) No $Q^u$</td>
<td>0.777</td>
<td>0.240</td>
<td>0.021</td>
<td>0.004</td>
</tr>
<tr>
<td>(vi) Acyclical skill risk</td>
<td>0.743</td>
<td>0.079</td>
<td>0.105</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Table 2: Optimal policies under alternative specifications. Note: with flexible prices, $x_t$ is pinned down by the firm’s first order condition and we find $|dx/db| < 3 \times 10^5$ across the state space. Moments of $dW/dx$ and $dx/db$ are computed at the optimal policy for the baseline model.

that there is a substantial welfare cost from inefficient fluctuations. A large value of StDev $\left( \frac{dx}{db} \right)$ indicates that unemployment benefits are effective in stabilizing the economy.

Rows (i) and (ii) of Table 2 summarize the baseline economy with and without aggregate shocks. Row (iii) of the table shows that there is almost no role for automatic stabilizers when prices are flexible. This confirms the important role of aggregate demand and inefficient business cycles.

Row (iv) increases the coefficient on inflation in the monetary policy rule to 2.50 from 1.66. With this more aggressive monetary policy rule there is less of a need for fiscal policy to manage aggregate demand. Therefore, stabilization plays a smaller role in the design of the optimal social insurance system than in the baseline calibration. Both flexible prices and aggressive monetary
policy make the real interest rate respond strongly to changes in slack, which stabilizes the economy. The elasticity of slack with respect to the social program, \(dx/db\), is therefore small as \(x\) is already stabilized. This is reflected in a small value for \(\text{StDev}(dx/db)\).

Rows (v) and (vi) illustrate two central mechanisms in our quantitative analysis: UI stabilizes the economy by dampening the cyclical fluctuations in the precautionary savings motive generated by unemployment risk and stabilizing the economy is beneficial because of the cyclical dynamics of idiosyncratic skill risk. Row (v) documents the role of the precautionary savings motive in the analysis. Let \(Q^u_t = E_t \left[1 + \nu(1 - q_{t+1}M_{t+1})\left(b^{-1} - 1\right)\right]\) be the component of the Euler equation that reflects unemployment risk. Row (v) considers a monetary policy rule

\[
I_t = \bar{I} \pi_t^{\omega_s} x_t^{\omega_s} (1/Q^u_t),
\]

which neutralizes this precautionary savings motive. Under this policy, aggregate demand is quite insensitive to the UI benefit as shown by \(\text{StDev}(dx/db)\). In this case, a main mechanism through which the UI benefit can help stabilize the economy has been removed. Row (vi) fixes the distribution of skill risk at its steady state distribution. Here we see that \(\text{StDev}(dW/dx)\) is much lower than in the baseline because cyclical skill risk contributes a considerable amount to the welfare cost of business cycles. The combination of these two mechanisms create the rationale for increasing unemployment benefits for stabilization purposes.

6.5 Extensions

Our model makes some special assumptions for tractability and we now evaluate the role of these assumptions in the analysis. We begin by considering alternative specifications of the wage rule before turning to the role of savings. We then evaluate an extension with time-varying UI benefits.

6.5.1 The wage rule

There are two features of wage determination that are especially important for the effectiveness of the stabilizers. The first is the cyclicality of wages, since this affects the amplitude of the business cycle. The second is whether there is a direct response of wages to changes in policy, as changes in benefits or take-home pay affect the bargaining power of workers.

In our wage rule, the parameter \(\zeta\) captures the elasticity of wages with respect to slack. Making
the wage more cyclical makes the intensive margin of labor supply more volatile while the unemployment rate (the extensive margin) becomes less volatile. With less variability in the unemployment rate there is less amplification through the precautionary savings motive, and so less value in using the automatic stabilizers to stabilize the business cycle. When we double $\zeta$, the optimal $b$ is now 0.79 rather than 0.81 under our baseline specification. While this change reduces the role of the automatic stabilizers it implies a counterfactually low level of unemployment volatility.

In the baseline specification, the steady state wage is essentially constant changes across policy regimes. This specification is consistent with empirical work that fails to find an effect of unemployment benefits on wages. However, some studies have found large effects of unemployment benefits on the equilibrium unemployment rate possibly reflecting general equilibrium effects operating through wages (Hagedorn et al., 2013). We explore how our analysis is affected when the steady state wage is increasing in the unemployment benefit. In our baseline specification the elasticity of the unemployment rate with respect to benefits is 0.5. We consider an alternative in which the steady state wage is increasing in $b$ such that the elasticity of unemployment to benefits is 1. This alternative specification is intended to show the sensitivity of our results to this type of modification.

With a positive wage elasticity, the greater sensitivity of the steady state unemployment rate to the unemployment benefit leads to a lower optimal benefits in the absence of aggregate shocks: 0.478 as opposed to 0.746 in our baseline. However, when there are aggregate shocks, low benefits lead to strong de-stabilizing dynamics because of the precautionary savings motive. Because there is little social insurance, the precautionary savings motive is stronger and more cyclical, which leads to stronger internal amplification of shocks so fluctuations become larger and therefore costlier. The automatic-stabilizer nature of unemployment benefits is substantially stronger as a result. In this case we find that aggregate shocks lead to a large change in the optimal benefit relative to what is optimal in steady state: the optimal $b$ rises from 0.478 to 0.788 with aggregate shocks. Therefore, a positive wage elasticity with respect to benefits leads to lower unemployment benefits overall but a much larger effect of the business cycle on the optimal policy.

30 The steady state wage only changes due to the costs of hiring in the term $1 - J_t/Y_t$, which is quite small in practice.
31 Card et al. (2007), Van Ours and Vodopivec (2008), Lalive (2007), and Johnston and Mas (2015) find no evidence that UI generosity affects earnings upon re-employment.
6.5.2 Aggregate borrowing and saving

Next, we evaluate the assumption that there are no assets in gross supply. This was important so far to keep both the analytical and the numerical models tractable, since we did not have to keep track of a changing wealth distribution. The key new assumption is that now there is a constant and policy-invariant positive stock of government debt that private households can hold to self insure against unemployment shocks, so now there is a distribution of wealth that changes over time and responds to policy. The level of taxes adjusts to keep this stock of debt constant, so there are no deficits as in our baseline.

Our model with savings focuses on unemployment risk and the role of UI benefits. We omit time-varying idiosyncratic skill risk to keep the computation manageable, but instead incorporate a simpler form of heterogeneity in skills, patience and job-loss rates. Specifically, we assume that households take one of three types, which we associate with phases of the life cycle so we call them “young”, “middle-aged,” and “old.” Each age group, indexed by $\epsilon$, has its own time-preference rate $\beta_\epsilon$, labor productivity $\alpha_\epsilon$, and job-loss rate $\upsilon_\epsilon$. An individual starts young with $\epsilon = 1$ and stochastically ages to $\epsilon = 2$ and so on. We calibrate to be consistent with the moments of the distribution of liquid asset holdings, earnings, and unemployment across age groups. Appendix E.4 provides a detailed description of the equilibrium conditions of the extended model and its calibration. We solve the extended model using the Reiter (2009) method, which gives a non-linear solution with respect to idiosyncratic state variables, but a linearized solution with respect to aggregate states.

The key implication of positive savings is that unemployment benefits are not the only source of insurance against unemployment spells. As a result, the precautionary savings motive is not as sensitive to changes in the level of benefits and the tradeoff between the stabilization benefit of UI and incentives is less favorable than in our baseline analysis. Panel A of Figure 4 shows how the volatility of the unemployment rate varies with steady state unemployment rate. The variation along the horizontal axis is induced by changes in $b$. Compare the lines labeled “No Savings” and “Savings.” The slopes of these lines reveal how much stabilization is achieved for a given increase in distortion to the steady state, which is an important consideration in the policy tradeoff. The line with savings is flatter than that without savings meaning that it is more costly to stabilize the economy with savings.
A. Stabilization vs. steady state distortion

Figure 4: A. Standard deviation of unemployment rate as function of steady state unemployment rate. B. Insurance as function of steady state unemployment rate. In both cases, the variation along the horizontal axis is induced by changing unemployment benefit generosity. Insurance is defined as the percentage difference between $\mathbb{E}[u'(c)|\text{unemployed}]$ and $\mathbb{E}[u'(c)|\text{employed}]$ in steady state.
Ideally we would evaluate the welfare benefits of varying $b$ in the model with savings, but this is not computationally feasible because an accurate assessment of the welfare cost of business cycles requires the time-varying distribution of skill risk and a non-linear solution. As an alternative, we construct a stand-in economy that has no savings, but mimics the policy tradeoffs in the model with savings. This stand-in model is our baseline model but with a low separation rate, $\nu = 0.028$ rather than the baseline value of 0.094. The “Low-Separation-Rate” economy reduces the importance of the precautionary savings motive in the business cycle because unemployment risk is less salient in the Euler equation of the employed households because they have a smaller change of becoming unemployed. UI benefits have a smaller stabilization benefit because the destabilizing force of the counter-cyclical precautionary savings motive is weaker. We choose the separation rate to match the slope of the Savings economy in Panel A of Figure 4.

Panel B of Figure 4 shows that the Low-Separation-Rate economy also does decently at matching the insurance-distortion tradeoff from the Savings economy. Specifically the figure shows the difference between the average marginal utilities of the unemployed and employed: $E[u'(c_{it})|n_{it} = 0] - E[u'(c_{it})|n_{it} = 1]$. The slopes of the lines in this figure show the insurance benefit of increasing UI benefits per unit of distortion.

In the Low-Separation-Rate model we find that the optimal $b$ rises from 0.789 without business cycles to 0.827 with aggregate shocks while the optimal $\tau$ falls from 0.267 to 0.252. These imply a replacement rate that rises from 45% to 55%. This 10 p.p. increase compares to an 18 p.p. increase for our baseline model. While the Low-Separation-Rate model has a smaller role for automatic stabilizers, a 10 p.p. change in replacement rates is still a substantial change in policy.

6.5.3 Cylical UI Policy

Our baseline model assumes that the UI benefit is a constant fraction $b$ of the earnings of an employed household with the same level of skills. Our focus has been on this constant policy in line with the view of UI as an automatic stabilizer. However, the generosity of UI benefits, notably the maximum duration, varies over the cycle, which can also serve to stabilize the economy. Does this cyclical generosity obviate the need for more generous benefits on average?
Figure 5 shows an empirical measure of UI generosity, measured by

\[ b_t = \left( \frac{1}{2} + \frac{1}{2} \frac{\text{Total UI Payments}}{\text{Continuing claims for UI DPI per capita trend}} \right)^{1-\tau}. \]

At the heart of this estimate is the average UI payment per recipient, which is then normalized by the trend in disposable personal income per capita. We then convert from a replacement rate to a level of insurance as we have done previously. This measure makes no attempt to control for the skill composition of the pool of unemployed and therefore likely overstates the cyclicity of UI generosity because the unemployed pool becomes more high-skill in recessions (see Mueller, 2017). The figure shows clear counter-cyclical spikes in benefit generosity.

Regressing the time series for benefit generosity shown in Figure 5 on the unemployment rate gives a coefficient of \( b_{\text{cyc}} \equiv 1.88 \). We consider an extension to the model in which \( b_t = \bar{b} + b_{\text{cyc}}(u-\bar{u}) \), where \( \bar{b} \) and \( \bar{u} \) are steady state values. Our analysis continues to focus on the optimal value of the constant \( \bar{b} \). We find that the optimal \( \bar{b} \) rises from 0.746 without cycles to 0.780 with them, while tax progressivity falls from 0.248 to 0.234. The implied replacement rate rises from 35% to 44%. The observed cyclicality of UI generosity reduces the need for more generous benefits on average, but nevertheless macroeconomic stabilization still pushes for a higher average level of benefits.
7 Conclusion

Policy debates take as given that there are stabilizing benefits of unemployment insurance and income tax progressivity, but there are few systematic studies of what factors drive these benefits and how large they are. In contrast, the study of these social programs in the academic literature rarely takes into account this macroeconomic stabilization role, instead treating it as a fortuitous side benefit.

This paper tries to remedy this situation. Our study provides a theoretical characterization of automatic stabilizers. In general terms, we view an automatic stabilizer as a fixed policy for which there is a positive covariance between the effect of slack on welfare, and the effect of the policy on slack. If a policy tool has this property of stimulating the economy more in recessions when slack is inefficiently high, then its role in stabilizing the economy calls for expanding the use of the policy beyond what would be appropriate in a stationary environment. Overall, we found that the role of social insurance programs as automatic stabilizers affects their optimal design and, in the case of unemployment insurance, it can lead to substantial differences in the generosity of the system.

Our focus on the automatic stabilizing nature of existing social programs led us to take a Ramsey approach to the ex ante design of fiscal policy. Future work might explore how these forces affect the design of the social insurance system from a Mirrleesian perspective. Another question is how these fiscal policy programs can adjust to the state of the business cycle, taking into account measurement difficulties, time inconsistency, political economy, and other challenges of implementing state-dependent stabilization policy. There is already some progress on these two topics, and hopefully our analysis will provide some insights to guide their further development.
References


Appendix

This appendix contains five sections, which: describe the role of the wage rule introduced in section 2; provide auxiliary steps to some results stated in section 3; prove the propositions in sections 4; prove the propositions in section 5; and describe the methods used to solve the model in section 6.

A The wage rule

This section first describes a bargaining protocol that gives rise to the wage rule we assumed, and next modifies our analytical results to allow for a non-isoelastic wage rule.

A.1 Nash bargaining protocol and our wage rule

This appendix presents a Nash bargaining protocol that gives rise to our wage rule. To define a Nash bargaining solution we have to define the outside options of the firm and the worker. If bargaining breaks down, we assume that the firm can obtain a different worker by paying the hiring cost again and the worker can meet another firm but loses some of the period’s time endowment. Let $z_t$ be the amount of time lost if bargaining breaks down. One could assume that $z_t$ is a constant parameter (e.g. it takes a given amount of time to meet a new firm once bargaining breaks down) or one could assume that it is easier to find a new firm when the labor market is tight or more productive $z_t = \bar{z}A^{-1}M_t^{-\epsilon}$. The worker chooses total market time, $h$, before bargaining begins. If bargaining breaks down, we assume the next employer pays the equilibrium wage (i.e. we do not consider bargaining in that relationship). We assume that employed workers of a given skill are able to pool the gains from bargaining between themselves (in equilibrium they all negotiate the same wage so there are no transfers, but this means that bargaining does not take into account the curvature of the individual utility function or the non-linearity of the tax system).

The surplus to a worker of an offer that pays $w$ when the market wage is $\bar{w}$ is

$$V(w, \bar{w}, \alpha) = \mu_V(\alpha)\alpha [wh - \bar{w}(h - z)],$$

where $\mu_V(\alpha)$ is the marginal valuation of resources by the employed workers of skill $\alpha$. The surplus
to the firm is
\[ J(w, \bar{w}, \alpha) = \mu_J \alpha \left[ \psi_1 M^{\psi_2} + \bar{w} h - w h \right], \]

where \( \mu_J \) is the marginal valuation of resources by the firm. In the firm’s surplus we assume hiring costs are proportional to \( \alpha \), which is consistent with our assumption that hiring an effective unit of labor costs \( \psi_1 M^{\psi_2} \) because the average \( \alpha \) in the employed and unemployed populations is always constant at one.

The generalized Nash bargaining solution satisfies
\[
\max_w V(w, \bar{w}, \alpha)^N J(w, \bar{w}, \alpha)^{1-N}
\]
where \( N \) is the worker’s bargaining power. The first order condition at \( w = \bar{w} \) is
\[
\frac{N}{1-N} \mu_J \psi_1 M^{\psi_2} h \mu_V(\alpha) \alpha^2 = \mu_J h \mu_V(\alpha) w z \alpha^2
\]
\[
\frac{N}{1-N} \psi_1 M^{\psi_2} = wz
\]
Substituting \( z = A^{-1} M^{-\xi} \)
\[
w = \frac{1}{\bar{z}} \frac{N}{1-N} \psi_1 A M^{\psi_2+\xi} \tag{27}
\]
Now set \( \bar{z} \) and \( N \) such that \( \frac{1}{\bar{z}} \frac{N}{1-N} \psi_1 = \bar{w} M^{-\zeta} \) and set \( \xi \) such that \( \psi_2 + \xi = \zeta \). The result is that this bargaining protocol gives rise to our wage rule except for the \((1 - J/Y)\) term, which is quantitatively innocuous. If \( z_t \) fluctuates in a more complicated manner, we can derive a wage rule that includes the \((1 - J/Y)\) term.

### A.2 The role of the wage rule in our analysis

We assumed that the wage was determined by equation (5). A more general specification uses a generic wage function:
\[
w_t = w(\eta_t, x_t, b, \tau),
\]
that maps the aggregate shocks, slack (or labor market tightness), and policy parameters into a wage.

In this more general case, we can write $h_t$ as:

$$h_t = \left\{ (1 - \tau) \left[ \frac{\eta_t^A}{S(x_t)} \left( 1 - \frac{J_t}{Y_t} \right) \right]^{-1} w(\eta_t^A, x_t, b, \tau) \right\}^{1/(1+\gamma)}$$

where $S(x_t)$ is the level of price dispersion associated with that level of slack. Using the generic wage rule and equations (3), (4), (18), (19), (35), and (37), we can write $J_t/Y_t$ as a function of $(\eta_t, x_t, b, \tau)$ to yield:

$$h_t = \{(1 - \tau)H(\eta_t, x_t, b, \tau)\}^{1/(1+\gamma)},$$

where $H_t \equiv \left[ \frac{\eta_t}{S(x_t)} \left( 1 - \frac{J_t}{Y_t} \right) \right]^{-1} w(\eta_t, x_t, b, \tau)$. The purpose of assuming the wage rule in equation (5) in the paper is that $H_t$ simplifies to $x_t^\ell$.

However, not making this simplification, and so carrying the new $H(.)$ term along only adds to our results a few additional terms. To start, in addition to those terms that appear in equation (21) (Proposition 1), we would now need to add:

$$E_0 \sum_{t=0}^{\infty} \beta^t (1 - u_t) \left[ \frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dH_t} \frac{\partial H_t}{\partial b} \bigg|_{x}.$$

Similarly, we would need to modify equation (22) (Proposition 2) to include a term:

$$E_0 \sum_{t=0}^{\infty} \beta^t (1 - u_t) \left[ \frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dH_t} \frac{\partial H_t}{\partial \tau} \bigg|_{x}.$$

Both of these terms relate to the adjustment of hours on the intensive margin. Intuitively, the wage plays two roles in the economy: it determines the marginal cost of the firms and therefore the incentives for hiring and it determines the incentives for labor supply on the intensive margin. The first effect is captured in our analysis through $dx_t/db$. Using our wage rule, the second effect is captured through $\frac{dh_t}{d\tau} \frac{dx_t}{db}$. A more general wage rule brings in these new effects of policy on intensive margin labor supply. A different wage rule would also result in different values for $dx_t/db$ and $dx_t/d\tau$, but these are not qualitatively different considerations than the ones we focus on.
B Additional steps for deriving the results in section 3

B.1 The value of employment

In equilibrium, $a_{i,t} = 0$, and search effort is determined by comparing the value of working and not working according to equation (12). This section of the appendix derives the two key steps that make this difference independent of the household’s type, so that all households choose the same search effort.

Lemma 3. The household’s value function has the form

$$V(\alpha, n, S) = V^\alpha(\alpha, S) + nV^n(S)$$

for some functions $V^\alpha$ and $V^n$ where $S$ is the aggregate state. The choice of search effort is then the same for all searching households regardless of $\alpha$.

Proof: The household’s search problem is

$$V^s(\alpha, S) = \max_q \left\{ MqV(\alpha, 1, S) + (1 - Mq)V(\alpha, 0, S) - \frac{q^{1+\kappa}}{1 + \kappa} \right\}.$$ 

Substitute for the value functions to arrive at

$$V^s(\alpha, S) = \max_q \left\{ Mq [V^\alpha(\alpha, S) + V^n(S)] + (1 - Mq) [V^\alpha(\alpha, S)] - \frac{q^{1+\kappa}}{1 + \kappa} \right\}$$

$$V^s(\alpha, S) = \max_q \left\{ MqV^n(S) - \frac{q^{1+\kappa}}{1 + \kappa} \right\} + V^\alpha(\alpha, S)$$

where we have brought $V^\alpha(\alpha, S)$ outside the max operator as it appears in an additively separable manner. As there is no $\alpha$ inside the max operator, the optimal $q$ is independent of $\alpha$. Note that we can write $V^s$ as $V^s(\alpha, S) = g(S) + V^\alpha(\alpha, S)$ where $g$ is the solution to the maximization problem above.

Suppose that the value function is of the form given in (28). We will establish that the Bellman equation maps functions in this class into itself, which implies that the fixed point of the Bellman
equation is in this class by the contraction mapping theorem. The Bellman equation for employed and unemployed are

\[
V^\alpha(\alpha, S) + V^n(S) = \log \left[ \lambda (\alpha (wh + d))^{1-\tau} \right] - \frac{h^{1+\gamma}}{1 + \gamma} + \beta \mathbb{E} \left[ (1 - v) \left( (V^\alpha(\alpha', S') + V^n(S')) + vV^s(\alpha', S') \right) \right]
\]

\[
V^\alpha(\alpha, S) = \log \left[ \lambda b (\alpha (wh + d))^{1-\tau} \right] - \xi + \beta \mathbb{E} \left[ V^s(\alpha', S') \right],
\]

(31)

where we have used the solution for consumption and the result that \( h \) is independent of \( \alpha \). Taking the difference yields

\[
V^n(S) = - \log(b) - \frac{h^{1+\gamma}}{1 + \gamma} + \xi + \beta (1 - v) \mathbb{E} \left[ V^n(S') - g(S') \right]
\]

(32)

and plugging \( V^s(\alpha, S) = g(S) + V^\alpha(\alpha, S) \) into the continuation value of the unemployed in (31) gives

\[
V^\alpha(\alpha, S) = \log \left[ \lambda b (\alpha (wh + d))^{1-\tau} \right] - \xi + \beta \mathbb{E} \left[ g(S') + V^\alpha(\alpha', S') \right].
\]

\[\square\]

B.2 Proof of lemma 2

First, the Euler equation for a household is

\[
\frac{1}{c_{i,t}} \geq \beta R_t \mathbb{E} \left[ \frac{1}{c_{i,t+1}} \right].
\]

as usual. Using (13) we have

\[
\left[ \alpha_{i,t}^{1-\tau} (n_{i,t} + (1 - n_{i,t}) b) \tilde{c}_i \right]^{-1} \geq \beta R_t \mathbb{E} \left[ \left[ \alpha_{i,t+1}^{1-\tau} (n_{i,t+1} + (1 - n_{i,t+1}) b) \tilde{c}_{t+1} \right]^{-1} \right].
\]
Notice that \( E \left[ \frac{\alpha_{i,t}^{1-\alpha}}{\alpha_{i,t+1}} \right] = E \left[ \epsilon_{i,t+1}^{\tau-1} \right] \) is common across households and is known at date \( t \). Now consider the two cases for \( n_{i,t} \) and use the EU and UU transition probabilities to arrive at

\[
\tilde{c}_{i,t}^{-1} \geq \beta R_t E \left[ [1 + \nu (1 - q_{t+1} M_{t+1}) (b^{-1} - 1)] \tilde{c}_{i,t+1}^{-1} \right] E \left[ \epsilon_{i,t}^{\tau-1} \right] \quad (33)
\]

\[
\tilde{c}_{i,t}^{-1} \geq \beta R_t E \left[ b [1 + (1 - q_{t+1} M_{t+1}) (b^{-1} - 1)] \tilde{c}_{i,t+1}^{-1} \right] E \left[ \epsilon_{i,t}^{\tau-1} \right]. \quad (34)
\]

The right-hand side of these inequalities is larger for the employed (we establish this formally below), so there are two possibilities: the Euler equation of the employed holds with equality or both inequalities are strict. Here we follow Krusell et al. (2011), Ravn and Sterk (2017), and Werning (2015) in assuming that the Euler equation of the employed/high-income household holds with equality. This household is up against its constraint \( a' = 0 \) so there could be other equilibria in which the Euler equation does not hold with equality. The equilibrium we focus on is the limit of the unique equilibrium as the borrowing limit approaches zero from below. See Krusell et al. (2011) for further discussion of this point. (33) with equality yields the desired result.

To establish that the right-hand side of (33) weakly exceeds that of (34) if

\[
q_{t+1} M_{t+1} + \frac{\nu}{b} (1 - q_{t+1} M_{t+1}) \geq 0
\]

and notice that \( qM \) is a job finding rate \( \in [0,1] \) and \( \nu \in [0,1] \) and \( b \in [0,1] \).

### B.3 Optimal hours worked and search effort

We start by deriving equation (17). Using labor market clearing and the definition of \( A_t \) we can write the aggregate production function as

\[
Y_t = A_t h_t (1 - u_t).
\]

Output net of hiring costs is paid to employed workers in the form of wage and dividend payments. As the average \( \alpha_{i,t} \) is equal to one we have

\[
Y_t - J_t = (w_t h_t + d_t) (1 - u_t).
\]

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Multiply both sides of (35) by \((Y_t - J_t)/Y_t\) and substitute for \(Y_t - J_t\) using (36) to give:

\[
w_t h_t + d_t = A_t h_t \frac{Y_t - J_t}{Y_t}.
\]

Substitute this for \(w_t h_t + d_t\) in equation (11) to arrive at

\[
h_t^\gamma = \frac{(1 - \tau) w_t}{A_t h_t \frac{Y_t - J_t}{Y_t}}.
\]

Finally, use the wage rule in equation (5) to arrive at (17).

We turn next to derive optimal search effort in equation (18). We use the results of Lemma 3, specifically equations (30) and (32), using \(v = 1\). This leads to

\[
\max_q \left\{ Mq \left[ \log \frac{1}{b} - \frac{h^{1+\gamma}}{1 + \gamma} + \xi \right] - \frac{q^{1+\kappa}}{1 + \kappa} \right\}.
\]

The first order condition yields equation (18).

### B.4 Equilibrium definition

We first state the intermediate firm’s problem and some addition equilibrium conditions and then state a definition of an equilibrium.

**Firm’s problem and inflation with Calvo pricing.** The intermediate firm’s problem is

\[
\max_{p_t^*: \{y_{j,s}, n_{j,s} \}_{s=t}^{\infty}} \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1 - \theta) \left[ \frac{p_t^*}{P_s} y_{j,s} - n_{j,s} h_{j,s} w_s - \psi_1 M_{s}^{\psi_2} v_{j,s} \right]
\]

subject to

\[
\begin{align*}
y_{j,s} &= (p_t^*/P_s)^{\mu/(1-\mu)} Y_s \\
y_{j,s} &= \eta_h h_s n_{j,s} \\
n_{j,s} &= (1 - v)n_{j,s-1} + v_{j,s}.
\end{align*}
\]
The solution to this problem satisfies

\[
\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1 - \theta)^{s-t} \left( \frac{p_s}{p_t^*} \right)^{\mu/(1-\mu)} Y_s \mu \left( w_s h_s + \psi_1 M_{s+1}^{\psi_2} - R_{s+1}^{-1} (1 - \theta) (1 - \nu) \psi_1 M_{s+1}^{\psi_2} \right)}{(\eta_s A h_s)}. \]

(38)

As is standard, inflation and price dispersion evolve according to:

\[
\pi_t = \left[ (1 - \theta) / \left[ 1 - \theta \left( \frac{p_t^*}{p_t} \right)^{1/(1-\mu)} \right] \right]^{1-\mu} \]

(39)

\[
S_t = (1 - \theta) S_{t-1}^{-\mu/(1-\mu)} + \theta \left( \frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)}. \]

(40)

where \(p_t^*/p_t\) is the relative price chosen by firms that adjust their price in period \(t\).

**Firm’s problem and inflation with sticky information.** Under the assumption of a unit separation rate, the marginal cost of the firm is:

\[
\frac{w_t + \psi_1 M_1^{\psi_2}/h_t}{\eta_t A}. \]

Marginal costs are the sum of the wage paid per effective unit of labor and the hiring costs that had to be paid, divided by productivity. The price-setting first order condition is

\[
p_t^* = \mu \left( \frac{w_t h_t + \psi_1 M_t^{\psi_2}}{A h_t} \right) \]

(41)

for firms with full information and \(\mathbb{E}_{t-1}(p_t^*)\) for others. Inflation satisfies

\[
\frac{1}{p_t^{1-\mu}} = \theta p_t^* \left( \frac{1}{1-\mu} \right) + (1 - \theta) \mathbb{E}_{t-1}(p_t^*)^{1/(1-\mu)} \]

(42)

and price dispersion evolves according to

\[
S_t = \left( \frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)} \left[ \theta + (1 - \theta) \left( \frac{\mathbb{E}_{t-1} p_t^*}{p_t^*} \right)^{\mu/(1-\mu)} \right]. \]

(43)
Equilibrium  The aggregate resource constraint is:

\[ Y_t - J_t = C_t + G_t. \]  

(44)

The Fisher equation is:

\[ R_t = I_t / \mathbb{E}_t [\pi_{t+1}]. \]  

(45)

The link between \( \tilde{c}_t \) and \( C_t \) depends on \( E_i \left[\alpha_{i,t}^{1-\tau}\right] \). This evolves according to:

\[ E_i \left[\alpha_{i,t}^{1-\tau}\right] = (1 - \delta) E_i \left[\alpha_{i,t-1}^{1-\tau}\right] E_i \left[\epsilon_{i,t}^{1-\tau}\right] + \delta. \]  

(46)

An equilibrium of the economy can be calculated from a system equations in 17 variables and three exogenous processes. The variables are

\[ C_t, \tilde{c}_t, u_t, E_i \left[\alpha_{i,t}^{1-\tau}\right], Q_t, R_t, I_t, \pi_t, Y_t, G_t, h_t, w_t, S_t, \frac{p_t}{p_t}, J_t, q_t, M_t. \]

And the equations are: (3), (4), (5), (6), (7), (14), (15), (16), (17), (18), (35), (41), (42), (43), (44), (45), and (46). The exogenous processes are \( \eta_t^A \), \( \eta_t^C \), and \( \eta_t^I \).

In the quantitative model with Calvo pricing, we replace (41) with (38), (42) with (39), and (43) with (40). Moreover, with persistent employment we replace (18) with (12) and we must keep track of the value of employment in excess of unemployment, which is forward-looking, independent of \( \alpha \) and can be calculated from the proof of Lemma 3.

C  Proofs for section 4

C.1  Skill dispersion without mortality

Lemma 4. Under no mortality, \( \delta = 0 \):

\[
E_0 \sum_{t=0}^{\infty} \beta^t E_i \left[ \log \left( \alpha_{i,t}^{1-\tau} \right) - \log \left( E_i \left[\alpha_{i,t}^{1-\tau}\right]\right) \right] \\
= \frac{1}{1 - \beta} \left[ E_i \log \left( \alpha_{i,0}^{1-\tau} \right) - \log \left( E_i \left[\alpha_{i,0}^{1-\tau}\right]\right) \right] + E_0 \sum_{t=0}^{\infty} \beta^{t+1} \left[ E_i \log \left( \epsilon_{i,t+1}^{1-\tau} \right) - \log \left( E_i \left[\epsilon_{i,t+1}^{1-\tau}\right]\right) \right].
\]
Proof: When there is no mortality, $\delta = 0$, we can compute the cumulative welfare effect of a change in $F(\epsilon_{i,t+1}, x_t)$ including the effects on current and future skill dispersion. In particular

$$\mathbb{E}_i \log(\alpha_{i,t}^{1-\tau}) = \mathbb{E}_i \log(\alpha_{i,t-1}^{1-\tau} \epsilon_{i,t}^{1-\tau})$$

$$= \mathbb{E}_i \log(\alpha_{i,t}^{1-\tau} \epsilon_{i,1}^{1-\tau} \cdots \epsilon_{i,t}^{1-\tau})$$

$$= \mathbb{E}_i \log(\alpha_{i,0}^{1-\tau}) + \mathbb{E}_i \log(\epsilon_{i,1}^{1-\tau}) + \cdots + \mathbb{E}_i \log(\epsilon_{i,t}^{1-\tau}).$$

Similarly

$$\log(\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]) = \log(\mathbb{E}_i [\alpha_{i,t-1}^{1-\tau} \mathbb{E}_i [\epsilon_{i,t}^{1-\tau}]])$$

$$= \log(\mathbb{E}_i [\alpha_{i,0}^{1-\tau}] \mathbb{E}_i [\alpha_{i,1}^{1-\tau}] \cdots \mathbb{E}_i [\epsilon_{i,t}^{1-\tau}])$$

$$= \log(\mathbb{E}_i [\alpha_{i,0}^{1-\tau}]) + \log(\mathbb{E}_i [\alpha_{i,1}^{1-\tau}]) + \cdots + \log(\mathbb{E}_i [\epsilon_{i,t}^{1-\tau}]).$$

Notice that in this no-mortality case, the date-$t$ loss from skill dispersion can be written as:

$$\mathbb{E}_i \log(\alpha_{i,t}^{1-\tau}) - \log(\mathbb{E}_i [\alpha_{i,t}^{1-\tau}]) = \mathbb{E}_i \log(\alpha_{i,0}^{1-\tau}) - \log(\mathbb{E}_i [\alpha_{i,0}^{1-\tau}]) + \sum_{s=1}^{t} \mathbb{E}_i \log(\epsilon_{i,s}^{1-\tau}) - \log(\mathbb{E}_i [\epsilon_{i,s}^{1-\tau}]).$$

Finally, take the expected discounted sum of this expression and rearrange to prove the result. □

Lemma 5. For a random variable $X$,

$$\frac{d}{d\tau} \{ \mathbb{E} [\log(X^{1-\tau})] - \log(\mathbb{E} [X^{1-\tau}]) \} = \frac{\text{Cov}(X^{1-\tau}, \log X)}{\mathbb{E} [X^{1-\tau}]}$$

Proof:

$$\frac{d}{d\tau} \{ \mathbb{E} [\log(X^{1-\tau})] - \log(\mathbb{E} [X^{1-\tau}]) \} = -\mathbb{E} [\log(X)] + \frac{\mathbb{E} [X^{1-\tau} \log X]}{\mathbb{E} [X^{1-\tau}]}$$

$$= -\mathbb{E} [\log(X)] + \frac{\mathbb{E} [X^{1-\tau}] \mathbb{E} [\log X] + \text{Cov}(X^{1-\tau}, \log X)}{\mathbb{E} [X^{1-\tau}]}$$

$$= \frac{\text{Cov}(X^{1-\tau}, \log X)}{\mathbb{E} [X^{1-\tau}]}$$

□
C.2 Proof of Proposition 1

For this proof, in addition to the social welfare function, (20), the relevant equations of the model are (3), (35), (4), (44), (18), and (17).

The first-order condition of the social welfare function with respect to $b$ is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{u_t}{b} - \frac{u_t}{1 - u_t + u_t b} + \frac{dW_t}{du_t} \frac{\partial u_t}{\partial b} \bigg|_x - vz_i \frac{\partial q_t}{\partial b} \bigg|_x + \frac{dW_t}{dx_t} \frac{\partial b}{db} \right\} = 0. \quad (47)$$

The first two terms in (47) can be expressed as

$$\frac{u_t}{b} - \frac{u_t}{1 - u_t + u_t b} = u_t \left( \frac{1}{b} - 1 + 1 - \frac{1}{1 - u_t + u_t b} \right) = u_t \left( \frac{1}{b} - 1 \right) \left( 1 - \frac{u_t b}{1 - u_t + u_t b} \right)$$

and note that

$$\frac{\partial \log (\breve{b}_t)}{\partial \log b} \bigg|_{x,q} = \frac{\partial}{\partial \log b} \log \left( \mathbb{E}_t \left[ \alpha_i^{1-\tau} \right] \left( 1 - u_t + u_t b \right) \right) \quad (48)$$

where the partial derivative on the right hand side of (48) is with respect to $b$ alone.$^{32}$ So, we have:

$$\frac{u_t}{b} - \frac{u_t}{1 - u_t + u_t b} = u_t \left( \frac{1}{b} - 1 \right) \frac{\partial \log (\breve{b}_t)}{\partial \log b} \bigg|_{x,q} \quad (49)$$

For the third and fourth terms in (47), start by noting that:

$$\frac{dW_t}{du_t} = \log b + \frac{1 - b}{1 - u_t + u_t b} - \frac{A_i h_t}{C_t} + \frac{\psi_1 M_i^{\psi_2}}{C_t} + \frac{h_t^{1+\gamma}}{1 + \gamma} - \xi \quad (50)$$

$$= \left( \log b + \frac{h_t^{1+\gamma}}{1 + \gamma} - \xi \right) + \frac{1}{\tilde{c}_t} \frac{\partial \breve{c}_t}{\partial u_t} \bigg|_x \quad (51)$$

$^{32}$ As $\mathbb{E}_t \left[ \alpha_i^{1-\tau} \right]$ is an endogenous state that depends on the history of $x$, we are taking the partial derivative holding fixed this history.
where
\[
\frac{1}{\tilde{c}_t} \frac{\partial \tilde{c}_t}{\partial u_t} \bigg|_x = \frac{1 - b}{1 - u_t + u_t b} - \frac{A_t h_t}{C_t} + \frac{\psi_1 M_t^{\psi_2}}{C_t}.
\]

Then, it follows that:
\[
\frac{dW_t}{du_t} \frac{\partial u_t}{\partial b} \bigg|_x - v_{q_t} \frac{\partial q_t}{\partial b} \bigg|_x = \left[ \left( \log b + \frac{h_t^{1+\gamma}}{1 + \gamma} - \xi \right) + \frac{1}{\tilde{c}_t} \frac{\partial \tilde{c}_t}{\partial u_t} \bigg|_x \right] \frac{\partial u_t}{\partial b} \bigg|_x - v_{q_t} \frac{\partial q_t}{\partial b} \bigg|_x.
\]

Using equation (18), this becomes
\[
\frac{dW_t}{du_t} \frac{\partial u_t}{\partial b} \bigg|_x - v_{q_t} \frac{\partial q_t}{\partial b} \bigg|_x = \left[ \left( \log b + \frac{h_t^{1+\gamma}}{1 + \gamma} - \xi \right) + \frac{1}{\tilde{c}_t} \frac{\partial \tilde{c}_t}{\partial u_t} \bigg|_x \right] \frac{\partial u_t}{\partial b} \bigg|_x + \left( - \log b - \frac{h_t^{1+\gamma}}{1 + \gamma} + \xi \right) \frac{du_t}{dq_t} \frac{\partial q_t}{\partial b} \bigg|_x \bigg|_x.
\]

Substituting (49) and (52) into (47) yields the result.

C.3 Proof of Proposition 2

First we use Lemma 4 to substitute for \( E_0 \sum_{i=0}^{\infty} \beta^t E_i \left[ \log \left( \alpha_{i,t}^{1-\tau} \right) - \log \left( E_i \left[ \alpha_{i,t}^{1-\tau} \right] \right) \right] \) in the social welfare function. Then we proceed as in the proof of Proposition 1. The first-order condition of the social welfare function with respect to \( \tau \) is:
\[
\text{Cov} \left( \alpha_{i,0}^{1-\tau}, \log \alpha_{i,0} \right) + E_0 \sum_{i=0}^{\infty} \beta^t \left\{ \frac{\beta}{1 - \beta} \text{Cov} \left( \epsilon_{i,t+1}^{1-\tau}, \log \epsilon_{i,t+1}^{1-\tau} \right) \right\} + \frac{dW_t}{dh_t} \frac{\partial h_t}{\partial \tau} \bigg|_x + \frac{dW_t}{dx_t} \frac{dx_t}{d\tau} = 0,
\]

where we have used Lemma 5 twice. From (20), (35), and (44) we have
\[
\frac{dW_t}{dh_t} = \frac{dW_t}{du_t} \frac{du_t}{dq_t} \frac{dq_t}{dh_t} + \frac{A_t(1 - u_t)}{C_t} - (1 - u_t) h_t^{\gamma} - v_{q_t} \frac{dq_t}{dh_t}.
\]
Using (18), (51), and \(du_t/dq_t = -vM_t\) we arrive at

\[
\frac{dW_t}{dh_t} \frac{\partial h_t}{\partial \tau} = \frac{dW_t}{du_t} \frac{du_t}{dx_t} \frac{dq_t}{dh_t} \frac{\partial h_t}{\partial \tau} + \frac{A_t(1 - u_t)}{C_t} \frac{\partial h_t}{\partial \tau} \bigg|_x - (1 - u_t)h_t^\gamma \frac{\partial h_t}{\partial \tau} \bigg|_x - vq_t^\kappa \frac{dq_t}{dh_t} \frac{\partial h_t}{\partial \tau} \bigg|_x
\]

\[
= \frac{1}{\tilde{c}_t} \frac{\partial \tilde{c}_t}{\partial u_t} \bigg|_x \frac{du_t}{dx_t} \frac{dq_t}{dh_t} \frac{\partial h_t}{\partial \tau} + \left[ \frac{A_t(1 - u_t)}{C_t} - (1 - u_t)h_t^\gamma \right] \frac{\partial h_t}{\partial \tau} \bigg|_x
\]

\[
= \frac{\partial \log \tilde{c}_t}{\partial \log u_t} \bigg|_x \frac{\partial \log u_t}{\partial \tau} + \left[ \frac{A_t}{C_t} - h_t^\gamma \right] \frac{\partial h_t}{\partial \tau} \bigg|_x.
\]

Substituting this into (53) yields the desired result.

**D Proofs for section 5**

### D.1 Proof of Proposition 3

Proceeding as in the proof of Proposition 2 we have

\[
\frac{dW_t}{dx_t} = \frac{dW_t}{du_t} \frac{du_t}{dx_t} - vq_t^\kappa \frac{dq_t}{dx_t} \bigg[ \frac{A_t(1 - u_t)}{C_t} - (1 - u_t)h_t^\gamma \right] \frac{dh_t}{dx_t} \bigg|_x - vM_t \frac{dM_t}{dx_t}
\]

where \(dW_t/du_t\) is given by (51). We rearrange (54) to arrive at the desired result. First, note that:

\[
\frac{du_t}{dx_t} = -vq_t \frac{dM_t}{dx_t} - vM_t \frac{dq_t}{dx_t}
\]

Using this, (18), and (51), equation (54) then becomes

\[
\frac{dW_t}{dx_t} = \left( -\left( -\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi \right) + \frac{1}{\tilde{c}_t} \frac{\partial \tilde{c}_t}{\partial u_t} \bigg|_x \right) \left(-vq_t \frac{dM_t}{dx_t} - vM_t \frac{dq_t}{dx_t}\right) - vM_t \left(-\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi \right) \frac{dq_t}{dx_t}
\]

\[
+ \left[ \frac{A_t(1 - u_t)}{C_t} - (1 - u_t)h_t^\gamma \right] \frac{dh_t}{dx_t} - \frac{1}{C_t} \frac{\partial J_t}{\partial x_t} \bigg|_u - \frac{\beta}{1 - \beta} (1 - \gamma) \frac{d}{dx_t} \frac{\sigma^2(x_t)}{2} - \frac{Y_t}{C_t S_t} \frac{dS_t}{dx_t}
\]

\[
= vq_t \left( -\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi \right) \frac{dM_t}{dx_t} - \frac{Y_t}{C_t S_t} \frac{dS_t}{dx_t} + \frac{1}{\tilde{c}_t} \frac{\partial \tilde{c}_t}{\partial u_t} \bigg|_x \frac{du_t}{dx_t}
\]

\[
+ \left[ \frac{A_t(1 - u_t)}{C_t} - (1 - u_t)h_t^\gamma \right] \frac{dh_t}{dx_t} - \frac{1}{C_t} \frac{\partial J_t}{\partial x_t} \bigg|_u + \frac{\beta}{1 - \beta} \frac{d}{dx_t} \left[ E_i \log \left( \epsilon_i,t+1 \right) - \log \left( E_i \left[ \epsilon_i,t+1 \right] \right) \right].
\]
This can be rearranged to the desired result by making use of

\[ \frac{1}{c_t} \left. \frac{\partial c_t}{\partial u_t} \right|_{x} = \frac{1 - b}{1 - u_t + u_t b} - \frac{A_t h_t}{C_t} + \frac{\psi_1 M_t^{\psi_2}}{C_t} = \left. \frac{1}{C_t} \frac{\partial C_t}{\partial u_t} \right|_{x} + \frac{1 - b}{1 - u_t + u_t b}, \]

\[ E_i \log \left( \epsilon_{i,t+1}^{1-\tau} \right) - \log \left( E_i \left[ \epsilon_{i,t}^{1-\tau} \right] \right) = \int \log \left( \frac{\epsilon_{i,t}^{1-\tau}}{\int \epsilon_{i,t}^{1-\tau} dF(\epsilon, x_t)} \right) dF(\epsilon, x_t), \]

and

\[ -u_q t \frac{dM_t}{dx_t} = \left. \frac{\partial u_t}{\partial x_t} \right|_{q}. \]

**D.2 Deriving the special case in section 5.2.3**

We can normalize \( E_{t-1} p_t = 1 \). A fraction \( \theta \) of firms set the price \( p_t^* \) and the remaining set the price \( E_{t-1} p_t^* = 1 \). From the definition of the price index we can solve for

\[ \left( \frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)} = \left( \frac{1 - (1 - \theta) p_t^{(1/(1-\mu))}}{\theta} \right)^\mu. \]

Substituting into (19) we arrive at

\[ S_t = (1 - \theta) p_t^{\mu/\mu-1} + \theta^{1-\mu} \left( 1 - (1 - \theta) p_t^{1/(\mu-1)} \right)^\mu. \]

This makes clear that \( S_t \) is a function of \( p_t \). Differentiating this function we arrive at

\[ S'(1) = (1 - \theta) \frac{\mu}{\mu - 1} \left[ p_t^{1/\mu-1} - \theta^{1-\mu} \left( 1 - (1 - \theta) p_t^{1/(\mu-1)} \right)^{\mu-1} \right] = 0 \]

\[ S''(1) = \frac{1 - \theta}{\theta} \frac{\mu}{\mu - 1}. \]

Next, rewrite the welfare function as

\[ W_t = \log (C_t) - (1 - u_t) \frac{h_t^{1+\gamma}}{1+\gamma} + \chi \log (G_t) + \text{t.i.p.}, \]

\[ C_t = Y_t - J_t - G_t, \]

\[ Y_t = \frac{\eta_t^A}{S(p_t)} h_t (1 - u_t) \]

\[ G_t = \chi C_t, \]

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Observe that \( u_t \) and \( J_t \) are exogenous in this case as \( M_t \) and \( q_t \) are exogenous. Use the production function to rewrite \( h_t \) in terms of \( Y_t \)

\[
h_t = \frac{Y_t S(p_t)}{\eta_t^A (1 - u_t)}. \]

Now rewrite \( W_t \) in terms of \( Y_t \) and \( p_t \)

\[
W_t = (1 + \chi) \log \left( \frac{Y_t - J_t}{1 + \chi} \right) - 1 - u_t \left( \frac{Y_t S(p_t)}{\eta_t^A (1 - u_t)} \right)^{1+\gamma}.
\]

With flexible prices we have \( p = 1 \) and \( S = 1 \). The flexible-price, socially-efficient level of output, \( Y^* \), satisfies

\[
\eta_t^A Y_t^* - J_t - G_t = \left( \frac{Y_t^*}{\eta_t^A (1 - u_t)} \right)^\gamma.
\]

Treating \( W_t \) as a function of \( Y_t \) and \( p_t \), we take a second-order Taylor approximation around the point \((Y_t^*, 1)\). The first-derivatives are

\[
W_Y(Y_t^*, 1) = \frac{1}{Y_t - J_t - G_t} - \left( \frac{Y_t S(p_t)}{\eta_t^A (1 - u_t)} \right)^\gamma \frac{S(p_t)}{\eta_t^A},
\]

\[
W_p(Y_t^*, 1) = -(1 - u_t) \left( \frac{Y_t S(p_t)}{\eta_t^A (1 - u_t)} \right)^\gamma \frac{Y_t}{\eta_t^A (1 - u_t)} S'(p_t)
\]

and both are zero, the former because \( Y^* \) is optimal and the latter because \( S'(1) = 0 \). The second derivatives are

\[
W_{YY}(Y_t^*, 1) = -\frac{1}{(C^*)^2} - \gamma (Y_t^*)^{\gamma - 1} \left( \frac{S(p_t)}{\eta_t^A} \right)^{1+\gamma} \left( \frac{1}{1 - u_t} \right)^\gamma,
\]

\[
W_{YP}(Y_t^*, 1) = -\left( \frac{Y_t^*}{1 - u_t} \right)^\gamma \left( \frac{1}{\eta_t^A} \right)^{1+\gamma} (1 + \gamma) S(p_t)^\gamma S'(p_t),
\]

\[
W_{PP}(Y_t^*, 1) = -(1 - u_t) \left( \frac{Y_t^*}{\eta_t^A (1 - u_t)} \right)^{1+\gamma} \left[ \gamma S(p_t)^{\gamma - 1} S'(p_t) + S(p_t)^\gamma S''(p_t) \right].
\]

Using \( S(1) = 1, S'(1) = 0 \), the expression for \( S''(1) \) above, and the optimality condition for \( Y_t^* \) we
arrive at

\[ W_{YY}(Y_t^*, 1) = -\frac{1}{(C^*)^2} \left[ 1 + \frac{C_t^*}{Y_t^*} \right], \]

\[ W_{Yp}(Y_t^*, 1) = 0, \]

\[ W_{pp}(Y_t^*, 1) = -\frac{Y_t^*}{C_t^*} \frac{1 - \theta}{\theta} \frac{\mu}{\mu - 1}. \]

By Taylor’s theorem we can write

\[ W(Y_t, p_t) \approx \frac{1}{2} W_{YY}(Y_t^*, 1)(Y_t - Y_t^*)^2 + \frac{1}{2} W_{pp}(Y_t^*, 1)(p_t - 1)^2 \]

Observe that \( Y \) and \( p \) are functions of \( x \). So when we differentiate with respect to \( x \) we arrive at

\[ \frac{dW_t}{dx_t} \approx W_{YY}(Y_t^*, 1)(Y_t - Y_t^*) \frac{dY_t}{dx_t} + W_{pp}(Y_t^*, 1)(p_t - 1) \frac{dp_t}{dx_t}. \]

Substituting for \( W_{YY} \) and \( W_{pp} \) and rearranging yields the result.

## E Description of methods for section 6

### E.1 Estimated income process

The material in this appendix describes the estimation of the time-varying skill risk process following McKay (2017). The income process is as follows: \( \alpha_{i,t} \) evolves as in (2). Earnings are given by \( \alpha_{i,t}w_t \) when employed and zero when unemployed. Notice that here we normalize \( h_t = 1 \) and subsume all movements in \( h_t \) into \( w_t \). While this gives a different interpretation to \( w_t \) it does not affect the distribution of earnings growth rates apart from a constant term. The innovation distribution is given by

\[ \epsilon_{i,t+1} \sim F(\epsilon; x_t) = \begin{cases} 
N(\mu_{1,t}, \sigma_1) & \text{with prob. } P_1, \\
N(\mu_{2,t}, \sigma_2) & \text{with prob. } P_2, \\
N(\mu_{3,t}, \sigma_3) & \text{with prob. } P_3 \\
N(\mu_{4,t}, \sigma_4) & \text{with prob. } P_4 
\end{cases} \]
The tails of \( F \) move over time as driven by the latent variable \( x_t \) such that

\[
\begin{align*}
\mu_{1,t} &= \bar{\mu}_t, \\
\mu_{2,t} &= \bar{\mu}_t + \mu_2 - x_t, \\
\mu_{3,t} &= \bar{\mu}_t + \mu_3 - x_t, \\
\mu_{4,t} &= \bar{\mu}_t.
\end{align*}
\]

where \( \bar{\mu}_t \) is a normalization such that \( \mathbb{E}_t[exp(\epsilon_{i,t+1})] = 1 \) in all periods.

The model period is one quarter. The parameters are selected to match the median earnings growth, the dispersion in the right tail (P90 - P50), and the dispersion in the left-tail (P50-P10) for one, three, and five year earnings growth rates computed each year using data from 1978 to 2011. In addition we target the kurtosis of one-year and five year earnings growth rates and the increase in cross-sectional variance over the life-cycle. The moments are computed from the Social Security Administration earnings data as reported by Guvenen et al. (2014) and Guvenen et al. (2015). Our objective function is a weighted sum of the squared difference between the model-implied and empirical moments.

The estimation procedure simulates quarterly data using the observed job-finding and -separation rates and then aggregates to annual income and computes these moments. To simulate the income process, we require time series for \( x_t \) and \( w_t \). We assume that these series are linearly related to observable labor market indicators (for details see McKay, 2017). Call the weights in these linear relationships \( \beta \). We then search over the parameters \( P, \mu, \sigma \), and \( \beta \) subject to the restrictions \( P_2 = P_3 \) and \( \sigma_2 = \sigma_3 \).

Guvenen et al. (2014) emphasize the pro-cyclicality in the skewness of earnings growth rates. The estimated income process does an excellent job capturing this as shown in the top panel of figure 6. The estimated \( \beta \) implies a time-series for \( x_t \) which shifts the tails of the earnings distribution and gives rise the pro-cyclical skewness shown in figure 6. We regress this time-series on the unemployment rate and find a coefficient of 16.7.\textsuperscript{33} The fourth component of the mixture distribution occurs with very low probability, and in our baseline specification we set it to zero. This choice is not innocuous, however, because the standard deviation \( \sigma_4 \) is estimated to be very

\textsuperscript{33} We regress this estimated time series \( x_t \) on the unemployment rate, which we smooth with an HP filter with smoothing parameter 100,000. If we call this regression function \( f \), we then proceed with \( F(\epsilon'; f(u)) \).
large and this contributes to the high kurtosis of the earnings growth distribution. In particular, omitting this component leads to a substantially smaller $\tau$ as a result of having less risk in the economy. We prefer to omit this from our baseline calibration because the interpretation of these high-kurtosis terms is unclear and we are not entirely satisfied with modeling them as permanent shocks to skill.

The resulting income process that we use in our computations is as follows: The innovation distribution is given by

$$
\epsilon_{i,t+1} \sim F(\epsilon; x_t) = \begin{cases} 
N(\mu_{1,t}, 0.0403) & \text{with prob. 0.9855}, \\
N(\mu_{2,t}, 0.0966) & \text{with prob. 0.00727}, \\
N(\mu_{3,t}, 0.0966) & \text{with prob. 0.00727}
\end{cases}
$$

with

$$
\mu_{1,t} = \bar{\mu}_t, \\
\mu_{2,t} = \bar{\mu}_t + 0.266 - 16.73(u_t - u^*), \\
\mu_{3,t} = \bar{\mu}_t - 0.184 - 16.73(u_t - u^*),
$$

where $u^*$ is the steady state unemployment rate in our baseline calibration. The bottom panels of figure 6 show the density of $\epsilon$ and how it changes with an increase in the unemployment rate.

### E.2 Global solution method

As a first step, we need to rewrite the Calvo-pricing first-order condition recursively:

$$
\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1 - \theta)^{s-t} \left( \frac{p_s}{p_s} \right)^{\mu/(1-\mu)} Y_{s\mu} \ell_s}{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1 - \theta)^{s-t} \left( \frac{p_s}{p_s} \right)^{1/(1-\mu)} Y_s},
$$

where

$$
\ell_s \equiv h_s w_s + \psi_1 M_s^{\psi_2} - R_{s+1}^{-1} (1 - \theta)(1 - \nu) \mathbb{E}_s \psi_1 M_{s+1}^{\psi_2}
$$

with

$$
\ell_s \equiv h_s w_s + \psi_1 M_s^{\psi_2} - R_{s+1}^{-1} (1 - \theta)(1 - \nu) \mathbb{E}_s \psi_1 M_{s+1}^{\psi_2}
$$
Figure 6: Properties of $F(\epsilon)$. 
is a measure of real marginal cost. Define $p^A_t$ as

$$ p^A_t = \mathbb{E}_t \sum_{s=t}^{\infty} R^{-1}_{t,s} (1-\theta)^{s-t} \left( \frac{p_t}{p_s} \right)^{\mu/(1-\mu)} Y_s \mu \ell_s $$

and $p^B_t$ as

$$ p^B_t = \mathbb{E}_t \sum_{s=t}^{\infty} R^{-1}_{t,s} (1-\theta)^{s-t} \left( \frac{p_t}{p_s} \right)^{1/(1-\mu)} Y_s $$

such that

$$ p^*_t p^A_t = p^A_t p^B_t. $$

$p^A_t$ and $p^B_t$ can be rewritten as

$$ p^A_t = \mu Y_t \ell_t + (1-\theta) \mathbb{E}_t \left[ \left( \frac{I_t}{\pi_{t+1}} \right)^{-1} \pi_{t+1}^{-\mu/((1-\mu) A} p^A_{t+1} \right] $$

$$ p^B_t = Y_t + (1-\theta) \mathbb{E}_t \left[ \left( \frac{I_t}{\pi_{t+1}} \right)^{-1} \pi_{t+1}^{-1/(1-\mu)} p^B_{t+1} \right]. $$

The procedure we use builds on the method proposed by Maliar and Maliar (2015) and their application to solving a New Keynesian model. We first describe how we solve the model for a given grid of aggregate state variables and then describe how we construct the grid.

There are seven state variables that evolve according to

$$ \mathbb{E}_t \left[ \alpha_{t+1}^{1-\tau} \right] = (1-\delta) \mathbb{E}_t \left[ \alpha_{t}^{1-\tau} \right] \mathbb{E}_t \left[ \epsilon_{t,t+1}^{1-\tau} u_t \right] + \delta $$

$$ \mathbb{E}_t \left[ \log \alpha_{t,t+1} \right] = (1-\delta) \left[ \mathbb{E}_t \left[ \log \alpha_{t,t} \right] + \mathbb{E}_t \left[ \log \epsilon_{t} \mid u_t \right] \right] $$

$$ S^A_{t+1} = S_t $$

$$ \text{Lag}_{u_{t+1}} = u_t $$

$$ \log \eta^A_{t+1} = \rho^A \log \eta^A_t + \varepsilon^A_{t+1} $$

$$ \log \eta^G_{t+1} = \rho^G \log \eta^G_t + \varepsilon^G_{t+1} $$

$$ \log \eta^I_{t+1} = \rho^I \log \eta^I_t + \varepsilon^I_{t+1}, $$

where $S^A$ is the level of price dispersion in the previous period and the $\varepsilon$ terms are i.i.d. normal
innovations.

There are six variables that we approximate with complete second-order polynomials in the state: \((1/C_t, p_t^A, p_t^B, J_t, V^n_t)\), where \(V^n_t\) is the value of being employed and \(V_t\) is the value of the social welfare function. We use (14) and (15) to write the Euler equation in terms of \(C_t\) and \(V_t\) pins down \(1/C_t\). \(p_t^A\) and \(p_t^B\) satisfy (56) and (57). \(V^n_t\) satisfies (32). \(V_t\) satisfies

\[
V_t = W_t + \beta \mathbb{E}_t [V_{t+1}].
\]

\(J_t\) satisfies \(J_t = \psi_1 M_t^{\psi_2}(v - u_t)\). Abusing language slightly, we will refer to these variables that we approximate with polynomials as “forward-looking variables.”

The remaining variables in the equilibrium definition can be calculated from the remaining equations and all of which only involve variables dated \(t\). We call these the “static” variables.

To summarize, let \(S_t\) be the state variables, \(X_t\) be the forward-looking variables, and \(Y_t\) be the static variables. The three blocks of equations are

\[
\begin{align*}
S' &= \mathcal{G}^S(S, X, Y, \varepsilon') \\
X &= \mathbb{E} \mathcal{G}^X(S, X, Y, S', X', Y') \\
Y &= \mathcal{G}^Y(S, X)
\end{align*}
\]

where \(\mathcal{G}^S\) are the state-transition equations, \(\mathcal{G}^X\) are the forward-looking equations and \(\mathcal{G}^Y\) are the state equations. Let \(X \approx \mathcal{F}(S, \Omega)\) be the approximated solution for the forward-looking equations for which we use a complete second-order polynomial with coefficients given by \(\Omega\). We then operationalize the equations as follows: given a value for \(S\), we calculate \(X = \mathcal{F}(S, \Omega)\) and \(Y = \mathcal{G}^Y(S, X)\). We then take an expectation over \(\varepsilon'\) using Gaussian quadrature. For each value of \(\varepsilon'\) in the quadrature grid, we compute \(S' = \mathcal{G}^S(S, X, Y, \varepsilon')\), \(X' = \mathcal{F}(S', \Omega)\) and \(Y' = \mathcal{G}^Y(S', X')\). We can now evaluate \(\mathcal{G}^X(S, X, Y, S', X', Y')\) for this value of \(\varepsilon'\) and looping over all the values in the quadrature grid we can compute \(\hat{X} = \mathbb{E} \mathcal{G}^X(S, X, Y, S', X', Y')\). \(\hat{X}\) will differ from the value of \(X\) that was obtained initially from \(\mathcal{F}(S, \Omega)\). We repeat these steps for all the values of \(S\) in our grid for the aggregate state space. We then adjust the coefficients \(\Omega\) part of the way towards those implied by the solutions \(\hat{X}\). We then iterate this procedure to convergence of \(\Omega\).

Evaluating some of the equations of the model involves taking integrals against the distribution...
of idiosyncratic skill risk $\epsilon_{i,t+1} \sim F(\epsilon_{i,t+1}, u_t - u^*)$. We do this using Gaussian quadrature within each of the components of the mixture distribution.

We use a two-step procedure to construct the grid on the aggregate state space. We have seven aggregate states so we choose the grid to lie in the region of the aggregate state space that is visited by simulations of the solution. We create a box of policy parameters $[b_L, b_H] \times [\tau_L, \tau_H]$. We then create a grid of twelve Sobol points on this box and for each pair $(b, \tau)$ we use the procedure of Maliar and Maliar (2015) to construct a grid on the aggregate state space and solve the model. This procedure iterates between solving the model and simulating the solution and constructing a grid in the part of the state space visited by the simulation. This gives us twelve grids, which we then merge and eliminate nearby points using the techniques of Maliar and Maliar (2015). This leaves us with one grid that we use to solve the model when we evaluate policies. Each of the grids that we construct have 100 points.

E.3 The policy trade-offs in the quantitative model

We now explain how the policy trade-offs documented in Proposition 1 and Proposition 3 can be calculated in the richer quantitative model.

The social welfare function is

$$V(\mathbb{E}_i [\alpha_i^{1-\tau}], A, S_{-1}, \eta^I, \eta^G, u_{-1}, \mathbb{E}_i \log (\alpha_i))$$

$$= (1 - \tau)\mathbb{E}_i \log (\alpha_i) - \log (\mathbb{E}_i [\alpha_i^{1-\tau}]) + u \log b - \log(1 - u + ub)$$

$$+ (1 + \chi) \log \left(\frac{A}{S}h(1 - u) - J\right) - (1 - u)\frac{h^{1+\gamma}}{1 + \gamma} - (u_{-1} + v (1 - u_{-1})) \frac{q^{1+\kappa}}{1 + \kappa} - u\xi$$

$$+ \beta \mathbb{E} \left[V(\mathbb{E}_i [\alpha_i^{1-\tau}]', A', S, \eta^I', \eta^G', u, \mathbb{E}_i \log (\alpha_i)')\right].$$
In addition we will use the following equations of the model

\[ u = [u_{-1} + v (1 - u_{-1})] (1 - qx) \]

\[ h = \left[ (1 - \tau)\bar{x}x^\gamma \right]^{1/(1+\gamma)} \]

\[ q^i = x V^n \left( \mathbb{E}i \left[ \alpha_{i}^{1-\tau} \right], A, S_{-1}, \eta', \eta'^G, u_{-1}, \mathbb{E}i \log (\alpha_{i,t}) \right) \]

\[ J = \psi_1 x^{\psi_2} [1 - u - (1 - v)(1 - u_{-1})] \]

\[ V^n (\ldots) = \left[ -\log (b) - \frac{h^{1+\gamma_1}}{1 + \gamma_1} + \xi \right] \]

\[ + \beta (1 - v) \mathbb{E} \left[ \left( 1 - \frac{\kappa}{1 + \kappa} q'x' \right) V^n \left( \mathbb{E}i \left[ \alpha_{i}^{1-\tau} \right]', A', S, \eta'', \eta'^G', u, \mathbb{E}i \log (\alpha_{i,t})' \right) \right] \]

**Insurance term.** Take the derivative of \( V \) with respect to \( b \) taking \( q \) and \( x \) as given

\[ V_{\text{Insur}} = \frac{u}{b} - \frac{u}{1 - u + ub} + \beta \mathbb{E} \left[ V'_{\text{Insur}} \right]. \]

**Incentives term.** Take the derivative of \( V \) with respect to \( q \) and multiply it by \( dq/db \) taking \( x \) as given:

\[ V_{\text{Incen}} = \frac{\partial W}{\partial u} \bigg|_x \left. \frac{\partial u}{\partial q} \bigg|_x \frac{\partial q}{\partial b} \bigg|_x \right. \]

\[ - (u_{-1} + v (1 - u_{-1})) q^i \frac{\partial q}{\partial b} \bigg|_x + \beta \mathbb{E} \left[ V'_{\text{Incen}} \right] + \beta \mathbb{E} \left[ V'_{u_{-1}} \right] \frac{\partial u}{\partial q} \bigg|_x \frac{\partial q}{\partial b} \bigg|_x \]
where

\[
\frac{\partial W}{\partial u} = \left\{ \left( \log b + \frac{1 - b}{1 - u + ub} \right) - \frac{1}{C} \left( 1 + \chi \frac{1}{\gamma} \right) \left( \frac{A}{S} h + \frac{\partial J}{\partial u} \right) + \frac{h^{1+\gamma}}{1 + \gamma} - \xi \right\}
\]

\[
\frac{\partial u}{\partial x} = -x [u_{-1} + v (1 - u_{-1})]
\]

\[
\frac{\partial q}{\partial b} = \frac{1}{\kappa} (x V^n)^{\frac{1}{\kappa} - 1} x \frac{\partial V^n}{\partial b} = \frac{1}{\kappa} V^n \frac{\partial V^n}{\partial b}
\]

\[
\frac{\partial V^n}{\partial b} = -\frac{1}{b} + \beta (1 - v) E \left[ \left( 1 - \frac{\kappa}{1 + \kappa} q' x' \right) \frac{\partial V^n}{\partial b} - \frac{\kappa}{1 + \kappa} x' \frac{\partial V^n}{\partial b} \right]
\]

\[
= -\frac{1}{b} + \beta (1 - v) E \left[ \left( 1 - \frac{\kappa}{1 + \kappa} q' x' \right) \frac{\partial V^n}{\partial b} - \frac{1}{1 + \kappa} x' \frac{\partial V^n}{\partial b} \right]
\]

\[
= -\frac{1}{b} + \beta (1 - v) E \left[ \left( 1 - q' x' \right) \frac{\partial V^n}{\partial b} \right]
\]

**Macro-stabilization term** Take the derivative of \( V \) with respect to \( x \) and multiply by the derivative of \( x \) with respect to \( b \)

\[
V_x = \left[ \left( \log b + \frac{1 - b}{1 - u + ub} \right) + \frac{h^{1+\gamma}}{1 + \gamma} - \xi \right] \frac{\partial u}{\partial x} \frac{dx}{db} - \left( u_{-1} + v (1 - u_{-1}) \right) q \frac{dq}{dx} \frac{dx}{db}
\]

\[
+ \left[ -\frac{1}{C} \left( 1 + \chi \frac{1}{\gamma} \right) \left( \frac{A}{S} h + \frac{\partial J}{\partial u} \right) \right] \frac{\partial u}{\partial x} \frac{dx}{db}
\]

\[
- \frac{1}{C} \left( 1 + \chi \frac{1}{\gamma} \right) \frac{A}{S} (1 - u) \frac{dx}{db}
\]

\[
+ \beta \mathbb{E} \left[ V_{u_{-1}}' \right] \frac{du}{dx} \frac{dx}{db}
\]

\[
+ \left[ \frac{1}{C} \left( 1 + \chi \frac{1}{\gamma} \right) \frac{A}{S} (1 - u) \frac{dx}{db} \right] \frac{h}{1 + \gamma} \frac{dx}{db}
\]

\[
- \frac{1}{C} \left( 1 + \chi \frac{1}{\gamma} \right) \frac{A}{S} h (1 - u) \frac{dx}{db} + \beta \mathbb{E} \left[ V_{S_{-1}}' \right] \frac{dS}{dx} \frac{dx}{db}
\]

\[
+ \beta \mathbb{E} \left[ V_{\xi_i^{\alpha_i^{-1}}} \right] \frac{dx}{db} \mathbb{E} \left[ \alpha_i^{1-\gamma} \right] \frac{dx}{db} + \beta \mathbb{E} \left[ V_{\xi, log(\alpha_i, t)}' \right] \frac{dx}{db} \mathbb{E} \left[ \log (\alpha_i, t) \right] \frac{dx}{db}
\]

The first line is unemployment risk; the second, third, and fourth lines are the Hosios terms as they reflect the gain in resources from reducing unemployment and the loss from tightening the labor market; the fifth line is the labor wedge; the sixth line is price dispersion; the seventh line is skill risk.

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We can compute $\frac{\partial u}{\partial x}$, $\frac{\partial q}{\partial x}$ and $\frac{dS}{dx}$ numerically from a monetary shock. We compute $\frac{dE}{db}$ from the finite difference across $b$.

The summary statistics of skill dispersion evolve according to:

$$E_i \left[ \alpha_{i,t}^{1-\tau} \right] = (1 - \delta) E_i \left[ \alpha_{i,t-1}^{1-\tau} \right] + \delta$$

$$\frac{d}{dx} E_i \left[ \alpha_{i,t}^{1-\tau} \right] = (1 - \delta) \frac{d}{dx} E_i \left[ \alpha_{i,t-1}^{1-\tau} \right] \frac{d}{dx} E_i \left[ \epsilon_{i,t}^{1-\tau} \right]$$

$$E_i \left[ \log (\alpha_{i,t}) \right] = (1 - \delta) E_i \left[ \log (\alpha_{i,t-1}) \right] + (1 - \delta) E_i \left[ \log (\epsilon_{i,t}) \right]$$

$$\frac{d}{dx} E_i \left[ \log (\alpha_{i,t}) \right] = (1 - \delta) \frac{d}{dx} E_i \left[ \log (\epsilon_{i,t}) \right].$$

$V_x$ reflects in part the change in value from how $x$ affects future state variables, which can be calculated from the envelope conditions:

$$V_{u_{-1}} = -(1 - \nu) \frac{q^{1+\kappa}}{1 + \kappa}$$

$$\left\{ \left( \log b + \frac{1 - b}{1 - u + ub} \right) - \frac{1 + \chi A}{C} \frac{h^{1+\gamma}}{S} + \frac{h^{1+\gamma}}{1 + \gamma} - \xi \right\} \frac{du}{du_{-1}} - \frac{1 + \chi}{C} \frac{dJ}{du_{-1}}$$

$$+ \left( \frac{1 + \chi A}{C} - h^{\gamma} \right) (1 - u) \frac{dh}{du_{-1}} - (u_{-1} + \nu(1 - u_{-1})) q^{\kappa} \frac{dq}{du_{-1}}$$

$$\beta E_i \left[ V'_{E_i \left[ \alpha_{i,t}^{1-\tau} \right]} \right] \frac{dE_i \left[ \alpha_{i,t}^{1-\tau} \right]}{du_{-1}} + V'_{S} \frac{dS}{du_{-1}} + V'_{u_{-1}} \frac{du}{du_{-1}}$$

$$V_{E_i \left[ \alpha_{i,t}^{1-\tau} \right]} = -\frac{1}{E_i \left[ \alpha_{i,t}^{1-\tau} \right]}$$

$$+ \left\{ \left( \log b + \frac{1 - b}{1 - u + ub} \right) - \frac{1 + \chi A}{C} \frac{h^{1+\gamma}}{S} + \frac{h^{1+\gamma}}{1 + \gamma} - \xi \right\} \frac{dE_i \left[ \alpha_{i,t}^{1-\tau} \right]}{du_{-1}} - \frac{1 + \chi}{C} \frac{dJ}{dE_i \left[ \alpha_{i,t}^{1-\tau} \right]}$$

$$+ \left( \frac{1 + \chi A}{C} - h^{\gamma} \right) (1 - u) \frac{dE_i \left[ \alpha_{i,t}^{1-\tau} \right]}{du_{-1}} - (u_{-1} + \nu(1 - u_{-1})) q^{\kappa} \frac{dE_i \left[ \alpha_{i,t}^{1-\tau} \right]}{du_{-1}}$$

$$\beta E_i \left[ V'_{E_i \left[ \alpha_{i,t}^{1-\tau} \right]} \right] \frac{dE_i \left[ \alpha_{i,t}^{1-\tau} \right]}{du_{-1}} + V'_{S} \frac{dS}{dE_i \left[ \alpha_{i,t}^{1-\tau} \right]} + V'_{u_{-1}} \frac{du}{dE_i \left[ \alpha_{i,t}^{1-\tau} \right]}.$$
\[ V_{E_i \log(\alpha_{i,t})} = 1 - \tau \]
\[ + \left\{ \left( \log b + \frac{1 - b}{1 - u + ub} \right) - \frac{1 + \chi A}{C} \frac{h}{S} + \frac{h^{1+\gamma}}{1+\gamma} - \xi \right\} \frac{du}{dE_i \log(\alpha_{i,t})} + \frac{1 + \chi}{C} \frac{dJ}{dE_i \log(\alpha_{i,t})} \]
\[ + \left( \frac{1 + \chi A}{C} \frac{1}{S} - h^\gamma \right) (1 - u) \frac{dh}{dE_i \log(\alpha_{i,t})} - (u_{-1} + v(1 - u_{-1})) q^* \frac{dq}{dE_i \log(\alpha_{i,t})} \]
\[ \beta \mathbb{E} \left[ V'_{E_i [\alpha_i^{1-\gamma}]} \frac{dE_i}{dE_i \log(\alpha_{i,t})} + V''_S \frac{dS}{dE_i \log(\alpha_{i,t})} + V'_{u-1} \frac{du}{dE_i \log(\alpha_{i,t})} \right] \]

\[ V_{S-1} = -\frac{1 + \chi Y}{C} \frac{dS}{S_{S-1}} \]
\[ + \left\{ \left( \log b + \frac{1 - b}{1 - u + ub} \right) - \frac{1 + \chi A}{C} \frac{h}{S} + \frac{h^{1+\gamma}}{1+\gamma} - \xi \right\} \frac{du}{dS_{S-1}} - \frac{1 + \chi}{C} \frac{dJ}{dS_{S-1}} \]
\[ + \left( \frac{1 + \chi A}{C} \frac{1}{S} - h^\gamma \right) (1 - u) \frac{dh}{dS_{S-1}} - (u_{-1} + v(1 - u_{-1})) q^* \frac{dq}{dS_{S-1}} \]
\[ \beta \mathbb{E} \left[ V'_{E_i [\alpha_i^{1-\gamma}]} \frac{dE_i}{dS_{S-1}} + V''_S \frac{dS}{dS_{S-1}} + V'_{u-1} \frac{du}{dS_{S-1}} \right] . \]

E.4 Extended model with savings

Define \( V(a, n, \epsilon) \) as the value of being employed \( (n = 1) \) or unemployed \( (n = 0) \) with assets \( a \) and in group \( \epsilon \). We omit aggregate states for simplicity of notation. The value of searching, \( V^s \), satisfies

\[ V^s(a, \epsilon) = \max_q \left\{ MqV(a, 1, \epsilon) + (1 - Mq)V(a, 0, \epsilon) - \frac{q^{1+\kappa}}{1 + \kappa} \right\} . \quad (58) \]

The decision problem of the employed household is

\[ V(a, 1, \epsilon) = \max_{c, h, a'} \left\{ \log(c) - \frac{h^{1+\gamma}}{1+\gamma} + \beta \epsilon (1 - v\epsilon) V(a', 1, \epsilon') + \beta \epsilon v\epsilon V^s(a', \epsilon') \right\} \quad (59) \]

subject to the budget constraint

\[ a' + c = \lambda (wh + d)^{1-\tau} \alpha^{1-\tau}_\epsilon + Ra. \]

The decision problem of the unemployed household is

\[ V(a, 0, \epsilon) = \max_{c, a'} \left\{ \log(c) - \xi + \beta \epsilon V^s(a', \epsilon') \right\} \quad (60) \]
subject to the budget constraint

\[ a' + c = b\lambda (wh(a, \epsilon) + d)^{1-\tau} a^{1-\tau}_c + Ra \]

where \( h(a, \epsilon) \) is the hours the household would have worked had they been employed.

The optimal \( q \) solves

\[ q(a, \epsilon)^\kappa = M (V(a, 1, \epsilon) - V(a, 0, \epsilon)) . \]

And substituting this into the definition of \( V^s_t \)

\[ V^s(a, \epsilon) = V(a, 0, \epsilon) + \frac{\kappa}{1+\kappa} (M [V(a, 1, \epsilon) - V(a, 0, \epsilon)])^{1+1/\kappa} . \]

The Euler equation for a household is

\[ \frac{1}{c} \geq \beta R_t E \left[ \frac{1}{c'} \right] . \]

The labor supply optimality condition is

\[ h^\gamma = \frac{1}{c} \lambda (1-\tau)w(wh + d)^{-\tau} a^{1-\tau}_c . \]

We track the distribution of wealth before the employment and group transitions occur at the start of the period, call the distribution \( \Gamma_t(a, n, \epsilon) \). For convenience, let \( \tilde{\Gamma}_t(a, n, \epsilon) \) be the distribution of wealth after transitions have occurred. The two are related according to

\[ \tilde{\Gamma}_t(a, 1, \epsilon') = \sum_{\epsilon} \Pr(\epsilon' | \epsilon) \{ (1 - v_\epsilon) \Gamma_t(a, 1, \epsilon) + M_t q_t(a, \epsilon) [\Gamma_t(a, 0, \epsilon) + v_\epsilon \Gamma_t(a, 1, \epsilon)] \} . \]

We then have average labor supply among workers of

\[ \mathcal{H}_t = \int h_t(a) a d\tilde{\Gamma}_t(a, 1, \epsilon) / \int d\tilde{\Gamma}_t(a, 1, \epsilon) . \]

and aggregate consumption is given by

\[ \sum_n \int c_t(a, n, \epsilon) d\tilde{\Gamma}_t(a, n, \epsilon) . \]
The government’s receipts are from labor income taxes

\[ \int \alpha \epsilon [w_t h_t(a, \epsilon)] + d_t - \lambda_t (w_t h_t(a, \epsilon) + d_t)^{1-\tau} \alpha^{1-\tau} d \Gamma_t(a, 1, \epsilon) \]

and its outlays are \( G_t \), interest \((R_t - 1)B\) where \( R_t \) is the ex post real return on bonds \( R_t = (1 + i_{t-1})/\pi_t \), and UI payments

\[ b \lambda_t \int \alpha^{1-\tau} (w_t h_t(a, \epsilon) + d_t)^{1-\tau} d \Gamma_t(a, 0, \epsilon). \]

The firm’s problem is the same as in the baseline economy with the exception that we replace \( h_t \) with the skill-weighted average work effort among employed households, denoted \( \mathcal{H}_t \).

An equilibrium of the economy can be calculated from a system equations that is similar to the baseline economy, but with aggregate work effort and consumption replaced by the equations above. As those equations depend on the policy rules and distribution of wealth, we also require equations that dictate how the distribution of wealth evolves, and how the policy rules are determined. The Reiter (2009) method the Euler equation, labor supply condition, and search effort first-order condition, and Bellman equations, at many points for the individual states and interpolates the policy rules between them. The distribution of wealth is approximated as a histogram that evolves according to the idiosyncratic shocks and the policy rules.

To calibrate the model, we think of an age group as approximately 12 years of life and set the probability of stochastically aging to \(1/48\). We use the 2001 SCF and divide the sample into age groups 25 to 36, 37 to 48 and 49 to 60. For each group we compute the median liquid assets, median earnings, and unemployment rate. We then set the values of \( \beta, \alpha, \) and \( \nu \) to target these moments. Liquid assets are defined as the sum of liquid accounts ("liq" in the SCF extracts sums checking, savings, and money market accounts), directly held mutual funds, stocks, and bonds less revolving debt. Following Kaplan et al. (2014), liquid account holdings are scaled by 1.05 to reflect cash holdings.