

# The structure of simple ‘New Economic Geography’ models (or, On identical twins)

Frédéric Robert-Nicoud\*

## Abstract

This paper shows that the mathematical structure of the most widely used New Economic Geography models is identical, irrespective of the underlying agglomeration mechanism assumed (factor migration, input-output linkages, endogenous capital accumulation). This enables us to provide analytical proofs to four important and related results in the field. First, standard models display at most two interior steady states beyond the symmetric one. Second, when interior, asymmetric steady-states exist they are unstable. Third, location displays hysteresis. Finally, with forward looking agents a shock to expectations might trigger an equilibrium switch. This paper also stresses the empirical implications of the most important results derived in this study.

**Keywords:** New Economic Geography, natural state space, number of steady-states, stability, hysteresis, expectations

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## 1. Introduction

The notion of space and its relation to economic activities has been studied as early as in the nineteenth century with the work of von Thünen in 1826 and others. More recent work by Krugman (1991a,b) and others revived the mainstream economists’ interest in the topic. This work relies heavily on the monopolistic competition framework put forth by Dixit and Stiglitz (1977) (DS hereafter). This new avatar of spatial economics became known as the ‘New economic geography’, or NEG for short. Three recent monographs assess the progress made in this new paradigm since the seminal work of Paul Krugman. These, together with the recently born *Journal of Economic Geography*, attest the vivid interest for this topic.<sup>1</sup> More generally, the book by Fujita and Thisse (2002) is probably the most exhaustive source on the topic for it unifies the spatial economics field from von Thünen to Krugman.

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\* Département d’Economie Politique, University of Geneva, Bd. Du Pont d’Arve 40, 1211 Geneva 4, Switzerland; and CEPR.

email <Robert@ecopo.unige.ch>

1 Fujita et al. (1999) deals with several positive issues; Brakman, Garretsen, and van Marrewijk (2001) is a non-technical textbook; and Baldwin, Forslid, Martin, Ottaviano, and Robert-Nicoud (2003a) apply NEG models to various policy issues. Peter Neary (2001), who critically evaluates the progress made in the field in his *JEL* paper, might add: ‘what next? The T-shirt? The Movie?’ A good guess is probably a book devoted to empirical evidence; see Overman et al. (2003) and Head and Mayer (2004) for two recent surveys of empirical work in Geography and Trade.

The main insight of the NEG comes from its formalization of agglomeration mechanisms based on endogenous market size.<sup>2</sup> Various trade models predict that sectors characterized by increasing returns to scale, imperfect competition, and transportation costs will be disproportionately active in locations with good market access (Krugman, 1980). In a simple two-country model, this ‘home market effect’ implies that the country with the larger *demand* for the good produced by such sector will end up *exporting* that good (this result sharply contrasts with the predictions of the Ricardian and Heckscher-Ohlin-Viner models of trade). NEG models add cumulative causation to this effect: hosting a larger share of increasing returns activities increases local demand and profitability. If there is factor mobility of sorts, then more of these factors will in turn locate themselves in the already large market, and the cycle repeats under certain conditions. Endogenous agglomeration results from this mechanism. Therefore, initially symmetric regions might end up hosting very different sectors as increasing returns activities have a tendency to locate in few places (Fujita and Thisse, 2002). As for the transmission mechanism whereby agglomeration occurs, the NEG exploits in a spatial setting the kind of circularity causality recurrent in models of monopolistic competition (Matsuyama, 1995).<sup>3</sup>

The beauty of the neoclassical trade theory stems for a good part on its ability to provide systematic predictions relying on few assumptions and robust to the choice of functional form (e.g., the two-factor, two-sector model of Jones, 1965). By contrast, to this day there is no unified NEG theory and there might not be any for years to come. Instead, various authors have provided a gallery of illuminating examples based on specific functional forms. Until recently, all contributions were exclusively based on the DS model of monopolistic competition.<sup>4</sup> Moreover, the cumulative causation makes these models cumbersome to analyse. As a result, they were initially explored numerically (see Maffezzoli and Trionfetti, 2002, for recent methodological improvements in numerical simulations of Krugman’s model). The problem with this is that, though these numerical examples are insightful, we cannot be sure that they provide an exhaustive description of the model’s characteristics (Baldwin et al., 2003a,b).

Fortunately, many results became available by analytical means over time. The first contribution of this paper is in the continuation of this task. To evaluate this first contribution, it is useful to have an idea of the structure of equilibria of DS-based NEG models. In such models, locations are typically *ex ante* identical (identical preferences, same technology, and same endowment of immobile factors). As a result, there always exists a *symmetric equilibrium* where locations have equal shares of mobile factors and economic activities (in particular the increasing returns sector). However, this equilibrium may or may not be stable. There may also be an equilibrium in which all

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2 Its emphasis on market size makes this literature at the crossroad of the ‘new international trade theory’ (Helpman and Krugman, 1985) and spatial economics.

3 See also Matsuyama and Takahasi (1998) on this issue in a different spatial model. These authors conduct welfare analysis and show how the coordination failures between migration decisions of individuals and entry decisions of firms typically result in inefficiencies at the long run equilibrium location.

4 Ottaviano et al. (2002) propose an alternative framework based on quadratic subutilities rather than CES. This model is much simpler to handle than the models based on DS monopolistic competition and displays pro-competitive effects that are absent in the DS framework. However, its dynamic properties are less rich than in the latter framework. See Ottaviano and Thisse (2004) and Baldwin et al. (2003a) for detailed analysis of the differences and similarities between these two frameworks.

increasing returns production is clustered in a single region. The first analytical result in the NEG literature was to derive the set of parameters (summarized by the ‘sustain point’ in the NEG lexicon) under which this *agglomeration equilibrium* exists (Krugman, 1991b). The second analytical result is due to Puga (1999). Among other things, Diego Puga’s paper shows that it is possible to linearize the model around the symmetric equilibrium and to derive a closed-form solution for the set of parameters under which the equilibrium is (locally) stable, that is, for the so-called ‘break point’. Finally, Baldwin (2001) showed that the informal methods developed in Krugman (1991b) and subsequent papers, as well as the ad hoc law of motions, can be rationalized. In particular, he shows that the conditions under which the equilibria are locally stable are also valid to assess their global stability.

However, two results remain unproven. First, the literature studies only one interior equilibrium (the symmetric one) even though there exist other interior equilibria for some parameter values. Hence, the focus of analysis on a very particular interior equilibrium and the full agglomeration equilibria of the current literature would be incomplete if there existed other stable equilibria. In this paper I prove that there are at most three interior equilibria and that, whenever they exist, the interior asymmetric equilibria are unstable.<sup>5</sup> This establishes that the disdain of the literature for the latter is justified.

Second, several recent important contributions of the literature rely on the possibility of location hysteresis. This is notably the case for some results in the burgeoning literature on tax competition in a NEG framework. This literature questions the received (neoclassical) wisdom in spectacular ways.<sup>6</sup> Location hysteresis implies that there are agglomeration rents that can be taxed without inducing the mobile factor (the tax base) to move out. To understand the nature of these rents, it is important to realize that agglomeration generates inertia: to put it colloquially, people and firms are there because other people and firms are there, too. So people are willing to move out of the agglomeration only if a large chunk of other people are willing to do so as well. Therefore, a large shift in the policy environment is needed to give incentives to a large number of people to move and thus shake inertia (see Baldwin et al., 2003a, for details).

It turns out that these agglomeration rents are bell-shaped in transportation costs, which in turn implies that there might be a ‘race to the top’ (Baldwin and Krugman, 2004) in the taxation of the mobile factor when transportation costs decline (a short-cut for economic integration or ‘globalization’). Most of the models to which the analysis of the current paper applies (listed in Table 1) have a range of parameters where both the symmetric and the agglomeration equilibria are stable. The existence of this range implies that self-fulfilling expectations are a possibility when workers are forward looking. In other words, shocks to expectations can trigger a jump between the symmetric equilibrium and the agglomeration outcome (Baldwin, 2001, Baldwin et al., 2003a). In any case, the issue of the existence of this range pertains to ranking the break and sustain points.<sup>7</sup> In this paper, I prove that the set of parameters such that both kind of equilibria are simultaneously stable is non empty. Following an early manuscript version of this paper, this important result has been incorporated in Neary (2001) and Fujita and Thisse (2002) (here I generalize it to all models of Table 1).

5 This last result is quite specific to the models studied in the current paper. I will get back to this in Section 4.

6 See e.g., Andersson and Forslid (2003), Baldwin et al. (2003a), Kind et al. (2000), and Ludema and Wooton (2000). On the political economics of regional subsidies, see Robert-Nicoud and Sbergami (2004).

7 This is by no means a trivial task because the expression for the sustain point is a non-linear, implicit equation.

**Table 1.** Three classes and 13 years of economic geography models

	Underlying model Krugman (1980)	Flam and Helpman (1987)
Factor migration	<b>CP</b> (core-periphery) Krugman (1991b)	<b>FE</b> (Footloose Entrepreneur) Forslid and Ottaviano (2003)
Input-output linkages (a.k.a. vertical linkages)	<b>CPVL</b> (core-periphery, vertical linkages) Krugman and Venables (1995)	<b>FCVL</b> (footloose capital, vertical linkages) Robert-Nicoud (2002) <b>FEVL</b> (Footloose entrepreneur, vertical linkages) Ottaviano (2002) and Ottaviano and Robert-Nicoud (2003)
Factor accumulation		<b>CC</b> (constructed capital) Baldwin (1999)

The proof of the first result works by showing that the most common variants of the core model in this literature are isomorphic at equilibrium in a certain, economically meaningful, state space. For this reason, I will dub it as the ‘natural’ state space. This allows us to prove results in the most tractable version and to apply those in the most intractable variants. In other words, the method of proof also tells us something deep about NEG models: even though they are a collection of illuminating examples (to paraphrase Jacques Thisse), these examples convey some degree of generality. Before turning to the analysis proper, let me digress a bit on this last point.

### 1.1. On the generality of the properties of the Krugman model

The common point between most NEG models is the combination of Dixit-Stiglitz monopolistic competition with iceberg transportation costs (Samuelson, 1952) and some sort of factor mobility.<sup>8</sup> Turn to Table 1, which classifies some contributions in this paradigm along two dimensions. Start with the vertical differentiation. Models in the left column add endogenous market size to the original trade model due to Krugman (1980). Papers that belong to the second column do likewise using an alternative functional form put forth by Flam and Helpman (1987). The horizontal classification discriminates among models according to the mechanism whereby the market size is endogenous. The seminal paper by Krugman (1991b) combines the former functional form with *migration* of the factor used intensively in the increasing returns sector. The resulting core-periphery (or CP) model is extraordinarily difficult to work with, which partly explains why it took so many steps to get a more or less comprehensive

8 More specifically, preferences take an upper-tier Cobb-Douglas form over an index of a homogeneous good and the increasing returns good and a lower-tier CES form among the increasing returns’ varieties. By contrast, in Ottaviano et al. (2002) preferences are quasi-linear for the upper-tier and quadratic for the lower tier. Pflüger (2003) conveys some cross-breeding: he assumes quasi-linear upper-tier preferences (like Ottaviano et al., 2002) and CES sub-utility (like Krugman, 1991b). Interestingly, asymmetric interior equilibria are stable when they exist in his setting.

list of its properties. Its closest cousin is the ‘footloose entrepreneur’ (or FE) model due to Forslid and Ottaviano (2003) (the terminology is borrowed from Baldwin et al., 2003a). This model is very similar to Paul Krugman’s original version but is much easier to manipulate because its use of the Flam-Helpman functional form allows for closed-form solutions unachievable in the Krugman framework.

In Krugman and Venables (1995), which simplified and popularized the mechanism put forth by Venables (1994, 1996), firms belonging to the increasing returns sector buy each others’ output as intermediate inputs (hence the name *vertical linkages*). Workers, though immobile across regions, move freely across sectors. This way, the market size relevant to the increasing sector is endogenous. Again, analytical results in this framework are rare and very difficult to obtain. For this reason, Ottaviano (2002) and Robert-Nicoud (2002) developed alternative vertical linkage models, each using the Flam-Helpman specification.<sup>9</sup> The latter model combines perfect (physical) capital mobility with vertical linkages. See Appendix 2 for a sketch of this model.

Finally, Baldwin (1999) proposes a model in which endogenous (human) *capital accumulation* makes the market size endogenous. Like all analyst-friendly models, Richard Baldwin’s uses the Flam-Helpman functional form (see also Appendix 2).

It is worth mentioning that Puga (1999) combines factor migration and vertical linkages using Krugman’s (1980) functional form whereas Faini (1984) combines vertical linkages and endogenous factor accumulation, albeit in a different setting.

With the exception of Faini (1984), *all papers mentioned above and enlisted in Table 1 are isomorphic in a certain, economically meaningful, state space*. An analogy with identical twins is helpful to understand the depth of the similarities among these apparently different models. Like identical twins, they share the same genome (or intrinsic properties). But even identical twins look a bit different and have different characters. Also, they are different individuals who might dress up differently.

This is a remarkable result in itself. Indeed, it shows that the stability properties of these models are *identical* despite their using of different functional forms and agglomeration mechanisms. This falls short of having a general theory, but nevertheless suggests that the main results derived by Krugman (1991b) and others have some degree of generality and should be pervasive to changes of functional forms.<sup>10</sup> For lack of space, I will show that this is the case for only three of the models in Table 1: the CP model by Krugman (1991b), the FE model due to Forslid and Ottaviano (2003), and, in Appendix 2, the FCVL model due to Robert-Nicoud (2002). In a related paper, Ottaviano and Robert-Nicoud (2003) convey the same message for better known models based on vertical linkages.<sup>11</sup> Nevertheless, I will mention in passing and in the conclusion how this method applies to such VL models as well as to Baldwin’s (1999) growth model.

The remainder of the paper is organized as follows. The section immediately after this introduces the notation, spells out the canonical CP model, and derives the equilibrium conditions in the new state space. Section 3 then does the same for the FE model and shows that the CP and FE models are isomorphic. Section 4 formally characterizes the set of steady-states and derives their stability properties: In particular, Proposition 3

9 See also Baldwin et al. (2003a, ch.8) and Ottaviano and Robert-Nicoud (2003).

10 See also footnote 4 on this.

11 Ottaviano and Robert-Nicoud (2003) also convey the welfare analysis of VL models borrowing from the methodology developed by Charlot et al. (2003) for the CP model.

establishes that there are at most three interior equilibria. Proposition 5 establishes that these models open the possibility for shocks to expectations to generate a switch from one equilibrium to another. Finally, Section 5 concludes in briefly sketching how other standard NEG models also fit into the natural state space and equations, and hence how the proof developed in this paper easily extends to VL and Baldwin's (1999) models. This final section also addresses the empirical implications of the main results derived in the paper.

Since the paper and the method of proof are slightly long, it might be useful to have a road map to which the reader can refer when she feels lost. In Figure 1 I have sketched the various steps that are necessary to derive the various Lemmas and Propositions of the paper.

## 2. The 'natural' state space for migration-based models

This section introduces the notation and establishes the state space in which the migration-based CP and FE models are isomorphic. This is the first necessary intermediate step on our way to establish the main results of Section 4.

### 2.1. The common structure

Common to virtually all NEG models are two regions (indexed by  $j = 1, 2$ ) and two sectors. The background sector  $A$  (agriculture) produces a homogenous good under constant returns to scale (or CRS) in a perfectly competitive environment using unskilled workers (whose wages are denoted by  $w_U$ ) only; its output is freely traded; consumers spend a share  $1 - \mu$  of their expenditure on  $A$ . The 'manufacturing' sector  $M$  produces a differentiated product under increasing returns to scale (or IRS) in a monopolistically competitive environment a-la Dixit and Stiglitz (1977) using skilled workers only (whose wages are denoted by  $w$ ). Denote the elasticity of substitution between any two varieties by  $\sigma > 1$ . Shipping this good into the other region involves 'iceberg' transportation costs:  $T > 1$  units need to be shipped so that 1 units arrives at destination; the rest,  $T - 1$ , melts in transit. Consumers spend a share of their expenditure  $0 < \mu < 1$  on the composite  $M$ .

Specifically, tastes are described by the following function:

$$U = C_A^{1-\mu} C_M^\mu, \quad C_M = \left( \int_{i=0}^{n^w} c(i)^{1-1/\sigma} di \right)^{\frac{1}{1-1/\sigma}}, \quad 0 < \mu < 1 < \sigma \quad (1)$$

where  $C_A$  is the consumption of good  $A$  and  $C_M$  is the (index) consumption of  $M$ . This good comes in  $n^w$  different varieties indexed by  $i$  ( $n^w$  will be determined at equilibrium);  $c(i)$  is the quantity consumed of variety  $i$ . Furthermore, in models based on factor migration under consideration here, each region is endowed with  $L_U/2$  unskilled workers. Both regions also share  $L$  skilled workers, with  $\lambda$  (respectively  $1 - \lambda$ ) of them living in region 1 (resp. 2);  $\lambda$  is the endogenous variable of interest.

In short, we have:

$\mu$ —the expenditure share on  $M$  goods

$n^w$ —the mass (or the number) of available  $M$ -varieties

$\sigma$ —the elasticity of substitution among the  $n^w$  varieties

$L_U/2$ —the mass of unskilled worker in region  $j \in \{1, 2\}$

$\lambda L$ —the mass of skilled workers in region 1.

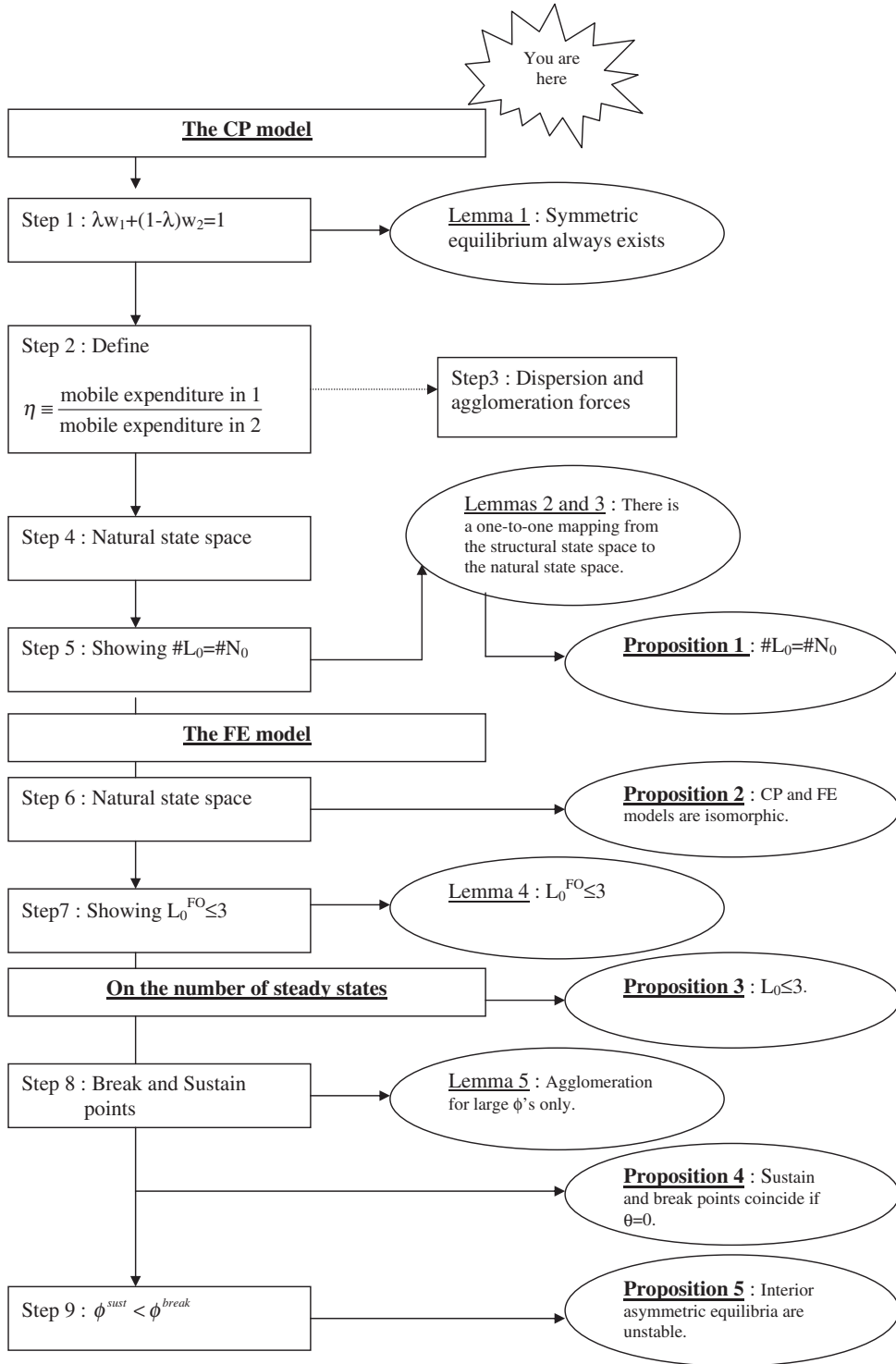


Figure 1. The road map.

## 2.2. The CP model

In Krugman's (1991b) CP model, each factor is specific to a different sector: the background (resp. manufacturing) sector uses unskilled (skilled) workers only. In particular, the cost function of the typical  $M$ -firm takes the following form:

$$C(x(i)) = w[F + \rho x(i)] \quad (2)$$

where  $x(i)$  is the firm output,  $F$  is the fixed labour requirement,  $\rho$  is the variable labour requirement, and  $w$  is the (skilled) labour wage; for clarifying purposes,  $w$  will be indexed by a subscript 1 or 2 so as to distinguish between regions when necessary.

## 2.3. Normalizations

Following Fujita et al. (1999), let us make the following normalizations.<sup>12</sup> First, we take  $A$  as the numéraire. Second, we can choose units in which manufacturing output and labour are measured. As stressed by Puga (1999), these involve choosing a value for  $\rho$  and  $F$ , respectively:

The variable labour requirement in (2):  $\rho = 1 - 1/\sigma$

The fixed labour requirement in (2):  $F = \mu/\sigma$

The fact that we choose a value for both the marginal labour requirement  $\rho$  and the fixed labour requirement  $F$  might worry some readers. These normalizations are perfectly valid, though, because the cost function (2) is homothetic. This assumption, together with free-entry, ensures that total input requirement is independent of marginal input requirement; the variable that adjusts is the total number of firms.

By the same token, we can choose units in which output in sector  $A$  is measured:

Sector  $A$  labour-output requirement is 1

The latter normalization implies  $w_U = 1$  in each region, whereas the normalization on the variable labour requirement in the  $M$  sector simplifies the equilibrium pricing expressions. The final choices of units we make concern the exogenous supply of primary factors. Because these are continuous variables, we can measure them in anyway we want. Following Krugman (1991a,b) we choose  $L$  and  $L_U$  so that nominal wages are equalized at the symmetric equilibrium. Assuming that  $\lambda = 1/2$  implies  $w = w_U = 1$  requires:

Labour endowments:  $L_U/L = (1 - \mu)/\mu$

It turns out that this normalization also implies that nominal wages are equalized throughout in the agglomerated equilibria, a priori when  $\lambda = 0, 1$ . Going slightly beyond the model, one might justify this assumption by saying that, deep down, there should be no persistent difference in the rewards workers might earn, regardless of the sector in which they work.

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12 Some of these normalizations might look a bit weird to the reader unfamiliar with the NEG (see Neary, 2001). However, since we are assuming a continuum of varieties we are given an extra degree of freedom that can be absorbed in an extra normalization. In other words, the normalizations below entail no loss of generality. See Baldwin et al. (2003a, box 2.2) on this point.

## 2.4. Instantaneous equilibrium in the CP model

With all these assumptions at hand, we can solve the model treating  $\lambda$  as a parameter to get the so-called ‘instantaneous’ (or ‘short run’) equilibrium. Such an equilibrium is defined for a given value of  $\lambda$  as a situation where all markets clear and trade is balanced. Let us start with the demand side. Maximization of (1) under the constraint that expenditure must not exceed disposable income  $Y$  yields:

$$C_{Aj} = (1 - \mu)Y_j; \quad c_j(i) = \frac{p_j(i)^{-\sigma}}{G_j} \mu Y_j, \quad G_j \equiv \left( \int_{i=0}^{n^w} p_j(i)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \quad (3)$$

where  $p_j(i)$  is the consumer price of variety  $i$  in region  $j$  and  $G_j$  is the true manufacturing price index. By (2) and our normalization for  $\rho$ , it is readily established that operating profits are equal to:

$$\pi_j(i) = \frac{p_j(i)c_j(i)}{\sigma} + T \frac{p_k(i)c_k(i)}{\sigma}, \quad k \neq j \quad (4)$$

Note that the iceberg formulation for transportation costs implies that the firm does actually ship  $Tc_k(i)$  units of  $i$  so that  $c(i)$  units arrive in the region  $k \neq j$ .

We now turn to the supply side. First, profit maximization of (2)–(4) gives the following optimal consumer pricing policy for the typical variety  $i$  produced in region  $j$ :

$$p_j(i) = \frac{\rho w_j}{1 - 1/\sigma} = w_j; \quad p_k(i) = \frac{\rho w_j}{1 - 1/\sigma} T = T w_j, \quad k \neq j \quad (5)$$

where the second equality in both expressions above stems from our choice of units and normalizations. Note that (5) implies markup pricing (that is, each manufacturer is a monopolist in his own variety). This markup is constant (a function of taste parameters only) and transportation costs are fully passed onto consumers (f.o.b. pricing is typical of DS monopolistic competition).

Plugging (5) into (4), we obtain:

$$\pi_j(i) = p_j(i)^{1-\sigma} \frac{\mu Y_j}{\sigma G_j^{1-\sigma}} + (p_j(i)T)^{1-\sigma} \frac{\mu Y_k}{\sigma G_k^{1-\sigma}}, \quad k \neq j \quad (6)$$

A region’s total disposable income is defined as the labour income of its various inhabitants. With a proportion  $\lambda$  of skilled labour living in region 1, this yields (remember the normalizations):

$$Y_1 = \frac{1 - \mu}{2} + \lambda \mu, \quad Y_2 = \frac{1 - \mu}{2} + (1 - \lambda) \mu \quad (7)$$

Next, we assume that there is free entry and exit in the  $M$  sector. As a result, operating profits must be equal to fixed costs at equilibrium for any active firm. Dropping the variety index for now, this reads:

$$\pi_j \equiv x_j [p_j - \rho w_j] = w_j F \Leftrightarrow x_j = x \equiv \frac{F(\sigma - 1)}{\rho} = \mu \quad (8)$$

that is, the free-entry condition pins down the equilibrium firm size, given the pricing policy (5). Note that this firm size is a function of parameters only (in particular, it is invariant in wages). Also, the last equality in the expression above stems from the normalizations.

Third, we invoke the full employment condition of skilled labour to pin down the equilibrium number (mass) of available varieties.<sup>13</sup> Labour supply is exogenously given by  $L = \mu$ . Labour demand is equal to:

$$n^w(F + \rho x) = n^w \left( \frac{\mu}{\sigma} + \rho \mu \right) = n^w \mu \quad (9)$$

where I have successively made use of (8) and of the normalizations. As a result, full employment of labour implies:

$$n^w = 1 \quad (10)$$

This completes the analysis of the ‘instantaneous’ equilibrium of the model. However, it is impossible to get a closed-form solution for most endogenous variables of this model. Instead, the short-run equilibrium is typically depicted by the following system of equation that summarizes the results derived thus far (this system is identical to Fujita et al., 1999, p.65):

$$\begin{aligned} Y_1 &= \mu \lambda w_1 + \frac{1 - \mu}{2}, & Y_2 &= \mu(1 - \lambda)w_2 + \frac{1 - \mu}{2} \\ G_1^{1-\sigma} &= \lambda w_1^{1-\sigma} + (1 - \lambda)(T w_2)^{1-\sigma}, & G_2^{1-\sigma} &= \lambda(T w_1)^{1-\sigma} + (1 - \lambda)w_2^{1-\sigma} \\ w_1^\sigma &= \frac{Y_1}{G_1^{1-\sigma}} + T^{1-\sigma} \frac{Y_2}{G_2^{1-\sigma}}, & w_2^\sigma &= T^{1-\sigma} \frac{Y_1}{G_1^{1-\sigma}} + \frac{Y_2}{G_2^{1-\sigma}} \end{aligned} \quad (11)$$

The first line is a simple restatement of (7): income in region  $j$ ,  $Y_j$ , is defined as the sum of skilled workers’ income (whose wage is  $w_j$ ) and unskilled workers’; income (whose wage is unity by choice of numéraire). To get the expression for the price indices in the second line, plug (5) and (10) into the definition of  $G_j$  in (3). Finally, the expressions in the third line combine (6) and (8). These are the free-entry-and-exit conditions (also called the ‘wage equations’ in the jargon): firms in region  $j$  break even if they pay the wage  $w_j$ .

## 2.5. Long run equilibrium in the CP model

In any instantaneous equilibrium, the expressions in (11) must hold simultaneously. A long run equilibrium (or steady state) is defined as an instantaneous equilibrium in which, in addition, the mobile factor has no incentive to migrate. Define the real (skilled labor) wage in region  $j$  as  $\omega_j$ , viz:

$$\omega_j \equiv w_j G_j^{-\mu} \quad (12)$$

Hence, the system in (11) can be viewed as providing an implicit solution for  $\omega_1 - \omega_2$  as a function of  $\lambda$  (and the parameters); accordingly, we write (11) as:

$$F(\omega_1 - \omega_2, \lambda) = 0 \quad (13)$$

for short. Skilled labor is assumed to move from one region to the other so as to eliminate current differences in real wages according to the following ad hoc law of motion:

$$\dot{\lambda} = \gamma \lambda (1 - \lambda) (\omega_1 - \omega_2) \quad (14)$$

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13 Since there are no economies of scope, we can identify each variety with one firm.

where  $\gamma > 0$  is a parameter. Clearly, this means that expectations are static.<sup>14</sup> A long run (or steady-state) equilibrium is defined as a combination of endogenous variables (that now include  $\lambda$ ) that solves (11) and such that (14) holds for  $\dot{\lambda} = 0$ . Two kinds of interior steady states might occur. First are the corner solutions  $\lambda \in \{0, 1\}$ , namely, all mobile workers are agglomerated in one region. In the jargon, such steady states are referred to as ‘core-periphery’ outcomes.<sup>15</sup> Second, there are the interior solutions whereby  $\lambda \in (0, 1)$  and  $\omega_1 = \omega_2$ . Given the symmetry of the model,  $\lambda = 1/2$  is always part of an interior steady state (see Lemma 1 in the immediate sequel). This particular case is usually referred to as the ‘symmetric equilibrium’. There might also exist other interior equilibria. Typically, these equilibria are disregarded in the literature.

The issue of interest for the remainder of the section is the characterization of the number of *interior* solutions, i.e. the number of  $\lambda$ 's such that  $\omega_1 = \omega_2$ . Call the set of these  $\lambda$ 's  $L_0$ , with  $\lambda_0$  being the typical element of  $L_0$ . That is:

$$L_0 \equiv \{\lambda \in [0, 1] : F(0, \lambda) = 0\} \tag{15}$$

Using (11), it is straightforward to show that the symmetric equilibrium always exists (whereby an even repartition of skilled labour between the two regions entails identical real wages) and that, whenever there exists an equilibrium in which real wages are equalized for an uneven repartition of skilled workers, then the opposite, symmetric repartition must also be part of a long run equilibrium (this is obvious because it only involves a re-labelling of the two regions). In mathematical symbols, we write these properties as the following lemma.

*Lemma 1:* First,  $\lambda = 1/2 \in L_0$  is always true. Also,  $\lambda_0 \in L_0$  if, and only if,  $1 - \lambda_0 \in L_0$

*Proof:* Immediate by virtue of the symmetry of the model. QED.

So far all this is very well known, and we cannot really go further than this. Hence, the next task is to rewrite (11) in an alternative state space that will prove very useful to move on.

## 2.6. The CP model in the ‘natural’ state space

**Step 1:** First, I claim that aggregate (nominal) expenditure of mobile (that is, skilled) workers is constant for all  $\lambda$ :

$$\lambda w_1 + (1 - \lambda)w_2 = 1 \tag{16}$$

To see this, multiply both sides of the wage equations in (11) by  $w_j^{1-\sigma}$  and the result follows readily by using the income and price index equations. This property stems from the Cobb-Douglas functional form of the upper-tier utility function.

**Step 2:** Next, define the parameter  $\phi$  as:

$$\phi \equiv T^{1-\sigma} \in [0, 1) \tag{17}$$

14 Baldwin (2001), however, shows that this is merely an assumption of convenience. Indeed, allowing for rational expectations and sufficiently large quadratic migration costs does not alter the ‘break’ and ‘sustain’ points defined below nor the local stability properties of the long run equilibria. Yet, expectations can be self-fulfilling when migration costs are low enough, as in Matsuyama (1991) or Krugman (1991c).

15 For instance, take  $\lambda = 1$ . In this case, all skilled workers and manufacturing firms are clustered in region 1—the core—and region 2 is void of manufacturing and, as such, is dubbed as the periphery.

or the ‘free-ness’ of trade ( $\phi$  is decreasing in  $T$ ).<sup>16</sup> We now move on by using the definition of  $\omega_j$  ( $j = 1, 2$ ) and plugging it into (11):

$$\begin{aligned} Y_1 &= \mu\lambda w_1 + \frac{1-\mu}{2}, & Y_2 &= \mu(1-\lambda)w_2 + \frac{1-\mu}{2} \\ G_1^{1-\sigma} &= \lambda w_1 (\omega_1 G_1^\mu)^{-\sigma} + \phi(1-\lambda)w_2 (\omega_2 G_2^\mu)^{-\sigma}, \\ G_2^{1-\sigma} &= \phi\lambda w_1 (\omega_1 G_1^\mu)^{-\sigma} + (1-\lambda)w_2 (\omega_2 G_2^\mu)^{-\sigma} \\ (\omega_1 G_1^\mu)^\sigma &= \frac{Y_1}{G_1^{1-\sigma}} + \phi \frac{Y_2}{G_2^{1-\sigma}}, & (\omega_2 G_2^\mu)^\sigma &= \phi \frac{Y_1}{G_1^{1-\sigma}} + \frac{Y_2}{G_2^{1-\sigma}} \end{aligned} \quad (18)$$

As is obvious from the expression above, operating profits are an increasing function of income  $Y_j$ . Also, the relative profitability of firms in each region depends, among other things, on the relative income (expenditure) in each region. Moreover, this relative income is endogenous in the model: it varies with the relative mobile expenditure. Indeed, when skilled workers migrate they earn a different income from before and this, as much as the fact that they migrate, changes market size. As we will see, this mobile expenditure will play a crucial role in our task of rewriting the model in an alternative state space. Accordingly, define the variable  $\eta$  as the ratio of the mobile factor’s expenditure in 1 to mobile expenditure in 2 (which is thus a measure of the relative market size of region 1), namely:

$$\eta \equiv \frac{\lambda w_1}{(1-\lambda)w_2} \quad (19)$$

By virtue of (16), there is a one-to-one relationship between  $\eta$  on the one hand and  $\lambda w_1$  and  $(1-\lambda)w_2$  on the other. As we shall see,  $\eta$  plays a fundamental role in the analysis.

**Step 3:** The third step involves the parameterization of some crucial forces at work.

## 2.7. Agglomeration and dispersion forces

A digression on the agglomeration and dispersion forces is useful at this stage because it boosts economic intuition. This discussion also makes sense of the definition of some collections of parameters that are recurrent in the analysis to come.

### 2.7.1. Agglomeration force #1: Backward linkage

The discussion in the paragraph above points to the first agglomeration force present in the model, known as the *backward linkage* (think about it as a demand linkage): when a firm moves from, say 2, to region 1 the labour force increases in the destination region and shrinks in the source region. As a result, the market size increases in the former and decreases in the latter, thus, all things equal, making firms relatively more profitable in region 1 (and higher profits translate into higher nominal wages given that firms break even by assumption), so the cycle repeats. Of course, this agglomeration force is stronger

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16 When  $\phi = 0$  transportation costs are infinite and trade free-ness is nil. When  $\phi = 1$  transportation costs are absent ( $T = 1$ ) and trade free-ness is total.

if skilled workers are numerous ( $L$  is large). To parameterize this agglomeration force, define:

$$\chi \equiv \frac{L_U/2}{L + L_U/2} < 1 \quad (20)$$

that is, assuming all mobile workers are agglomerated in a single region ( $\lambda \in \{0, 1\}$ ),  $\chi$  is defined as the GDP of the region empty of manufacturing workers (the ‘periphery’) over the GDP of the region in which all skilled workers are agglomerated (the ‘core’). Hence,  $\chi$  is inversely related to the *backward linkage*. If  $\chi = 1$ , no expenditure is mobile. If  $\chi = 0$ , all expenditure is mobile; in such a case agglomeration would always occur. To make the problem interesting, this possibility is always ruled out by imposing a ‘lumpy-feet’ assumption  $\chi > 0$ .<sup>17</sup> In the current model, given our normalizations  $\chi$  is equal to  $(1 - \mu)/(1 + \mu)$ .

### 2.7.2. Agglomeration force #2: forward linkage

The second agglomeration force is the so-called *forward linkage* (best thought of as a supply linkage and also known as the price-index effect). When a firm moves from region 2 to region 1 residents of the latter face a decreasing price index because they now save on transportation costs (one more variety is being produced in their domestic market). All things equal, a lower cost of living in 1 induces more workers (and hence more firms) to follow, so, again, the cycle repeats. This Price-index effect is larger, the larger the share of expenditure that is spent on the manufacturing good (a priori the larger is  $\mu$ ) and the lower the elasticity of substitution among varieties (a priori the lower is  $\sigma$ ). Accordingly, define  $\theta$  as the following collection of parameters:

$$\theta \equiv \frac{\mu}{\rho} = \frac{\mu\sigma}{\sigma - 1} \quad (21)$$

A large value for  $\theta$  is associated with strong forward linkages.<sup>18</sup> Following Fujita et al. (1999, p.59), I impose the ‘no-black-hole’ condition  $\theta < 1$  (otherwise agglomeration occurs for all parameter values and the problem is uninteresting).<sup>19</sup> The reason for which  $\theta$  is precisely defined that way will become clear in the sequel.

### 2.7.3. Dispersion force

Against the forward and backward linkages acts a *dispersion force*, the so-called *market crowding effect* (Baldwin et al. 2003a). When a firm moves from 2 to 1 it enjoys a larger market share in the destination region and a lower market share in the region it is leaving than before the move. Also, it makes market 1 more crowded and market 2 less so. Other things equal, this gives other firms the incentive to move in the exact opposite direction. Hence, the market crowding effect acts as a stabilizing force. Mathematically, the

17 More generally, when  $\chi$  is close to zero, the difference in market sizes between the core and the periphery is huge. This acts as a strong agglomeration force because firms prefer to locate in the larger market (*ceteris paribus*) so as to save on transportation costs.

18 Interestingly, there are no forward linkages (i.e.  $\theta = 0$ ) in Baldwin (1999). I will come back to this in section 4.

19 When either the ‘lumpy-feet’ or the ‘no-black-hole’ condition fails, the symmetric equilibrium is never stable. If the no-black-hole condition is violated, then both the break and sustain points defined below are equal to zero.

domestic and export market share of a typical firm established in 1 are given by (remember that  $n^w = 1$  at equilibrium):

$$s_1^1 = \frac{w_1^{1-\sigma}}{G_1^{1-\sigma}}, \quad s_1^2 = \phi \frac{w_1^{1-\sigma}}{G_2^{1-\sigma}} \quad (22)$$

where I am following the notational convention  $s_{\text{from}}^{\text{to}}$ ; foreign market shares are defined symmetrically. It can be shown that  $s_1^1$  increases relative to  $s_1^2$  when  $\lambda$  increases, hence substantiating the claim above. By definition of  $G_j$  in (11), it is clear that  $\lambda s_1^1 + (1-\lambda) s_2^1 \equiv 1$ .

**Step 4:** Using (19)–(21), we can transform (18) further to simplify both the resolution of the problem and the algebra. To this aim, yet another definition is useful: transform the price index in a way that is analogous to the mapping from  $T$  to  $\phi$  in (17), namely:

$$\Delta_j \equiv G_j^{1-\sigma}, \quad \frac{\partial \Delta_j}{\partial G_j} < 0 \quad (23)$$

Crucially, the entire problem can be rewritten in terms of ratios. Indeed, in this two-region world, only relative sizes matter. The main reason for this being that preferences and marginal costs are homothetic. This means that, say,  $Y_1$  conveys little information about the market size of region 1 in itself; however, the variable  $Y_1/Y_2$  is more informative in that it compares  $Y_1$  against a natural benchmark: the nominal income in the other region. Accordingly, define  $\omega$ ,  $Y$ , and  $\Delta$  as:

$$\omega \equiv \frac{\omega_1}{\omega_2}, \quad Y \equiv \frac{Y_1}{Y_2}, \quad \Delta \equiv \frac{\Delta_1}{\Delta_2} \quad (24)$$

and  $q$  as:

$$q \equiv \omega^\sigma \quad (25)$$

As we shall see shortly,  $q$  is the ‘natural’ benchmark to gauge whether migration is profitable. Given the law of motion (14), the model reaches an interior steady state whenever  $q = 1$ .

There we are: we now have everything at hand to write the instantaneous equilibrium of the CP model in its ‘natural’ state-cum-parameter space. This space is natural in that it points to the forces that are at work in the model. It also stresses the combinations in which the structural parameters actually matter in the determination of the endogenous variables, and in which way the latter are actually affected.

Using (24) and substituting for the definitions of the variables  $\eta$  (13),  $\omega_j$  (12), and  $\Delta_j$  (23) and of the parameters  $\phi$  (17),  $\chi$  (20), and  $\theta$  (21) into (18), we obtain the following system:

$$Y = \frac{\eta + \chi}{1 + \eta\chi}; \quad \eta\Delta^\theta = q \frac{\Delta - \phi}{1 - \Delta\phi}; \quad q = \Delta^\theta \frac{Y + \Delta\phi}{Y\phi + \Delta} \quad (26)$$

The model in (26) is identical to the model in (11), as we will show shortly. To recall, the endogenous variables  $Y$ ,  $\Delta$ ,  $q$  are functions of the ‘natural’ parameters  $\phi$ ,  $\chi$ ,  $\theta$  and of the state variable  $\eta$ . In particular,  $\theta$  and  $\chi$  are directly related to the no-black-hole and lumpy-feet conditions.

All three expressions in (26) have an intuitive economic interpretation (here I basically emphasize how to illustrate the agglomeration forces using the natural state space).

First, incomes are naturally increasing in the local share of skilled workers' expenditure, i.e. the mobile expenditure. From the first expression, we have  $\partial Y/\partial \eta > 0$ .

Next, together with the expression for  $Y$ , the last expression illustrates the *backward linkage* whereby increasing the mass of firms (and hence of skilled workers) in one region also increases the market size for the good they are producing. Mathematically, it is straightforward to see that  $\partial q/\partial Y > 0$ . Of course, the absence of such linkages (i.e.  $\chi = 1$ ) implies  $Y = 1$  for all  $\eta$ , and hence  $\partial q/\partial \eta = 0$ .

Finally, the middle expression provides an implicit definition for  $\Delta$  and illustrate the *forward linkage* whereby increasing the mass of skilled workers (and hence of firms) in one region also decreases the (relative) price index in that region because less varieties have to be imported. To clarify the illustration of this point, assume real wages are equalized, so that  $q = 1$ . It can be shown fairly easily that  $0 \leq \theta < 1$  implies  $\partial \Delta/\partial \eta > 0$ , the price index in 1 falls (relatively to the price index in 2) with the share of mobile expenditure in 1.<sup>20</sup>

By analogy with (13), we can rewrite (26) in compact form as:

$$f(q, \eta) = 0 \tag{27}$$

With this notation, an interior steady-state is defined as a value for  $\eta$  such that  $f(1, \eta) = 0$ . By analogy with (14), define  $N_0$  as:

$$N_0 \equiv \{\eta \geq 0 : f(1, \eta) = 0\} \tag{28}$$

We denote the typical element of  $N_0$  by  $\eta_0$ . It is readily seen using (26) that  $\eta = 1$  is always an element of  $N_0$ , i.e.  $\eta = 1$  is consistent with  $q = 1$  (this is a corollary of Lemma 1).

**Step 5:** We can infer the exact number of interior-steady states  $\#L_0$  of the problem (14) from  $\#N_0$  if the two are related, e.g., if  $\#N_0 = \#L_0$ . Proposition 1 below shows that there indeed exists a one-to-one mapping from  $N_0$  to  $L_0$ . We show this by a succession of Lemmas. The first lemma in the immediate sequel says that the mapping from the original (or structural) state variable  $\lambda$  to the natural state variable  $\eta$  is a surjection, namely, to all positive  $\eta$  corresponds at least one  $\lambda$  in  $[0, 1]$ . I have relegated the proofs of intermediate results to Appendix 1.

*Lemma 2:* Define the function  $\mathbf{M}: [0, 1] \leftrightarrow [0, +\infty)$  where  $\eta = \mathbf{M}(\lambda)$ . Then  $\mathbf{M}$  is a surjection (i.e. onto). Moreover,  $\mathbf{M}'(\cdot) > 0$  at the symmetric steady state  $\lambda = 1/2$ .

*Proof:* See Appendix 1.

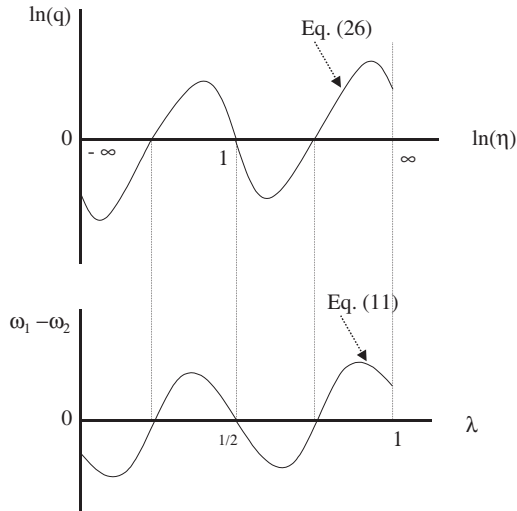
The next lemma says that expenditure of skilled workers in, say, region 1, increases with the share of such workers there, even taking the (potentially) depressing effect of  $\lambda$  on  $w_1$ . In other words, the elasticity term  $(\partial w_1/\partial \lambda)/(w_1/\lambda)$  is larger than  $-1$ , and possibly positive.<sup>21</sup>

*Lemma 3:* (a)  $\mathbf{M}$  is a bijection; (b) In particular,  $\eta$  is increasing in  $\lambda$ .

*Proof:* See Appendix 1.

20 To see this, note first that the price-index equations in (11) imply  $\Delta_1 - \phi \Delta_2 = (1 - \phi^2)\lambda w_1^{1-\sigma} > 0$ . By the same token,  $\Delta_2 - \phi_1 \Delta > 0$ , hence  $\phi \leq \Delta \leq 1/\phi$ . Now, total differentiation of the second expression in (26) yields  $d \ln \eta = d \ln q + [-\theta + 1/(\Delta - \phi) + \phi/(1 - \Delta\phi)]$ . Finally, note that  $d \ln q = 0$  by assumption and that the term in the square bracket is positive by the results derived in the first step.

21 This latter claim can be made rigorously fairly easily. Since this is nowhere needed for our purposes, however, I omit the proof.



**Figure 2.** Instantaneous equilibrium

Turn to Figure 2 (instantaneous equilibrium). This figure plots  $\ln(q)$  as a function of  $\ln(\eta)$  in the upper panel, namely eq. (26), and  $\omega_1 - \omega_2$  as a function of  $\lambda$  in the lower panel, namely eq. (11).<sup>22</sup> Lemma 3 above says that to each  $\lambda$  on the horizontal axis in the lower panel corresponds exactly one  $\eta$  on the horizontal axis of the upper panel. The figure also prefigures the result of Proposition 1. This proposition is essentially a corollary of Lemma 3 in that it shows that the number of solutions for  $\lambda$  to the problem (15) is identical to the number of solutions to the problem (28), viz.  $\#L_0 = \#N_0$ . This intermediate result is fundamental, since it is much easier to characterize  $q$  in (26) than  $\omega_1 - \omega_2$  in (11). In mathematical language:

*Proposition 1:* Let  $\mathbf{M}_0: L_0 \rightarrow N_0$  be the mapping such that  $\eta_0 = \mathbf{M}_0(\lambda_0)$ . Then  $\mathbf{M}_0$  is a bijection, implying  $\#L_0 = \#N_0$ .

*Proof:*  $\mathbf{M}_0$  is identical to  $\mathbf{M}$ , except for its range and domain that are subsets of those of  $\mathbf{M}$ . Therefore, to each solution for  $\eta$  in (28) corresponds exactly one value for  $\lambda$  (and reciprocally). This must obviously correspond to a solution in (15). Indeed,  $\lambda_0 \in L_0$  implies  $\omega_1 = \omega_2$ , or  $q = (\omega_1/\omega_2)^\sigma = 1$ , so  $\mathbf{M}(\lambda_0) \in N_0$ . A parallel argument shows that  $\forall \eta_0 \in N_0$ ,  $\exists \lambda_0 \in L_0$  such that  $\lambda_0 \in \mathbf{M}^{-1}(\eta_0) = \mathbf{M}_0^{-1}(\eta_0)$ ; by Lemma 3, this  $\lambda_0$  is unique. QED.

## 2.8. Discussion

To repeat, showing that there are at most three interior equilibria in Krugman's (1991a) model is a task most easily established indirectly. This property is easily established using another model (the FE model) and showing that both models are isomorphic in some specific state-cum-parameter space.

22 Note that the mapping from  $\ln(\eta)$  to  $\ln(q)$  is symmetric.

In this section, Proposition 1 established that there is a one-to-one mapping between the solution sets in the two state spaces for Krugman’s CP model. In the terms of the ‘natural’ state-cum-parameter space, we are left showing that the curve  $q - 1$  plot against  $\eta$  crosses the horizontal axis at most thrice, a case Figure 1 illustrates. A sufficient condition for this to be true is that the curve  $q - 1$  admits at most two flat points when plotted against  $\lambda$  or  $\eta$ .

Showing this directly proved to be a task beyond the reach of my patience. Rather, there is a more elegant—albeit indirect—way of doing it. This method first involves showing that the alternative model of migration-driven agglomeration, the FE model, can be fully described by the system in (26); in other words, the FE and CP models are isomorphic. This is a key result of the paper in itself.

Moreover, the FE model does admit at most three interior equilibria so this property immediately applies to the CP model as well.

These two results are formally established in Propositions 2 and 3 of the following section.

### 3. The Footloose entrepreneur model

The goal here is to derive the characteristics of the set of instantaneous equilibria of the FE model so as to export them into the CP model. Theoretically, in this section I could follow all the steps of the previous section to show how the instantaneous equilibrium of this alternative model based on factor migration can be rewritten as (26). (The law of motion and the long run equilibrium are still depicted by (14) and (28), respectively.) In fact, I will only point to the differences between the two models, implying that whenever I am being silent about a variable or a parameter it remains defined as in the previous section.

In the FE model, skilled workers are specific to the M-sector, but unskilled workers are employed in both sectors; specifically, the cost function is no-longer homothetic as in (2) but takes the following functional form instead:

$$C(x(i)) = wF + w_U \rho x(i) \tag{29}$$

where  $w_U$  is the unskilled labor wage rate and is equal to unity by our choices of units and normalizations.<sup>23</sup> That is, the unskilled wage is equal to 1 because the A-sector transforms one unit of unskilled labour into one unit of A (the numéraire) under perfect competition and its output is freely traded. Since the M sector also uses unskilled workers as the sole variable inputs, the optimal prices are now function of the parameters only. Hence, (5) has to be replaced by:

$$p_j(i) = \frac{\rho}{1 - 1/\sigma} = 1; \quad p_k(i) = \frac{\rho}{1 - 1/\sigma} T = T, \quad k \neq j \tag{30}$$

and the following expressions have to be updated accordingly. The key to the tractability of this model is that the definition of the price indices and the wages equations are no-longer recursive. Before substantiating that claim, we have to make one more change.

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23 This apparently innocuous change has a strong implication: in the CP model, the size of firms is fixed by free-entry and the variable of adjustment is the number of firms; in the FE model, exactly the opposite is true (specifically, the number of firms varies with the prevailing nominal wage). Nevertheless, the two models are isomorphic.

### 3.1. Normalizations

We keep all the normalizations of the CP model, but one. We take:

$$\text{Labour endowments: } L_U/L = (\sigma - \mu)/\mu;$$

so that (16) holds as before (again, this normalization implies that nominal wages are equalized throughout at any stable long run equilibrium).

### 3.2. Instantaneous equilibrium in the FE model

**Step 6:** I now show that the CP and FE models are isomorphic in the natural state space. Following the same steps as in the previous section, it is clear that (10) still holds. However, the instantaneous equilibrium is now being characterized by the following set of equations:

$$\begin{aligned} Y_1 &= \frac{\mu\lambda w_1}{\sigma} + \frac{\sigma - \mu}{2\sigma}, & Y_2 &= \frac{\mu(1 - \lambda)w_2}{\sigma} + \frac{\sigma - \mu}{2\sigma} \\ G_1^{1-\sigma} &= \lambda + (1 - \lambda)\phi, & G_2^{1-\sigma} &= \lambda\phi + (1 - \lambda) \\ w_1 &= \frac{Y_1}{G_1^{1-\sigma}} + \phi \frac{Y_2}{G_2^{1-\sigma}}, & w_2 &= \phi \frac{Y_1}{G_1^{1-\sigma}} + \frac{Y_2}{G_2^{1-\sigma}} \end{aligned} \quad (31)$$

This is a simpler model in that the price-indices now depends on  $\lambda$  and  $\phi$  only—in particular, they depend on no ‘short-run’ endogenous variable; this way explicit solutions for  $\omega_1$  and  $\omega_2$  are available (see Forslid and Ottaviano 2003 and Appendix 1 in the present paper).

By virtue of (16), we can define  $\eta$  as the ratio of the mobile factor’s expenditure in 1 to mobile expenditure in 2, as before (19). Also,  $\phi$  is still defined by (17) and  $\chi$  is still defined by (20); with the new normalization, we have  $\chi = (\sigma - \mu)/(\sigma + \mu)$ . Also, the parameter capturing the forward linkages (21) has to be redefined as:

$$\theta \equiv \frac{\mu}{\sigma - 1} \quad (32)$$

The interpretation of  $\theta$  and the no-black-hole condition  $\theta < 1$  remain the same. Finally, the definition of  $q$  (25) has to be replaced by:

$$q \equiv \frac{\omega_1}{\omega_2} \quad (33)$$

As an aside, and to ease the step from (31) to (35), note that (12), (19), and (24) together imply:

$$\omega = \left(\frac{G_1}{G_2}\right)^{-\mu} \eta \frac{1 - \lambda}{\lambda} \quad (34)$$

Now, solve for  $\lambda/(1 - \lambda)$  and plug the result into (31); use also the definition of  $\Delta_j$  (23) and the ratio notation to get:

$$Y = \frac{\eta + \chi}{1 + \eta\chi}; \quad \eta\Delta^\theta = q \frac{\Delta - \phi}{1 - \Delta\phi}; \quad q = \Delta^\theta \frac{Y + \Delta\phi}{Y\phi + \Delta} \quad (35)$$

Thus we have shown the first central result of this paper:

*Proposition 2:* The CP and FE model are isomorphic in the natural state space.<sup>24</sup>

*Proof:* Clearly, (35) and (26) are identical, hence the CP and FE models are isomorphic. QED.

Consequently, we can use Figure 2 for the FE model, too, in which case the bottom and upper panels correspond to Eq. (31) and Eq. (35), respectively. We define the set of interior long run solutions to (35) for  $\eta$  and  $\lambda$  as  $N_0^{FO}$  and  $L_0^{FO}$ , respectively:

$$N_0^{FO} \equiv \{\eta > 0 : f(1, \eta) = 0\}, \quad L_0^{FO} \equiv \{\lambda > 0 : F^{FO}(0, \lambda) = 0\} \quad (36)$$

these are the equivalent of  $N_0$  in (28) and  $L_0$  in (15).

At this stage it is worth pausing to get the intuition for the remainder of the proof of the main result in this paper (Proposition 3 below). Since (35) is identical to (26), the sets  $N_0$  and  $N_0^{FO}$  are identical as well, though generically the sets  $L_0$  and  $L_0^{FO}$  are not. (In particular, asymmetric steady-states differ). What is important, however, is that the number of steady states in each model is identical.

**Step 7:** A final intermediate result is needed before we can turn to the proof proper:

*Lemma 4:* The FO model admits at most 3 interior steady-states, viz.  $\#L_0^{FO} \leq 3$ .

*Proof:* See Appendix 1.

We have at last everything at hand to prove the main result of this paper, namely that  $\#L_0$  is no larger than 3 for the CP model as well.

## 4. On the number of steady-states and location hysteresis

This section contains the remaining results of the paper. Proposition 3 establishes that the CP model admits at most three interior steady-states and that location hysteresis happens for some parameter values. Proposition 4 proves that the existence of forward linkages is a prerequisite to the possibility of equilibrium switching generated by shocks to expectations; of course, this can only happen in versions of the model in which agents are forward-looking, as in Baldwin (2001) and Ottaviano (2001). Finally, Proposition 5 establishes that asymmetric interior steady-states are always unstable (if they exist at all).

### 4.1. Threshold effects, hysteresis, and shocks to expectations

Technically speaking, location displays hysteresis whenever several stable equilibria coexists under the same parameter values. In the current models, there is a region in the parameter space (to be defined precisely shortly) under which two equilibria are (locally and globally) stable: the agglomeration equilibrium where agglomeration occurs in region 1 and the agglomeration equilibrium where agglomeration occurs in region 2. Given the law of motion (14), it is clear that location is path dependent. This form

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24 Appendix 2 shows that the FCVL model is also a perfect twin of the CP and FE models.

of hysteresis is sufficient to generate agglomeration rents that are at the core of the literature on NEG and tax competition; see Baldwin et al. (2003a, Part IV) for a comprehensive analysis.

In the current models, there even exists a region of the parameter space for which the symmetric and agglomeration equilibria are all three stable. This configuration is interesting because it means that two spatial configurations very different in nature—full spatial dispersion and concentration—can emerge as a long run equilibrium. It also implies that shocks to expectations may trigger an equilibrium switch (that is to say, the basins of attraction of several steady-states overlap; see Baldwin, 2001). The models of Baldwin (1999) and Ottaviano et al. (2002) do not exhibit this property.

It is useful to start by determining the generic number of steady-states.

### 4.2. On the number of steady-states

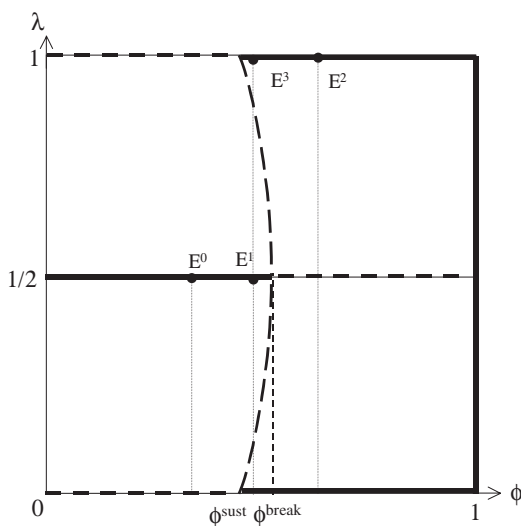
The next proposition is the second central result of this paper.

*Proposition 3:* The CP model admits at most three interior steady-states.

*Proof:* Start with the FE model. Using Lemmas 3 and 4, we have  $\#N_0^{FO} \leq 3$ . Since the (26) is identical to (35), it must be that  $N_0 = N_0^{FO}$ . Invoking Proposition 1, we find  $\#L_0 = \#N_0 = \#N_0^{FO} = \#L_0^{FO}$ . This implies  $\#L_0 \leq 3$  and establishes the result.

QED.

Figure 3 illustrates a possible configuration of equilibria that obeys this proposition. Subsequent analysis (Lemma 5 and Proposition 5) will show that the set of equilibria has indeed the shape as shown and no other. This figure is the by now famous ‘tomahawk bifurcation diagram’ (Krugman, 1991b). It plots  $\lambda$  on the vertical axis against  $\phi$  and pictures the stable steady-states in plain lines and the unstable ones in dotted lines. We



**Figure 3.** The tomahawk diagram.

infer from the figure that when transportation costs are important ( $\phi < \phi^{sust}$ ) the only stable steady-state is the symmetric one ( $\lambda = 1/2$ ).<sup>25</sup> In particular, the core-periphery structure is said not to be ‘sustainable’ (hence the name of the threshold  $\phi^{sust}$ ). When trade integration is deep enough ( $\phi > \phi^{break}$ ) the only stable configuration is the core-periphery structure ( $\lambda \in ]0, 1[$ ); the stability of the symmetric steady-state is said to be ‘broken’ (hence the name of the threshold  $\phi^{break}$ ). For intermediate values of  $\phi$ , both the dispersed and the core-periphery outcomes are stable because  $\phi^{sust} < \phi^{break}$  in this model (see Proposition 4 below). Over this parameter range, there are also two interior, asymmetric steady-states. These are always unstable (Proposition 5 below). This form of multiplicity of equilibria thus implies that location is path-dependent and that expectations alone might move the economy from one equilibrium to another.

To see why the model displays hysteresis, pick three equidistant values for  $\phi$  in Figure 3, say  $\phi^0$ ,  $\phi'$  and  $\phi''$  such that  $\phi^0 < \phi^{sust} < \phi' < \phi^{break} < \phi''$ . Starting from  $\phi = \phi^0$ , it is clear that only the symmetric equilibrium is stable, hence the economy finds itself at equilibrium  $E^0$  on Figure 3. A mild increase in openness to  $\phi'$  does not change the location equilibrium, given (14), hence the new equilibrium is  $E^1$ . By contrast, a similar increase from  $\phi'$  to  $\phi''$  generates a spectacularly different outcome: indeed, when  $\phi = \phi''$ , only the core-periphery patterns are stable; for the sake of illustration, say that the manufacturing activities take place in location 1, hence the new equilibrium is at  $E^2$ . Note in passing that this illustrates a very general insight: in a NEG setting, policy may have dramatically non-linear effects.<sup>26</sup> Now, imagine that the last policy change is reversed, i.e. trade openness decreases from  $\phi''$  to  $\phi'$ . Given (14), all firms and skilled workers stay in region 1, even though this policy makes location 1 less desirable than location 2 (because serving market 2 is more expensive). Hence, the final equilibrium is  $E^3$ . This illustrates another important insight of the NEG: a policy reversal does not necessarily yield location reversal.

**Step 8:** In order to establish that the bifurcation diagram has the shape drawn in Figure 2, we need to define the ‘break’ and ‘sustain’ points and to rank them.<sup>27</sup>

### 4.3. The break and sustain points

Two local stability tests are usually applied in the NEG literature.<sup>28</sup> A first question we may ask is, ‘is the core-periphery structure *sustainable*?’ To answer this question we solve for the values of  $\phi$  in  $[0, 1]$  such that it is (weakly) unprofitable to produce in the periphery.<sup>29</sup> Put differently, we ask: under which conditions no skilled worker has any incentive to locate in region 1 given that all skilled workers are clustered in 2? Using the

25 The parameters  $\phi^{break}$  and  $\phi^{sust}$  will be introduced shortly.

26 This is a ‘threshold effect’, in the terms of Baldwin et al. (2003a, chapter 9).

27 As an aside, it should be noted that doing so using the ‘natural’ state-space, viz. eq. (26) or (35), is much simpler than it is using (11) or (31).

28 Generally, these test are also valid to asses the global stability of the system. See footnote 14.

29 When  $\phi = 1$  the two regions are perfectly integrated into a single entity and location is therefore irrelevant. In particular,  $\phi = 1$  implies  $\omega_1 = \omega_2$  for all  $\lambda$  (or  $q = 1$  for all  $\eta$ ).

structural state space, this is equivalent to finding the set of values for  $\phi$  (call it  $\Phi^{sust}$ ) such that  $\lambda = 0$  implies  $\omega_1 \leq \omega_2$ :

$$\Phi^{sust} \equiv \{\phi \in [0, 1] : \lambda = 0 \text{ and } \omega_1 \leq \omega_2\} \quad (37)$$

Given Proposition 3, this is the same as finding the values of  $\phi$  such that  $\eta = 0$  implies  $q \leq 1$ , or:

$$\Phi^{sust} = \{\phi \in [0, 1] : \eta = 0 \text{ and } q \leq 1\} \quad (38)$$

To answer the question above, impose  $\eta = 0$  and  $q \leq 1$  in (26) or (35). Thus (35) becomes:

$$Y = \chi; \quad \Delta = \phi; \quad 1 \geq \Delta^\theta \frac{Y + \Delta\phi}{Y\phi + \Delta} \quad (39)$$

It is readily verified that the inequality is violated for low values of  $\phi$  and holds trivially for  $\phi = 1$ . As Lemma 5 below establishes, the set of  $\phi$  such that (39) holds,  $\Phi^{sust}$ , is compact (i.e. closed and bounded). Hence, we are left finding the lowest value of  $\phi$ —call it  $\phi^{sust}$ —such that (39) holds with equality. Solving for  $\phi$ , it is readily verified that the ‘sustain point’  $\phi^{sust}$  is implicitly defined as:

$$(1 + \chi)(\phi^{sust})^{1-\theta} - (\phi^{sust})^2 - \chi = 0 \quad (40)$$

To be more precise,  $\phi^{sust}$  is the lowest root of the above polynomial ( $\phi = 1$  is the other real root). This expression holds for the CP and the FE models alike (remember, they are isomorphic by Proposition 2). The corner steady-states are stable whenever  $\phi \geq \phi^{sust}$ .

We may also ask an alternative question: When is the symmetric equilibrium unstable? Answering this question involves signing the first derivative of  $\omega_1 - \omega_2$  at  $\lambda = 1/2$  in the structural state space or the first derivative of  $\ln(q)$  with respect to  $\ln(\eta)$  in the natural state space.<sup>30</sup> The economic rationale for asking this question is as follows. Start with the symmetric equilibrium (in which real wages are always equalized) and imagine that a shock were to perturb this equilibrium slightly in the form of moving ‘one’ skilled worker from region 2 to region 1. If the resulting real wage in 1 is larger than in 2, then the initial shock is destabilizing in the sense that additional skilled workers have the tendency to leave region 2 to region 1. Thus, formally, we want to find the set of values for  $\phi$  (call it  $\Phi^{break}$ ) such that:

$$\begin{aligned} \Phi^{break} &\equiv \left\{ \phi \in [0, 1] : \frac{d}{d\lambda}(\omega_1 - \omega_2) \Big|_{\lambda=1/2} \geq 0 \right\} \\ &= \left\{ \phi \in [0, 1] : \frac{d}{d \ln \eta} \ln q \Big|_{\eta=1} \geq 0 \right\} \end{aligned} \quad (41)$$

So let us normalize (25) around the symmetric equilibrium. In the natural state space, this is facilitated by noting that  $\eta = 1$  implies  $Y = \Delta = q = 1$  because of the symmetry of the model and by definition of the ratio variables (24). This is handy because in that case  $d \ln \eta = d\eta$ ,  $d \ln Y = dY$ ,  $d \ln \Delta = d\Delta$  and  $d \ln q = dq$ . Given this, it is straightforward to show

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30 This first-order approximation technique was introduced by Puga (1999).

that total differentiation of the system in (35) can be written in matrix form as:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \theta - Z^{-1} & -1 \\ -Z & Z - \theta & 0 \end{bmatrix} \begin{bmatrix} dY \\ d\Delta \\ dq \end{bmatrix} = \begin{bmatrix} (1 - \chi)/(1 + \chi) \\ -1 \\ 0 \end{bmatrix} d\eta, \quad Z \equiv \frac{1 - \phi}{1 + \phi} \quad (42)$$

(where  $Z$  is monotonically decreasing in  $\phi$ , with  $Z = 0$  if  $\phi = 1$  and  $Z = 1$  if  $\phi = 0$ ).

Applying Cramer's rule on (42), we find:

$$\begin{aligned} 0 \leq \frac{dq}{d\eta} &\equiv -Z \frac{Z(\theta - Z^{-1})(1 - \chi) + (Z - \theta)(1 + \chi)}{(1 - Z^2)(1 + \chi)} \\ &= \frac{-Z^2[\theta(1 - \chi) + (1 + \chi)] + Z(1 - \chi) + \theta(1 + \chi)}{(1 - Z^2)(1 + \chi)} \end{aligned} \quad (43)$$

The numerator of the last expression above is quadratic in  $Z$  and, clearly,  $Z = 0$  ( $\phi = 1$ ) is one root of this polynomial. (As we shall see in Lemma 5,  $\Phi^{\text{break}}$  is compact and includes 1.) Hence, we are left finding a lower bound for this set and hence to find the value of  $\phi < 1$  such that the inequalities in (41) and (43) hold with equality.

Define the non-zero root of this second-order polynomial for  $Z > 0$  as  $Z^{\text{break}}$ . The corresponding value for  $\phi$  is:<sup>31</sup>

$$\phi^{\text{break}} \equiv \chi \frac{1 - \theta}{1 + \theta} \quad (44)$$

which is in  $(0,1)$  by the no black-hole and the lumpy-feet conditions. The symmetric steady-state is unstable whenever  $\phi \geq \phi^{\text{break}}$

Using the definitions of  $\chi$  and  $\theta$ , (40) and (44) are of course equivalent to Eq. (5.17) and Eq. (5.28) in Fujita et al. (1999, pp.70, 74) and Eqs. (25) and (26) in Forslid and Ottaviano (2003). I sum up these results in the following Lemma. (Part of its proof is needed for Proposition 4, hence I develop it in the main text rather than in the appendix.)

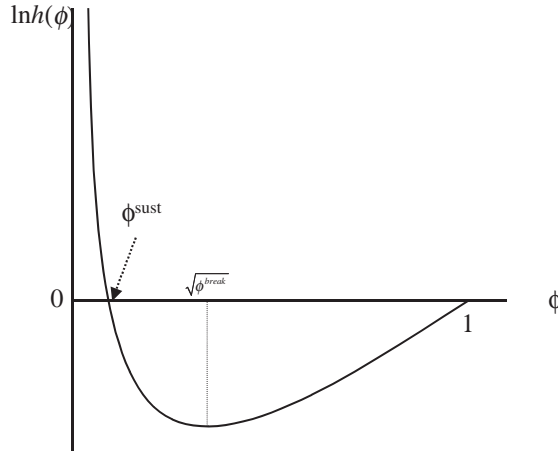
*Lemma 5:* The sets  $\Phi^{\text{sust}}$  and  $\Phi^{\text{break}}$  are compact.

*Proof:* Start with  $\Phi^{\text{break}}$ . Clearly, the numerator of the last expression in (43) is concave in  $Z$  everywhere on the unit interval. Also, it is nil when  $Z = 0$  and negative when  $Z \approx 1$ . Hence, the set of  $Z$ 's for which the inequality in (43) is satisfied is compact. Since  $Z(\cdot)$  is a monotonic transformation of  $\phi$ ,  $\Phi^{\text{break}}$  is also compact. Turn now to  $\Phi^{\text{sust}}$ . To this end, I characterize how the natural logarithm of the function  $h(\phi) \equiv [\phi^2 + \chi]/[\phi^{1-\theta}(1 + \chi)]$ , which is just a transformation of (40), changes with  $\phi$ .<sup>32</sup> In particular,  $\ln(h)$  and the left-hand side of (40) have the same zeroes. Given this, we have:

$$\begin{aligned} \ln h(\phi) &= \ln(\phi^2 + \chi) - (1 - \theta) \ln \phi - \ln(1 + \chi) \\ \frac{d}{d\phi} \ln h(\phi) &= \frac{2\phi}{\phi^2 + \chi} - \frac{1 - \theta}{\phi} \\ \frac{d^2}{d\phi^2} \ln h(\phi) &= \frac{2(\chi - \phi^2)\phi^2 + (1 - \theta)(\phi^2 + \chi)^2}{\phi^2(\phi^2 + \chi)^2} \end{aligned} \quad (45)$$

31 For computational purposes, note that  $\phi = (1 - Z)/(1 + Z)$  by definition of  $Z$ .

32 As Baldwin et al. (2003a) show,  $h(\cdot)$  has an economic interpretation as the inverse of agglomeration rents.



**Figure 4.** Proving the  $\phi^{break} > \phi^{sust}$ .

therefore

$$\begin{aligned} \lim_{\phi \rightarrow 0} \ln h(\phi) &= +\infty, & \lim_{\phi \rightarrow 0} \frac{d}{d\phi} \ln h(\phi) &= -\infty \\ \ln h(1) &= 0, & \frac{d}{d\phi} \ln h(1) &= \frac{2}{1+\chi} - (1-\theta) > 0 \\ \frac{d}{d\phi} \ln h(\phi) &= 0 \Leftrightarrow \phi = \sqrt{\phi^{break}} \\ \frac{d^2}{d\phi^2} \ln h\left(\sqrt{\phi^{break}}\right) &= \frac{2\chi \frac{2\theta}{1+\theta} \phi^{break} + (1-\theta)(\phi^{break} + \chi)^2}{\phi^2(\phi^2 + \chi)^2} > 0 \end{aligned} \tag{46}$$

that is,  $h(\cdot)$  is decreasing and has a vertical asymptote at  $\phi = 0$ ,  $h(\cdot)$  is equal to zero and is increasing at  $\phi = 1$ , and  $h(\cdot)$  has a unique minimum at  $\sqrt{\phi^{break}}$  (why exactly the square root of  $\phi^{break}$  remains a mystery to me). All these properties are illustrated in Figure 4, in which I plot  $\ln h$  as a function of  $\phi$ . As can be seen on the graph, the properties of  $h$  derived in (46) taken together imply that  $h$  has a unique root between zero and unity, namely,  $\phi^{sust}$ . Thus the inequality in (39) is satisfied for all  $\phi$  larger than  $\phi^{sust}$ , that is,  $\Phi^{sust}$  is compact. QED.

#### 4.4. A digression on Baldwin (1999)

In Baldwin (1999), agglomeration stems from endogenous accumulation of human capital rather than skilled worker migration. The functional forms are otherwise similar to the FE model (in particular, the reward to human capital is  $\pi$ , the Ricardian surplus of the firm). Therefore, when making their investment decision, these workers compare the stream of future (operating) profits with the current cost of acquiring skills (this investment is made in units of the numéraire). Therefore, the costs-of living or price indices have no impact on either the investment benefit or the investment cost. In other words, there are no forward linkage (or price index effect) in this

model, that is  $\theta = 0$ . Interestingly, when  $\theta = 0$  the break and sustain points coincide and are equal to  $\chi$ , as a quick look at (40) and (44) reveals.

It turns out that if the would-be migrants in the CP and FE models based their migration decisions on *nominal* wages differences rather than on real wages differences, then the same result would obtain, namely, the break and sustain points would coincide. Hence, it is because people care about the cost-of-living that the sustain point comes before the break point. The more they do so (the higher is  $\theta$ ), the larger is the overlap.

I summarize this finding in the following proposition:

*Proposition 4:* When the forward linkage is absent ( $\theta = 0$ ), as in Baldwin (1999) or in the CP and FE models if skilled workers based their migration decision on nominal wages differences, then the break and sustain points coincide.

This implies that the very possibility that shocks to expectations may make the economy jump from one equilibrium to another relies on the existence of forward linkages. Also, without such linkages, the dynamics of the model is less rich in that steady-states of different nature do not coexist in any region of the parameter space.

#### 4.5. Stability of the asymmetric interior steady-states

I now show that the break point is larger than the sustain point (a priori  $\Phi^{break} \subseteq \Phi^{sust}$ ) and that the asymmetric interior steady-states are unstable whenever they exist. These facts imply that shocks to expectations are sufficient for the economy to switch steady state.

**Step 9:** I claim that whenever they exist asymmetric steady-states are unstable. Therefore, an incremental change of the parameters near the thresholds  $\phi^{break}$  and  $\phi^{sust}$  results in a discrete jump from the symmetric equilibrium to the a corner one (or the other way round).<sup>33</sup>

*Proposition 5:* If they exist, interior asymmetric equilibria are unstable.

*Proof:* We have to show that  $\phi^{sust} < \phi^{break}$ , as a look at Figure 3 confirms.<sup>34</sup> We do this in two steps. Step1: Since  $\ln h(\phi)$  is negative if, and only if,  $\phi < \phi^{sust}$ , it is sufficient to evaluate the sign of  $\ln h(\phi^{break})$  (which should be negative). Therefore, define the function  $G(\theta, \chi)$  as:

$$G(\theta, \chi) \equiv \ln h(\phi^{break}) \tag{47}$$

We know from Proposition 4 that the break and sustain points coincide when  $\theta = 0$ , that is,  $G(0, \chi) = 0$  for all  $\chi \in [0, 1]$ . So we are left to showing that the presence of vertical linkages makes the break point strictly larger than the sustain point. A sufficient condition

33 The ‘bang-bang’ property of the model is due to the assumption that potential migrants are homogenous. Tabuchi and Thisse (2002) and Murata (2003) show that taste heterogeneity has a strong implication on the relationship between the location equilibrium and transportation costs: in particular, it is continuously differentiable, unlike Figure 1.

34 The early manuscript version of this proof, which inspired the corresponding proofs in Neary (2001) and Fujita and Thisse (2002), was valid for the CP model only. Here, I generalize it to all models of Table 1.

for this to be true is that  $\partial G/\partial\theta \leq 0$  holds for all  $\chi$  and  $\theta$  in  $[0,1]$ . Step 2: To this aim, note that  $G(\cdot, \chi)$  is concave:

$$\frac{d^2}{d\theta^2} G(\theta, \chi) = -4 \frac{\left(\frac{1+\theta}{1-\theta} - \chi\right) \left[\left(\frac{1+\theta}{1-\theta}\right)^2 - \chi\right]}{(1+\theta)(1-\theta)^2 \left[\left(\frac{1+\theta}{1-\theta}\right)^2 + \chi\right]^2} < 0 \quad (48)$$

because  $0 < \chi, \theta < 1$ . Hence, a sufficient condition for  $G(\cdot, \chi) \leq 0$  to hold is  $G(0, \chi) = 0$  (which we know to be true from Proposition 4) and the first derivative of  $G$  with respect to  $\theta$  to be non-positive when evaluated at  $\theta = 0$ . This is indeed the case, as I now show. Define:

$$g_0(\chi) \equiv \frac{d}{d\theta} G(\theta, \chi) \Big|_{\theta=0} = \frac{\partial}{\partial\phi} \ln h(\phi; \theta) \Big|_{\phi=\phi^{break}} \frac{d}{d\theta} \phi^{break} \Big|_{\theta=0} + \frac{\partial}{\partial\theta} \ln h(\theta; \phi) \Big|_{\phi=\phi^{break}, \theta=0} \quad (49)$$

Using (44) and (45), it takes little algebra to see that  $g_0(\chi) \leq 0$  holds for all  $\chi$  in  $[0, 1]$ :

$$g_0(\chi) = 2 \frac{1-\chi}{1+\chi} + \ln \chi \leq 0 \quad (50)$$

To get the inequality above, note that  $g_0(\chi)$  is concave on  $[0, 1]$  and that both  $g_0 = 0$  and  $\partial g_0/\partial\chi = 0$  hold when  $\chi = 1$ .<sup>35</sup> These facts imply that  $G(\theta, \chi)$  is non-positive over the whole parameter space. The point of all this is that the upper bound of  $G(\theta, \chi)$ , and thus the upper bound of  $h(\phi^{break})$ , is zero. We know, therefore, that for all permissible values of  $\chi$  and  $\theta$ ,  $\phi^{sust} \leq \phi^{break}$ , as claimed. QED.

This completes the proof and implies that the asymmetric interior steady-states are unstable.

#### 4.6. Further discussion

As we saw, without forward linkages (or price-index effect), the break and sustain points coincide. This is *not* a general result. To see this, note first that the break point without forward linkage is defined as the (limiting) set of parameter for which a marginal migration *raises* the nominal wage in the host region and decreases it in the source region; that is, we are talking about signing a first derivative. Second, the sustain point without forward linkage is defined as the (limiting) set of parameters for which the nominal wages are larger in region 1 than in region 2 if and only if all mobile workers are settled in region 1; that is, we are talking about a difference in levels. There is no reason a priori why these two limiting values should coincide. In fact, they do so in the present setting because of Cobb-Douglas preferences in particular. Adding the price-index effect

35 I am grateful to Maarten Bosker who pointed out to me that a previous version of this paper contained algebraic mistakes in (49) and (50) (CEPR discussion paper no. 4326). Though these mistakes required minor alterations to those expressions, these alterations do not change the inequalities in (49) and (50).

to this setting results in the sustain point coming before the break point. In sharp contrast, Pflüger (2003) shows that the break point comes before the sustain point when preferences are quasi-linear (linear in  $A$  and logarithmic in  $M$ ).

Closer to the actual setting, Puga (1999) shows that the ‘bang-bang’ property of the model occurs because wage costs do not rise fast enough when firms are clustering. To understand the mechanism at work, start at the symmetric equilibrium. Now imagine that the parameters of the model take values so that  $\phi > \phi^{\text{break}}$ . When the first firm moves out of the symmetric equilibrium, wage costs increase by less than revenue for equal real wages so the move is worthy (Puga 1999 has shown that this alternative way of modeling the dynamics of the model yields the same analytical results). Hence, a second firm follows suit; this, too, is worthy, and so on, all the way to full agglomeration (because no stable asymmetric equilibrium exists).

When the local labour supply is relatively inelastic then this ‘bang-bang’ property may vanish. In Krugman and Venables (1995), manufacturing (or skilled) workers are immobile across regions and must be pulled from the traditional A sector. In Puga (1999), skilled workers can also move across regional borders, like in the CP and FE models. The point in the Krugman, Venables and Puga settings being that, if there are decreasing returns in labour in the A sector, then manufacturing firms face an less-than-fully elastic labour supply.

This has two important consequences. First, the agglomerated steady state is no longer stable when transportation costs are very low. To see this, observe that nominal wages are much higher in the region in which the manufacturing sector is clustered than in the periphery because skilled labour demand is higher in the former than in the latter. If some firms were to locate in the periphery, they would have nearly as good an access to the large market in the core than firms located there (when transportation costs are low) but with much lower labour costs. Hence, the agglomeration equilibrium is sustained for intermediate values of  $\phi$  only (it can be shown that  $\Phi^{\text{sust}}$  and  $\Phi^{\text{break}}$  both remain compact, though they no longer include 1). In terms of Figure 3, and assuming that the ‘bang-bang’ properties of the model are unaffected, the diagram would look like a double-edged tomahawk.

The second important consequence is that these bang-bang properties may disappear altogether. Indeed, when firms are clustering wage costs might rise fast enough in a way that makes the location equilibrium a smooth function of  $\phi$  (with the asymmetric equilibria now being stable). In terms of Figure 3, the diagram would look like a double-pitchfork instead. Whether it takes the shape of a double-pitchfork or a double-tomahawk depends on the parameter values of the model.

## 5. Concluding remarks

This paper has shown that the original core-periphery (CP) model by Krugman (1991a,b) and the alternative footloose entrepreneur (FE) model by Forslid and Ottaviano (2003) can be entirely characterized by the same set of equations in the appropriate state-cum-parameter space; in other words, the CP and FE models are isomorphic—indeed, expressions (26) and (35) are *identical*. An implication of this fact is that it is sufficient to describe the properties of either model to know the stability properties of both. This is the strategy that I have pursued in this paper. It is worth emphasizing that the natural state variable of these models is mobile expenditure. This implies that the relevant variable to

look at in empirical studies (at least those explicitly based on NEG models) is the spatial distribution of expenditure or income of mobile factors (rather than population).

In both the CP and the FE models agglomeration is driven by (skilled) labour migration. Other agglomeration mechanisms have been put forward, too. On the one hand are the models based on input-output (or ‘vertical’) linkages among firms. The ‘footloose-capital’ model with vertical linkages (FCVL model in Table 1) developed in Robert-Nicoud (2002) is isomorphic to the CP and FE models, hence the proof developed in this paper generalizes to that model as well, as Appendix 2 shows. However, the original model with vertical linkages (see e.g., Fujita et al., 1999, section 14.2), referred to as the CPVL model in Table 1 (CPVL stands for ‘core-periphery, vertical linkages’), has two state variables, not one (as does its tractable cousin, the FEVL due to Ottaviano, 2002). Indeed, in this model factor owners are immobile across regions but labour moves from one sector to the other so that *nominal* wages are equalized within each region and across sectors. As it turns out, a simple extension of the method developed in this paper allows an exhaustive analysis of the equilibrium properties of the CPVL model, but this would take us too far apart here (remarkably, (26) drives the behaviour of one state variable in both the CPVL and FEVL models). The interested reader can refer to Ottaviano and Robert-Nicoud (2003) for details.

On the other hand are models based on factor accumulation; the simplest among these models (Baldwin, 1999) can be written as a special case of (26)—specifically,  $\theta = 0$ . This implies that this model generically admits at most one interior equilibrium—the symmetric one. Hence, the break and sustain point coincide. (The methodology developed in this paper can be used to show both of these facts but it is much easier to show these more directly; see Baldwin, 1999.)

### 5.1. Common features of popular NEG models

Given that all these models are isomorphic (and nearly identical), it is unsurprising that they all share the same features. Here are some of them. First, location displays hysteresis when transportation costs are low enough (i.e. when trade openness, as parameterized by  $\phi$ , is large enough). Specifically, agglomeration is self-sustaining so that it generates location-specific economic rents. These rents can be taxed to some extent without losing the tax base, a fact that the new literature on taxation of mobile factors exploits intensively. Second, policy potentially has very non-linear effects. In the *ex ante* symmetric region-case that the NEG typically assumes, policy has no impact on location unless it goes beyond some threshold. When it does so, the impact is spectacular and discrete (typically the equilibrium jumps from a symmetric outcome to a core-periphery one or the other way round). Finally, since all the models of Table 1 but one (Baldwin, 1999) have a range of parameters where both the symmetric and the agglomeration equilibria are stable, shocks to expectations can generate a jump between the symmetric equilibrium and the agglomeration outcome when migrants are forward looking.

### 5.2. Implications for empirical work

The threshold and hysteresis results have stark implications for empirical work (Baldwin et al., 2003a). First, policy potentially has non-linear effects, hence empirical studies that impose linearity are potentially miss-specified. Second, past experience can be a poor

guide to assess the effect of a particular policy. Also, a bad policy might not easily be reverted if there are ‘lock-in’ (hysteresis) effects—but see Davis and Weinstein (2002, 2004).

Moreover, as we saw at great lengths, all NEG models of Table 1 are isomorphic irrespective of the agglomeration mechanism they are assuming. This in itself brings good and bad news alike for the empirical work. The good news first. The predictions of the model are robust to the assumption about the underlying agglomeration mechanism. This implies that the results of an empirical investigation based on the reduced form of any model are more general and could be derived from any other structural model, the only difference being in the interpretation of the structural parameters. This conveys bad news, too, because under these conditions it is at best very difficult to identify the channel by which agglomeration operates. In particular, all three mechanisms listed in the rows of Table 1 probably play a role in the real world. Since they all show up the same way in the reduced form, it might be difficult to assess the magnitude of their respective roles.

It is my hope that the present paper is helpful in understanding why these models are so strikingly similar and how the agglomeration and dispersion forces interact in such a way that no more than five steady-states ever exist. Common functional forms are necessary for the isomorphism result. However, it is extremely remarkable that they are sufficient as well, since the mechanisms driving agglomeration differ from one model to the next, as does the interpretation of the parameters. This allows for some cautious optimism as for the generality of the predictions of the NEG theory in its current form.

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## Appendix 1: Proofs of lemmas

In this appendix I restate the Lemmas and prove them. (The proofs for Lemma 1 and 5 are in the main text.)

*Lemma 2:* Define the function  $\mathbf{M}: [0, 1] \rightarrow [0, +\infty)$  where  $\eta = \mathbf{M}(\lambda)$ . (a) Then  $\mathbf{M}$  is a surjection (i.e. onto). (b) Moreover,  $\mathbf{M}'(\cdot) > 0$  at the symmetric steady state  $\lambda = \frac{1}{2}$ .

*Proof:* (a) We have to show that  $\forall \eta \in [0, \infty)$  there exists a  $\lambda \in [0, 1]$  for which  $\eta = \mathbf{M}(\lambda)$ . Start with  $\lambda = 0$ ; by (16) we know that  $w_2 = 1$  and by definition of  $\eta$ , viz.  $\eta \equiv \lambda w_1 / ((1 - \lambda)w_2)$ , this implies  $\eta = 0$ . By symmetry, we know that  $\lambda = 1$  implies  $\eta = \infty$ . In other words,  $\mathbf{M}$  maps the least (greatest) element of the domain into the least (greatest) element of the range. Consequently, if  $\mathbf{M}(\cdot)$  is continuous then to any  $\eta$  in the range  $[0, \infty)$  corresponds at least one  $\lambda$  in the domain of  $\mathbf{M}(\cdot)$ . Next, if we can show that  $w_1$  and  $w_2$  are continuous in  $\lambda$ , then  $\eta$  is continuous in  $\lambda$ , too. By (16),  $w_1(\lambda)$  is continuous if, and only if,  $w_1(\lambda)/w_2(\lambda)$  is continuous, as long as both  $w_1$  and  $w_2$  are strictly positive. Define  $\Lambda \equiv \lambda/(1 - \lambda)$  and  $w \equiv w_1/w_2$ , and rewrite (11) using the ratio notation to get:

$$Y = \frac{\Lambda w + \chi}{1 + \Lambda w \chi}; \quad \Delta = \frac{\Lambda w^{1-\sigma} + \phi}{\phi \Lambda w^{1-\sigma} + 1}; \quad w^\sigma = \frac{Y + \Delta \phi}{Y \phi + \Delta} \quad (51)$$

Three things can readily be noted from (51). First, whenever,  $\phi > 0$ ,  $\lambda = \Lambda = 0$  implies,  $w^\sigma = (\chi/\phi + \phi)/(\chi + 1)$ , a real, finite and positive number. More generally,  $w$  is finite and positive, viz.  $w \notin \{0, \infty\}$ , for all  $\Lambda$ . To see this, set  $w = 0$  or  $w = \infty$  in (1) and observe that this implies a contradiction in the third equation whenever  $\phi > 0$ , viz.  $0 = (\chi + 1)/(1/\phi + \chi\phi)$ . Finally, no denominator in the three expressions of (1) nor  $w^{1-\sigma}$  is ever non-positive. These facts imply that each endogenous variable in (51) viz.  $Y$ ,  $\Delta$  and  $w$ , is a continuous function of  $\Lambda$  (and of  $\lambda$  in turn). We can back up this result for  $w_1(\lambda)$  and  $w_2(\lambda)$  to claim that these two functions are continuous, too. As we saw, this in turn implies that  $\mathbf{M}(\cdot)$  is continuous, and, together with the fact that  $\mathbf{M}(0) = 0$  and  $\mathbf{M}(1) = \infty$ , that  $\mathbf{M}(\cdot)$  is a surjection. (b) Differentiate (51) at the symmetric

steady-state to get:

$$\begin{bmatrix} 1 & 0 & -(1-\chi)/(1+\chi) \\ 0 & 1 & (\sigma-1)Z \\ -Z & Z & \sigma \end{bmatrix} \begin{bmatrix} dY \\ d\Delta \\ dw \end{bmatrix} = \begin{bmatrix} (1-\chi)/(1+\chi) \\ Z \\ 0 \end{bmatrix} d\Lambda \quad (52)$$

Solve for  $dw/d\Lambda$  and plug this into  $d\eta \equiv dw + d\Lambda$  to get:

$$\left. \frac{d\eta}{d\Lambda} \right|_{\lambda=1/2} = \phi \frac{\sigma(1+\chi)}{\phi^2 + (2\sigma-1)(1+\chi)\phi + \chi} \quad (53)$$

this is unambiguously positive, as was to be shown.

QED.

*Lemma 3:* (a)  $\mathbf{M}$  is a bijection; (b)  $\eta$  is increasing in  $\lambda$ .

*Proof:* (a) To each  $\eta$  corresponds at least one  $\lambda$ , by Lemma 1. So we are left to show that to each  $\eta$  corresponds at most one  $\lambda$  (namely,  $\mathbf{M}$  is an injection, too). The proof is by contradiction. Assume that to some ratio of expenditure  $\eta$  correspond two instantaneous equilibrium configurations,  $\lambda$  and  $\lambda' \neq \lambda$ . Formally, that is to say that there exist two tuples  $\{\lambda, w_1, w_2, Y_1, Y_2, G_1, G_2\}$  and  $\{\lambda', w'_1, w'_2, Y'_1, Y'_2, G'_1, G'_2\}$  that solve (28) and such that  $\eta' = \eta$ , where  $\eta = \lambda w_1 / ((1-\lambda)w_2)$  and  $\eta' = \lambda' w'_1 / ((1-\lambda')w'_2)$ . Let  $\Delta'$  and  $Y'$  be the equivalent to  $\Delta$  and  $Y$  in (26) for the second tuple. The first thing to stress is that  $\eta' = \eta$  implies  $Y' = Y$  by the first expression in (26). Second,  $\eta' = \eta$  and  $\lambda' \neq \lambda$  imply  $w'_j \neq w_j, j=1, 2$  by (19). This, together with the definitions of  $\omega$  (12)),  $\theta$  (21),  $\Delta$  (23) and (25), imply  $q'(\Delta')^{-\theta} \neq q\Delta^{-\theta}$  (this is obvious because  $w^\sigma = q\Delta^\theta$ ). Using the second or third expression in (26), this in turn implies:

$$\Delta' \neq \Delta \quad (54)$$

Finally, we can use the second and third expressions in (26) to eliminate  $q$  and  $\Delta^\theta$ . Thus we get:

$$\eta = \left( \frac{Y + \Delta\phi}{Y\phi + \Delta} \right) \left( \frac{\Delta - \phi}{1 - \phi\Delta} \right), \quad \eta' = \left( \frac{Y' + \Delta'\phi}{Y'\phi + \Delta'} \right) \left( \frac{\Delta' - \phi}{1 - \phi\Delta'} \right) \quad (55)$$

Since  $\eta' = \eta$  and  $Y' = Y$ , (55) implies:

$$\Delta' = \Delta \quad (56)$$

which contradicts (54)—even though (55) has two roots in  $\Delta$ , one of these roots is negative (and hence has no economic meaning). Hence, to each  $\eta$  corresponds only one  $\lambda$ . (b) The proof immediately follows from Lemma 2 and (a) above.

QED.

We note that the Lemma 3 also holds for the FE model.

*Lemma 4:* The FO model admits at most 3 interior steady-states, viz.  $\#L_0^{FO} \leq 3$ .

*Proof:*<sup>36</sup> From (31), it is possible to get analytical solutions for  $w_1$  and  $w_2$ . Using the definition of  $\omega$ , and the ratio notation, we get:

$$\omega = \frac{\lambda\phi + (1-\lambda)\Phi}{(1-\lambda)\phi + \lambda\Phi} \left[ \frac{\lambda + (1-\lambda)\phi}{\lambda\phi + (1-\lambda)} \right]^\theta \quad (57)$$

where  $\Phi \equiv (\chi + \phi^2)(\sigma + \mu)/(2\sigma)$  is a collection of parameters. Simple algebra reveals that  $\omega$  generically admits two flat points. Indeed, the numerator of  $d \ln \omega / d\lambda$  is a second-order polynomial in  $\lambda$ , as the following expression shows:

$$\frac{d \ln \omega}{d\lambda} = \frac{\theta(1-\phi^2)[\lambda\phi + (1-\lambda)\Phi][\lambda\Phi + (1-\lambda)\phi] - (\Phi^2 - \phi^2)[\lambda\phi + (1-\lambda)][\lambda + (1-\lambda)\phi]}{[\lambda\phi + (1-\lambda)\Phi][\lambda\Phi + (1-\lambda)\phi][\lambda\phi + (1-\lambda)][\lambda + (1-\lambda)\phi]} \quad (58)$$

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36 Forslid and Ottaviano (2003) independently developed the proof for this result.

This implies that  $d\omega/d\lambda$  equals zero at most twice which, in turn, implies that the set in (15) admits at most three zeroes for the FO model. Hence we write  $\#L_0^{FO} \leq 3$ . *QED*.

## Appendix 2: the FCVL and CC models

### 2.1. The FCVL model

The ‘Footloose capital vertical linkages’ model (or FCVL model for short) combines two empirically important phenomena: capital is mobile and firms buy each others’ output as intermediate inputs. Hence, this model mixes factor mobility of sorts with vertical linkages, hence its name. This appendix provides a very brief description of the key expressions of this model due to Robert-Nicoud (2002), which is also exposed at more length in chapter 8 of Baldwin et al. (2003a).

There are two primary factors: capital  $K$  and (unskilled) labour  $L_U$ .  $K$  replaces skilled labour in the FE and CP models.  $K$  is mobile across regions but  $L_U$  is not. Region 1 owns a share  $\lambda$  of world endowment  $K$  and an exogenous 50% share of world  $L_U$ . Capital is a disembodied factor. Capital ownership, like unskilled labour, is evenly spread between the two regions. Preferences are still described by (1). The cost function resembles (29) in the FE model:

$$C(x(i)) = wF + \rho w_U^{1-\alpha} G^\alpha x(i), \quad 0 < \alpha < 1 \quad (59)$$

where  $w$  is the capital reward. In this model we take  $F = 1$ , hence a firm can be identified with the unit of capital needed to start production; there are thus  $K$  active firms worldwide. The only difference with (29) is that the variable input is a Cobb-Douglas composite of unskilled labour and of the  $M$  composite itself, defined as  $C_M$  in (1).  $\alpha$  is the share of the variable costs spent on manufacturing intermediate inputs.

Since capital is disembodied, capital owners search for the highest *nominal* return and move capital according to the law of motion:

$$\dot{\lambda} = \gamma \lambda (1 - \lambda) (w_1 - w_2) \quad (60)$$

where subscripts indicate the region in which the firm is operating. We normalize  $K$  to unity and choose units so that  $L_U = [(1 - \alpha)\sigma + \alpha - \mu]/\mu$  and  $\rho = 1 - 1/\sigma$ .

With all these at hand, it is a matter of a few lines of standard algebra to establish that the instantaneous equilibrium of the model is fully described by the following expressions:

$$\begin{aligned} E_1 &= \mu \frac{L_U + \bar{w}}{2} + \alpha(\sigma - 1)\lambda w_1, & E_2 &= \mu \frac{L_U + \bar{w}}{2} + \alpha(\sigma - 1)(1 - \lambda)w_2 \\ \Delta_1 &= \lambda \Delta_1^\alpha + \phi(1 - \lambda)\Delta_2^\alpha, & \Delta_2 &= \phi\lambda \Delta_1^\alpha + (1 - \lambda)\Delta_2^\alpha \\ \sigma w_1 &= \Delta_1^\alpha \left( \frac{E_1}{\Delta_1} + \phi \frac{E_2}{\Delta_2} \right), & \sigma w_2 &= \Delta_2^\alpha \left( \phi \frac{E_1}{\Delta_1} + \frac{E_2}{\Delta_2} \right) \end{aligned} \quad (61)$$

where  $\bar{w} \equiv \lambda w_1 + (1 - \lambda)w_2 = 1$  is the (constant) average operating profit in the world economy and  $E_j$  is region  $j$ 's expenditure on manufacturing products. Expenditure consists on final demand (the first term in the right hand side expression of the first lines in (61) above) and on intermediate demand (the second term in those expressions). By our choice of  $L_U$ , we have  $\bar{w} = 1$  and  $E_1 + E_2 = \sigma$  for all  $\lambda$ .

To write the model in the natural state space, define the natural variables as:

$$\eta \equiv \frac{\lambda w_1}{(1 - \lambda)w_2}, \quad Y \equiv \frac{E_1}{E_2}, \quad \Delta \equiv \frac{\Delta_1}{\Delta_2}, \quad q \equiv \frac{w_1}{w_2} \quad (62)$$

and the natural parameters as:

$$\theta = \alpha, \quad \chi = \frac{\sigma - \alpha(\sigma - 1)}{\sigma + \alpha(\sigma - 1)} \quad (63)$$

Using (62) and (63), it is easy to see that (61) reduces to (26) or (35), hence the FCVL model has the same dynamic properties as, and is isomorphic to, the FE and CP models. Ottaviano and Robert-Nicoud (2003) show that better known models with vertical linkages (e.g., the CPVL in section 2

of Fujita et al., 1999, ch.14) also share these properties, so I refer the reader to this paper for details.

## **2.2. The CC model**

The ‘Constructed capital’ model (or CC model for short) is due to Baldwin (1999) and is one of the most tractable models of the NEG family. It combines the simple functional forms of the FE model and endogenous human capital accumulation. There are two factors, unskilled labour and (human) capital. As usual, the labour stock is exogenously given and evenly spread between the two regions. Capital, in contrast to the FE and FCVL models, is spatially immobile and endogenously accumulated using unskilled labour for its fabrication.

The interested reader is invited to refer to Baldwin (1999) or to chapter 6 of Baldwin et al. (2003a) for details.

It is a relatively straightforward exercise to show that the equations depicting the instantaneous equilibrium of the model can be rewritten in the natural state space as (26) or (35). The reader who carries out this exercise will realize that  $\theta = 0$  in this setting, as claimed in the text of the present paper. Hence the CC model is a degenerate twin of the CP, FE and FCVL models.