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Assignment reversals: Trade, skill allocation and wage inequality ☆

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Abstract

The allocation of skilled labor across industries shapes inter-industry wage differences and wage inequality. This paper shows the ranking of industries by workforce skill differs between developed and developing countries and develops a multi-sector assignment model to understand the causes and consequences of this fact. Heterogeneous agents leverage their ability through their span of control over an homogeneous input. In equilibrium, higher skill agents sort into sectors where the cost per efficiency unit of input is lower. Consequently, skill allocation is endogenous to country-sector specific variation in input productivity and costs and when the ranking of sectors by effective input costs differs across countries there is an assignment reversal. Assignment reversals between North and South have novel implications for how trade affects wages because they imply the Stolper–Samuelson theorem does not hold. Instead, each country has a comparative advantage in its high skill sector and output trade integration causes the relative wage of high skill workers, and wage inequality within the high skill sector, to increase in both countries.

JEL classification: F11; F16; J24; J31

Keywords: International trade; Wage inequality; Labor assignment

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1. Introduction

For over half a century the Stolper and Samuelson (1941) theorem dominated analysis of the effects of trade on wage inequality. In a Stolper–Samuelson world inter-industry trade raises wage inequality between skilled and unskilled workers in relatively skill abundant countries and lowers inequality in relatively unskilled abundant countries.[1] Contrary to this prediction many developing countries that liberalized trade in the 1980s and 1990s experienced increases in wage inequality (Goldberg and Pavcnik, 2007). This observation has cast doubt on the empirical relevance of the Stolper–Samuelson theorem and led to the emergence of a literature documenting alternative channels through which trade may affect wage inequality.[2] The mechanisms identified in this literature are not driven by inter-industry output trade and could, in principle, co-exist with Stolper–Samuelson effects. For example, Burstein and Vogel (2015) quantify the effects of international trade on the skill premium in a model where trade induces both Stolper–Samuelson effects and increased demand for skill within industries. By contrast, this paper challenges the logic underlying the Stolper–Samuelson theorem and shows why North–South trade between developed and developing countries does not necessarily cause Stolper–Samuelson effects.

The Stolper–Samuelson theorem relies on the assumption that the ranking of sectors by skill intensity is the same in all countries. In the two country, two sector model this assumption guarantees that if one country has a comparative advantage in the skill intensive sector, then the other country's comparative advantage must lie in unskilled labor intensive production. Variation in workforce skill across sectors is usually explained by invoking cross-sector differences in production technologies that affect the demand for skill. Both traditional multi-sector models, such as the Heckscher–Ohlin model, and the more recent comparative advantage assignment literature (Sattinger, 1975; Ohnsorge and Trefler, 2007; Costinot and Vogel, 2010; Acemoglu and Autor, 2011) follow this approach. Open economy applications of these models further assume there is no cross-country technology variation, at least in those parts of the technology that affect the demand for skill. Consequently, the ranking of sectors by workforce skill is constant across countries.

However, industry level data implies the ranking of sectors by workforce skill varies systematically across countries. Define the "wage rank correlation" to be the rank correlation of a country's industry wages with industry wages in the US. Fig. 1 shows wage rank correlations plotted against per capita income.[3] Although the correlation is always positive, it is strongly increasing in income per capita. While industrialized countries have similar industry wage structures to the US, the industry wage ranking varies substantially between low and high income countries. Section 2.1 shows that the correlation observed in Fig. 1 is a robust feature of industry wage data sets. Under the assumption that inter-industry wage differences primarily reflect

[1] Although originally derived in a canonical two country, two sector, two factor Heckscher–Ohlin model, variants of the Stolper–Samuelson theorem have been obtained in many different environments. See Costinot and Vogel (2010) for a recent example.

[2] Channels that have been highlighted in the literature include: intra-industry trade (Manasse and Turrini, 2001; Yeaple, 2005; Sampson, 2014); offshoring (Feenstra and Hanson, 1996); capital trade (Csillag and Koren, 2009; Parro, 2013; Burstein et al., 2013), and; trade-induced expansion of skill intensive R&D activity (Dinopoulos and Segerstrom, 1999).

[3] The wage data is from the UNIDO Industrial Statistics database and covers 42 countries and 127 ISIC 4 digit manufacturing industries in 2000. Income per capita is from the Penn World Tables 6.3. See Section 2.1 and Appendix C for a complete description of the data.

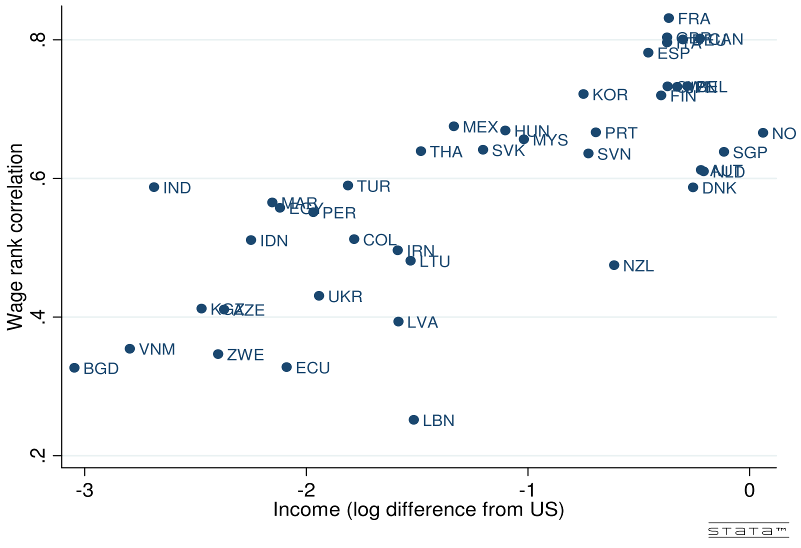

Fig. 1. Wage rank correlations – UNIDO 2000.

workforce skill,[4] Fig. 1 implies the existence of an important phenomenon: assignment reversals. I define an assignment reversal to occur whenever the ranking of sectors by workforce skill differs across countries.[5]

To explain assignment reversals Section 3 develops an assignment model in which worker sorting results from variation in the effective cost of non-labor inputs across both countries and industries. This approach is motivated by Section 2.3 which shows that, controlling for country and industry fixed effects, average wages are higher in industries with lower capital costs. To isolate plausibly exogenous variation in the cost of capital, I combine data on countries' geographic proximity to major capital exporters and on the composition of capital investment by industry to construct a measure of the cost of capital imports that varies across industries within a country. The results imply that when an industry has cheaper access to capital equipment, it employs higher skilled workers.

The assignment model builds upon the idea that skilled workers leverage their ability through their span of control over other production inputs. This idea has motivated work on the allocation and pay of managerial talent (Rosen, 1982; Garicano and Rossi-Hansberg, 2006; Gabaix and Landier, 2008; Terviö, 2008), but has not been used to study the allocation of skill across sectors. Formally, the model marries Roy (1951) to Becker (1973) by allowing for both multiple sectors and matching between two factors of production with non-zero opportunity costs: heterogeneous labor and an homogeneous non-labor input. This assignment problem is new to the literature.[6]

[4] Krueger and Summers (1986) and Abowd et al. (1999) document that inter-industry wage differences are mostly due to variation in workforce composition. Cross-country data on workforce skill is not available at the level of disaggregation used in Fig. 1. However, Section 2.1 shows there exists cross-country variation in the ranking of industries by workforce education at a more aggregate level.

[5] Kurokawa (2011) documents the existence of an assignment reversal between the US and Mexico.

[6] Sattinger (1979) considers the problem of matching heterogeneous workers to machines of different quality when all worker-machine pairs produce the same output good and machines are in fixed supply. However, in existing models with multiple sectors either production uses a single input as in comparative advantage assignment models such

Consider a competitive economy with a continuum of agents who differ along a single dimension of heterogeneity called skill and sort across a finite number of sectors. In comparative advantage assignment models the production technology is assumed to take the form:

$$y(\theta, k) = g(\theta)F(\theta, k),$$

where y is the output of a skill θ agent working in sector k. Provided F is log-supermodular there is positive assortative matching of high skill agents to high k sectors. I extend this framework by allowing agents to work with an additional input which can be interpreted as materials, land or capital. A production team comprising one agent working with input quantity x in sector k produces output:

$$y(\theta, k) = g(\theta)F(\theta, Q_k x),$$

where Q_k represents input productivity in sector k, g is strictly increasing in θ and F exhibits constant returns to scale. Variation in Q induces changes in the effective input cost and higher Q is equivalent to a lower input price. The restriction on g implies the existence of increasing returns to skill. Importantly, the input quantity used by each agent is endogenous and chosen to maximize profits.

Solving the assignment problem shows that, in equilibrium, a log-submodular production function implies positive assortative matching between skill and sectoral input productivity. Interestingly, this result reverses the condition on F required for positive assortative matching in comparative advantage assignment models. The switch is a consequence of making input quantity endogenous. The choice of x determines an agent's span of control. Since there are increasing returns to skill, higher skill agents leverage their abilities by having larger spans of control. F is strictly log-submodular if and only if the elasticity of substitution between inputs exceeds unity and this substitutability implies agents with greater spans of control produce relatively more in sectors where the effective input cost is lower. Consequently, positive assortative matching is the efficient allocation. This is an example of the scale of operations effect discussed in Sattinger (1993). By contrast, if each agent must work with the same input quantity, substitutability mandates that high skill agents work with low productivity inputs and log-submodularity of F implies negative assortative matching.[7]

Assignment reversals occur when the ranking of sectors by input costs or productivity varies across countries. What could cause such variation? First, Ricardian non-labor input augmenting technology differences at the country-sector level. For example, in France land is more productive for producing wine than rice, while in Bangladesh the reverse is true. Similarly, countries with better contracting institutions are likely to have higher relative productivity in sectors that use contract intensive inputs. Second, input price variation. Due to trade costs the price of imported inputs increases with distance from the exporting country, which raises relative input costs in sectors that use imported inputs intensively. The evidence in Section 2.3 is consistent with this channel affecting the equilibrium labor assignment. Since existing multi-sector models of skill allocation assume an invariant ranking of sectors by workforce skill they do not admit the possibility of assignment reversals.[8] In the Heckscher–Ohlin model the key assumption is

as Sattinger (1975) and Costinot and Vogel (2010) or production combines different types of labor in fixed quantities (Grossman and Maggi, 2000; Grossman, 2004).

[7] Similarly, if the production function is strictly log-supermodular the equilibrium assignment exhibits positive assortative matching if input quantity is fixed and negative assortative matching if it is endogenous.

[8] An exception is Murphy et al. (1991) who discuss the possibility of cross-country assignment reversals in the allocation of talent between rent seeking and entrepreneurial activities.

that there are no factor intensity reversals. Consequently, Heckscher–Ohlin skill intensity reversals could explain observed assignment reversals. However, skill intensity reversals result from cross-country variation in the skill premium and in Section 2.2 I find no evidence the assignment reversals evident in Fig. 1 are driven by differences in the skill premium.

The equilibrium assignment also has interesting implications for the distribution of wages:

1. Labor's share of output is decreasing in worker skill and, therefore, in wages – a correlation that is observed empirically.
2. Holding skill constant, the returns to skill (the elasticity of wages with respect to skill) are higher in sectors with greater input productivity or lower input cost. Consistent with this prediction Gibbons et al. (2005) estimate the returns to skill are greater in occupations that employ higher skill workers.

Moreover, the span of control is a sufficient statistic for both wage inequality and labor's share of output. Scale and skill are complements and whenever agents' spans of control increase wage inequality rises and labor's output share falls. Consequently, it is straightforward to use the model to study the causes of variation in wage inequality.

Section 4 embeds the assignment problem in general equilibrium in a two sector closed economy and Section 5 introduces trade between two countries. The fact the wage rank correlation in Fig. 1 is strongly increasing in income per capita implies assignment reversals are rare between developed countries, but relatively common when comparing developed and developing countries. Therefore, in the open economy I consider two cases: North–North trade where the ranking of sectors by input productivity and, consequently, workforce skill is the same in both countries, and; North–South trade where there is an assignment reversal across countries. North–North trade leads to a Stolper–Samuelson effect – trade raises wage inequality between high and low skill workers in the country that has a comparative advantage in the high skill sector and reduces wage inequality between high and low skill workers in the other country.

However, in the North–South case with an assignment reversal I find that: (i) both countries have a comparative advantage in their high skill, high wage sector regardless of their relative factor endowments; (ii) trade liberalization causes the high skill sector to expand in both countries and both countries export the output of their high skill sector, and; (iii) in both countries trade liberalization causes wage levels and wage inequality to increase in the high skill sector and decrease in the low skill sector implying both countries experience wage polarization and increased inequality between high and low skill workers. Therefore, assignment reversals offer a new explanation for why trade liberalization has led to increased wage inequality not only in the relatively skill abundant North, but also in the relatively skill scarce South.

Unlike alternative mechanisms through which trade has been linked to increases in wage inequality, the assignment reversals channel cannot co-exist with Stolper–Samuelson effects since assignment reversals imply the ranking of sectors by workforce skill differs across countries which overturns the logic behind the Stolper–Samuelson theorem. Since the inter-industry trade share and skill endowment differences are greater between less similar countries, in the absence of assignment reversals Stolper–Samuelson effects should be most important in North–South trade. An important consequence of North–South assignment reversals is that they explain why Stolper–Samuelson effects may not occur even as a consequence of inter-industry output trade between North and South.

I also use the assignment model to study the effects of technical change and trade in the non-labor input. Technical change increases the returns to skill whenever it reduces the effective input cost allowing agents to increase their spans of control. Input trade liberalization allows importers to purchase the non-labor input at lower cost. Thus, it is equivalent to input cost reducing technical change and causes an increase in within sector returns to skill and wage inequality in the importing country. This prediction is consistent with the findings of Csillag and Koren (2009) and Parro (2013) who show that capital imports increase the relative wage of high skill labor.

This paper demonstrates how endogenizing the sectoral skill allocation leads to a new perspective on the determinants of the wage distribution and the effects of output trade. In related work, Grossman et al. (2015) develop a model in which there are two heterogeneous labor types and agents both sort across sectors and match to form production teams within sectors. However, Grossman et al. (2015) do not consider how cross-sector variation in effective input costs affects sorting and do not allow for the production technology to vary across countries. Consequently, they do not address the causes and consequences of assignment reversals.

2. Assignment reversals evidence

The allocation of skill across sectors can be inferred from data on industry wages and workforce characteristics. Since, for most countries, industry wage data is available at a more disaggregated level than measures of workforce skill, this section starts by treating an industry's mean wage per employee as a measure of the average skill of the industry's workforce. Under this assumption, I demonstrate that industry wage data implies the existence of cross-country assignment reversals which are systematically related to countries' income levels. I then show that using available data on observable measures of workforce skill leads to the same conclusion. Next, I consider the possibility the assignment reversals found in the data result from Heckscher–Ohlin skill intensity reversals. I find no support for this hypothesis. Finally, I provide evidence that variation in the cost of capital equipment across countries and industries leads to assignment reversals.

2.1. International wage structure comparisons

The assumption that inter-industry wage differences primarily reflect differences in workforce skill, rather than variation in industry specific rents is supported by the empirical literature on inter-industry wage differences. Krueger and Summers (1986) find that observable worker characteristics alone account for around half of inter-industry wage differences in the US. Moreover, once panel data is used to also control for unobservable worker characteristics, the explanatory power of workforce composition rises further (Abowd et al., 1999; Abowd et al., 2002).

Studies of inter-industry wage differences have generally concluded that the pattern of industry wages is highly correlated across countries. For example, Krueger and Summers (1986) find that in eight of the thirteen countries they consider the correlation of log wages with the US exceeds 0.8,[9] leading them to conclude that the "wage structure is amazingly parallel in looking at data for different countries" (p. 1). However, the consensus found in the literature has emerged primarily from comparisons between industrialized economies. Noting that four of the five countries with correlations below 0.8 are non-industrialized economies Krueger and Summers (1986) caution that the wage structure in mature capitalist economies is "different from that of Communist or less developed economies" (p. 2).

[9] The correlations are calculated using wage data for around 20 manufacturing industries in 1981 or 1982.

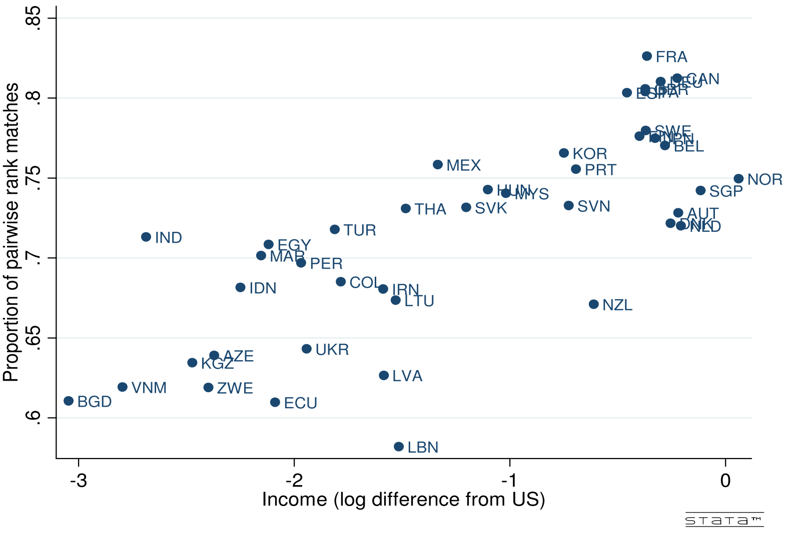

Fig. 2. Pairwise industry wage rank matches – UNIDO 2000.

Fig. 1, discussed previously in the introduction, shows that this claim continues to hold when looking at industry wage data for a broader sample of countries than considered by Krueger and Summers (1986). Remember that Fig. 1 shows wage rank correlations (the correlation between the ranking of industries by wage levels in a given country and the ranking in the US) plotted against income per capita (expressed as log differences from US per capita income). Regressing the wage rank correlation on the income difference gives a significant slope coefficient of 0.13 (Table 1, column a). The positive association is robust to weighting observations by industry employment shares when calculating the wage rank correlations (column b) and to computing correlations using wages instead of wage ranks (columns c and d). An alternative approach to quantifying the similarity between a country's inter-industry wage structure and that of the US is to compute the proportion of industry pairs in which the ranking of industries by wage levels is the same as in the US. Fig. 2 shows that for a country such as France the proportion exceeds 80%, but for the poorest country in the sample, Bangladesh, it is only 61%. The relationship between income per capita and the proportion of pairwise rank matches is positive and significant (column e). Industry wage data also implies that income convergence with the US is associated with convergence towards the US inter-industry wage structure. Regressing the change in the wage rank correlation on the change in income per capita relative to the US for 70 countries between 1965 and 1995 gives a significant slope of 0.12 (column f).[10]

If poorer countries report less reliable data, these findings could be caused by measurement error. To allay this concern Fig. 3 shows wage rank correlations plotted against income per capita using industry wage data for 1995 taken from the EU KLEMS database. The EU KLEMS database is designed to provide accurate industry level data for use in growth accounting exercises. The database covers 29 countries (the EU-25 plus Australia, Japan, South Korea and the US) and, at its most disaggregated level, 29 manufacturing industries. Again, the wage rank correlation is strongly increasing in per capita income, but the slope of the relationship is larger than

[10] The wage data covers 28 ISIC 3 digit manufacturing industries. See Appendix C for further details.

Table 1
International wage structure comparisons.

Dependent variable:	Wage rank correlation		Wage correlation		Proportion pairwise rank matches	Δ Wage rank correlation	Wage rank correlation
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Income per capita	0.13***	0.075***	0.14***	0.089***	0.055***		0.26***
	(0.016)	(0.024)	(0.028)	(0.031)	(0.0068)		(0.064)
Δ Income per capita						0.12**	
						(0.052)	
Constant	0.75***	0.79***	0.70***	0.76***	0.78***	0.0049***	0.69***
	(0.025)	(0.037)	(0.042)	(0.045)	(0.011)	(0.0012)	(0.036)
R^2	0.57	0.19	0.36	0.19	0.58	0.06	0.41
N	42	42	42	42	42	70	25

Robust standard errors in parentheses.
Income per capita is expressed as the log difference from US per capita income.
*Indicates coefficient statistically significant at 10% level; **at 5% level, and; ***at 1% level.
Columns (a)–(e) use UNIDO Industrial Statistics wage data covering 127 4 digit manufacturing industries in 2000.
Columns (b) and (d) use correlations calculated using industry employment shares as weights.
Column (f) uses UNIDO Industrial Statistics wage data covering 28 3 digit manufacturing industries and calculates changes between 1965 and 1995.
Column (g) uses EU Klems wage data covering 29 manufacturing industries in 1995.

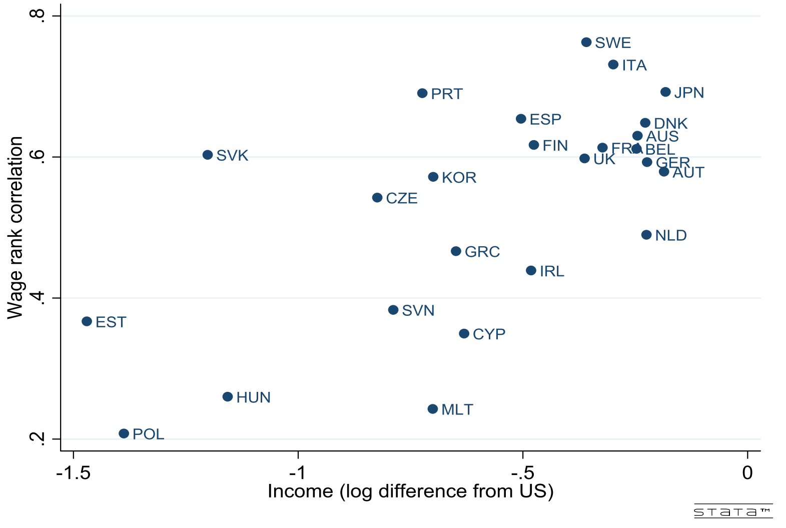

Fig. 3. Wage rank correlations – EU KLEMS 1995.

in the UNIDO data. Regressing the wage rank correlation on the income per capita difference gives a slope of 0.26.

The results in Table 1 support Krueger and Summers' (1986) hypothesis that while developed countries have strikingly similar industry wage structures, this similarity does not extend to developing economies. Under the maintained assumption that inter-industry wage differences stem from variation in workforce skill, the cross-country variation in wage rank correlations implies that assignment reversals exist and occur more frequently between countries at different stages of development than between countries with similar income per capita levels.

Unfortunately, cross-country data on industry workforce skill is not available at the same level of disaggregation as industry wage data. However, the IPUMS-International database of individual-level censuses does report both respondents' industry (at approximately the 1 digit level) and their educational attainment. From this data I calculated the share of workers in each country-industry pair who had completed secondary school and used this measure of industry skill intensity to compute the "skill rank correlation" of each country with the US. There is a positive association between skill rank correlations and income per capita, although the slope of 0.018 is smaller than for the wage rank correlations considered above (Table 2, column a). A stronger positive association is found if tertiary education completion shares are used (column b). These results are consistent with the industry wage data, but more disaggregated workforce skill data is needed to directly observe assignment reversals.

2.2. *Skill intensity reversals?*

Could the cross-country variation in wage rank correlations be caused by Heckscher–Ohlin skill intensity reversals?[11] Consider a multi-sector Heckscher–Ohlin economy in which pro-

[11] See Minhas (1962) and Leontief (1964) for analysis of the conditions under which factor intensity reversals may occur and a debate over their existence. The extensive literature on factor intensity reversals tends to conclude that capital intensity reversals are of limited empirical relevance, but has largely overlooked skill intensity reversals.

Table 2
International industry skill comparisons.

Dependent variable:	Skill rank correlation	
	Secondary (a)	Tertiary (b)
Income per capita	0.018*	0.046**
	(0.010)	(0.020)
Constant	0.89***	0.93***
	(0.015)	(0.024)
R^2	0.11	0.33
N	32	32

Robust standard errors in parentheses.
Income per capita is expressed as the log difference from US per capita income.
*Indicates coefficient statistically significant at 10% level; **at 5% level, and; ***at 1% level.
Education data is from IPUMS-International and covers 15 industries.

duction uses two types of labor: skilled and unskilled. In each industry the skill intensity of production will depend on the skill premium and the elasticity of substitution between skilled and unskilled labor, while in each country the skill premium will depend on output prices and the supply of skilled relative to unskilled labor. If skill intensity reversals occur, then industries in which the elasticity of substitution is relatively high will be skilled labor intensive in countries with low skill premia and unskilled labor intensive in countries with high skill premia. In particular, if all industries use constant elasticity of substitution production technologies it is simple to show that the number of skill intensity reversals between any two countries is an increasing function of the difference between their skill premia.[12] Therefore, if variation in wage rank correlations is caused by Heckscher–Ohlin skill intensity reversals, it will be strongly correlated with variation in skill premia.

Internationally comparable measures of the skill premium are not available for the majority of the countries in the UNIDO sample used in Fig. 1. However, differences in skill premia across countries are well explained by variation in human capital levels.[13] Therefore, to crudely examine whether cross-country differences in the inter-industry wage structure are due to Heckscher–Ohlin skill intensity reversals I regress the wage rank correlations shown in Fig. 1 on countries' stocks of physical and human capital per capita.[14] There is a strong positive association between the capital stock and the wage rank correlation, but the human capital variable is insignificant (Table 3, column a). Similar results are obtained when the secondary school enrollment rate, which is available for a larger sample of countries, is used as a proxy for the skill premium (column b). These findings do not support the conjecture that Heckscher–Ohlin skill intensity reversals are driving cross-country variation in wage rank correlations.

2.3. Equipment costs and the inter-industry wage structure

To motivate the assignment model developed in Section 3, I next provide evidence that differences in non-labor input costs can generate cross-country variation in the inter-industry wage

[12] See Reshef (2007) for a theoretical analysis of the causes and consequences of skill intensity reversals in such a model.

[13] See, for example, Fernandez et al. (2005) and Brambilla et al. (2012).

[14] The physical and human capital variables are computed using the methodology of Caselli (2005) and are expressed as the log difference from US physical and human capital per capita, respectively. See Appendix C for details.

Table 3
Wage structure and factor endowments.

Dependent variable:	Wage rank correlation	
	(a)	(b)
Human capital per capita	−0.0029	
	(0.14)	
Secondary enrollment rate		−0.022
		(0.066)
Physical capital per capita	0.085***	0.095***
	(0.028)	(0.015)
Constant	0.72***	0.71***
	(0.038)	(0.025)
R^2	0.53	0.54
N	32	41

Robust standard errors in parentheses.
Human capital, secondary enrollment rate and physical capital are expressed as the log difference from their respective US values.
*Indicates coefficient statistically significant at 10% level; **at 5% level, and; ***at 1% level.
UNIDO Industrial Statistics wage data covers 127 4 digit manufacturing industries in 2000.

structure resulting in assignment reversals. Specifically, I show that controlling for country and industry fixed effects, the mean wage is higher in industries where the cost of capital equipment is lower. In order to obtain a measure of the cost of capital equipment that varies across countries and industries and is plausibly exogenous to industry wages, I exploit the interaction between differences in the composition of equipment investment across industries and the geographic distribution of equipment production.

Suppose capital is produced as a Cobb–Douglas aggregate of I equipment varieties with expenditure shares that vary by industry. Then the cost of capital equipment P_{kc} in industry k of country c is given by:

$$P_{kc} \propto \prod_{i=1}^{I} p_{ic}^{\alpha_i^k} \tag{1}$$

where p_{ic} is the price of equipment variety i in country c and α_i^k is the share of industry k's capital expenditure allocated to equipment variety i. Note that the expenditure shares do not vary across countries and the equipment variety prices do not vary across industries. I divide equipment into $I = 15$ varieties and use capital expenditure data for US industries to compute the expenditure shares.[15]

Equipment variety prices are not available for most countries and, even if available, may be endogenous to industry outcomes. To overcome these difficulties I make use of three empirical regularities documented by Eaton and Kortum (2001) and Caselli and Wilson (2004). First, equipment production is highly concentrated in a handful of countries that invest heavily in research and development. Second, equipment imports from the major equipment producers

[15] See Appendix C for a detailed description of the data used in this section.

Table 4
Revealed advantage in equipment exports.

Exporter	Revealed advantage			
	Mean	Std. dev.	Min.	Max.
Canada	6.7%	4.6%	2.4% (Electrical apparatus)	16.7% (Railroad equipment)
China	8.1%	9.5%	0.1% (Aircraft)	31.8% (Furniture and fixtures)
France	8.4%	4.7%	3.2% (Metalworking machinery)	22.8% (Aircraft)
Germany	16.8%	6.8%	8.4% (Computers, office and accounting equipment)	27.9% (Agricultural machinery)
Italy	7.8%	5.3%	1.8% (Computers, office and accounting equipment)	18.1% (Furniture and fixtures)
Japan	18.7%	14.2%	1.5% (Furniture and fixtures)	53.0% (Ships and boats)
United Kingdom	7.4%	3.9%	3.7% (Furniture and fixtures)	16.8% (Aircraft)
United States	26.1%	10.0%	5.0% (Ships and boats)	44.7% (Engines and turbines)

Revealed advantage computed for 15 equipment types in 2000.
Revealed advantage defined as the exporter's share of total exports of the equipment type by the eight exporters listed in the table.
The equipment types in which each country has its minimum and maximum revealed advantage are listed in parentheses.

account for over half of equipment investment in most countries.[16] Third, trade costs generate variation in the cost of equipment across countries. In addition, I document below that there is substantial variation in the market shares of major equipment producing countries across equipment varieties. This implies the relative production costs of different producers varies across equipment varieties. Based on these four facts, I conjecture that the price of each equipment variety is lower, ceteris paribus, in countries that are geographically close to the major exporters of that equipment variety. Therefore, in each country the cost of capital equipment will be relatively low in industries that use intensively equipment varieties for which the country is geographically close to exporters with large export market shares. If valid, this conjecture provides a source of variation in the cost of capital equipment that is both measurable and exogenous to industry wage outcomes.

To implement this idea I define the major equipment exporters to be the eight largest equipment exporters between 1995 and 2000: US, Japan, Germany, France, UK, Canada, Italy and China. Each of these countries accounted for more than 3.5% of world exports of the 15 equipment varieties between 1995 and 2000 and collectively they accounted for 64% of equipment exports. Let the revealed advantage RA_{id} of exporter d in equipment variety i be d's share of total exports of equipment variety i by the 8 major equipment exporters. Table 4 shows summary statistics on revealed advantages in 2000. There is substantial within-exporter variation in revealed advantage. The average coefficient of variation across the eight exporters is 64% and each exporter has a revealed advantage below 9% in at least one equipment variety and above

16 In fact, Caselli and Wilson (2004) argue that "for most countries, imports of capital of a certain type are an adequate proxy for overall investment in that type of equipment" (p. 2).

Table 5
Imports, investment and the cost of capital equipment.

Dependent variable:	Imports	Investment per worker	
	(a)	(b)	(c)
log Cost of imported equipment	−3.395** (1.672)		
log Cost of imported capital		−9.818*** (3.019)	−9.396*** (3.277)
Capital interaction			0.014 (0.024)
Skill interaction			−0.080 (0.136)
Contract interaction			0.242 (0.151)
Equipment variety dummies	Yes		
Industry dummies		Yes	Yes
Country fixed effects	Yes	Yes	Yes
R^2	0.74	0.33	0.33
N	514	2891	2707

Standard errors in parentheses. In column (a) standard errors are clustered by country. In columns (b) and (c) standard errors are clustered by country and 2 digit industry.
All dependent variables expressed in logs.
The R^2 statistic reports the within R^2.
*Indicates coefficient statistically significant at 10% level; **at 5% level, and; ***at 1% level.
In column (a) sample includes 15 equipment types in 36 countries in 2000.
In columns (b) and (c) sample includes 127 ISIC 4 digit manufacturing industries in 36 countries in 2000.

16% in some other equipment variety. I define the cost of imported equipment *CIE* for equipment variety i in country c as:

$$CIE_{ic} = \left(\sum_{d=1}^{8} \frac{RA_{id}}{GD_{cd}} \right)^{-1},$$

where *GD* measures the gravity-adjusted distance between country c and country d. The gravity-adjusted distance is computed as:

$$GD_{cd} = -\hat{\eta}_0 \log dist_{cd} - \hat{\eta}_1\, lang_{cd} - \hat{\eta}_2\, bord_{cd} - \hat{\eta}_3\, col_{cd},$$

where *dist* denotes distance and *lang*, *bord* and *col* are dummy variables indicating whether countries c and d share a common language, share a border or were ever in a colonial relationship, respectively. The $\hat{\eta}$ coefficients are obtained by estimating a gravity model of equipment trade including *dist*, *lang*, *bord* and *col*, together with importer and exporter fixed effects, as regressors.[17] If geographic proximity to a major equipment exporter lowers the relative cost of equipment varieties in which the exporter has a high revealed advantage, then it should also increase imports of such equipment varieties. Regressing imports by equipment variety in 2000 on the cost of imported equipment I do indeed find that imports are higher when the cost of imported equipment is lower (Table 5, column a).

I use the cost of imported equipment CIE_{ic} as a proxy for the equipment variety price p_{ic} in (1) and define the cost of imported capital *CIC* in industry k and country c as:

[17] The estimated coefficients are: $\hat{\eta}_0 = -1.21$; $\hat{\eta}_1 = 0.59$; $\hat{\eta}_2 = 0.54$; $\hat{\eta}_3 = 0.91$.

$$CIC_{kc} = \prod_{i=1}^{I} CIE_{ic}^{\alpha_i^k}.$$

Columns (b) and (c) of Table 5 show that industry level investment per worker is higher when the cost of imported capital is lower. These results support the hypothesis that *CIC* captures variation in industries' capital equipment costs.

To analyze how the cost of capital equipment affects the inter-industry wage structure I estimate the following equation:

$$\log \omega_{kc} = \gamma \log CIC_{kc} + \phi X_{kc} + \alpha_k + \delta_c + \epsilon_{kc}, \tag{2}$$

where ω is the mean industry wage, X is a vector of controls, α_k is an industry dummy and δ_c is a country fixed effect. As controls I use the logarithms of industry level measures of capital, skill and contract intensity computed from US data interacted with the logarithms of country level measures of capital abundance, skill abundance and the rule of law, respectively.[18] The sample covers 36 countries and 127 ISIC 4 digit industries. The eight major equipment exporters are excluded from the sample since the cost of imported capital is endogenous in these countries.

The results of estimating (2) are shown in Table 6. The mean industry wage is higher when the cost of imported capital is lower and this effect is observed regardless of whether the capital, skill and contract interactions are excluded (column a) or included (column b). In order to generate assignment reversals, variation in the cost of imported capital should affect the industry wage ranking. When the dependent variable is an industry's percentile rank in the wage distribution, the cost of imported capital is insignificant if it is the only explanatory variable (column c), but remains significant when the interaction controls are included (column d).

A concern with these results is the possibility of reverse causality. Suppose higher wage industries use capital more intensively. The resulting demand for equipment imports could generate reverse causality through its effect on the revealed advantages of equipment exporters. The demand effect would be strongest between neighboring countries and when the importing country is economically large. Another issue is measurement error. The relationship between the cost of capital equipment P_{kc} and the cost of imported capital CIC_{kc} is likely to be weaker in richer countries that produce a larger share of their capital equipment domestically, leading to measurement error that is systematically correlated with per capita income. To mitigate these concerns I restrict the sample to countries with low income per capita. In the restricted sample measurement error should be reduced. In addition, the estimates are less likely to be biased by reverse causality since the low income per capita countries are on average smaller economies and also geographically more distant from the major equipment exporters. Columns (e)–(h) of Table 6 repeat the regressions shown in columns (a)–(d), but with the sample restricted to countries with income per capita below the sample median. A lower cost of imported capital raises both the industry wage and the industry's rank in the wage distribution. The magnitude of the estimates is roughly four times larger than in columns (a)–(d), which is consistent with the full sample estimates suffering from attenuation bias. Based on the estimate in column (e) of Table 6, a one standard deviation increase in the log cost of imported capital causes a 3.3% increase in the industry wage.[19] Collectively, the results in Table 6 show wages are higher in industries that have

[18] To understand the choice of controls note, for example, that in the absence of factor price equalization the Heckscher–Ohlin model predicts that relatively skill abundant countries will have relatively lower industry wages in relatively skill intensive industries.

[19] The standard deviation is calculated using the low income sample after demeaning the log cost of imported capital by country and industry.

Table 6
Cost of imported capital and the inter-industry wage structure.

Dependent variable:	Full sample				Low income per capita sample			
	Wage		Wage rank percentile		Wage		Wage rank percentile	
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
log Cost of imported capital	−3.151***	−3.639***	−0.565	−2.234**	−12.169***	−12.673***	−8.772***	−8.770***
	(1.080)	(1.235)	(0.841)	(0.891)	(3.459)	(3.599)	(2.009)	(2.146)
Capital interaction		−0.011		0.035***		0.020		0.031***
		(0.009)		(0.005)		(0.016)		(0.008)
Skill interaction		−0.071		0.099***		−0.009		−0.010
		(0.056)		(0.036)		(0.108)		(0.059)
Contract interaction		0.087*		0.073***		0.129		0.098**
		(0.045)		(0.024)		(0.088)		(0.048)
Industry dummies	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.39	0.39	0.46	0.48	0.39	0.40	0.40	0.42
N	3656	3444	3668	3454	1899	1832	1899	1832

Standard errors, clustered by country and 2 digit industry, in parentheses.
All dependent variables, except Wage rank percentile, expressed in logs.
The R^2 statistic reports the within R^2.
*Indicates coefficient statistically significant at 10% level; **at 5% level, and; ***at 1% level.
In columns (a)–(d) the sample includes 127 ISIC 4 digit manufacturing industries in 36 countries in 2000.
In columns (e)–(h) the sample is restricted to the 18 countries with income per capita below the sample median.

access to relatively cheap sources of imported capital. This suggests the cost of capital equipment plays a significant role in shaping the inter-industry wage structure and generating assignment reversals.

3. Assignment model

To understand how heterogeneous workers sort across sectors and the causes of assignment reversals this section develops a multi-sector labor assignment model and characterizes the equilibrium assignment. Motivated by the evidence in Section 2.3, the model focuses on how non-labor input costs affect sorting.

3.1. Assignment problem

Consider an economy facing the following assignment problem. There exists an heterogeneous factor that differs along a single dimension of heterogeneity indexed by θ. To be concrete, suppose the factor is labor and there are a continuum of agents with differing skill levels θ. Let $M(\theta)$ be the mass of agents with skill less than or equal to θ and suppose M has support on $(0, \bar{\theta}]$. Bounded support is the only restriction on the skill distribution M required to obtain the main results of the paper.

The economy comprises K productive activities in which labor can be employed. The interpretation of these productive activities depends on how the assignment problem is embedded in general equilibrium. For consistency with the general equilibrium model in Section 4 I will refer to the productive activities as sectors, but they could also be tasks or occupations. Each sector produces a different good and the production technology varies across sectors. The assignment problem is to characterize the mapping of agents to sectors.

Suppose in all sectors output is produced by production teams, each of which consists of one agent working with an homogeneous non-labor input.[20] In particular, let the output of a skill θ agent working with x units of input in sector k, $y_k(\theta, x)$, be given by:

$$y_k(\theta, x) = g(\theta) F(\theta, Q_k x), \tag{3}$$

where g is non-negative, differentiable and strictly increasing and F is a twice differentiable, constant returns to scale function that is strictly increasing in both its arguments, strictly concave and satisfies $\lim_{\theta \to 0} \frac{\partial F}{\partial \theta} = \lim_{x \to 0} \frac{\partial F}{\partial x} = \infty$. Within a sector all production teams produce the same output good.

Four features of the production function are particularly noteworthy. First, the labor input to production is indivisible. If, instead, agents with different skill levels were perfect substitutes within production teams, θ would simply measure an agent's efficiency units of labor and there would be no assignment problem. Second, skill enters production symmetrically in every sector. Holding $Q_k x$ fixed, the marginal effect of skill on output is constant across sectors. Third, g captures the existence of increasing returns to ability. Fourth, Q_k is an input augmenting productivity term which captures technology variation across sectors. This is the only source of cross-sector heterogeneity.[21] Note that $Q_k x$ can be interpreted as the quantity of input used measured in ef-

[20] The model does not speak to where the boundaries of the firm may lie, so I will refer to the basic unit of activity as a production team.

[21] Appendix B generalizes the production function to allow for multiple sources of cross-sector heterogeneity and demonstrates that the effect of input productivity on the equilibrium assignment is robust to this extension.

ficiency units and $1/Q_k$ is the cost per efficiency unit of input. Thus, the effective input cost is decreasing in Q_k. I assume sectors are ordered such that Q_k is increasing in k with sector one being the least technologically advanced and sector K the most.

Assume there is perfect competition in all markets, that all sectors must produce positive aggregate output and that the homogeneous input is in perfectly elastic supply at cost p. Provided x is a choice variable there is no loss of generality in assuming all sectors use the same input since allowing for variation in input cost across sectors is equivalent to varying Q_k. We can now solve the assignment problem in partial equilibrium taking the input cost and the existence of positive aggregate demand for each sector's output as given. The general equilibrium model developed in Section 4 shows that the partial equilibrium assignment patterns are robust to endogenizing input supply and explicitly specifying output demand.

Formally, the production function in (3) is similar to that used by Rosen (1982) in a single sector model of firm hierarchies. In theory, the input could represent materials, land, capital or an homogeneous labor input. Cross-sector technology heterogeneity may result from either sector specific input augmenting technology investments or from sector specific differences in the input price. Following Rosen (1982) the form of the production function can be motivated by assuming a skill θ agent supplies θ units of labor input and produces output of quality $g(\theta)$. $Q_k x$ denotes the quantity of input used, measured in efficiency units, and diminishing returns to x result from spreading a fixed labor input over an increasing quantity of non-labor input. However, the fact higher skill agents produce higher quality output means there are increasing returns to skill.

3.2. Equilibrium assignment

The solution to the assignment problem depends crucially on whether or not the quantity of input used x is endogenous. Suppose for now x is endogenous. Then an agent with skill θ working in sector k chooses x to maximize her wage $w_k(\theta)$ which equals the value of output produced by her production team less the input cost:

$$w_k(\theta) = \max_{x \geq 0} \{\psi_k y_k(\theta, x) - px\},$$

where ψ_k is the price of sector k output. Remembering F has constant returns to scale it is useful to express the production technology in intensive form. Define $f(s) = F(1, s)$ where $s \equiv \frac{Q_k x}{\theta}$ denotes the agent's span of control. The span of control measures the efficiency units of input used per unit of skill and captures the extent to which an agent leverages her ability by working with large amounts of the input. Note that f is strictly increasing and strictly concave. With this change of variables:

$$y_k(\theta, s) = \theta g(\theta) f(s), \tag{4}$$

and the agent's maximization problem is:

$$w_k(\theta) = \max_{s \geq 0} \left\{ \theta \left[\psi_k g(\theta) f(s) - \frac{p}{Q_k} s \right] \right\}. \tag{5}$$

The elasticity of output with respect to the span of control will play a key role in determining the equilibrium assignment. From (4) we see that this elasticity equals the elasticity of f with respect to the span of control $\epsilon^f(s)$. I will refer to $\epsilon^f(s)$ as the output elasticity. The properties of the output elasticity are given by the following lemma. The proofs of all lemmas and propositions are in Appendix A.

Lemma 1. *The following are equivalent: (i) F is strictly log-submodular; (ii) F has elasticity of substitution greater than one; (iii) $\epsilon^f(s)$ is strictly increasing in the span of control s.*

Similarly, strict log-supermodularity of F is equivalent to F having elasticity of substitution σ less than one and to $\epsilon^f(s)$ being strictly decreasing in s, while if $\sigma = 1$ then $\epsilon^f(s)$ is independent of s.[22] Following Acemoglu (2002) I will refer to labor and the homogeneous input as gross complements if $\sigma < 1$ and gross substitutes if $\sigma > 1$. Note that σ need not be constant, but any restrictions on σ are assumed to hold globally.

Solving the maximization problem (5) implies the optimal span of control $s_k(\theta)$ satisfies:

$$f'[s_k(\theta)] = \frac{p}{\psi_k Q_k g(\theta)}. \tag{6}$$

Henceforth, I will suppress the dependence of s_k on θ unless its inclusion is necessary to avoid confusion. Since f is strictly increasing and strictly concave, f' is positive and strictly decreasing implying that the span of control is strictly decreasing in the effective input cost p/Q_k, but strictly increasing in the output price ψ_k and the agent's skill θ. The span of control is increasing in θ only because $g' > 0$, that is because there exist increasing returns to ability. Substituting (6) back into (5) implies the sector k wage function $w_k(\theta)$ is given by:

$$w_k(\theta) = \psi_k \theta g(\theta) \left[f(s_k) - s_k f'(s_k) \right]. \tag{7}$$

The above analysis solves the agent's optimization problem conditional on her sector of employment, but how do agents sort across sectors? In equilibrium each agent chooses to work in the sector k that maximizes her wage:

$$w(\theta) = \max_{k=1,\dots,K} \{w_k(\theta)\}.$$

Agents choose sectors taking output prices as given, but since all sectors produce positive aggregate output by assumption, output prices must be such that a positive mass of agents sorts into every sector.

Consider an agent choosing between two sectors k and l with $Q_k > Q_l$. To ensure wages in sector k are not strictly higher than wages in sector l at all skill levels we must have $\psi_k < \psi_l$.[23] The intuition is straightforward – if sector k has both a better technology and a higher output price than sector l then all agents will prefer to work in sector k. Similarly, to guarantee some workers prefer sector k we must have $\psi_k Q_k > \psi_l Q_l$, which implies $s_k(\theta) > s_l(\theta)$. An agent's span of control is greater in the more technologically advanced sector.

Now by differentiating (7) we obtain:

$$\frac{d}{d\theta}\left[\frac{w_k(\theta)}{w_l(\theta)}\right] \propto \epsilon^g(\theta) \left[\epsilon^f(s_k) - \epsilon^f(s_l)\right], \tag{8}$$

where $\epsilon^g(\theta) > 0$ is the elasticity of g with respect to skill. From Lemma 1 we know that if F is strictly log-submodular then the output elasticity is strictly increasing in the span of control.

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22 See Costinot (2009) for a definition and discussion of log-supermodularity and log-submodularity. In particular, I use the fact that F is strictly log-submodular if and only if $\frac{\partial^2 \log F}{\partial\theta\partial x} < 0$. Though implicit in Sattinger (1975) and Kugler and Verhoogen (2012), I am not aware of previous work that demonstrates explicitly the link between log-supermodularity, log-submodularity and the elasticity of substitution of a constant returns to scale production function.

23 Suppose $\psi_k \geq \psi_l$. Then since $Q_k > Q_l$ we have $\psi_k Q_k > \psi_l Q_l$. Noting that $f(s_k) - s_k f'(s_k)$ is strictly increasing in s_k and, therefore, in $\psi_k Q_k$ it follows that $w_k(\theta) > w_l(\theta)$ for all θ, which cannot occur.

Since $s_k(\theta) > s_l(\theta)$ this implies the right hand side of (8) is positive meaning the wage in sector k relative to sector l is strictly increasing in skill. Therefore, to ensure neither sector dominates the other there must exist a threshold $\tilde{\theta} \in (0, \bar{\theta}]$ such that agents with skill below $\tilde{\theta}$ strictly prefer sector l and agents with skill above $\tilde{\theta}$ strictly prefer sector k. On the other hand, if F is log-supermodular the output elasticity is strictly decreasing in the span of control and the sorting pattern is reversed.

The preceding discussion considers only two sectors. However, by comparing all pairs of sectors it is straightforward to extend the results to encompass K sectors. The ranking of sectors by productivity Q_k fully determines the ranking of output prices ψ_k and of $\psi_k Q_k$. With $Q_K > Q_{K-1} > \ldots > Q_1$, then in any equilibrium such that all sectors produce positive aggregate output:

(i) $\psi_1 > \psi_2 > \ldots > \psi_K$;
(ii) $\psi_1 Q_1 < \psi_2 Q_2 < \ldots < \psi_K Q_K$.

These orderings hold regardless of whether or not F is log-submodular. However, when F is strictly log-submodular we also have:

(iii) $\exists\, 0 = \theta_0 \leq \theta_1 \leq \ldots \leq \theta_{K-1} \leq \theta_K = \bar{\theta}$ such that only agents with skill $\theta \in [\theta_{k-1}, \theta_k]$ are employed in sector k.[24]

This means that in equilibrium agents are partitioned by skill and there is positive assortative matching of higher skill agents into more technologically advanced sectors. If F is strictly log-supermodular the sorting pattern is reversed and there is negative assortative matching. Proposition 1 summarizes the equilibrium assignment of agents to sectors when the quantity of input used is endogenous.

Proposition 1 *(Endogenous input quantity). If the production function is strictly log-submodular then the equilibrium assignment of agents to sectors exhibits positive assortative matching. High skill agents are assigned to sectors with high levels of technology. If the production function is strictly log-supermodular then in equilibrium there is negative assortative matching.*

Proposition 1 shows that sorting is driven by cross-sector differences in the effective cost of non-labor inputs. This motivation for sorting is not found in the existing assignment literature which only allows for a single factor of production. I explore below the implications of this result, but one consequence is immediately apparent. Assignment reversals occur when the ranking of sectors by input productivity or, equivalently, effective input cost differs across countries. The importance of assignment reversals for understanding the effects of international trade on wage inequality is discussed in Section 5.

Why does a log-submodular production function lead to positive assortative matching? When F is log-submodular labor and non-labor inputs are gross substitutes. This substitutability means the output elasticity is increasing in the span of control. Since the span of control is increasing in both θ and Q_k, the highest skill agents sort into the sector with the best technology in order to maximize their leverage by using the input with the lowest effective cost. This is an exam-

[24] The inequalities in (iii) will be strict if there are no mass points in the distribution of θ.

ple of a scale of operations effect (Sattinger, 1993). However, if F is log-supermodular having a greater span of control reduces the output elasticity because the complementarity between factors diminishes the value of working with large quantities of input when the labor input is fixed. Consequently, higher skill agents reduce their spans of control by working in sectors with lower Q_k.

When the elasticity of substitution between labor and the non-labor input equals one, the wage function is the same in all sectors and there is no sorting. From (8) we also see that $g' > 0$ is necessary for Proposition 1 to hold. If $g' = 0$ (implying constant returns to ability) span of control is independent of skill by (6) and, in equilibrium, all agents are indifferent between sectors and there is no sorting.

It is useful to compare Proposition 1 with the predictions of the comparative advantage assignment literature (Sattinger, 1975; Ohnsorge and Trefler, 2007; Costinot, 2009; Costinot and Vogel, 2010; Acemoglu and Autor, 2011). In this literature there is a single heterogeneous factor of production, the production function is Ricardian and a log-supermodular production function leads to positive assortative matching between the heterogeneous factor and sectors. For example, if the factor is labor then log-supermodularity of labor productivity in skill and some variable that indexes sectors implies that in equilibrium more skilled labor is assigned to sectors where the marginal effect of skill on labor productivity is greater. By contrast, Proposition 1 implies that log-submodularity of the production function implies positive assortative matching. To reconcile this apparent contradiction we must interpret the production function used in comparative advantage assignment models as a reduced form representation of the revenue function net of all non-labor input costs. This net revenue function is equivalent to the wage function $w_k(\theta)$ discussed above and differentiation of (7) shows that the wage is log-supermodular in θ and Q_k if and only if output is log-submodular and there are increasing returns to ability. An important contribution of this paper is to show how the properties of the net revenue function used in previous assignment models are related to the properties of the underlying production technology when there are two factors of production.[25]

The key assumption under which log-submodularity implies positive assortative matching is not the existence of an homogeneous input, but that the input level x is endogenously chosen. Suppose instead each agent must work with a fixed input quantity $\tilde{x}$. In this case each agent works in the sector where she generates the greatest revenue, exactly as happens in the comparative advantage assignment literature. Wages are given by:

$$w_k(\theta) = \psi_k g(\theta) F(\theta, Q_k \tilde{x}) - p\tilde{x},$$

and comparing sectors k and l with $Q_k > Q_l$ we have that when $w_k(\theta) = w_l(\theta)$:

$$\frac{d}{d\theta}\left[\frac{w_k(\theta)}{w_l(\theta)}\right] \propto \epsilon^f(s_l) - \epsilon^f(s_k), \tag{9}$$

which is negative if F is strictly log-submodular and positive if F is strictly log-supermodular by Lemma 1. Therefore, when the input quantity is exogenously fixed the equilibrium assignment is reversed and a log-submodular production function implies negative assortative matching.

[25] Note that when production uses non-labor inputs a distinction must be made between the primitive production function given in (3) and the equilibrium output function $y_k(\theta, Q_k) = \theta g(\theta) f\left[s_k(\theta)\right]$ which gives output conditional on the optimal input choice. When F is log-submodular, the equilibrium output function can be either log-submodular or log-supermodular. However, the wage function will always be log-supermodular, which ensures positive assortative matching.

Proposition 2 *(Exogenous input quantity). If the production function is strictly log-submodular then the equilibrium assignment exhibits negative assortative matching between skill and input productivity. If the production function is strictly log-supermodular then in equilibrium there is positive assortative matching.*

To understand why fixing x reverses the sorting pattern remember that when F is log-submodular the inputs are gross substitutes. If input quantity is fixed, efficiency requires matching high skill agents with low technology sectors to take advantage of this substitutability. By contrast, if input choice is endogenous high skill agents leverage their ability by using more of the input. When there are increasing returns to skill and the inputs are gross substitutes, the leveraging effect is sufficiently strong that the equilibrium assignment features positive assortative matching.

The switch between positive and negative assortative matching triggered by allowing for input adjustability has potentially interesting implications for how institutional development affects the labor market. For example, consider an economy with a log-submodular production function. Suppose initially financial institutions are under-developed and borrowing constraints force all agents to work with a fixed input quantity. Under these circumstances high skill agents will work in low technology sectors. However, if credit markets develop to the point where agents can pledge some fraction of their income as collateral then more skilled agents will be able to work with greater quantities of input, sorting will reverse and financial development will precipitate dramatic changes in the labor market and the distribution of income.

Since in observed production technologies non-labor input quantity is usually adjustable, I assume x is endogenous for the remainder of this paper. Motivated by the evidence in Section 2.3, I also assume the production technology is log-submodular to ensure that industries with a lower effective input cost employ more skilled workers and pay higher wages. Formally, I make the following assumption.

Assumption 1. (i) The production function is strictly log-submodular in labor skill θ and input quantity x. (ii) Input quantity x is a choice variable.

3.3. Wage distribution

The equilibrium assignment has two important implications for the wage structure. First, from (4) and (7), labor's share of output is given by:

$$\frac{w_k(\theta)}{\psi_k y_k(\theta)} = 1 - \epsilon^f(s_k). \tag{10}$$

Span of control is increasing in skill and under Assumption 1 the output elasticity is increasing in the span of control. It follows that labor's share of output is decreasing in skill, or equivalently wages, both within and across sectors. Moreover, input expenditure equals $\psi_k \theta g(\theta) f(s_k) \epsilon^f(s_k)$ which is increasing in θ, implying input expenditure per worker is increasing in wages and decreasing in labor's share of output both within and across sectors. Proposition 3 summarizes these results.

Proposition 3. *Both across sectors and across production teams within sectors: (i) labor's share of output is strictly decreasing in skill and wages, and; (ii) input expenditure per worker is strictly increasing in skill and wages.*

By contrast, if input quantity is fixed labor's share of output is increasing in θ within and across sectors. While if input quantity is endogenous and the production function is log-supermodular labor's share of output is increasing in skill within sectors, but has discontinuous downward jumps at the thresholds for sector assignment, meaning the cross-sector correlation is in general ambiguous.[26] This shows that when there are multiple factors of production, the pattern of intra-sectoral variation in income shares can be used to infer properties of the production technology. For example, regressing the log of labor's share of value-added on log wages and industry fixed effects using Colombian manufacturing plant data from 1985 gives a slope of -0.18 which is significant at the 1% level.[27] This correlation is consistent with Proposition 3, but not with the existence of a log-supermodular production function.

The second important property of the equilibrium assignment comes from differentiating (7) to obtain the returns to skill:

$$\epsilon^{w_k}(\theta) = 1 + \frac{\epsilon^g(\theta)}{1 - \epsilon^f(s_k)}, \tag{11}$$

where $\epsilon^{w_k}(\theta) \equiv \frac{\theta w_k'(\theta)}{w_k(\theta)}$. Equation (11) implies that, holding θ constant, the span of control is a sufficient statistic for the returns to skill. Moreover, under Assumption 1 the output elasticity is increasing in s_k, meaning a higher span of control raises the returns to skill. Since the span of control is increasing in $\psi_k Q_k$, it follows that the returns to skill are strictly increasing in $\psi_k Q_k$. Intuitively, when labor and the non-labor input are gross substitutes, high skill agents are better able than low skill agents to take advantage of positive technology or output price shocks to increase production levels by working with greater input quantities. Across sectors, $\psi_k Q_k > \psi_l Q_l$ if and only if $Q_k > Q_l$ implying the returns to skill are higher in more technologically advanced sectors. Consistent with this prediction Gibbons et al. (2005) find that returns to skill are higher in more skilled occupations.

The wage distribution depends on both the wage function $w(\theta)$ and the distribution of skill across agents. The model places no restrictions on the shape of the skill distribution, but equation (11), in combination with Lemma 2 below, allows us to characterize how shocks, such as technical change and trade liberalization, affect wage inequality when the skill distribution is held constant.

Lemma 2. *Let $w(\theta)$ and $\tilde{w}(\theta)$ be wage functions such that $\epsilon^w(\theta) > \epsilon^{\tilde{w}}(\theta)\ \forall\, \theta \in (\theta_a, \theta_b) \subseteq \left(0, \bar{\theta}\right]$. Then wage inequality among any subset of agents with skill levels in $[\theta_a, \theta_b]$ is higher under $w(\theta)$ than under $\tilde{w}(\theta)$ for any measure of inequality that respects scale independence and second-order stochastic dominance.*

Lemma 2 tells us that within-group wage inequality rises whenever both the returns to skill increase at all skill levels and membership of the group is unchanged. Adapting an approach used by Helpman et al. (2010) the proof of Lemma 2 shows that, after a change in means, the wage distribution implied by $\tilde{w}(\theta)$ second-order stochastically dominates the distribution implied

[26] Part (ii) of Proposition 3 is unchanged if the production function is log-supermodular.

[27] The data is from Roberts and Tybout (1996). The estimate controls for 4 digit ISIC industry fixed effects. Plant wages are calculated as the sum of salaries and benefits divided by total employment and are instrumented by their two year lag to correct for measurement error. Standard errors are clustered by 4 digit industry. The estimated wage coefficient is negative and significant for every available year (1979–1991) when either labor's share of value-added or labor's share of output is used as the dependent variable.

by $w(\theta)$. Combining Lemma 2 with equation (11) implies that the sign of the change in wage inequality within any group of agents is fully determined by variation in the span of control. This result will be used repeatedly below to characterize how wage inequality is affected by trade.

4. General equilibrium

This section embeds the assignment model developed above in general equilibrium in a closed economy. Section 5 then extends the general equilibrium model to allow for international trade and analyzes the consequences of assignment reversals in an open economy.

To develop a general equilibrium version of the model I need to specify the input production technology and the source of demand for each sector's output. The assignment problem is sufficiently tractable to permit multiple alternative general equilibrium settings. For example, the productive activities agents undertake could be tasks, occupations or industries. Likewise, the input could represent land, materials, capital or homogeneous labor. For the remainder of this paper I assume each productive activity constitutes a separate sector and there exists an aggregate output good that can be used either for consumption or to produce the homogeneous input. These assumptions are chosen primarily for their simplicity, allowing the paper to focus on identifying the new insights that arise from the assignment model. However, in Appendix B I show that the main results continue to hold in a more complex model where agents are assigned to tasks and task outputs are used as factor inputs in a Heckscher–Ohlin model. This alternative set-up gives a version of the Heckscher–Ohlin model in which the ranking of industries by workforce skill is endogenous to the distribution of input productivity across tasks.

4.1. Assumptions

Suppose there are two sectors, $K = 2$, with $Q_2 > Q_1$ and assume the skill distribution has continuous support on $\left(0, \bar{\theta}\right]$ and no mass points.[28] Output from the two sectors is combined to produce a final good using a Cobb–Douglas technology:

$$Z = \left(\frac{Y_1}{\beta}\right)^{\beta} \left(\frac{Y_2}{1-\beta}\right)^{1-\beta}, \qquad \beta \in (0, 1), \tag{12}$$

where Z is final good output and Y_k is aggregate output of sector k:

$$Y_k = \int_{\theta_{k-1}}^{\theta_k} \theta g(\theta) f(s_k) dM(\theta). \tag{13}$$

This technology guarantees all sectors must produce positive aggregate output. The final good can be used either for consumption or to produce the homogeneous input. Each unit of final output can be transformed into γ units of the homogeneous input. This completes the specification of the economy. The use of a Cobb–Douglas final good production technology simplifies solving the model, but all the closed and open economy results obtained below continue to hold if the final good is produced using a general constant returns to scale technology. See Appendix B for details.

[28] This assumption is for ease of exposition. It is straightforward to solve the model when the skill distribution is discrete, but the notation is more cumbersome due to the necessity of keeping track of where agents work when they are indifferent between sectors.

4.2. Equilibrium

Given Assumption 1 we know there is positive assortative matching between agents and sectors. Therefore, there exists a skill threshold θ_1 such that agents with skill below θ_1 work in sector one and agents with skill above θ_1 work in sector two.

To solve the model it is convenient to let the final good be the numeraire. This immediately implies $p = \frac{1}{\gamma}$ and that the effective input cost in sector k is $\frac{1}{Q_k \gamma}$. From (12) the final good producers' unit cost minimization problem is:

$$\min_{Y_1 \geq 0, Y_2 \geq 0} \{\psi_1 Y_1 + \psi_2 Y_2\} \text{ subject to } \left(\frac{Y_1}{\beta}\right)^{\beta} \left(\frac{Y_2}{1-\beta}\right)^{1-\beta} = 1,$$

and solving this problem implies:

$$1 = \psi_1^{\beta} \psi_2^{1-\beta}. \tag{14}$$

Since $Q_2 > Q_1 \Rightarrow \psi_2 < \psi_1$ we must have $\psi_2 < 1 < \psi_1$. In addition, (14) implies $\frac{d\psi_1}{d\psi_2} < 0$. If the price of sector two output rises, then the price of sector one output falls. Cost minimization and (12) also give the market clearing equations:

$$\beta Z = \psi_1 Y_1, \qquad (1-\beta) Z = \psi_2 Y_2. \tag{15}$$

Equations (6), (7), (13), (14) and (15) are sufficient to reduce the equilibrium to a system of two equations in the two unknowns, θ_1 and ψ_2. First, the wage equalization (WE) condition requires that an agent with ability θ_1 be indifferent between the two sectors. From (7) and (14) this implies:

$$f\left[s_1(\theta_1)\right] - s_1(\theta_1) f'\left[s_1(\theta_1)\right] = \psi_2^{\frac{1}{\beta}} \left(f\left[s_2(\theta_1)\right] - s_2(\theta_1) f'\left[s_2(\theta_1)\right]\right). \tag{WE}$$

Second, the output markets must clear. Using (13), (14) and (15) gives the market clearing (MC) condition:

$$\int_0^{\theta_1} \theta g(\theta) f(s_1) dM(\theta) = \frac{\beta}{1-\beta} \psi_2^{\frac{1}{\beta}} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta). \tag{MC}$$

In both equilibrium conditions s_1 and s_2 are defined by (6) and depend implicitly on ψ_2.

Fig. 4 shows the (WE) and (MC) conditions in θ_1–ψ_2 space. The (WE) curve is downward sloping because an increase in ψ_2 makes sector two more profitable and, since $\frac{w_2(\theta)}{w_1(\theta)}$ is increasing in θ, this decreases the skill level at which agents are indifferent between sectors. The (MC) curve is upward sloping because a higher ψ_2 reduces the relative demand for sector two output, which necessitates the reallocation of labor to sector one. Together the two conditions define a unique equilibrium – see the proof of Proposition 4 for details.

Proposition 4. *There exists a unique closed economy equilibrium with a threshold skill θ_1 such that agents with skill above θ_1 work in the high technology sector and agents with skill below θ_1 work in the low technology sector.*

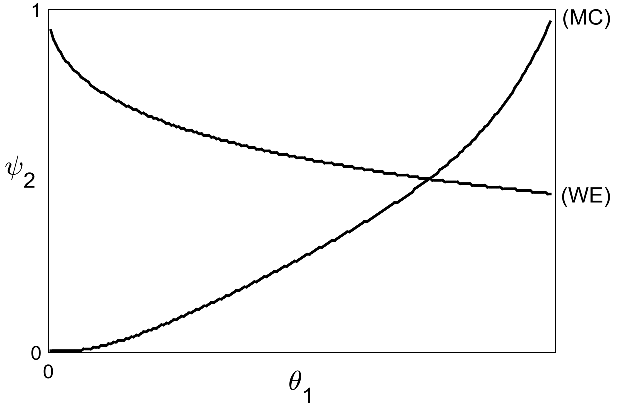

Fig. 4. Closed economy equilibrium.

4.3. Technical change

How does input augmenting productivity growth affect the closed economy equilibrium? The most dramatic effect occurs when technical change switches the ranking of sectors by input productivity. For example, an increase in input productivity in sector one from $Q_1 < Q_2$ to $Q_1' > Q_2$ precipitates an assignment reversal making sector one the high skill, high wage sector.

However, regardless of whether technological progress changes the sector technology ranking the equilibrium conditions imply[29]:

$$\frac{d\left[\psi_k Q_k\right]}{dQ_j} > 0, \qquad \frac{d\psi_j}{dQ_j} < 0, \qquad \frac{d\psi_l}{dQ_j} > 0, \qquad j, k, l = 1, 2, \quad l \neq j. \tag{16}$$

Unsurprisingly, productivity growth in a sector leads to a price decline in that sector and a price rise in the other sector. More importantly, productivity growth in either sector always increases $\psi_k Q_k$ in both sectors. Remembering equations (6) and (11) this implies the span of control $s_k(\theta)$ and returns to skill $\epsilon^{w_k}(\theta)$ rise in both sectors. Appealing to Lemma 2, the higher returns to skill increase within-group wage inequality among any group of agents who all work in the same sector and who do not switch sectors following the technology shock. Input augmenting productivity growth raises the returns to skill in both sectors because it causes all agents to increase their spans of control, which disproportionately benefits high skill agents for whom the elasticity of output with respect to the span of control is greater.

Technical change can also take the form of an increase in the productivity γ of input production. From (6) we see that span of control depends on the effective input cost $\frac{1}{Q_k\gamma}$, meaning that an increase in γ is equivalent to equiproportional increases in both Q_1 and Q_2. Therefore, we have:

$$\frac{d\left[\psi_k \gamma\right]}{d\gamma} > 0, \qquad k = 1, 2,$$

implying that when γ increases the returns to skill rise in both sectors. Proposition 5 summarizes the effects of technical change.

[29] See the proof of Proposition 5 for details.

Proposition 5. *Technical change that reduces the effective non-labor input cost in either sector raises the returns to skill in both sectors. Consequently, wage inequality increases within any group of agents who all work in the same sector and who do not switch sectors.*

Proposition 5 shows that in the assignment model technical change is skill-biased when it is either input augmenting or reduces the input cost. However, it's worth noting that such technical change is complementary to skill in two distinct senses. First, any sector which experiences a sufficiently large positive technology shock becomes the high skill sector, regardless of the skill level of its workers prior to the shock. Second, technical change increases the returns to skill within both sectors. This prediction is consistent with evidence that skill-biased technical change contributed to the increased inequality that was observed throughout the US wage distribution during the 1980s (Autor et al., 2006). Since technical changes raises agents' spans of control, equation (10) implies that it also reduces labor's share of output. Labor's share of output in the US has declined over the past twenty-five years, but there is not yet a consensus about the causes of this decline (Elsby et al., 2013).

Without placing restrictions on the shape of the skill distribution, or the functional form of the production technology, the effect of technical change on the skill threshold θ_1 and on inequality within groups that include agents who are induced to switch sectors by the technological shock is, in general, ambiguous. In particular, at skill levels such that agents switch from the high skill to the low skill sector following a technology shock the returns to skill can decrease. However, I show in the proof of Proposition 5 that whenever technical change causes the high skill sector to expand on the extensive margin ($d\theta_1 < 0$) wage inequality increases within all subgroups of the population.

5. Open economy

Let us now extend the model to include two countries: home and foreign. I will use an asterisk to denote foreign variables. I assume the two countries are identical along all dimensions except: (i) the cost per efficiency unit of input; (ii) the skill distribution, and; (iii) population size. By Assumption 1 the production function F is log-submodular in both countries. The aim of this section is to understand the consequences of international trade when effective input costs differ across both sectors and countries.

The effective input cost depends on both input augmenting productivity Q_k and the productivity of input production γ. Variation in effective input costs across countries and sectors result from differences in country-sector specific knowledge stocks and technical capabilities and from cross-country differences in input costs. Fig. 1 and Section 2.1 showed that assignment reversals occur more frequently between developed Northern countries and developing Southern countries than between two Northern countries. In this section I consider two cases: North–South trade with an assignment reversal, and; North–North trade without an assignment reversal.

In an open economy both sectoral outputs and the homogeneous input may be traded. To separate the effects of output and input trade I will start by analyzing output trade assuming the input is non-tradable and then proceed to allow input trade. Assuming each sector's output is freely traded implies sectoral output prices are equalized across countries meaning that, in the open economy, industries are defined by the output good they produce rather than by any labeling based on industries' input productivity levels. The final good price is also equalized across countries and, as above, I let the final good be the numeraire. I also assume each country's skill distribution has continuous bounded support, but I allow the functional form and upper bound of the skill distribution to differ across countries. When comparing the closed and open economy equilibria I will use a tilde to denote autarky outcomes.

5.1. Assignment reversals

Let us start by considering the North–South case where there is an assignment reversal across countries and the homogeneous input is non-traded. In particular, suppose home has higher productivity in sector two, $Q_1 < Q_2$, but foreign has higher productivity in sector one, $Q_1^* > Q_2^*$. This means in autarky sector two is the high skill, high wage sector at home, while sector one is the high skill, high wage sector abroad. In addition, diversified production requires $\tilde{\psi}_2 < 1 < \tilde{\psi}_1$ and $\tilde{\psi}_1^* < 1 < \tilde{\psi}_2^*$ meaning:

$$\frac{\tilde{\psi}_2}{\tilde{\psi}_1} < 1 < \frac{\tilde{\psi}_2^*}{\tilde{\psi}_1^*},$$

which implies home has a comparative advantage in sector two and foreign has a comparative advantage in sector one. Therefore, when the ranking of sectors by input productivity differs across countries, each country has a comparative advantage in its high productivity sector, which is also its high skill, high wage sector.

We know from Section 3 that if $\psi_2 \geq \psi_1$ in the open economy equilibrium then in the home country sector two offers a strictly higher wage than sector one at all skill levels. Similarly, if $\psi_2 \leq \psi_1$ then sector one is strictly preferred to sector two by all foreign agents. Since free trade equalizes output prices across countries it follows that in the open economy at least one of the countries must specialize in its high skill sector. Without loss of generality, let us suppose that $\psi_2 \leq \psi_1$.[30] Then foreign specializes in sector one and equation (14) implies $\psi_2 \leq 1 \leq \psi_1$.

In the open economy output prices must satisfy (14) and equilibrium spans of control and wages are given by (6), (7) and their foreign equivalents. As in the closed economy, the open economy equilibrium reduces to a system of two equations in two unknowns, θ_1 and ψ_2. The wage equalization (WE) condition, which determines the skill threshold above which home agents select into sector two, is unchanged from the closed economy case. The difference is that output markets clear at the global, not the national, level. From (15) and its foreign equivalent global output market clearing requires:

$$Y_1 + Y_1^* = \frac{\beta}{1-\beta}\psi_2^{\frac{1}{\beta}}(Y_2 + Y_2^*),$$

and using (13), (14) and that foreign is specialized in sector one we obtain the open economy market clearing (MC′) condition:

$$\int_0^{\theta_1} \theta g(\theta) f(s_1) dM(\theta) + \int_0^{\bar{\theta}^*} \theta g(\theta) f(s_1^*) dM^*(\theta) = \frac{\beta}{1-\beta}\psi_2^{\frac{1}{\beta}} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta). \quad \text{(MC}'\text{)}$$

The only difference from the closed economy market clearing condition is the second term on the left hand side of (MC′), which represents foreign's sector one output. As in the closed economy, the (WE) curve is downward sloping and the (MC′) curve is upward sloping in θ_1–ψ_2 space and together they define a unique equilibrium. However, foreign production shifts the

30 Equation (17) below gives a necessary and sufficient condition for this to be the equilibrium outcome.

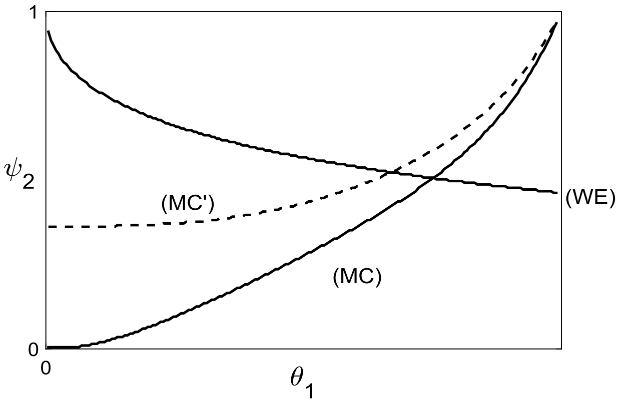

Fig. 5. Open economy equilibrium.

(MC′) curve upwards relative to the (MC) curve in the closed economy (see Fig. 5). Therefore, globalization reduces the skill threshold above which home agents work in sector two, $\theta_1 < \tilde{\theta}_1$ and increases the home price of sector two output, $\psi_2 > \tilde{\psi}_2$.

For $\psi_2 \le 1 \le \psi_1$ to be the equilibrium outcome we must have that when $\psi_1 = \psi_2 = 1$, which implies both countries are specialized in their high productivity sector, there is an excess supply of good two. Consequently, a necessary and sufficient condition for output prices to satisfy $\psi_2 \le 1 \le \psi_1$ in equilibrium is:

$$\int_0^{\bar{\theta}^*} \theta g(\theta) f(s_1^*) dM^*(\theta) \le \frac{\beta}{1-\beta} \int_0^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta), \tag{17}$$

where the spans of control are defined by (6) with $\psi_1 = \psi_2 = 1$. This condition tells us that if foreign is economically "small" relative to home then in the open economy equilibrium foreign specializes in its high productivity sector. In this context, an economy's size depends on how much output it can produce in its high productivity sector and smallness can result from having a relatively low population, relatively unskilled agents or relatively low input productivity in the high technology sector. Proposition 6 summarizes the structure of production in the open economy equilibrium.

Proposition 6. *When there is an assignment reversal across countries there exists a unique open economy equilibrium such that: (i) each country exports the output of its high skill sector; (ii) the smaller economy specializes in its high skill sector, and; (iii) compared to autarky the skill threshold above which agents select into the high skill sector is lower in both countries.*

Since each country has a comparative advantage in its high technology sector and high skill agents are matched to the high technology sector, the model predicts the export sector is the high skill sector in both countries. This prediction is absent from models that do not include assignment reversals.[31] Brambilla et al. (2012) use data from sixteen Latin American countries

[31] Matsuyama (2007) presents a model in which export sectors are always more skill intensive than import sectors because, by assumption, export production uses a more skill intensive technology than production for domestic consumption.

to estimate skill premiums at the country-industry level. They find that the skill premium is higher in industries with a greater exports to output ratio which implies there is an incentive for more skilled workers to sort into export industries.

Comparing the open economy equilibrium to autarky we have $\tilde{\psi}_2 < \psi_2 < \tilde{\psi}_2^*$ and $\tilde{\psi}_1^* < \psi_1 < \tilde{\psi}_1$. Therefore, following trade integration each country experiences an increase in the price of its high skill sector and a decrease in the price of its low skill sector. From (6) and (7), these price changes increase the wages of agents in the high skill sector and decrease the wages of agents in the low skill sector. Whether agents who switch into the high skill sector following globalization obtain a higher wage than in autarky is ambiguous, but in each country there exists a skill threshold such that, following trade liberalization, the wage of all agents with skill below the threshold falls and the wage of all agents with skill above the threshold rises.[32] Thus, trade liberalization benefits high skill labor in both countries.

From equation (11) and Lemma 2 the price changes triggered by globalization increase the returns to skill in the high skill sector and decrease the returns to skill in the low skill sector. Consequently, in both countries, moving from autarky to free trade increases wage inequality within any group of agents employed in the high skill sector following trade liberalization and decreases wage inequality within any group of agents employed in the low skill sector following trade liberalization. Since the smaller economy specializes in its high skill sector it experiences a pervasive rise in wage inequality – wage inequality increases within any subset of the population containing at least two agents with different skill levels. In addition, if equation (17) holds with equality, meaning the two economies are the same size, then both countries are fully specialized in the open economy equilibrium and trade integration causes a pervasive increase in wage inequality in both countries. Proposition 7 summarizes the effects of output trade on wages.

Proposition 7. *When there is an assignment reversal across countries moving from autarky to free trade causes each country to experience an increase in the price of its high skill good and a decrease in the price of its low skill good. Consequently, in both countries, wage levels and wage inequality increase in the high skill sector and decrease in the low skill sector.*

Contrary to the Stolper–Samuelson theorem, Proposition 7 predicts that when there is an assignment reversal trade raises wage inequality both in the North and in the South. This prediction is supported by the experience of many developing countries that undertook trade liberalizations in the 1980s and 1990s.[33] Mechanisms that have been invoked to explain how trade integration can raise inequality in both developed and developing countries include offshoring (Feenstra and Hanson, 1996), trade induced intra-industry input quality upgrading (Verhoogen, 2008; Kugler and Verhoogen, 2012), trade in capital (Csillag and Koren, 2009; Parro, 2013; Burstein et al., 2013), higher skill intensity of export production (Matsuyama, 2007) and intra-industry selection of high skill or high wage firms into exporting (Manasse and Turrini, 2001; Yeaple, 2005; Helpman et al., 2010; Monte, 2011; Sampson, 2014). All these mechanisms could co-exist with Stolper–Samuelson effects. In contrast, assignment reversals explain not only why trade raises wage inequality in both North and South, but also why Stolper–Samuelson effects are

[32] See the proof of Proposition 7 for details.

[33] See, for example, Hanson and Harrison (1999) on Mexico, Han et al. (2012) on China and Goldberg and Pavcnik (2007) for a summary of the empirical literature. Not all developing countries have experienced increases in wage inequality following trade liberalization. Gonzaga et al. (2006) find that Brazil's 1990s trade liberalization reduced the skill premium in a manner consistent with the Stolper–Samuelson theorem.

not observed.[34] In the absence of assignment reversals, the relevance of the Stolper–Samuelson prediction for understanding the effects of trade on wage inequality should be greatest when considering inter-industry output trade between dissimilar countries with very different relative endowments of skilled labor. Importantly, the evidence in Section 2.1 shows it is in exactly these circumstances that there is most likely to be an assignment reversal between countries.

5.2. No assignment reversals

Now consider trade between two Northern countries where the ranking of sectors by effective input costs is the same in both countries, meaning there are no assignment reversals. Assume in both countries input productivity is higher in sector two than in sector one. In this case, the pattern of comparative advantage across countries will depend on the effective input costs and skill distributions of the two countries according to the autarky equilibrium conditions (WE) and (MC) and their foreign equivalents. The autarky price of sector two output is lower, ceteris paribus, in the country with: (i) higher relative productivity in sector two; (ii) lower absolute costs per efficiency unit of input, or; (iii) a greater proportion of high skill agents. Without loss of generality, assume $\tilde{\psi}_2 < \tilde{\psi}_2^*$, meaning home has a comparative advantage in sector two, while foreign has a comparative advantage in sector one. Therefore, in the open economy equilibrium home exports output from its high skill sector, while foreign exports output from its low skill sector.

Since the ordering of sectors by workforce skill is invariant across countries, trade-induced price changes cannot increase the price of the high skill good in both countries. Only the country with a comparative advantage in the high skill sector experiences an increase in the price of its high skill output. Open economy market clearing requires $\tilde{\psi}_2 < \psi_2 < \tilde{\psi}_2^*$ and from (14) this also implies $\tilde{\psi}_1^* < \psi_1 < \tilde{\psi}_1$. At home trade liberalization has similar effects to those experienced by both countries when there is an assignment reversal: the high skill sector expands on the extensive margin and the price changes benefit high skill labor. However, in foreign the price of high skill output declines, the price of low skill output increases and the low skill sector expands on the extensive margin. Consequently, trade liberalization benefits low skill labor – there exists a threshold such that the wage of all foreign agents with skill below the threshold is higher in the open economy than in autarky and the wage of all foreign agents with skill above the threshold is lower. Moreover, the returns to skill increase in the low skill sector and decrease in the high skill sector, meaning that trade liberalization increases wage inequality within any group of foreign agents employed in the low skill sector in autarky and decreases wage inequality within any group of foreign agents employed in the high skill sector following integration.

Proposition 8. *When the ranking of sectors by input productivity is the same in both countries there exists a unique open economy equilibrium such that: (i) one country exports its high skill, high productivity good and the other country exports its low skill, low productivity good, and; (ii) in both countries moving from autarky to free trade increases wage levels and wage inequality in the export sector and decreases wage levels and wage inequality in the import sector.*

Proposition 8 shows that in the absence of assignment reversals the effects of output trade integration on wage inequality between high and low skill agents are the same as those predicted

[34] Working within the Heckscher–Ohlin model, Davis (1996) argues that the existence of multiple cones of diversification can overturn the Stolper–Samuelson prediction for some countries.

by the Heckscher–Ohlin model. Inequality increases in the country which has a comparative advantage in the high skill good and decreases in the other country. However, in contrast to the Heckscher–Ohlin model, in the assignment model trade also affects within-group wage inequality. In particular, Propositions 7 and 8 show that in the assignment model trade liberalization increases returns to skill in the export sector and decreases returns to skill in the import sector of both countries, regardless of patterns of input productivity or comparative advantage.

Propositions 7 and 8 show that regardless of whether there is an assignment reversal across countries, trade moves wage levels and wage inequality in the same direction in both sectors of both countries.[35] Suppose the US has a comparative advantage in its high technology, high skill sector.[36] Then the assignment model predicts trade liberalization raises the level and inequality of wages in the high skill sector in the US and lowers the level and inequality of wages in the low skill sector. This is consistent with observed wage polarization in the US where, during the 1990s and 2000s, wage levels and inequality have increased rapidly in the upper half of the wage distribution, while both the level and inequality of wages have risen less quickly or perhaps declined in the lower half of the distribution (Autor et al., 2006).

It is also interesting to note the similarities between the assignment model and a specific factors trade model. For infra-marginal workers who do not switch sectors following trade liberalization, trade benefits workers in the export-oriented sector and hurts workers in the import-competing sector – exactly as predicted by a specific factors model with sector-specific labor. However, because workers are heterogeneous and the equilibrium assignment is endogenous, the assignment model can also address questions that lie outside the specific factors framework, such as how workers sort across sectors, which workers switch sectors following a shock and what determines within-sector wage inequality.

5.3. Input trade

Finally, let us consider the effects of input trade. In the absence of trade the input price is $\frac{1}{\gamma}$ at home and $\frac{1}{\gamma^*}$ in the foreign country. Without loss of generality, assume foreign has higher productivity in input production than home implying $\gamma^* > \gamma$. Then home will import the input from foreign and moving from an equilibrium in which the input is non-tradable to free input trade is equivalent to increasing home's productivity in the input production sector from γ to γ^*.

An increase in γ reduces the effective input cost in both sectors by raising $\gamma\psi_k$, $k = 1, 2$. Consequently, spans of control increase in both sectors and, applying equation (11) and Lemma 2, this raises the returns to skill and wage inequality within each sector for agents whose sector of employment does not change. Thus, liberalization of input trade increases within sector wage inequality in the country that imports the input by allowing workers to better leverage their skills, which disproportionately benefits more skilled agents. This result holds regardless of whether there are assignment reversals across countries and regardless of whether sectoral outputs are

[35] Equations (6), (7) and (11) imply that following an output price change the wage level and returns to skill in any sector must move in the same direction. Alternative shocks, such as technical change, do not necessarily generate this comovement.

[36] Consistent with this assumption, Romalis (2004) provides evidence that relatively skill abundant countries, such as the US, have a comparative advantage in relatively skill intensive sectors where sectoral skill intensity is measured using US data.

traded or non-traded.[37] By contrast, the effect of input trade on wages in the input exporter depends on the how ψ_2 changes, which is ambiguous at this level of generality.

Proposition 9. *Input trade raises the returns to skill in both sectors in the country that imports the input. Consequently, wage inequality increases within any group of agents who all work in the same sector and who do not switch sectors.*

Comparing Proposition 9 to Proposition 5 above shows that, for an importer, input trade has the same effects on wage inequality as an increase in the productivity of input production. This comparison highlights the fact that in the assignment model inputs embody cross-country technology differences and input trade is equivalent to a technology transfer that raises the productivity of the importing country.

If the non-labor input is interpreted as capital, the model predicts that reductions in the cost of capital imports increase within-sector returns to skill. This prediction is supported by two recent papers that estimate the impact of capital imports on wages. Csillag and Koren (2009) undertake structural estimation of a single sector model of worker assignment, similar to Sattinger (1979), using a rich matched employer–employee–imports data set from Hungary. They find that on average imported machines are more productive than domestic machines and are matched with higher skill workers. In addition, the returns to skill on the median productivity imported machine are 26% higher than on the median productivity domestic machine. Parro (2013) estimates the impact of capital imports on the skill premium using a calibrated version of the Eaton and Kortum (2002) model in which production uses skilled labor, unskilled labor and capital and there is capital-skill complementarity. The paper finds that from 1990–2007 reductions in capital trade costs and productivity growth in capital production each increased the skill premium by around 2 percentage points on average across countries.

For ease of exposition I have assumed throughout the paper there is a unique non-labor input, implying the input price is constant across sectors within a country. However, it is simple to generalize the assignment model to allow for sector specific inputs. Suppose one unit of the final good can be transformed into γ_k units of the input used in sector k implying $p_k = \frac{1}{\gamma_k}$ is the input price in sector k. Then the effective input cost in sector k is $\frac{Q_k}{\gamma_k}$ and, provided the input is non-tradable, variation in γ_k across sectors is equivalent to variation in Q_k. For example, the equilibrium assignment is as in Proposition 1, except sectors are ranked by $\frac{Q_k}{\gamma_k}$ instead of Q_k.

However, this equivalence breaks down when the input is tradable because while input augmenting productivity Q_k is disembodied, variation in γ_k is embodied in the input which can be traded across countries. Each country-sector pair will source its input from the lowest cost supplier. In the extreme case where there is no within-country variation in input-augmenting productivity (i.e. $Q_k = Q_l \, \forall k, l$), free trade in inputs implies the ranking of sectors by effective input costs is the same in all countries. Consequently, there are no assignment reversals. When all cross-sector technology differences are embodied in inputs, trade and technology transfer are perfect substitutes and input trade leads to global convergence in the ranking of sectors by workforce skill.

[37] When sectoral outputs are non-traded I assume the final good is freely tradable to ensure the trade balance condition holds.

6. Conclusions

The Stolper–Samuelson prediction for how trade affects wage inequality is premised on the assumption that the ranking of sectors by skill intensity is invariant across countries. However, industry wage and educational attainment data imply the sectoral skill allocation differs dramatically between developed and developing economies. To explain this observation, the paper has developed a new labor assignment model in which cross-sector differences in the cost and productivity of non-labor inputs shape labor sorting and cross-country variation in the ranking of sectors by effective non-labor input costs generate assignment reversals. Embedding the assignment model in an open economy shows that when there is an assignment reversal across countries, the Stolper–Samuelson theorem does not hold and trade raises wage inequality in both the developed North and the developing South.

To illustrate the consequences of assignment reversals this paper has worked with a simple two sector, two country open economy model. An important agenda for future work is to evaluate the quantitative importance of assignment reversals compared to other channels linking trade and wage inequality by allowing for assignment reversals in a multi-country, multi-sector model with both inter-industry and intra-industry trade. A multi-sector model would also allow for continuous variation in the correlation between countries' sector skill rankings in line with the evidence presented in Section 2.1, whereas in the two sector model assignment reversals are a binary phenomenon. Differences in the relative prevalence of assignment reversals between countries may explain why the effects of trade integration on inequality have varied across developing countries.

The theoretical tools for solving assignment problems that have been developed in this paper could be applied to address a wide range of questions that feature multiple productive activities and matching between two factors of production with non-zero opportunity costs of forming a match. For example, if the homogeneous input is interpreted as homogeneous unskilled labor, the model can be reinterpreted as a model of firm hierarchies. Consequently, the assignment model could be used to extend the single sector literature on globalization and firm hierarchies (Antràs et al., 2006; Burstein and Monge-Naranjo, 2009) to a multi-sector world. It would also be interesting to allow for endogenous technical change in input productivity and analyze the conditions under which profit maximizing R&D leads to assignment reversals. Finally, the increasing availability of matched employer–employee data opens new opportunities for testing assignment models and estimating the role of non-labor inputs in determining labor sorting. Such estimates would also help to discipline attempts to quantify the relevance of alternative sorting mechanisms.

Appendix A. Proofs

A.1. Proof of Lemma 1

Since F is twice differentiable it is strictly log-submodular if and only if $\frac{\partial^2 \log F}{\partial\theta\partial x} < 0$. Differentiating F gives:

$$
\begin{aligned}
\frac{\partial^2}{\partial\theta\partial x}\log F(\theta, Q_k x) &= \frac{Q_k}{F^2}(F F_{\theta x} - F_\theta F_x),\\
&= \frac{Q_k}{F^2} F F_{\theta x}(1-\sigma),
\end{aligned}
$$

where the second line uses the fact that the elasticity of substitution of a twice differentiable, constant returns to scale function F is given by $\sigma = \frac{F_\theta F_x}{F F_{\theta x}}$. Since F has constant returns to scale and is strictly concave we must have $F_{\theta x} > 0$. Therefore, F is strictly log-submodular if and only if $\sigma > 1$.

Finally, to prove that $\sigma > 1$ is equivalent to $\epsilon^f(s)$ being strictly increasing in s differentiate $\epsilon^f(s)$ to obtain:

$$\begin{aligned}\frac{\partial}{\partial s}\epsilon^f(s) &= \frac{1}{f^2}(ff' + sff'' - sf'^2),\\ &= \frac{1}{f^2}(F_\theta F_x - F F_{\theta x}),\\ &= \frac{F F_{\theta x}}{f^2}(\sigma - 1).\end{aligned}$$

A.2. *Proof of Proposition 1*

Consider the case where F is strictly log-submodular. For any $k \in \{1, \dots, K\}$ the requirement that sector k produces positive aggregate output implies there exists $\theta \in (0, \bar{\theta}]$ such that agents with skill θ weakly prefer sector k to any other sector. Suppose the equilibrium assignment does not exhibit positive assortative matching. Then there exists $l' < l$ and $\theta_a, \theta_b \in (0, \bar{\theta}]$ with $\theta_b > \theta_a$ such that $w_l(\theta_a) \geq w_k(\theta_a)\,\forall k$ and $w_{l'}(\theta_b) \geq w_k(\theta_b)\,\forall k$.

However, $l > l' \Rightarrow Q_l > Q_{l'} \Rightarrow \psi_l Q_l > \psi_{l'} Q_{l'} \Rightarrow s_l > s_{l'}$. Since F is strictly log-submodular, $\epsilon^f(s)$ is strictly increasing in s and, therefore, it follows from equation (8) above that $s_l > s_{l'} \Rightarrow \frac{d}{d\theta}\left[\frac{w_l(\theta)}{w_{l'}(\theta)}\right] > 0\,\forall\theta$. Consequently, $w_l(\theta_a) \geq w_{l'}(\theta_a) \Rightarrow w_l(\theta_b) > w_{l'}(\theta_b)$, which contradicts the assumption that there is not positive assortative matching.

An analogous argument can be used to prove that there is negative assortative matching when F is strictly log-supermodular.

A.3. *Proof of Proposition 2*

The proof follows the same reasoning used to prove Proposition 1 except $\frac{d}{d\theta}\left[\frac{w_k(\theta)}{w_l(\theta)}\right]$ is given by (9) instead of (8).

A.4. *Proof of Proposition 3*

Part (i) follows immediately from equation (10) and Lemma 1 after remembering that span of control is strictly increasing in θ.

To prove part (ii) let $x_k(\theta)$ denote input use by a skill θ agent working in sector k. From optimal input choice (6) we have:

$$px_k(\theta) = \psi_k \theta g(\theta) f(s_k)\epsilon^f(s_k), \tag{18}$$

which is strictly increasing in θ for given k. Now suppose $w_k(\theta) = w_l(\theta)$ and $Q_k > Q_l$. Then substituting (7) into (18) gives:

$$\begin{aligned}\frac{px_k(\theta)}{px_l(\theta)} &= \frac{\epsilon^f(s_k)}{\epsilon^f(s_l)}\frac{1-\epsilon^f(s_l)}{1-\epsilon^f(s_k)},\\ &> 1,\end{aligned}$$

where the inequality follows from Lemma 1 and $s_k(\theta) > s_l(\theta)$. Thus, input expenditure per worker increases discontinuously at the skill thresholds that separate sectors. It follows that input expenditure per worker is strictly increasing in θ.

A.5. Proof of Lemma 2

Let Ω be an arbitrary subset of agents with skill levels in $[\theta_a, \theta_b]$. If the mass of agents in Ω is concentrated at a single point, then there is no inequality between members of Ω. Assume this is not the case and let $\theta_{\min} = \inf\{\theta \in \Omega\}$ and $\theta_{\max} = \sup\{\theta \in \Omega\}$. Clearly, $\theta_{\max} > \theta_{\min}$.

Let $\hat{w}(\theta) = C\tilde{w}(\theta)$ where C is chosen to ensure $\mathbb{E}_\Omega \hat{w}(\theta) = \mathbb{E}_\Omega w(\theta)$ and $\mathbb{E}_\Omega$ denotes an expectation taken over the subset Ω. Obviously, $\epsilon^{\hat{w}}(\theta) = \epsilon^{\tilde{w}}(\theta)\,\forall\theta$. Since $\epsilon^{w}(\theta) > \epsilon^{\hat{w}}(\theta)\,\forall\theta \in (\theta_{\min}, \theta_{\max})$ we have that if $w(\theta') = \hat{w}(\theta')$ with $\theta' \in \Omega$ then $w(\theta) > \hat{w}(\theta)\,\forall\theta > \theta', \theta \in \Omega$ and $w(\theta) < \hat{w}(\theta)\,\forall\theta < \theta', \theta \in \Omega$. Remembering that $\mathbb{E}_\Omega \hat{w}(\theta) = \mathbb{E}_\Omega w(\theta)$ it immediately follows that $w(\theta)$ and $\hat{w}(\theta)$ satisfy a single-crossing property on $[\theta_{\min}, \theta_{\max}]$ with $w(\theta_{\min}) < \hat{w}(\theta_{\min})$ and $w(\theta_{\max}) > \hat{w}(\theta_{\max})$.

Consequently, the wage distribution over Ω induced by $\hat{w}(\theta)$ second-order stochastically dominates the distribution induced by $w(\theta)$. Since $\hat{w}(\theta)$ and $\tilde{w}(\theta)$ are identical up to a change in scale it follows that for any measure of inequality that respects scale independence and second-order stochastic dominance wage inequality among members of Ω is higher when wages are given by $w(\theta)$ than when wages are given by $\tilde{w}(\theta)$.

A.6. Proof of Proposition 4

Since the skill distribution has no mass points the (MC) condition implies that $\psi_2 \to 0$ as $\theta_1 \to 0$ and $\psi_2 \to \infty$ as $\theta_1 \to \bar{\theta}$. The (WE) condition implies that $\psi_2 < 1\,\forall\theta_1 \in \left(0, \bar{\theta}\right]$ since if $\psi_2 \geq 1$ all agents obtain a strictly higher wage in sector two than in sector one. Differentiating the (WE) condition gives:

$$\left(\psi_2^{\frac{1}{\beta}} f\left[s_2(\theta_1)\right] - f\left[s_1(\theta_1)\right]\right)\epsilon^{g}(\theta_1)\hat{\theta}_1 - s_1(\theta_1) f'\left[s_1(\theta_1)\right]\left(\hat{Q}_1 + \hat{\gamma}\right) =$$
$$-\psi_2^{\frac{1}{\beta}} s_2(\theta_1) f'\left[s_2(\theta_1)\right]\left(\hat{\psi}_2 + \hat{Q}_2 + \hat{\gamma}\right) - \left(f\left[s_1(\theta_1)\right] - \beta s_1(\theta_1) f'\left[s_1(\theta_1)\right]\right)\frac{\hat{\psi}_2}{\beta}, \tag{19}$$

where $\hat{\theta}_1 \equiv \frac{d\theta_1}{\theta_1}$ and analogous definitions hold for other variables. Differentiating the (MC) condition gives:

$$C_1\hat{\theta}_1 = C_2\left(\frac{1-\beta}{\beta}\hat{\psi}_2 - \hat{Q}_1 - \hat{\gamma}\right) + C_3\left(\hat{\psi}_2 + \hat{Q}_2 + \hat{\gamma}\right) + C_4\hat{\psi}_2, \tag{20}$$

where:

$$C_1 \equiv \left(f\left[s_1(\theta_1)\right] + \frac{\beta\psi_2^{\frac{1}{\beta}}}{1-\beta} f\left[s_2(\theta_1)\right]\right)\theta_1^2 g(\theta_1) dM(\theta_1) > 0,$$

$$C_2 \equiv \int_0^{\theta_1} \theta g(\theta) \frac{f'\left[s_1(\theta)\right]^2}{-f''\left[s_1(\theta)\right]} dM(\theta) > 0,$$

$$C_3 \equiv \frac{\beta \psi_2^{\frac{1}{\beta}}}{1-\beta} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) \frac{f'\left[s_2(\theta)\right]^2}{-f''\left[s_2(\theta)\right]} dM(\theta) \geq 0,$$

$$C_4 \equiv \frac{\psi_2^{\frac{1}{\beta}}}{1-\beta} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f\left[s_2(\theta)\right] dM(\theta) \geq 0. \tag{21}$$

The derivations of (19) and (20) use $\hat{\psi}_1 = -\frac{1-\beta}{\beta}\hat{\psi}_2$, which follows from differentiating (14). Note that (19) and (20) allow for variation in Q_1, Q_2 and γ. This is not necessary to prove Proposition 4, but will be needed for the proof of Proposition 5.

Let $\hat{Q}_1 = \hat{Q}_2 = \hat{\gamma} = 0$. Note that: (i) $\frac{\psi_2^{\frac{1}{\beta}} f[s_2(\theta_1)]}{f[s_1(\theta_1)]} = \frac{1-\epsilon^f[s_1(\theta_1)]}{1-\epsilon^f[s_2(\theta_1)]} > 1$ since the span of control is higher in sector two, and; (ii) $f\left[s_1(\theta_1)\right] > s_1(\theta_1) f'\left[s_1(\theta_1)\right]$. Therefore, it follows from (19) that the (WE) curve is strictly downwards sloping on $\left(0, \bar{\theta}\right]$. In addition, equation (20) implies that the (MC) curve is strictly upward sloping on $\left(0, \bar{\theta}\right]$. Combining these results with the boundary conditions above proves that the (WE) and (MC) curves have a unique intersection on $\left(0, \bar{\theta}\right)$.

A.7. Proof of Proposition 5

Let $\hat{\gamma} = 0$. Suppose $\hat{Q}_1 = 0$, but $\hat{Q}_2 > 0$. Then, if $\hat{\psi}_2 \geq 0$ equation (19) implies $\hat{\theta}_1 < 0$, but equation (20) implies $\hat{\theta}_1 > 0$ – a contradiction. Therefore, we must have $\hat{\psi}_2 < 0 \Rightarrow \hat{\psi}_1 > 0$. Now suppose $\hat{\psi}_2 < 0$ and $\hat{\psi}_2 + \hat{Q}_2 \leq 0$. Then equation (19) implies $\hat{\theta}_1 > 0$, but equation (20) implies $\hat{\theta}_1 < 0$ giving a contradiction. Therefore, we must have $\hat{\psi}_2 + \hat{Q}_2 > 0$. Similar reasoning shows that if $\hat{Q}_1 > 0$ and $\hat{Q}_2 = 0$ then (19) and (20) together imply $\hat{\psi}_2 > 0$, $\hat{\psi}_1 < 0$ and $\hat{\psi}_1 + \hat{Q}_1 = -\frac{1-\beta}{\beta}\hat{\psi}_2 + \hat{Q}_1 > 0$. This proves the claims made in equation (16).

Given $\frac{d\left[\psi_k Q_k\right]}{dQ_j} > 0$, $j,k = 1,2$ equations (6) and (11) together imply that $\frac{d\epsilon^{w_k}(\theta)}{dQ_j} > 0$ $\forall\theta$, $j,k = 1,2$. Lemma 2 is then sufficient to conclude that technological progress increases within-group inequality among any group of agents who all work in the same sector and do not switch sectors following the technology shock.

If $\frac{d\theta_1}{dQ_j} < 0$, agents switch from sector one to sector two following an increase in Q_j. Since $s_2(\theta) > s_1(\theta)\,\forall\theta$, equation (11) implies $\epsilon^{w_2}(\theta) > \epsilon^{w_1}(\theta)\,\forall\theta$. Remembering that $\frac{d\epsilon^{w_k}(\theta)}{dQ_j} > 0\,\forall\theta$, $j,k = 1,2$ this means that an increase in Q_j unambiguously increases $\epsilon^w(\theta)$ at any value of θ such that agents switch from sector one to sector two following the shock. It immediately follows that $\frac{d\epsilon^w(\theta)}{dQ_j} > 0\,\forall\theta$. Therefore, Lemma 2 implies that income inequality increases among all subsets of agents.

Finally, observe from (19) and (20) that the case where $\hat{\gamma} = \chi \neq 0$ is equivalent to having $\hat{\gamma} = 0$ and $\hat{Q}_1 = \hat{Q}_2 = \chi$. It immediately follows that $\frac{d\left[\psi_k \gamma\right]}{d\gamma} > 0$, $k = 1,2$ implying that an increase in γ has the same qualitative effects on the returns to skill and wage inequality as an increase in either Q_1 or Q_2.

A.8. Proof of Proposition 6

The (WE) condition is the same as in the closed economy. It is a strictly downward sloping curve on $\left(0, \bar{\theta}\right]$. Let ψ_2^{WE} be the value of ψ_2 at which the (WE) curve intersects the $\theta_1 = 0$ axis. Obviously, $\psi_2^{WE} \leq 1$. The (MC′) condition implies $\psi_2 \to \infty$ as $\theta_1 \to \bar{\theta}$. Differentiating the (MC′) condition gives:

$$C_1\hat{\theta}_1 = C_2\left(\frac{1-\beta}{\beta}\hat{\psi}_2 - \hat{Q}_1 - \hat{\gamma}\right) + C_3\left(\hat{\psi}_2 + \hat{Q}_2 + \hat{\gamma}\right) + C_4\hat{\psi}_2 + C_5\left(\frac{1-\beta}{\beta}\hat{\psi}_2 - \hat{Q}_1^* - \hat{\gamma}^*\right), \tag{22}$$

where C_1, C_2, C_3 and C_4 are defined by (21) and:

$$C_5 \equiv \int_0^{\bar{\theta}^*} \theta g(\theta) \frac{f'\left[s_1^*(\theta)\right]^2}{-f''\left[s_1^*(\theta)\right]} dM^*(\theta) > 0.$$

Equation (22) implies that the (MC′) curve is strictly upward sloping on $\left(0, \bar{\theta}\right]$. Let $\psi_2^{MC'}$ be the value of ψ_2 at which the (MC′) curve intersects the $\theta_1 = 0$ axis. Equation (17) implies $\psi_2^{MC'} \leq 1$. If $\psi_2^{MC'} < \psi_2^{WE}$ then the (WE) condition and the (MC′) condition must have a unique intersection on $\left(0, \bar{\theta}\right)$ and this gives the open economy equilibrium. If $\psi_2^{MC'} \geq \psi_2^{WE}$ then equilibrium is given by $\theta_1 = 0$ and $\psi_2 = \psi_2^{MC'}$ and in equilibrium both countries specialize in their high productivity sector. This proves the existence of a unique open economy equilibrium.

The remainder of Proposition 6 follows immediately from the discussion in the main body of the paper.

A.9. Proof of Proposition 7

Consider the home country and assume home is not specialized in equilibrium. Since $\tilde{\psi}_2 < \psi_2$ and $\tilde{\psi}_1 > \psi_1$, equations (6) and (7) imply that $w_2(\theta) > \tilde{w}_2(\theta)$ and $w_1(\theta) < \tilde{w}_1(\theta)\,\forall\theta$. In addition, $0 < \theta_1 < \tilde{\theta}_1$ and the continuity of w and $\tilde{w}$ imply $w(\theta_1) < \tilde{w}(\theta_1)$ and $w(\tilde{\theta}_1) > \tilde{w}(\tilde{\theta}_1)$. Moreover, $\epsilon^{w}(\theta) > \epsilon^{\tilde{w}}(\theta)\,\forall\theta \in (\theta_1, \tilde{\theta}_1)$. Therefore, invoking continuity once more, w and $\tilde{w}$ must intersect exactly once on $(\theta_1, \tilde{\theta}_1)$. Trade liberalization reduces the wage of all agents with skill below the intersection and increases the wage of all agents with skill above the intersection.

From (6) we have that $\tilde{\psi}_1 > \psi_1 \Rightarrow \tilde{s}_1(\theta) > s_1(\theta)\,\forall\theta$. Equation (11) then implies $\epsilon^{\tilde{w}}(\theta) > \epsilon^{w}(\theta)\,\forall\theta < \theta_1$. Applying Lemma 2 this means that wage inequality within any subset of agents who work in sector one after trade liberalization is lower in the open economy than in the closed economy. By contrast, $\tilde{\psi}_1 Q_1 < \tilde{\psi}_2 Q_2 < \psi_2 Q_2 \Rightarrow \epsilon^{\tilde{w}}(\theta) < \epsilon^{w}(\theta)\,\forall\theta > \theta_1$. Consequently, trade liberalization increases wage inequality within any subset of agents who work in sector two in the open economy.

Similar reasoning can be used to prove the analogous results for the home country when $\theta_1 = 0$ and for the foreign country.

A.10. Proof of Proposition 8

Equilibrium is defined by the wage equalization (WE) condition and its foreign equivalent, which are the same as in autarky, and by the global output market clearing condition:

$$\int_0^{\theta_1} \theta g(\theta) f(s_1) dM(\theta) + \int_0^{\theta_1^*} \theta g(\theta) f(s_1^*) dM^*(\theta) =$$

$$\frac{\beta}{1-\beta} \psi_2^{\frac{1}{\beta}} \left[\int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta) + \int_{\theta_1^*}^{\bar{\theta}^*} \theta g(\theta) f(s_2^*) dM^*(\theta) \right].$$

From the foreign wage equalization condition, θ_1^* is strictly decreasing in ψ_2. Given this relationship it is easy to differentiate the market clearing condition, as was done in the proofs of Propositions 4 and 6, and show that it defines a strictly upward sloping relationship between θ_1 and ψ_2. The market clearing condition also implies that when $\theta_1 = \bar{\theta}$, $\psi_2 > \tilde{\psi}_2^* > \tilde{\psi}_2$ implying that in θ_1–ψ_2 space the market clearing curve sits above the home (WE) curve when $\theta_1 = \bar{\theta}$. Let ψ_2^{WE} be the value of ψ_2 at which the home (WE) curve intersects the $\theta_1 = 0$ axis. Let $\psi_2^{MC''}$ be the value of ψ_2 at which the market clearing curve intersects the $\theta_1 = 0$ axis. If $\psi_2^{MC''} < \psi_2^{WE}$ then the home (WE) condition and the market clearing condition must have a unique intersection on $(0, \bar{\theta})$ and this gives the open economy equilibrium. If $\psi_2^{MC''} \geq \psi_2^{WE}$ then equilibrium is given by $\theta_1 = 0$ and $\psi_2 = \psi_2^{MC''}$. This proves the existence of a unique open economy equilibrium.

In addition, since the global market clearing condition is simply the sum of the home autarky market clearing condition (MC) and its foreign equivalent we cannot have $\psi_2 \leq \tilde{\psi}_2$ or $\psi_2 \geq \tilde{\psi}_2^*$. In the former case there is excess global supply of good one, and in the later there is excess global supply of good two. Therefore, $\tilde{\psi}_2 < \psi_2 < \tilde{\psi}_2^*$. The remainder of the proof follows from the discussion in the main body of the paper and from using reasoning analogous to that applied in the proof of Proposition 7 to characterize the effect of moving from autarky to free trade on wage levels and wage inequality.

A.11. Proof of Proposition 9

First, consider the case when sectoral outputs are non-traded. In this case equilibrium is given, as in autarky, by the (WE) and (MC) conditions at home and their foreign equivalents abroad, but with $p = \frac{1}{\gamma^*}$ in both countries. Therefore, for home, input trade is equivalent to experiencing an increase in γ in autarky. The result then follows immediately from applying Proposition 5.

When the sectoral outputs are traded there are multiple cases to consider depending on whether there exists an assignment reversal and whether production in each country is diversified. However, since the same reasoning applies in each case I will only give the proof for the case considered in Section 5.1 where an assignment reversal exists and foreign is specialized in producing good one. In this case equilibrium is given by the (WE) and (MC′) conditions and differentiating these conditions gives (19) and (22). Input trade implies $\hat{\gamma} > 0$ while $\hat{Q}_1 = \hat{Q}_2 = \hat{\gamma}^* = 0$. Now, if $\hat{\psi}_2 \geq 0$ equation (19) implies $\hat{\theta}_1 < 0$ and equation (22) then implies $\hat{\psi}_1 + \hat{\gamma} > 0$. Alternatively, if $\hat{\psi}_2 < 0$ and $\hat{\psi}_2 + \hat{\gamma} \leq 0$ then equation (19) implies $\hat{\theta}_1 > 0$, but equation (22) implies $\hat{\theta}_1 < 0$ giving a contradiction. It follows that if $\hat{\psi}_2 < 0$ then $\hat{\psi}_2 + \hat{\gamma} > 0$. Therefore, in both cases we have $\hat{\psi}_k + \hat{\gamma} > 0$, $k = 1, 2$. Proposition 9 then follows immediately from applying equations (6) and (11) and Lemma 2.

Appendix B. Theoretical extensions

B.1. Cross-sector heterogeneity

It is straightforward to modify the production technology in (3) to allow for sources of cross-sector heterogeneity other than differences in input productivity. Suppose production in sector k requires a team of N_k workers and if each worker has skill θ output is given by[38]:

$$y_k(\theta, x) = g(\theta) A_k \left[\lambda_k (B_k \theta)^{\frac{\sigma-1}{\sigma}} + (1 - \lambda_k) (Q_k x)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{1'}$$

Given Assumption 1 we have $\sigma > 1$. This formulation allows for cross-sector heterogeneity in team size N_k, Hicks-neutral productivity A_k, labor augmenting productivity B_k, non-labor input productivity Q_k and the labor intensity of production λ_k. I restrict the production function to be a constant elasticity of substitution (CES) technology in order to introduce the CES parameter λ_k. If λ_k is not included in the analysis then the results below hold without imposing functional form restrictions on F.

The same reasoning employed to derive Proposition 1 can be used to characterize the equilibrium assignment when output is given by (1′). The structure of equilibrium is unchanged, but agents sort across sectors based not on the ranking of sectors by Q_k, but on the ranking of sectors by V_k where:

$$V_k \equiv \left(\frac{1 - \lambda_k}{\lambda_k} \right)^{\frac{\sigma}{\sigma-1}} \frac{N_k Q_k}{B_k}.$$

Higher ability agents are assigned to sectors with higher V_k. Consequently, skill levels and wages are higher, ceteris paribus, in sectors with: (i) higher non-labor input productivity; (ii) lower labor augmenting productivity; (iii) larger production teams, and; (iv) lower labor intensity.

Interestingly, different forms of technical change have contrasting implications for sorting across sectors. Whereas increases in non-labor input productivity tend to draw more skilled workers into a sector, labor augmenting technical change has the opposite effect. To understand this result, remember that when Assumption 1 holds and output is given by (3) higher ability agents sort into sectors with higher spans of control. If we redefine the span of control to equal the number of efficiency units of input used per efficiency unit of skill, $s_k(\theta) \equiv \frac{Q_k x}{B_k \theta}$, this insight remains true under the production technology (1′). Labor augmenting technical change reduces an agent's optimal span of control and, therefore, has the opposite effect to increases in non-labor input productivity. Similarly, higher labor intensity is equivalent to a simultaneous rise in labor augmenting productivity and fall in non-labor input productivity and decreases the optimal span of control. Meanwhile, higher team size increases the output price by raising labor costs, thereby leading to a greater optimal span of control. Finally, the equilibrium sorting pattern does not depend on Hicks-neutral productivity A_k because A_k is multiplicatively separable from the production function.

B.2. Generalized final good technology

Suppose instead of equation (12), the final good production function is given by:

[38] This specification assumes that in equilibrium all members of a team have the same skill level. This will necessarily be the case if, for example, a team inherits the skill level of its least able member.

$$Z = H(Y_1, Y_2),$$

where H is a constant returns to scale function that is strictly increasing in both its arguments, strictly concave and satisfies $\lim_{Y_k \to 0} \frac{\partial H}{\partial Y_k} = \infty$, $k = 1, 2$. Obviously, introducing this final good technology does not affect the existence of positive assortative matching between high skill agents and high technology sectors.

Let $\zeta \equiv \frac{Y_2}{Y_1}$. Then cost minimization in final good production and the choice of the final good as numeraire together imply $\frac{d\psi_2}{d\psi_1} = -\frac{1}{\zeta} < 0$ and:

$$\frac{h'(\zeta)}{h(\zeta) - \zeta h'(\zeta)} = \frac{\psi_2}{\psi_1}, \tag{23}$$

where $h(\zeta) \equiv H(1, \zeta)$. Since H is strictly concave, (23) implies ζ is a strictly decreasing function of $\frac{\psi_2}{\psi_1}$.

As in the Cobb–Douglas case, equilibrium reduces to a wage equalization condition and a market clearing condition. The wage equalization condition is still given by equation (WE) above, while the market clearing condition is:

$$\int_0^{\theta_1} \theta g(\theta) f(s_1) dM(\theta) = \frac{1}{\zeta} \int_{\theta_1}^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta).$$

By differentiating this expression and using $\zeta'\left(\frac{\psi_2}{\psi_1}\right) < 0$, it is straightforward to show the market clearing condition defines an upward sloping curve in θ_1–ψ_2 space and that Propositions 4 and 5 continue to hold.

To solve the open economy model note that the open economy market clearing condition is:

$$Y_1 + Y_1^* = \frac{1}{\zeta}(Y_2 + Y_2^*),$$

where ζ is given by (23). In addition, when there is an assignment reversal across countries foreign will specialize in good one if and only if:

$$\bar{\zeta} \int_0^{\bar{\theta}^*} \theta g(\theta) f(s_1^*) dM^*(\theta) \le \int_0^{\bar{\theta}} \theta g(\theta) f(s_2) dM(\theta)$$

where $\frac{h'(\bar{\zeta})}{h(\bar{\zeta}) - \bar{\zeta} h'(\bar{\zeta})} = 1$ and $\psi_1 = \psi_2 = h'(\bar{\zeta})$.

Using these expressions we can solve for the open economy equilibrium following the same reasoning applied in the Cobb–Douglas case and Propositions 6, 7, 8 and 9 continue to hold.

B.3. Heckscher–Ohlin assignment model

Consider the following variant of the Heckscher–Ohlin model. There are two industries and two factors of production and each industry has a Cobb–Douglas technology:

$$Z_j = \left(\frac{Y_{1j}}{\mu_j}\right)^{\mu_j} \left(\frac{Y_{2j}}{1 - \mu_j}\right)^{1-\mu_j}, \qquad \mu_j \in (0, 1), \quad j = 1, 2,$$

where Z_j is output of industry j and Y_{kj} is the quantity of factor k used in industry j. Assume $\mu_1 > \mu_2$ meaning industry one is relatively intensive in factor one. Now, suppose the factors of production do not represent the economy's endowments, but must be produced. Factor k is the output of task k and task production is governed by the assignment problem in Section 3. Finally, suppose output from the two industries is combined to produce a final good, which can either be consumed or used as the input in task production. Output of the final good is given by:

$$Z = \left(\frac{Z_1}{\beta}\right)^{\beta}\left(\frac{Z_2}{1-\beta}\right)^{1-\beta}, \qquad \beta \in (0,1).$$

In this set-up factor supplies are endogenous to the equilibrium of the assignment problem. Suppose task two has higher input productivity than task one, $Q_2 > Q_1$. Then, given Assumption 1, high skill agents will be assigned to task two and low skill agents will perform task one.

Following the same logic used to solve for equilibrium in Section 4.2, it is easy to show the closed economy equilibrium of this Heckscher–Ohlin assignment model can be characterized by the same (WE) and (MC) conditions derived in Section 4.2, except the parameter β is replaced by $\mu_1\beta + \mu_2(1-\beta)$. Consequently, the model has a unique closed economy equilibrium featuring positive assortative matching between agents and tasks and the effects of technical change on the returns to skill and wage inequality are as described in Section 4.3.

In the baseline model all workers in the high productivity sector have higher skill than any worker in the low technology sector. However, in this Heckscher–Ohlin variant each industry employs both high skill workers to perform task two and low skill workers to perform task one. The equilibrium wage function ensures employers are indifferent between all workers assigned to a particular task. Therefore, I will assume the skill distribution of workers employed in each task is the same in both industries. Under this assumption the average wage w_j in industry j is:

$$w_j = \frac{\bar{w}_1 + \nu_j \bar{w}_2}{1+\nu_j},$$

where $\bar{w}_k$ is the average wage of agents assigned to task k and:

$$\nu_j \equiv \frac{1-\mu_j}{\mu_j}\,\frac{\mu_1\beta + \mu_2(1-\beta)}{1-\mu_1\beta-\mu_2(1-\beta)}\,\frac{M(\bar{\theta})-M(\theta_1)}{M(\theta_1)}.$$

Unsurprisingly, the mean industry wage is a weighted average of the mean task wages. Note that $\mu_1 > \mu_2 \Rightarrow \nu_1 < \nu_2$. Therefore, the mean industry wage is higher in the industry that is intensive in the high skill task. As in the baseline model, shocks to input productivity which switch the productivity ranking across tasks will reverse the ranking of industries by average wages and average employee skill. It can also be shown that labor's share of output is lower in the industry that is intensive in the high skill task.

Appendix C. Data

UNIDO's Industrial Statistics database contains employment and compensation data for 127 ISIC Revision 3 manufacturing industries at the 4 digit level. The database starts in 1990, but country coverage varies over time. The wage variable is defined as the ratio of Wages and salaries to Employment. The sample used in the paper is selected as follows: (i) for each country the data used is from the latest year between 1995 and 2000 for which wage data is reported; (ii) all industries reporting negative wages and salaries, or with fewer than 10 employees, were dropped;

(iii) only countries with data on at least 60% of industries were included.[39] The final sample covers 43 countries including the US. The sample countries are: Austria, Azerbaijan, Bangladesh, Belgium, Canada, Colombia, Denmark, Ecuador, Egypt, Finland, France, Germany, Hungary, India, Indonesia, Iran, Italy, Japan, Kyrgyzstan, Latvia, Lebanon, Lithuania, Malaysia, Mexico, Morocco, Netherlands, New Zealand, Norway, Peru, Portugal, Singapore, Slovakia, Slovenia, South Korea, Spain, Sweden, Thailand, Turkey, Ukraine, United Kingdom, US, Vietnam and Zimbabwe. Wage data for the US is available from 1997–2000. The statistics shown in Fig. 1 and Fig. 2 are calculated using US data for the same year in which a country reported data, unless the data is from 1995 or 1996, in which case US data from 1997 is used.

UNIDO's Industrial Statistics database does not include long time series of industry data at the 4 digit level. Consequently, changes in wage rank correlations are computed using wage data for 3 digit ISIC Revision 2 manufacturing industries. The 3 digit data covers 28 industries and I drop country-year observations with wage data for fewer than 80% of industries. I use data from 1965–1995 and compute annualized changes between the first and the last year in which a country is included in the data set. Only countries for which the first and the last year are at least 10 years apart are included.

The EU KLEMS data is taken from the March 2008 release of the database. The industry wage rate is defined as the ratio of Compensation of employees to Total hours worked by employees. The data for 1995 covers 29 countries and, at the most disaggregated level available, 29 manufacturing industries. I use the NAICS-based data for the US and drop Luxembourg from the sample since it has a higher income per capita than the US.

The IPUMS-International data includes all 34 countries for which a census taken between 1995 and 2005 is available. I use the internationally harmonized educational attainment and industry of employment variables and drop all respondents for whom either educational attainment or industry is unknown. I drop the industry labeled "Other industry, n.e.c.", leaving 15 industries covering the entire economy. Mali is not included in the regressions reported in Section 2.1 because its extremely low skill rank correlations make it a clear outlier.

Capital stock per capita is computed from the Penn World Tables 6.3 using the perpetual inventory method as implemented by Caselli (2005). Human capital per capita is computed from the Barro and Lee (2001) educational attainment data set. Average years of schooling for the population 25 and over is converted to human capital following the methodology in Caselli (2005).

The fifteen equipment types used to compute the cost of imported capital are: Computers, office and accounting equipment; Communication equipment; Instruments and medical equipment; Fabricated metal products; Engines and turbines; Metalworking machinery; Special industry machinery, n.e.c.; General industrial equipment; Electrical equipment; Autos and trucks; Aircraft; Ships and boats; Railroad equipment; Furniture and fixtures, and; Agricultural machinery. The 1997 US capital flow table gives equipment investment for 53 manufacturing industries which I map to ISIC 4 digit industries by combining the concordance from capital flow industries to NAICS industries in the capital flow table and a concordance from NAICS industries to ISIC industries from the US Census Bureau. For most industries the capital expenditure shares only vary at the 2 or 3 digit level. Consequently, I estimate equation (2) with the standard errors clustered by country-2 digit industry groups.

[39] Informal data examination suggests there is substantial noise in the Industrial Statistics database. The 60% coverage cut-off is designed to select for countries that produce relatively comprehensive industrial statistics, since such countries are likely to report higher quality data. It also reduces the selection bias that may arise if there is non-randomness in which industries report data. The results in the paper do not depend on the exact value of the cut-off.

Equipment trade data is from the NBER-United Nations world trade data set and I use a concordance from SITC Rev. 2 product categories to US BEA industries obtained from the Center for International Data to calculate trade in each of the 15 equipment varieties. The geographic variables used to estimate the gravity equation are from CEPII. The population weighted arithmetic mean distance between major cities is used to measure distance. The gravity equation is estimated using equipment trade in 2000.

Investment in Table 5 and wages in Table 6 are from UNIDO's Industrial Statistics database. The capital, skill and contract intensity variables are defined as the capital stock per worker, the share of non-production workers in employment and the fraction of inputs neither sold on an exchange nor reference priced, respectively. The capital and skill intensities are computed using the NBER manufacturing database for 2000, while contract intensity is taken from Nunn (2007) and is based on US input-output tables in 1997. To obtain measures of capital, skill and contract intensity for ISIC industries I used concordances between NAICS and ISIC Revision 3 industries from the US Census Bureau and Statistics Canada to construct a concordance that mapped each NAICS manufacturing industry to its primary ISIC counterpart. Capital abundance is defined as capital stock per capita computed from the Penn World Tables 6.3, skill abundance is defined as the secondary school enrollment rate from the World Bank's World Development Indicators and the rule of law is taken from the World Bank's World Governance Indicators for 2000. The countries in the low income per capita sample are: Azerbaijan; Bangladesh; Colombia; Ecuador; Egypt; India; Indonesia; Iran; Kyrgyzstan; Latvia; Lebanon; Morocco; Peru; Thailand; Turkey; Ukraine; Vietnam, and; Zimbabwe.

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