DYNAMIC SELECTION: AN IDEA FLOWS THEORY OF ENTRY, TRADE, AND GROWTH*

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This article develops an idea flows theory of trade and growth with heterogeneous firms. Entrants learn from incumbent firms, and the diffusion technology is such that learning depends not on the frontier technology, but on the entire distribution of productivity. By shifting the productivity distribution upward, selection causes technology diffusion, and in equilibrium this dynamic selection process leads to endogenous growth without scale effects. On the balanced growth path, the productivity distribution is a traveling wave with a lower bound that increases over time. The free entry condition implies trade liberalization must increase the dynamic selection rate to offset the profits from new export opportunities. Consequently, trade integration raises long-run growth. Dynamic selection is a new source of gains from trade not found when firms are homogeneous. Calibrating the model implies dynamic selection approximately triples the gains from trade compared to heterogeneous firm economies with static steady states. *JEL* Codes: F12, O33, O41.

I. INTRODUCTION

Understanding the gains from trade is central to evaluating the costs and benefits of globalization. Building on the finding that only high-performing firms participate in international trade (Bernard and Jensen 1995) recent work has studied the implications of firm heterogeneity for the gains from trade. The existence of substantial productivity differences between firms producing very similar products (Syverson 2011) introduces two channels for aggregate productivity gains that are absent when all firms produce on the technology frontier: cross-firm resource reallocation from less to more productive firms (Melitz 2003; Hsieh and Klenow 2009) and technology diffusion between firms (Luttmer 2007; Lucas and Moll 2014).

The literature on firm heterogeneity and trade has, with few exceptions, focused on the reallocation channel and studied economies with static steady states (Melitz 2003; Atkeson and Burstein 2010; Arkolakis, Costinot, and Rodríguez-Clare 2012;

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Melitz and Redding 2013).[1] However, abstracting from technology diffusion overlooks a dynamic complementarity between selection-induced reallocation and technology diffusion. Selection on productivity causes less productive firms to exit and shifts the productivity distribution of incumbent firms upwards. When knowledge spillovers depend on the entire distribution of technologies used in an economy, an upward shift in the productivity distribution leads to technology diffusion. Thus, selection causes technology diffusion. Moreover, since technology diffusion raises average productivity, it leads to low-productivity firms becoming unprofitable and generates further selection. To understand the consequences of this complementarity, I incorporate technology diffusion into a dynamic open economy with heterogeneous firms. The combination of selection and technology diffusion creates a new channel through which trade increases growth and generates a new source of dynamic gains from trade.

To introduce technology diffusion, I develop a dynamic version of Melitz (2003) that features knowledge spillovers from incumbent firms to entrants. In most endogenous growth theory, the engine of growth is either knowledge spillovers that reduce the relative cost of entry in an expanding varieties framework (Romer 1990) or productivity spillovers that allow entrants to improve the frontier technology in a quality ladders framework (Aghion and Howitt 1992; Grossman and Helpman 1991). However, Bollard, Klenow, and Li (2013) find that entry costs do not fall relative to the cost of labor as economies grow. Moreover, the persistence of large within-industry productivity differences and the fact that most entrants do not use frontier technologies imply that in addition to innovation, the diffusion of existing technologies is important for aggregate productivity growth. Motivated by this observation recent work on idea flows has studied technology diffusion by assuming agents learn from meetings with other randomly chosen agents in an economy (Alvarez, Buera, and Lucas 2008; Lucas and Moll 2014; Perla and Tonetti 2014).

I build on the idea flows literature and assume (i) spillovers affect productivity, but not the cost of entry relative to labor costs;

1. Throughout this article I use the term “static steady-state economies” to refer to both static models and papers such as Melitz (2003) and Atkeson and Burstein (2010) that incorporate dynamics, but do not allow for growth and consequently have a steady state that is constant over time.

and (ii) spillovers depend not on the frontier technology but on the entire distribution of productivity. To be specific, each firm has both a product and a process technology. Product ownership gives a firm the monopoly right to produce a particular variety and is protected by an infinitely lived patent.[2] A firm's process technology determines its productivity and is nonrival and partially nonexcludable. When a new product is created, an entering firm adopts a process technology by learning from incumbent firms. In this manner knowledge about how to organize, manage, and implement production diffuses between firms. However, learning frictions such as information asymmetries and adoption capacity constraints mean not all entrants learn from the most productive firms. Instead, knowledge spillovers depend on the average productivity of all producers and spillovers increase as the distribution of incumbent firm productivity improves. This formalization of knowledge spillovers is consistent with evidence that the productivity distributions of entrants and incumbent firms move together over time (Aw, Chen, and Roberts 2001; Foster, Haltiwanger, and Krizan 2001; Disney, Haskel, and Heden 2003).

In the language of the Melitz model, the knowledge spillover process implies that instead of drawing productivity from an exogenous distribution, entrants sample from a distribution that is endogenous to the productivity distribution of incumbent firms. Consequently, when selection increases the productivity cutoff below which firms exit, it also generates spillovers that improve the productivity draws of future entrants. Entry then causes further selection due to increased competition. In equilibrium the positive feedback between selection and technology diffusion leads to endogenous growth driven by a dynamic selection mechanism.[3] On the balanced growth path, the firm size distribution is stationary and the productivity distribution of incumbent firms is a traveling wave that shifts upward over time as the exit cutoff grows.[4]

2. For a theory of product technology diffusion, see product cycle models such as those considered by Grossman and Helpman (1991).

3. This article uses "dynamic selection" to refer to long-run growth resulting from growth in the exit cutoff. Constantini and Melitz (2008), Atkeson and Burstein (2010), and Burstein and Melitz (2011) study the dynamics of selection along the transition path between static steady states, but do not allow for long-run growth.

4. Luttmer (2010) notes that the U.S. firm employment distribution appears to be stationary. König, Lorenz, and Zilibotti (2012) show using European firm data

In the open economy firms face both fixed and variable trade costs. Only high-productivity firms export and selection increases the exit cutoff and shifts the productivity distribution of incumbent firms upward as in Melitz (2003). Consequently, trade liberalization generates technology diffusion and the expected productivity of future entrants rises. Unsurprisingly, this technology diffusion magnifies the rise in average productivity following trade liberalization. More important, it leads to a permanent increase in the long-run growth rate. To understand why, consider the free entry condition. In equilibrium, the cost of entry must equal an entrant's expected discounted lifetime profits. In the absence of technology diffusion, free entry implies an increase in the expected profits from exporting is offset by a reduced probability of survival leading to the static selection effect found in Melitz (2003). However, with technology diffusion an increase in the level of the exit cutoff does not change the distribution of entrants' productivity relative to the exit cutoff. Instead, I show that free entry requires an increase in the growth rate of the exit cutoff, which raises the rate at which a successful entrant's technology becomes obsolete and reduces entrants' expected discounted lifetime profits.[5] This dynamic selection effect of trade increases the growth rate of average productivity and, consequently, consumption per capita. Thus, the complementarity between selection and technology diffusion implies trade liberalization raises growth.[6]

How does higher growth affect the gains from trade? In static steady-state economies such as Melitz (2003), the equilibrium exit cutoff and export threshold are efficient, implying that any adjustments in their levels following changes in trade costs generate welfare gains absent from homogeneous firm models (Melitz and Redding 2013). However, Atkeson and Burstein (2010) find these welfare gains are small relative to increases in average firm

that the observed firm productivity distribution behaves like a traveling wave with increasing mean.

5. Atkeson and Burstein (2010) also highlight the role played by the free entry condition in determining the general equilibrium gains from trade. However, while in a static steady-state economy the free entry condition limits the gains from static selection, in this article free entry is critical in ensuring dynamic gains from trade.

6. The empirical literature on trade and growth faces the twin challenges of establishing causal identification and separating level and growth effects. However, the balance of evidence suggests a positive effect of trade on growth. See, for example, Frankel and Romer (1999) or Wacziarg and Welch (2008).

productivity since in general equilibrium the gains from selection and reallocation are offset by reductions in entry and technology investment. Similarly, Arkolakis, Costinot, and Rodríguez-Clare (2012) argue firm heterogeneity is not important for quantifying the aggregate gains from trade. In particular, they show that in both the homogeneous firms model of Krugman (1980) and a version of Melitz (2003) with a Pareto productivity distribution, the gains from trade can be expressed as the same function of two observables: the import penetration ratio and the elasticity of trade with respect to variable trade costs (the trade elasticity). By raising the growth rate, the dynamic selection effect generates a new source of gains from trade not found in either static steady-state economies with heterogeneous firms or dynamic economies with homogeneous firms. However, given the findings of Atkeson and Burstein (2010) and Arkolakis, Costinot, and Rodríguez-Clare (2012) it is natural to ask whether the benefits from an increase in the dynamic selection rate are offset by other general equilibrium effects.

To answer this question, the article decomposes the welfare effects of trade into two terms: a static term that is identical to the gains from trade in Melitz (2003) (assuming a Pareto productivity distribution) and has the same calibration using the import penetration ratio and the trade elasticity as the gains from trade in Arkolakis, Costinot, and Rodríguez-Clare (2012), and a dynamic term that depends on trade only through the growth rate of consumption per capita. The dynamic term is strictly increasing in the growth rate because selection generates a positive externality by raising the productivity of future entrants.[7] Since trade raises growth, the welfare decomposition implies the gains from trade in this article are strictly higher than in Melitz (2003). Conditional on the observed import penetration ratio and trade elasticity, the gains from trade are also strictly higher than in the class of static steady-state economies studied by Arkolakis, Costinot, and Rodríguez-Clare (2012).[8] It follows that the combination of firm

7. Starting from the decentralized equilibrium a social planner can raise welfare by increasing the dynamic selection rate through either subsidizing entry or taxing the fixed production cost.

8. An important distinction here is that the predicted import penetration ratio and trade elasticity are the same functions of underlying parameters as in Melitz (2003). However, they differ from the predictions made by other models considered by Arkolakis, Costinot, and Rodríguez-Clare (2012). See Melitz and Redding (2013) for further discussion of this point.

heterogeneity and technology diffusion raises the gains from trade.

To assess the magnitude of the gains from trade-induced dynamic selection I calibrate the model using U.S. data. As in Arkolakis, Costinot, and Rodríguez-Clare (2012) the import penetration ratio is a sufficient statistic for the level of trade integration and the welfare effects of trade can be calculated in terms of a small number of observables and parameters. In addition to the import penetration ratio and trade elasticity, the calibration uses the rate at which new firms are created, the population growth rate, the intertemporal elasticity of substitution, the discount rate, and the elasticity of substitution between goods. The baseline calibration implies U.S. growth is 11% higher than it would be under autarky. More important, the increase in the dynamic selection rate triples the gains from trade compared to the static steady-state economies considered by Arkolakis, Costinot, and Rodríguez-Clare (2012). The finding that dynamic selection is quantitatively important for the gains from trade is extremely robust. For plausible variations in the parameter values, the dynamic selection effect always at least doubles the gains from trade.

In addition to contributing to the debate over the gains from trade, this article is related to the endogenous growth literature. Open economy endogenous growth theories with homogeneous firms find that the effects of trade on growth in a single-sector economy are driven by scale effects and international knowledge spillovers (Grossman and Helpman 1991; Rivera-Batiz and Romer 1991). By contrast, neither scale effects nor international knowledge spillovers are necessary for trade to raise growth through dynamic selection. To highlight the novelty of the dynamic selection mechanism, I assume there are no international knowledge spillovers and I show the equilibrium growth rate is independent of population size, that is, there are no scale effects. Thus, this article implies neither the counterfactual prediction that larger economies grow faster (Jones 1995b) nor the semi-endogenous growth prediction that population growth is the only source of long-run growth (Jones 1995a). There are no scale effects in this article because both the productivity distribution and the mass of varieties produced are endogenous and knowledge spillovers only depend on the distribution of productivity. In equilibrium a larger population leads to an increase in the mass of varieties produced (unlike in quality ladders growth models), but since the creation of new varieties does not lower the cost of future entry (unlike in expanding varieties growth models) the growth rate is unaffected.

Selection-based growth in closed economies has been studied in recent work on idea flows by Luttmer (2007, 2012), Alvarez, Buera, and Lucas (2008), Lucas and Moll (2014), and Perla and Tonetti (2014). Most closely related to this paper is Luttmer (2007), who allows for free entry and spillovers from incumbents to entrants, but focuses on how postentry productivity shocks shape the equilibrium productivity distribution and does not give a complete characterization of the balanced growth path or analyze the effects of trade. By abstracting from postentry productivity shocks this article identifies the determinants of aggregate growth and shows the free entry condition is central in determining the relationship between trade and growth. Moreover, the specification of knowledge spillovers introduced here provides a more tractable way to model technology diffusion than is found in previous work on idea flows. In Section V and Appendix B I take advantage of this tractability by extending the technology diffusion model to allow for international knowledge spillovers, frontier technology growth, and firm-level productivity dynamics. The finding that trade raises growth by increasing the dynamic selection rate is robust to these extensions.

The effects of trade on growth and selection are considered by Baldwin and Robert-Nicoud (2008), Alvarez, Buera, and Lucas (2011), Impullitti and Licandro (2012), and Perla, Tonetti, and Waugh (2015). Baldwin and Robert-Nicoud (2008) incorporate firm heterogeneity into an expanding variety growth model and find that whether trade raises growth depends on the extent of international knowledge spillovers. However, since knowledge spillovers affect entry costs instead of entrants' productivity, the model has three counterfactual implications. First, the equilibrium productivity distribution is time invariant. Second, entry costs decline relative to labor costs as the economy grows. Third, average firm size decreases as the economy grows. Alvarez, Buera, and Lucas (2011) show international knowledge spillovers increase growth in an Eaton and Kortum (2002) trade model, but assume the rate of technology diffusion is independent of agents' optimization decisions and do not model firm-level behavior. Impullitti and Licandro (2012) study an oligopolistic economy with innovation by incumbent firms and find trade increases growth because the procompetitive effect of trade leads to lower mark-ups, which raises innovation. By contrast, in this article mark-ups are constant and the engine of growth is the dynamic complementarity between selection and knowledge spillovers. Perla, Tonetti, and Waugh (2015) develop an open economy extension of Perla and Tonetti (2014) in which growth is driven by technology diffusion between incumbent firms, but the mass of firms is fixed. They find trade can raise or lower growth depending on how the costs of searching for a better technology are specified, but since the mass of firms is exogenous, they do not include the free entry condition which, in this article, ensures a positive effect of trade on growth.

The remainder of the article is organized as follows. Section II introduces the technology diffusion model, while Section III solves for the balanced growth path equilibrium and analyzes the effect of trade on growth. Section IV characterizes household welfare on the balanced growth path and then calibrates the model to quantify the gains from trade. Finally, Section V demonstrates the robustness of the paper's results to relaxing some of the simplifying assumptions made in the baseline model, before Section VI concludes. An online Technical Appendix provides additional details on the derivations of some of the equations used in the article.

II. Technology Diffusion Model

Consider a world made up of $J+1$ symmetric economies. When $J=0$ there is a single autarkic economy, whereas for $J>0$ we have an open economy model. Time t is continuous and the preferences and production possibilities of each economy are as follows.

II.A. Preferences

Each economy consists of a set of identical households with dynastic preferences and discount rate ρ. The population L_t at time t grows at rate $n \geq 0$ where n is constant and exogenous. Each household has constant intertemporal elasticity of substitution preferences and seeks to maximize:

$$U = \int_{t=0}^{\infty} e^{-\rho t} e^{nt} \frac{c_t^{1-\frac{1}{\gamma}} - 1}{1 - \frac{1}{\gamma}} dt, \tag{1}$$

where c_t denotes consumption per capita and $\gamma > 0$ is the intertemporal elasticity of substitution. The numeraire is chosen so that the price of the consumption good is unity. Households can lend or borrow at interest rate r_t, and a_t denotes assets per capita. Consequently, the household's budget constraint expressed in per capita terms is:

$$\dot{a}_t = w_t + r_t a_t - c_t - n a_t, \tag{2}$$

where w_t denotes the wage. Note that households do not face any uncertainty.

Under these assumptions and a no Ponzi game condition, the household's utility maximization problem is standard[9] and solving gives the Euler equation:

$$\frac{\dot{c}_t}{c_t} = \gamma(r_t - \rho), \tag{3}$$

together with the transversality condition:

$$\lim_{t\to\infty} \left\{ a_t exp\left[-\int_0^t (r_s - n) ds \right] \right\} = 0. \tag{4}$$

II.B. Production and Trade

Output is produced by monopolistically competitive firms each of which produces a differentiated good. Labor is the only factor of production and all workers are homogeneous and supply one unit of labor per period. There is heterogeneity across firms in labor productivity θ. A firm with productivity θ at time t has marginal cost of production $\frac{w_t}{\theta}$ and must also pay a fixed cost f per period to produce. The fixed cost is denominated in units of labor. The firm does not face an investment decision, and firm productivity remains constant over time.[10] The final consumption good is produced under perfect competition as a constant elasticity of substitution aggregate of all available goods with elasticity of substitution $\sigma > 1$ and is nontradable.[11]

Firms can sell their output both at home and abroad. However, as in Melitz (2003) firms that select into exporting

9. See, for example, chapter 2 of Barro and Sala-i-Martin (2004).

10. Appendix B analyzes extensions of the model that include firm-level productivity dynamics.

11. This is equivalent to assuming households have constant elasticity of substitution preferences over differentiated goods.

face both fixed and variable costs of trade. Exporters incur a fixed cost f_x per export market per period denominated in units of domestic labor, while variable trade costs take the iceberg form. To deliver one unit of output to a foreign market a firm must ship $\tau \geq 1$ units. I assume $\tau^{\sigma-1}f_x > f$, which is a necessary and sufficient condition to ensure that in equilibrium not all firms export. Since I consider a symmetric equilibrium, all parameters and endogenous variables are invariant across countries.

Conditional on the distribution of firm productivity, the structure of production and demand in this economy is equivalent to that in Melitz (2003) and solving firms' static profit maximization problems is straightforward. Firms face isoelastic demand and set factory gate prices as a constant mark-up over marginal costs. Firms only choose to produce if their total variable profits from domestic and foreign markets are sufficient to cover their fixed production costs and firms only export to a given market if their variable profits in that market are sufficient to cover the fixed export cost. Variable profits in each market are strictly increasing in productivity and since $\tau^{\sigma-1}f_x > f$ the productivity above which firms export exceeds the minimum productivity for entering the domestic market. In particular, there is a cutoff productivity θ_t^* such that firms choose to produce at time t if and only if their productivity is at least θ_t^*.[12] This exit cutoff is given by:

$$\theta_t^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left(\frac{fw_t^{\sigma}}{c_t L_t}\right)^{\frac{1}{\sigma-1}}. \tag{5}$$

In addition, there is a threshold $\tilde{\theta}_t > \theta_t^*$ such that firms choose to export at time t if and only if their productivity is at least $\tilde{\theta}_t$. The export threshold is:

$$\tilde{\theta}_t = \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \tau \theta_t^*. \tag{6}$$

12. In equilibrium θ_t^* will be strictly increasing over time. Since firm productivity remains constant over time, it follows that for firms with productivity below θ_t^* there is zero option value from continuing to operate in the hope of future profits. Consequently, firms' exit decisions depend only on their static profit maximization problems and θ_t^* is obtained by setting static profits equal to 0.

Firms can lend or borrow at interest rate r_t and the market value $V_t(\theta)$ of a firm with productivity θ is given by the present discounted value of future profits:

$$V_t(\theta) = \int_t^{\infty} \pi_v(\theta) exp\left(-\int_t^v r_s ds\right) dv, \tag{7}$$

where π_v denotes the profit flow from both domestic and export sales at time v net of fixed costs and $\pi_v(\theta) = 0$ if the firm does not produce.

In what follows, it will be convenient to use the change of variables $\phi_t \equiv \frac{\theta}{\theta_t^*}$, where ϕ_t is firm productivity relative to the exit cutoff. I will refer to ϕ_t as a firm's relative productivity. Let $W_t(\phi_t)$ be the value of a firm with relative productivity ϕ_t at time t. Obviously, only firms with $\phi_t \geq 1$ will choose to produce and only firms with $\phi_t \geq \tilde{\phi} \equiv \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} \tau$ will choose to export. For these firms prices, employment and profits are given by:

$$p_t^d(\phi_t) = \frac{\sigma}{\sigma-1} \frac{w_t}{\phi_t \theta_t^*}, p_t^x(\phi_t) = \tau p_t^d(\phi_t),$$

$$l^d(\phi_t) = f\left[(\sigma-1)\phi_t^{\sigma-1} + 1\right], \; l^x(\phi_t) = f\tau^{1-\sigma}\left[(\sigma-1)\phi_t^{\sigma-1} + \tilde{\phi}^{\sigma-1}\right], \tag{8}$$

$$\pi_t^d(\phi_t) = f w_t\left(\phi_t^{\sigma-1} - 1\right), \; \pi_t^x(\phi_t) = f\tau^{1-\sigma} w_t\left(\phi_t^{\sigma-1} - \tilde{\phi}^{\sigma-1}\right), \tag{9}$$

where I have used d and x superscripts to denote the domestic and export markets, respectively. Observe that employment is a stationary function of relative productivity and that, conditional on relative productivity ϕ_t, both domestic and export profits are proportional to the fixed cost of production. Since there are J export markets, total firm employment is given by $l(\phi_t) = l^d(\phi_t) + Jl^x(\phi_t)$ and total firm profits are $\pi_t(\phi_t) = \pi_t^d(\phi_t) + J\pi_t^x(\phi_t)$.

II.C. Knowledge Spillovers and Entry

To invent new goods, entrants must employ workers to undertake research and development (R&D). Employing $R_t f_e$ R&D workers produces a flow R_t of innovations where $f_e > 0$ is an entry cost parameter. Each innovation generates both an idea for a new good (product innovation) and a production technology for producing the good (process innovation). Product ownership is protected by an infinitely lived patent, but knowledge spillovers occur because firms' process technologies are nonrival and partially nonexcludable. Consequently, innovators can learn from the production techniques (technologies, managerial methods, organizational forms, input choices, etc.) used by existing firms.[13] However, due to frictions that limit knowledge diffusion such as information asymmetries and absorption capacity constraints not all entrants learn from the most productive firms.[14] Instead, knowledge spillovers depend on the entire distribution of technologies used by incumbent firms.

Formally, I model knowledge spillovers by assuming that the productivity of entrants is given by:

$$\theta = x_t \psi, \tag{10}$$

where x_t is the average productivity of firms that operate at time t and ψ is a stochastic component drawn from a time-invariant sampling distribution with cumulative distribution function $F(\psi)$. Knowledge spillovers are captured by variation in x_t. To understand the knowledge spillover process observe that x_t has the following three properties. First, x_t is a location statistic such that if $G_t(\theta)$ is the cumulative productivity distribution function for firms that produce at time t and $G_{t_1}(\theta) = G_{t_0}(\theta/\kappa)$ then $x_{t_1} = \kappa x_{t_0}$. Thus, if G_t shifts to the right by a proportional factor κ then x_t increases by the same factor κ. Second, holding $G_t(\theta)$ constant, x_t is independent of the mass of incumbent firms. This ensures x_t is independent of the size of the economy. Third, conditional on the mean productivity, x_t does not depend on the maximum of the incumbent firm productivity distribution. In particular, knowledge spillovers are driven not by the frontier technology but by shifts in the entire productivity distribution. Modeling entrants' productivity draws using equation (10) implies that the cumulative

13. A large literature documents the importance of learning from other producers in agricultural technology diffusion (e.g., Foster and Rosenzweig 1995; Bandiera and Rasul 2006; Conley and Udry 2010). Robertson, Swan, and Newell (1996) discuss the role of information networks in shaping the adoption of computer-aided production management in UK manufacturing firms. See Baptista (1999) for an overview of the literature on process technology diffusion.

14. Conley and Udry (2010) find that pineapple farmers learn from other producers even when those producers use suboptimal input levels.

distribution function of entrants' productivity $\tilde{G}_t$ is given by: $\tilde{G}_t(\theta) = F(\theta/x_t)$. This implication is consistent with the observations that (i) there is substantial productivity heterogeneity within an entering cohort, and (ii) the productivity distributions of entrants and incumbents move together closely over time.[15]

The specification of knowledge spillovers introduced above differs in important ways from that used in either expanding variety (Romer 1990) or quality ladders (Grossman and Helpman 1991; Aghion and Howitt 1992) growth models. In expanding variety models, knowledge accumulation lowers entry costs relative to labor costs and average firm employment falls as the economy grows. However, observed variation in firm sizes is inconsistent with these predictions. Bollard, Klenow, and Li (2013) use cross-country, cross-industry data on the number and size of firms to infer that entry costs are approximately proportional to labor costs and do not fall with development. In addition, the U.S. firm employment distribution is roughly stable over time (Luttmer 2010). In quality ladders models, entrants learn from frontier technologies and are more productive than incumbent firms. Yet empirical studies find that most entrants do not use frontier technologies (Foster, Haltiwanger, and Krizan 2001). In contrast to expanding variety models, the knowledge spillovers studied in this article affect productivity not entry costs, while in contrast to quality ladders models the spillovers are a function of not only frontier technologies but all technologies used in the economy.

The structure of knowledge spillovers embodied in equation (10) builds on epidemic models of technology diffusion, the search model of technological change developed by Kortum (1997), and recent work on idea flows (Luttmer 2007; Alvarez, Buera, and Lucas 2008; Lucas and Moll 2014; Perla and Tonetti 2014). In epidemic models of technology diffusion the rate at which a new

15. For evidence, see Foster, Haltiwanger, and Krizan (2001) for the United States; Aw, Chen, and Roberts (2001) for Taiwan; and Disney, Haskel, and Heden (2003) for the United Kingdom. For example, Aw, Chen and Roberts (2001), p. 71, conclude that "the productivity distributions of entering firms and incumbents shift over time in similar ways." Selection effects could rationalize this finding without requiring any knowledge spillovers, but selection alone is insufficient to generate endogenous long-run growth.

technology spreads depends on the proportion of the population that uses the technology (Griliches 1957; Mansfield 1961). Epidemic models explain the lags in technology diffusion and why the rate at which a new technology is adopted is S-shaped over time (Stoneman 2001), but do not consider the case where there are a continuum of productivity levels rather than a binary technology use variable. Kortum (1997) analyzes a closed economy, quality ladders model where knowledge spillovers cause improvements in the productivity distribution from which new ideas are drawn and the strength of spillovers depends on the stock of R&D. By contrast, in this article only R&D that causes shifts in the firm productivity distribution leads to knowledge spillovers.

The idea flows literature studies the evolution of the productivity distribution when agents learn from meeting other agents with higher knowledge. Since meetings result from random matching between agents, the technology diffusion process depends on the distribution of knowledge in an economy. Applied to the economy studied in this article, learning through random matching would imply that the productivity distribution of entrants was identical to the productivity distribution of incumbent firms. As in the idea flows literature, I model knowledge spillovers as a function of the entire productivity distribution, but instead of assuming random matching equation (10) takes a reduced-form approach in which the productivity of entrants depends on the average of the incumbent firm productivity distribution and a random component. Consequently, the productivity distributions of entrants and incumbents may differ. For the baseline model considered in Sections III and IV this difference is relatively unimportant. I show in Appendix B that if knowledge spillovers result from random matching between entrants and incumbent firms, the balanced growth path and the effects of trade integration obtained in the baseline model are unaffected. However, equation (10) provides a more tractable representation of knowledge spillovers than random matching. In Section V I discuss how to take advantage of this tractability and relax some of the simplifying assumptions made in the baseline model.

A final observation concerning equation (10) is that knowledge spillovers are intranational not international in scope. Section V analyzes an extension of the model with international knowledge spillovers, but in the baseline model entrants only learn from domestic firms.

There is free entry into R&D, implying that in equilibrium the expected cost of innovating equals the expected value of creating a new firm:

$$f_e w_t = \int_\theta V_t(\theta) d\tilde{G}_t(\theta). \tag{11}$$

Entry is financed by a competitive and costless financial intermediation sector which owns the firms and thereby enables investors to pool the risk faced by innovators. Consequently, each household effectively owns a balanced portfolio of all firms and R&D projects.[16]

How does the relative productivity distribution evolve over time? Let H_t and $\tilde{H}_t$ be the cumulative distribution functions of relative productivity ϕ for existing firms and entrants, respectively. Given the structure of knowledge spillovers we must have $\tilde{H}_t(\phi) = F\left(\frac{\phi\,\theta_t^*}{x_t}\right)$. To characterize the intertemporal evolution of H_t I first formulate a law of motion for $H_t(\phi)$ between t and $t+\Delta$ assuming time is discrete with periods of length Δ and then take the limit as $\Delta \to 0$. Let M_t be the mass of producers in the economy at time t and assume the exit cutoff is strictly increasing over time.[17] Then the mass of firms with relative productivity less than ϕ at time $t+\Delta$ is:

$$\begin{aligned} M_{t+\Delta} H_{t+\Delta}(\phi) = M_t\left[H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\phi\right) - H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\right)\right] \\ + \Delta R_t\left[F\left(\frac{\phi\,\theta^*_{t+\Delta}}{x_t}\right) - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right)\right]. \end{aligned} \tag{12}$$

Since $\phi_{t+\Delta} \le \phi \Leftrightarrow \phi_t \le \frac{\theta^*_{t+\Delta}}{\theta^*_t}\phi$ the first term on the right-hand side is the mass of time t incumbents that have relative productivity less than ϕ, but greater than one, at time $t+\Delta$. $M_t H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\phi\right)$ gives the mass of time t producers with relative productivity

16. Since countries are symmetric it does not matter whether asset markets operate at the national or global level. For completeness, I assume asset markets are national.

17. When solving the model, I restrict attention to balanced growth paths on which θ_t^* is strictly increasing in t. In an economy with a declining exit cutoff, equilibrium would depend on whether exit from production was temporary or irreversible. I abstract from these issues in this article.

less than ϕ at time $t+\Delta$, while $M_t H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\right)$ is the mass of time t incumbents that exit at time $t+\Delta$ because their productivity falls below the exit cutoff. The second term on the right-hand side gives the mass of entrants at time t whose relative productivity lies between 1 and ϕ at time $t+\Delta$.

Letting $\phi \to \infty$ in equation (12) implies:

$$M_{t+\Delta} = M_t\left[1 - H_t\left(\frac{\theta^*_{t+\Delta}}{\theta^*_t}\right)\right] + \Delta R_t\left[1 - F\left(\frac{\theta^*_{t+\Delta}}{x_t}\right)\right], \tag{13}$$

and taking the limit as $\Delta \to 0$ gives:[18]

$$\frac{\dot{M}_t}{M_t} = -H'_t(1)\frac{\dot{\theta}^*_t}{\theta^*_t} + \left[1 - F\left(\frac{\theta^*_t}{x_t}\right)\right]\frac{R_t}{M_t}. \tag{14}$$

This expression illustrates the two channels that affect the mass of incumbent firms. R&D generates a flow R_t of innovations, but a fraction $F\left(\frac{\theta^*_t}{x_t}\right)$ of innovators receive a productivity draw below the exit cutoff and choose not to produce. In addition, as the exit cutoff increases, firms' relative productivity levels decline and a firm exits when its relative productivity falls below 1. The rate at which firms exit due to growth in the exit cutoff depends on the density of the relative productivity distribution at the exit cutoff $H'_t(1)$.

Now using equation (13) to substitute for $M_{t+\Delta}$ in equation (12), rearranging and taking the limit as $\Delta \to 0$ we obtain the following law of motion for $H_t(\phi)$:

$$\begin{aligned}\dot{H}_t(\phi) = {} & \left\{\phi H'_t(\phi) - H'_t(1)[1 - H_t(\phi)]\right\}\frac{\dot{\theta}^*_t}{\theta^*_t} \\ & + \left\{F\left(\frac{\phi\,\theta^*_t}{x_t}\right) - F\left(\frac{\theta^*_t}{x_t}\right) - H_t(\phi)\left[1 - F\left(\frac{\theta^*_t}{x_t}\right)\right]\right\}\frac{R_t}{M_t}.\end{aligned} \tag{15}$$

Thus, the evolution of the relative productivity distribution is driven by growth in the exit cutoff and the entry of new firms. When $\dot{H}_t(\phi) = 0$ for all $\phi \geq 1$ the relative productivity distribution is stationary.

18. In obtaining both this expression and equation (15) I assume θ^*_t is differentiable with respect to t and $H_t(\phi)$ is differentiable with respect to ϕ. Both conditions will hold on the balanced growth path considered later. The online Technical Appendix provides further details on the derivation of equations (14) and (15).

II.D. Equilibrium

In addition to consumer and producer optimization, equilibrium requires the labor and asset markets to clear in each economy in all periods. Labor market clearing implies:

$$L_t = M_t \int_\phi l(\phi) dH_t(\phi) + R_t f_e, \tag{16}$$

while asset market clearing requires that aggregate household assets equal the combined worth of all firms:

$$a_t L_t = M_t \int_\phi W_t(\phi) dH_t(\phi). \tag{17}$$

Finally, as an initial condition I assume that at time 0 there exists in each economy a mass $\hat{M}_0$ of potential producers with productivity distribution $\hat{G}_0(\theta)$. We can now define the equilibrium.

An equilibrium of the world economy is defined by time paths for $t \in [0, \infty)$ of consumption per capita c_t, assets per capita a_t, wages w_t, the interest rate r_t, the exit cutoff θ_t^*, the export threshold $\tilde{\theta}_t$, average firm productivity x_t, firm values $W_t(\phi)$, the mass of firms in each economy M_t, the flow of innovations in each economy R_t and the relative productivity distribution $H_t(\phi)$ such that (i) households choose c_t to maximize utility subject to the budget constraint (2) implying the Euler equation (3) and the transversality condition (4); (ii) producers maximize profits implying the exit cutoff satisfies equation (5), the export threshold satisfies equation (6), and firm value is given by equation (7); (iii) free entry into R&D implies equation (11); (iv) the exit cutoff is strictly increasing over time and the evolution of M_t and $H_t(\phi)$ are governed by equations (14) and (15); (v) labor and asset market clearing imply equations (16) and (17), respectively; and (vi) at time 0 there are $\hat{M}_0$ potential producers in each economy with productivity distribution $\hat{G}_0(\theta)$.

III. Balanced Growth Path

I will solve for a balanced growth path equilibrium of the world economy. On a balanced growth path $c_t, a_t, w_t, \theta_t^*, \tilde{\theta}_t, x_t, W_t(\phi), M_t$ and R_t grow at constant rates, r_t is constant, and the distribution of relative productivity ϕ is stationary, meaning

$\dot{H}_t(\phi) = 0 \,\forall\, t, \phi$. When solving for the balanced growth path I impose the following assumption on the sampling distribution F from which the stochastic component of an entrant's productivity is drawn.

ASSUMPTION 1.

(i) The sampling productivity distribution F is Pareto: $F(\psi) = 1 - (\frac{\psi}{\psi_{\min}})^{-k}$ for $\psi \geq \psi_{\min}$ with $k > \max\{1, \sigma - 1\}$.
(ii) The lower bound of the sampling productivity distribution satisfies: $x_t \psi_{\min} \leq \theta_t^*$.

The first part of Assumption 1 simply states that F is a Pareto distribution with scale parameter $\psi_{\min}$ and shape parameter k.[19] The second part implies not all entrants draw productivity levels above the exit cutoff and provided the inequality is strict some entrants receive productivity draws below the exit cutoff and choose not to produce.

Using Assumption 1 to substitute for F in equation (15), setting $\dot{H}_t(\phi) = 0$ and solving the resulting first-order differential equation for $H(\phi)$ implies that the unique stationary relative productivity distribution is a Pareto distribution with scale parameter 1 and shape parameter k.

LEMMA 1. Given Assumption 1 there exists a unique stationary relative productivity distribution: $H(\phi) = 1 - \phi^{-k}$.

The proof of Lemma 1 is in Appendix A. Lemma 1 implies that on any balanced growth path the productivity distribution has a stable shape and resembles a traveling wave which shifts upward as the exit cutoff grows. Aw, Chen, and Roberts (2001) find that industry level productivity distributions tend to maintain stable shapes as they shift upwards in Taiwan, while König, Lorenz, and Zilibotti (2012) show that the productivity distribution of western European firms behaves like a traveling wave. An immediate corollary of Lemma 1 is that the upper tails of the firm employment, revenue and profit distributions follow Pareto distributions and that the employment distribution is stationary.[20]

19. Appendix B characterizes the balanced growth path when there are no functional form restrictions on F.

20. It is well known that the upper tails of the distributions of firm sales and employment are well approximated by Pareto distributions (Luttmer 2007). Axtell (2001) argues that Pareto distributions provide a good fit to the entire sales and employment distributions in the United States.

Lemma 1 implies that on a balanced growth path the productivity distribution of incumbent firms $G_t(\theta)$ is Pareto with shape parameter k and scale parameter θ_t^*. Consequently, the average productivity of incumbents is $x_t = \frac{k}{k-1}\theta_t^*$ implying that increases in the exit cutoff generate knowledge spillovers. Suppose we define $\lambda \equiv \frac{x_t \psi_{\min}}{\theta_t^*} = \frac{k}{k-1}\psi_{\min}$. λ is a measure of the strength of knowledge spillovers. To satisfy part (ii) of Assumption 1, I assume $\lambda \leq 1$, meaning $\psi_{\min} \leq \frac{k-1}{k}$. On a balanced growth path the fraction of entrants that draw productivity levels below the exit cutoff is $F(\frac{\psi_{\min}}{\lambda})$ and the relative productivity distribution of entrants is:

$$\tilde{H}(\phi) = F\left(\frac{\phi\,\psi_{\min}}{\lambda}\right) = H\left(\frac{\phi}{\lambda}\right).$$

Thus, entrants' relative productivity is drawn from a distribution that has the same functional form as the incumbents' relative productivity distribution, but is shifted inward by a factor of $\frac{1}{\lambda}$. If $\lambda = 1$ then entrants and incumbents have identical productivity distributions.

Now let $\frac{\dot{c}_t}{c_t} = q$ be the growth rate of consumption per capita. Then the household budget constraint (2) implies that assets per capita and wages grow at the same rate as consumption per capita:

$$\frac{\dot{a}_t}{a_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{c}_t}{c_t} = q,$$

while the Euler equation (3) gives:

$$q = \gamma(r - \rho), \tag{18}$$

and the transversality condition (4) requires:

$$r > n + q \Leftrightarrow \frac{1-\gamma}{\gamma}q + \rho - n > 0, \tag{19}$$

where the equivalence follows from equation (18). This inequality is also sufficient to ensure that household utility is well defined. Since all output is consumed each period and economies are symmetric, output per capita is always equal to consumption per capita.

Next, differentiating equation (5), which defines the exit cutoff, implies:

$$q = g + \frac{n}{\sigma - 1}, \tag{20}$$

where $g = \frac{\dot{\theta}_t^*}{\theta_t^*}$ is the rate of growth of the exit cutoff and, therefore, the rate at which the productivity distribution shifts upwards. From equation (6) the export threshold is proportional to the exit cutoff, meaning that g is also the growth rate of the export threshold and since each firm's productivity θ remains constant over time g is the rate at which a firm's relative productivity ϕ_t decreases.

Equation (20) makes clear that there are two sources of growth in this economy. First, productivity growth resulting from dynamic selection as the exit cutoff grows. Growth in the exit cutoff is driven by the dynamic complementarity between selection and technology diffusion. To understand the dynamic complementarity note that the productivity distribution of potential producers at time zero equals the exogenous distribution $\hat{G}_0(\theta)$. Due to the fixed cost of entry potential producers with productivity below θ_0^* choose to exit immediately generating selection as in Melitz (2003). This selection effect improves the average productivity draw of entrants by increasing the knowledge spillovers variable x_t. As new firms enter competition becomes tougher leading to further selection and additional knowledge spillovers that raise the average productivity of future entrants. In this way the combination of selection and knowledge spillovers sustains long-run productivity growth. As the exit cutoff grows, the reallocation of resources to more productive firms raises average labor productivity and output per capita. This effect is the dynamic analogue of the static selection effect that results from changes in the level of the exit cutoff. Henceforth, I will refer to g as the dynamic selection rate. Understanding what determines the dynamic selection rate is the central concern of this paper.

The second source of growth is population growth. Using the employment function (8), the labor market clearing condition (16) simplifies to:

$$L_t = \frac{k\sigma + 1 - \sigma}{k + 1 - \sigma} M_t f \left[1 + J\tau^{-k} \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right] + R_t f_e. \tag{21}$$

Consequently, on a balanced growth path we must have that the mass of producers and the flow of innovations grow at the same rate as population:

$$\frac{\dot{L}_t}{L_t} = \frac{\dot{M}_t}{M_t} = \frac{\dot{R}_t}{R_t} = n.$$

Thus, the link between population growth and consumption per capita growth arises because when the population increases, the number of varieties produced grows and, since the final good production technology exhibits love of varieties, this raises consumption per capita.

To solve for the dynamic selection rate, we can now substitute the profit function (9) and $\phi_t = \frac{\theta}{\theta_t^*}$ into equation (7) and solve for the firm value function. Since $\frac{\dot{\phi}_t}{\phi_t} = -g$, a firm that has relative productivity ϕ_t at time t exits at $t + \frac{log\phi_t}{g}$. Moreover, if $\phi_t > \tilde{\phi}$ the firm stops exporting at $t + \frac{log\left(\phi_t/\tilde{\phi}\right)}{g}$. Therefore, we obtain:[21]

$$
\begin{aligned}
V_t(\theta) &= W_t(\phi_t), \\
&= fw_t\Big[\frac{\phi_t^{\sigma-1}}{(\sigma-1)g + r - q}\left(1 + I[\phi_t \geq \tilde{\phi}]\frac{Jf_x}{f}\tilde{\phi}^{1-\sigma}\right) \\
&\qquad + \frac{(\sigma-1)g}{r-q}\frac{\phi_t^{\frac{q-r}{g}}}{(\sigma-1)g + r - q}\left(1 + I\big[\phi_t \geq \tilde{\phi}\big]\frac{Jf_x}{f}\tilde{\phi}^{\frac{r-q}{g}}\right) \\
&\qquad - \frac{1}{r-q}\left(1 + I\big[\phi_t \geq \tilde{\phi}\big]\frac{Jf_x}{f}\right)\Big],
\end{aligned} \tag{22}
$$

where $I[\phi_t \geq \tilde{\phi}]$ is an indicator function that takes value 1 if a firm's relative productivity is greater than or equal to the export threshold and 0 otherwise. Thus, the value of a firm with relative productivity ϕ grows at rate q. Substituting equation (22) into the free entry condition (11), using $\tilde{G}_t(\theta) = \tilde{H}(\phi) = H\left(\frac{\phi}{\lambda}\right)$ and integrating to obtain the expected value of an innovation implies:

$$
q = kg + r - \frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k}{f_e}\left(f + Jf_x\tilde{\phi}^{-k}\right). \tag{23}
$$

Together with equations (18) and (20), equation (23) gives us three equations for the three unknowns q, g, and r. Solving we obtain the growth rate of consumption per capita:

$$
q = \frac{\gamma}{1+\gamma(k-1)}\left[\frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{f_e}\left(1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right) + \frac{kn}{\sigma-1} - \rho\right], \tag{24}
$$

21. The online Technical Appendix provides further details on the derivation of equations (22) and (23).

where $\lambda = \frac{k}{k-1}\psi_{\min}$. Given equation (24), we can use equation (18) to obtain r and equation (20) to obtain g.

Finally, recall that when characterizing the evolution of the relative productivity distribution in Section II.C I assumed $g > 0$. To ensure this condition is satisfied and the transversality condition (19) holds, I impose the following parameter restrictions.

Assumption 2. The parameters of the world economy satisfy:

$$\frac{\sigma-1}{k+1-\sigma}\left(\frac{k}{k-1}\psi_{\min}\right)^{k}\frac{f}{f_e} > \rho + \frac{1-\gamma}{\gamma}\frac{n}{\sigma-1},$$

$$\frac{(1-\gamma)(\sigma-1)}{k+1-\sigma}\left(\frac{k}{k-1}\psi_{\min}\right)^{k}\frac{f}{f_e}\left[1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]$$
$$> \gamma k(n-\rho) - (1-\gamma)\frac{k+1-\sigma}{\sigma-1}n.$$

The first inequality ensures that $g > 0$ holds for any $J \geq 0$, while the second inequality is implied by the transversality condition.

This completes the proof that the world economy has a unique balanced growth path. Note that the proof holds for any non-negative value of J including the closed economy case where $J = 0$.

Proposition 1. Given Assumptions 1 and 2, the world economy has a unique balanced growth path equilibrium on which consumption per capita grows at rate:

$$q = \frac{\gamma}{1+\gamma(k-1)}\left[\frac{\sigma-1}{k+1-\sigma}\left(\frac{k}{k-1}\psi_{\min}\right)^{k}\frac{f}{f_e}\left(1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right) + \frac{kn}{\sigma-1} - \rho\right].$$

Remembering that Assumption 1 imposes $k > \max\{1, \sigma-1\}$, we immediately obtain a corollary of Proposition 1 characterizing the determinants of the growth rate.

Corollary 1. The growth rate of consumption per capita is strictly increasing in the fixed production cost f, the scale parameter of the productivity sampling distribution $\psi_{\min}$, the intertemporal elasticity of substitution γ, the population growth rate n, and the number of trading partners J, but is strictly decreasing in the entry cost f_e, the fixed export cost f_x, the variable trade cost τ, and the discount rate ρ.

To understand Proposition 1 and Corollary 1, let us start by considering how trade integration affects growth. The equilibrium growth rate is higher in the open economy than in autarky. Moreover, either increasing the number of countries J in the world economy, reducing the variable trade cost τ or reducing the fixed export cost f_x raises growth. To see why openness raises growth, consider the free entry condition (11). Using equation (7) and $\tilde{G}_t(\theta) = H(\frac{\phi}{\lambda})$ the free entry condition on the balanced growth path can be rewritten as:

$$f_e w_t = \int_\phi \left[\int_t^\infty \pi_v(\phi_v) e^{-(v-t)r} dv \right] dH\left(\frac{\phi}{\lambda}\right). \tag{25}$$

The cost of entry on the left-hand sides equals the expected present discounted value of entry on the right-hand side. Since $\pi_t(\phi)$ is proportional to w_t by equation (9), the free entry condition (25) is independent of the level of wages. Conditional on a firm's relative productivity and the wage level, equation (9) shows that domestic profits are independent of trade integration, while trade increases the profits of exporters. Therefore, the new export opportunities that follow trade liberalization raise the value of entry, ceteris paribus. In addition, trade liberalization does not change entrants' relative productivity distribution $\tilde{H}(\phi) = H(\frac{\phi}{\lambda})$. Consequently, trade liberalization causes an increase in the flow of entrants relative to the mass of incumbent firms $\frac{R_t}{M_t}$, which raises the dynamic selection rate g. To see this, note that since M_t grows at rate n, the exit cutoff θ_t^* grows at rate g, $H_t'(1) = k$ and $F\left(\frac{\theta_t^*}{x_t}\right) = 1 - \lambda^k$, equation (14) implies that on a balanced growth path:

$$\frac{R_t}{M_t} = \frac{n + gk}{\lambda^k}. \tag{26}$$

As the dynamic selection rate rises, firms' relative productivity levels decline at a faster rate, and this reduces a firm's expected future profits and its expected life span. In equilibrium, the negative effect of increased dynamic selection on future profits exactly offsets the increase in expected profits from exporting.

Thus, free entry mandates that trade liberalization raises growth because faster dynamic selection is required to offset the value of improved export opportunities.[22]

It is useful to compare Proposition 1 with the effects of trade liberalization when new entrants receive a productivity draw from an exogenously fixed distribution and there are no productivity spillovers as in Melitz (2003). In the absence of knowledge spillovers trade liberalization still creates new export profit opportunities that increase the value of entry, ceteris paribus. However, in static steady state models such as Melitz (2003), the offsetting negative profit effect, which ensures the free entry condition is satisfied, comes from an increase in the level of the exit cutoff. A higher exit cutoff reduces both entrants' probability of obtaining a productivity draw above the exit cutoff and entrants' expected relative productivity conditional on successful entry. By contrast, in this article knowledge spillovers imply that shifts in the level of the exit cutoff do not affect the relative productivity distribution of entrants. On the balanced growth path entrants draw relative productivity from a stationary distribution $H(\frac{\phi}{\lambda})$ that is unaffected by trade liberalization. Thus, although free entry implies that trade generates selection with and without knowledge spillovers, when entrants learn from incumbents trade has a dynamic selection effect.

To obtain Proposition 1, I used the assumption that x_t equals average incumbent firm productivity. However, the balanced growth path depends on x_t only through λ. Consequently, assuming any alternative specification of knowledge spillovers such that $x_t \propto \theta_t^*$ when the productivity distribution is Pareto would alter neither the properties of the balanced growth path nor the effects of trade liberalization. For example, if we assume x_t equals the minimum, median or 63rd percentile of the incumbent productivity distribution then the value of λ changes, but the balanced growth path of the world economy is otherwise identical to that characterized in Proposition 1.

Two additional implications of Proposition 1 are particularly noteworthy. First, growth is independent of population size,

22. Note that this analysis holds both for comparisons of the open economy with autarky and for the consequences of a partial trade liberalization resulting from an increase in J or a reduction in either τ or f_x.

meaning there are no scale effects. Second, growth is increasing in the fixed production cost.[23] Let us consider each of these findings in turn. Scale effects are a ubiquitous feature of the first generation of endogenous growth models (Romer 1990; Grossman and Helpman 1991; Aghion and Howitt 1992) where growth depends on the size of the R&D sector which, on a balanced growth path, is proportional to population. However, Jones (1995b) documents that despite continuous growth in both population and the R&D labor force, growth rates in developed countries have been remarkably stable since World War II.[24] This finding prompted Jones (1995a) to pioneer the development of semi-endogenous growth models in which the allocation of resources to R&D remains endogenous, but there are no scale effects because diminishing returns to knowledge creation mean that population growth is the only source of long-run growth. Semi-endogenous growth models have in turn been criticized for attributing long-run growth to a purely exogenous factor and understating the role of incentives to perform R&D in driving growth.[25]

There are three features of the technology diffusion model that lead to the absence of scale effects. First, the mass of goods produced is endogenous. In quality ladders growth models, the number of goods produced is constant and, consequently, the profit flow received by innovators is increasing in population, which generates a scale effect. In this article, population growth increases the mass of goods produced. Thus, in larger economies producers face more competitors and the incentive to innovate does not depend on market size. Second, unlike in expanding varieties growth models, the creation of new goods does not reduce the cost of R&D for future innovators, implying that population growth does not generate horizontal knowledge spillovers. Third and most important, knowledge spillovers depend on the distribution of productivity among all incumbent firms. In

23. Luttmer (2007) also finds that the consumption growth rate is increasing in $\frac{f}{f_e}$ when there are productivity spillovers from incumbents to entrants.

24. Although see Kremer (1993) for evidence that scale effects may be present in the very long run.

25. Jones (2005) draws a distinction between strong scale effects where the scale of an economy affects output growth and weak scale effects where scale affects the level of output. Using this terminology, the technology diffusion model in this paper features weak scale effects (see equations (32) and (33)), but not strong scale effects.

particular, I assumed in Section II.C that the variable x_t, which captures knowledge spillovers, equals the average productivity of incumbent firms, but is independent of the mass of incumbent firms. Together these three features ensure that an increase in scale does not generate knowledge spillovers and therefore does not affect the equilibrium growth rate. Instead, an increase in population simply raises the mass of goods produced since the additional competition from new firms exactly offsets the fall in the real cost of entry caused by increasing the labor force. As equation (26) makes clear, the dynamic selection rate depends not on the innovation rate, which is proportional to population, but on the innovation rate relative to the mass of producers, which is scale independent.

A related model featuring endogenous growth without scale effects is developed by Young (1998), who allows for R&D to raise both the quality and the number of goods produced, but assumes knowledge spillovers only occur along the vertical dimension of production. However, in Young (1998) there is no selection on productivity, implying that the dynamic selection effect analyzed in this paper is missing.

Since growth is independent of population size, holding the global population $(J+1)L_t$ fixed but increasing J raises the growth rate. Increasing the number of export markets creates new profit opportunities for firms with productivity above the export threshold and increases the total fixed costs Jf_x paid by exporters. The combination of these two effects raises the expected present discounted value of entry, ceteris paribus, and for the free entry condition to hold the dynamic selection rate increases leading to higher growth.

While Proposition 1 holds when trade costs are sufficiently high that $\tau^{\sigma-1}f_x > f$, the absence of scale effects implies the free trade growth rate is the same as the autarky growth rate. Moving from autarky to free trade is equivalent to increasing the size of the economy and therefore does not affect growth. In addition, the positive effects of trade liberalization on growth in Proposition 1 occur if and only if the maintained assumption $\tau^{\sigma-1}f_x > f$ holds and there is selection into exporting. Moving from autarky to an equilibrium in which all firms export has the same effect on growth as increasing the autarky fixed production cost from f to

$f + Jf_x$. It follows that when all firms export, the growth rate q is independent of τ and strictly increasing in f_x.[26]

Early work on the effects of trade in endogenous growth models found that global integration increases growth via the scale effect provided knowledge spillovers are sufficiently international in scope (Rivera-Batiz and Romer 1991; Grossman and Helpman 1991).[27] More recent publications have shown that if firm heterogeneity is included in standard expanding variety (Baldwin and Robert-Nicoud 2008) or quality ladders (Haruyama and Zhao 2008) models, the relationship between trade and growth continues to depend on the extent of international knowledge spillovers. In models without scale effects such as Young (1998) and the semi-endogenous growth model of Dinopoulos and Segerstrom (1999) the long-run growth rate is independent of an economy's trade status because trade is equivalent to an increase in scale. By contrast, in this article growth is driven by selection, not scale, and the dynamic selection mechanism through which trade increases growth does not require the existence of scale effects or international knowledge spillovers. Instead, it requires a combination of firm heterogeneity, export selection, and intranational technology diffusion.

A higher fixed production cost increases growth through a similar mechanism to trade integration. From the profit function (9) we see that for a given relative productivity ϕ and wage w_t, profits are proportional to f. Since on the balanced growth path entrants' relative productivity distribution is independent of f it follows that the expected initial profit flow received by a new entrant (relative to the wage) is increasing in f. However, the free entry condition (11) implies that in equilibrium the expected value of innovating (relative to the wage) is independent of f. Therefore, to satisfy the free entry condition, the increase in an entrant's expected initial profits generated by a rise in f must be offset by a fall in the entrant's expected future profits, which requires that relative productivity ϕ declines at a faster rate and the

26. The online Technical Appendix includes further details on the balanced growth path under free trade or when there are trade costs, but all firms export.

27. A complementary line of research examines how trade integration affects the incentives of asymmetric countries with multiple production sectors to undertake R&D (Grossman and Helpman 1991).

firm's expected lifespan falls. Thus, higher f increases the rate of dynamic selection g which raises the growth rate q. Substituting (26) back into the labor market clearing condition implies:

$$(27)\quad M_t = \left[\frac{k\sigma+1-\sigma}{k+1-\sigma} f\left(1+J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right)+(n+gk)\frac{f_e}{\lambda^k}\right]^{-1} L_t.$$

It follows that raising f reduces the mass of goods produced. This reduction in competition among incumbents leads to higher profits conditional on ϕ.

The effects of the remaining parameters on the growth rate are unsurprising. Increasing the entry cost by raising f_e must, in equilibrium, lead to an increase in the expected value of innovating and this is achieved through lower growth which increases firms' expected life spans. Similarly, growth is strictly increasing in the lower bound of the sampling productivity distribution $\psi_{\min}$ because an increase in $\psi_{\min}$ raises the strength of knowledge spillovers λ and when spillovers are stronger an entrant's expected initial relative productivity is higher. Consequently, to ensure the free entry condition (11) holds the dynamic selection rate must increase to offset the rise in initial profits. A higher intertemporal elasticity of substitution or a lower discount rate raise growth by making households more willing to invest now and consume later, while, as discussed above, population growth raises consumption per capita growth through its impact on the growth rate of the mass of producers M_t. The elasticity of substitution σ and the Pareto shape parameter k have ambiguous effects on growth.

III.A. Transition Dynamics

Lemma 1 shows that there exists a unique stationary relative productivity distribution, but in equilibrium does $H_t(\phi)$ converge to this distribution? The answer depends on the properties of the initial productivity distribution $\hat{G}_0(\theta)$. As the exit cutoff increases the functional form of the relative productivity distribution $H_t(\phi)$ depends on the right-tail properties of $\hat{G}_0(\theta)$ and the sampling distribution F. When productivity is sufficiently high, whichever distribution has the thicker right tail dominates, and if F has the thicker right tail then as t becomes large $H_t(\phi)$ inherits the functional form of F and converges to a Pareto distribution.

Formally, suppose $\hat{G}_0(\theta)$ satisfies the following assumption.

ASSUMPTION 3. The sampling distribution F has a weakly thicker right tail than the initial productivity distribution $\hat{G}_0(\theta)$:

$$\lim_{\theta\to\infty} \frac{1-\hat{G}_0(\theta)}{\theta^{-k}} = \kappa,$$

where $\kappa \geq 0$.

Note that any bounded initial productivity distribution satisfies Assumption 3 with $\kappa = 0$. Assumption 3 is a necessary and sufficient condition to ensure that the relative productivity distribution converges to the balanced growth path distribution whenever there is dynamic selection.

PROPOSITION 2. When Assumption 1 holds and the exit cutoff θ_t^* is unbounded as $t \to \infty$ then in equilibrium $\lim_{t\to\infty} H_t(\phi) = 1 - \phi^{-k}$ if and only if Assumption 3 is satisfied.

The proof of Proposition 2 is in Appendix A. The requirement that $\theta_t^* \to \infty$ is necessary to ensure that for large t only the right-tail properties of $\hat{G}_0$ and F matter.

In the Technical Appendix, I show that the symmetric balanced growth path described in Proposition 1 is locally stable to asymmetric perturbations of the initial conditions.

IV. GAINS FROM TRADE

Both static and dynamic selection create new sources of gains from trade that do not exist when firms are homogeneous. However, as shown by Atkeson and Burstein (2010) and Arkolakis, Costinot, and Rodríguez-Clare (2012), in general equilibrium the welfare gains generated by the static selection effect are offset by lower entry. Are the gains from dynamic selection offset by other general equilibrium effects? To answer this question we must move beyond simply considering the equilibrium growth rate and solve for the welfare effects of trade.

IV.A. Balanced Growth Path Welfare

The state variables of the technology diffusion model are the productivity distribution $G_t(\theta)$ and the mass of incumbent firms M_t. Lemma 1 implies the stationary relative productivity distribution is independent of trade. In addition, equation (27) shows trade liberalization reduces the mass of incumbent firms.

This decline in M_t occurs instantaneously following a trade liberalization as a consequence of an upward jump in the exit cutoff θ_t^*. It follows that if the economy is on a balanced growth path when trade liberalization occurs, the equilibrium jumps instantaneously to the new balanced growth path and there are no transition dynamics. Therefore, to characterize the welfare effects of trade liberalization, it is sufficient to compare welfare on the preliberalization and postliberalization balanced growth paths.[28]

Suppose that at time 0 the productivity distribution of potential producers $\hat{G}_0(\theta)$ is Pareto with shape parameter k and scale parameter $\hat{\theta}_0^*$ and the mass of potential producers $\hat{M}_0$ is such that in equilibrium some firms have productivity below the exit cutoff at time 0 and choose to exit immediately. This refinement of the initial condition assumed in Section II.D ensures the economy is always on a balanced growth path.

Substituting $c_t = c_0 e^{qt}$ into the household welfare function (1) and integrating implies:

$$U = \frac{\gamma}{\gamma - 1}\left[\frac{\gamma c_0^{\frac{\gamma-1}{\gamma}}}{(1-\gamma)q + \gamma(\rho - n)} - \frac{1}{\rho - n}\right]. \tag{28}$$

Therefore, household welfare depends on both the consumption growth rate q and the level of consumption c_0. From the household budget constraint (2), the Euler equation (3), and the transversality condition (19), we can write the initial level of consumption per capita c_0 in terms of initial wages and assets as:[29]

$$c_0 = w_0 + \left(\frac{1-\gamma}{\gamma}q + \rho - n\right)a_0, \tag{29}$$

where $\frac{1-\gamma}{\gamma}q + \rho - n$ is the marginal propensity to consume out of wealth, which is positive by the transversality condition.

28. Transition dynamics may arise following a reduction in trade integration (a fall in J, an increase in τ or an increase in f_x) since in this case $\frac{M_t}{L_t}$ increases by equation (27), but a downward jump in θ_t^* does not necessitate an instantaneous increase in M_t. The details of the adjustment process will depend on whether firm exit is assumed to be irreversible. In this article I abstract from these considerations and focus on balanced growth path welfare.

29. This is a textbook derivation. See, for example, Barro and Sala-i-Martin (2004), pp. 93–94.

Now using equation (22) to substitute for $W_t(\phi)$ in the asset market clearing condition (17), integrating the right-hand side to obtain average firm value and using equation (23) gives:

$$a_t L_t = \frac{f_e}{\lambda^k} w_t M_t, \tag{30}$$

which has the intuitive interpretation that the value of the economy's assets at any given time equals the expected R&D cost of replacing all incumbent firms.

Next, using the initial condition given above, the time 0 exit cutoff θ_0^* is given by:

$$\theta_0^* = \hat{\theta}_0^* \left(\frac{\hat{M}_0}{M_0} \right)^{\frac{1}{k}}. \tag{31}$$

We can now solve for initial consumption per capita by combining this expression with equations (5), (20), (24), (27), (29), and (30) to obtain:

$$\begin{aligned} c_0 = {} & A_1 f^{-\frac{k+1-\sigma}{k(\sigma-1)}} \left[1 + J\tau^{-k} \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right]^{\frac{1}{k}} \\ & \times \left[1 + \frac{\sigma-1}{k\sigma+1-\sigma} \frac{n+gk}{n+gk+\frac{1-\gamma}{\gamma}q+\rho-n} \right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}}, \end{aligned} \tag{32}$$

where:

$$A_1 \equiv (\sigma-1) \left(\frac{k}{k+1-\sigma} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{k+1-\sigma}{k\sigma+1-\sigma} \right)^{\frac{k\sigma+1-\sigma}{k(\sigma-1)}} \hat{\theta}_0^* \hat{M}_0^{\frac{1}{k}} L_0^{\frac{k+1-\sigma}{k(\sigma-1)}} > 0. \tag{33}$$

Remember that Assumption 2 ensures $g > 0$ and $\frac{1-\gamma}{\gamma}q + \rho - n > 0$. Thus, both the numerator and the denominator of the final term in equation (32) are positive.

Armed with the equilibrium growth rate in equation (24) and the initial consumption level in equation (32) we can now analyze the welfare effects of trade integration. Observe that trade affects both growth and the consumption level only through the value of $T \equiv J\tau^{-k} \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}}$. T measures the extent of trade integration between countries. T is strictly increasing in the number of countries J in the world economy and the fixed production cost f, but strictly decreasing in the variable trade cost τ and the fixed export cost f_x. When calibrating the model in Section IV.B I show that the import penetration ratio is a sufficient statistic for T and that T is monotonically increasing in the import penetration ratio.

Trade affects welfare through two channels. First, trade raises welfare by increasing c_0 for any given growth rate. These static gains from trade z^s are given by the term:

$$z^s = \left[1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]^{\frac{1}{k}} = (1+T)^{\frac{1}{k}}$$

in equation (32). The static gains from trade result from the net effect of increased access to imported goods, a reduction in the number of goods produced domestically, and average productivity gains caused by an increase in the level of the exit cutoff. Interestingly, the static gains equal the total gains from trade in comparable economies with firm heterogeneity, but without knowledge spillovers. Thus, both in static steady-state economies such as the variant of Melitz (2003) considered by Arkolakis, Costinot, and Rodríguez-Clare (2012) where entrants draw productivity from a Pareto distribution and also in a version of the model where innovators draw productivity from a time-invariant Pareto distribution (in this case the exit cutoff is constant on the balanced growth path and trade does not affect the consumption growth rate) the gains from trade equal z^s.

Second, trade affects welfare by raising the growth rate through the dynamic selection effect. I refer to the change in welfare caused by trade-induced variation in the growth rate as the dynamic gains from trade. From equation (28) we see that increased growth has a direct positive effect on welfare, but equation (32) shows that it also affects the level of consumption. The level effect is made up of two components. First, there is the increase in $n+gk$ which from equation (26) occurs because trade raises the innovation rate relative to the mass of producers. This requires a reallocation of labor between production and R&D that decreases the consumption level. Second, variation in q changes households' marginal propensity to consume out of wealth $\frac{1-\gamma}{\gamma}q+\rho-n$. Provided the intertemporal elasticity of substitution $\gamma<1$ this effect raises the marginal propensity to consume and increases c_0. In general, the net effect of higher growth on c_0 can be either positive or negative and substituting $g = q - \frac{n}{\sigma-1}$ into equation (32) and differentiating with respect to q shows that higher growth increases c_0 if and only if:

$$n\left(1-\frac{1}{k}\frac{1-\gamma}{\gamma}\frac{k+1-\sigma}{\sigma-1}\right)>\rho.$$

However, regardless of the sign of the level effect, substituting for c_0 using equation (32) and then differentiating equation (28) with respect to growth shows that the dynamic gains from trade are positive. Thus, the direct positive effect of growth on welfare always outweighs any negative indirect effect resulting from variation in c_0. Proposition 3 summarizes the welfare effects of trade. The proposition is proved in Appendix A.

Proposition 3. Trade integration resulting from either an increase in the number of trading partners J, a reduction in the fixed export cost f_x, or a reduction in the variable trade cost τ increases welfare through two channels: (i) by raising the level of consumption for any given growth rate (static gains); and (ii) by raising the growth rate of consumption per capita (dynamic gains). The static gains equal the total gains from trade obtained in a Melitz (2003) economy with a Pareto productivity distribution.

Two observations follow immediately from Proposition 3. First, since both the static and dynamic gains from trade are positive, trade is welfare improving. Second, since the dynamic gains are positive, the total gains from trade in this article are strictly larger than in a static steady-state economy such as Melitz (2003). This demonstrates that dynamic selection leads to a new source of gains from trade, which is not offset by other general equilibrium effects. In contrast to the findings of Atkeson and Burstein (2010) and Arkolakis, Costinot, and Rodríguez-Clare (2012), in this article firm heterogeneity matters for the gains from trade.[30]

30. This result is related to the literature that studies the gains from trade in economies not covered by Arkolakis, Costinot, and Rodríguez-Clare (2012). Ossa (2012) shows that cross-sectoral heterogeneity in trade elasticities increase the gains from trade relative to Arkolakis, Costinot, and Rodríguez-Clare (2012)'s estimates, but his argument applies regardless of whether there is firm-level heterogeneity. Edmond, Midrigan, and Xu (2012) and Impullitti and Licandro (2012) find that when there are variable mark-ups procompetitive effects can substantially increase the gains from trade, although Arkolakis et al. (2012) show that this will not always be the case. By contrast, this article focuses on understanding whether

To understand why the higher growth resulting from trade liberalization is welfare improving, consider the efficiency of the decentralized equilibrium. An increase in the exit cutoff generates knowledge spillovers that cause the productivity distribution of entrants to shift upward. However, neither exiting firms nor entrants internalize the social value of these spillovers. Consequently, in the decentralized equilibrium there is too little entry and exit and the dynamic selection rate is inefficiently low. Increasing the flow of entrants relative to the mass of incumbent firms $\frac{R_t}{M_t}$ raises the dynamic selection rate by equation (26) and exploits the knowledge spillovers externality. I show in Appendix B that a benevolent government can raise welfare using either a R&D subsidy or a tax on the fixed production cost since both policies incentivize entry relative to production and increase the dynamic selection rate.[31] Similarly, since trade raises growth by increasing $\frac{R_t}{M_t}$, it necessarily leads to dynamic welfare gains because of the knowledge spillovers externality.

IV.B. *Quantifying the Gains from Trade*

How large are the dynamic gains from trade? This section evaluates the importance of the dynamic selection effect in determining the overall gains from trade. To quantify the gains from trade, I start by calibrating the model using U.S. data and then perform robustness checks against this baseline, but it should be remembered when interpreting the calibration results that the theory assumes symmetry across countries. The key to the calibration is showing that the gains from trade can be expressed in terms of a small number of observables and commonly used parameters. In particular, it is not necessary to specify values of J, f, f_x, f_e, or λ.

Define the gains from trade z in equivalent variation terms as the proportional increase in the autarky level of consumption required to obtain the open economy welfare level. Thus, z satisfies $U\left(zc_0^A, q^A\right) = U(c_0, q)$ where U, q, and c_0 are defined by equations

firm heterogeneity matters for the gains from trade in a dynamic single sector economy with constant mark-ups.

31. I assume the policies are financed by lump-sum transfers to households. Acemoglu et al. (2013) also find that it is welfare improving to tax fixed production costs, but for a different reason. In their model, exit induced by taxing the fixed cost of production reduces competition for skilled workers to perform R&D. By contrast, in this article exit induced by the tax leads to knowledge spillovers and increases the dynamic selection rate.

(28), (24), and (32), respectively, and A superscripts denote autarky values.[32] From equation (28) we have:

$$z = \frac{c_0}{c_0^A}\left[\frac{(1-\gamma)q^A + \gamma(\rho - n)}{(1-\gamma)q + \gamma(\rho - n)}\right]^{\frac{\gamma}{\gamma-1}}.$$

Observe that if $q = q^A$ the gains from trade are given by the increase in the initial consumption level, which from equation (32) equals the static gains from trade z^s. I define the dynamic gains from trade z^d to be $z^d \equiv \frac{z}{z^s}$.

The static gains from trade depend only on the import penetration ratio (IPR) and the trade elasticity (TE). To see this, first calculate import expenditure in each country (IMP) which is given by:

$$IMP_t = \frac{k\sigma}{k+1-\sigma} M_t w_t f J \tau^{-k} \left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}. \tag{34}$$

Equation (34) shows that k equals the trade elasticity (the elasticity of imports with respect to variable trade costs). Now divide equation (34) by total domestic sales $c_t L_t$ to obtain:

$$z^s = \left(\frac{1}{1-IPR}\right)^{\frac{1}{TE}}. \tag{35}$$

This expression is identical to the formula for calibrating the gains from trade obtained by Arkolakis, Costinot, and Rodríguez-Clare (2012). It follows that the calibrated static gains from trade in the technology diffusion model developed in this article equal the calibrated total gains from trade in the class of static steady state economies studied by Arkolakis, Costinot, and Rodríguez-Clare (2012). Models covered by Arkolakis, Costinot, and Rodríguez-Clare (2012) include Anderson (1979), Krugman (1980), and Eaton and Kortum (2002) in addition to the version of Melitz (2003) with a Pareto productivity distribution.

The U.S. IPR for 2000, defined as imports of goods and services divided by gross output, equals 0.081.[33] Anderson and Van

32. In this section I focus on comparing welfare at observed levels of trade with autarky welfare. However, the same methodology can be used to compare welfare in two equilibria with different levels of trade integration.

33. Imports of goods and services are from the World Development Indicators (April 2012) and gross output is from the OECD STAN Database for Structural Analysis (vol. 2009).

Wincoop (2004) conclude based on available estimates that the trade elasticity is likely to lie between 5 to 10. I set $k = 7.5$ for the baseline calibration, while in the robustness checks I allow k to vary between 2 and 10. This interval includes the trade elasticity of 4 estimated by Simonovska and Waugh (2014).

To calibrate the dynamic gains from trade, we can express $\frac{\lambda^k f}{f_e}$, which is needed to calculate the growth rate in equation (24), as a function of n, k, σ, γ, ρ, IPR, and the entry rate of new firms relative to the mass of existing firms (NF). Since a fraction λ^k of innovations lead to the creation of new firms we have $NF = \lambda^k \frac{R_t}{M_t}$ and using equations (20), (24), and (26) gives:

$$\frac{\lambda^k f}{f_e} = \frac{k+1-\sigma}{\gamma k(\sigma-1)}(1-IPR)\left\{[1+\gamma(k-1)](NF-n)+\frac{k(1-\gamma)}{\sigma-1}n+\gamma k\rho\right\}.$$

The U.S. Small Business Administration reports an entry rate of 11.6% per annum in 2002 (Luttmer 2007). Therefore, I set $NF = 0.116$. For the population growth rate I use $n = 0.011$ based on average annual U.S. population growth from 1980 to 2000 as reported in the World Development Indicators.

Finally, there are three parameters to calibrate: σ, γ, and ρ. To calibrate σ observe that the right tail of the firm employment distribution is a power function with index $\frac{-k}{\sigma-1}$. Luttmer (2007) shows that for U.S. firms in 2002 the right tail index of the employment distribution equals –1.06. Therefore, I let the elasticity of substitution $\sigma = \frac{k}{1.06}+1$ implying $\sigma = 8.1$. Note that $k > \max\{1, \sigma-1\}$ as required by Assumption 1. Helpman, Melitz, and Yeaple (2004) use European firm sales data to estimate $k+1-\sigma$ at the industry level, obtaining estimates that mostly lie in the interval between 0.5 and 1, implying $\sigma \in \left[k, k+\frac{1}{2}\right]$. In the robustness checks I allow σ to vary over a range that includes this interval.

Although controversy exists over the value of the intertemporal elasticity of substitution, estimates typically lie between 0.2 and 1.[34] Following García-Peñalosa and Turnovsky (2005) I let $\gamma = \frac{1}{3}$ in the baseline calibration. A low intertemporal elasticity of substitution will tend to reduce the dynamic gains from trade by making consumers less willing to substitute consumption over time. I also follow García-Peñalosa and Turnovsky (2005) in

34. See, for example, Hall (1988), Vissing-Jorgensen (2002), Yogo (2004), and Guvenen (2006).

TABLE I
CALIBRATION OBSERVABLES AND PARAMETERS

Observable/parameter		Value	Source
Import penetration ratio	IPR	0.081	U.S. import penetration ratio in 2000
Firm creation rate	NF	0.116	U.S. Small Business Administration 2002
Population growth rate	n	0.011	U.S. average 1980–2000
Trade elasticity	k	7.5	Anderson and Van Wincoop (2004)
Elasticity of substitution across goods	σ	8.1	$\sigma = \frac{k}{1.06} + 1$ to match right tail index of employment distribution
Intertemporal elasticity of substitution	γ	0.33	García-Peñalosa and Turnovsky (2005)
Discount rate	ρ	0.04	García-Peñalosa and Turnovsky (2005)

choosing the discount rate and set $\rho = 0.04$. In the robustness checks I allow γ to vary between 0.2 and 1 and ρ to vary between 0.01 and 0.15. Table I summarizes the data and parameter values used for the baseline calibration. Assumption 2 is satisfied both for the baseline calibration and in all the robustness checks.

Table II shows the calibration results. Consumption per capita growth is 10.7% higher at observed U.S. trade levels than in a counterfactual autarkic economy. Due to the dynamic welfare gains resulting from higher growth, the total calibrated gains from trade are 3.2 times higher than the static gains. Thus, dynamic selection is quantitatively important when calculating the total gains from trade.[35]

The results imply that at the calibrated equilibrium, the elasticity of the gains from trade to the import penetration ratio is 0.038 and this elasticity is 3.2 times higher than the elasticity of the static gains from trade. The semi-elasticity of the gains from trade to a 1 percentage point increase in the import penetration ratio is 0.47, which is also 3.2 times higher than the semi-

35. The calibration predicts a growth rate of 1.56% per annum. The average annual U.S. GDP per capita growth rate for 1980–2000 in the World Development Indicators is 2.07%. The difference may reflect the fact that sources of growth such as physical and human capital accumulation and technology upgrading by incumbent firms are absent from the technology diffusion model. Foster, Haltiwanger, and Krizan (2001) find that around one-quarter of total factor productivity growth in U.S. manufacturing from 1977 to 1987 can be attributed to entry and exit. The model can be calibrated to match the U.S. growth rate by setting $k = 5.6$. In this case trade raises consumption per capita growth by 10.6% and the total gains from trade are 2.9 times higher than the static gains.

TABLE II
CALIBRATION RESULTS

Outcome		Value
Growth rate, trade	q	0.0156
Growth rate, autarky	q^A	0.0141
Growth (trade versus autarky)	$\frac{q}{q^A}$	1.107
Consumption level (trade versus autarky)	$\frac{c_0}{c_0{}^A}$	1.010
Static gains from trade	z^s	1.011
Dynamic gains from trade	z^d	1.025
Total gains from trade	z	1.036
Gains from trade (total versus static)	$\frac{z-1}{z^s-1}$	3.2

elasticity of the static gains from trade. A 1 percentage point increase in the import penetration ratio raises the consumption per capita growth rate q by 0.02 percentage points.

Next I consider the robustness of these results, first with respect to the import penetration ratio. Unsurprisingly, the gains from trade are higher when trade integration is greater (Figure I). Increasing the import penetration ratio from 0.051 (Japan) to 0.36 (Belgium) raises welfare gains from 2.2% to 19.2%. More important, the ratio of the total gains to the static gains, which measures the proportional increase in the gains from trade due to dynamic selection, remains approximately constant as the import penetration ratio varies. Figure II plots the growth rate under trade relative to the autarky growth rate on the left-hand axis and the total gains from trade relative to the static gains from trade on the right-hand axis. The total gains are a little over three times larger than the static gains for all levels of the import penetration ratio between 0 and 0.5.[36]

Finally, Figure III shows the consequences of varying the other inputs to the calibration. I plot the growth effect of trade (left-hand axis) and the ratio of the total gains from trade to the static gains (right-hand axis) as a function of each variable in

36. The elasticity and semi-elasticity of the gains from trade to the import penetration ratio increase as the import penetration ratio rises. Increasing the import penetration ratio from 0.051 to 0.36 raises the elasticity from 0.023 to 0.25 and the semi-elasticity from 0.45 to 0.67. However, in both cases the overall effect relative to the static effect remains close to 3.2.

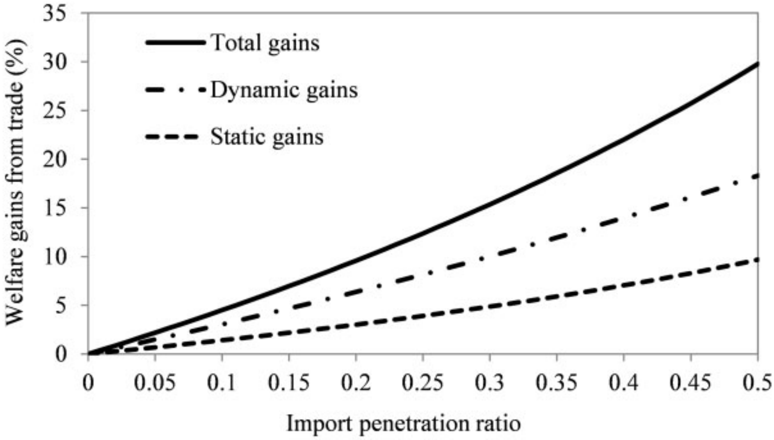

FIGURE I

Import Penetration Ratio and the Gains from Trade

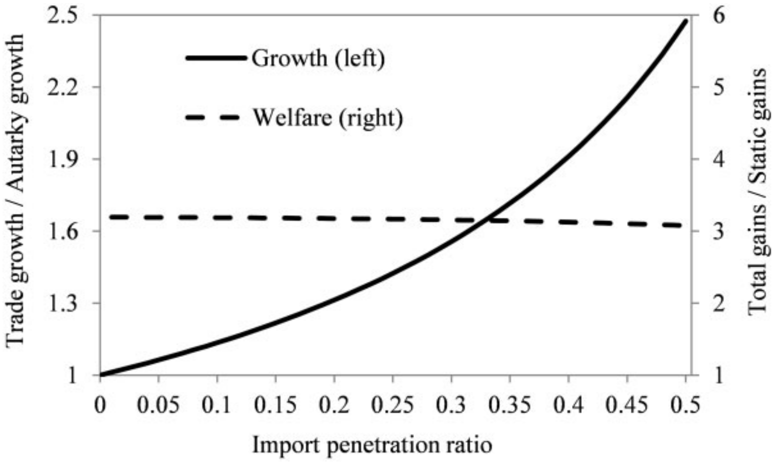

FIGURE II

Import Penetration Ratio and the Dynamic Gains from Trade

turn, while holding the other inputs fixed at their baseline values.[37] In all cases the dynamic gains from trade are

37. The sole exception is Figure IIIc, where I adjust σ to ensure $\sigma = \frac{k}{1.06} + 1$ always holds as the trade elasticity varies.

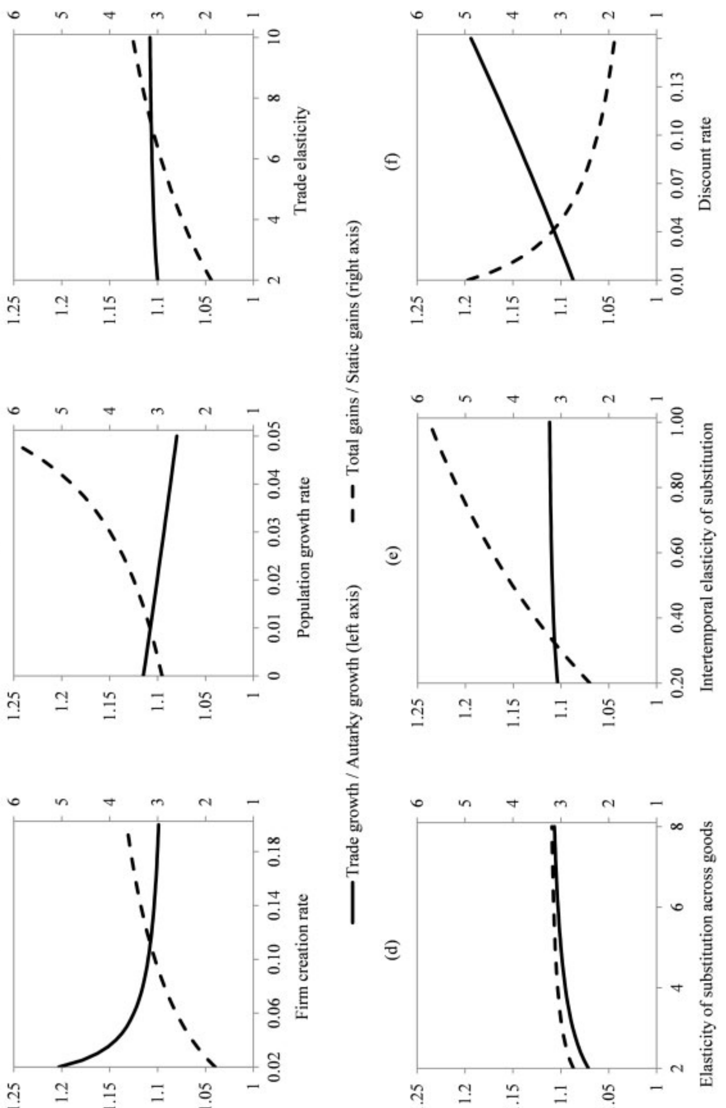

FIGURE III

Dynamic Gains from Trade

quantitatively important, and the results suggest that dynamic selection at least doubles the gains from trade. For example, either lowering the intertemporal elasticity of substitution or raising the discount rate reduces the dynamic gains from trade because it lowers the value of future consumption growth. However, even if the intertemporal elasticity of substitution is reduced to 0.2, the gains from trade are 2.4 times higher with dynamic selection, while the discount rate must exceed 14% before the total gains from trade are less than double the static gains.

V. International Knowledge Spillovers

In the baseline model there are no international knowledge spillovers. Suppose instead entrants learn not only from domestic firms but also from foreign firms that sell in the domestic market. To formalize this idea, let x_t, which affects entrant productivity through equation (10), equal the average productivity of all firms that sell in a given market. Otherwise the model is unchanged. This extension can be solved using the same reasoning applied before. There exists a unique balanced growth path on which the stationary relative productivity distribution is Pareto by Lemma 1 and the equilibrium growth rate is still given by equation (24). The only difference from the baseline model is the value of λ, which measures the strength of knowledge spillovers. By definition, λ equals $\frac{x_t \psi_{\min}}{\theta_t^*}$. Calculating the average productivity of all firms selling in a market gives that on the balanced growth path:

$$x_t = \frac{k\theta_t^*}{k-1}\frac{1+J\tilde{\phi}^{1-k}}{1+J\tilde{\phi}^{-k}},$$

which implies:

$$\lambda = \frac{k\psi_{\min}}{k-1}\tilde{\lambda} \text{ where } \tilde{\lambda} \equiv \frac{1+J\tilde{\phi}^{1-k}}{1+J\tilde{\phi}^{-k}}. \tag{36}$$

In the absence of international knowledge spillovers $\tilde{\lambda} = 1$ and λ is independent of trade integration. With international knowledge spillovers $\tilde{\lambda} > 1$ whenever not all firms are exporters (i.e., whenever $\tilde{\phi} > 1$). Thus, international knowledge spillovers

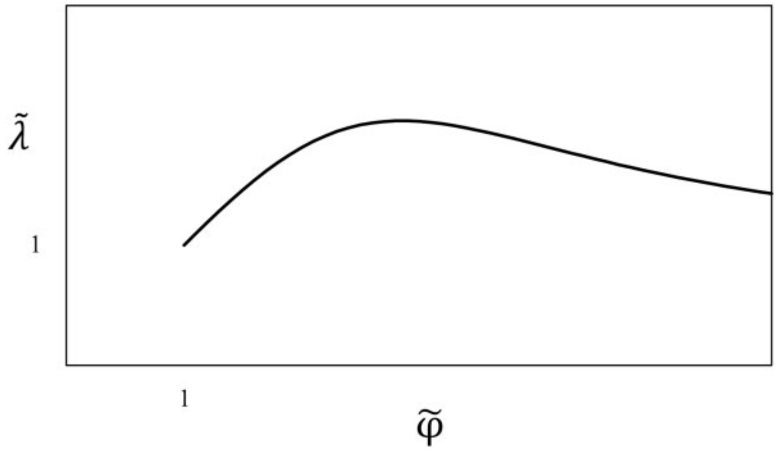

FIGURE IV
Trade Integration and International Knowledge Spillovers

increase the strength of knowledge spillovers by raising λ. This increase in λ occurs because exporters are a select group of high-productivity firms. Consequently, entrants learn more from the average foreign firm than from the average domestic firm.

Differentiating the expression for $\tilde{\lambda}$ gives:

$$\frac{d\tilde{\lambda}}{d\tilde{\phi}} \propto \frac{k}{\tilde{\phi}} + \frac{J}{\tilde{\phi}^k} - (k-1).$$

Therefore, $\tilde{\lambda}$ is inverse U-shaped as a function of $\tilde{\phi}$ as shown in Figure IV and knowledge spillovers are strongest for some interior $\tilde{\phi} \in (1, \infty)$. If we define $\tilde{\lambda}_{\max} \equiv \max_{\tilde{\phi} \geq 1} \tilde{\lambda}$ then $\tilde{\lambda} \in \left[1, \tilde{\lambda}_{\max}\right]$. To ensure the transversality condition holds when there are international knowledge spillovers requires not only Assumption 2 but also the following parameter restriction.

ASSUMPTION 4. The parameters of the world economy satisfy:

$$\frac{(1-\gamma)(\sigma-1)}{k+1-\sigma}\left(\frac{k}{k-1}\psi_{\min}\right)^k \frac{\tilde{\lambda}_{\max}^k f}{f_e}\left[1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]$$

$$> \gamma k(n-\rho) - (1-\gamma)\frac{k+1-\sigma}{\sigma-1}n.$$

Note that if $\gamma \leq 1$ Assumption 2 implies Assumption 4, but Assumption 4 allows for the possibility $\gamma > 1$.

Inspection of the equilibrium growth rate in equation (24) shows that growth is increasing in λ. Therefore, trade has a larger effect on growth when there are international knowledge spillovers because in addition to the direct positive effect of trade on growth identified in Section III, there is an indirect positive effect caused by the rise in λ. In addition, the consumption level c_0 is independent of λ by equation (32). It follows immediately that international knowledge spillovers do not affect the static gains from trade, but increase the dynamic gains from trade. Proposition 4 summarizes these results.

PROPOSITION 4. Suppose Assumptions 1, 2, and 4 hold and not all firms are exporters. Compared to autarky, trade raises growth and welfare by more when there are international knowledge spillovers than if there are only domestic knowledge spillovers.

By exposing domestic entrants to the superior technologies used by foreign exporters, international knowledge spillovers open a second channel through which trade increase the dynamic selection rate. This channel operates if and only if the average foreign exporter is more productive than the average domestic firm. When all firms are exporters, this condition is not satisfied and the effects of trade are identical to the baseline model. Similarly, if we assume an alternative specification for international knowledge spillovers in which entrants learn from all domestic and foreign firms and x_t equals the average productivity of all incumbent firms anywhere in the world, then symmetry across economies implies that λ and the effects of trade are unchanged from the baseline model.[38]

Proposition 4 compares the open economy equilibrium to autarky. The effects of marginal reductions in trade costs are more

38. Developing a technology diffusion model with asymmetric economies is beyond the scope of this article, but it is reasonable to expect international knowledge spillovers will play a particularly important role in shaping the gains from North-South trade where the productivity distribution differs across countries.

subtle. Differentiating equation (36) shows that $\tilde{\lambda}$ is strictly increasing in J, an increase in the number of trading partners strengthens knowledge spillovers. However, the effect or reducing either τ or f_x on $\tilde{\lambda}$ is in general ambiguous. Lower variable or fixed trade costs reduce the export threshold $\tilde{\phi}$ and this has two offsetting effects. First, the fraction of firms that are exporters increases, which raises $\tilde{\lambda}$ because exporters are more productive than nonexporters. Second, the average productivity of exporters decreases which reduces $\tilde{\lambda}$. As shown in Figure IV, when $\tilde{\phi}$ is high and most firms do not export the first effect dominates and a reduction in $\tilde{\phi}$ increases $\tilde{\lambda}$. However, for low $\tilde{\phi}$ the second effect dominates and $\tilde{\lambda}$ falls as trade integration increases. The effect of marginal reductions in either τ or f_x on growth and the dynamic gains from trade is also ambiguous since for sufficiently low $\tilde{\phi}$ the negative effect of lower trade costs on $\tilde{\lambda}$ can outweigh the direct positive effect of lower trade costs on the growth rate. Thus, although international knowledge spillovers imply higher gains from trade, they also imply that marginal reductions in trade costs may reduce growth when initial trade costs are low.

V.A. *Other Extensions*

This article shows that incorporating technology diffusion into an otherwise standard open economy model with heterogeneous firms leads to a new source of dynamic gains from trade. The baseline model makes a number of simplifying assumptions. In particular, I assume entrants sample from a Pareto productivity distribution and I assume firm productivity remains constant after entry. These assumptions make it possible to solve the model in closed form, but they are not responsible for the finding that trade raises growth. In Appendix B I study the consequences of relaxing these simplifying assumptions. I consider three extensions of the baseline model. First, I allow for firms to experience postentry productivity growth and for entrants to draw productivity from distributions other than the Pareto distribution. Second, I consider knowledge spillovers that benefit both entrants and incumbent firms. Third, I introduce an alternative specification of the R&D technology which allows for decreasing returns to scale in R&D and congestion in the technology diffusion process. The finding that trade raises growth by increasing the dynamic selection rate is broadly robust across these alternative specifications. The intuition is the same in each case. When knowledge spillovers link the productivity distribution of entrants to that of incumbent firms, the free entry condition mandates that trade integration must increase the dynamic selection rate to offset the profits from new export opportunities.

VI. CONCLUSIONS

A complete understanding of the welfare effects of trade integration must account for the relationship between trade and growth. Yet existing work on open economies with heterogeneous firms has mostly overlooked dynamic effects. By incorporating knowledge spillovers into a dynamic version of the Melitz model this article shows that the interaction of firm heterogeneity and technology diffusion has novel and important implications for understanding the dynamic consequences of trade.

Motivated by evidence that there is substantial productivity dispersion within entering cohorts of firms and that the productivity distributions of entrants and incumbents move together over time, the article assumes entrants learn not only from frontier technologies but from the entire distribution of technologies used in an economy. This generates a dynamic complementarity between selection and technology diffusion that leads to endogenous growth through dynamic selection. Growth through dynamic selection is only possible when firms are heterogeneous. The balanced growth path equilibrium is consistent with empirical work showing that the firm size distribution is stable over time, while the firm productivity distribution shifts to the right as a traveling wave.

Trade liberalization raises growth by increasing the rate of dynamic selection. Faster dynamic selection is required to offset higher export profits and ensure the free entry condition is satisfied. The dynamic selection effect is a new channel for gains from trade and I prove that it strictly increases the gains from trade compared to static steady state economies with heterogeneous firms. Calibrating the model shows the dynamic gains are quantitatively important and at least double the overall gains from trade.

The specification of knowledge spillovers used in this article builds on the idea flows literature, but introduces a reduced-form learning technology that enables tractable, open economy, general equilibrium analysis. In contrast to expanding variety and quality ladders growth models, the article finds that when spillovers depend on the entire technology distribution in an economy, growth does not feature scale effects. By linking the productivity distribution of entrants to that of incumbents, the knowledge spillover process also generates a rich set of predictions about technology diffusion which can be tested using firm-level data sets. For example, testing the impact of shocks to the incumbent firm productivity distribution on the productivity of entrants would shed further light on the nature of knowledge spillovers.

This article has focused primarily on within-country technology diffusion with symmetric economies. However, the framework it develops to model technology diffusion could also be used to study cross-country technology diffusion with asymmetric economies or to analyze geographic variation in spillovers within countries. In this way it should further contribute to advancing our understanding of the dynamic consequences of globalization.

Appendix A: Proofs

Proof of Lemma 1

The relative productivity distribution is stationary if and only if $\dot{H}_t(\phi) = 0 \, \forall \, \phi$. Setting $\dot{H}_t(\phi) = 0$ in equation (15) and substituting for F using Assumption 1 gives:

$$0 = \left\{\phi H'(\phi) - H'(1)[1 - H(\phi)]\right\} \frac{\dot{\theta}^*_t}{\theta^*_t} + \left[1 - \phi^{-k} - H(\phi)\right] \left(\frac{\theta^*_t}{x_t \psi_{\min}}\right)^{-k} \frac{R_t}{M_t}. \tag{37}$$

By substitution, $H(\phi) = 1 - \phi^{-k}$ solves equation (37) and gives a stationary relative productivity distribution for any values of $\frac{\dot{\theta}^*_t}{\theta^*_t}$, and $\left(\frac{\theta^*_t}{x_t \psi_{\min}}\right)^{-k} \frac{R_t}{M_t}$. To prove uniqueness, note that on any balanced growth path $\frac{\dot{\theta}^*_t}{\theta^*_t}$ and $(\frac{\theta^*_t}{x_t \psi_{\min}})^{-k} \frac{R_t}{M_t}$ are constant. Therefore, on any balanced growth path equation (37) defines a first-order differential equation for $H(\phi)$. Since H must satisfy the boundary condition $H(1) = 0$ and equation (37) is continuous in ϕ and Lipschitz continuous in H whenever ϕ is positive, the Picard-Lindelöf theorem implies there exists a unique solution. It follows that $H(\phi) = 1 - \phi^{-k}$ is the unique stationary relative productivity distribution.

Proof of Proposition 2

A necessary and sufficient condition for Proposition 2 to hold is $\frac{1-G_t(\theta_1)}{\theta_1^{-k}} \frac{\theta_2^{-k}}{1-G_t(\theta_2)} \to 1$ as $t \to \infty$ for all $\theta_1, \theta_2 > \theta_t^*$ since this ensures $G_t(\theta)$ converges to a Pareto distribution with shape parameter k as $t \to \infty$.

Let $Z_t(\theta)$ denote the mass of firms with productivity greater than or equal to θ at time t where $\theta > \theta_t^*$. Since incumbent firm productivity remains constant and there is a flow R_t of entrants who draw productivity from distribution $\tilde{G}_t(\theta) = F\left(\frac{\theta}{x_t}\right)$ we have:

$$Z_{t+\Delta}(\theta) = Z_t(\theta) + \Delta R_t \left[1 - F\left(\frac{\theta}{x_t}\right)\right].$$

Taking the limit as $\Delta \to 0$ and using the functional form of F from Assumption 1 gives:

$$\dot{Z}_t(\theta) = R_t \left(\frac{\theta}{x_t \psi_{\min}}\right)^{-k},$$

and solving this differential equation implies:

$$Z_t(\theta) = Z_0(\theta) + \left(\frac{\theta}{\psi_{\min}}\right)^{-k} \int_0^t R_s x_s^k ds.$$

Now substituting $Z_t(\theta) = M_t[1 - G_t(\theta)]$ into this equation implies that for all $\theta_1, \theta_2 > \theta_t^*$ we have:

$$\frac{1 - G_t(\theta_1)}{\theta_1^{-k}} \frac{\theta_2^{-k}}{1 - G_t(\theta_2)} = \frac{M_0 \frac{1-\hat{G}_0(\theta_1)}{\theta_1^{-k}} + \psi_{\min}^k \int_0^t R_s x_s^k ds}{M_0 \frac{1-\hat{G}_0(\theta_2)}{\theta_2^{-k}} + \psi_{\min}^k \int_0^t R_s x_s^k ds}.$$

As $t \to \infty$ we know $\theta_t^* \to \infty$. Therefore, a necessary and sufficient condition for the right-hand side of the above equation to converge to 1 for all $\theta_1, \theta_2 > \theta_t^*$ is that $\frac{1-\hat{G}_0(\theta)}{\theta^{-k}} \to \kappa$ as $\theta \to \infty$ for some $\kappa \geq 0$, that is, that Assumption 3 holds.

Proof of Proposition 3

To show that the dynamic gains from trade are positive substitute equations (32) and (20) into equation (28) and differentiate with respect to q to obtain:

$$\begin{aligned}\frac{dU}{dq} &\propto -(k\sigma+1-\sigma)\gamma D_1\left(kD_1-\frac{1-\gamma}{\gamma}D_2\right)\\ &\quad + k\gamma(D_1+D_2)[k\sigma(D_1+D_2)-(\sigma-1)D_1],\\ &= k^2\gamma\sigma D_2^2 + D_1D_2\left[k^2\gamma\sigma+(k\sigma+1-\sigma)(1+\gamma(k-1))\right],\\ &> 0\end{aligned}$$

where $D_1 \equiv \frac{1-\gamma}{\gamma}q+\rho-n$ and $D_2 \equiv n+gk$. In the first line of the expression, the first term on the right-hand side captures the indirect effect of higher growth on welfare through changes in c_0, while the second term captures the direct effect. The final inequality comes from observing that Assumption 2 implies both $D_1 > 0$ and $D_2 > 0$.

To obtain a version of the model developed in this article without productivity spillovers assume new entrants draw productivity from a Pareto distribution with scale parameter 1 and shape parameter k. Thus, $\tilde{G}(\theta) = 1-\theta^{-k}$ is independent of t. Assuming the baseline model is otherwise unchanged, the same reasoning used in Section II.C implies:

$$\frac{\dot{M}_t}{M_t} = -k\frac{\dot{\theta}_t^*}{\theta_t^*} + \frac{R_t}{M_t}\theta_t^{*-k}.$$

It immediately follows that on a balanced growth path the exit cutoff must be constant implying $g = 0$. Consumer optimization and the solution for the exit cutoff in equation (5) then give $q = \frac{n}{\sigma-1}$ meaning that the growth rate is independent of trade integration. With this result in hand it is straightforward to solve the remainder of the model and show $c_0 \propto z^s$.

Proof of Proposition 5

(Propositions 5–8 and Assumptions 5–7 appear in Appendix B.) The proof has two parts. First, I show that trade integration (an increase in J, a reduction in τ or a reduction in f_x) raises the growth rate q on any balanced growth path. Second, I prove there exists a unique balanced growth path by showing there exists a unique stationary relative productivity distribution.

Define $E(\phi) \equiv \frac{W_t(\phi)}{w_t}$. On a balanced growth path $E(\phi)$ is given by equation (22). Differentiating gives:

$$\frac{\partial E(\phi)}{\partial \tau} \propto -(\sigma-1)\left[\left(\frac{\phi}{\tilde{\phi}}\right)^{\sigma-1} - \left(\frac{\phi}{\tilde{\phi}}\right)^{\frac{q-r}{g}}\right]\frac{d\tilde{\phi}}{d\tau} \quad \forall\, \phi > \tilde{\phi},$$

$$< 0,$$

where the second line follows because $\frac{d\tilde{\phi}}{d\tau} > 0$ and the transversality condition implies $r > q$. Similarly we have:

$$\frac{\partial E(\phi)}{\partial f_x} \propto -1 + \left(\frac{\phi}{\tilde{\phi}}\right)^{\frac{q-r}{g}} \quad \forall\, \phi > \tilde{\phi},$$

$$< 0,$$

where the second line again follows from the transversality condition. We also have $\frac{\partial E(\phi)}{\partial J} > 0$ for all $\phi > \tilde{\phi}$ and $\frac{\partial E(\phi)}{\partial \tau} = \frac{\partial E(\phi)}{\partial f_x} = \frac{\partial E(\phi)}{\partial J} = 0$ for all $1 \leq \phi \leq \tilde{\phi}$.

Next, write $E(\phi) = E^d(\phi) + E^x(\phi)$ where E^d denotes the present discounted value from domestic sales relative to the wage (i.e., set $J = 0$ in equation (22) to obtain E^d) and E^x denotes the value created by exporting. Using equations (18) and (20) to substitute for g and r in equation (22) and differentiating gives:

$$\frac{\partial^2 E^d(\phi)}{\partial \phi \partial q} \propto -\left(\rho + \frac{1-\gamma}{\gamma}\frac{n}{\sigma-1}\right)\frac{\log\phi}{g^2}\phi^{\frac{q-r}{g}-1} - \frac{(\sigma-1)+\frac{1-\gamma}{\gamma}}{(\sigma-1)g+r-q}\frac{1}{\phi}\left(\phi^{\sigma-1} - \phi^{\frac{q-r}{g}}\right),$$

which is negative for all $\phi > 1$ by part (iii) of Assumption 6 and the transversality condition. Since $\frac{\partial E^d(\phi)}{\partial q} = 0$ when $\phi = 1$, it follows that $\frac{\partial E^d(\phi)}{\partial q} < 0$ for all $\phi > 1$. Similar reasoning shows that $\frac{\partial E^x(\phi)}{\partial q} < 0$ for all $\phi > \tilde{\phi}$ and obviously $\frac{\partial E^x(\phi)}{\partial q} = 0$ for all $1 \leq \phi \leq \tilde{\phi}$. Thus, higher growth reduces $E(\phi)$ whenever $\phi > 1$. Combining this result with the effects of trade integration on $E(\phi)$ obtained above implies that for the free entry condition (40) to hold we must have:

$$\frac{dq}{d\tau} < 0, \frac{dq}{df_x} < 0, \frac{dq}{dJ} > 0.$$

For the second part of the proof, showing that there exists a unique stationary relative productivity distribution is equivalent to proving that the differential equation (41) has a unique solution. Suppose $H'(1) = \chi$. Under the initial condition $H(1) = 0$, equation (41) is a first-order ordinary differential equation for $H(\phi)$. Since (41) is continuous in ϕ and Lipschitz continuous in H, the Picard-Lindelöf theorem implies that there exists a unique solution $H(\phi; \chi)$ with $H'(1; \chi) = \chi$.

$H'(\phi; \chi)$ is strictly increasing in both $H(\phi)$ and χ. Consequently, $\frac{\partial H(\phi;\chi)}{\partial \chi} > 0$ for all $\phi > 1$. Moreover, $H'(1; 0) < 0$ and $H''(1; 0) < 0$ meaning $H(\phi; 0) < 0$ for all $\phi > 1$. In addition, for any $\phi > 1$, $H(\phi; \chi)$ can be made arbitrarily large by choosing a sufficiently high χ. It follows that there exists a unique $\chi^* > 0$ such that $H(\phi_{\max}; \chi^*) = 1$.[39] The unique solution to equation (41) is $H(\phi) = H(\phi; \chi^*)$.

Proof of Proposition 6

Under Assumption 7, the free entry condition (11) is replaced on a balanced growth path by:

$$f_e = \int_{(\phi,\zeta)} E(\phi, \zeta) d\tilde{H}(\phi, \zeta),$$

where $E(\phi, \zeta) \equiv \frac{W_t(\phi,\zeta)}{w_t}$ and $W_t(\phi, \zeta)$ denotes firm value at time t conditional on (ϕ, ζ). Using equation (9) and observing that a firm with relative productivity ϕ at time 0 and future productivity growth ζ has relative productivity $\phi e^{\int_0^t \zeta_s ds} e^{-gt}$ at time t, we have that $E(\phi, \zeta)$ is given by:

$$E(\phi, \zeta) = I[\phi_t \geq 1] f \int_0^\infty \left[\left(\phi e^{\int_0^t \zeta_s ds} \right)^{\sigma-1} e^{-(\sigma-1)gt} - 1 \right] e^{-(r-q)t} dt$$
$$+ I[\phi_t \geq \tilde{\phi}] f J \tau^{1-\sigma} \int_0^\infty \left[\left(\phi e^{\int_0^t \zeta_s ds} \right)^{\sigma-1} e^{-(\sigma-1)gt} - \tilde{\phi}^{\sigma-1} \right] e^{-(r-q)t} dt.$$

We can now differentiate this expression and show (i) holding q constant, $E(\phi, \zeta)$ is strictly increasing in J and strictly decreasing in τ and f_x for all $\phi \geq \tilde{\phi}$ and independent of these variables for all $1 \leq \phi \leq \tilde{\phi}$; and (ii) $E(\phi, \zeta)$ is strictly decreasing in q given

39. If $\phi_{\max} = \infty$ this should be interpreted as meaning there exists a unique χ^* such that $lim_{\phi \to \infty} H(\phi; \chi^*) = 1$.

that part (ii) of Assumption 7 holds and that g and r satisfy equations (18) and (20) on a balanced growth path. The proposition then follows from the free entry condition.

Proof of Proposition 7

Let us solve for a balanced growth path taking Assumptions 1 and 8 as given. Since $\phi_t = \frac{x_t \psi}{\theta_t^*}$ we have $\frac{\dot{\phi}_t}{\phi_t} = \frac{\dot{x}_t}{x_t} - \frac{\dot{\theta}_t^*}{\theta_t^*}$. Assuming each firm's relative productivity is nonincreasing in t (i.e., $\dot{\phi}_t \leq 0$), equations (14) and (15), which govern the evolution of the mass of firms and the relative productivity distribution, respectively, are replaced by:

$$\frac{\dot{M}_t}{M_t} = -H_t'(1)\left(\frac{\dot{\theta}_t^*}{\theta_t^*} - \frac{\dot{x}_t}{x_t}\right) + \left[1 - F\left(\frac{\theta_t^*}{x_t}\right)\right]\frac{R_t}{M_t},$$

$$\dot{H}_t(\phi) = \left\{\phi H_t'(\phi) - H_t'(1)[1 - H_t(\phi)]\right\}\left(\frac{\dot{\theta}_t^*}{\theta_t^*} - \frac{\dot{x}_t}{x_t}\right)$$
$$+\left\{F\left(\frac{\phi\,\theta_t^*}{x_t}\right) - F\left(\frac{\theta_t^*}{x_t}\right) - H_t(\phi)\left[1 - F\left(\frac{\theta_t^*}{x_t}\right)\right]\right\}\frac{R_t}{M_t}.$$

Setting $\dot{H}_t(\phi) = 0$ implies that the unique stationary relative productivity distribution is $H(\phi) = 1 - \phi^{-k}$. It immediately follows that $x_t = \frac{k}{k-1}\theta_t^*$ implying $\frac{\dot{\theta}_t^*}{\theta_t^*} = \frac{\dot{x}_t}{x_t}$ and $\dot{\phi}_t = 0$. We also have that $\lambda = \frac{k}{k-1}\psi_{\min}$ and $\tilde{H}(\phi) = H(\frac{\phi}{\lambda})$ as in the baseline model.

On a balanced growth path equations (18)–(21) continue to hold, but instead of (22) the firm value function is given by:

$$W_t(\phi) = \frac{fw_t}{r-q}\left\{\left(\phi^{\sigma-1} - 1\right) + I\left[\phi \geq \tilde{\phi}\right]\frac{Jf_x}{f}\left[\left(\frac{\phi}{\tilde{\phi}}\right)^{\sigma-1} - 1\right]\right\}.$$

Integrating to obtain the expected value of entry and using the free entry condition, equations (18) and (20) imply there is a unique balanced growth path with growth rate:

$$q = \frac{\gamma}{1-\gamma}\left[\frac{\sigma-1}{k+1-\sigma}\frac{\lambda^k f}{f_e}\left(1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right) - \rho\right].$$

Assumption 8 ensures the transversality condition is satisfied and $g > 0$. Note that $\gamma < 1$ implies q is strictly increasing in J and strictly decreasing in τ and f_x.

Welfare on the balanced growth path is given by equation (28) and solving for c_0 we obtain:

$$c_0 = A_1 f^{-\frac{k+1-\sigma}{k(\sigma-1)}}\left[1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right]^{\frac{1}{k}}\left[1 + \frac{\sigma-1}{k\sigma+1-\sigma}\frac{n}{\frac{1-\gamma}{\gamma}q+\rho}\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}}, \tag{38}$$

where A_1 is given by equation (33). Observe that the static gains from trade are the same as in the baseline model. Since $\gamma < 1$, equation (38) implies $\frac{\partial c_0}{\partial q} > 0$. Therefore, higher growth is welfare increasing since it raises both q and c_0. It follows that the dynamic gains from trade are strictly positive.

Proof of Proposition 8

To prove the proposition, I need to show that q is strictly increasing in $T = J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}$. The result can be proved by taking the total derivatives of equations (42) and (43) (see Appendix B) and rearranging to obtain $\frac{dq}{dT}$, but here is a simpler argument. Suppose T increases, but q does not. Then equation (42) implies that $\omega\left(\frac{M_t}{R_t}\right)$ must decrease which requires a fall in $\frac{M_t}{R_t}$. From the definition of ω we have that $\frac{R_t}{M_t}\omega\left(\frac{M_t}{R_t}\right) = \Omega\left(\frac{R_t}{M_t}, 1\right)$ which increases when $\frac{M_t}{R_t}$ falls. Therefore, we must have that the left-hand side of equation (43) increases, while the right-hand side does not, giving a contradiction. It follows that an increase in T must lead to an increase in q.

Appendix B: Extensions of the Technology Diffusion Model

Knowledge Spillovers through Random Matching

Suppose instead of equation (10), knowledge spillovers result from random matching between entrants and incumbents. I assume that each entrant searches for a process technology to use and is randomly matched with an incumbent firm whose technology she imperfectly imitates. Formally, this implies that the productivity distribution of entrants is a scaled version of the productivity distribution of incumbent firms where the scaling parameter $\hat{\lambda}$ measures the strength of spillovers.

Thus, $\tilde{G}_t(\theta) = G_t(\frac{\theta}{\hat{\lambda}})$ where $\hat{\lambda} \in (0, 1]$. Note that although $\hat{\lambda}$ is closely related to the variable λ, which captures the strength of knowledge spillovers in the baseline model, there is a subtle difference. In the baseline model $\lambda \equiv \frac{x_t \psi_{\min}}{\theta_t^*}$ is an endogenous function of average incumbent productivity and equals $\frac{k}{k-1}\psi_{\min}$ on any balanced growth path. However, with random matching $\hat{\lambda}$ is an exogenous parameter that measures the effectiveness with which an entrant imitates an incumbent's technology.

Under this specification of knowledge spillovers, equation (14) for the growth rate of the mass of firms becomes:

$$\frac{\dot{M}_t}{M_t} = -H_t'(1)\frac{\dot{\theta}_t^*}{\theta_t^*} + \left[1 - H_t\left(\frac{1}{\hat{\lambda}}\right)\right]\frac{R_t}{M_t},$$

and equation (15), which characterizes the dynamics of the relative productivity distribution, is replaced by:

$$\begin{aligned} \dot{H}_t(\phi) = {} & \left\{\phi H_t'(\phi) - H_t'(1)[1 - H_t(\phi)]\right\}\frac{\dot{\theta}_t^*}{\theta_t^*} \\ & + \left\{H_t\left(\frac{\phi}{\hat{\lambda}}\right) - H_t\left(\frac{1}{\hat{\lambda}}\right) - H_t(\phi)\left[1 - H_t\left(\frac{1}{\hat{\lambda}}\right)\right]\right\}\frac{R_t}{M_t}. \end{aligned} \tag{39}$$

All other equations in Section II are unchanged.

Now observe that if ϕ has a Pareto distribution at time t then by equation (39) $\dot{H}_t(\phi) = 0$, implying that the Pareto distribution is a stationary relative productivity distribution.[40] Instead of Assumption 1, I impose the following initial condition.

Assumption 5. The productivity distribution at time 0 is Pareto: $G_0(\theta) = 1 - \left(\frac{\theta}{\theta_t^*}\right)^{-k}$ for $\theta \geq \theta_t^*$ with $k > \max\{1, \sigma - 1\}$.

40. More generally, solving equation (39) with $\dot{H}_t(\phi) = 0$ implies:

$$H(\phi) = 1 - \phi^{-k} + \phi^{-k}\int_1^{\phi} b(s)s^{k-1}ds,$$

where $k > 0$ and $b(\phi)$ satisfies:

$$b'(\phi)\phi\frac{\dot{\theta}_t^*}{\theta_t^*} = b(\phi)\frac{\dot{M}_t}{M_t} - b\left(\frac{\phi}{\hat{\lambda}}\right)\frac{R_t}{M_t},$$

with $b(1) = 0$. Obviously, $b(\phi) = 0$ solves this equation and implies ϕ has a Pareto distribution, but it is not known whether other solutions exist.

When Assumption 5 holds, there exists a unique stationary relative productivity distribution $H(\phi) = 1 - \phi^{-k}$. It follows that on a balanced growth path the relative productivity distributions of both entrants and incumbents are the same as in Section III provided $\lambda = \hat{\lambda}$. Having established this result, the same reasoning used in Sections III and IV shows that under Assumptions 2 and 5, Propositions 1 and 3 continue to hold. Therefore, on the balanced growth path, the equilibrium growth rate and the effects of trade on growth and welfare are the same with random matching as when knowledge spillovers are given by equation (10).

Frontier Expansion and Postentry Productivity Growth

In the baseline model firms draw productivity from a Pareto distribution and each firm's productivity θ does not change over time. By simplifying the model and ensuring the existence of a closed-form solution for the balanced growth path, these assumptions facilitate a clear exposition of the dynamic selection mechanism through which trade raises growth. In this section I relax these assumptions and show that neither assumption is necessary to obtain the paper's finding that trade increases growth.

First, let us generalize F to allow for non-Pareto sampling distributions. Instead of Assumptions 1 and 2, and the knowledge spillovers process used in the baseline model, I make the following assumption, which allows for entrants to sample productivity from any differentiable distribution.

ASSUMPTION 6.

(i) F is a differentiable cumulative distribution function with support $[\psi_{\min}, \psi_{\max}]$.

(ii) Knowledge spillovers are given by: $x_t = x\theta_t^*$ where x is a constant that satisfies $x\psi_{\min} \leq 1$ and $x\psi_{\max} > 1$.

(iii) The parameters of the world economy satisfy: $\rho + \frac{1-\gamma}{\gamma}\frac{n}{\sigma-1} > 0$ and $(\sigma - 1) + \frac{1-\gamma}{\gamma} > 0$.

(iv) The transversality condition is satisfied and the dynamic selection rate $g > 0$.

Since $\psi_{\max}$ can be infinite, Assumption 6 allows for the sampling distribution F to be either bounded or unbounded above.

On any balanced growth path the value of a firm with relative productivity ϕ is given by equation (22). Note that $\frac{W_t(\phi)}{w_t}$ is a stationary function of ϕ. By differentiating this function we can show: (i) $\frac{W_t(\phi)}{w_t}$ is strictly increasing in J and strictly decreasing in τ or f_x for all $\phi > \tilde{\phi}$, but is independent of J, τ, and f_x for all $\phi \leq \tilde{\phi}$; and (ii) $\frac{W_t(\phi)}{w_t}$ is strictly decreasing in q for all $\phi > 1$ provided part (iii) of Assumption 6 holds.[41] Thus, holding relative productivity constant, trade integration increases the present discounted value (relative to the wage) of all exporters. In addition, the parameter restrictions in part (iii) of Assumption 6 are sufficient to ensure higher growth decreases the present discounted value (relative to the wage) of firms at all relative productivity levels. A sufficient condition for the parameter restrictions in part (iii) to hold is $\gamma \leq 1$ and, as discussed in Section IV.B, empirical studies generally estimate $\gamma < 1$.

Using Assumption 6 the free entry condition (11) can be written as:

$$f_e = \int_{\phi} \frac{W_t(\phi)}{w_t} dF\left(\frac{\phi}{x}\right). \tag{40}$$

Part (ii) of Assumption 6 ensures that entrants' relative productivity distribution is stationary and independent of trade integration. Therefore, using the properties of $\frac{W_t(\phi)}{w_t}$ derived above, the free entry condition implies that trade integration (a rise in J, a reduction in τ or a reduction in f_x) strictly increases the growth rate q by raising the dynamic selection rate g. Higher growth is required to offset the increase in the expected value of entry caused by higher expected export profits. Intuitively, the functional form of F does not matter because the impacts of trade integration and q on $\frac{W_t(\phi)}{w_t}$ do not change sign as ϕ varies.

Neither parts (ii) or (iii) of Assumption 6 are necessary for trade to increase growth (for example, see Proposition 1), but they allow us to characterize the effects of trade integration on q without imposing any structure on the sampling distribution F. If part (ii) does not hold and F is not Pareto, then entrants' relative productivity distribution $\tilde{H}(\phi) = F\left(\frac{\phi\theta_t^*}{x_t}\right)$ is endogenous to trade integration through $\frac{\theta_t^*}{x_t}$. This endogeneity could either reinforce or weaken the positive effect of trade on growth depending on how trade affects the incumbent firm relative productivity distribution $H(\phi)$ and how knowledge spillovers x_t are specified.

41. See the proof of Proposition 5 in Appendix A for details.

Analyzing this effect would be an interesting topic for future research.

I have established that trade integration leads to higher growth on any balanced growth path. To show that a balanced growth path exists I also need to prove the existence of a stationary relative productivity distribution. Setting $\dot{H}_t(\phi) = 0$ in equation (15) and using equation (14) to eliminate $\frac{R_t}{M_t}$ gives:

$$(41) \qquad \phi H'(\phi) = \frac{n}{g}\left[H(\phi) - \frac{F\left(\frac{\phi}{x}\right) - F\left(\frac{1}{x}\right)}{1 - F\left(\frac{1}{x}\right)}\right] + H'(1)\frac{1 - F\left(\frac{\phi}{x}\right)}{1 - F\left(\frac{1}{x}\right)}.$$

The proof of Proposition 5 shows that this differential equation has a unique solution, implying the existence of a unique stationary relative productivity distribution $H(\phi)$. The solution $H(\phi)$ depends on trade integration through the dynamic selection rate g. Whenever $\psi_{\max} < \infty$, relative productivity is bounded above with $\phi_{\max} = x\psi_{\max}$. In this case growth is driven both by the diffusion of existing technologies as in the baseline model and by the expansion of the technology frontier as $\theta_{\max} = \theta_t^* \phi_{\max}$ increases. Thus, it is not necessary for the productivity distribution to have an unbounded right tail for trade to raise growth by increasing the dynamic selection rate. Combining the results above gives Proposition 5.

PROPOSITION 5. Given Assumption 6, the world economy has a unique balanced growth path equilibrium for any sampling distribution F. Trade integration raises the growth rate of consumption per capita on the balanced growth path.

Without imposing any functional form restrictions on F it is not possible to characterize how trade affects welfare analytically, but the equilibrium conditions in Section IV could be used to solve for balanced growth path welfare numerically for any given F.

Now let us extend the baseline model to allow for firms' productivity levels to change over time. Developing a theory to explain the postentry dynamics of firm productivity is not the purpose of this article. Instead, I show the dynamic selection effect of trade is robust to allowing for general firm-level productivity dynamics that are independent of trade integration.[42]

42. Lileeva and Trefler (2010), Atkeson and Burstein (2010), and Bustos (2011) analyze economies in which trade affects incumbent firms' incentives to undertake technology investment. To the extent that trade-induced technology upgrading

Suppose Assumptions 1 and 2 and the specification of knowledge spillovers used in the baseline model are replaced by the following assumption.

Assumption 7.

(i) Entrants at time t draw both an initial relative productivity ϕ and a set of productivity growth rates $\zeta = \{\zeta_s\}_{s \geq t}$ where $\zeta_s = \frac{\dot{\theta}_s}{\theta_s}$ from a stationary distribution $\tilde{H}(\phi, \zeta)$ that is independent of trade integration.
(ii) The intertemporal elasticity of substitution satisfies $\gamma \leq 1$.
(iii) The transversality condition is satisfied and the dynamic selection rate $g > 0$.

Part (i) of Assumption 7 implies entrants draw not a productivity level but a productivity path. The assumption allows for an arbitrary distribution of post entry productivity dynamics at the firm level,[43] and firms that enter with the same productivity may have different postentry growth rates. Implicit in part (i) is also the assumption that the structure of knowledge spillovers is such that trade integration does not change entrants' sampling distribution.

When firm productivity changes over time, firms with relative productivity $\phi < 1$ may have an option value from continuing to operate in the expectation of future productivity growth. To abstract from option values, I assume firms may choose to cease production temporarily and costlessly. Consequently, each firm produces if and only if its relative productivity $\phi \geq 1$.

When Assumption 7 holds, analogous reasoning to that used for Proposition 5 shows that trade integration and higher growth have opposite effects on the expected value of entry. Consequently, the free entry condition gives Proposition 6. The proof is in Appendix A.

Proposition 6. Given Assumption 7, trade integration raises the growth rate of consumption per capita on any balanced

raises export profits it is likely to magnify the positive effect of trade on growth identified in this paper.

43. Although restrictions must be placed on the distribution $\tilde{H}(\phi, \zeta)$ to guarantee the equilibrium is well defined. For example, explosive productivity growth must be ruled out to ensure the expected present discounted value of entry is finite.

growth path equilibrium of the world economy with post-entry firm level productivity dynamics.

Part (ii) of Assumption 7 is sufficient to ensure Proposition 6 holds for an arbitrary sampling distribution $\tilde{H}(\phi, \zeta)$, but it is not necessary. Note also that Proposition 6 does not prove there exists a balanced growth path equilibrium. To prove existence requires showing there exists a stationary relative productivity distribution, which is not possible without imposing greater structure on $\tilde{H}(\phi, \zeta)$.

Propositions 5 and 6 show that the dynamic selection mechanism through which trade increases growth in the baseline model is also present when the productivity distribution is not Pareto and when firms' productivity levels vary over time. In both cases the result is driven by the same logic: trade integration raises export profits and free entry then requires an offsetting increase in the dynamic selection rate.

Learning by Incumbent Firms

The focus of this article is on knowledge spillovers from incumbent firms to entrants. However, existing firms may also benefit from knowledge spillovers. A simple way to incorporate learning by incumbents into the model is to assume firm productivity at time t is given by:

$$\theta_t = x_t \psi,$$

where x_t equals the average productivity of incumbent firms as in the baseline model and ψ continues to be a stochastic component drawn at entry from the sampling distribution F. The only difference between this specification and the baseline model is that each firm's productivity depends on the current value of x_t. Thus, upward shifts in the productivity distribution generate spillovers that raise the productivity of both entrants and incumbents. The remainder of the model is unchanged except Assumption 2 is replaced by the following assumption.

ASSUMPTION 8. The parameters of the world economy satisfy:

$$1 > \gamma > \left(1 + \left[\frac{\sigma - 1}{k + 1 - \sigma}\left(\frac{k}{k-1}\psi_{\min}\right)^k \frac{f}{f_e}\left(1 + J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right) - \rho\right]\frac{\sigma - 1}{n}\right)^{-1},$$

$$n < \frac{\sigma-1}{k+1-\sigma}\left(\frac{k}{k-1}\psi_{\min}\right)^{k}\frac{f}{f_e}\left(1+J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right).$$

The first inequality guarantees $g > 0$, and the second inequality ensures the transversality condition holds. Evidence supporting the assumption $\gamma < 1$ was discussed in Section IV.B.

We can now solve for a balanced growth path equilibrium following the same steps used in Sections III and IV. The stationary relative productivity distribution is Pareto as in Lemma 1 and since $x_t = \frac{k}{k-1}\theta_t^*$ the relative productivity of each firm remains constant over time. Proposition 7 shows that on the balanced growth path trade integration raises the growth rate leading to positive dynamic gains from trade. The proof is in Appendix A.

PROPOSITION 7. Given Assumptions 1 and 8, when knowledge spillovers raise the productivity of both entrants and incumbents, the world economy has a unique balanced growth path equilibrium on which consumption per capita grows at rate:

$$q = \frac{\gamma}{1-\gamma}\left[\frac{\sigma-1}{k+1-\sigma}\left(\frac{k}{k-1}\psi_{\min}\right)^{k}\frac{f}{f_e}\left(1+J\tau^{-k}\left(\frac{f}{f_x}\right)^{\frac{k+1-\sigma}{\sigma-1}}\right)-\rho\right].$$

Trade integration increases the growth rate of consumption per capita. The positive effect of trade on growth increases the welfare gains from trade.

When knowledge spillovers to incumbents are sufficiently strong that each firm's relative productivity remains constant over time, an increase in the dynamic selection rate does not affect firms' expected life spans. Instead, under the maintained assumption that $\gamma < 1$, higher growth decreases the expected value of entry by raising $r - q$ and reducing the present discounted value of future profits. Thus, the channel through which an increase in the dynamic selection rate lowers the value of innovating differs from the baseline model, but free entry continues to imply that trade integration raises growth. Moreover, the positive effect of trade on growth leads to dynamic welfare gains that are additional to the gains from trade in static steady-state economies. Allowing for weaker knowledge spillovers to incumbent firms such that relative productivity is declining in g gives a hybrid between the baseline model and the variant considered in this section. Unsurprisingly, the effect of trade integration on growth remains positive.

R&D Technology

The baseline model features constant returns to scale in R&D. In this section I generalize the R&D technology to allow for congestion in technology diffusion. Assume that when $R_t f_e$ workers are employed in R&D the flow of new innovations is $\Omega(R_t, M_t)$ where Ω is homogeneous of degree 1, strictly increasing in R_t, weakly increasing in M_t, and $\Omega(0, 0) = 0$.[44] Ω gives the mass of innovators who successfully learn from incumbents' production techniques. Allowing Ω to depend on M_t introduces decreasing returns to scale in R&D investment and implies R&D is more productive when there are more incumbent firms to learn from.

Given this R&D technology, we can solve for a balanced growth path equilibrium following the same reasoning applied already. Modifying the R&D technology does not affect households' welfare maximization problem or firms' static profit maximization problem meaning that equations (18) and (20) continue to hold. In addition, the stationary relative productivity distribution is unchanged and Lemma 1 still holds. However, the free entry condition now implies:

$$q = kg + r - \frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left[1 + J\tau^{-k} \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right] \omega\left(\frac{M_t}{R_t} \right),$$

where $\omega\left(\frac{M_t}{R_t}\right) \equiv \Omega\left(1, \frac{M_t}{R_t}\right) = \frac{1}{R_t}\Omega(R_t, M_t)$. Combining this expression with equations (18) and (20) gives:

$$q = \frac{\gamma}{1 + \gamma(k-1)} \left[\frac{\sigma - 1}{k + 1 - \sigma} \frac{\lambda^k f}{f_e} \left(1 + J\tau^{-k} \left(\frac{f}{f_x} \right)^{\frac{k+1-\sigma}{\sigma-1}} \right) \omega\left(\frac{M_t}{R_t} \right) + \frac{kn}{\sigma - 1} - \rho \right]. \tag{42}$$

Comparing equation (42) with the baseline economy growth rate given by equation (24), the only difference is the inclusion of $\omega\left(\frac{M_t}{R_t}\right)$. To obtain the equilibrium value of $\frac{R_t}{M_t}$ note that in this

44. The baseline model corresponds to the case $\Omega(R_t, M_t) = R_t$. Assuming Ω is homogeneous of degree 1 ensures the existence of a balanced growth path equilibrium.

version of the model equation (14), which gives the rate at which new firms are created, becomes:

$$\frac{R_t}{M_t}\omega\left(\frac{M_t}{R_t}\right)=\frac{1}{\lambda^k}\left(kq-\frac{k+1-\sigma}{\sigma-1}n\right). \tag{43}$$

Equations (42) and (43) define a system of two equations in the two unknowns q and $\frac{R_t}{M_t}$. It is not possible to solve for q explicitly. However, assuming the transversality condition holds and $g>0$, the proof of Proposition 8 shows that q is higher under trade than in autarky and is strictly increasing in J, strictly decreasing in τ, and strictly decreasing in f_x. Thus, as in the baseline model, trade raises growth by increasing the rate of dynamic selection. Moreover, solving for the initial consumption level shows c_0 is still given by equation (32). It follows that even with decreasing returns to scale in R&D there exist dynamic gains resulting from the progrowth effects of trade and these dynamic gains increase the total gains from trade relative to static steady-state versions of the model. Proposition 8 summarizes these results.

PROPOSITION 8. Given Assumption 1, when there is congestion in technology diffusion the world economy has a unique balanced growth path equilibrium. Trade integration raises the growth rate of consumption per capita. The positive effect of trade on growth increases the welfare gains from trade.

To calibrate the model with congestion in technology diffusion let $\Omega(R_t, M_t)=R_t^{\alpha}M_t^{1-\alpha}$ where $\alpha\in(0,1]$ parameterizes the returns to scale in R&D. Figure V shows how trade affects growth and welfare as α varies between 0 and 1 with other observables and parameters held constant at their baseline values from Table I.[45] Reducing the returns to scale in R&D lowers the

45. The balanced growth path equilibrium conditions do not imply the existence of an observable that can be used to calibrate α directly, and I am not aware of any empirical work that estimates the returns to scale in R&D when R&D is aimed at learning about existing technologies. Allowing for congestion in technology diffusion does not affect the calibration of the static gains from trade.

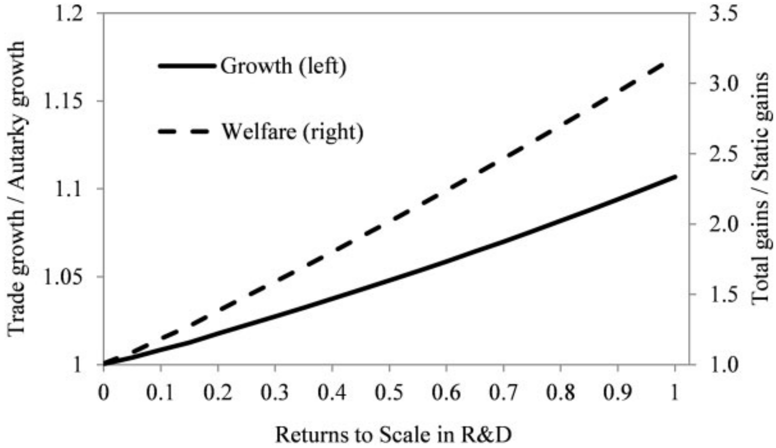

FIGURE V

Returns to Scale in R&D and the Dynamic Gains from Trade

dynamic gains from trade as reallocating labor from production to R&D has a smaller effect on growth. However, provided the returns to scale exceed 1/2, the dynamic selection effect of trade at least doubles the gains from trade.

Tax Policy

A complete optimal policy analysis of the dynamic selection model lies beyond the scope of this article. However, to better understand its welfare properties, we can analyze the effects of linear taxes on fixed costs and R&D. Consider a single autarkic economy in which the government taxes the fixed cost of production at rate υ and subsidizes R&D at rate υ_e.[46] Thus, each firm must pay $w_t f(1+\upsilon)$ per period to produce, and employing an R&D worker costs $w_t(1-\upsilon_e)$. Also, assume the government balances its budget through lump-sum transfers to households and the R&D technology takes the form introduced in the previous section, which allows for the possibility of congestion in technology diffusion.

46. The closed economy results derived below generalize immediately to the open economy model, but only if we abstract from strategic policy interactions across countries by imposing symmetric taxes in all economies.

Under these assumptions it is straightforward to solve for the balanced growth path equilibrium using reasoning analogous to that applied in Section III. Provided $g > 0$ and the transversality condition holds there exists a unique balanced growth path equilibrium on which:

$$(44)\quad q = \frac{\gamma}{1+\gamma(k-1)}\left[\frac{\sigma-1}{k+1-\sigma}\left(\frac{k}{k-1}\psi_{\min}\right)^k \frac{f}{f_e}\omega\left(\frac{M_t}{R_t}\right)\frac{1+v}{1-v_e}+\frac{kn}{\sigma-1}-\rho\right],$$

$$c_0 = A_1(k\sigma+1-\sigma)^{\frac{k\sigma+1-\sigma}{k(\sigma-1)}} f^{-\frac{k+1-\sigma}{k(\sigma-1)}}(1+v)$$
$$*\left[(k+1-\sigma)+k(\sigma-1)(1+v)+(\sigma-1)\frac{1+v}{1-v_e}\frac{n+gk}{n+gk+\frac{1-\gamma}{\gamma}q+\rho-n}\right]^{-\frac{k\sigma+1-\sigma}{k(\sigma-1)}},$$

(45)

where $\frac{R_t}{M_t}$ satisfies equation (43) as before. Observe that either taxing the fixed production cost or subsidizing entry leads to higher growth by increasing the ratio of fixed costs to entry costs and raising the dynamic selection rate. Also, by comparing equation (44) with equation (24) and equation (45) with equation (32) we see that while tax policy can mimic the growth effect of trade integration it cannot simultaneously replicate the effect of trade on the level of consumption.

Household welfare on the balanced growth path still depends on q and c_0 through equation (28). Therefore, to analyze the welfare effects of tax policy we can substitute equations (44) and (45) into equation (28) and then differentiate with respect to v and v_e while using equation (43) to account for the endogeneity of $\frac{R_t}{M_t}$. For the sake of brevity the resulting algebra is omitted, but there are two main findings.

First, when $v = v_e = 0$ welfare is strictly increasing in v. Moreover, provided $\Omega(R_t, M_t) = R_t$ welfare is also strictly increasing in v_e. This means that in the baseline model with constant returns to scale in R&D either taxing the fixed cost of production or subsidizing entry raises welfare relative to an economy without taxes. In each case the policy is welfare improving because it increases the firm creation rate $\lambda^k \frac{R_t}{M_t}$, which is inefficiently low in the decentralized equilibrium since innovators do not internalize the knowledge spillovers that entry generates.

Second, if the government chooses v and v_e simultaneously to maximize welfare the optimal tax rates satisfy:[47]

$$v = \left(\frac{k}{k-1}\psi_{\min}\right)^{-k}\frac{f_e}{f}\frac{n+gk}{\omega\left(\frac{M_t}{R_t}\right)},$$

$$v_e = 1 - \frac{\omega\left(\frac{M_t}{R_t}\right)}{\omega\left(\frac{M_t}{R_t}\right) - \frac{M_t}{R_t}\omega'\left(\frac{M_t}{R_t}\right)}\left[1 + \frac{k+1-\sigma}{\sigma-1}\frac{v}{1+v}\frac{\omega\left(\frac{M_t}{R_t}\right)}{\omega\left(\frac{M_t}{R_t}\right) - \frac{M_t}{R_t}\omega'\left(\frac{M_t}{R_t}\right)}\right]^{-1}.$$

It immediately follows that the government sets $v > 0$, implying a tax on the fixed costs of production. In addition, in the baseline model with constant returns to scale in R&D we have $v_e > 0$, meaning entry is subsidized. However, when there is congestion in technology diffusion, R&D may either be subsidized or taxed depending on the shape of Ω.

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SUPPLEMENTARY MATERIAL

An Online Appendix for this article can be found at QJE online (qje.oxfordjournals.org).

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47. These results hold assuming the government's maximization problem is concave. In general concavity is not guaranteed, but a sufficient condition that ensures concavity is $\Omega(R_t, M_t) = R_t^{\alpha}M_t^{1-\alpha}$ with $\alpha \in (0, 1]$ and $(1-\gamma)(k+1-\sigma) > \alpha\gamma k(\sigma-1)$.

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