

Balanced Growth Despite Uzawa[†]

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The evidence for the United States points to balanced growth despite falling investment-good prices and a less-than-unitary elasticity of substitution between capital and labor. This is inconsistent with the Uzawa Growth Theorem. We extend Uzawa's theorem to show that the introduction of human capital accumulation in the standard way does not resolve the puzzle. However, balanced growth is possible if education is endogenous and capital is more complementary with schooling than with raw labor. We present a class of aggregate production functions for which a neoclassical growth model with capital-augmenting technological progress and endogenous schooling converges to a balanced growth path. (JEL E22, E24, I26, J24, O33, O41, O47)

Some key facts about economic growth have become common lore. Among those famously cited by Kaldor (1961) are the observation that output per worker and capital per worker have grown steadily, while the capital-output ratio, the real return on capital, and the shares of capital and labor in national income have remained fairly constant. Jones (2016, p. 5) updates these facts using the latest available data. He reports that real per capita GDP in the United States has grown “at a remarkably steady average rate of around two percent per year” for a period of nearly 150 years, while the ratio of physical capital to output has remained nearly constant. The shares of capital and labor in total factor payments were very stable from 1945 through about 2000.¹

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¹As is well known from Piketty (2014) and others before him and since, the capital share in national income has been rising, and that of labor falling, since around 2000: see, for example, Elsby, Hobijn, and Sahin (2013); Karabarbounis and Neiman (2014); and Lawrence (2015). It is not clear yet whether this is a temporary fluctuation around the longstanding division, part of a transition to a new steady-state division, or perhaps (as Piketty asserts) a permanent departure from stable factor shares.

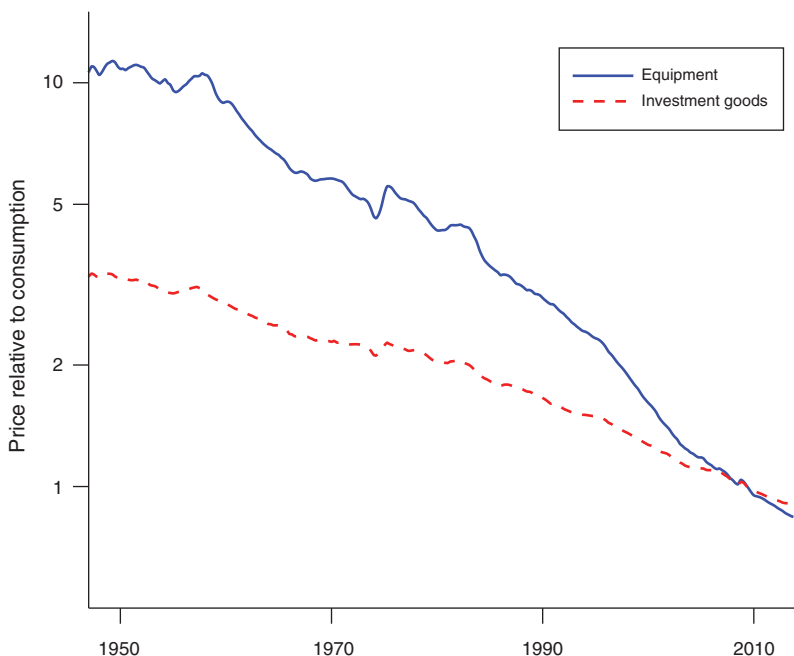


FIGURE 1. US RELATIVE PRICE OF EQUIPMENT, 1947–2013

Source: Federal Reserve Bank Economic Data (FRED), series PIRIC and PERIC

These facts suggest to many the relevance of a *balanced growth path* and thus the need for models that predict sustained growth of output, consumption, and capital at constant rates. Indeed, neoclassical growth theory was developed largely with this goal in mind. Apparently, it succeeded. As Jones and Romer (2010, p. 225) conclude: “There is no longer any interesting debate about the features that a model must contain to explain [the Kaldor facts]. These features are embedded in one of the great successes of growth theory in the 1950s and 1960s, the neoclassical growth model.”

Alas, “all is not well,” as Hamlet might say. Jones (2016) highlights yet another fact that was noted earlier by Gordon (1990), Greenwood, Hercowitz, and Krusell (1997), and others: the relative price of capital equipment, adjusted for quality, has been falling steadily and dramatically since at least 1960. Figure 1 reproduces two series from the FRED database.² In the period from 1947 to 2013, the relative price of investment goods declined at a compounded average rate of 2.0 percent per annum. The relative price of equipment declined at an even faster annual rate of 3.8 percent.

The observation of falling capital prices rests uncomfortably with features of the economy thought to be needed for balanced growth. As Uzawa (1961) pointed out, and Schlicht (2006) and Jones and Scrimgeour (2008) later clarified, a balanced growth path in the two-factor neoclassical growth model with a constant and

²The Federal Reserve Economic Data (FRED) are maintained by the Federal Reserve Bank of St. Louis. Their investment and equipment prices are based on updates of Gordon’s (1990) series by Cummins and Violante (2002) and DiCecio (2009).

exogenous rate of population growth and a constant rate of labor-augmenting technological progress requires either an aggregate production function with a unitary elasticity of substitution between capital and labor or else an absence of capital-augmenting technological progress. The size of the elasticity of substitution is much debated and still controversial; yet, a preponderance of the evidence suggests an elasticity well below 1.³ And the decline in quality-adjusted prices of investment goods (and especially equipment) relative to final output suggests that capital-augmenting technological progress—embodied, for example, in each new generation of equipment—has been occurring.⁴

The Uzawa Growth Theorem rests on the impossibility of getting an endogenous rate of capital accumulation to line up with an exogenous growth rate of effective labor in the presence of capital-augmenting technological progress, unless the aggregate production function takes a Cobb-Douglas form. The *problem*, it would seem, stems from the model's assumption of an inelastic supply of effective labor that does not respond to capital deepening, even over time. If human capital could be accumulated via, for example, investments in schooling, then perhaps effective labor growth would fall into line with growth in effective capital, and a balanced growth path would be possible in a broader set of circumstances. Seen in this light, another fact about the US growth experience is encouraging. We reproduce—as did Jones (2016)—a figure from Goldin and Katz (2007). Figure 2 shows the average years of schooling measured at age 30 for all cohorts of native American workers born between 1876 and 1982.⁵ Clearly, educational attainment has been rising steadily for more than a century. Put differently, there has been ongoing investment in *human capital*. Indeed, Uzawa (1965), Lucas (1988), and others have established existence of balanced growth paths in neoclassical growth models that incorporate standard treatments of human capital accumulation, albeit in settings that lack capital-augmenting technological progress.⁶

Unfortunately, the usual formulation of human capital does not do the trick. In the next section, we prove an extended version of the Uzawa Growth Theorem that allows for education. We specify an aggregate production function that has effective capital (the product of physical capital and a productivity-augmenting technology term) and human capital as arguments. Human capital can be any increasing function of technology-augmented *raw labor* and a variable that measures cumulative investments in schooling. In this setting, we show that balanced growth again requires either a

³Chirinko (2008, p. 671), for example, who surveyed and evaluated a large number of studies that attempted to measure this elasticity, concluded that “the weight of the evidence suggests a value of [the elasticity of substitution] in the range of 0.4 to 0.6.” In research conducted since that survey, Karabarbounis and Neiman (2014) estimate an elasticity of substitution greater than 1, but Chirinko, Fazzari, and Meyer (2011); Oberfield and Raval (2014); Chirinko and Mallick (2014); Herrendorf, Herrington, and Valentinyi (2013); and Lawrence (2015) all estimate elasticities below 1.

⁴Motivated by Uzawa's Growth Theorem, Acemoglu (2003) and Jones (2005) develop theories of directed technical change in order to provide an explanation for the absence of capital-augmenting technical change. To be consistent with balanced growth, both look for restrictions that would lead endogenous technical change to be entirely labor-augmenting. Neither attempts to reconcile capital-augmenting technical change with balanced growth.

⁵We are grateful to Larry Katz for providing the unpublished data that allowed us to extend his earlier figure.

⁶Uzawa (1965) studies a model with endogenous accumulation of human capital in which education augments *effective labor supply* so as to generate convergence to a steady state. Lucas (1988) incorporates an externality in his measure of human capital, a possibility that we do not consider here. Acemoglu (2009, pp. 371–74) characterizes a balanced growth path in a setting with overlapping generations.

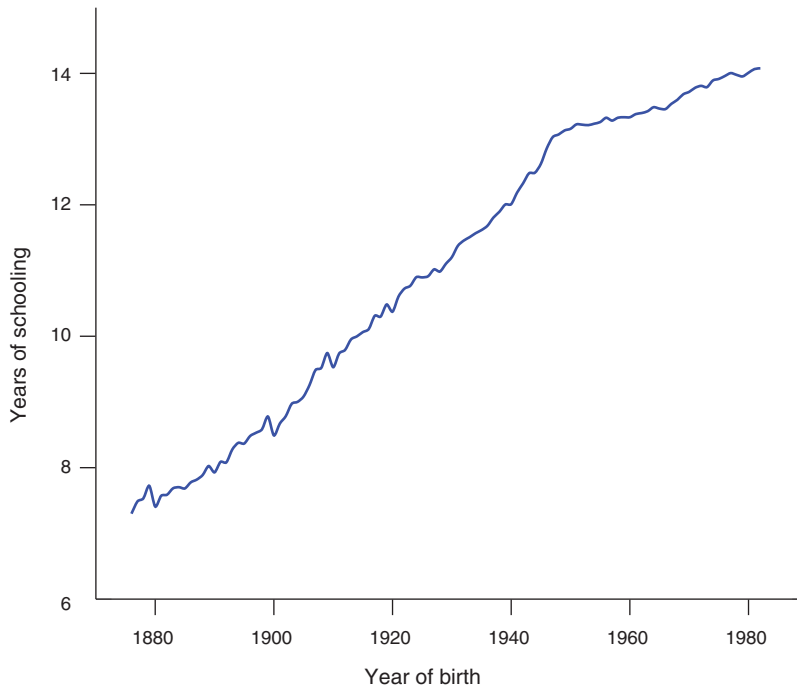


FIGURE 2. US EDUCATION BY BIRTH COHORT, 1876–1982

Source: Goldin and Katz (2007) and additional data from Lawrence Katz

unitary elasticity of substitution between physical capital and human capital or an absence of capital-augmenting technological progress. The intuition is similar to that provided by Jones and Scrimgeour for the original Uzawa theorem. Along a balanced growth path, the value of physical capital that is produced from final goods inherits the trend in output.⁷ But the growth rate of final output is a weighted average of the growth rates of effective capital and effective labor, with factor shares as weights. If these shares are to remain constant along a balanced growth path with an aggregate production function that is not Cobb-Douglas, then effective capital and effective labor must grow at common rates. So, the growth rate of output also mirrors the growth rate of effective capital. If the growth rate of output must be equal to both the growth rate of physical capital and that of effective capital, then there is no room for capital productivity to improve or for the cost of investment goods to fall.

But our findings in Section I also suggest a resolution to the puzzle. Ongoing increases in education potentially can reconcile the existence of a balanced growth path with a sustained rise in capital or investment productivity and an elasticity of substitution between capital and labor less than unity, provided that schooling enters the aggregate production function in a particular way. If the production technology is such that investments in schooling offset the change in the capital share that results from capital deepening, balanced growth can emerge. To be more precise, suppose

⁷ If the price of investment goods relative to consumption can change—something Jones and Scrimgeour did not consider—the analogous requirement is that the *value* of the capital stock inherits the growth rate of output.

that $F(K, L, s; t)$ is the output that can be produced with the technology available at time t by L units of raw labor and K units of physical capital, when the economy has an education level summarized by the scalar measure s . The measure might reflect, for example, the average years of schooling in the workforce or the relative supplies of skilled to unskilled hours. Suppose that $F(\cdot)$ has constant returns to scale in K and L and that $\sigma_{KL} < 1$, where $\sigma_{KL} \equiv F_L F_K / F F_{LK}$ is the elasticity of substitution between capital and labor, holding schooling constant. We will show that a balanced growth path with constant factor shares, positive capital-augmenting technological progress, and a rising index of educational attainment can emerge if and only if the ratio of the marginal product of schooling to the marginal product of labor rises as the capital stock grows: i.e., $\partial(F_s/F_L)/\partial K > 0$. Clearly, this precludes a production function of the form $F(K, H; t)$, where $H = G(L, s)$ is a standard measure of human capital, because then F_s/F_L is independent of K . A necessary condition for balanced growth in the presence of capital-augmenting technological progress and a non-unitary elasticity of substitution is a sufficient degree of complementarity between capital and education. Of course, many researchers have noted the empirical relevance of *capital-skill complementarity* (see, most prominently, Krusell et al. 2000 and Autor, Katz, and Krueger 1998), albeit with varying interpretations of the word “skill” and of the word “complementarity.” Our analysis makes clear that the appropriate sense of complementarity is a relative one: growth in the capital stock must raise the marginal product of schooling relatively more than it does the marginal product of raw labor.

The fact that schooling gains *can* offset the effects of capital-augmenting technological progress on the capital share does not of course mean that they *will* do so in a reasonable model of education decisions. We proceed in Section II to introduce optimizing behavior. We first solve a social planner’s problem that incorporates a reduced-form specification of the trade-off between an index of an economy’s education level and its labor supply. A simplifying assumption is that an economy’s schooling can be represented by a scalar measure that can jump from one moment to the next. Under this assumption, when the aggregate production function belongs to a specified class, the optimal growth trajectory converges to a balanced-growth path with constant rates of growth of output, consumption and capital, and constant factor shares. Following the presentation of the planner’s problem, we describe two distinct models in which the market equilibrium shares the dynamic properties of the efficient solution. In both models, the economy is populated by a continuum of similar dynasties, each comprising a sequence of family members who survive for only infinitesimal lifespans. In the *time-in-school* model of Section IIB, each individual decides what fraction of her brief existence to devote to schooling, thereby determining her productivity in her remaining time as a worker. Firms allocate capital to their various employees as a function of their productivity levels and therefore their schooling. In the *manager-worker* model of Section IIC, individuals instead make a discrete educational choice. Those who spend a fixed fraction of their life in school are trained to work as managers with their remaining time. Those who do not opt for management training have their full lives to serve as production workers. In this case, our measure of the economy’s education level is the ratio of manager hours to worker hours. We take the productivity of a production unit (workers combined with equipment) as increasing in this ratio due to improved monitoring.

In both models the economy converges to a balanced-growth path for a specified class of production functions, all of whose members are characterized by stronger complementarity between capital and schooling than between capital and raw labor.

The class of production functions that we describe in our Assumption 1 is not only sufficient for the emergence of balanced growth, but (essentially) necessary as well.⁸ The endogenous gains in education must not only counteract the decline in capital share that would otherwise result from capital-augmenting technological progress with $\sigma_{KL} < 1$, but they must do so exactly. The requirements for balanced growth remain strong, but they are not obviously at odds with the empirical evidence. Moreover, the restrictions on technology are no stronger than those relating to preferences that are known to be needed for balanced growth. Importantly, our simplifying assumptions about demographics and education are not essential to the argument; we show in our working paper (Grossman et al. 2016) that balanced growth can emerge in an overlapping-generations model with finite lives, wherein the economy's educational state is characterized by a distribution of schooling levels. The key assumption there is analogous to Assumption 1 and relates to how capital affects the productivity of education relative to that of raw labor.

In the concluding section, we discuss how our findings relate to the large and still-growing literature on the long-run implications of investment-specific technological change.

I. The Extended Uzawa Growth Theorem and a Possible Way Out

In this section, we state and prove a version of the Uzawa Growth Theorem, using methods adapted from Schlicht (2006) and Jones and Scrimgeour (2008). We extend the theorem to allow for falling investment-good prices and the possible accumulation of human capital. We also show how investments in schooling can loosen the straitjacket of the theorem, but only if capital accumulation boosts the marginal product of education proportionally more than it does the marginal product of raw labor.

Let $Y_t = F(A_t K_t, B_t L_t, s_t)$ be a standard neoclassical production function with constant returns to scale in its first two arguments, where, as usual, Y_t is output, K_t is capital, L_t is labor, and where A_t and B_t characterize the state of (disembodied) technology at time t , augmenting respectively physical capital and raw labor.⁹ We take s_t to be some scalar measure of the prevailing education level in the economy that is independent of the economy's size. For example, s_t might be the average years of schooling among workers, or the fraction of the labor force with a college degree, or the ratio of trained managers to production-line workers. The labor force L_t grows at some constant rate, g_L , that can be positive, negative, or zero.

⁸More precisely, we show in the online Appendix that balanced growth in the presence of ongoing capital-augmenting technological progress requires that the technology has a representation with the form indicated in Assumption 1.

⁹For ease of exposition and for comparability with the literature, we treat technology as a combination of components that augment physical capital and raw labor. However, as we show in the online Appendix, our Proposition 1 can readily be extended to any constant returns to scale production function with the form $F(K_t, L_t, s_t; t)$. Indeed, Uzawa (1961) originally proved his theorem (without the education variable s_t) in this more general form.

At time t , the economy can convert one unit of output into q_t units of capital. Growth in q_t represents what Greenwood, Hercowitz, and Krusell (1997) have called *investment-specific technological change*. This is a form of *embodied* technical change—familiar from the earlier work of Johansen (1959), Solow (1960), and others—inasmuch as new capital goods require less foregone consumption than did prior vintages of capital. The economy's resource constraint can be written as

$$Y_t = C_t + I_t/q_t,$$

where C_t is consumption and I_t is the number of newly installed units of capital. Investment in new capital augments the capital stock after replacing depreciation, which occurs at a fixed rate δ : i.e.,

$$\dot{K}_t = I_t - \delta K_t.$$

We begin with a lemma that extends slightly the one proved by Jones and Scrimgeour (2008) so as to allow for investment-specific technological progress. Define a balanced growth path (BGP) as a trajectory along which the economy experiences constant proportional rates of growth of Y_t , C_t , and K_t . Let $g_X = \dot{X}/X$ denote the growth rate of the variable X along a BGP.

LEMMA 1: *Suppose g_q is constant. Then, along any BGP with $0 < C_t < Y_t$, $g_Y = g_C = g_K - g_q$.*

The proof, which closely follows Jones and Scrimgeour, is relegated to the online Appendix. The lemma states that the growth rates of consumption and capital mirror that of total output. However, with the possibility of investment-specific technological progress, it is the *value* of the capital stock measured in units of the final good (and the resources used in investment) that grows at the same rate as output.¹⁰

Now define $\gamma_K \equiv g_A + g_q$. This can be viewed as the *total rate of capital-augmenting technological change*, combining the rate of disembodied progress (g_A) and the rate of embodied progress (g_q). Also, define, as we did before, $\sigma_{KL} \equiv (F_L F_K)/(F_{LK} F)$ to be the elasticity of substitution between capital and labor holding fixed the education index. In the online Appendix, we prove the following proposition.

PROPOSITION 1: *Suppose q grows at constant rate g_q . If there exists a BGP along which factor shares are constant and strictly positive when the factors are paid their marginal products, then*

$$(1) \quad (1 - \sigma_{KL}) \gamma_K = \sigma_{KL} \frac{F_L}{F_K} \frac{\partial(F_s/F_L)}{\partial K} \dot{s}.$$

The proposition stipulates a relationship between the combined rate of capital-augmenting technological progress and the change in the education index

¹⁰When capital goods are valued, their price p_t in terms of final goods must equal the cost of new investment, i.e., $p_t = 1/q_t$.

that is needed to keep factor shares constant as the value of the capital stock and output grow at common rates.

We can now revisit the two cases that are familiar from the literature. First, suppose that there are no opportunities for investment in schooling, so that s remains constant. This is the setting considered by Uzawa (1961). Setting $\dot{s} = 0$ in (1) yields Corollary 1.

COROLLARY 1 (Uzawa): *Suppose that s is constant. Then a BGP with constant and strictly positive factor shares can exist only if $\sigma_{KL} = 1$ or $\gamma_K = 0$.*

As is well known, balanced growth in a neoclassical economy with exogenous population growth and no investments in human capital requires either a Cobb-Douglas production function or an absence of capital-augmenting technological progress.¹¹

Second, suppose that (effective) labor and schooling can be aggregated into an index of human capital, $H(BL, s)$, such that net output can be written as a function of effective physical capital and human capital, as in Uzawa (1965); Lucas (1988); or Acemoglu (2009). Denote this production function by $\tilde{F}[AK, H(BL, s)] \equiv F(AK, BL, s)$. Then $F_s/F_L = H_s/H_L$, which is independent of K . Setting $\partial(F_s/F_L)/\partial K = 0$ in (1) yields Corollary 2.

COROLLARY 2 (Human Capital): *Suppose that there exists a measure of human capital, $H(BL, s)$, such that $F(AK, BL, s) \equiv \tilde{F}[AK, H(BL, s)]$. Then a BGP with constant and strictly positive factor shares can exist only if $\sigma_{KL} = 1$ or $\gamma_K = 0$.*

In this case, ongoing accumulation of human capital cannot perpetually neutralize the effects of capital deepening on the factor shares.

However, Proposition 1 suggests that balanced growth with constant factor shares might be possible despite a non-unitary elasticity of substitution between capital and labor and the presence of capital-augmenting technological progress, so long as $\dot{s} \neq 0$ and $\partial(F_s/F_L)/\partial K \neq 0$. Suppose, for example, that $\sigma_{KL} < 1$, as seems most consistent with the evidence. Suppose further that educational attainment grows over time, again in line with observation. Then the existence of a BGP with constant factor shares requires $\partial(F_s/F_L)/\partial K > 0$; i.e., an increase in the capital stock must raise the marginal product of schooling by proportionally more than it does the marginal product of raw labor. In looser parlance, the technology must be characterized by capital-skill complementarity.¹²

¹¹ Our Proposition 1 is predicated on constant and interior factor shares. But, in the Uzawa case, log differentiation of the production function with respect to time, holding s constant, implies

$$g_Y = \theta_K(g_A + g_K) + (1 - \theta_K)(g_B + g_L),$$

where $\theta_K = KF_K/Y$ is the capital share in national income. In a steady state in which Y and K grow at constant rates in response to constant rates of growth of A, B, L , and q , θ_K must be constant as well. Note that Jones and Scrimgeour do not assume constant factor shares in their statement and proof of the Uzawa Growth Theorem.

¹² Some might ask why we interpret s as *schooling*, rather than some other variable that evolves over time and affects factor productivity. First, we need s to be endogenous, otherwise it could be subsumed into the technology. Second, we want s to be something that econometricians have used as a control variable when estimating the elasticity of substitution, σ_{KL} , inasmuch as we rely on those estimates when assuming $\sigma_{KL} < 1$. Most recent estimates of the elasticity of substitution use quality-adjusted measures of labor and wages that control for schooling (e.g.,

The results in this section use only resource constraints and the assumption that factors are paid their marginal products. We have, as yet, provided no model of savings or of schooling decisions. Moreover, we have shown that a BGP with constant factor shares *might exist*, but not that one *does exist* under some reasonable set of assumptions about individual behavior and a reasonable specification of the aggregate production function. In the next section, we study a simple economy in which the level of education can be summarized by a scalar variable that can jump discretely from one moment to the next. In Grossman et al. (2016), we consider a more realistic setting in which individuals' education accumulates slowly over time and the distribution of schooling levels in the economy evolves gradually.

II. Balanced Growth with Short Lifespans

We begin by posing a social planner's problem that incorporates a reduced-form treatment of schooling choice. In Section IIA, the planner designs a time path for a scalar variable that summarizes the education level in the workforce. The planner faces a trade-off between the level of schooling and the labor available for producing output. The economy experiences both labor-augmenting and capital-augmenting technological progress, and the elasticity of substitution between capital and labor in aggregate production is less than 1. Here we show that the planner's allocation converges to a unique BGP for a specified class of production functions and under certain parameter restrictions. Moreover, if the efficient allocation can be characterized by balanced growth after some moment in time, then the technology *must* have a representation with a production function in the specified class. We derive the steady-state growth rate of output for the planner's solution and the associated (and constant) factor shares.

In the succeeding subsections, we develop a pair of models of individual behavior and aggregate production that generate education functions that exhibit the form posited in Section IIA. At the end of the section, we discuss briefly the results in Grossman et al. (2016) that can be derived from a more realistic model of schooling choice with overlapping generations.¹³

A. A Planner's Problem with a Reduced-Form Education Function

The economy comprises a continuum of identical family dynasties of measure 1. Each family has a continuum N_t of members alive at time t , where N_t grows at the exogenous rate n . Dynastic utility at some time t_0 is given by

$$(2) \quad u(t_0) = \int_{t_0}^{\infty} N_t e^{-\rho(t-t_0)} \frac{c_t^{1-\eta} - 1}{1-\eta} dt,$$

where c_t is consumption per family member at time t and ρ is the subjective discount rate.

Antràs 2004; Klump, McAdam, and Willman 2007; Oberfield and Raval 2014) or focus on cross-sectional variation across industries so that schooling choices do not vary (Chirinko, Fazzari, and Meyer 2011).

¹³In our working paper, we also describe how the model can be extended to include directed technical change, in the manner suggested by Acemoglu (2003). We show that the equilibrium of such a model generally exhibits both capital-augmenting and labor-augmenting technical change.

Consider the problem facing a social planner who seeks to maximize utility for the representative dynasty subject to a resource constraint, an evolving technology, and an ongoing trade-off between some measure of the economy's education level and the contemporaneous labor supply. Write this trade-off in reduced form as $L_t = D(s_t)N_t$, with $D'(s_t) < 0$ for all s_t , where L_t measures the raw labor that produces output at time t and s_t is a scalar index that summarizes the distribution of schooling levels among those workers. The production function takes the form $Y_t = F(A_t K_t, B_t L_t, s_t)$, where A_t again converts physical capital to *effective capital* in view of the disembodied technology available at time t , and similarly B_t converts raw labor to effective labor. We assume that $F(\cdot)$ has constant returns to scale in its first two arguments, i.e., that doubling the physical inputs doubles output for any education level and any state of technology. The economy can convert one unit of the final good into q_t units of capital at time t . Capital depreciates at the constant rate δ and labor-augmenting technological progress takes place at the constant rate $\gamma_L \equiv \dot{B}_t/B_t$.

We assume that the technology can be represented by a member of a class of aggregate production functions that take the following form.

ASSUMPTION 1: *The production function can be written as $F(AK, BL, s) = \tilde{F}[D(s)^a AK, D(s)^{-b} BL]$, with $a, b > 0$, where*

- (i) $h(z) \equiv \tilde{F}(z, 1)$ is strictly increasing, twice differentiable, and strictly concave for all z ; and
- (ii) $\sigma_{KL} \equiv F_L F_K / F F_{LK} < 1$.

Assumption 1 immediately implies that $\partial(F_s/F_L)/\partial K > 0$.¹⁴ Therefore, the technology satisfies the prerequisites for the existence of a BGP, per Proposition 1, provided that the planner's optimal choice of schooling is rising over time.

We also impose some parameter restrictions. Let $\mathcal{E}_h(z) \equiv zh'(z)/h(z)$ be the elasticity of the $h(\cdot)$ function. Note that $\mathcal{E}_h(z)$ is strictly decreasing under Assumption 1.¹⁵

ASSUMPTION 2: (i) $\lim_{z \rightarrow 0} \mathcal{E}_h(z) < \frac{b}{a+b}$; (ii) $\lim_{z \rightarrow \infty} \mathcal{E}_h(z) < \frac{b-1}{a+b-1} < \lim_{z \rightarrow 0} \mathcal{E}_h(z)$; (iii) $\rho > n + (1 - \eta) \left[\gamma_L + \frac{b-1}{a} \gamma_K \right]$.

Part (i) of Assumption 2 ensures that the marginal product of schooling is nonnegative for all levels of K, L , and s .¹⁶ Part (ii) guarantees that the optimal

¹⁴See the proof in the online Appendix. We also prove that, under Assumption 1, $\sigma_{KL} < 1$ if and only if $F(AK, BL, s)$ is strictly log supermodular in K and s , which is another way of expressing capital-skill complementarity.

¹⁵To see this, note that $d \ln \mathcal{E}_h(z) / d \ln z = [1 - \mathcal{E}_h(z)] (\sigma_{KL} - 1) / \sigma_{KL}$, which is negative when $\sigma_{KL} < 1$.

¹⁶Assumption 1 implies $F_s(AK, BL, s) = [D'(s)/D(s)] [aKF_K(AK, BL, s) - bLF_L(AK, BL, s)]$. The assumption that $F(AK, BL, s)$ is constant returns to scale implies $F(AK, BL, s) = KF_K(AK, BL, s) + LF_L(AK, BL, s)$. Combining these two equations, we see that $F_s > 0$ for all AK, BL , and s if and only if $\lim_{z \rightarrow 0} \mathcal{E}_h(z) < b/(a+b)$.

schooling choice is positive, as we will see below. It also implies, with Assumption 1 and Assumption 2(i), that $b > 1$.¹⁷ Part (iii) ensures that utility in (2) is finite.

The planner’s problem has two separable components, one static and one dynamic. The static problem is to choose the education level and the labor force at every moment in time so as to maximize output Y_t , subject to the inverse relationship between the two. The dynamic problem is to allocate consumption over time so as to maximize dynastic utility in (2), subject to the aggregate capital accumulation equation, $\dot{K}_t = q_t(Y_t - N_t c_t) - \delta K_t$. The solution to the dynamic problem is standard and features the familiar Euler equation. We provide the details in the online Appendix. Here we focus on the static problem, which captures how the planner’s choice of education, s_t , relates to the state of technology, as summarized by $\{A_t, B_t, q_t\}$, and the momentary capital stock, K_t .

In the light of Assumption 1, the planner’s static problem boils down to choosing s_t and L_t at every moment in time to maximize $Y_t = \tilde{F}[D(s_t)^a A_t K_t, D(s_t)^{-b} B_t L_t]$, subject to the resource constraint, $L_t = D(s_t) N_t$. Once we substitute the constraint into the maximand, we have

$$\begin{aligned} Y_t &= \max_{s_t} \tilde{F}[D(s_t)^a A_t K_t, D(s_t)^{1-b} B_t N_t] \\ &= \max_{s_t} D(s_t)^{1-b} B_t N_t \tilde{F}\left[\frac{D(s_t)^{a+b-1} A_t K_t}{B_t N_t}, 1\right]. \end{aligned}$$

Now, make a change of variables, using $z_t \equiv D(s_t)^{a+b-1} A_t K_t / B_t N_t$, and recall the definition of $h(z) \equiv \tilde{F}(z, 1)$. Then the static problem can be rewritten as

$$(3) \quad Y_t = \max_{z_t} (B_t N_t)^{1-\theta} (A_t K_t)^\theta z_t^{-\theta} h(z_t),$$

where $\theta \equiv (b - 1)/(a + b - 1)$. The first-order condition for this problem implies

$$(4) \quad \mathcal{E}_h(z_t) = \theta \quad \text{for all } t \geq t_0.$$

In other words, the planner chooses education so that $z_t \equiv D(s_t)^{a+b-1} A_t K_t / B_t N_t$ remains constant over time; $z_t = z^* = \mathcal{E}_h^{-1}(\theta)$. In this sense, the planner offsets capital deepening with increased schooling. Part (ii) of Assumption 2 ensures that there exists a strictly positive solution for z^* and the fact that $\mathcal{E}_h(z)$ is strictly decreasing implies that the solution is unique.¹⁸ Once z_t is chosen optimally with $z_t = z^*$, (3) implies that output is a Cobb-Douglas function of effective capital and technology-augmented population, with exponents θ and $1 - \theta$, respectively.

We will not rehearse the details of the transition path; these are familiar from neo-classical growth theory. In the online Appendix, we show that the planner chooses the initial per capita consumption level, c_{t_0} , so as to put the economy on the unique

¹⁷ Assumption 1(i) implies $\lim_{z \rightarrow \infty} \mathcal{E}_h(z) \geq 0$. So, Assumption 2(ii) requires $(b - 1)/(a + b - 1) > 0$. Thus, if $a + b > 1$, then $b > 1$. Suppose $a + b < 1$ and $b < 1$. Then Assumption 2(i) and Assumption 2(ii) imply $b(a + b - 1) < (a + b)(b - 1)$ or $b < (b - 1)$, which cannot hold. Thus, we must have $b > 1$.

¹⁸ In the online Appendix, we show that the second-order condition is satisfied at $z_t = z^*$ under Assumption 1. Moreover, we show that the second-order condition would be violated if the elasticity of substitution between capital and labor were to exceed 1.

saddle path that converges to a steady state. On the BGP, consumption and output grow at constant rate g_Y and the capital stock grows at constant rate g_K .

We can readily calculate the growth rates of output and consumption along the BGP. From $z_t \equiv D(s_t)^{a+b-1} A_t K_t / B_t N_t$ and the fact that $z_t = z^*$ along an optimal trajectory, we have

$$(a + b - 1)g_D + g_A + g_K = \gamma_L + n$$

for all $t \geq t_0$. By setting $z_t = z^*$ in (3) and then log differentiating with respect to time, we also find that

$$(a + b - 1)g_Y = a(\gamma_L + n) + (b - 1)(g_A + g_K)$$

along the optimal path. Finally, combining these two equations and using Lemma 1—which requires that $g_Y = g_K - g_q$ along any BGP—we can solve for g_D and g_Y . Proposition 2 reports the results.

PROPOSITION 2: *Suppose there is a trade-off between labor supply and a summary index of economy-wide education given by $L_t = D(s_t) N_t$. Let Assumptions 1 and 2 hold. Then along the optimal trajectory from any initial capital stock, K_{t_0} , the economy converges to a BGP. On the BGP,*

- (i) *aggregate output and aggregate consumption grow at the common rate $g_Y = n + \gamma_L + \frac{b-1}{a}\gamma_K$;*
- (ii) *the index of education grows according to $\dot{s} = -\frac{\gamma_K D(s)}{aD'(s)}$, so that $g_D = -\frac{\gamma_K}{a}$.*

The growth of per capita income is increasing in the rate of labor-augmenting technological progress, just as in the neoclassical growth model without endogenous schooling. But now a BGP exists even when there is ongoing capital-augmenting technological progress or when the price of investment goods is falling at a constant rate. The fact that $b > 1$ implies that the growth rate of per capita income also is increasing in γ_K , the combined rate of embodied and disembodied capital-augmenting progress.

We have not as yet introduced any market decentralization, which we will do only for the specific models described in Sections IIB and IIC below. However, in anticipation that capital will be paid its marginal product in a competitive equilibrium, we can define the capital share in national income at time t as $\theta_{Kt} = (\partial Y_t / \partial K_t) K_t / Y_t$. Using (3) with $z_t = z^*$, we see that $\theta_{Kt} = (b - 1) / (a + b - 1) \equiv \theta$ for all $t \geq t_0$. The labor share, which includes the return to education, equals $1 - \theta$. That is, the planner chooses the trajectories for the capital stock and schooling such that the factor shares remain constant, both along the transition path and in the steady state. Notice that the growth rate and the capital share both are increasing in b and decreasing in a ; in this sense, fast growth and a high capital share go hand in hand.

We offer some remarks about the role of Assumption 1 and the intuition for our BGP. With $Y_t = \tilde{F}[A_t K_t D(s_t)^a, B_t L_t D(s_t)^{-b}]$, the effect of schooling on the relationship between inputs and output is akin to that of factor-biased technical progress.

Hicks (1932) described the bias in technical progress according to its impact on relative factor demands at given relative factor prices. Technical progress is labor saving (or, equivalently, capital using) if it causes an increased relative demand for capital at the initial wage-to-rental ratio. In our setting, and under Assumption 1, added schooling does exactly that; it tilts the unit isoquants in (K, L) space in such a way that the cost-minimizing technique shifts toward capital.¹⁹ We can say, therefore, that the productivity gains associated with schooling are capital using.

Capital-augmenting technological progress expands the relative supply of effective capital. In our model, it also induces investment in education. This increases the relative demand for capital. With our functional form assumption, the extra demand just absorbs the excess supply. To see that this is so, notice that $D(s_t)^a A_t q_t$ is constant along the BGP. In short, the optimal schooling choice generates extra demand for equipment that neutralizes the effect of the capital-augmenting progress and the declining investment-good prices on the growth of the effective capital stock.²⁰

Effectively, there is a horse race between the effects of capital deepening and of education on the factor shares which, with the multiplicative way that $D(s)$ interacts with the two inputs and the constant elasticities on this variable, ends in a dead heat. As capital accumulates and becomes more productive due to technical progress, the less-than-unitary elasticity of substitution between capital and labor exerts downward pressure on the capital share. Meanwhile, complementarity between effective capital and schooling means that capital accumulation raises the return to education. The planner responds by investing more in schooling, which depresses the education-plus-technology augmented capital stock relative to the education-plus-technology augmented labor force. This exerts upward pressure on the capital share. With the functional form specified in Assumption 1, the two forces just balance.

Needless to say, Assumption 1 describes a broad class of technologies. For concreteness, we offer one example. Consider²¹

$$(5) \quad Y_t = (B_t L_t)^{\frac{a}{a+b}} \left\{ (A_t K_t)^\alpha + \left[D(s_t)^{-(a+b)} B_t L_t \right]^\alpha \right\}^{\frac{b/(a+b)}{\alpha}}.$$

¹⁹Following Takayama (1974), define $\varpi(k, s, t)$ as the ratio of the marginal product of labor to the marginal product of capital, where $k \equiv AK/BL$. Under Assumption 1,

$$\varpi(k, s, t) = \frac{Bk}{A} \left[\frac{1}{\mathcal{E}_h(D(s)^{a+b} k)} - 1 \right].$$

Since $D(s)$ is strictly decreasing in s and $\mathcal{E}_h(z)$ is strictly decreasing in z , it follows that $\varpi_s < 0$. This means that schooling is Hicks labor-saving in Takayama's terminology.

²⁰Violante (2008) defines skill-biased technical change as a technology change that, ceteris paribus, raises the marginal product of skilled labor relative to that of unskilled labor in the formation of an aggregate labor input. By analogy, we might also say that education under our Assumption 1 is capital biased; growth in s raises F_K/F_L at a given input ratio.

²¹Our example makes use of the fact (shown in the online Appendix), that, whenever the marginal product of schooling is positive, Assumption 1 is formally equivalent to assuming that $F(AK, BL, s)$ can be written as

$$F(AK, BL, s) = (BL)^{\frac{a}{a+b}} G(AK, D(s)^{-(a+b)} BL)^{\frac{b}{a+b}}$$

where $G(\cdot)$ is constant returns to scale, strictly increasing in both its arguments, $G(z, 1)$ is twice differentiable and strictly concave for all z , and $\sigma_{KL}^G \equiv \frac{G_K G_L}{G G_{KL}} < 1$. Written in this form, the basis for the complementarity between capital and schooling is clear. The example in (5) is the special case of this formulation in which $G(\cdot)$ has a constant elasticity of substitution between its two arguments.

Then output at time t can be expressed as a function of $D(s_t)^a A_t K_t$ and $D(s_t)^{-b} B_t L_t$. With $a > 0$, $b > 1$, and $\alpha < 0$, Assumptions 1 and 2 are both satisfied. Here, the negative value of α generates the required complementarity between capital and schooling.²²

One might wonder whether we are able to dispense with the functional-form restriction of Assumption 1. The answer to this question is no. In the online Appendix, we prove that if $L_t = D(s_t) N_t$ and if the solution to the social planner's problem exhibits balanced growth after some time T with increasing schooling and a constant capital share $\theta_K \in (0, 1)$, then either there is no capital-augmenting technological progress ($\gamma_K = 0$) or else the technology can be represented along the equilibrium trajectory by a production function with the form $\tilde{F}[A_t K D(s)^a, B_t L D(s)^{-b}]$, with $a > 0$ and $b = 1 + a\theta_K/(1 - \theta_K) > 1$. In other words, Assumption 1 is not only sufficient for the existence of a BGP with $\gamma_K > 0$ and $\sigma_{KL} < 1$, but it is essentially necessary as well. As with any model that generates balanced growth, knife-edge restrictions are required to maintain the balance; our model is no exception to this rule.

To demonstrate the flexibility of our approach, we next present two examples of market economies that generate the reduced form described above. The discussion of the two models in the main text is brief; details are in the online Appendix.

B. *Balanced Growth in a "Time-in-School" Model*

As above, the representative family has a continuum N_t of members at time t . Each life is fleetingly brief; an individual attends school for the first fraction of her momentary existence and then joins the workforce for the remainder of her life. The variable s_t now represents the fraction of life that the representative member of the generation alive at time t devotes to education; she spends the remaining fraction $1 - s_t$ working. In this case, $D(s) = 1 - s$, so that the family's labor supply is $L_t = N_t(1 - s_t)$. Given the brevity of life, there is no discounting of an individual's wages relative to her time in school. But dynasties do discount the earnings (and well being) of subsequent generations relative to those currently alive. Every new cohort starts from scratch with no schooling.

Each individual chooses her consumption, savings, and schooling to maximize total dynastic utility, which at time t_0 is given by (2). Each individual supposes that other family members in her own and subsequent generations will behave similarly. Savings are used to purchase units of physical capital, which are passed on within the family from one generation to the next. The N_t members of the representative dynasty collectively inherit K_t units of capital at time t , considering that the aggregate capital stock is fully owned by the population and there is a unit continuum of dynasties in the economy.

Firms produce output using capital, labor, and the technology available to them at the time. Each firm rents capital on a competitive market and allocates it to its employees, taking into consideration their levels of education. A firm's output is the

²²It is possible to interpret (5) in terms of a two-task production process. Suppose each worker contributes a joint input of educated and raw labor (*brains* and *brawn*). The firm combines the educated labor (defined as $D(s_t)^{-(a+b)} L_t$) with effective capital to complete one task. In so doing, the two have a constant elasticity of substitution of $1/(1 - \alpha) < 1$. Meanwhile, the input of raw labor addresses the second task. Finally, the two tasks enter the overall production function in Cobb-Douglas form.

sum of what is produced by its various workers. As usual, the profit-maximizing choices for the firm equate the marginal product of each unit of capital to the competitive rental rate and the marginal product of each type of worker to her competitive wage. The equilibrium determines a wage schedule, $W_t(s)$, which gives the wage of a worker with schooling s at time t . Even for those schooling options that are not actually chosen in equilibrium at time t , we can calculate a worker's marginal product and thus what the wage would be, based on the prevailing technology and the capital that a firm would allocate to such a worker at the prevailing rental rate.

Schooling choices have no persistence for the family. Therefore, an individual alive at time t who seeks to maximize dynastic utility should choose s to maximize her own wage income, $(1 - s)W_t(s)$. The first-order condition for this problem requires $(1 - s_t)W_t(s_t) = W_t(s_t)$. We show in the online Appendix that this first-order condition for privately optimal schooling choice implies that in the competitive equilibrium

$$\mathcal{E}_h\left((1 - s_t)^{a+b-1}\frac{A_t K_t}{B_t N_t}\right) = \frac{b - 1}{a + b - 1}.$$

Evidently, the individual's income-maximizing choice of schooling matches the planner's path for s_t in (4), once we recognize that $D(s) = 1 - s$. Part (ii) of Proposition 2 then implies

$$\dot{s}_t = (1 - s_t)\frac{\gamma_K}{a}.$$

On a BGP, schooling rises over time, but at a declining rate; the complementary time spent working, $D(s) = 1 - s$, falls at a constant exponential rate, $\dot{D}(s)/D(s) = -\gamma_K/a$.

It comes as no surprise that the market equilibrium with perfect competition and complete markets mimics the planner's solution. The point we wish to emphasize is that the time-in-school model converges to a BGP and that the wage schedule $W_t(s)$ gives the family members the appropriate incentives to extend their time in school from one generation to the next. The returns to schooling rise with the accumulation of effective capital, thanks to the assumed capital-schooling complementarity, and the extra schooling is exactly what is needed to maintain balanced growth of the two inputs to production.

C. Balanced Growth in a "Manager-Worker" Model

Now, we present an entirely different model that yields a similar reduced form. We imagine teams that combine *managers* and *production workers*. Firms allocate capital equipment to teams according to their productivity. Only production workers are directly responsible for operating equipment and thus for generating output. But the productivity of a team depends on the ratio of its managers to workers, as in the hierarchical models of management proposed by Beckmann (1977), Rosen (1982), and others.

The family structure, demographics, and preferences are the same as before. Lifespans are short. Each individual decides whether to devote a fixed fraction m of

her potential working life to school. If she opts to do so, she will acquire the skills needed to serve as a manager and she will have $1 - m$ units of time remaining to perform this function. Those who do not go for management training are employed as production workers. They will use all of their available time to earn unskilled wages.

Let L_t be the time units supplied by production workers at time t and let M_t be the time units supplied by managers. Since production workers devote all of their time to their jobs, L_t is also the number of production workers. Managers are in school a fraction m of their time, so the number of managers is $M_t/(1 - m)$. The population divides between workers and managers, so

$$(6) \quad L_t + \frac{M_t}{1 - m} = N_t.$$

This time, we take $s_t = M_t/L_t$ to be our index of schooling. This is the ratio of manager hours to worker hours (or of skilled to unskilled labor) and the inverse of the typical manager's *span of control*. With this definition, (6) implies $L_t + L_t s_t/(1 - m) = N_t$, so that $D(s) = [1 + s/(1 - m)]^{-1}$ in this model.

Monitoring makes the workers and their equipment more productive. In particular, we suppose that the production function at time t can be written as $\tilde{F}[D(s)^a A_t K, D(s)^{-b} B_t L]$, with $\tilde{F}(\cdot)$ homogeneous of degree 1 in its two arguments. With $s = M/L$, this implies that output is a constant returns to scale function of the three inputs, $A_t K$, $B_t L$, and $B_t M$.

In this model, the education decision for the representative individual born at time t is simple: pursue schooling if lifetime earnings of a manager exceed those of a worker and not otherwise. In an equilibrium with $M_t > 0$, every individual must be indifferent between the two occupations, so that $(1 - m) W_{Mt} = W_{Lt}$, where W_{Mt} and W_{Lt} are the wages per unit time of managers and workers, respectively. Over time, accumulation of effective capital exerts upward pressure on the skill premium, because the functional form of Assumption 1 ensures that capital is more complementary with managers than it is with production workers. This provides the incentive for a greater fraction of each new generation to gain skills. The expanding relative supply of managers to workers restores the equality in earnings.

In the online Appendix, we show that equalization of lifetime earnings of workers and managers implies

$$\mathcal{E}_h \left(\left[1 + \frac{s_t}{1 - m} \right]^{-(a+b)} \frac{A_t K_t}{B_t L_t} \right) = \frac{b - 1}{a + b - 1}.$$

This gives the same education index as in the planner's solution (4). It follows that the economy converges to a BGP, with a constant rate of output growth given by part (i) of Proposition 2, and with a constant capital share and an ever increasing ratio of manager hours to worker hours.

D. *Balanced Growth with Overlapping Generations*

The models described in Sections IIB and IIC are rather stylized, because they assume that an economy's education can be described by a scalar variable that can

jump from one moment to the next. In reality, schooling investments take time and an economy's distribution of education levels adjusts slowly. In our working paper, Grossman et al. (2016), we develop an overlapping-generations (OLG) model that has these features. Here, we describe briefly the additional insights and predictions that emerge from that analysis.

In the OLG model, individuals experience finite but stochastic lifespans. Births and deaths occur with constant hazard rates. An individual devotes the first part of her existence to school. She chooses the target length of time to remain in school before entering the labor force. If the individual survives to adulthood, she spends the second phase of life working, with a productivity that depends on her educational attainment, her experience, and on technology at the time. Firms allocate capital to their workers as a function of these characteristics, and a firm's total output is the sum of what is produced by its various workers. Productivity rises with experience early in a worker's career, but falls with experience subsequently. If a worker survives until her productivity falls to zero, she retires.

Analogous to Assumption 1, we assume in the OLG model that if L workers with s years of schooling and u years of experience are allocated K units of capital, they can produce $\tilde{F}(e^{-as}A_tK, e^{bs}B_tL, u)$ units of output at time t . In the equilibrium, every birth cohort chooses a different educational target. The labor force comprises workers with different schooling levels and different years of experience who work with different amounts of capital. Despite this richness, the economy-wide distributions of schooling and experience evolve in a relatively simple way that permits aggregation.

Like the short-lifespan models of Sections IIB and IIC, the OLG economy has a unique BGP. Along the BGP, educational attainment increases *linearly* over time, much like the patterns depicted in Figure 2 for long stretches of US history. The wage structure at every moment takes a Mincerian form (see Mincer 1974), with log wages that vary in the cross section with schooling and experience. Finally, the model predicts declining labor-force participation, consistent with the postwar evidence for men in the United States.

One important difference between the OLG model and the short-lifespan models is worth emphasizing. In the OLG model of Grossman et al. (2016), factor shares are neither constant along the transition path nor independent of the rates of technological progress in the long run. Our numerical analysis suggests that a permanent slowdown in the rate of capital-augmenting technological progress will induce an increase in the capital share. In fact, with plausible parameter values, a one percentage point decline in the annual rate of investment-specific technical change—such as has been measured by the International Monetary Fund (2014, p.89) for the period after 2000—might account for much or all of the rise in the capital share in US national income that has been witnessed in those years.

III. Relationship to the Literature on Investment-Specific Technical Change

By way of concluding remarks, it might be useful for us to relate our results to the large literature that has studied the long-run implications of investment-specific technological change. In his seminal paper on embodied technical progress, Solow (1960) did not close his model to solve for a steady state, but he indicated how

this could be done. However, Solow employed a Cobb-Douglas production function throughout this paper, and his discussion about closing the model relies on this assumption. Sheshinski (1967) demonstrated convergence to a BGP in an extended version of the Johansen (1959) model with both embodied and disembodied technological progress. Although he does not restrict attention to any particular production function, he does insist that both forms of progress are Harrod-neutral, i.e., they augment the productivity of labor. So, the technology gains in Sheshinski's paper, while embodied in vintages of capital, are nonetheless assumed to be labor-augmenting. These findings are echoed in Greenwood, Hercowitz, and Krusell (1997), who resurrected the literature on technological improvements that are embodied in new equipment. They studied an economy that has no opportunities for schooling in which two types of capital (*equipment* and *structures*) and labor are combined to produce consumption goods. Unlike Sheshinski, they do not assume that embodied progress is Harrod-neutral and, consequently, they are led to conclude that a Cobb-Douglas production function is necessary to generate balanced growth, in keeping with the dictates of the Uzawa Growth Theorem.

Krusell et al. (2000) posit a technology with capital-skill complementarity according to which output is produced with equipment, structures, and two types of labor (skilled and unskilled). Leaving aside their distinction between equipment and structures, their model is one with capital and two types of labor, much like our manager-worker model in Section IIC above. Although their production function incorporates capital-skill complementarity, it does not satisfy the dictates of our Assumption 1. Nor do they endogenously determine the supplies of skilled and unskilled workers. They, and much of the substantial literature that has adopted their production function, do not address the prospects for balanced growth with ongoing declines in investment-good prices and endogenous schooling, but instead focus on the transition dynamics that result from a specified sequence of relative price changes and of factor supplies. Two recent papers do try to generate balanced growth in models of investment-specific technological progress that is not Harrod-neutral. He and Liu (2008) introduce endogenous schooling into the Krusell et al. (2000) model, so that the relative supplies of skilled and unskilled labor are determined in the general equilibrium. They define a BGP to be an equilibrium trajectory along which equipment, structures, and output all grow at constant rates and the fraction of skilled workers converges to a constant. With this definition, they conclude (see their Proposition 1) that balanced growth is consistent with ongoing investment-specific technological change only when the aggregate production function takes a Cobb-Douglas form. Maliar and Maliar (2011) study a similar environment, but assume instead that the stocks of skilled and unskilled labor grow at constant and exogenous rates. They show that balanced growth requires $g_A < 0$ to offset the investment-specific technology gains, such that (in our notation) $\gamma_K = 0$. In contrast to these papers, we have shown that balanced growth is in fact compatible with a falling relative price of capital, nonnegative growth in capital productivity, and $\sigma_{KL} \neq 1$, provided that capital and schooling are sufficiently complementary. Our result requires that the aggregate production function falls into the class defined by Assumption 1 and that an appropriate index of the economy's educational outcome is rising over time.

The basic mechanism in our model is straightforward: over time, growing stocks of effective capital raise the returns to schooling, which induces individuals to spend more time in school. Inasmuch as capital and labor are complements, capital accumulation tends to lower capital's share in national income, but this is offset by the subsequent rise in schooling, because capital and schooling are also complements. When capital and schooling are more complementary than capital and labor, the second effect can neutralize the first. Although the presence of these offsetting forces is natural enough, restrictions on how schooling enters the production function are needed to maintain exact balance along an equilibrium trajectory. The restrictions are in a sense analogous to those usually imposed on preferences in a dynamic model in order to generate balanced growth. Specifically, while it may be natural to assume that income and substitution effects offset one another as wages rise, the intratemporal utility function must be specified in a particular way so as to maintain perfect balance along an equilibrium trajectory. Just as balanced-growth preferences are consistent with a range of intertemporal elasticities of substitution and labor-supply elasticities, so too are the restrictions we impose on the production function consistent with a range of elasticities of substitution between capital and labor and between capital and schooling.

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