SALES AND COLLUSION IN A MARKET WITH STORAGE

Francesco Nava
London School of Economics

Pasquale Schiraldi
London School of Economics

Abstract
Sales are a widespread and well-known phenomenon documented in several product markets. This paper presents a novel rationale for sales that does not rely on consumer heterogeneity, or on any form of randomness to explain such periodic price fluctuations. The analysis is carried out in the context of a simple repeated price competition model, and establishes that firms must periodically reduce prices in order to sustain collusion when goods are storable and the market is large. The largest equilibrium profits are characterized at any market size. A trade-off between the size of the industry and its profits arises. Sales foster collusion, by magnifying the inter-temporal links in consumers' decisions. (JEL: L11, L12, L13, L41)

1. Introduction

The occurrence of periodic price reductions, or sales, is a pervasive and well-known microeconomic phenomenon that has been documented in several product markets. Typically, a high price is charged in most periods, but occasionally the price is cut to supply more units to a potentially larger group of consumers. Product markets in which this phenomenon is prominent are, for instance, brand name paper towels, soft drinks, or canned goods. The regular occurrence of such circumstances suggests that sales cannot be entirely explained by random variations in supply, demand, or the aggregate price level. Moreover, certain sale periods are traditional and so well publicized that it is difficult to justify them as devices to separate informed from uninformed consumers. A growing empirical literature also, documents that the majority of periodic sales take place for products that are fairly storable, and that heterogeneity in storage capacity explains part of the responsiveness of consumers to changes in prices (Bell and Hilber 2006; Hendel and Nevo 2006, 2011; Erdem, Keane, and Imai 2003; Seiler 2013). Such evidence highlights the primary role that storage capacity may play in determining consumers' purchasing behavior and thus, retailers' pricing decisions.

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E-mail: f.nava@lse.ac.uk (Nava); p.schiraldi@lse.ac.uk (Schiraldi)

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Our analysis shows why sales can foster collusion in markets in which goods are storable, and provides a novel motive for firms to engage in sales. While factors such as informational differences and heterogeneity in willingness to search or pay have received notable attention, the theoretical literature on storage constraints remains scarce despite empirical relevance documented by a growing literature. Notable exceptions are Salop and Stiglitz 1982; Hong, McAfee, and Nayyar 2002; Hendel, Lizzeri, and Roketskiy 2012.

We consider an industry in which in every period, n firms produce a homogeneous storable good, and sell it to a mass of consumers with unit demand in every period. We restrict attention to economies in which at least a fraction of consumers (but possibly all consumers) have access to a fixed storage capacity S. In this context, we examine the effects of storage on firms' incentives to hold periodic sales to support a greater degree of cooperation in a repeated price competition setting. Sale strategies are characterized by a regular price, by a price mark-down (on the regular price), and by the frequency of sales. We show that periodic sales strategies (sustained by grim trigger punishments) allow firms to collude on significant profit levels, even when the number of competitors is so large as to prevent collusion on any strategy without sales. Sales can strengthen collusion, as storage intertemporally links consumer demand and thus reduces the short-run gains from a deviation. In any equilibrium with sales, firms will charge in any period of sales a big enough discount to induce all consumers with open storage capacity to stockpile a quantity sufficient to satisfy all their demand until the next period of sales. Such behavior however, can reduce incentives to deviate—both in regular price periods, as only consumers without units already stored would purchase units from a deviant firm, and in periods with sales both because a lower price is charged in such periods, and because consumers with storage would reduce their demand if a deviation were observed in the wake of the imminent price war (implied by the grim trigger punishments).

The analysis begins by identifying consumer demand for the proposed environment, and by characterizing the set of sales strategies which can be used to collude when market size is too large to sustain collusion on any strategy without sales. Then for environments in which all consumers have access to storage, results establish that sales strategies are profit maximizing for the industry, and characterize the most profitable equilibrium strategy for any given number of competitors. A trade-off emerges between industry profits and industry size (which we often refer to as stability). When the number of competitors is small, the monopoly profit can be sustained in classical trigger strategies. However, as the market grows large, no strategy without sales remains incentive compatible, and the largest equilibrium profit is pinned down by a strategy with sales. The trade-off between cartel profits and cartel size is explicitly characterized. Less frequent sales reduce aggregate profits (since larger discounts must be offered to induce more storage), but increase the incentives to comply with equilibrium pricing (since deviations attract a smaller fraction of the equilibrium demand). Most conclusions extend to environments with heterogeneous consumers. The analysis characterizes the largest number of competitors that can collude on positive profits, and shows that sales are necessary to collude when the number of competitors is large. Access to multi-unit storage technologies is shown to mitigate, but not eliminate the profit–size trade-off. For economies with unit storage, the trade-off between cartel profits and cartel size can again be explicitly characterized. Comparative statics establish how the trade-off is affected by changes in the environment.

Although the results are presented in the context of a model with fully rational consumers and possibly heterogeneous storage technologies, alternative interpretations are possible. In particular, the results developed here would also apply to a model in which some consumers never expect prices to rise sharply (and thus purchase a single unit in every period), while the remaining consumers understand price dynamics in the market and purchase the optimal number of units given the expected future price path. In this reading of the model, myopic buyers may be seen as consumers whose opportunity cost of investing time in understanding future prices is high. This interpretation would be better suited to match evidence suggesting that high-income households are less responsive to sales (Griffith et al. 2009; Gauri, Sudhir, and Talukdar 2008). The optimal behavior of rational buyers in our model implies that consumers would curtail their demand if an unexpected price cut were to take place in a period of sales, as prices would remain low in the future (due to the retaliatory nature of the trigger punishments). Results however, are robust to numerous alternative specifications which relax the rationality of consumer behavior. The last part of the analysis shows that the profit–size trade-off persists even in economies in which consumers expect pricing to revert to the equilibrium path after any number of deviations. In these environments sales can still benefit collusion, as deviations from strategies without sales might induce consumers to purchase multiple units if prices are expected to remain high in the future.

Within the proposed framework collusion will always be strengthened at the expense of aggregate profits, since profits always decline with sales when consumers are homogeneous in their willingness to pay. If however, consumers with higher storage capacity had a lower willingness to pay, a sale strategy might achieve higher profits than any no-sale strategy (by price discriminating different types of consumers), and thus foster collusion even further. We elected to keep valuations homogeneous across consumers in order to display more transparently the effects of the intertemporal link in consumer demand.

Literature Review. One of the first theoretical explanations for sales relates consumer search behavior to price discrimination. Two prominent examples in this literature are Varian (1980) and Salop and Stiglitz (1982). Varian argues that, with heterogeneously informed consumers, retail price variations can arise as a natural outcome of mixed-strategy equilibrium in which firms price discriminate consumers with different information. Salop and Stiglitz instead consider a model with search costs in which consumers are imperfectly informed about the prices charged by stores and differ in their ability to stockpile. In such framework the authors show that stores have incentives to hold unannounced sales to induce consumers to purchase future consumption. Both models, however, are essentially static models and cannot account for correlation in prices. Even though the random sales feature remains a compelling explanation for some erratic price behavior, it appears less suited to account for many of the documented retail markdowns that are predictable, publicly known, and take place in most stores simultaneously (Pesendofer 2002; Warner and Barsky 1995).

The appealing fashion/clearance paradigm for sales (Lazear 1986; Pashigian 1988; Pashigian and Bowen 1991) can also hardly explain a wide variety of retail items for which the fashion hypothesis appears a priori less appropriate (either because the items are homogeneous, or because styles change little over time).

A final relevant literature has motivated sales as a form of intertemporal price discrimination (Conlisk, Gerstner, and Sobel 1984; Hendel and Nevo 2011; Hong, McAfee, and Nayyar 2002; Narasimhan and Jeuland 1985; Sobel 1984). Conlisk, Gerstner, and Sobel (1984), and in particular Sobel (1984) study the incentives to hold cyclical simultaneous sales as a means of price discrimination in a durable-good environment. In most periods, prices are kept high to extract surplus from high-value consumers, but periodically prices are decreased in order to sell to a larger group of consumers with lower reservation values. A key assumption to generate such price cycles is the constant inflow of new heterogeneous consumers in the market. Hendel and Nevo (2011), Hong, McAfee, and Nayyar (2002), and Jeuland and Narasimhan (1985) study the incentives to hold periodic sales in a market with storable goods and heterogeneous consumers. In this setup, the incentives to price-discriminate consumers over time with sales are fully explained by the positive correlation between storage costs and consumers' willingness to pay. We complement these papers by offering a novel rationale for the existence of sale in a dynamic storable goods model in which the incentives to hold periodic sales arise even in the absence of any form of consumer heterogeneity. Moreover, as in Sobel (1984), we characterize the optimal timing for sales.

Other related studies on dynamic pricing in storable goods markets differ significantly from ours in their aims (Anton and Das Varma 2005; Ariga, Matsui, and Watanabe 2010; Su 2010). For instance, Anton and Das Varma study the quantity competition in a two-period model with storable goods. The authors show that when consumers are sufficiently patient (and thus storage costs are low), firms have a strong short-term incentive to capture future market shares from their rivals. As a result, equilibrium prices increase in the second period ($p_1 = \delta p_2$) and there is rational in-advance purchase by buyers with perfect foresight. The two-period model raises the important issue of long-run market dynamics: rising prices cannot continue indefinitely. Thus, equilibrium price and associated storage (inventory) cycles become an important possibility which we analyze in the current framework. Both frameworks entail dynamic inefficiencies, as production costs are incurred prior to consumption for every unit stored. Our study is also related to the work of Hendel, Lizzeri, and Roketskiy (2012), which analyzes the nonlinear pricing problem of a monopolist facing a large number of consumers with access to storage. Within this context, the authors show that consumers may store units to limit the monopolist's surplus extraction ability, and that periodic sales may raise the monopoly profits by limiting the intertemporal arbitrage opportunities of consumers.

The spirit of the paper is similar to other studies that have analyzed the relationship between intertemporal linking in decisions and collusive behavior (Ausubel and

Deneckere 1987; Dana and Fong 2011; Gul 1987; Schiraldi and Nava 2012). These studies differ significantly in their goals and setups. However, all of them exploit some intertemporal link in decisions to enhance the incentives to collude. Ausubel and Deneckere (1987) and Gul (1987) develop oligopoly models of durable goods pricing, and show that the Coase conjecture (Coase 1972) fails whenever multiple firms operate in the market, since firms' ability to collude improves. Within this durable good framework, Schiraldi and Nava compare the ability to collude with and without secondary markets, and shows how a second-hand market may further expand the ability to collude. Dana and Fong argue that intertemporal bundling along with staggered long-term contracts may facilitate collusion. One of the key novelties in the analysis developed here is that the intertemporal link in decisions emerges endogenously as a consequence the pricing strategies of firms. In particular, sales induce consumers to store thereby creating demand cycles, and consequently the link in consumer demand which may be exploited to enhance the incentives to collude. Rotemberg and Saloner (1986) study a similar phenomenon, but in a model with exogenous demand cycles and without storage. Our analysis shows that when storage is possible such demand cycle might arise endogenously as a result of strategic pricing along the equilibrium path.

Roadmap. Section 2 introduces the model, defines the relevant class of sale strategies, and presents several preliminary results comparing different sale strategies in terms of industry profits and industry size. Section 3 restricts attention to economies in which all consumers are homogeneous. The main results of the section establish when sales strategies are profit maximizing for the industry, and characterize the profit–size trade-off that different strategies entail. Section 4 extends the baseline model and results by allowing consumer heterogeneity. The maximal industry profits are determined for any market size. The relationship between frequency and depth of sales, and market size is discussed. Comparative static results show how the trade-off is affected by changes in patience, in profitability, and in the fraction of consumers with access to storage. Section 5 departs from the previous analysis by assuming that buyers' beliefs about future prices are not affected by deviations. The analysis shows that the profit–size trade-off is resilient to such a change in beliefs. Section 6 presents an alternative interpretation of the model, discusses the robustness of the results developed, and ties behavior in the model to empirical evidence on promotions. All proofs are relegated to the Appendix. An Online Appendix contains some derivations, and a few additional results omitted from the main text for the sake of clarity.

2. A Model with Storable Goods

This section introduces a simple economy with storage, defines the class of sale strategies that will be analyzed throughout the paper, and develops several preliminary results.

A Simple Economy with Storage

Consider an infinite-horizon discrete-time model with infinitely lived producers and consumers. Suppose that two goods are traded in the model which we shall refer to as consumption q and money m. In each period, all consumers are endowed with a large amount of money and with no consumption. The preferences of a consumer purchasing q units of consumption in exchange of m units of money are determined by the map

$$u(q,m) = \begin{cases} v - m & \text{if } q \geq 1, \\ -m & \text{if } q < 1. \end{cases}$$

Hence, the marginal value of consumption is v for the first unit consumed and 0 for any additional unit consumed. All consumers discount the future at a common factor δ, and the present-discounted value of a sequence of utilities $\{u_t\}_{t=0}^{\infty}$ is thus proportional to

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t u_t.$$

There is a unit measure of consumers. Consumers differ only in their ability to store the consumption good. In particular, assume that a fraction α_0 of the consumers is unable to store goods, while a fraction α_S can store up to S additional units of consumption. Such units do not depreciate when stored and can be consumed in any future period. The analysis also applies to economies in which all consumers are homogeneous, as the fraction of consumers with storage is allowed to take any value in the interval [0, 1].

A finite set of firms N with cardinality n supplies a consumption good to this market. All firms have a common constant marginal cost of producing the consumption good, c. For any vector of prices $\mathrm{p} = (p_1, \ldots, p_n)$ set by the n firms on the units of consumption sold, let $d(\mathrm{p})$ denote the aggregate demand at such prices, and let $n^*(\mathrm{p})$ denote the number of players posting the minimal price,

$$n^*(\mathrm{p}) = |\arg\min_{j\in N} p_j|.$$

As customary, the aggregate demand is split equally between firms quoting the lowest price. Thus, the individual demand faced by firm i satisfies

$$d_i(\mathrm{p}) = \begin{cases} d(\mathrm{p})/n^*(\mathrm{p}) & \text{if } p_i = \min_{j\in N} p_j, \\ 0 & \text{if } p_i > \min_{j\in N} p_j. \end{cases}$$

The stage game profit of firm $i \in N$ given any price vector p satisfies

$$\pi_i(\mathrm{p}) = (p_i - c)d_i(\mathrm{p}).$$

All firms discount the future at a common factor δ. The present-discounted value of a sequence of profits $\{\pi_{it}\}_{t=0}^{\infty}$ is for any firm i proportional to

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t \pi_{it}.$$

Sale Strategies and Equilibrium

Firms and consumers observe all the prices quoted in the market in any previous period. Denote by H the set of possible price histories in the game.[1] A firm's strategy maps histories into a price quoted at a given date. Consumers use information about past quoted prices and about the equilibrium strategies to form beliefs about future prices in the economy. Since consumers are small, we assume that their decisions cannot be observed by any other individual. Consumers will thus choose how many units to purchase from the firms quoting the lowest price in order to maximize their individual payoff. For any sequence of future prices $\mathrm{p}^t = \{\mathrm{p}_z\}_{z=t}^{\infty}$, let $p_z^* = \min_{i \in N} \{p_{iz}\}$ denote the market price in period z, and let $T(\mathrm{p}^t)$ denote the number of periods that an individual has to wait before discounted future market price falls below the current price. That is,

$$T(\mathrm{p}^t) = \min z \text{ subject to } p_t^* > \delta^z p_{t+z}^*.$$

If there were no storage constraints, $T(\mathrm{p}^t)$ would implicitly determine how many units would be purchased by an individual with no units stored. This would be the case, since consumers with access to storage would purchase multiple units only when they perceive the storage cost $(1-\delta)c$ to be smaller than the cost of future price increases.[2] In general, however, individual demand would never exceed $S+1$ units, as only S units can be stored. The next remark formalizes these observations, and derives the demand of every consumer when faced with a future price stream p^t. Denote the individual demand of a consumer respectively with and without access to storage by d_S and d_0.

REMARK 1. *If $p_z^* \leq v$ in any period z, the individual demand for consumption good at time t:*

1. *by consumers without storage technology satisfies $d_0(\mathrm{p}^t) = 1$;*
2. *by consumers with storage technology and with s units already in storage satisfies*

$$d_S(s, \mathrm{p}^t) = \max\{\min\{T(\mathrm{p}^t), S+1\} - s, 0\}.$$

The remark immediately implies that all consumers of the same type purchase the same number of units in every period. Hence, aggregate demand in a period in which all consumers with access to storage have the same number of units s satisfies

$$d(s, \mathrm{p}^t) = \alpha_0 + \alpha_S d_S(s, \mathrm{p}^t).$$

The equilibrium strategies analyzed throughout the paper discipline deviations as trigger strategies would. However, equilibrium prices vary along the equilibrium path. In particular, strategies prescribe that firms set a regular markup μ in periods without

1. Formally, the set of possible histories in the induced game satisfies $H = \{\emptyset\} \cup \{\cup_{t=1}^{\infty} [\times_{s=1}^{t} \mathbb{R}_+^n]\}$.

2. The storage cost coincides with $(1-\delta)c$, as the rate of time preferences represents the opportunity cost of spending money sooner to stock units.

sales, and periodically reduce the markup to $\mu\sigma$, for $\sigma \in [0, 1]$, every $\varkappa$ periods. Formally, we consider strategies in which all firms set prices along the equilibrium path so that for some $\varkappa \in \{2, 3, \ldots\}$

$$\bar{p}_t = \begin{array}{ll} (1+\mu)c & \text{if } \operatorname{mod}(t, \varkappa) \neq 0, \\ (1+\mu\sigma)c & \text{if } \operatorname{mod}(t, \varkappa) = 0, \end{array}$$

where $\operatorname{mod}(t, \varkappa) \neq 0$ denotes the $\varkappa$ modulo of the time period t. Such strategies may be interpreted as a cyclical sales policy in which all firms jointly reduce prices every $\varkappa$ periods.[3] Deviations from the equilibrium path are punished via reversion to competitive pricing in each future time period. Thus, for any history $h^t \in H$ of length t, a sale strategy γ prescribes to set prices so that

$$\gamma(h^t) = \begin{array}{ll} \bar{p}_t & \text{if } p_{iz} = \bar{p}_z \text{ for any i and any } z \leq t, \\ c & \text{otherwise.} \end{array}$$

The equilibrium punishment strategy is Nash in any subgame in which a deviation has already occurred, since no firm can benefit from a unilateral deviation when all the other firms are pricing competitively. Therefore, the incentives to comply with a sale strategy are pinned down entirely by looking only at deviations from the equilibrium path. Let $\Pi_t(\gamma)$ denote the present-discounted value of aggregate profits on the equilibrium path at time t. Similarly, let $\Delta_t(\gamma)$ denote the present discounted-value of the most profitable deviation from the equilibrium path. Finally, let the ratio of equilibrium to deviation profits, Π_t/Δ_t, be denoted by R_t. The next result characterizes the upper-bound on market size for which a sale strategy constitutes a subgame perfect equilibrium (SPE).

REMARK 2. *A sale strategy γ is a SPE of the infinite repetition of the game if and only if*

$$n \leq R_t(\gamma) \text{ for any } t \geq 0. \tag{1}$$

The result holds as the fraction of aggregate profits that a firm earns on the equilibrium path, Π_t/n, never exceeds the gains from a unilateral deviation, Δ_t, when condition 1 holds. Condition 1 imposes an upper-bound on the largest number of firms that can collude on a strategy γ in a SPE. Such a bound is defined by the map $n(\gamma) = \min_{t \geq 0} R_t(\gamma)$, and as expected, for any strategy without sales (and thus without storage) condition 1 simplifies to the common requirement $n \leq (1/1-\delta)$.

Throughout, we refer to the strategies defined in this section as sale strategies. Any one of these strategies will be completely pinned down by the three parameters: the regular markup μ, the sales discount $1-\sigma$, and the sale frequency $\varkappa$. Throughout, the analysis restricts attention to sale strategies for which the regular markup in periods without sales is bounded above by the monopoly markup $\bar{\mu} = (v - c/c)$. This implies that consumers without access to storage are always willing to purchase units. Such a restriction is imposed only because any strategy with a higher markup would reduce profits to no avail. The set of possible sale strategies is denoted by $\mathcal{S} = (0, \bar{\mu}] \times [0, 1] \times \{2, 3, \ldots\}$.

3. The Online Appendix shows how results can be generalized to allow for asynchronized sales.

Preliminary Results on Stability and Profits

This section develops several preliminary results that compare different sales strategies in $\mathcal{S}$ in terms of profits and stability. Throughout the analysis, the stability of a strategy will be evaluated by the largest number of competitors that can collude on a given strategy γ in a SPE, $n(\gamma) = \min_{t\geq 0} R_t(\gamma)$. The analysis shows that all strategies without sales are equally stable, and that strategies with sales can be used to sustain larger cartels at the expense of profits. Necessary and sufficient conditions for the existence of strategies with sales that are more stable than any strategy without sales are presented. These results are essential for the characterization of the trade-off between cartel profits and cartel size that will be developed in the following sections.

The following two definitions clarify the intent of our analysis. For a fixed discount factor δ, the stability of a strategy γ will be determined by the largest number of firms $n(\gamma)$ that can collude on γ. The profitability of a strategy will instead be determined by the present discounted value of profits at time zero.

DEFINITION 1. *A sale strategy γ is said to be more stable than strategy γ' if $n(\gamma) \geq n(\gamma')$.*

DEFINITION 2. *A sale strategy γ is said to be more profitable than strategy γ' if $\Pi_0(\gamma) \geq \Pi_0(\gamma')$.*

An alternative, but similar, definition of stability may involve the floor of the map $n(\gamma)$. But similar conclusions would hold.[4]

In order to compare the stability of any two sale strategies, it is convenient to express equilibrium and deviation profits in terms of the parameters of the strategy. Since no consumer has any units stored at the beginning of the game ($s_0 = 0$), it is possible to recursively define the equilibrium demand d_t and storage s_t in each period $t \geq 0$ as follows:

$$d_t = d(s_t, \mathrm{p}^t),\ d_{St} = d_S(s_t, \mathrm{p}^t),\ \text{and}\ s_{t+1} = s_t + d_{St} - 1.$$

The next remark shows that, since any sale strategy is cyclical, it is without loss of generality to consider only the first $\varkappa$ periods to characterize the entire stream of payoffs. In particular, the result shows that no consumer has units stored in periods of sales, and that the evolution of aggregate demand is cyclical.

REMARK 3. *The following claims must hold:*

1. *no consumer has stored units in periods of sales: if* $\mathrm{mod}(t, \varkappa) = 0$*,* $s_t = 0$*;*
2. *demand is constant at congruent dates of the cycle: if* $\mathrm{mod}(t, \varkappa) = \mathrm{mod}(z, \varkappa)$*,* $d_t = d_z$*.*

4. Alternatively, the analysis could have focused on the lowest possible discount factor $\delta(\gamma)$ needed to collude on strategy γ in a population of fixed size n. But doing so would have complicated the analysis without affecting our conclusions.

The claim follows from the properties of the equilibrium pricing path p^* and of the map $d_S(s_t, \mathrm{p}^t)$, and implies that consumers have the same demand at congruent dates in the cycle. Let $S(t)$ denote the number of periods after date t that have to elapse before the next sale takes place,

$$S(t) = \begin{cases} 0 & \text{if } \mathrm{mod}(t, \varkappa) = 0, \\ \varkappa - \mathrm{mod}(t, \varkappa) & \text{if } \mathrm{mod}(t, \varkappa) \neq 0. \end{cases}$$

For any sale strategy γ and for any period $t \in \{1, \ldots, \varkappa - 1\}$, equilibrium aggregate profits satisfy

$$\begin{aligned} \Pi_t(\gamma) &= \frac{1-\delta}{1-\delta^{\varkappa}} \sum\nolimits_{z=0}^{\varkappa-1} \delta^z \pi_{t+z}(\mathrm{p}^{t+z}) \\ &= \frac{1-\delta}{1-\delta^{\varkappa}} \left[\left[\sum\nolimits_{z=0}^{\varkappa-1} \delta^z d_{t+z} \right] - (1-\sigma)\delta^{S(t)} d_0 \right] \mu c, \end{aligned}$$

where the first term of the final expression computes profits as if no sales ever took place, while the second term adjusts profits for the markdown offered in periods of sales. Since a unilateral deviation to price $y \neq \bar{p}_t$ at stage t implies reversion to competitive pricing in any future period, for $y < \bar{p}_t$ the deviation payoffs at each stage are determined by the surplus that a firm can extract during the deviation period, and satisfy

$$\Delta_t(y, \gamma) = (1-\delta)(y-c) d(s_t, \mathrm{y}^t).$$

For convenience, define three relevant classes of sale strategies for which preliminary results are developed. The first class $\mathcal{N}$ consists of all those sale strategies for which no discount is ever offered along the equilibrium path. The second class $\mathcal{C}$ comprises all those strategies in which consumers with access to storage purchase only in periods with sales. The third class $\mathcal{E} \subset \mathcal{C}$ will be the main focus of our analysis, as we will establish that only strategies belonging to this subset of $\mathcal{C}$ can foster collusion when compared to strategies that do not employ sales. Formally, define the three sets of strategies as follows.

DEFINITION 3. *Let $\mathcal{N} \subset \mathcal{S}$ denote those strategies such that $\sigma = 1$.*

Let $\mathcal{C} \subset \mathcal{S}$ denote those strategies for which $\varkappa \leq S + 1$ and $(1 + \mu\sigma) \leq \delta^{\varkappa-1}(1+\mu)$.

Let $\mathcal{E} \subset \mathcal{C}$ denote those strategies for which

$$\frac{\alpha_0}{\varkappa \alpha_S + \alpha_0} \leq \sigma.$$

The two conditions in the definition of $\mathcal{C}$ imply respectively that the sales' frequency cannot exceed the storage capacity, and that any consumer with access to storage purchases units only in periods of sales. The latter restriction is often referred to as the *storage constraint*, and is often expressed as

$$\sigma \le \delta^{\varkappa-1}\left(1+\frac{1}{\mu}\right)-\frac{1}{\mu}.$$

The remainder of this section shows that the additional constraint in the definition of the set $\mathcal{E}$ requires revenues in periods of sales to exceed revenues in periods without sales. This observation will then be exploited to establish that $\mathcal{E}$ consists of all those strategies with equilibrium sales which are more stable than any strategy without sales.

The next proposition establishes several introductory results on the stability and on the profitability of different sale strategies. In particular, it shows that any two sales strategies in which no sales take place are equally stable, and that any sale strategy in which consumers with storage purchase units during no-sales periods is dominated both in terms of profits and in terms of stability by a strategy in which sales never take place. The latter observation considerably simplifies the analysis, and will be exploited to characterize the set of sale strategies that sustain collusion in industries in which the number of competitors is too large to collude on the revenue-maximizing no-sale strategy, $\mu = \bar{\mu}$ and $\sigma = 1$.[5]

PROPOSITION 1. *The following claims must hold:*

1. *if* $\sigma = 1$, *all consumers purchase units every period, thus* $\mathcal{N} \subseteq \mathcal{S}\backslash\mathcal{C}$;
2. *any strategy in* $\mathcal{N}$ *is more stable than any strategy in* $\mathcal{S}\backslash\mathcal{C}$;
3. *any strategy that sets* $\mu = \bar{\mu}$ *and* $\sigma = 1$ *is profit maximizing within* $\mathcal{S}$;
4. *for any strategy in* $\mathcal{C}$, $s_t = S(t)$ *and*

$$d_t = \begin{cases} \alpha_0 + \varkappa\alpha_S & \textit{if}\ \mathrm{mod}(t,\varkappa) = 0, \\ \alpha_0 & \textit{if}\ \mathrm{mod}(t,\varkappa) \neq 0. \end{cases}$$

The first two parts of the proposition establish that no-sales strategies are more stable than any other sale strategy in which consumers with access to storage purchase units in a period without sales (i.e. strategies in $\mathcal{S}\backslash\mathcal{C}$), and imply that all no-sale strategies (i.e. strategies in $\mathcal{N}$) are equally stable. These results obtain, since a deviation is more profitable in periods of high demand when no markdown takes place, as a firm deviating by an infinitesimal discount would be able to supply the entire market at the regular price. This should also clarify why sales were assumed to take place in the initial period, since any sale strategy violating such a requirement would necessarily belong to $\mathcal{S}\backslash\mathcal{C}$. The third part of the proposition is trivial, and establishes that no-sales monopoly pricing is revenue maximizing among all the sales strategies. The last part of the proposition can be used to simplify equilibrium and deviation payoffs for any strategy in $\mathcal{C}$, as it implies that consumers with access to storage purchase exactly $\varkappa$

5. Recall that $\bar{\mu} = (v - c/c)$ denotes the monopoly markup in our framework.

units in every period of sales, and none otherwise. In particular, for any strategy $\gamma \in \mathcal{C}$ equilibrium payoffs satisfy

$$\Pi_t(\gamma) = \left[\alpha_0 + \frac{1-\delta}{1-\delta^{\varkappa}}\delta^{S(t)}\left[\sigma(\varkappa\alpha_S + \alpha_0) - \alpha_0\right]\right]\mu c,$$

where the first term consists of the revenue made by selling only to consumers without storage at the regular markup, and the second term accounts for the revenues made in periods with sales adjusted by the discount. The revenue-maximizing deviation payoffs for any strategy $\gamma \in \mathcal{C}$ instead satisfy

$$\Delta_t(\gamma) = \max_y \Delta_t(y, \gamma) = \begin{cases} (1-\delta)\sigma\mu c & \text{if } \mathrm{mod}(t, \varkappa) = 0, \\ (1-\delta)\alpha_0\mu c & \text{if } \mathrm{mod}(t, \varkappa) \neq 0. \end{cases}$$

For any sale strategy in $\mathcal{C}$ deviation payoffs decline (when compared to a similar strategy, but with no markdown, $1 - \sigma = 0$) both in periods with no sales (as fewer consumers purchase units upon observing a deviation) and in periods with sales (as no consumer purchases more than a single unit at a discount in the wake of an imminent price war). The corresponding profit ratios for any strategy $\gamma \in \mathcal{C}$ thus satisfy

$$R_t(\gamma) = \begin{cases} \dfrac{1}{1-\delta}\dfrac{\alpha_0}{\sigma} + \dfrac{1}{1-\delta^{x}}\left[(\varkappa\alpha_S + \alpha_0) - \dfrac{\alpha_0}{\sigma}\right] & \text{if } \mathrm{mod}(t, \varkappa) = 0, \\ \dfrac{1}{1-\delta} + \dfrac{1}{1-\delta^{\varkappa}}\delta^{S(t)}\left[(\varkappa\alpha_S + \alpha_0)\dfrac{\sigma}{\alpha_0} - 1\right] & \text{if } \mathrm{mod}(t, \varkappa) \neq 0. \end{cases}$$

Sale strategies in $\mathcal{C}$ will improve stability whenever deviation profits decline more than equilibrium profits. For such strategies, the critical ratios $R_t(\gamma)$ are unaffected by the regular markup μ, but decrease with σ in periods with sales, and increase with σ otherwise.

The previous observations identify $\mathcal{E}$ as the set of sale strategies that belong to $\mathcal{C}$ for which revenues in periods of sales exceed revenues in periods without sales. This requirement imposes a lower bound on the sales discount that guarantees that deviation profits decline more than equilibrium profits, when compared to strategies that do not display equilibrium path sales. By exploiting this observation, the next proposition establishes that sales strategies in $\mathcal{E}$ are more stable than any strategy outside $\mathcal{E}$, and consequently more stable than strategies without sales. Thus, a strategy in $\mathcal{E}$, although less profitable than the revenue-maximizing no-sale strategy, may be desirable as more competitors may collude on it. Moreover, since all such strategies belong to $\mathcal{C}$, sales take place along the equilibrium path.

PROPOSITION 2. *Any strategy in $\mathcal{E}$ is more stable than any strategy in $\mathcal{S}\backslash\mathcal{E}$. Moreover, $\mathcal{E}$ contains a strategy with a cycle of length $\varkappa \in \{2, \ldots, S+1\}$ if and only if*

$$\delta^{\varkappa-1} \geq \frac{v-c}{v}\frac{\alpha_0}{\varkappa - \alpha_0(\varkappa-1)} + \frac{c}{v}. \tag{2}$$

The proposition is proven by establishing that whenever a strategy belongs to $\mathcal{C}$ it is without loss to ignore all but the first two periods, in order to characterize its stability.[6] Such a conclusion, coupled with the observation that the maximal cartel size $n(\gamma)$ is independent of μ and single peaked in σ for any given $\varkappa$, implies that all strategies in $\mathcal{E}$ must be more stable than any without sales. Whenever the set $\mathcal{E}$ has a non-empty interior sale strategies exist that are strictly more stable than any strategy without sales. If so, sales are necessary to sustain collusion along the equilibrium path when the market is large. Consequently, a strategy $\gamma \in \mathcal{E}$ will be strictly more stable than a strategy without sales if and only if

$$\sigma \in \left(\frac{\alpha_0}{\alpha_S \varkappa + \alpha_0}, \frac{\mu + 1}{\mu}\delta^{\varkappa - 1} - \frac{1}{\mu}\right].$$

This requirement is exploited in the second part of the argument to derive necessary and sufficient conditions for the set $\mathcal{E}$ to be non-empty. Such conditions jointly discipline all the free parameters of the model—namely, the fraction of consumers with storage α_S, the profitability of the market $v - c$, and the discount factor δ. It is easy to verify that, at a given a cycle of length $\varkappa$, more sale strategies will belong to $\mathcal{E}$ whenever either α_S, or $v - c$, or δ increase.[7] Furthermore, as the bound on the discount factor arises from the storage constraint, condition 2 would only discipline the time-preferences of the consumers if those were to differ from the time-preferences of the firms. However, even when the two coincide, the restriction imposed on δ remains independent of the number of firms in the market. A consequence of the result is that $\mathcal{E}$ is non-empty whenever

$$\delta \geq \frac{v - c}{v}\frac{\alpha_0}{2 - \alpha_0} + \frac{c}{v}.$$

Before proceeding to the next section, for notational convenience, define $\sigma(\varkappa)$ as the unique positive root of the following quadratic equation:

$$R_1(\sigma(\varkappa), \varkappa) = R_0(\sigma(\varkappa), \varkappa),$$

if such a solution exists in [0, 1], and set $\sigma(\varkappa) = 1$ otherwise. The analysis will show that such a markdown can maximize stability at a given sale frequency $\varkappa$, when consumers with access to storage purchase units only during periods of sales. The details of the derivation of $\sigma(\varkappa)$ and the proof of uniqueness are deferred to the Online Appendix. Further, define $\kappa(\varkappa)$ as the smallest sales discount for which consumers with access to storage would purchase $\varkappa$ units in periods of sales when the regular markup is set at the monopoly level, $\bar{\mu} = (v - c)/c$,

$$\kappa(\varkappa) = \frac{v}{v - c}\delta^{\varkappa - 1} - \frac{c}{v - c}.$$

6. This is the case, since in any period without sales, deviation profits coincide, whereas equilibrium profits increase as the sales loom closer, by definition of $\mathcal{E}$.

7. A proof of this observation can be found in the Online Appendix.

3. Homogeneous Consumers

To highlight the nature of the profit–size trade-off, we begin by restricting attention to simple economies in which all consumers are homogeneous and have access to storage, $\alpha_S = 1$. Two main results are presented within this stylized framework. The first establishes that sales strategies in $\mathcal{S}$ are always profit maximizing for the industry when storage capacity is unbounded. The second result instead compares sale strategies in $\mathcal{S}$ in terms of cartel profits and cartel size (stability), and shows that a trade-off emerges between the two. The existence of such a trade-off in environments in which consumers are homogeneous provides a novel rationale for sales which, in contrast to the vast majority of the literature, does not rely on consumer heterogeneity. These results are central to the analysis, and establish why sales may foster collusion in large markets.

The analysis carried out in the previous section only compared sale strategies within $\mathcal{S}$. However, within this simplified setup the restriction to periodic sales strategies is without loss. In fact, the next proposition shows that, when $S = \infty$, for any aggregate profit level that obtains in a SPE of the game, there exists a (weakly) higher profit level can be supported in an equilibrium in which strategies belong to $\mathcal{S}$. The result is proven only for trigger strategies in which defections are punished via reversion to competitive pricing. This restriction, however, has no bite here, as a straightforward extension of arguments developed by Abreu (1988) could be used show to any SPE payoff can be sustained in trigger strategies.

PROPOSITION 3. *If $\alpha_S = 1$ and $S = \infty$, for any SPE trigger strategy γ with profits $\Pi(\gamma)$ there exists a SPE sale strategy $\hat{\gamma} \in \mathcal{S}$ such that $\Pi(\hat{\gamma}) \geq \Pi(\gamma)$.*

The argument is proven by considering the interval between two periods of positive demand in which the strategy γ yields the highest per-period profits; and by showing that a sale strategy $\hat{\gamma} \in \mathcal{S}$ supports even higher per-period profits in equilibrium throughout the game. The proof is simplified by the two assumptions invoked, because an infinite storage capacity implies that consumers only purchase units when they have none stored, and because per-period profits simplify when consumers are homogeneous.[8]

The previous observation and Proposition 2 imply that it is without loss to restrict attention to strategies in $\mathcal{E}$ in order to characterize the profit-maximizing sale strategy for an industry with any number of competitors. For any such strategy, firms cannot gain from a deviation when storage is positive for all consumers, as no units would be purchased upon observing a deviation. The incentives to comply with a sale strategy are therefore pinned down by the deviation gains during periods of sales, which always attract unit demand, as consumers expect lower prices in the future due to the retaliatory

8. The result and its proof can also be extended to the general framework $\alpha_0 > 0$ and $S < \infty$. However, several technical complications arise.

nature of the trigger punishments. When $\alpha_S = 1$, aggregate profits and stability of any sale strategy $\gamma \in \mathcal{E}$ respectively simplify to

$$\Pi(\gamma) = \frac{(1-\delta)\varkappa}{1-\delta^{\varkappa}}\sigma\mu c \quad \text{and} \quad n(\gamma) = \frac{\varkappa}{1-\delta^{\varkappa}}.$$

The stability of a given sale strategy is independent of the markup charged during periods of sales. However, the maximal number of firms that can collude on a given sale strategy increases with $\varkappa$, which implies that strategies with infrequent sales may foster collusion. Aggregate profits instead increase in the sale markup $\sigma\mu$.

The next proposition highlights the specific nature of the trade-off between cartel profits and cartel size that different sale strategies entail. In particular, the result explicitly characterizes for any storage capacity S the largest number of firms $N(\Pi)$ that could enter the market at a given profit level Π without it unraveling to a competitive equilibrium. The maximal cartel size always decreases as equilibrium profits increase. In this context, the proposition establishes that the strategy $(\bar{\mu}, \kappa(\varkappa), \varkappa)$ is profit maximizing among all sale strategies with frequency $\varkappa$, provided that

$$\kappa(\varkappa) = \frac{\delta^{\varkappa-1}v - c}{v-c} \geq 0.$$

It then proceeds to show that the industry-maximizing aggregate profits $\Pi_0(\bar{\mu}, \kappa(\varkappa), \varkappa)$ decline with $\varkappa$, as less frequent sales must be met by larger discounts to attract the desired demand. A trade-off between cartel profits and cartel size emerges, because infrequent sales lower profits, but increase the number of competitors that can collude on a given strategy. The result also establishes that consumer heterogeneity is not essential in explaining periodic sales. For convenience, let $\hat{\varkappa}$ denote the largest value of $\varkappa$such that $\kappa(\varkappa) \geq 0$ if such value does not exceed $S+1$, and let $\hat{\varkappa} = S+1$ otherwise. Also, denote equilibrium profits when the markdown is set to the storage constraint by

$$\Pi^{\varkappa} = \Pi_0(\bar{\mu}, \kappa(\varkappa), \varkappa) = \frac{(1-\delta)\varkappa}{1-\delta^{\varkappa}}(\delta^{\varkappa-1}v - c).$$

Observe that $\Pi^1 = v - c$ coincides with the monopoly profit.

PROPOSITION 4. *If $\alpha_S = 1$, for any profit level $\Pi \in (0, \Pi^1]$ the maximal number of firms that can collude on Π while employing a sale strategy satisfies*

$$N(\Pi) = \begin{cases} \dfrac{\varkappa}{1-\delta^{\varkappa}} & \text{if } \Pi \in (\Pi^{\varkappa+1}, \Pi^{\varkappa}] \text{ for } \varkappa \in \{1, \ldots, \hat{\varkappa}-1\}, \\ \dfrac{\hat{\varkappa}}{1-\delta^{\hat{\varkappa}}} & \text{if } \Pi \in (0, \Pi^{\hat{\varkappa}}]. \end{cases}$$

Moreover, strategy $(\bar{\mu}, \kappa(\varkappa), \varkappa) \in \mathcal{S}$ is profit maximizing among all sale strategies with frequency $\varkappa$.

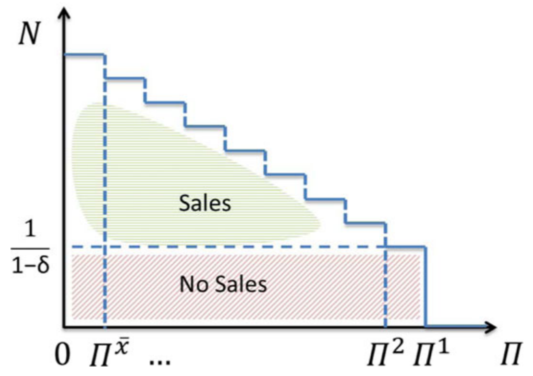

FIGURE 1. The profit–stability trade-off $N(\Pi)$ characterized in Proposition 4.

When all consumers are homogeneous, the profit–size trade-off is pinned down by a decreasing step function in which maximal cartel size decreases as profits increase (see figure 1). Price fluctuations along the equilibrium path thus reduce the incentives to deviate. These results obtain, as the drop in demand in response to deviations is most pronounced when consumers anticipate purchasing large number of units. Thus, periodic sales may be exploited to collude on positive profits even when constant pricing is not incentive compatible. Because all consumers are homogeneous, units are sold only during period of sales. The next sections depart from some of the stark assumptions invoked here, and show how results extend to environments in which players differ either in their ability to store units, or in their ability to predict future prices. In these more realistic setups, units will be sold in every period along the equilibrium path.

4. Heterogeneous Consumers

This section considers environments in which some consumers do not have access to storage $\alpha_0 > 0$. Results again compare different sale strategies in $\mathcal{S}$ both in terms of cartel profits and in terms of cartel size. The first two propositions characterize two relevant strategies in $\mathcal{E}$. The former will be the most stable sale strategy, while the latter will be the most profitable of all the sale strategies in $\mathcal{E}$. Whenever the two strategies do not coincide, a trade-off between cartel profits and stability will emerge even within $\mathcal{E}$. Such results establish why sales may be necessary to collude in large markets even when only a fraction of consumers responds to price fluctuations. The second part of the section instead presents an explicit characterization of the profit–size trade-off for economies in which $S = 1$. The restriction to economies with unit-storage is imposed only for sake of tractability, as numerous qualitative features of the trade-off are not affected by this assumption.

General Storage

We begin with the characterization of the sale strategy that maximizes the number of competitors that can collude in equilibrium. To do so, we argue that, whenever $\sigma(\varkappa)$ satisfies the storage constraint, a discount of $1-\sigma(\varkappa)$ maximizes stability at given sale frequency $\varkappa$. For convenience, identify two particular sale frequencies, $\bar{\varkappa}$ and $\breve{\varkappa}$. In particular, let $\bar{\varkappa}$ denote the frequency of sales that maximizes time-zero stability among all frequencies that satisfy the storage constraint, $\kappa(\varkappa) \geq \sigma(\varkappa)$; and let $\breve{\varkappa}$ denote the smallest frequency for which the storage constraint is violated at discount $1-\sigma(\varkappa)$. Formally, let

$$\begin{aligned} \bar{\varkappa} &= \arg\max_{\varkappa \in \{2,\dots,S+1\}} R_0(\sigma(\varkappa), \varkappa) \qquad && \text{s.t. } \kappa(\varkappa) \geq \sigma(\varkappa), \\ \breve{\varkappa} &= \arg\min_{\varkappa \in \{2,\dots,S+1\}} \varkappa && \text{s.t. } \kappa(\varkappa) < \sigma(\varkappa). \end{aligned}$$

Now consider three alternative scenarios: (i) $\kappa(\varkappa) > \sigma(\varkappa)$ for any frequency $\varkappa \in \{2, \dots, S+1\}$; (ii) $\kappa(\varkappa) < \sigma(\varkappa)$ for any $\varkappa \in \{2, \dots, S+1\}$; and (iii) otherwise. The first scenario describes environments in which the stability-maximizing markdown $1-\sigma(\varkappa)$ meets the storage constraint at any sale frequency $\varkappa$; the second scenario describes environments in which $1-\sigma(\varkappa)$ always fails to meet the storage constraint; while the third scenario covers the remaining cases. The next result characterizes the sale strategy that maximizes equilibrium cartel size in each of these three cases. Such a strategy requires firms to price at the monopoly markup in periods without sales, and uniquely pins down the optimal discount for the remaining periods. In the first scenario, the discount coincides with the stability-maximizing markdown $1-\sigma(\bar{\varkappa})$, and sales occur every $\bar{\varkappa}$ periods in order to maximize stability in periods of sales. In the second scenario instead, the discount is determined by the storage constraint (it amounts to $1-\kappa(\breve{\varkappa})$), and sales occur every $\breve{\varkappa}$ periods so to maximize stability in periods without sales. In the third and intermediate scenario, the strategy coincides with one of the two previous cases.

PROPOSITION 5. *Assume that $\mathcal{E} \neq \emptyset$ and $\alpha_0 > 0$. If* (i) *holds, or if* (iii) *holds and $R_0(\sigma(\bar{\varkappa}), \bar{\varkappa}) > R_1(\kappa(\breve{\varkappa}), \breve{\varkappa})$, then no strategy in $\mathcal{S}$ is strictly more stable than strategy $\gamma^* \in \mathcal{E}$:*

μ^*	σ^*	$\varkappa^*$
$\bar{\mu}$	$\sigma(\bar{\varkappa})$	$\bar{\varkappa}$

,

otherwise, no strategy in $\mathcal{S}$ is strictly more stable than strategy $\gamma^* \in \mathcal{E}$:

μ^*	σ^*	$\varkappa^*$
$\bar{\mu}$	$\kappa(\breve{\varkappa})$	$\breve{\varkappa}$

.

Moreover, γ^* is the most profitable of all the strategies in $\mathcal{S}$ with equal stability.

The most stable strategy often displays infrequent sales, $\varkappa^* > 2$. Infrequent sales can benefit stability, as the incentives to deviate can decline when larger discounts are offered to induce players to store multiple units. The stability-maximizing discount and frequency of sales crucially depend on the fraction of consumers with storage, on the discount rate, and on the monopoly markup in the economy.[9] Strategy γ^* is, however, independent of the number of firms in the market. Thus, the largest number of firms willing to collude on a strategy in $\mathcal{S}$ can be found by looking at $n(\gamma^*)$. The monopoly markup is charged in periods without sales since μ has no effect on the stability (cartel size). The optimal sales discount $1 - \sigma^*$ is instead chosen to maximize $n(\gamma)$ within the set of markdowns for which the strategy belongs to $\mathcal{E}$. When $S = 1$, the most stable sale strategy simplifies to $\gamma^* = (\bar{\mu}, \min\{\sigma(2), \kappa(2)\}, 2)$.[10] If so, the stability-maximizing discount minimizes deviation gains across the two periods of the pricing cycle. When the storage constraint of the consumers does not bind $\sigma(2) < \kappa(2)$, such a markdown equalizes deviation gains across the two periods of the cycle.

Even though strategy γ^* maximizes the size of a cartel in equilibrium, more profitable strategies may exist in $\mathcal{E}$. The next proposition formally establishes this insight by characterizing the most profitable sale strategy within $\mathcal{E}$. As in the previous proposition, the strategy requires firms to set collusive markups in periods without sales. The sale markdown is now, however, uniquely pinned down by the consumer's storage constraint, as larger discounts always hurt equilibrium profits. Similarly, sale frequency is uniquely pinned to 2 as infrequent sales hurt profits due to the cost of anticipating production.

PROPOSITION 6. *If $\mathcal{E} \neq \emptyset$, no strategy in $\mathcal{E}$ is strictly more profitable than strategy $\gamma^+ \in \mathcal{E}$:*

μ^+	σ^+	$\varkappa^+$
$\bar{\mu}$	$\kappa(2)$	2

The profit maximizing discount can obviously be smaller than that of the most stable strategy γ^*, and no longer depends on the fraction of consumers with storage in the economy. Consequently, the upper bound on the number of firms that can collude on strategy γ^+ may be strictly smaller than for γ^*, $n(\gamma^+) \leq n(\gamma^*)$.

The previous two propositions highlighted the trade-off arising between cartel profits and cartel size in environments in which $\alpha_0 > 0$. The first result established that strategies displaying equilibrium path sales could be used to increase cartel size at the expense of aggregate profits. Strategy γ^* was proven to be more stable than any other strategy without sales, but less profitable than full collusion. Similarly, γ^* was clearly more profitable and less stable than the competitive outcome (i.e. the Nash equilibrium of the stage game). The second result instead showed that even within $\mathcal{E}$ profit–size trade-offs would persist, provided that γ^+ and γ^* differed. An additional

9. When $\breve{\varkappa}$ is defined, the sale strategy γ^* often simplifies to $(\bar{\mu}, \sigma(\breve{\varkappa} - 1), \breve{\varkappa} - 1)$ if $\breve{\varkappa} > 2$, and to $(\bar{\mu}, \kappa(2), 2)$ if $\breve{\varkappa} = 2$.

10. If $S = 1$, $\sigma(2) = \min\{1, \theta\}$ where θ is the unique positive root of the quadratic equation

$$\theta^2 (2 - \alpha)\delta - \theta\alpha(1 - \alpha) - \alpha^2\delta = 0.$$

conclusion of such propositions implied that infrequent sales would occasionally increase cartel size compared to strategies with more frequent sales, without hurting profits.

To conclude this part of the analysis, consider an economy in which $\delta = 0.95$, $\alpha_0 = 0.15$, $v = 10$, and $c = 1$. If $S = 1$, the maximal equilibrium cartel size grows from 20 to 28 when firms switch from the monopoly strategy γ^m to the most stable sale strategy γ_1^*. But when $S = 30$, the maximal cartel size is attained by a strategy with infrequent sales which take place every 21 periods. Maximal cartel size grows to 37 when firms collude on the most stable sale strategy γ_{30}^*. Since sales occur less frequently smaller discounts are necessary to sustain the maximal cartel size. Thus, the profits of the most stable strategy γ_{30}^* can be larger than those associated with the most stable strategy γ_1^* in an economy in which at most a single unit can be stored $S = 1$. The profit–size trade-off can therefore decline when consumers gain access to more efficient storage technologies. The following table reports all the relevant variables for the example discussed.

	n	Π	σ	μ	$\varkappa$
γ^m	20.0	9.00	1.00	9	$\forall$
γ^+	20.5	8.72	0.94	9	2
γ_1^*	28.6	1.96	0.15	9	2
γ_{30}^*	37.4	4.62	0.27	9	21

Unit Storage and the Profit–Size Trade-Off

When at most one unit can be stored, a more stringent characterization of the profit–stability trade-off is possible. In particular, observe that the previous propositions implied that increasing the regular markup μ would increase cartel profits and not decrease cartel size, and that decreasing the sales markdown $1 - \sigma$ would increase profits and decrease cartel size if and only if $\sigma \in [\sigma(2), \kappa(2)]$. With a slight abuse of notation, for any strategy $\gamma = (\bar{\mu}, \sigma, 2)$ let $n(\sigma) = n(\gamma)$ denote the maximal number of firms that can collude on strategy γ in equilibrium, and let $\pi(\sigma) = \Pi_0(\gamma)$ denote the profits raised by the strategy γ. Since the profits $\pi(\sigma)$ strictly increase in the sale markdown σ, it is possible to compute the maximal number of firms $N(\Pi)$ that can collude on profit level Π while employing a sale strategy. In particular, for any cartel profit level $\Pi \in \mathbb{R}_+$ it follows that

$$N(\Pi) = n(\pi^{-1}(\Pi)).$$

A trade-off emerges between cartel profits and cartel size for any markdown $\sigma \in [\sigma(2), \kappa(2)]$. The maximal number of firms that can sustain a collusive sale strategy declines as profits increase. This is the case as profits obviously decrease in the sale markdown $1 - \sigma$, while cartel size increases. Within the range $[\sigma(2), \kappa(2)]$, the maximal number of firms that can collude on a strategy $\gamma = (\bar{\mu}, \sigma, 2)$ declines with σ, as deviations become more profitable in periods of sales,[11]

$$\frac{dN(\Pi)}{d\Pi} = \frac{dn/d\sigma}{d\pi/d\sigma}(\pi^{-1}(\Pi)) = \frac{dR_0/d\sigma}{d\Pi_0/d\sigma}(\bar{\mu}, \pi^{-1}(\Pi), 2) \leq 0.$$

Moreover, any sale strategy $\gamma = (\bar{\mu}, \sigma, 2)$ raising more profits by setting $\sigma > \kappa(2)$ would be outperformed both in terms of profits and in terms of cartel size by a strategy without sales in which all units are sold at the monopoly markup (as no player would store units in either scenario). Similarly, any strategy setting $\sigma < \sigma(2)$ would simultaneously reduce both cartel size and profits when compared to strategy γ^*.

For convenience let $\Pi^m = v - c$ denote the monopoly profit, and let Π^* and Π^+ respectively denote the profits of the most stable and of the most profitable strategies in $\mathcal{E}$:

$$\begin{aligned} \Pi^* &= \left[\tfrac{\delta\alpha_0}{1+\delta} + \tfrac{2-\alpha_0}{1+\delta}\sigma^*\right](v-c), \\ \Pi^+ &= \left[\tfrac{\delta\alpha_0}{1+\delta} + \tfrac{2-\alpha_0}{1+\delta}\sigma^+\right](v-c). \end{aligned}$$

The next proposition highlights the specific nature of the trade-off between cartel profits and cartel size that different sale strategies entail. In particular, the result explicitly characterizes the largest number of firms $N(\Pi)$ that could enter the market at a given profit level Π without it unraveling to a competitive equilibrium. The maximal cartel size always decreases as equilibrium profits increase. Furthermore, in the interval $(\Pi^*, \Pi^+]$, $N(\Pi)$ is strictly decreasing and convex provided that $\alpha_0 > 0$, and by construction satisfies $N(\Pi) > 1/(1-\delta)$. When profits fall below Π^* however, the maximal cartel size is no longer affected by further reduction in profits, and coincides with the maximal cartel size of the most stable sales strategy $n(\gamma^*)$. Similarly, when profits exceed Π^+, the cartel size no longer responds to further increases in profits, and coincides with the maximal cartel size of any no-sale strategy $1/(1-\delta)$.

PROPOSITION 7. *If $\mathcal{E} \neq \emptyset$, for any profit level $\Pi \in (0, \Pi^m]$ the maximal number of firms that can collude on Π while employing a sale strategy satisfies*

$$N(\Pi) = \begin{cases} \dfrac{1}{1-\delta} & \textit{if} \quad \Pi \in (\Pi^+, \Pi^m], \\ \dfrac{1}{1-\delta}\dfrac{(2-\alpha_0)\Pi}{(1+\delta)\Pi - \delta\alpha_0(v-c)} & \textit{if} \quad \Pi \in (\Pi^*, \Pi^+], \\ \dfrac{1}{1-\delta}\dfrac{(2-\alpha_0)\Pi^*}{(1+\delta)\Pi^* - \delta\alpha_0(v-c)} & \textit{if} \quad \Pi \in (0, \Pi^*]. \end{cases}$$

The profit–stability trade-off emerges because larger sale discounts occasionally favor stability, but certainly hurt profits. In the limit case in which all consumers have access to storage, $\alpha_0 = 0$, the trade-off persists as a simple step function, since

11. A formal derivation of these observations appears in the Online Appendix.

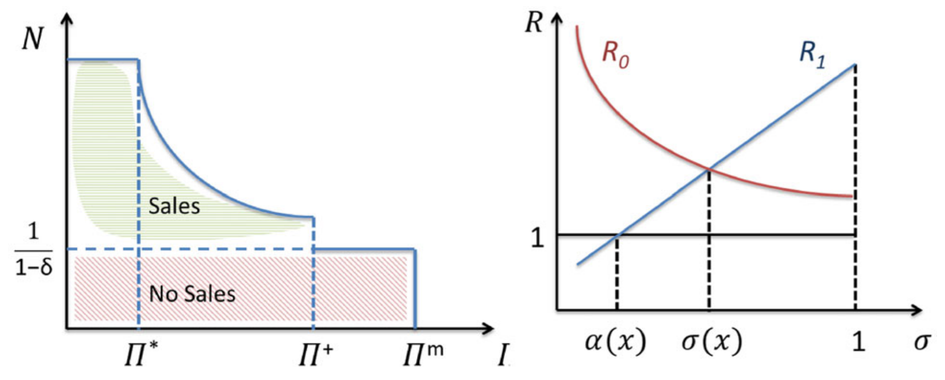

FIGURE 2. Left: $R_1(\gamma)$ and $R_0(\gamma)$ as functions of σ, where $\alpha(\varkappa) = \alpha_0/(\alpha_S \varkappa + \alpha_0)$. Right: the profit–stability trade-off $N(\Pi)$.

$N(\Pi) = 2/(1-\delta^2)$ for any $\Pi \leq \Pi^+$. The right plot of Figure 2 depicts $R_1(\gamma)$ and $R_0(\gamma)$ and shows that for any value of $\varkappa$ a unique discount $\sigma(\varkappa)$ exists which maximizes the size of the cartel. The left plot depicts the profit–stability trade-off $N(\Pi)$ derived in the previous proposition.

The last result of the section shows how changes in the environment might affect the profit–size trade-off. Comparative statics are developed for the four relevant free parameters of the model: δ, α_0, v, and c. Results are discussed in detail after the statement of the proposition.

PROPOSITION 8. *If $\mathcal{E} \neq \emptyset$, for any profit level $\Pi \in (0, \Pi^m]$ the maximal number of firms that can collude on Π while employing a sale strategy satisfies*

$dN(\Pi)$	$d\delta$	$d\alpha_0$	dv	dc
$\Pi \in (\Pi^+, \Pi^m]$	+	0	0	0
$\Pi \in (\Pi^*, \Pi^+]$	+	?	+	−
$\Pi \in (0, \Pi^*] \cap \Pi^* \neq \Pi^+$	+	−	0	0
$\Pi \in (0, \Pi^*] \cap \Pi^* = \Pi^+$	+	−	+	−

where $dN(\Pi)/d\alpha_0 > 0$ if and only if $\delta > \Pi/(2\Pi^m - \Pi)$. Moreover, the cut-off profit levels Π^* and Π^+ and the maps σ and κ evaluated at $\varkappa = 2$ and $\sigma(2) \leq \kappa(2)$ further satisfy

	$d\delta$	$d\alpha_0$	dv	dc
$d\Pi^+$	+	+	+	−
$d\Pi^*$	−	+	+	−
$d\sigma$	−	+	0	0
$d\kappa$	+	0	+	−

The result shows that increasing patience (i.e. reducing storage costs) can lead to larger equilibrium cartels at any profit level, and to the persistence of the profit–size

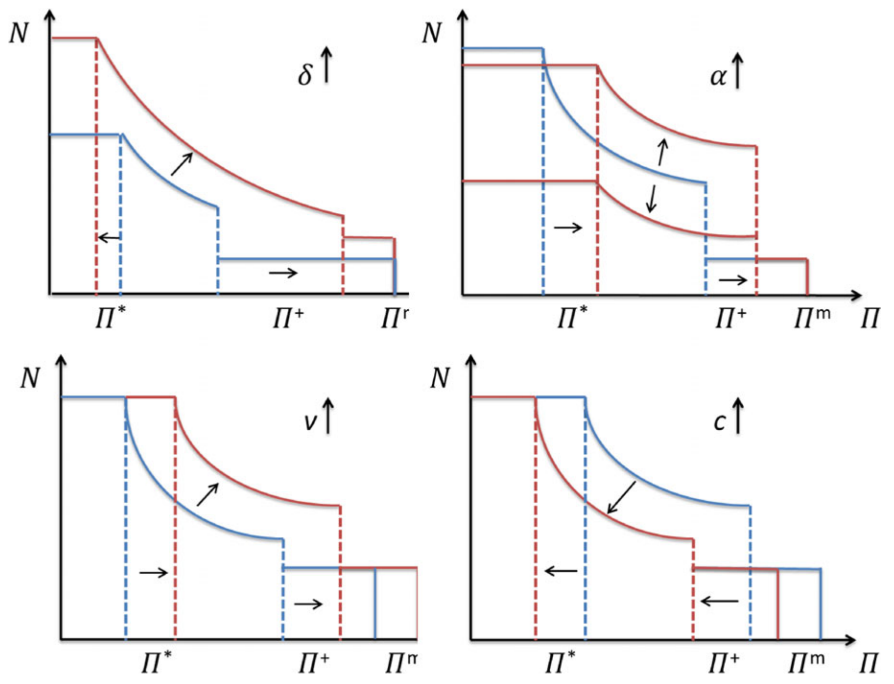

FIGURE 3. Comparative statics on the trade-off $N(\Pi)$ with respect to an increase: in δ (top left), in α_0 (top right), in v (bottom left), and in c (bottom right).

trade-off on a larger range of profits. On the other hand, increasing the profitability of the market (i.e. $v - c$) increases the maximal equilibrium cartel size, but only for intermediate profit levels, since the stability $n(\gamma)$ is independent of values and costs when evaluated both at the most stable strategy γ^* and at the monopoly strategy γ^m. The result also establishes that increasing the fraction of consumers without storage (i.e. α_0) reduces the stability of the most stable sale strategy γ^* as intertemporal linking between decisions declines. The effect of such a change on the maximal cartel size at intermediate profit values is ambiguous, as a large fraction of consumers with storage may lead to lower equilibrium profits due to the cost of anticipating production. Clearly, the same change would have no effect on the stability of strategies without sales, since no consumer would store units.

The range of profits for which the profit–stability trade-off persists declines as the number of consumers without access to storage grows, and eventually vanishes at some value $\bar{\alpha} < 1$ (i.e. when sufficiently few consumers have access to storage). The discount offered during a period with sales (i.e. $1 - \sigma$) in the most profitable policy γ^+ declines with patience and profitability, but is unaffected by the fraction of consumers with storage. On the other hand, the discount offered in the most stable strategy γ^* (when such strategy does not coincide with γ^+) grows with patience and the fraction of consumers with storage, but is unaffected by profitability. Figure 3 provides a visual characterization of the comparative statics results presented in the previous proposition.

5. Buyers and Beliefs

Two features of the model played an important role in the construction of the profit–size trade-off, namely consumer beliefs about future prices, and the trigger punishments. In particular, the rationality imposed on consumers storing units implied that such buyers would always understand how deviations would affect future prices. The results developed however, are robust to numerous alternative specifications provided that at least some consumers understand that unexpected price cuts might lead with some positive probability to lower prices for some time in the future. This section establishes that an intertemporal link in demand persists even if no consumer is able to revise his beliefs about future prices upon observing a deviation. In this scenario, the stability of strategies both with and without sales declines, because consumers may prefer to purchase multiple units upon observing a deviation if they expect prices to remain high in the future. However, even in such environments strategies with sales may display a higher stability than strategies without sales, as unexpected price cuts might attract a smaller demand whenever consumers have units already stored.

To make this point, again consider environments in which $\alpha_0 > 0$ and $S = 1$. But now suppose that buyers understand price dynamics on the equilibrium path, but not off the equilibrium path. If so, no player revises its beliefs about future prices when a deviation is observed. In such environments, given any equilibrium price path p, and any profile of posted prices in period t, $\bar{p}_t$, consumer demand amounts to $d_t(s_t, \{\bar{p}_t, \mathrm{p}^{t+1}\})$. The change in beliefs never affects equilibrium profits, but affects deviation gains. The next and final result establishes that even within this framework, collusion may be fostered by strategies that display sales along the equilibrium path. The result obtains as deviating firms can exploit the lack of foresight from buyers to increase profits by selling more units when undercutting the equilibrium price. Therefore, sales may foster collusion as demand is constrained by storage capacity in periods in which high prices are charged.

In these environments, deviation payoffs for any strategy without sales, $\gamma = (\mu, 1, \varkappa) \in \mathcal{N}$, satisfy

$$\Delta(\gamma) = (1 - \delta) \max\{(2 - \alpha_0)\hat{\mu}(\gamma), \mu\}c,$$

where $\hat{\mu}(\gamma)$ denotes the largest markup at which consumers with access to storage would stockpile units when a deviation takes place, $\hat{\mu}(\gamma) = \delta(1 + \mu) - 1$. Firms may exploit buyers' beliefs to sell more than a single unit despite the price war that would arise when a deviation takes place. Thus, the stability of a strategy $\gamma \in \mathcal{N}$ satisfies

$$n(\gamma) = \frac{1}{1-\delta} \min\left\{1, \frac{\mu}{(2-\alpha_0)\hat{\mu}(\gamma)}\right\}.$$

A trade-off arises even among strategies in $\mathcal{N}$, as high markups increase profits, but reduce the stability of strategies without sales. The maximal number of firms that can collude on a strategy without sales is bounded above by the previous critical threshold

$1/(1-\delta)$. For a periodic sale strategy $\gamma \in \mathcal{C}$ instead, deviation payoffs satisfy

$$\Delta_t(\gamma) = \begin{array}{ll} (1-\delta)(2-\alpha_0)\sigma\mu c & \text{if } \mathrm{mod}(t,\varkappa) = 0, \\ (1-\delta)\max\left\{\tilde{\mu}(\gamma), \alpha_0\mu\right\} c & \text{if } \mathrm{mod}(t,\varkappa) = 1, \end{array}$$

where $\tilde{\mu}(\gamma)$ denotes the largest markup at which consumers with positive storage would purchase an additional unit when a deviation takes place in a period without sales, $\tilde{\mu}(\gamma) = \delta(1+\sigma\mu) - 1$. Given buyers' beliefs, deviations now attract the full supply in periods of sales. However, in periods without sales deviation gains may increase when firms benefit from selling to consumers with already units in storage. Consequently, the stability of any strategy $\gamma \in \mathcal{C}$ is given by

$$n(\gamma) = \frac{1}{1-\delta^2} \min\left\{ \frac{\left[\delta\alpha_0 + (2-\alpha_0)\sigma\right]}{(2-\alpha_0)\sigma}, \frac{\left[\alpha_0 + \delta(2-\alpha_0)\sigma\right]\mu}{\max\left\{\tilde{\mu}(\gamma), \alpha_0\mu\right\}} \right\}.$$

A trade-off emerges even among strategies in $\mathcal{C}$, as high markups in periods of sales increase profits, but reduce the stability. The maximal number of firms that can collude on a strategy of sales remains bounded above by $\frac{1}{1-\delta}$. The next proposition characterizes the profit–stability trade-off for strategies with and without sales, and shows why sales can foster collusion when consumers are sufficiently patient. For convenience, we define two relevant pay-off thresholds,

$$\Pi^n = \frac{(2-\alpha_0)(1-\delta)}{(2-\alpha_0)\delta - 1} c \quad \text{and} \quad \Pi^s = \alpha_0(v-c).$$

PROPOSITION 9. *If δ is sufficiently high, the following claims must hold.*

1. *The maximal number firms that can collude on Π while employing a strategy in $\mathcal{N}$ satisfies*

$$N_n(\Pi) = \begin{cases} \dfrac{1}{1-\delta} & \text{if } \Pi \in (0, \Pi^n], \\ \dfrac{1}{1-\delta} \dfrac{\Pi}{(2-\alpha_0)[\delta(\Pi + c) - c]} & \text{if } \Pi \in (\Pi^n, \Pi^m]. \end{cases}$$

2. *The maximal number firms that can collude on Π while employing a strategy in $\mathcal{C}$ satisfies*

$$N_s(\Pi) = \begin{cases} \dfrac{1}{1-\delta} & \text{if } \Pi \in (0, \Pi^s], \\ \dfrac{1}{1-\delta} \dfrac{\Pi}{(1+\delta)\Pi - \delta\alpha_0(v-c)} & \text{if } \Pi \in (\Pi^s, \Pi^+]. \end{cases}$$

3. *There exists a range of profits (Π^n, Π°) on which $N_n(\Pi) < N_s(\Pi)$ for some $\Pi^\circ \in (\Pi^s, \Pi^+]$.*

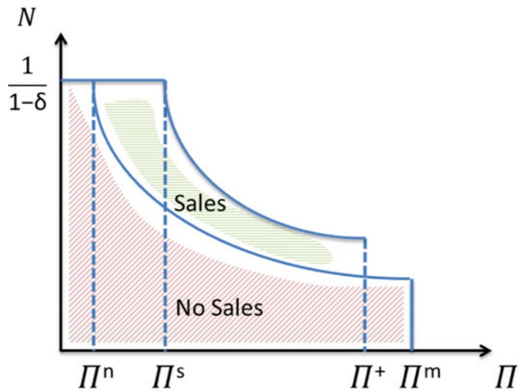

FIGURE 4. The profit–stability trade-off $N(\Pi)$.

Figure 4 characterizes the results described in Proposition 9. In these environments, sales may foster collusion at intermediate profit levels. This is the case as deviation from a strategy with no sales can attract a demand of $(2-\alpha_0)$ provided that sufficiently big discount is offered, whereas deviation from a strategy with sales can attract such demand only when the price is already low (and no consumer has units stored). The change in consumer rationality also implies that no sale strategy sustains a cartel size larger than $1/(1-\delta)$, in contrast to the results derived for rational buyers where strategies with sales exceeded such bound. In this scenario, $N(\Pi) = \max\{N_n(\Pi), N_s(\Pi)\}$ when $\Pi < \Pi^\circ$, and $N(\Pi) = N_n(\Pi)$ otherwise.

The result highlights why the trade-off between profits and stability is resilient to alternative assumptions on consumer rationality, and shows why sales may be used to foster collusion even in such environments.

6. Comments and Conclusions

The analysis introduced a novel rationale for sales in an industry in which a homogeneous storable good is produced by n firms, and sold to consumers with access to storage. In this context, the paper has examined the effects of storage on firms' incentives to hold periodic sales to support a greater degree of collusion. Sales were proven to strengthen the incentives to collude, as storage would intertemporally link consumer demand and thus reduce the short-run gains from a deviation. In particular, the analysis showed that if a market was sufficiently large, firms had to periodically reduce prices in order to sustain collusion. Such behavior was proven to reduce the incentives to deviate both in regular price periods (as only consumers without storage would purchase units), and in periods with sales (as consumers with storage would curtail their demand, if a deviation were observed in the wake of an imminent price war).

The first part of the analysis characterized consumer demand and the set $\mathcal{E}$ of sale strategies which allowed for a greater degree of collusion than any strategy without sales. The second part of the analysis identified the sale strategies maximizing cartel size and cartel profits within the set $\mathcal{E}$. A trade-off was shown to emerge between cartel profits and size. The trade-off and its dependence on the environment were explicitly characterized in several scenarios. The relationship between the optimal sale markdown and the environment was also explored. The third and final part of the analysis established instead that the trade-off would persist even in environments in which deviations would not affect consumer beliefs about future prices. In such setups, however, the nature of the profit–size trade-off differed significantly, as unexpected price reduction would never cause demand to decline.

Critical Comments, Robustness, Evidence

Asynchronous Sales. Although the analysis has focused on periodic sales strategies with synchronized sales, numerous conclusions extend to strategies in which firms do not coordinate their behavior. A section of the Online Appendix tackles such concerns explicitly.

Storage, Income, and Rationality. Results have been presented in the context of a model with rational consumers and heterogeneous storage technologies. Alternative interpretations are however, possible. Notably, the results developed in Section 4 would immediately apply to a model in which some consumers simply do not expect prices to rise sharply in future periods, and thus purchase a single unit in every period in which the price does not exceed their value,[12] while the remaining consumers understand price dynamics in the market and purchase the optimal number of units given the expected future price path. If so, myopic buyers may also be seen as consumers whose opportunity cost of investing time in understanding future price dynamics is higher. This interpretation could be exploited to match evidence suggesting that middle-income households are more responsive to sales, as very low-income households are constrained in their ability to store, while high-income households have a higher value for time, and thus do not necessarily invest in taking advantage of sales (Griffith et al. 2009; Gauri, Sudhir, and Talukdar 2008).

In the rational consumer model, along the equilibrium path demand expands to its maximum level in any period of sales. In such periods, however, retaliatory punishments and rationality imply that demand would contract if the sale discount $1-\sigma$ was unexpectedly high. Although such behavior appears hard to test as it relies on what consumers perceive as an unexpected price cut, the empirical literature appears to have found some evidence which is at least consistent with this type of behavior. In particular, Blattberg, Briesh, and Fox (1995), survey the empirical results on sales and promotions, and report as one of their findings that markdowns, which differ from the

12. These consumers always expect $p_t > \delta p_{t+1}$, and therefore always demand a single unit in every period of the game even if deviations are observed.

common price dynamics, reduce prices expected by buyers in the future. In turn, such beliefs could in principle reduce demand, as consumers expect prices to remain low for some time in the future. In this spirit, Mela, Jedidi, and Bowman (1998) hypothesize that households develop price expectations on the basis of their prior exposure to promotions over a long period of time, and argue that these expectations, coupled with the costs of inventorying products, affect consumer purchase timing and purchase quantity decisions. In particular, evidence suggests that increasing expectations of future promotions can lead to a reduced likelihood of purchase incidence on a given shopping trip. Although the evidence is not conclusive, empirical papers on promotions consistently show that unexpected pricing behavior of firms does in fact affect consumer beliefs about future pricing behavior. Our model presents a simple framework in which the intertemporal link in consumer beliefs can be exploited to enhance collusion. More elaborated models in which irrationality and brand loyalty play a more prominent role may be better suited to explain the numerous regularities that appear in the empirical literature on promotions. However, in all these models a link in demand can emerge if expected future prices affect optimal stockpiling decisions, and in turn such a link might be exploited to enhance collusion by endogenously generating demand cycles along the equilibrium path.

Unknown Storage Capacity. The analysis was developed for economies in which the storage capacity of consumers was known to firms. However, all results on single-unit storage would equivalently apply to a model in which α_0 denotes the fraction of players without access to storage, while α_S denotes the fraction of players that can store at least one unit (but possibly more). Clearly, such a change in the setup would not alter expected demand if employing sale strategies that cut prices every two periods, as no consumer would ever purchase more than two units in any given period. Thus, although the results of Section 4 would no longer characterize the optimal punishments, they would still show how sales may foster collusion at the expense of profits.

Sales Strategies, and Bundling. The analysis has restricted attention to cyclical sale strategies in $\mathcal{S}$. Alternative equilibrium pricing strategies in which sales do not take place at congruent dates might also enhance collusion by creating an intertemporal link in demand. However, no such strategy could be as stable as the optimal sale strategy in $\mathcal{S}$. In fact, if no units were stored along the equilibrium path the incentives to collude would decline by an argument equivalent to Proposition 2. But if units were stored, the incentives to collude would decline in one of the periods in which consumers purchase multiple units, as the evolution of future prices would differ from the optimal strategy characterized here. The restriction to sales strategies $\mathcal{S}$ also implied that firms were confined to setting a linear price in every period at which to sell all the units demanded. Thus, bundling and nonlinear pricing were ruled out of the model by assumption. However, it would be possible to show that allowing firms to sell bundles of different sizes at different prices would not alter the profit–size trade-off within the proposed framework, as the profit-maximizing strategy at any cartel size would require pricing all bundles at a constant marginal price.

Appendix: Proofs

Proof of Remark 1

Part (1) of the claim is trivial. To prove (2) notice that by construction $d_S(s, \mathrm{p}^t) \in [0, S+1-s]$. The upper bound must hold, since no consumer can store more than S units. The lower bound must hold because no player can benefit by disposing already purchased units given that $p_z^* \geq 0$ for any z. Also notice that only profiles of demand that guarantee a consumption stream of a unit in every period can be optimal, since prices satisfy $p_z^* \leq v$. Thus, payoff streams can be compared by looking only at the total expenditure on consumption good.

Then, consider the case in which $T(\mathrm{p}^t) \leq S+1$. By contradiction consider a profile of demand for the successive $T(\mathrm{p}^t)$ periods, $\{d_{t+z}\}_{z=0}^{T(\mathrm{p}^t)}$ and suppose that $d_t \neq \max\{T(\mathrm{p}^t) - s, 0\}$. If so, there exists a profile of demands $\{d'_{t+z}\}_{z=0}^{T(\mathrm{p}^t)}$ that costs less and that leaves the consumer with exactly as many units stored in period $t + T(\mathrm{p}^t)$. In fact, consider

$$d'_{t+z} = \begin{cases} \max\{T(\mathrm{p}^t) - s, 0\} & \text{if } z = 0, \\ 0 & \text{if } z \in \{1, \ldots, T(\mathrm{p}^t) - 1\}, \\ \sum_{z=0}^{T(\mathrm{p}^t)} d_{t+z} - d'_t & \text{if } z = T(\mathrm{p}^t). \end{cases}$$

By construction the profile leaves the consumer with exactly as many units stored in period $t + T(\mathrm{p}^t)$. Moreover d' costs less, since

$$\begin{aligned} \sum_{z=0}^{T(\mathrm{p}^t)} \delta^z \bar{p}_{t+z} d_{t+z} &= \sum_{z=0}^{T(\mathrm{p}^t)-1} \delta^z \bar{p}_{t+z} d_{t+z} + \delta^{T(\mathrm{p}^t)} p_{t+T(\mathrm{p}^t)} d_{t+T(\mathrm{p}^t)} \\ &\geq \bar{p}_t \sum_{z=0}^{T(\mathrm{p}^t)-1} d_{t+z} + \delta^{T(\mathrm{p}^t)} p_{t+T(\mathrm{p}^t)} d_{t+T(\mathrm{p}^t)} \\ &= \bar{p}_t d'_t + \bar{p}_t [d'_{t+T(\mathrm{p}^t)} - d_{t+T(\mathrm{p}^t)}] + \delta^{T(\mathrm{p}^t)} p_{t+T(\mathrm{p}^t)} d_{t+T(\mathrm{p}^t)} \\ &= p_t d'_t + \delta^{T(\mathrm{p}^t)} p_{t+T(\mathrm{p}^t)} d'_{t+T(\mathrm{p}^t)} + \left[\delta^{T(\mathrm{p}^t)} p_{t+T(\mathrm{p}^t)} - \bar{p}_t\right] \\ &\quad \times \left[d_{t+T(\mathrm{p}^t)} - d'_{t+T(\mathrm{p}^t)}\right] \geq p_t d'_t + \delta^{T(\mathrm{p}^t)} p_{t+T(\mathrm{p}^t)} d'_{t+T(\mathrm{p}^t)} \\ &= \sum_{z=0}^{T(\mathrm{p}^t)} \delta^z \bar{p}_{t+z} d'_{t+z} \end{aligned}$$

given that (i) $\sum_{z=0}^{T(\mathrm{p}^t)-1} d_z \geq d'_t$ & $d_{t+T(\mathrm{p}^t)} \leq d'_{t+T(\mathrm{p}^t)}$, since that consumers consume one unit in every period, and (ii) $\delta^{T(\mathrm{p}^t)} p_{t+T(\mathrm{p}^t)} < \bar{p}_t \leq \delta^z \bar{p}_{t+z}$ for any $z \in (0, T(\mathrm{p}^t))$. Thus a contradiction is established. A very similar and omitted argument works also for the case in which $T(\mathrm{p}^t) > S+1$ and establishes the claim.

Proof of Remark 2

The proof of the result is trivial. No player benefits from a deviation along the equilibrium path if

$$\frac{\Pi_t(\gamma)}{n} \geq \Delta_t(\gamma),$$

where $\Delta_t(\gamma)$ denotes the most profitable deviation. Such condition is exploited to pin down the requirement on the critical discount rate. Moreover, no deviation can be profitable off the equilibrium path, since all players make at most zero profits when all competitors quote prices at marginal cost.

Proof of Remark 3

The first claim is proven by induction. Note that $\mathrm{mod}(0, \varkappa) = 0$ and $s_0 = 0$. Next we show that if $s_t = 0$ for any $t \leq T$ such that $\mathrm{mod}(t, \varkappa) = 0$, then it also the case that $s_t = 0$ for any $t \leq T + \varkappa$ such that $\mathrm{mod}(t, \varkappa) = 0$. In fact consider the largest date τ such that $\tau \leq T$ and $\mathrm{mod}(\tau, \varkappa) = 0$. Such date exists by the initial condition and the induction hypothesis. At such date the demand of an individual with storage satisfies

$$d_S(0, \mathrm{p}^\tau) = \min\{T(\mathrm{p}^\tau), S + 1\}.$$

Moreover, $T(\mathrm{p}^\tau) \leq \varkappa$, since $(1 + \mu\sigma) > \delta^\varkappa(1 + \mu\sigma)$. Hence, $d_S(0, \mathrm{p}^\tau) \leq \varkappa$ and $s_{\tau+1} < \varkappa$ given that one unit will be consumed. Moreover, in any period $z \in \{t + 1, \ldots, t + \varkappa - 1\}$, since $(1 + \mu) > \delta(1 + \mu) > \delta(1 + \mu\sigma)$, we have that $T(\mathrm{p}^z) \leq 1$ and consequently

$$d_S(s_z, \mathrm{p}^z) = \begin{cases} 0 & \text{if } s_z > 0 \\ 1 & \text{if } s_z = 0 \end{cases} \Rightarrow s_{z+1} = \begin{cases} s_z - 1 & \text{if } s_z > 0 \\ 0 & \text{if } s_z = 0, \end{cases}$$

which establishes that $s_{\tau+\varkappa} = 0$, since $s_\tau < \varkappa$. The second claim follows immediately, since from the previous part of the proof it is straightforward to observe that

$$d_S(s_t, \mathrm{p}^t) = \begin{cases} d_S(0, \mathrm{p}^0) & \text{if } \mathrm{mod}(t, \varkappa) = 0 \\ \max\{0, 1 - s_t\} & \text{if } \mathrm{mod}(t, \varkappa) \neq 0. \end{cases} \tag{A.1}$$

Proof of Proposition 1

Note that claim (1) is immediate, because $\sigma = 1$ implies $d_S(s_t, \mathrm{p}^t) = 1$ for any t as $\bar{p}_{t+1} = \bar{p}_t$ requires $\bar{p}_t > \delta \bar{p}_{t+1}$. To prove claim (2), first observe that all strategies in $\mathcal{N}$ are equally stable. Note that, by the proof of claim (1), for any strategy $\gamma \in \mathcal{N}$ equilibrium payoffs simplify to $\Pi_t(\gamma) = \mu c$. Thus, a deviating player can capture at most such a profit by undercutting the price marginally. Any deviation to a price $y \in (c, (1 + \mu)c)$ must satisfy $d(s_t, \mathrm{y}^t) \leq 1$, since $\bar{p}_t > \delta c$ and therefore

$\Delta_t(y,\gamma) \leq (1-\delta)(y-c)$. Hence, $\Delta_t(\gamma) = (1-\delta)\mu c$ and $R_t(\gamma) = 1/(1-\delta)$ for any $t \in \{0,1,\ldots\}$ and any $\gamma \in \mathcal{N}$. Now, consider a strategy $\gamma \in \mathcal{S}\backslash\mathcal{C}$ and a period t in which $d_t > \alpha_0$ and $\bar{p}_t = (1+\mu)c$. Note that such conditions imply that $s_t = 0$ and $d_t \geq 1$. If so, by $\bar{p}_t = (1+\mu)c$, we get that

$$\Delta_t(y,\gamma) = (1-\delta)(y-c)d(s_t,\mathrm{y}^t) = (1-\delta)\,(y-c)$$

and $\Delta_t(\gamma) = (1-\delta)\mu c$. Moreover, if such a period exists, it must be that

$$d_{S0} = \min\{T(\mathrm{p}^0), S+1\} < \varkappa$$

because of the evolution of savings and demand discussed in the previous remark (condition A.1). In turn this requires that $\delta^{t-1}(1+\mu) \geq (1+\mu\sigma) > \delta^t(1+\mu)$ for some $t \in \{1,\ldots,\varkappa-1\}$. If so, pick the smallest t for which $(1+\mu\sigma) > \delta^t(1+\mu)$ and notice that

$$\begin{aligned}\Pi_t(\gamma) &= \frac{(1-\delta)}{1-\delta^\varkappa}\left[\left[\textstyle\sum_{z=0}^{\varkappa-1}\delta^z d_{t+z}\right] - (1-\sigma)\delta^{S(t)}d_0\right]\mu c\\ &= \alpha_0\Pi_{0t}(\gamma) + \alpha_S\Pi_{St}(\gamma) \leq \mu c,\end{aligned}$$

where the last inequality must hold since

$$\begin{aligned}\Pi_{0t}(\gamma) &= \frac{\left(1-\delta^{S(t)}+\delta^{S(t)+1}-\delta^\varkappa\right)+\sigma(\delta^{S(t)}-\delta^{S(t)+1})}{1-\delta^\varkappa}\mu c \leq \mu c\\ \Pi_{St}(\gamma) &= \frac{\left(1-\delta^{\varkappa-t}\right)+\sigma t(\delta^{\varkappa-t}-\delta^{\varkappa-t+1})}{1-\delta^\varkappa}\mu c \leq \mu c \quad\Leftrightarrow\quad \sigma \leq \frac{\left(1-\delta^t\right)}{t(1-\delta)}.\end{aligned}$$

The inequality bounding $\Pi_{0t}(\gamma)$ must hold, since it cannot be profitable to cut prices on consumers that do not alter their demand. The inequality bounding $\Pi_{St}(\gamma)$ must hold instead, since firms prefer to delay production costs and because $\delta^{t-1}(1+\mu) \geq (1+\mu\sigma)$ requires

$$\sigma \leq \frac{1+\mu\sigma}{1+\mu} \leq \delta^{t-1} \leq \frac{\sum_{z=0}^{t-1}\delta^z}{t} = \frac{\left(1-\delta^t\right)}{t(1-\delta)}.$$

Hence, a strategy $\gamma \in \mathcal{S}\backslash\mathcal{C}$ cannot be more stable than a strategy in $\mathcal{N}$, since $R_t(\gamma) \leq 1/(1-\delta)$.

The proof of claim (3) is trivial. The proposed strategy raises a profit of $v-c$, since $d_t = 1$ for any t. No strategy in which $d_t = 1$ for any t can do better, since v is the highest price that a buyer willing pay for a unit of consumption. But any other strategy such that $d_t \neq 1$ for some t must satisfy $d_{S0} > 1$, by the properties of the demand function derived in condition (A.1). In turn, if $d_{S0} > 1$, it must be that $(1+\mu\sigma) \leq \delta(1+\mu)$. Thus by (A.1), we get that profits can be expressed as follows for some $d_{S0} \in (1,\varkappa]$:

$$\begin{aligned}\Pi_0(\gamma) &= \frac{(1-\delta)}{1-\delta^\varkappa}\left[\left[\textstyle\sum_{z=0}^{\varkappa-1}\delta^z d_z\right] - (1-\sigma)d_0\right]\mu c\\ &= \frac{(1-\delta)}{1-\delta^\varkappa}\left[\left[\alpha_0\textstyle\sum_{z=1}^{d_{S0}-1}\delta^z + \sum_{z=d_{0S}}^{\varkappa-1}\delta^z\right] + \sigma(\alpha_0+\alpha_S d_{S0})\right]\mu c.\end{aligned}$$

An argument similar to the one developed in the previous part of the proof shows that $\Pi_0(\gamma) \leq \mu c$. In particular, write profits as $\Pi_0(\gamma) = \alpha_0 \Pi_{00}(\gamma) + \alpha_S \Pi_{S0}(\gamma)$ and notice that for the same reason as described in part (2), $\Pi_{00}(\gamma) \leq \mu c$. Then let $t = d_{0s}$ and notice that

$$\Pi_{S0}(\gamma) = \frac{\left(\delta^t - \delta^\varkappa\right) + \sigma t(1-\delta)}{1-\delta^\varkappa}\mu c \leq \mu c \quad \Leftrightarrow \quad \sigma \leq \frac{\left(1-\delta^t\right)}{t(1-\delta)},$$

where the inequality bounding $\Pi_{S0}(\gamma)$ is established by $\delta^{t-1}(1+\mu) \geq (1+\mu\sigma)$ as in part (2). This establishes claim (3), since $\mu c \leq v - c$ is necessary for profits to be maximal by the properties of the demand function. Part (4) follows trivially from condition (A.1) and the demand functions of both types of consumers discussed in the text.

Proof of Proposition 2

First we establish that a strategy in $\mathcal{E}$ is more stable than a strategy in $\mathcal{N}$. Consider a strategy $\gamma \in \mathcal{E}$. By definition of $\mathcal{E}$,

$$\sigma \geq \frac{\alpha_0}{\varkappa \alpha_S + \alpha_0}.$$

If so, $R_1(\gamma) \geq 1/(1-\delta)$ and for any $t \in \{1, 2, \ldots, \varkappa - 2\}$,

$$R_t(\gamma) = \frac{1}{1-\delta} + \frac{1}{1-\delta^\varkappa}\delta^{\varkappa - t}\left[\frac{\varkappa\alpha_S + \alpha_0}{\alpha_0}\sigma - 1\right] \leq R_{t+1}(\gamma),$$

since $S(t) = \varkappa - t$, if $t \leq \varkappa - 1$. Hence, the stability of a strategy in $\gamma \in \mathcal{E}$ will be pinned down by the minimum between $R_0(\gamma)$ and $R_1(\gamma)$. Moreover, $R_0(\gamma) > 1/(1-\delta)$ since for any μ and for any $\varkappa > 1$,

$$\begin{aligned} R_0(\gamma) &= \frac{1}{1-\delta}\frac{\alpha_0}{\sigma} + \frac{1}{1-\delta^x}\left[(\varkappa\alpha_S + \alpha_0) - \frac{\alpha_0}{\sigma}\right] \geq \frac{1}{1-\delta}\left[\alpha_0 + \frac{(1-\delta)\varkappa}{1-\delta^x}\alpha_S\right] \\ &= \frac{1}{1-\delta}\left[\alpha_0 + \frac{\varkappa}{1+\delta+\cdots+\delta^{x-1}}\alpha_S\right] > \frac{1}{1-\delta}, \end{aligned}$$

where the first inequality holds since $dR_0(\gamma)/d\sigma < 0$. Which establishes that if a strategy γ belongs to $\mathcal{E}$ then it must be more stable than any strategy in $\mathcal{N}$, since $\min_{t\geq 0} R_t(\gamma) \geq 1/(1-\delta)$. Since any strategy in $\mathcal{N}$ is more stable than any strategy in $\mathcal{S}\backslash\mathcal{C}$, what remains to be proven is that any strategy in $\mathcal{E}$ is more stable than strategies in $\mathcal{C}\backslash\mathcal{E}$. But this is immediate since $\gamma \in \mathcal{C}\backslash\mathcal{E}$ implies

$$\sigma < \frac{\alpha_0}{\varkappa\alpha_S + \alpha_0},$$

and thus $R_1(\gamma) < 1/(1-\delta)$.

Next we establish that Proposition (2) implies the existence of a strategy with a cycle of length $\varkappa \in \{2, \ldots, S+1\}$ in $\mathcal{E}$. Let the constraint of Proposition (2) hold for some $\varkappa \in \{2, \ldots, S+1\}$. Take any strategy that sets

$$\mu = \frac{v}{c} - 1$$

and

$$\sigma \in \left[\frac{\alpha_0}{\alpha_S \varkappa + \alpha_0}, \delta^{\varkappa - 1}\left(1 + \frac{1}{\mu}\right) - \frac{1}{\mu}\right] \tag{A.2}$$

at the given value $\varkappa$. The strategy obviously belongs to $\mathcal{E}$. Moreover, such a strategy exists since the interval in which σ was chosen is non-empty, whenever the constraint of Proposition (2) holds at $\varkappa$.

Next we establish the necessity of the constraint of Proposition (2). Any strategy in $\mathcal{E}$ must satisfy (A.2) by construction. Consider any one of these strategies, and notice that

$$\left[\frac{\alpha_0}{\alpha_S \varkappa + \alpha_0}, \delta^{\varkappa - 1}\left(1 + \frac{1}{\mu}\right) - \frac{1}{\mu}\right] \subseteq \left[\frac{\alpha_0}{\alpha_S \varkappa + \alpha_0}, \delta^{\varkappa - 1}\frac{v}{v-c} - \frac{c}{v-c}\right].$$

Since the non-emptiness of the bigger interval is equivalent to the constraint of Proposition (2), we get that the constraint of Proposition (2) being violated prevents the existence of a policy with cycle length $\varkappa$ in $\mathcal{E}$. This establishes the necessity.

Proof of Proposition 3

Consider any SPE trigger strategy γ, with an equilibrium price path p. Observe that for any deterministic equilibrium price path $d(s, \mathrm{p}^t) = 0$ whenever $s > 0$, $\alpha_0 = 0$ and $S = \infty$. This follows because in the previous period in which players purchased any units (call this period $t - z$), it had to be the case that $p_{t-z} < \delta^z p_t$, which implies players would have preferred to buy more units in period $t - z$ rather than buying any units in period t.

Consider the implied pattern of demand $d_t(\mathrm{p}^t)$ implied by equilibrium prices p, and recall that SPE profits satisfy

$$\Pi_t(\gamma) = (1-\delta)\textstyle\sum_{z=0}^{\infty}\delta^z \pi_{t+z}(\mathrm{p}^{t+z}) = (1-\delta)\textstyle\sum_{z=0}^{\infty}\delta^z \left(p_{t+z} - c\right) d_{t+z}(\mathrm{p}^{t+z}).$$

Let $D(\mathrm{p}) = \left\{z | d_z(\mathrm{p}^z) > 0\right\}$ and $D_t(\mathrm{p}) = \left\{z \geq t | d_z(\mathrm{p}^z) > 0\right\}$. For any period $t \in D(\mathrm{p})$, let $f(t) \in D(\mathrm{p})$ denote the following period in which units are sold. That is $d_{f(t)}(\mathrm{p}^{f(t)}) > 0$, and $d_z(\mathrm{p}^z) = 0$ for any $z \in \{t+1, \ldots, f(t) - 1\}$. For any period $\tau \in D(\mathrm{p})$ let

$$\begin{aligned}\Pi_\tau^{f(\tau)}(\gamma) &\equiv \frac{(1-\delta)}{(1-\delta^{f(\tau)-\tau})}\textstyle\sum_{z=\tau}^{f(\tau)-1}\delta^{z-\tau}\left(p_z - c\right) d_z(\mathrm{p}^z) \\ &= \frac{(1-\delta)}{(1-\delta^{f(\tau)-\tau})}\left(p_\tau - c\right) d_\tau(\mathrm{p}^\tau).\end{aligned}$$

Further observe that

$$\Pi_t(\gamma) = (1-\delta)\sum\nolimits_{z\in D_t(\mathrm{p})}\delta^z\left(p_z - c\right)d_z(\mathrm{p}^z) = \sum\nolimits_{z\in D_t(\mathrm{p})}(\delta^z - \delta^{f(z)})\Pi_z^{f(z)}(\gamma).$$

Thus, $\Pi(\gamma) = \Pi_0(\gamma)$ is a weighted average of 0 and $\Pi_z^{f(z)}(\gamma)$ for any $z \in D(\mathrm{p})$, since $(\delta^z - \delta^{f(z)}) \geq 0$ for any $z \in D(\mathrm{p})$ and since $\sum_{z\in D(\mathrm{p})}(\delta^z - \delta^{f(z)}) \leq 1$. Thus, a period $\bar{\tau} \in D(\mathrm{p})$ always exists in which $\Pi_{\bar{\tau}}^{f(\bar{\tau})}(\gamma) \geq \Pi(\gamma)$. In particular, pick $\bar{\tau} \in \arg\max_{z\in D(\mathrm{p})} \Pi_z^{f(z)}(\gamma)$. A necessary condition for strategy γ to be SPE is that $R_{\bar{\tau}}(\gamma) = \Pi_{\bar{\tau}}(\gamma)/\Delta_{\bar{\tau}}(\gamma) \geq n$.

To conclude the argument, we now establish the existence of a sale strategy $\hat{\gamma} \in \mathcal{S}$ for which (i) $\Pi(\hat{\gamma}) \geq \Pi(\gamma)$, and (ii) $n(\hat{\gamma}) \geq R_{\bar{\tau}}(\gamma)$. This would conclude the proof as condition (ii) would establish that $\hat{\gamma}$ is SPE by Remark 2, as $n(\hat{\gamma}) \geq R_{\bar{\tau}}(\gamma) \geq n$. In particular, consider the sale strategy $\hat{\gamma} = ((v-c)/c, (p_{\bar{\tau}} - c)/(v-c), f(\bar{\tau}) - \bar{\tau})$. The profits of such a sale strategy $\hat{\gamma}$ satisfy the condition (i), since

$$\Pi(\hat{\gamma}) = \left[\frac{1-\delta}{1-\delta^{\varkappa}}\right]\sigma\mu c\varkappa = \left[\frac{1-\delta}{1-\delta^{f(\bar{\tau})-\bar{\tau}}}\right](p_{\bar{\tau}} - c)\,(f(\bar{\tau}) - \bar{\tau}) \geq \Pi_{\bar{\tau}}^{f(\bar{\tau})}(\gamma) \geq \Pi(\gamma),$$

where the first inequality follows by $f(\bar{\tau}) - \bar{\tau} \geq d_{\bar{\tau}}(\mathrm{p}^{\bar{\tau}})$ (which must hold as $s_{f(\bar{\tau})} = 0$, by $d_{f(\bar{\tau})}(\mathrm{p}^{f(\bar{\tau})}) > 0$) and where the rest follows trivially from previous arguments. To establish condition (ii) instead, observe that $\Delta_{\bar{\tau}}(\gamma) = \Delta_0(\hat{\gamma}) = (1-\delta)(p_{\bar{\tau}} - c)$, as no consumer has units stored in such periods. Moreover, as $\bar{\tau} \in \arg\max_{z\in D(\mathrm{p})} \Pi_z^{f(z)}(\gamma)$, it must be that $\Pi(\hat{\gamma}) \geq \Pi_{\bar{\tau}}(\gamma)$, since $\Pi_{\bar{\tau}}(\gamma)$ is a weighted average of numbers smaller than $\Pi_{\bar{\tau}}^{f(\bar{\tau})}(\gamma)$. This immediately implies that

$$R_0(\hat{\gamma}) = \frac{\Pi(\hat{\gamma})}{\Delta_0(\hat{\gamma})} \geq \frac{\Pi_{\bar{\tau}}(\gamma)}{\Delta_{\bar{\tau}}(\gamma)} = R_{\bar{\tau}}(\gamma) \geq n.$$

Moreover, observe that $\Delta_1(\hat{\gamma}) = 0$ implies $R_1(\hat{\gamma}) = \infty$. Thus the result obtains, as $n(\hat{\gamma}) = \min_{t\geq 0} R_t(\hat{\gamma}) = R_0(\hat{\gamma}) \geq n$ suffices to establish that strategy $\hat{\gamma}$ is SPE.

Proof of Proposition 4

If $\alpha_S = 1$, deviation profits in any period without sales are equal to zero. Thus, no firm ever wishes to deviate in such periods (i.e. $R_1(\gamma) = \infty$ if $\gamma \in \mathcal{E}$). Moreover, notice that in this scenario

$$R_0(\gamma) = \frac{\varkappa}{1-\delta^{\varkappa}}.$$

Hence, any two strategies in $\mathcal{E}$ with the same sale frequency are equally stable, provided that both satisfy the storage constraint, $\sigma \leq \kappa(\varkappa)$. Because $R_0(\gamma)$ is increasing in $\varkappa$, the stability $n(\gamma)$ of a sale strategy increases with $\varkappa$. To derive the trade-off then observe that, for any sale frequency $\varkappa$, setting $\sigma = \kappa(\varkappa)$ and $\mu = \bar{\mu}$ maximizes profits with no consequences on stability. Thus, the trade-off characterized in the proposition must hold, since $n(\gamma)$ increases in $\varkappa$ while $\Pi^{\varkappa} = \Pi_0(\bar{\mu}, \kappa(\varkappa), \varkappa)$ decreases in $\varkappa$ since

$$\begin{aligned}\frac{\partial \Pi_0(\mu, \kappa(\varkappa), \varkappa)}{\partial \varkappa} &= \frac{c(1-\delta)}{(1-\delta^{\varkappa})}\left[(\delta^{\varkappa-1}(1+\mu)-1) + \delta^{\varkappa-1}((1+\mu)-\delta)\frac{\log\delta^{x}}{(1-\delta^{\varkappa})}\right] \\ &\leq \frac{c(1-\delta)}{(1-\delta^{\varkappa})}\left[(\delta^{\varkappa-1}(1+\mu)-1) - \delta^{\varkappa-1}((1+\mu)-\delta)\right] \\ &= -\frac{c(1-\delta)^2}{(1-\delta^{\varkappa})} < 0,\end{aligned}$$

where the inequality holds as $\log\delta^{x} \leq \delta^{x} - 1$, and the rest is simple algebra.

Proof of Proposition 5

To prove the claim it suffices to show that γ^* is more stable than any other strategy in $\mathcal{E}$. Consider any other strategy $\gamma = (\mu, \sigma, \varkappa) \in \mathcal{S}$. First let us establish that if $\gamma \in \mathcal{E}$, then the sale strategy $\gamma(\varkappa) = (\bar{\mu}, \min\{\kappa(\varkappa), \sigma(\varkappa)\}, \varkappa)$ also belongs to $\mathcal{E}$ and is more stable than γ. Note that

$$\alpha(\varkappa) \equiv \frac{\alpha_0}{\alpha_S\varkappa + \alpha_0} \leq \delta^{\varkappa-1}\left(1+\frac{1}{\mu}\right) - \frac{1}{\mu} \leq \kappa(\varkappa),$$

where the first inequality holds since $\gamma \in \mathcal{E}$, and the second since $\mu \leq \bar{\mu}$. Moreover,

$$\min\{\kappa(\varkappa), \sigma(\varkappa)\} \in [\alpha(\varkappa), \kappa(\varkappa)],$$

since $\sigma(\varkappa) \geq \alpha(\varkappa)$, as $R_1(\gamma) < 1/(1-\delta) < R_0(\gamma)$ for any $\sigma < \sigma(\varkappa)$. Thus, $\gamma(\varkappa) \in \mathcal{E}$. To prove that $\gamma(\varkappa)$ is more stable than γ, first note that the markup μ does not affect $\min_{t\geq 0} R_t(\gamma)$ and increases $\kappa(\varkappa)$, which in turn implies that setting μ to its upper bound cannot reduce the stability. Then note that $\min_{t\geq 0} R_t(\sigma, \varkappa)$ is single peaked in $\sigma \in [0, 1]$, since $R_0(\sigma, \varkappa)$ decreases in σ, since $R_1(\sigma, \varkappa)$ increases in σ, and since $R_0(0, \varkappa) > R_1(0, \varkappa)$ (see the Online Appendix for details). Also notice that the peak $\min_{t\geq 0} R_t(\sigma, \varkappa)$ with respect to σ is achieved exactly at $\sigma = \sigma(\varkappa)$. Thus, if $\sigma(\varkappa) \leq \kappa(\varkappa)$, no strategy with the same cycle length can be more stable than $\gamma(\varkappa)$. If however, $\sigma(\varkappa) > \kappa(\varkappa)$, the most stable strategy must satisfy $\sigma = \sigma(\varkappa)$, since $\min_{t\geq 0} R_t(\sigma, \varkappa)$ increases in σ for $\sigma < \sigma(\varkappa)$.

Next observe that by the implicit function theorem we get that

$$\sigma'(\varkappa) = -\frac{R_{1\varkappa} - R_{0\varkappa}}{R_{1\sigma} - R_{0\sigma}} \geq 0.$$

Note that the denominator is trivially positive (see Online Appendix), and that the numerator is negative since at $\sigma = \sigma(\varkappa)$,

$$R_{0\varkappa} - R_{1\varkappa} = \frac{\alpha_S}{1-\delta^{\varkappa}}\left(1 - \delta^{\varkappa-1}\frac{\sigma}{\alpha_0}\right) + \frac{\delta^{\varkappa-1}\log\delta}{(1-\delta^{\varkappa})^2}\left(\frac{\delta\alpha_0}{\sigma} - 1\right)\left(\frac{\sigma}{\alpha(\varkappa)} - 1\right) \geq 0,$$

where the first term is positive since $\sigma(\varkappa) \leq \alpha_0/\delta^{\varkappa-1}$, and where the second term is positive since $\sigma(\varkappa) \geq \min\left\{\alpha(\varkappa), \alpha_0\right\}$ (see the Online Appendix for details). Hence, since $\kappa'(\varkappa) < 0$, there exists a unique value $\bar{\varkappa}$ such that $\sigma(\bar{\varkappa}) = \kappa(\bar{\varkappa})$.

Let $f(\varkappa) = (\kappa(\varkappa)/\alpha(\varkappa)) - 1$. Note that for a sale strategy to be more stable than a strategy without sales it must be that $f(\varkappa) \geq 0$. Furthermore,

$$\begin{aligned}\frac{\partial}{\partial \varkappa} R_1(\kappa(\varkappa), \varkappa) &= \frac{\delta^{\varkappa-1}}{1-\delta^{\varkappa}}\left[f'(\varkappa) + \frac{\log \delta}{1-\delta^{\varkappa}} f(\varkappa)\right] \\ &\leq \frac{\delta^{\varkappa-1}}{1-\delta^{\varkappa}}\left[f'(\varkappa) - \frac{1-\delta}{1-\delta^{\varkappa}} f(\varkappa)\right] \\ &\leq \frac{\delta^{\varkappa-1}}{(1-\delta^{\varkappa})\,\varkappa}[\varkappa f'(\varkappa) - f(\varkappa)] \leq 0,\end{aligned}$$

where the first inequality holds since $\log \delta \leq \delta - 1$, and the second since $1 - \delta^{\varkappa} \leq \varkappa\,(1-\delta)$. The third inequality holds since

$$\begin{aligned}\varkappa f'(\varkappa) - f(\varkappa) &= \frac{1}{\alpha(\varkappa)}\left[\varkappa\kappa'(\varkappa) - \frac{\varkappa\alpha'(\varkappa)\kappa(\varkappa)}{\alpha(\varkappa)} - \kappa(\varkappa) + \alpha(\varkappa)\right] \\ &= \frac{1}{\alpha(\varkappa)}\left(1 + \frac{1}{\mu}\right)[\log \delta^{\varkappa}(\delta^{\varkappa-1}) + \alpha(\varkappa)(1-\delta^{\varkappa-1})] \\ &\leq \frac{1}{\alpha(\varkappa)}\left(1 + \frac{1}{\mu}\right)[(\delta^{\varkappa} - 1)\delta^{\varkappa-1} + \alpha(\varkappa)(1-\delta^{\varkappa-1})] \\ &\leq \frac{1}{\alpha(\varkappa)}\left(1 + \frac{1}{\mu}\right)[(\alpha(\varkappa) - \delta^{\varkappa-1})(1-\delta^{\varkappa-1})] < 0,\end{aligned}$$

where the first inequality holds, since $\log \delta^{\varkappa} \leq \delta^{\varkappa} - 1$, where the second holds trivially, and where the last inequality holds since $\kappa(\varkappa) \geq \alpha(\varkappa)$ is equivalent to

$$\delta^{\varkappa-1} \geq \alpha(\varkappa) + \frac{\mu+1}{\mu}(1-\alpha(\varkappa)) > \alpha(\varkappa).$$

The last few observations together established that if $\sigma(\varkappa) \geq \kappa(\varkappa)$ for some $\varkappa$, then increasing the cycle length would only reduce the stability of the sale strategy $\gamma(\varkappa)$. In turn this establishes that setting $\varkappa > \breve{\varkappa}$ cannot improve stability.

Finally, note that, if $\bar{\varkappa}$ exits, no strategy with period $\varkappa < \breve{\varkappa}$ can be more stable than $(\bar{\mu}, \sigma(\bar{\varkappa}), \bar{\varkappa})$ by definition of $\bar{\varkappa}$. Thus, the most stable sale strategy will be either $(\bar{\mu}, \sigma(\bar{\varkappa}), \bar{\varkappa})$ or $(\bar{\mu}, \kappa(\breve{\varkappa}), \breve{\varkappa})$ depending on the relative stability of the two.

The observation about profits follows trivially, since changing σ and $\varkappa$ would necessarily reduce stability by construction of γ^* and because

$$\mu^* = \frac{v-c}{c}$$

raises the highest profit and cannot lower stability.

Proof of Proposition 6

By the properties of the profit function discussed in the Online Appendix, profits at time 0 increase in μ, σ, and $\varkappa$. Thus, the most profitable strategy in $\mathcal{E}$ with a cycle of length $\varkappa$ must trivially satisfy

$$\mu = \frac{v-c}{c}$$

and $\sigma = \kappa(\varkappa)$, since

$$\delta^{\varkappa-1}\left(1+\frac{1}{\mu}\right)-\frac{1}{\mu}$$

increases in μ. Thus, the result follows immediately since $\varkappa$ is selected by definition so to maximize profits in $\mathcal{E}$ and since

$$\begin{aligned}
\frac{\partial\Pi_0(\mu,\kappa(\varkappa),\varkappa)}{\partial\varkappa} &= \alpha_0\frac{c(1-\delta)^2\delta^{\varkappa-1}(1+\mu)}{(1-\delta^{\varkappa})^2}\log\delta + \\
&\alpha_S\frac{c(1-\delta)}{(1-\delta^{\varkappa})^2}[(\delta^{\varkappa-1}(1+\mu)-1)(1-\delta^{\varkappa})+\delta^{\varkappa-1}((1+\mu)-\delta)\log\delta^{x}] \\
&\leq \alpha_S\frac{c(1-\delta)}{(1-\delta^{\varkappa})^2}[(\delta^{\varkappa-1}(1+\mu)-1)(1-\delta^{\varkappa})+\delta^{\varkappa-1}((1+\mu)-\delta)\log\delta^{x}] \\
&\leq -\alpha_S\frac{c(1-\delta)^2}{(1-\delta^{\varkappa})} < 0,
\end{aligned}$$

where the second inequality holds as $\log\delta^{x} \leq \delta^{x}-1$, and the rest is simple algebra.

Proof of Proposition 7

First note that if $\Pi > \Pi^+$ no strategy in $\mathcal{E}$ is more profitable than Π^+. Thus, no such profit level can be sustained by a sale strategy belonging to $\mathcal{E}$. If so, the most stable strategy is one without sales. However, all strategies in $\mathcal{N}$ are equally stable by Proposition 4 and thus, $N(\Pi) = 1/(1-\delta)$ for any such strategy.

Then suppose that $\Pi \leq \Pi^*$ and consider any strategy $(\mu,\sigma,2)$ with profits Π. Note that $\Pi \leq \Pi^*$ implies that either $\mu \leq \mu^*$ or $\sigma \leq \sigma^*$. Also note that a different strategy $(\mu',\sigma^*,2)$ exists which raises exactly the same profits, since any profit level $\Pi \leq \Pi^*$ can be obtained by picking $\mu' \in (0,\mu^*]$. Thus, observe that strategy $(\mu',\sigma^*,2)$ is equally stable to strategy $\gamma^* = (\mu^*,\sigma^*,2)$ and thus more stable than $(\mu,\sigma,2)$.

Finally consider the case in which $\Pi \in (\Pi^*,\Pi^+]$. Note that for this to be the case it must be that $\Pi^* < \Pi^+$, which in turn requires

$$\sigma^* = \sigma(2) < \kappa(2) = \sigma^+.$$

Note that setting $\mu = \mu^*$ is always optimal for both profits and stability. Thus, for any profit level $\Pi \in (\Pi^*,\Pi^+]$ a corresponding sales discount exists $\sigma(\Pi) \in (\sigma^*,\sigma^+]$ which sustains profit level Π. Such a discount is found by solving the following equality with respect to σ:

$$\Pi = \left[\alpha_0\delta + \sigma(2-\alpha_0)\right]\frac{\mu^* c}{1+\delta} \quad \Rightarrow \quad \sigma(\Pi) = \frac{1}{2-\alpha_0}\left[\frac{\Pi}{v-c}(1+\delta) - \alpha_0\delta\right].$$

The value of $N(\Pi)$ in such interval can then be found by computing

$$N(\Pi) = R_0(\mu^*, \sigma(\Pi), 2) = \frac{1}{1-\delta}\frac{\Pi}{\sigma(\Pi)\mu^* c} = \frac{1}{1-\delta}\frac{\left(2-\alpha_0\right)\Pi}{(1+\delta)\,\Pi - \delta\alpha_0\,(v-c)},$$

which establishes the desired result.

Proof of Proposition 8

First note when $\Pi \in (\Pi^+, \Pi^m]$ the sign of all the derivatives of $N(\Pi) = 1/(1-\delta)$ is trivial. Next, consider the case in which $\Pi \in (\Pi^*, \Pi^+]$. Note that within such interval $\sigma \in (\sigma^*, \sigma^+]$ and

$$\frac{dN(\Pi)}{d\alpha_0} = \frac{1}{1-\delta}\frac{\left[2\delta\,(v-c) - (1+\delta)\,\Pi\right]\Pi}{\left((1+\delta)\,\Pi - \delta\alpha_0\,(v-c)\right)^2} > 0 \quad \Leftrightarrow \quad 2\delta\,(v-c) > (1+\delta)\,\Pi$$

$$\frac{dN(\Pi)}{d\delta} = \frac{1}{(1-\delta)^2}\left[\frac{\left(2-\alpha_0\right)\left[2\delta\left(\Pi - \alpha_0\,(v-c)\right) + \alpha_0\,(v-c)\right]\Pi}{\left((1+\delta)\,\Pi - \delta\alpha_0\,(v-c)\right)^2}\right] > 0$$

$$\frac{dN(\Pi)}{dv} = -\frac{dN(\Pi)}{dc} = \frac{1}{1-\delta}\frac{\left(2-\alpha_0\right)\alpha_0\delta\Pi}{\left((1+\delta)\,\Pi - \delta\alpha_0\,(v-c)\right)^2} > 0.$$

The second inequality holds, since $\Pi > \sigma\,(v-c)$ and $\sigma > \alpha_0$ together imply $\Pi > \alpha_0\,(v-c)$ (where the first condition holds since $\Pi_0 > \Delta_0$ for the strategy to belong to $\mathcal{E}$, and where the second condition holds since $\sigma > \sigma^*$ and since the only positive root of $\sigma(2)$ satisfies $\sigma^* > \alpha_0$, as explained in the web-appendix).

Before we proceed final scenario $\Pi \in (0, \Pi^*]$, let us prove all the remaining results. First, observe that $d\sigma(2)/dv = d\sigma(2)/dc = 0$, since both R_0 and R_1 are independent of values and costs (see the Online Appendix). Further note that by the implicit function theorem applied to the map $\sigma(2)$,

$$\sigma_\delta = \frac{d\sigma(2)}{d\delta} = -\frac{R_{1\delta} - R_{0\delta}}{R_{1\sigma} - R_{0\sigma}} = -\frac{(1-\alpha_0)\alpha_0\sigma^*}{\delta\left[2\delta(2-\alpha_0)\sigma^* - (1-\alpha_0)\alpha_0\right]}$$

$$\sigma_\alpha = \frac{d\sigma(2)}{d\alpha_0} = -\frac{R_{1\alpha} - R_{0\alpha}}{R_{1\sigma} - R_{0\sigma}} = \frac{\sigma^*(\sigma^*\delta + 1) + 2\alpha_0(\delta - \sigma^*)}{\left[2\delta(2-\alpha_0)\sigma^* - (1-\alpha_0)\alpha_0\right]}.$$

Moreover, note that $d\sigma(2)/d\delta < 0$, since $2\delta(2-\alpha_0)\sigma^* > (1-\alpha_0)\alpha_0$ by definition of σ^*; and that in the only relevant scenario (i.e. $\kappa(2) > \sigma(2)$) $d\sigma(2)/d\alpha_0 > 0$, since $\delta > \kappa(2) > \sigma(2) = \sigma^*$. Also, note that $R_{1\delta} > 0$, $R_{0\delta} < 0$, $R_{1\alpha} < 0$ and $R_{0\alpha} < 0$. The sign of the derivatives of the map $\kappa(2)$ follow trivially from its definition.

Then note that Π^+ and its derivatives with respect to the relevant parameters satisfy

$$\Pi^+ = \frac{\delta\alpha_0}{1+\delta}(v-c) + \frac{2-\alpha_0}{1+\delta}(\delta v - c)$$
$$\frac{d\Pi^+}{d\delta} = \frac{2v+2(1-\alpha_0)c}{(1+\delta)^2} > 0 \ \& \ \frac{d\Pi^+}{d\alpha_0} = \frac{c\,(1-\delta)}{1+\delta} > 0$$
$$\frac{d\Pi^+}{dv} = \frac{2\delta}{1+\delta} > 0 \ \& \ \frac{d\Pi^+}{dc} = -\frac{2-\alpha_0(1-\delta)}{1+\delta} < 0.$$

To compute the derivatives of Π^*, consider the case in which $\kappa(2) > \sigma(2)$—or else, Π^* and Π^+ and their respective derivatives would coincide. If so,

$$\Pi^* = \left[\frac{\delta\alpha_0}{1+\delta} + \frac{2-\alpha_0}{1+\delta}\sigma(2)\right](v-c)$$
$$\frac{d\Pi^*}{d\delta} = \frac{1}{(1+\delta)^2}\left[\alpha_0 - \left(2-\alpha_0\right)\sigma(2) + \left(2-\alpha_0\right)(1+\delta)\frac{d\sigma(2)}{d\delta}\right](v-c) < 0$$
$$\frac{d\Pi^*}{d\alpha_0} = \left[\frac{\delta-\sigma(2)}{1+\delta} + \frac{2-\alpha_0}{1+\delta}\frac{d\sigma(2)}{d\alpha_0}\right](v-c) > 0$$
$$\frac{d\Pi^*}{dv} = -\frac{d\Pi^*}{dc} = \left[\frac{\delta\alpha_0}{1+\delta} + \frac{2-\alpha_0}{1+\delta}\sigma(2)\right] > 0,$$

where the first inequality holds since $\mathcal{E} \neq \emptyset$ implies $\alpha_0 - \left(2-\alpha_0\right)\sigma(2) \leq 0$, and the second inequality holds since $\kappa(2) > \sigma(2)$ implies $\delta > \sigma(2)$. At last, consider the case in which $\Pi \in (0, \Pi^*]$. Suppose that $\Pi^* \neq \Pi^+$. If so, $\kappa(2) > \sigma(2)$ and therefore

$$\frac{dN(\Pi)}{d\delta} = \frac{1}{(1-\delta)^2}\left[R_0 + (1-\delta)\left[R_{0\delta} + R_{0\sigma}\sigma_\delta\right]\right] > 0 \tag{A.3}$$

$$\frac{dN(\Pi)}{d\alpha_0} = \frac{1}{1-\delta}\left[R_{0\alpha} + R_{0\sigma}\sigma_\alpha\right] < 0 \tag{A.4}$$

$$\frac{dN(\Pi)}{dv} = \frac{dN(\Pi)}{dc} = 0, \tag{A.5}$$

where (A.5) holds trivially, where (A.3) is positive because $R_{0\sigma}\sigma_\delta > 0$ and because

$$R_0 + (1-\delta)\,R_{0\delta} = \frac{\alpha_0}{\sigma} + \frac{2\delta}{(1+\delta)^2}\left[(2-\alpha_0) - \frac{\alpha_0}{\sigma}\right] > 0,$$

and where (A.14) is negative since

$$R_{0\alpha} + R_{0\sigma}\sigma_\alpha = \frac{R_{1\sigma}R_{0\alpha} - R_{0\sigma}R_{1\alpha}}{R_{1\sigma} - R_{0\sigma}} = -\frac{\alpha_0\delta + \sigma\left(2-\alpha_0\right)}{R_{1\sigma} - R_{0\sigma}}\frac{\delta}{\sigma\alpha_0\,(1+\delta)^2} < 0.$$

Finally, consider the case in which $\Pi \in (0, \Pi^*]$ and $\Pi^* = \Pi^+$. If so, $\kappa(2) \leq \sigma(2)$ and

$$\frac{dN(\Pi)}{d\delta} = \frac{1}{(1-\delta^2)^2}\left[2\delta + \frac{2-\alpha_0}{\alpha_0}\left[(1+\delta^2)\kappa + (1-\delta^2)\kappa_\delta\right]\right] > 0$$

$$\frac{dN(\Pi)}{d\alpha_0} = -\frac{\delta}{1-\delta^2}\frac{2\kappa}{\alpha_0^2} < 0$$

$$\frac{dN(\Pi)}{dv} = \frac{\delta}{1-\delta^2}\frac{2-\alpha_0}{\alpha_0}\kappa_v > 0 \ \ \& \ \ \frac{dN(\Pi)}{dc} = \frac{\delta}{1-\delta^2}\frac{2-\alpha_0}{\alpha_0}\kappa_c < 0,$$

which concludes the proof.

Proof of Proposition 9

Assume that the discount factor δ satisfies

$$\delta > \frac{1 + \dfrac{\alpha_0}{(2-\alpha_0)}\bar{\mu}}{1 + \alpha_0\bar{\mu}}. \tag{A.6}$$

This implies that $(2-\alpha_0)\delta > 1$. To establish claim (1) consider any strategy $\gamma = (\mu, 1, \varkappa) \in \mathcal{N}$ and consider the strategy $\gamma = (\mu^n, 1, \varkappa) \in \mathcal{N}$ where

$$\mu^n = \frac{(2-\alpha_0)(1-\delta)}{(2-\alpha_0)\delta - 1}.$$

Condition (A.6) implies that $\mu^n > 0$. If $\mu \leq \mu^n$ (that is if $\Pi(\gamma) \leq \Pi^n$), then $(2-\alpha_0)\hat{\mu}(\gamma) \leq \mu$ and $n(\gamma) = \frac{1}{1-\delta}$. Alternatively, if $\mu > \mu^n$ (that is if $\Pi(\gamma) > \Pi^n$) and $(2-\alpha_0)\hat{\mu}(\gamma) > \mu$ and

$$n(\gamma) = \frac{1}{1-\delta}\frac{\mu}{(2-\alpha_0)[\delta(1+\mu) - 1]},$$

these observations then immediately yield the result of claim (1) since $\Pi(\gamma) = \mu c$.

To establish claim (2) consider any strategy $\gamma = (\mu, \sigma, 2) \in \mathcal{C}$ and recall that

$$\Pi(\gamma) = \frac{(2-\alpha_0)\sigma + \delta\alpha_0}{1+\delta}\mu c.$$

Hence, profits increase both in μ and in σ provided that the storage constraint is satisfied $\sigma \leq \kappa(2)$. Provided that $\delta(1+\sigma\mu) - 1 > 0$, simple algebra also establishes that

$$\frac{\left[\delta\alpha_0 + (2-\alpha_0)\sigma\right]}{(2-\alpha_0)\sigma} \leq \frac{\left[\alpha_0 + \delta(2-\alpha_0)\sigma\right]\mu}{\delta(1+\sigma\mu) - 1}.$$

Therefore, the incentives to deviate in periods of sales exceed those in periods without sales if the deviation entails selling to players with units already stored. The following simplification obtains:

$$n(\gamma) = \frac{1}{1-\delta^2} \min\left\{ \frac{\left[\delta\alpha_0 + (2-\alpha_0)\sigma\right]}{(2-\alpha_0)\sigma}, \frac{\left[\alpha_0 + \delta(2-\alpha_0)\sigma\right]}{\alpha_0} \right\}.$$

If $\sigma < \alpha_0/(2-\alpha_0)$, the first term in the minimum exceeds the second. The converse holds if $\sigma > \alpha_0/(2-\alpha_0)$, and the two terms coincide whenever $\sigma = \alpha_0/(2-\alpha_0)$. Thus, any strategy $\tilde{\gamma} = (\mu, \alpha_0/(2-\alpha_0), 2)$ satisfies $n(\tilde{\gamma}) = 1/(1-\delta)$ and $\Pi(\tilde{\gamma}) = \alpha_0\mu c$. Increasing the markup μ to its upper bound $\bar{\mu}$ increases profits with no effect on stability. Moreover, when $\mu = \bar{\mu}$, the strategy satisfies the storage constraint, because

$$\delta(1+\bar{\mu}) > \left(1 + \frac{\alpha_0}{2-\alpha_0}\bar{\mu}\right)$$

by (A.6). Any lower profit level can also be sustained by strategies with a lower regular markup μ and the same discount $\alpha_0/(2-\alpha_0)$, provided that consumers are sufficiently patient, so that $\alpha_0/(2-\alpha_0) \le \kappa(2)$. To conclude the proof of part (2) observe that whenever $\sigma > \alpha_0/(2-\alpha_0)$,

$$n(\gamma) = \frac{1}{1-\delta^2}\left[\frac{\delta\alpha_0 + (2-\alpha_0)\sigma}{(2-\alpha_0)\sigma}\right] = \frac{1}{1-\delta}\left[\frac{\Pi(\gamma)}{(1+\delta)\Pi(\gamma) - \alpha_0\mu c}\right],$$

where the equality obtains by definition of $\Pi(\gamma)$. Moreover, since increasing the markup μ to its upper-bound $\bar{\mu}$ increases profits with no effect on stability, we obtain the desired result,

$$N_s(\Pi) = \frac{1}{1-\delta}\left[\frac{\Pi}{(1+\delta)\Pi - \alpha_0(v-c)}\right] \text{ for any } \Pi \ge \Pi^s.$$

Clearly Π cannot exceed Π^+ or else the storage constraint would not be satisfied. Moreover, $\Pi^+ > \Pi^s$ is equivalent to

$$\delta > \frac{1}{1 + \dfrac{2-2\alpha_0}{2-\alpha_0}\bar{\mu}},$$

which holds always by condition (A.6).

To establish claim (3) observe that the largest profit at which $N_n(\Pi) = 1/(1-\delta)$, amounts to Π^n. Instead, the largest profit at which $N_s(\Pi) = 1/(1-\delta)$ amounts to Π^s. Moreover, condition (A.6) is equivalent to

$$\frac{(2-\alpha_0)(1-\delta)}{(2-\alpha_0)\delta - 1}c < \alpha_0(v-c),$$

and therefore $N_s(\Pi) > N_n(\Pi)$, whenever $\Pi \in (\Pi^n, \Pi^s]$. However, strategies in $\mathcal{C}$ remain more stable than strategies in $\mathcal{N}$ until some threshold $\Pi^\circ > \Pi^s$. In particular, if Π^- denotes the unique root to the equation $N_s(\Pi^-) = N_n(\Pi^-)$, we have that $\Pi^\circ = \min\{\Pi^+, \Pi^-\}$.

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Supporting Information

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Online Appendix