Preemption and the Efficiency of Entry*

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Abstract

This paper studies the timing of entry into a new market. Potential entrants differ in their efficiency. In an entry game with two firms, the more efficient firm always enters first in equilibrium. In a game with three firms instead, the equilibrium order of entry does not necessarily reflect the efficiency ranking. We characterize conditions on the primitives of the model that induce an inefficient order of entry.

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1 Introduction

The order of entry in a new market does not always reflect the relative efficiency of the entering firms. A more efficient firm earns higher per-period profits upon entry, hence one may conclude that it has a stronger incentive to enter than a less efficient firm. Therefore, it should enter earlier, even if this comes at a higher cost. This is not always the case.

In several examples of new markets, the firms who enjoyed a cost advantage due to economies of scope with their pre-existing activities were late entrants. Procter and Gamble, who dominated the market for granular detergents, entered the market for liquid laundry detergents with its brand Era only in 1972, following the pioneer Lever Brothers, who had introduced Wisk in 1956. In the wine cooler market, the pioneer was California Cooler in 1981, while Gallo and Seagram, who dominated the wine and distilled spirit industries, followed only a few years later. The firm who pioneered the mass market for sugar-free diet beverages was RC Cola in 1961, followed only years later by Pepsi (1964) and Coca Cola (1982).[1]

When planned economies opened markets to Western firms, the first entrants were not always the strongest firms. In 1995, the first entrant in the Bulgarian beer market was Interbrew, while the European market leader Heineken was second.[2] In 1992, four small foreign firms were present in the Chinese market ((San Miguel, Asia-Pacific Breweries, Pabst, and Beck's) while the global market leaders (Annheuser-Busch, Heineken, South African Breweries (SAB), Interbrew) only followed a few years later.[3]

One explanation of the fact that the order of entry in a market does not always reflect relative efficiency is that in some cases efficiency is endogenous. With ongoing technological progress, later entrants are endogenously more efficient because the choice to wait allows them to develop a better production process, or a higher-quality product. This explanation is analyzed by Dutta Lach and Rustichini (1995) for the case of costless technological progress, and by Hoppe and Lehman-Grube (2001) for the more general case of potentially

[1] See Tellis and Golder (1996).
[2] See Marinov and Marinova (1999).
[3] See Heracleous (2001).

costly R&D.

In this paper, we provide an alternative explanation that may be more suitable for markets for technologically "mature" products. For these products, asymmetries in productive efficiency and/or in product quality are already given at the time the new market opens. We construct a model of a dynamic entry game with entry cost declining over time. Our main result is that for this class of markets, the intuition that a more efficient firm has a stronger incentive to enter than a less efficient firm, hence it should enter earlier, is correct if the game is played by two firms but can fail when there are three firms: adding a third competitor qualitatively changes the equilibrium outcome.

In particular, we consider a game where three firms have to decide when to enter a new market. Post-entry profits are declining in the number of rival entrants, and the cost of entry declines exogenously over time. This is a *preemption game*: Firms want to delay entry in order to reduce the entry cost, but they also want to enter earlier than their rivals in order to earn higher flow profits before the following entries.

To show that with more than two potential entrants the order of entry does not necessarily reflect relative efficiency, we consider a tractable extension of the two-firm model: the case of three potential entrants. In our setup, the entry game is played by one efficient firm ("type A" firm) and two inefficient firms ("type B" firms). We show that with a general payoff structure, the unique equilibrium outcome may be such that the order of entry is $B - A - B$. That is, one of the inefficient firms enters first, the efficient firm follows strictly later, to be followed by the remaining inefficient firm. We also provide sufficient conditions on the primitives of the model, that is on firms' post-entry profits, that guarantee that this is indeed the equilibrium entry order. In the Appendix, we numerically obtain the range of parameters for which the equilibrium entry order is $B - A - B$ in an example where profits are derived from Cournot competition.

The intuition behind our result is as follows. When a firm considers entering first, it takes into account for how long it will earn monopoly profits. Thus the incentive to enter first, to preempt its rivals, depends on the timing of second entry, which in turn depends on the intensity of the preemption race in the ensuing two-player subgame. If firm A enters first, the resulting subgame among the two type B firms can involve a relatively intense preemption race, and thus second entry will occur relatively soon. If however a type B firm enters first, the resulting subgame among firm A and the remaining type B firm involves relatively weak preemption incentives: The second entrant is firm A, and it can afford to wait long and enter at a relatively low cost, because B is a weak competitor. As we show, it can hold that as a result the first entrant enjoys monopoly profits for a shorter period if it is of type A than if it is of type B. This shorter monopoly period can outweigh the fact that the monopoly profit flow earned would be higher for the more efficient firm. As a result, in equilibrium one of the inefficient type B firms will enter first. The exact timing of first entry will be determined by a preemption race among the two inefficient firms. Rents among the two inefficient type B firms are equalized.

The conditions on the primitives of the model that guarantee that the first entrant is one of the less efficient firms are clearly related to the above intuition: first, post-entry profits have to be such that the preemption race to be second rather than third is more intense when played by two type B firms than by A and a type B firm. Second, monopoly profits cannot be too much higher for A than for a B type firm, or else A would have a strong incentive to be the first entrant, even if the monopoly period were short.

Our model builds on the classic literature on two-player preemption games. We rely on Fudenberg and Tirole (1985) to derive the outcome of the two-player symmetric subgame played by two inefficient firms. From Riordan's (1992) analysis, we obtain the outcome of the two-player asymmetric subgame. It follows from Riordan (1992) that in a two-player asymmetric game, the more efficient firm always enters first. We contribute to this literature by showing that this order of entry may be reversed when adding a third firm.

Our result can also be relevant to the recent empirical literature on static entry games with asymmetric potential entrants. Following Berry (1992), it is often assumed that entry occurs in the order of profitability to solve the inherent multiplicity problem. Indeed, Quint and Einav (2005) show that this assumption can be rationalized by the outcome of a war of attrition where entry costs are sunk gradually: The entrants in the unique subgame perfect equilibrium in such a game are the most efficient firms. Our game shows that if the underlying game is a preemption game this result may be reversed: At any given point in time, the firms observed in the market are not necessarily the most efficient ones.

Similarly, our result may be important when evaluating potential anticompetitive effects of a merger. Section 3 of the Merger Guidelines[4] states that "A merger is not likely to create or enhance market power [...] if entry into the market is so easy that market participants [...] could not profitably maintain a price increase above premerger levels. Such entry likely will deter an anticompetitive merger [...]." Therefore, in evaluating the role of potential entry in deterring anticompetitive mergers, one needs to predict the efficiency of the firm most likely to enter first after the merger. Our result shows that this might not be the most efficient among the potential entrants.

2 Model

We model entry in a new market as an infinite horizon dynamic game in continuous time. Our assumptions correspond to those made by Fudenberg and Tirole (1985) and Riordan (1992), when specialized to the case of a new market, and with a third firm added to the model. In particular, we consider a model with one efficient firm, firm A, and two identical "type B" firms. Each firm has to decide whether and when to enter a new market. Before entry, it receives no profits. Upon entry, firm i (for $i = A, B$) earns flow profits $\pi_i(m, -i)$, where m is the total number of firms that have entered, hence $m \in \{1, 2, 3\}$, and $-i$ stands for the identity of rival firms that have entered. For example, $\pi_B(2, A)$ stands for the profits of a type B firm in duopoly if its rival is firm A. The following profits are relevant in our model:

	Firm A	Type B firms	
Monopoly	$\pi_A(1)$	$\pi_B(1)$	
Duopoly	$\pi_A(2)$	$\pi_B(2, A)$	$\pi_B(2, B)$
Triopoly	$\pi_A(3)$	$\pi_B(3)$	

In monopoly there are no rival firms and in triopoly the identity of rivals is uniquely identified by the identity of firm i. Similarly, in a duopoly firm A will always oppose a type

[4]See US Department of Justice and Federal Trade Commission (1992).

B firm. Hence, to economize on notation, we leave out the $-i$ term in the monopoly and triopoly cases as well as for firm A's duopoly profits.

All the above profits are positive. For a given firm, profits decline in the number of competitors. Moreover, firm A's higher efficiency is reflected in payoffs.[5] Firm A always earns higher profits than a type B firm, for a given number of competitors. Also, a type B firm earns lower profits if its opponent is firm A than if its opponent is the other type B firm, i.e. profits decline in the efficiency of rival firms. Formally:

Assumption 1

(i) $\pi_i(m, -i) > 0 \qquad \forall (m, -i)$

(ii)

$$\begin{aligned}
&\pi_A(1) > \pi_B(1) \\
&\pi_A(2) > \pi_B(2, B) > \pi_B(2, A) \\
&\pi_A(3) > \pi_B(3) \\
&\pi_A(1) > \pi_A(2) > \pi_A(3) \\
&\pi_B(1) > \pi_B(2, B) > \pi_B(2, A) > \pi_B(3)
\end{aligned}$$

In Appendix A we consider an example of post-entry competition that gives rise to a profit structure that satisfies Assumption 1: Firms compete *à la* Cournot with constant marginal costs, and firm A's marginal cost is strictly less then the type B firms' marginal cost.

The present value at time zero of entering the market at time t is $c(t)$. Following the literature,[6] we assume that it declines over time, at a decreasing rate:

Assumption 2

(i) $\left(c(t)\, e^{rt}\right)' < 0 \ \forall t$

(ii) $\left(c(t)\, e^{rt}\right)'' > 0 \ \forall t$

[5] We assume that payoffs are independent of the time of entry, which implies that there is no ongoing technological progress. Dutta, Lach and Rustichini (1995) and Hoppe and Lehman-Grube (2001) analyze the case where efficiency is endogenously determined by the time of entry.

[6] See Fudenberg and Tirole (1985) and Riordan (1992).

The payoff function for firm i, conditional upon a given entry order in which i is the j-th entrant, as a function of its own entry time t_j and the competitors' entry times t_{-j} is:

$$f_i\left(t_j, t_{-j}\right) \equiv \sum_{m=1}^{3} I\left[j \leq m\right] \cdot \int_{t_m}^{t_{m+1}} \pi_i(m, -i) e^{-rs} ds - c\left(t_j\right) \qquad (*)$$

where $I\left[\cdot\right]$ is the indicator function and $t_4 \equiv +\infty$. Before t_j, firm i receives zero profits. Then, it receives flow profits $\pi_i(m, -i)$ depending on the number and identity of the competitors present in the market. Finally, $c\left(t_j\right)$ denotes entry cost.

Assumption 3 normalizes time as to guarantee that entry at time 0 is not profitable.

Assumption 3

(i) $\frac{\pi_A(1)}{r} - c(0) < 0$

(ii) $-c'(0) > \pi_A(1)$

Part (i) implies that even if a firm could preempt all its rivals and earn monopoly profits forever, entry at time zero would be too costly. Part (ii) guarantees that $f_i\left(t_j\right)$ is increasing at time zero.

Assumption 4 instead guarantees that the entry cost eventually becomes so low, and slows down so much, that even entry with the lowest possible profits dominates staying out.

Assumption 4

(i) $\exists \tau$ *such that* $c\left(\tau\right) e^{r\tau} < \frac{\pi_B(3)}{r}$

(ii) $\lim_{t \to \infty} c'(t) e^{rt} \in \left(-\pi_B\left(3\right), 0\right]$

Next, to highlight the trade-offs faced by each firm, we extend the terminology in Katz and Shapiro (1987) and define the *stand-alone entry time* for the profit flow $\pi_i(m, -i)$. Consider the hypothetical problem of firm i, if it could act as a single decision maker and select the optimal time to make an investment which has cost $c\left(t\right)$ and guarantees flow payoff of $\pi_i(m, -i)$ forever, where $c\left(t\right)$ and $\pi_i(m, -i)$ satisfy assumptions 1 and 2. This firm would solve the following problem:

$$\max_t g_{i,m,-i}\left(t\right) \equiv \frac{\pi_i(m, -i)}{r} e^{-rt} - c\left(t\right). \qquad (1)$$

We denote the solution to this problem as $T_i^*(m,-i)$. Our assumptions guarantee that $g_{i,m,-i}(t)$ is strictly quasi-concave in t and admits a strictly positive finite maximum, which solves the first order condition:

$$-\pi_i(m,-i)e^{-rt} - c'(t) = 0.$$

The condition is easily interpreted: a marginal delay of entry implies foregone profits $\pi_i(m,-i)e^{-rt}$ and cost savings $c'(t)$. Given the quasiconcavity of $g_{i,m,-i}(t)$ in t_j, it follows that $T_i^*(m,-i)$ is decreasing in $\pi_i(m,-i)$. Hence, the following inequalities follow from assumption $1(ii)$:

$$\begin{aligned}
T_A^*(1) &< T_B^*(1) \\
T_A^*(2) &< T_B^*(2,B) < T_B^*(2,A) \\
T_A^*(3) &< T_B^*(3) \\
T_A^*(1) &< T_A^*(2) < T_A^*(3) \\
T_B^*(1) &< T_B^*(2,B) < T_B^*(2,A) < T_B^*(3)
\end{aligned}$$

It is clear that firm A's stand-alone entry time, *for a given rank in the entry order,* is always earlier than that of a less efficient firm: By delaying entry, A would forego a higher profit than a type B firm would.

In order to model entry as a preemption game of complete information in continuous time, we follow Hoppe and Lehmann-Grube (2005), who illustrate how to adopt the framework introduced by Simon and Stinchcombe (1989) for this class of games. As in Simon and Stinchcombe (1989), we restrict play to pure strategies and interpret continuous time as "discrete time, but with a grid that is infinitely fine."

Moreover, we need to address the issue of nonexistence of an equilibrium in pure strategies in preemption games, which is related to the possibility of coordination failures.[7] Since we adopt the Simon and Stinchcombe (1989) framework, we need to explicitly rule out the

[7] See the example in Simon and Stinchcombe (1989, p. 1178-1179).

possibility of coordination failures,[8] and we do so using a randomization device as in Katz and Shapiro (1987), Dutta, Lach and Rustichini (1995), and Hoppe and Lehmann-Grube (2005):

Assumption 5

If n firms invest at the same instant t (with $n \in [2, N]$), then only one firm, each with probability $\frac{1}{n}$, succeeds.

Assumption 5 rules out the possibility of coordination failures and thus ensures existence of an equilibrium in pure strategies.

3 Inefficient Entry

In this section, it is shown that while in a preemption race with only two asymmetric players the first firm to enter is always the most efficient one, this result can be reversed if the game is played by more than two players.

3.1 The Game with Two Asymmetric Firms.

Let us first consider a preemption game played by two asymmetric firms, that is one efficient firm of type A and one inefficient firm of type B. In such a game, it follows from the analysis in Riordan (1992) that the first entrant is A. Below, we illustrate the key mechanism behind Riordan's (1992) result in the context of our model.

Consider the payoff for each firm in case it is the "leader" in this game, i.e. the first entrant, or the "follower", i.e. the second entrant. Suppose that firm A enters first at time t. Provided that t is earlier than firm B's stand-alone entry time $T_B^*(2, A)$, B follows exactly at at $T_B^*(2, A)$. Hence, firm A receives payoff

$$\widetilde{L}_A(t) = \pi_A(1) \int_t^{T_B^*(2,A)} e^{-rs} ds + \pi_A(2) \int_{T_B^*(2,A)}^{\infty} e^{-rs} ds - c(t)$$

[8] This observation is due to Hoppe and Lehmann-Grube (2005).

and firm B receives payoff

$$\widetilde{F}_B(t) = \pi_B(2) \int_{T_B^*(2,A)}^{\infty} e^{-rs} ds - c(T_B^*(2,A)).$$

Suppose instead that firm B enters first at time t. Provided that t is earlier than firm A's stand-alone entry time $T_A^*(2)$, A follows exactly at $T_A^*(2)$. The leader's and follower's payoff are

$$\widetilde{L}_B(t) = \pi_B(1) \int_t^{T_A^*(2)} e^{-rs} ds + \pi_B(2,A) \int_{T_A^*(2)}^{\infty} e^{-rs} ds - c(t)$$

and

$$\widetilde{F}_A(t) = \pi_A(2) \int_{T_A^*(2)}^{\infty} e^{-rs} ds - c(T_A^*(2)).$$

respectively. Notice that, as we argue in section 2, the fact that $\pi_A(2) > \pi_B(2, A)$ implies that firm A's optimal entry time as a follower is earlier than firm B's: $T_A^*(2) < T_B^*(2, A)$.

Now consider the incentive for each firm to preempt the competitor and be the leader, rather than the follower. Firm A prefers to enter as a leader at time t whenever

$$\begin{aligned}
\widetilde{D}_A(t) &= \widetilde{L}_A(t) - \widetilde{F}_A(t) \\
&= \pi_A(1) \int_t^{T_B^*(2,A)} e^{-rs} ds - \pi_A(2) \int_{T_A^*(2)}^{T_B^*(2,A)} e^{-rs} ds + \\
&\quad -[c(t) - c(T_A^*(2))]
\end{aligned}$$

is positive. Similarly, firm B prefers to enter as leader at t if

$$\begin{aligned}
\widetilde{D}_B(t) &= \widetilde{L}_B(t) - \widetilde{F}_B(t) \\
&= \pi_B(1) \int_t^{T_A^*(2)} e^{-rs} ds + \pi_B(2,A) \int_{T_A^*(2)}^{T_B^*(2,A)} e^{-rs} ds \\
&\quad -[c(t) - c(T_B^*(2,A))]
\end{aligned}$$

is positive.

It follows immediately from Riordan (1992) that firm A is the leader in equilibrium. This result is based on the comparison between the two $\widetilde{D}_i(t)$ functions. More precisely, on the fact that for any time t earlier than $T_A^*(2)$, the incentive to preempt is always stronger for A than for B, that is $\widetilde{D}_A(t) > \widetilde{D}_B(t)$. The crucial observation to prove this inequality is that $T_A^*(2) < T_B^*(2,A)$. Consider the first term in $\widetilde{D}_A(t)$ and $\widetilde{D}_B(t)$. Monopoly profits are not only higher for A, but also earned for a longer period, that is until $T_B^*(2,A)$ rather than until $T_A^*(2)$. Consider then the third term in $\widetilde{D}_A(t)$ and $\widetilde{D}_B(t)$. Anticipating entry from $T_A^*(2)$ to t is cheaper than anticipating entry from $T_B^*(2,A)$ to t. To complete the argument, one only needs to show that the increase in duopoly profits for B and the decrease in duopoly profits for A do not offset the previous two effects. The intuition is as follows.[9] By preempting B, firm A delays the date from which it earns duopoly profits from $T_A^*(2)$ to $T_B^*(2,A)$. In the interval $[T_A^*(2), T_B^*(2,A))$ duopoly profits are replaced by monopoly profits, so the total effect is still positive. B instead, by preempting A, anticipates the date from which it earns duopoly profits, from $T_B^*(2,A)$ to $T_A^*(2)$. By definition of $T_B^*(2,A)$, anticipating entry as a duopolist to the left of this point is detrimental: extra duopoly profits are more than offset by the increase in entry cost.

Consider now the remaining information we have about $\widetilde{D}_A(t)$ and $\widetilde{D}_B(t)$. First, they are both negative for $t = 0$: by Assumption 2 preemption is too costly at time zero. Moreover, they are both strictly quasi-concave, because of the convexity of the cost function, and have a maximum in $T_i^*(1)$ for $i = A, B$ respectively. Finally, in $t = T_A^*(2)$, the function $\widetilde{D}_A(t)$ is strictly positive.

Following the argument in Riordan (1992), the equilibrium has the following features. First entry cannot take place for t very close to zero, because $\widetilde{D}_A(t)$ and $\widetilde{D}_B(t)$ are both negative. From some $\widetilde{T}_A^1 < T_A^*(2)$ onwards, $\widetilde{D}_A(t)$ becomes positive: firm A would rather be leader than follower, and ideally it would like to delay first entry until $T_A^*(1)$. If $\widetilde{D}_B(t)$ is negative in the interval $\left[\widetilde{T}_A^1, T_A^*(1)\right]$, firm B has no incentive to enter before $T_A^*(1)$, and firm A can therefore not only be the first to enter, but also enter at its preferred time. If instead $\widetilde{D}_B(t)$ is positive from some point $\widetilde{T}_B^1 \in \left(\widetilde{T}_A^1, T_A^*(1)\right)$ onwards, then A is forced to anticipate first entry to $t = \widetilde{T}_B^1$ by the threat of preemption.[10] Following the terminology in Riordan (1992), we refer to firm A as a “strong leader” if $T_A^*(1) < T_B^1$ and as a “weak

[9] For a formal proof of a similar argument, see the proof of Claim 4 in the Appendix.

[10] The fact that $D_A(t) > D_B(t)$ guarantees that $T_B(1) < T_A(1)$.

leader" otherwise.

In any case, B cannot enter first in equilibrium. In a candidate equilibrium with first entry by B at some time t, it has to hold that in t firm B strictly prefers the leader's payoff to the follower's payoff. But then $\widetilde{D}_A(t)$ is also positive, hence A can profitable deviate preempting B and entering at $(t-\varepsilon)$.

3.2 The Game with Three Firms.

Here we consider the game with one A firm and two B firms. We show that the efficient-entry result in the two-firm game can be reversed when there are three firms. More precisely, we show that the unique equilibrium outcome can be that the entry order is $B-A-B$ and provide sufficient conditions on post-entry profits to guarantee that this happens.

With a construction similar to the one presented in subsection 3.1, we first derive the payoff from being the leader, or one of the followers, for each firm. Then, we compute the preemption incentives, that is the difference between the leader's and the follower's payoff for each firm. Finally, we show under which circumstances the entry order in equilibrium will be $B-A-B$. We will show that the main difference with the two-firm case is that with three asymmetric firms it is not always the case that the most efficient firm has a strictly stronger preemption incentive.

If firm A enters first in this game, the ensuing subgame will be played by two firms of type B. We call this subgame the "BB-subgame." If on the other hand a firm of type B enters first, the ensuing subgame will be played by firm A and the remaining type B firm. We call this the "AB-subgame." The unique equilibrium outcomes of these two-player subgames are readily known from work by Fudenberg and Tirole (1985) for the BB-subgame and by Riordan (1992) for the AB-subgame. It follows from their analysis that the first entry times of both subgames are uniquely determined. We we will denote them as $t_2(ABB)$ and $t_2(BAB)$ respectively. They further show that in either subgame last entry is by a B firm, and occurs at $T^*_B(3)$, a type B's stand-alone time as a last entrant.

Consider the case in which the type A firm preempts its rivals and enters first at time t. The B firms will follow at $t_2(ABB)$ and $T^*_B(3)$ respectively, hence firm A will earn a leader payoff:

$$L_A(t) = \pi_A(1)\int_t^{t_2(ABB)} e^{-rs}ds + \pi_A(2)\int_{t_2(ABB)}^{T_B^*(3)} e^{-rs}ds + \pi_A(3)\int_{T_B^*(3)}^{\infty} e^{-rs}ds - c(t)$$

and each of the B firms will earn a follower payoff:

$$\begin{aligned} F_B(t) &= \pi_B(2,A)\int_{t_2(ABB)}^{T_B^*(3)} e^{-rs}ds + \pi_B(3)\int_{T_B^*(3)}^{\infty} e^{-rs}ds - c\left(t_2(ABB)\right) \\ &= \pi_B(3)\int_{T_B^*(3)}^{\infty} e^{-rs}ds - c\left(T_B^*(3)\right) \end{aligned}$$

where the l.h.s. of the last equality represents the payoff of the early entrant in the subgame, the r.h.s. represents the payoff of the late entrant, and the equality describes the "rent equalization" result in Fudenberg-Tirole (1985).

If instead a type B firm enters first, a two-player game with asymmetric firms ensues in which the type A firm enters at $t_2(BAB)$ and the remaining B firm at time $T_B^*(3)$. The early B firm obtains the leader payoff:

$$L_B(t) = \pi_B(1)\int_t^{t_2(BAB)} e^{-rs}ds + \pi_B(2,A)\int_{t_2(BAB)}^{T_B^*(3)} e^{-rs}ds + \pi_B(3)\int_{T_B^*(3)}^{\infty} e^{-rs}ds - c(t).$$

Firm A is preempted and thus earns the follower payoff:

$$F_A(t) = \pi_A(2)\int_{t_2(BAB)}^{T_B^*(3)} e^{-rs}ds + \pi_A(3)\int_{T_B^*(3)}^{\infty} e^{-rs}ds - c(t_2(BAB))$$

and the late B firm obtains follower's payoff $F_B(t)$.[11]

We can now write the incentive to be the first entrant in the game for an efficient and an inefficient firm, respectively.

[11] Observe that a B firm's follower payoff is independent of whether the subgame following first entry is between two type B firms or one type A and one type B firm.

Firm A would like to preempt its rivals whenever

$$\begin{aligned} D_A(t) &= L_A(t) - F_A(t) \\ &= \pi_A(1) \int_t^{t_2(ABB)} e^{-rs} ds + \pi_A(2) \int_{t_2(ABB)}^{t_2(BAB)} e^{-rs} ds - c(t) + c\left(t_2(BAB)\right) \end{aligned}$$

is positive. By preempting the rivals, firm A gains monopoly profits from time t until $t_2(ABB)$, achieves duopoly profits starting from $t_2(ABB)$ rather than $t_2(BAB)$, and finally sustains a higher entry cost because it enters earlier.

Similarly, a type B firm prefers to be the leader rather than the follower if

$$\begin{aligned} D_B(t) &= L_B(t) - F_B(t) \\ &= \pi_B(1) \int_t^{t_2(BAB)} e^{-rs} ds + \pi_B(2, A) \int_{t_2(BAB)}^{t_2(ABB)} e^{-rs} ds - c(t) + c\left(t_2(ABB)\right) \end{aligned}$$

is positive. By preempting the rivals, a B firm gains monopoly profits from t until $t_2(BAB)$, achieves duopoly profits starting from $t_2(BAB)$ rather than $t_2(ABB)$, and finally sustains a higher entry cost because it enters earlier.

In order to characterize a region of the primitives of the model for which the equilibrium entry order is $B - A - B$, we need to find conditions that guarantee that the preemption incentive as described by the functions $D_i(t)$ for $i = A, B$ is "stronger" for the type-B firms than for A, in a sense that we formalize in the next section.

3.3 The inefficient equilibrium.

The formal construction we present here is analogous to the ones presented in Fudenberg and Tirole (1985) and Riordan (1992). Consider the $D_i(t)$ functions, for $i = A, B$. In $t = 0$, they are both negative, because by Assumption 3 preemption is too costly at time zero. Moreover, they are both strictly quasi-concave because of the convexity of the cost function, and have a maximum in $T_i^*(1)$ for $i = A, B$ respectively.

Suppose that the earliest point in time in which a B firm weakly prefers to be a leader rather than a follower is even earlier than the first point in time in which A weakly prefers to be the leader. That is, the smallest point in which $D_B(t)$ intersects zero, is earlier than the earliest point in which $D_A(t)$ intersects zero. Then, the equilibrium outcome will be that one type-B firm enters first, exactly at the earliest time in which $D_B(t)$ intersects zero, and an AB subgame ensues, with firm A and the remaining type-B firm entering at $t_2(BAB)$ and $T_B^*(3)$ respectively.

Formally, let T_B^1 and T_A^1 be defined as:

$$T_B^1 = \begin{cases} \min\{\tau \text{ such that } D_B(\tau) = 0\} \text{ if } D_B(t) \text{ admits at least one zero} \\ +\infty \text{ otherwise} \end{cases}$$

$$T_A^1 = \begin{cases} \min\{\tau \text{ such that } D_A(\tau) = 0\} \text{ if } D_A(t) \text{ admits at least one zero} \\ +\infty \text{ otherwise} \end{cases}$$

The following Lemma holds:

Lemma 1 *If $T_B^1 < T_A^1$, the game admits a unique SPNE outcome, in which the entry order is $B - A - B$, and entry times are $t_1 = T_B^1$, $t_2 = t_2(BAB)$, $t_3 = T_B^*(3)$. In equilibrium, both inefficient firms achieve the same equilibrium payoff, and the efficient firm achieves a higher payoff.*

In equilibrium, first entry takes place exactly at T_B^1. Clearly, no firm has an incentive to enter earlier, because for $t < T_B^1$, each firm prefers to be the follower rather than a very early leader. Moreover, first entry cannot take place later than T_B^1 because in a right-neighbourhood of T_B^1, $D_B(t)$ is positive. If first entry took place at $t > T_B^1$, one of the two B firms would enter third at $T_B^*(3)$ and receive $F_B(t)$. That firm would rather deviate, preempt the rivals, and be a leader at $T_B^1 + \varepsilon$. Following a logic that is analogous to that in Fudenberg-Tirole (1985), the preemption race between the two B firms guarantees that first entry takes place exactly at T_B^1 so that there is rent equalization for the two B firms and neither has an incentive to preempt further. Moreover, $T_B^1 < T_A^1$ guarantees that in T_B^1 firm A strictly prefers to be follower rather than leader, hence has no incentive to deviate either.

3.4 Conditions for inefficient entry

In this subsection, we present our main result: sufficient conditions on the primitives of the model that guarantee that T_B^1 is indeed smaller than T_A^1.

Proposition 1 *If*

1) $\pi_B(2, A)$ *is sufficiently large*

2) $\pi_A(2)$, $\pi_B(2, B)$ *and* $\pi_B(3)$ *are sufficiently small*

3) $\pi_A(1) - \pi_B(1)$ *is sufficiently small,*

then $T_B^1 < T_A^1$ *and the equilibrium entry order in the game is* $B - A - B$.

There exists a range of payoffs such that assumption (1) *and the above conditions are satisfied.*

The first two conditions guarantee that second entry takes place earlier in a BB subgame than in an AB subgame, which is crucial to guarantee that there is inefficient entry in equilibrium. To see why this is the case, suppose instead that $t_2(BAB) < t_2(ABB)$. In this case, the preemption incentive to be the first rather than the second entrant would always be stronger for A than for a B-type firm, for the following reasons: the first entrant achieves monopoly profits for some time. For firm A, these profits are higher than for firm B, and they would be achieved for a longer time (until $t_2(ABB)$, rather than until $t_2(BAB)$). Moreover, by entering at t and being a leader rather than a follower, each firm sustains a higher entry cost. The cost would increase less for firm A, who would otherwise enter at $t_2(BAB)$, than for firm B, who would otherwise enter at $t_2(ABB)$. Finally, by being leader rather than follower, a firm changes the time from which it starts earning duopoly profits. For an A-type leader, this date would be delayed from $t_2(BAB)$ to $t_2(ABB)$. In that interval, duopoly profits would be replaced by monopoly profits, so the total effect would be positive. For B instead, this date would be anticipated from $t_2(ABB)$ to $t_2(BAB)$. Nonetheless, the extra duopoly profits earned in this period would be more than offset by the increase in entry cost, so the total effect would be negative.[12] In sum, if $t_2(BAB) < t_2(ABB)$, then the total

[12] For a formal argument, see the Proof of Claim 5.

preemption incentive to be first rather than second is stronger for firm A than for a type-B firm and inefficient entry cannot happen in equilibrium.

The intuition for why conditions (1) and (2) guarantee that second entry takes place earlier in a BB subgame than in an AB subgame is derived observing how the intensity of the preemption race in a BB subgame and in an AB subgame depend on the underlying parameters.

The difference $[\pi_B(2, A) - \pi_B(3)]$ is a measure of the intensity of the preemption race in a BB subgame: the larger this difference, the stronger the incentive to be second rather than third, after A enters first, hence the earlier is $t_2(ABB)$. Next, consider an AB subgame. If A is a "strong leader" in this subgame, $t_2(BAB)$ is equal to $T_A^*(2)$. Therefore, the smaller $\pi_A(2)$, the later is $t_2(BAB)$. Similarly, if A is a "weak leader", $t_2(BAB)$ is determined by the incentive B has to preempt A, which is increasing in both $\pi_B(2, B)$ and $\pi_B(3)$: by being second rather than third, B would receive duopoly profits for some time, and triopoly profits for a longer time (from $T_A^*(3)$ rather than from $T_B^*(3)$).

Finally, consider condition (3). Conditions (a) and (b) guarantee that $t_2(ABB) < t_2(BAB)$. Let us assume they hold, and compare the incentive to be first rather than second entrant for firm A and for a B firm, respectively.

- Per-period monopoly profits are higher for A than for B.

- If B is the leader, it achieves monopoly profits for a longer time, with respect to an $A-$type leader (until $t_2(BAB)$ rather than just until $t_2(ABB)$).

- In the interval from $t_2(ABB)$ to $t_2(BAB)$, a B-type leader earns monopoly rather than duopoly profits.

- For firm A, duopoly profits are earned from $t_2(ABB)$ if it is a leader, and from $t_2(BAB)$ if it is a follower. The latter is preferred, because in that interval the entry cost decreases at rate higher than $\pi_B(2, A)$.

- Finally, the cost increase due to the anticipation of entry to be first rather than second is higher for an A-type leader, who would otherwise enter at $t_2(BAB)$, than for a B-type leader, who would instead enter at $t_2(ABB)$.

The first of these effects contributes to a stronger preemption incentive for A than for B. Every other effect goes in the opposite direction. Therefore, if the difference in monopoly profits is sufficiently small, the first effect is sufficiently small with respect to the other effects that $D_B(t) > D_A(t)$ in some range, and in particular $T_B^1 < T_A^1$.

We conclude this section by discussing for which markets the conditions in Proposition 1 are likely to be satisfied.

Condition (3) requires that the difference in monopoly profits is sufficiently small. Given the assumption that $\pi_A(2) > \pi_B(2, A)$, conditions (1) and (2) imply that also the difference in duopoly profits between A and B in an AB duopoly has to be sufficiently small, although positive. Finally, given that $\pi_B(3)$ is bounded above by $\pi_A(3)$, the statement that $\pi_B(3)$ has to be sufficiently small also means that the difference in triopoly profits has to be sufficiently large. In sum, our conditions can be satisfied in markets where the impact of a given difference in efficiency on firms' profits is amplified as the number of competitors in the market increases.

It is easy to generate examples with this feature assuming that post-entry profits are derived from Cournot competition and firms produce at a constant marginal cost which is lower for A than for B. For example, in such a market, if demand is linear in prices or exhibits constant elasticity, the ratio of A's profits over B's profits is larger in an ABB triopoly than in an AB duopoly. Appendix A analyzes the case of Cournot competition with constant elasticity of demand using numerical methods. It presents the range of asymmetry values, as measured by the difference between the two marginal costs, for which the equilibrium entry order is $B - A - B$.

4 Conclusions

We presented a preemption game of entry into a new market with ex-ante asymmetric firms. It is well known from the literature that in a two-firm game the equilibrium entry order reflects the efficiency ranking. We show that this result can be reversed if the game is played by more than two firms and provide sufficient conditions on the parameters of the model which guarantee that the equilibrium is one with first entry by a less efficient firm. Our result provides an explanation for late entry by leading firms into new markets for technologically mature products.

Appendix A: Numerical Example

We now present an example to illustrate that a profit structure satisfying the conditions of Proposition (1) can be generated by a standard asymmetric oligopoly model for a range of parameter values.

Flow profits upon investment arise from Cournot competition. The inverse demand function is given by $P(Q) = Q^{-\eta}$, where Q is total output in the industry and $\eta \in (0,1)$ is the elasticity. Firms' cost functions are given by $K_i(q_i) = k_i q_i$ for $i = A, B$. We normalize the efficient firm's marginal cost k_A to 1. Marginal costs then satisfy: $k_B > k_A = 1$. In order to guarantee that all firms produce strictly positive quantities in all possible market structures,[13] we assume $k_B \in \left(1, \frac{1}{1-\eta}\right)$. The resulting profit structure satisfies Assumption 1. The present value cost of entry declines exponentially at rate α, satisfying Assumptions 2 and 4:[14] $c(t) = \overline{c}e^{-(r+\alpha)t}$.

We fix $r = 0.03$ and $\overline{c} = 20$. For a range of values of a, η and k_B for which Assumption 3 is satisfied as well we compute the equilibrium entry order. Table 1 shows that for some pairs (α, η), there exists a range of values of k_B for which the entry order in equilibrium is $B - A - B$.

Table 1: Ranges of k_B for which the equilibrium order of entry is B-A-B

	η			
α	0.2	0.4	0.6	0.8
0.02	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
0.03	$\emptyset$	$\emptyset$	$\emptyset$	(1.1031, 1.1165)
0.04	$\emptyset$	$\emptyset$	(1.0580, 1.0718)	(1.0636, 1.1336)
0.05	(1.0150, 1.0165)	(1.0298, 1.0405)	(1.0387, 1.0781)	(1.0264, 1.1451)
0.06	(1.0118, 1.0174)	(1.0204, 1.0428)	(1.0193, 1.0825)	(1.0000, 1.1534)
0.07	(1.0082, 1.0181)	(1.0107, 1.0445)	(1.0000, 1.0859)	(1.0000, 1.1596)
0.08	(1.0044, 1.0186)	(1.0000, 1.0458)	(1.0000, 1.0885)	(1.0000, 1.1646)

To relate these results to the conditions stated in Proposition 1, consider the effect of

[13] See Corchon (2007).

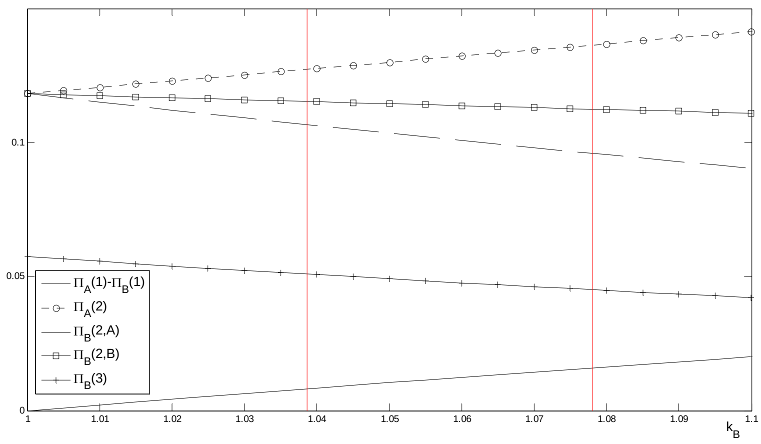
[14] This choice of cost function is motivated by the example given in Fudenberg and Tirole (1985).

a change in k_B on profits. An increase in k_B, ceteris paribus:

1. reduces $\pi_B(2, A)$,
2. increases $\pi_A(2)$, and reduces $\pi_B(2, B)$ and $\pi_B(3)$,
3. increases $\pi_A(1) - \pi_B(1)$.

These effects are illustrated in Figure 1.

Figure 1: The effect of k_B on firms' profits for $\eta = 0.6$, $\alpha = 0.05$.



As k_B increases, conditions 1 and 3 are less likely to be satisfied. This is one of the reasons why the range of values of k_B for which the entry order in equilibrium is $B - A - B$ is bounded above.

Moreover, consider condition 2. For sufficiently strong asymmetry, that is for sufficiently large k_B, firm A is a strong leader in an AB subgame. A further increase in k_B, by increasing $\pi_A(2)$, accelerates entry by firm A in such a subgame. The equilibrium entry order $B - A - B$ then becomes less likely. This provides a second reason why the range is bounded above.

On the other hand, for some pairs (α, η), the equilibrium entry order for values of k_B close to one is $A - B - B$ (e.g. for $\alpha = 0.05$, $\eta = 0.2$). However, a range of values of k_B for which the equilibrium entry order is $B - A - B$ still exists. For values of k_B very close to one, firm A is a weak leader in an AB subgame. A marginal increase in k_B

decreases $\pi_B(2, B)$ and $\pi_B(3)$. Thus, it weakens the preemption constraint on firm A in such a subgame, delaying its entry. The equilibrium entry order $B - A - B$ then becomes more likely.

Appendix B: Proofs

Proof of Lemma 1. We prove the result through a series of Claims. As in most preemption games, the equilibrium is constructed starting from the end of the game and moving backwards. This requires the analysis of a large set of off-equilibrium path subgames which makes a lenghty proof with several intermediate steps necessary. We first give an outline of the structure of the proof.

Outline of the proof.

Claims 1 and 2 establish that all firms must enter by $T_B^*(3)$. Claim 3 describes the equilibrium outcome of subgames with two B firms active, building on the results by Fudenberg and Tirole (1985). Similarly, Claim 4 describes the equilibrium outcome of subgames with A and one of the B firms active, building on the results by Riordan (1992). Claim 5 proves an important necessary condition: If $T_B^1 < T_A^1$, then it has to be the case that second entry takes place earlier if the first entrant is A, than if it is a B-type firm. Finally, Claims 6, 7 and 8 complete the result establishing the equilibrium outcome of subgames with all three firms active, starting late in the game (Claim 6), early in the game (Claim 7), and at time zero (Claim 8).

Claim 1 *In any subgame starting at time τ, with only one active firm i, firm i enters at* $\max\{\tau, T_i^*(3)\}$.

Proof. For $t \geq \tau$, the function

$$g_{i,3}(t) \equiv \int_t^{+\infty} \pi_i(3)\, e^{-rs} ds - c(t)$$

that is maximized at $T_i^*(3)$, represents firm i's payoff from entering last at time t. By assumptions 4(i) and 2(i) it is strictly positive for every t larger than some finite t'. Hence its maximum value is strictly positive. Therefore, if $\tau < T_i^*(3)$ firm i will wait until $T_i^*(3)$ and then enter, while if $\tau \geq T_i^*(3)$ then it will enter immediately. ■

An immediate consequence of Claim 1 is the following:

Claim 2 *In any subgame starting at time $\tau \geq T_B^*(3)$, with any number of active firms, all firms enter immediately.*

Proof. The proof is analogous to the proof of Claim 1 in Argenziano and Schmidt-Dengler (2009).■

The next Claim analyzes BB subgames starting at $\tau < T_B^*(3)$. It follows immediately from our assumptions and the analysis in Fudenberg and Tirole (1985) that, given the functions

$$\begin{aligned} L_B^{2,A}(t) &= \pi_B(2,A)\int_t^{T_B^*(3)} e^{-rs}ds - c(t) \\ F_B^{2,A}(t) &= \pi_B(3)\int_{T_B^*(3)}^{+\infty} e^{-rs}ds - c\left(T_B^*(3)\right) \\ D_B^{2,A}(t) &= L_B^{2,A}(t) - F_B^{2,A}(t) \end{aligned}$$

there exists a point $T_B^{2,A} \in (0, T_B^*(2,A))$ such that $D_B^{2,A}\left(T_B^{2,A}\right) = 0$ and that the following result holds:

Claim 3 *In any SPNE, in any BB subgame starting at time $\tau < T_B^*(3)$ there is a unique equilibrium outcome, such that:*
(i) entries take place at $t_2 = \max\left\{\tau, T_B^{2,A}\right\}$ and $t_3 = T_B^*(3)$.
(iii) If $\tau \leq T_B^{2,A}$, both B firms achieve payoff $F_B^{2,A}\left(T_B^*(3)\right)$, while if $\tau > T_B^{2,A}$ payoffs for the early and late entrant are $L_B^{2,A}(\tau)$ and $F_B^{2,A}(\tau) < L_B^{2,A}(\tau)$ respectively.

Claim 4 analyzes AB subgames starting at $\tau < T_B^*(3)$.

Consider the following function

$$D_B^{2,B}(t) = \pi_B(2,B)\int_t^{T_A^*(3)} e^{-rs}ds + \pi_B(3)\int_{T_A^*(3)}^{T_B^*(3)} e^{-rs}ds - \left[c(t) - c(T_B^*(3))\right] \tag{2}$$

It is strictly quasiconcave and admits a unique global maximum in $t = T_B^*(2,B) \in (T_A^*(2), T_B^*(3))$. It takes negative value at zero by assumptions 1 and 3(i), and in $t = T_A^*(3)$ by definition of $T_B^*(3)$. Hence, in the interval $t \in [0, T_A^*(3)]$ the following cases are possible:

Case 1 The function is negative everywhere

Case 2 The function has two (possibly coinciding) intersections with zero, $\left\{\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right\}$ such that $\underline{T}_B^{2,B} \leq \overline{T}_B^{2,B}$, and $T_A^*(2) \leq \underline{T}_B^{2,B}$

Case 3 The function has two (possibly coinciding) intersections with zero, $\left\{\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right\}$ such that $\underline{T}_B^{2,B} \leq \overline{T}_B^{2,B}$, and $\underline{T}_B^{2,B} < T_A^*(2)$.

Define $T_B^{2,B}$ as follows:

$$T_B^{2,B} = \begin{cases} +\infty \text{ in case 1} \\ \underline{T}_B^{2,B} \text{ in cases 2 and 3.} \end{cases}$$

The following result holds:

Claim 4 *In any SPNE, in any AB subgame starting at time $\tau < T_B^*(3)$, it holds that:*
(i)If $\tau \leq \min\left\{T_B^{2,B}, T_A^*(2)\right\}$, firm A enters first in the subgame, at $t_2 = \min\left\{T_B^{2,B}, T_A^*(2)\right\}$ and firm B enters later, at $t_3 = T_B^*(3)$
(ii)If $\tau > \min\left\{T_B^{2,B}, T_A^*(2)\right\}$:
- in case 1, firm A enters first in the subgame, at $t_2 = \tau$ and firm B enters later, at $t_3 = T_B^*(3)$.
- in cases 2 and 3, for $\tau \notin \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$, firm A enters first in the subgame, at $t_2 = \tau$ and firm B enters later, at $t_3 = T_B^*(3)$, while for $\tau \in \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$ either firm A enters first in the subgame, at $t_2 = \tau$ and firm B enters later, at $t_3 = T_B^*(3)$, or firm B enters first in the subgame, at $t_2 = \tau$ and firm A enters later, at $t_3 = T_A^*(3)$.

Proof. To prove this result, we first show that in our model, in an AB subgame, the condition $\overline{\Delta}_j(y_j, z_j, z_i) < \overline{\Delta}_i(y_i, z_i, z_j)$ in Theorem (1) in Riordan (1992) is satisfied, with the interpretation that $i = A$ and $j = B$. The equivalent of the condition $\overline{\Delta}_j(y_j, z_j, z_i) < \overline{\Delta}_i(y_i, z_i, z_j)$ in our model, for an AB subgame, is that $T_A^2 < T_B^{2,B}$, where T_A^2 is defined as the smallest value of t such that the following function is null:

$$D_A^2(t) = \pi_A(2) \int_t^{T_A^*(3)} e^{-rs} ds + [\pi_A(2) - \pi_A(3)] \int_{T_A^*(3)}^{T_B^*(3)} e^{-rs} ds - [c(t) - c(T_A^*(3))].$$

The function $D_A^2(t)$ is strictly quasi-concave in t, strictly negative for $t = 0$, it has strictly positive value for $t = T_A^*(3)$, and admits a unique global maximum in $t = T_A^*(2) < T_A^*(3)$. Hence, T_A^2 is well defined and belongs to the interval $(0, T_A^*(2))$. For $T_A^2 < T_B^{2,B}$ to hold, it is sufficient that $D_A^2(t) - D_B^{2,B}(t) > 0$ for every $t < T_A^*(3)$. To see that this condition holds, notice that $D_A^2(t) - D_B^{2,B}(t)$ can be rewritten as

$$\begin{aligned} &[\pi_A(2) - \pi_B(2,B)] \int_t^{T_A^*(3)} e^{-rs} ds + [\pi_A(2) - \pi_A(3)] \int_{T_A^*(3)}^{T_B^*(3)} e^{-rs} ds \\ &-\pi_B(2,B) \int_{T_A^*(3)}^{T_B^*(3)} e^{-rs} ds + c\left(T_A^*(3)\right) - c\left(T_B^*(3)\right). \end{aligned}$$

The first two terms are positive by assumption 1(ii) and the last one by definition of $T_B^*(3)$.

Given that $T_A^2 < T_B^{2,B}$, condition $\overline{\Delta}_j\left(y_j, z_j, z_i\right) < \overline{\Delta}_i\left(y_i, z_i, z_j\right)$ in Theorem (1) in Riordan (1992) is satisfied, and part (i) of the Lemma follows immediately from part (i) of Riordan's theorem. Moreover, part (ii) of the Lemma follows from the analysis in the Appendix of Riordan (1992), in particular from Lemma A3 and from the proof of Lemma A4, where $\widehat{\Delta_1}\left(z_1\right) \geq \widehat{\Delta_2}\left(z_2\right)$ is equivalent to $T_A^*(3) \leq T_B^*(3)$, $t\left(\widehat{\Delta_1}\left(x_1\right)\right)$ is equal to $T_A^*(2)$, $\overline{\Delta}_2\left(y_2, z_2, z_1\right) < \widehat{\Delta_1}\left(x_1\right) \leq \overline{\Delta}_1\left(y_1, z_1, z_2\right)$ is equivalent to $T_A^2 < T_A^*(2) \leq T_B^{2,B}$, $\overline{\Delta}_2\left(y_2, z_2, z_1\right) = \widehat{\Delta_1}\left(x_1\right) < \overline{\Delta}_1\left(y_1, z_1, z_2\right)$ is equivalent to $T_A^2 < T_A^*(2) = T_B^{2,B}$ and $\overline{\Delta}_2\left(y_2, z_2, z_1\right) > \widehat{\Delta_1}\left(x_1\right)$ is equivalent to$T_A^*(2) > T_B^{2,B}$.$\blacksquare$

Next, we prove that $T_B^1 < T_A^1$ implies $t_2(BAB) < t_2(ABB)$.

Consider subgames with three active firms starting at $\tau < \min\left\{T_B^{2,A}, T_A^{2,B}\right\}$. The functions $D_A(\cdot)$ and $D_B(\cdot)$ as defined in section 3.2 are negative at zero by assumptions 1 and 3(i), are strictly quasiconcave, and maximized at $T_A^*(1)$ and $T_B^*(1) > T_A^*(1)$ respectively. The following Claim holds:

Claim 5 *If $T_B^1 < T_A^1$, then it has to be the case that $T_B^{2,A} < \min\left\{T_A^*(2), T_B^{2,B}\right\}$.*

Proof. We prove the result by contradiction. Suppose $\min\left\{T_A^*(2), T_B^{2,B}\right\} \leq T_B^{2,A}$. Then, by Claims 3 and 4, for any $t \leq \min\left\{T_A^*(2), T_B^{2,B}\right\}$, $D_A(t)$ and $D_B(t)$ can be written as

$$\begin{aligned} D_A(t) &= \pi_A(1)\int_t^{\min\left\{T_A^*(2),T_B^{2,B}\right\}} e^{-rs}ds + [\pi_A(1)-\pi_A(2)]\int_{\min\left\{T_A^*(2),T_B^{2,B}\right\}}^{T_B^{2,A}} e^{-rs}ds \\ &\quad -c(t) + c\left(\min\left\{T_A^*(2), T_B^{2,B}\right\}\right), \end{aligned}$$

$$D_B(t) = \pi_B(1)\int_t^{\min\left\{T_A^*(2),T_B^{2,B}\right\}} e^{-rs}ds + \pi_B(2,A)\int_{\min\left\{T_A^*(2),T_B^{2,B}\right\}}^{T_B^{2,A}} e^{-rs}ds - c(t) + c\left(T_B^{2,A}\right).$$

Both functions are strictly quasiconcave and negative at zero. $D_A\left(\min\left\{T_A^*(2), T_B^{2,B}\right\}\right)$ is strictly positive, so it has to be the case that $T_A^1 < \min\left\{T_A^*(2), T_B^{2,B}\right\}$. Moreover, $D_B\left(\min\left\{T_A^*(2), T_B^{2,B}\right\}\right)$ is strictly negative because the function $\left[\pi_B(2,A)\int_{\min\left\{T_A^*(2),T_B^{2,B}\right\}}^{+\infty} e^{-rs}ds - c(t)\right]$ is strictly quasiconcave and maximized at $T_B^*(2,A) > T_B^{2,A}$,hence it is strictly increasing in $\left[\min\left\{T_A^*(2), T_B^{2,B}\right\}, T_B^{2,A}\right]$. It follows that either $T_B^1 > \min\left\{T_A^*(2), T_B^{2,B}\right\} > T_A^1$, in which case the statement follows, or $T_B^1 \leq \min\left\{T_A^*(2), T_B^{2,B}\right\}$. For the latter case, we show that $D_A(t) > D_B(t)$ for any $t \in \left[0, \min\left\{T_A^*(2), T_B^{2,B}\right\}\right]$ which in turn implies the result.

First, notice that by Assumption (1) the first term in $D_A(t)$ is greater than the first term in $D_B(t)$, and the second term in $D_A(t)$ is positive. Moreover,

$$-\pi_B(2,A)\int_{\min\left\{T_A^*(2),T_B^{2,B}\right\}}^{T_B^{2,A}} e^{-rs}ds + c\left(\min\left\{T_A^*(2), T_B^{2,B}\right\}\right) - c\left(T_B^{2,A}\right) > 0$$

by definition of $T_B^*(2,A)$. We can therefore conclude that even if $T_B^1 \leq \min\left\{T_A^*(2), T_B^{2,B}\right\}$, $D_A(t) > D_B(t)$ for any $t \in \left[0, \min\left\{T_A^*(2), T_B^{2,B}\right\}\right]$, which in turn implies it cannot be the case that $T_B^1 < T_A^1$. ■

Next, we assume that the condition $T_B^1 < T_A^1$ in the statement of the Proposition holds, and consider subgames starting at $\tau \in \left[T_B^{2,A}, T_B^*(3)\right)$ with all the three players still active. The analysis will rely on the implication derived in the previous Claim, namely that $T_B^{2,A} < \min\left\{T_A^*(2), T_B^{2,B}\right\}$.

At time τ, if all three firms are active and A enters first, it follows from Claim 3 that the two B firms follow at τ and $T_B^*(3)$ respectively. Payoffs are

$$L_A(\tau) = L_A^2(\tau) = \pi_A(2)\int_{\tau}^{T_B^*(3)} e^{-rs}ds + \pi_A(3)\int_{T_B^*(3)}^{\infty} e^{-rs}ds - c(\tau)$$

for A and a lottery between $L_B^{2,A}(\tau)$ and $F_B^{2,A}(\tau)$ for both B firms, with $L_B^{2,A}(\tau) > F_B^{2,A}(\tau)$.

If instead one of the B firms enters at τ, it follows from Claim 4 that if $\tau \in [T_B^{2,A}, \min\left\{T_B^{2,B}, T_A^*(2)\right\})$, then the entry order is B–A–B , entry times are $\left(\tau, \min\left\{T_B^{2,B}, T_A^*(2)\right\}, T_B^*(3)\right)$ and payoffs are $L_B(\tau)$, $L_A^2(\min\left\{T_B^{2,B}, T_A^*(2)\right\})$ and $F_B^{2,B}(\tau)$ for the first, second and third entrant, respectively.

If instead $\tau \in \left[\min\left\{T_B^{2,B}, T_A^*(2)\right\}, T_B^*(3)\right]$, then:

- in case 1, entry order is $B - A - B$, entry times are $(\tau, \tau, T_B^*(3))$ and payoffs are $L_B^{2,A}(\tau)$, $L_A^2(\tau)$ and $F_B^{2,B}(\tau)$ for the first, second and third entrant, respectively.

- in cases 2 and 3, for $\tau \notin \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$, entry order, entry times and payoffs are those described for case 1, while for $\tau \in \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$ either entry order, entry times and payoffs are those described for case 1, or entry order is $B - B - A$, entry times are $(\tau, \tau, T_A^*(3))$ and payoffs are $L_B^{2,B}(\tau)$ for the first two entrant and

$$F_A^2(\tau) \equiv \pi_A(3)\int_{T_A^*(3)}^{+\infty} e^{-rs}ds - c(T_A^*(3)).$$

for firm A.

The following Claim holds:

Claim 6 *In any SPNE of the game, the outcome of subgames with three active firms starting at* $\tau \in \left(T_B^{2,A}, T_B^*(3)\right)$ *is as follows:*
(i) If $\tau \in \left[T_B^{2,A}, \min\left\{T_B^{2,B}, T_A^*(2)\right\}\right)$*, one of the B firms enters at* $t_1 = \tau$*, the A firm enters at* $t_2 = \min\left\{T_B^{2,B}, T_A^*(2)\right\}$ *and the remaining B firm enters at* $t_3 = T_B^*(3)$*;*
(ii) If $\tau \in \left[\min\left\{T_B^{2,B}, T_A^*(2\right\}, T_B^*(3)\right)$*:*
(iia) for any τ *in the interval in case 1, and for any* τ *in the interval such that* $\tau \notin \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$ *in cases 2 and 3, the unique outcome is that firm A and one of the B firms*

enter at $t_1 = t_2 = \tau$ and the remaining B firm enters at $t_3 = T_B^*(3)$;

(iib) moreover, in cases 2 and 3, for $\tau \in \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$ the outcome is either that firm A and one of the B firms enter at $t_1 = t_2 = \tau$ and the remaining B firm enters at $t_3 = T_B^*(3)$, or that both B firms enter at $t_1 = t_2 = \tau$ and the A firm enters at $t_3 = T_A^*(3)$.

Proof. For simplicity, we develop the proof of this Claim under the following assumption: Suppose that at any time t, if a firm is indifferent between being the m-th investor at t and the $(m+1)$-th investor, then it invests at t. It is immediate to verify that even without this assumption the result still holds.

First, consider subgames with three active firms starting at $\tau \in [T_A^*(2), T_B^*(3))$ for case 1, or $\tau \in \left(\overline{T}_B^{2,B}, T_B^*(3)\right)$ for case 2. In equilibrium, at τ, it has to be the case that both B firms play Enter and A plays either Enter or Wait. Assumption 5 and Claims 3 and 4 then guarantee the result.

If firms play either of these action profiles, payoffs are $L_A^2(\tau)$ for the A firm, and a lottery between $L_B^{2,A}(\tau)$ and $F_B^{2,A}(\tau)$ for the B firms. By Claim 3, in this interval $L_B^{2,A}(\tau) > F_B^{2,A}(\tau)$ so no B firm has an incentive to deviate and receive $F_B^{2,A}(\tau)$ with probability one. By the same argument, there cannot be an equilibrium in which at τ only one of the type-B firms plays Enter, regardless of A's action, because the other one would rather deviate and play Enter. Consider now firm A. Given that both B firms play Enter, A's action does not affect its payoff, so A has no profitable deviation from either profile described above.

Next, we prove that there are no other action profiles at τ compatible with equilibrium. There cannot be an equilibrium in which only firm A plays Enter at τ, because each of the B firms would receive a lottery between $L_B^{2,A}(\tau)$ and $F_B^{2,A}(\tau)$ and would rather deviate and play Enter at τ as well, thus receiving a similar lottery but with higher probability to obtain $L_B^2(\tau)$. Moreover, there cannot be an equilibrium in which all three firms play Wait at τ. In such an equilibrium, the first entry would happen at some time t later than τ. By Claim 2 first entry would happen at some later $t \in (\tau, T_B^*(3)]$. From the arguments presented so far in the proof of part (ii) of this Claim, it could only be the case that the A firm and one of the B firms enter simultaneously at t and the remaining B firm follows at $T_B^*(3)$. Since the function $L_A^2(\tau)$ is strictly quasiconcave and maximized at $T_A^*(2) \leq \tau$, A would then have an incentive to deviate and preempt the rivals playing Enter at $(t-\varepsilon)$. So, in case 1, for $\tau \in [T_A^*(2), T_B^*(3))$ there cannot be an equilibrium in which all three firms play Wait at τ. Hence, we can conclude that for any τ in this interval the unique equilibrium outcome is the one described in part (iia) of the Claim.

Next, consider subgames with three active firms starting at $\tau \in \left[\underline{T}_B^{2,B}, \overline{T}_B^{2,B}\right]$ We prove that in equilibrium, all firms play Enter at τ. Assumption 5(i), together with Claims 3 and 4 then guarantee the result.

If firms play the above profile, A receives $L_A^2(\tau)$ with probability $\frac{2}{3}$ and $F_A^2(\tau)$ with probability $\frac{1}{3}$. By the proof of Claim 4, $D_A^2(\tau) = L_A^2(\tau) - F_A^2(\tau)$ is weakly positive in $[T_A^*(2), T_A^*(3)]$, hence in the interval we are considering. It follows that A has no incentive to deviate because it would then receive $L_A^2(\tau)$ or $F_A^2(\tau)$ with probabilities $\frac{1}{2}, \frac{1}{2}$. (By an analogous argument, there cannot be an equilibrium in which at τ A plays Wait and either one or both B firms play Enter). As for the B firms, if the above profile is played, each B firm receives $L_B^{2,B}(\tau)$, $L_B^{2,A}(\tau)$ and $F_B^{2,A}(\tau)$ with probabilities $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ while by deviating it would receive a similar lottery with probabilities $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right)$. Since in this interval $D_B^{2,B}(t) = L_B^{2,B}(\tau) - F_B^{2,B}(\tau) > 0$ and $L_B^{2,A}(\tau) > F_B^{2,A}(\tau)$, the deviation is not profitable. (By an analogous argument, there cannot be an equilibrium in which at τ A plays Enter, and one or both of the B firms play Wait).

Finally, there cannot be an equilibrium in which all three firms play Wait at τ. In such an equilibrium, by the argument presented above, the first entry would take place at some later time $t \leq \overline{T}_B^{2,B}$. If in t only one or two firms plays Enter, any firm who plays Wait has an incentive to deviate and play Enter at $(t-\varepsilon)$. Similarly, if in t all three firms play Enter, each of them has an incentive to deviate and play Enter at $(t-\varepsilon)$. Hence, we can conclude that for any τ in this interval the unique equilibrium outcome is the one described in part (iiib) of the Claim.

Next, for case 2, consider subgames with three active firms starting at $\tau \in \left[T_A^*(2), \underline{T}_B^{2,B}\right]$. Given part (iib) of this Claim, the equilibrium outcome of any such subgame must be that first entry happens weakly before $\underline{T}_B^{2,B}$. Then, the same arguments presented in the first part of this proof guarantee that in equilibrium, at τ both B firms play Enter and A plays either Enter or Wait, which in turn guarantees that for any τ in this interval the unique equilibrium outcome is the one described in part (iiia) of the Claim.

Finally, consider subgames with three active firms starting at $\tau \in \left[T_B^{2,A}, \min\left\{T_B^{2,B}, T_A^*(2)\right\}\right)$. In equilibrium, at τ, it has to be the case that the B firms play Enter and the A firm plays Wait. Then, Assumption 5(i), together with Claim 4, guarantees the result. If firms play the prescribed actions, A receives $F_A(\tau)$, and the B firms a lottery between $L_B(\tau)$ and $F_B^{2,A}(\tau) = F_B^{2,B}(\tau)$ with probabilities $\left(\frac{1}{2}, \frac{1}{2}\right)$ By Assumption 1 $L_B(\tau) > L_B^{2,A}(\tau)$ which is turn larger that $F_B^{2,A}(\tau)$. Therefore, no B firm has an incentive to deviate and receive $F_B^{2,A}(\tau)$ with probability 1. By the same argument, there cannot be an equilibrium in which at τ only one B firm plays Enter.

As for firm A, by deviating it would receive a lottery between $F_A(\tau)$ and $L_A^2(\tau)$. It is easy to verify that this deviation is not profitable, using the fact that the function

$$\pi_A(2) \int_t^{T_B^*(3)} e^{-rs} ds + \pi_A(3) \int_{T_B^*(3)}^{+\infty} e^{-rs} ds - c(t)$$

is strictly quasiconcave and maximized at $T_A^*(2) > \tau$. By an identical argument, a strategy profile in which the A firm and one or two B firms play Enter at τ, cannot be an equilibrium, since A would want to deviate and play Wait.

Finally, we prove that a profile in which all three firms play Wait at τ cannot be part of an equilibrium. By part (ii), first entry would then take place at some later time $t \leq \min\left\{T_B^{2,B}, T_A^*(2)\right\}$. In t, it holds that $L_B(t) > L_B^{2,A}(t) > F_B^{2,A}(t)$.

Suppose at time t only the two B firms play Enter. By continuity, regardless of what A plays at t, each of the B firms has a strict incentive to preempt the rival and enter at time $(t - \varepsilon)$. Similarly, if at time t only one of the B firm plays Enter, then the other B firm has an incentive to preempt and enter at time $(t - \varepsilon)$. Finally, if at time t only the A firm plays Enter, then each B firm has an incentive to preempt and enter at time $(t - \varepsilon)$. Therefore, there cannot be an equilibrium of the subgame starting at τ in which the first entry happens later than τ. ■

Next, we analyze subgames with three active firms starting at $\tau \in [0, T_B^{2,A})$. Consider again the functions $D_A(\tau)$ and $D_B(\tau)$. Evaluated at $T_B^{2,A}$, $D_B(\tau)$ is positive, because

$$D_B(T_B^{2,A}) = [\pi_B(1) - \pi_B(2,A)] \int_{T_B^{2,A}}^{\min\{T_A^*(2),T_B^{2,B})} e^{-rs} ds > 0$$

by Assumption (1). It follows that there exists one and only one point $T_B^1 \in \left(0, T_B^{2,A}\right)$ such that $D_B(T_B^1) = 0$. As for $D_A(\tau)$ instead,

$$D_A(T_B^{2,A}) = \pi_A(2) \int_{T_B^{2,A}}^{\min\{T_A^*(2),T_B^{2,B})} e^{-rs} ds - c(T_B^{2,A}) + c\left(\min\{T_A^*(2), T_B^{2,B})\right) < 0$$

because the function

$$\pi_A(2) \int_{t}^{\min\{T_A^*(2),T_B^{2,B})} e^{-rs} ds - c(t)$$

is strictly quasiconcave, maximized at $T_A^*(2)$, hence strictly increasing for $t \in \left[T_B^{2,A}, \min\{T_A^*(2), T_B^{2,B})\right]$. It follows that two cases are possible:

Case a $D_A(\tau) < 0 \ \forall \tau \in \left[0, T_B^{2,A}\right]$, and $T_A^1 = +\infty$;

Case b There exist two points, $\underline{T}_A^1$ and $\overline{T}_A^1$, with $0 < \underline{T}_A^1 \leq \overline{T}_A^1 < T_B^{2,A}$, in which $D_A(\tau)$ is null, and $T_A^1 = \underline{T}_A^1$.

Given the assumption $T_B^1 \leq T_A^1$, the following Claim holds:

Claim 7 *In any SPNE of the game, the outcome of subgames with three active firms starting at $\tau \in [0, T_B^{2,A})$ is as follows:*
(i) If $\tau \leq T_B^1$ one of the B firms enters at $t_1 = T_B^1$, the A firm enters at $t_2 = \min\left\{T_B^{2,B}, T_A^*(2)\right\}$ and the remaining B firm enters at $t_3 = T_B^*(3)$
(ii) If $\tau \in (T_B^1, T_B^{2,A})$:
(iia) for any τ in the interval in case a, and for any τ in the interval such that $\tau \notin \left[\underline{T}_A^1, \overline{T}_A^1\right]$ in case b, the unique outcome is that one of the B firms enters at $t_1 = \tau$, the A firm enters at $t_2 = \min\left\{T_B^{2,B}, T_A^*(2)\right\}$ and the remaining B firm enters at $t_3 = T_B^*(3)$
(iib) moreover, in case b, for $\tau \in \left[\underline{T}_A^1, \overline{T}_A^1\right]$ the outcome is either as in (iia), or that firm A enters at $t_1 = \tau$ and the B firms enter at $t_2 = T_B^{2,A}$ and $t_3 = T_B^*(3)$ respectively.

Proof. First, consider subgames with three active firms starting at $\tau \in [T_B^1, T_B^{2,A})$ for case a, or $\tau \in (\overline{T}_A^1, T_B^{2,A})$ for case b. In equilibrium, at τ, it has to be the case that A plays Wait, and the B firms play Enter. If firms play the above profile, A receives $F_A(\tau)$ and each B firm a lottery between $L_B(\tau)$ and $F_B(\tau)$. By deviating, A would receive $L_A(\tau)$ with positive probability and a B firm would receive $F_B(\tau)$. Then, the fact that $D_A(\tau) < 0$ and $D_B(\tau) > 0$ guarantees that no firm has an incentive to deviate. There cannot be an equilibrium in which firm A and one of the B firms play Wait, and the other B firm plays Enter, because the B firm which plays Wait would rather deviate and play Enter, thus exchanging $F_B(\tau)$ for a lottery between $L_B(\tau)$ and $F_B(\tau)$. There cannot be an equilibrium in which the A firm and at least one of the B firms play Enter, because the A firm would then receive a lottery between $L_A(\tau)$ and $F_A(\tau)$ and would rather deviate and receive $F_A(\tau)$. There cannot be an equilibrium in which the A firm plays Enter and both B firms play Wait, because both B firms would receive $F_B(\tau)$ and would rather deviate and receive a lottery between $L_B(\tau)$ and $F_B(\tau)$. Finally, there cannot be an equilibrium in which all three firms play Wait at τ. In such an equilibrium, by Claim 6 first entry would take place at some later time $t \leq T_B^{2,A}$. But this cannot be part of an equilibrium, because at t one of the following action profiles would have to be played:

- A plays Enter and either one or both B firms play Enter: then the A firm would rather deviate and play Wait.~

- A plays Enter and both B firms play Wait: then each B firm would rather deviate and play Enter

- A plays Wait and either one or both B firms play Enter: then each B firm would rather deviate and play Enter at $(t - \varepsilon)$.

Hence, we can conclude that for any τ in this interval the unique equilibrium outcome is the one described in part (iia) of the Claim.

Next, for case b, consider any subgame with three active firms starting at $\tau \in \left[\underline{T}_A^1, \overline{T}_A^1\right]$. In equilibrium, all firms play Enter at τ. If firms play the above profile, each firm i receives a lottery between $L_i(\tau)$ and $F_i(\tau)$, and the fact that in this interval $D_A(\tau) > 0$ and $D_B(\tau) > 0$ guarantees that there are no profitable deviations. Similarly, this fact guarantees that there cannot be an equilibrium in which either one or two firms only play Enter at τ, because in that case there is at least one firm which plays Wait and has an incentive to deviate and play Enter. Finally, there cannot be an equilibrium in which all three firms play Wait at τ. In such an equilibrium, by the argument presented above first entry would happen at some later time $t \leq \overline{T}_A^1$. If at t only one or two firms plays Enter, any firm who plays Wait has an incentive to deviate and play Enter at $(t-\varepsilon)$. Similarly, if in t all three firms play Enter, each of them has an incentive to deviate and play Enter at $(t-\varepsilon)$. Hence, we can conclude that for any τ in this interval the unique equilibrium outcome is the one described in part (iiib) of the Claim.

Next, for case b, consider any subgame with three active firms starting at $\tau \in \left[T_B^1, \underline{T}_A^1\right)$. Given part (iib), the equilibrium outcome of any such subgame must be that first entry happens weakly before $\underline{T}_A^1$, then the same arguments presented in the first part of this proof guarantee that in equilibrium, at τ, the A firm plays Wait and the B firms play Enter, which in turn guarantees that for any τ in this interval the unique equilibrium outcome is the one described in part (iiia) of the Claim.

Finally, consider subgames with three active firms starting at $\tau \leq T_B^1$. In equilibrium, all firms play Wait for any $t \in [\tau, T_B^1)$. If they do so, the outcome is the one described in part (i) of the Claim and firm i receives payoff $F_i(\tau)$. (Notice that each B firm receives a lottery between $L_B(T_B^1) = F_B(T_B^1)$ by definition of T_B^1, and $F_B(\tau) = F_B(T_B^1)$). The fact that in this interval $D_A(\tau) < 0$ and $D_B(\tau) < 0$ guarantees that there are no profitable deviations. By the same argument, there cannot be an equilibrium in which any number of firms plays Enter at τ, because then there would be at least one firm receiving $L_i(\tau)$ with positive probability, and this firm would rather deviate and receive $F_i(\tau)$ with probability one. Hence, we can conclude that for any τ in this interval the unique equilibrium outcome is the one described in part (i) of the Claim. ■

Finally, given the assumption $T_B^1 \leq T_A^1$, the following Claim holds:

Claim 8 *The unique SPNE outcome of the game is that one of the B firms enters at* $t_1 = T_B^1$*, the A firm enters at* $t_2 = \min\left\{T_B^{2,B}, T_A^*(2)\right\}$ *and the remaining B firm enters at* $t_3 = T_B^*(3)$.

Proof. The result is an immediate implication of Claim 7, because the game itself is a subgame with three active firms, starting at $\tau = 0$.■

The statement in Lemma 1 follows immediately from the above Claims. ■

Proof of Proposition 1

First, we prove that conditions (1) and (2) are sufficient for $t_2(ABB) < t_2(BAB)$. Consider the quantities $T_B^{2,B}$ and $T_B^{2,A}$defined in the proof of Lemma 1. In the limit case $\pi_A(2) = \pi_B(2, B) = \pi_B(2, A)$, it holds that $D_B^{2,A}(t) > D_B^{2,B}(t)$, hence $T_B^{2,A} < T_B^{2,B}$. Moreover, it follows from Fudenberg and Tirole (1985) that $T_B^{2,A} < T_B^*(2, A)$. Since $\pi_B(2) = \pi_A(2, A)$ implies $T_A^*(2) = T_B^*(2, A)$, it follows that $T_B^{2,A} < T_A^*(2)$. Therefore, it holds that $T_B^{2,A} < \min\left\{T_A^*(2), T_B^{2,B}\right\}$, that is $t_2(ABB) < t_2(BAB)$.

Next, we prove that $T_B^{2,A}$ is decreasing in $\pi_B(2, A)$ and increasing in $\pi_B(3)$, that $T_A^*(2)$ is decreasing in $\pi_A(2)$, and that $T_B^{2,B}$ is decreasing in both $\pi_B(2, B)$ and $\pi_B(3)$.

First, notice that $T_B^{2,A}$ is implicitly defined by

$$D_B^{2,A}\left(T_B^{2,A}\right) = \pi_B(2, A)\int_{T_B^{2,A}}^{T_B^*(3)} e^{-rs}ds - \left[c(T_B^{2,A}) - c\left(T_B^*(3)\right)\right] = 0.$$

By the implicit function theorem:

$$\begin{aligned}
\frac{\partial T_B^{2,A}}{\partial \pi_B(2, A)} &= -\frac{\left[e^{-rT_B^{2,A}} - e^{-rT_B^*(3)}\right]/r}{-\pi_B(2, A)\, e^{-rT_B^{2,A}} - c'\left(T_B^{2,A}\right)} < 0 \quad \text{and} \\
\frac{\partial T_B^{2,A}}{\partial \pi_B(3)} &= \frac{\partial T_B^{2,A}}{\partial T_B^*(3)} \cdot \frac{\partial T_B^*(3)}{\partial \pi_B(3)} = -\frac{\pi_B(2, A)\, e^{-rT_B^*(3)} + c'\left(T_B^*(3)\right)}{-\pi_B(2, A)\, e^{-rT_B^{2,A}} - c'\left(T_B^{2,A}\right)} \cdot \frac{\partial T_B^*(3)}{\partial \pi_B(3)} > 0
\end{aligned}$$

where the inequality in the first line holds because both the numerator and the denominator are positive, as $T_B^{2,A} < T_B^*(3)$, and $T_B^{2,A} < T_B^*(2, A)$. The inequality in the second line holds because $\frac{\partial T_B^*(3)}{\partial \pi_B(3)} < 0$, and both the numerator and the denominator of $\frac{\partial T_B^{2,A}}{\partial T_B^*(3)}$ are positive because $T_B^*(3) > T_B^*(2, A)$ and $T_B^{2,A} < T_B^*(2, A)$.

Next, notice that $T_A^*(2)$ is decreasing in $\pi_A(2)$, as we proved in Section 2. Moreover,

$T_B^{2,B}$ satisfies equation (2) and by the implicit function theorem:

$$\frac{\partial T_B^{2,B}}{\partial \pi_B\left(2,B\right)} = -\frac{\left[e^{-rT_B^{2,B}} - e^{-rT_B^*(3)}\right]/r}{-\pi_B\left(2,B\right)e^{-rT_B^{2,B}} - c'\left(T_B^{2,B}\right)} < 0$$

where the inequality holds because both the numerator and the denominator are positive, as $T_B^{2,B} < T_B^*(3)$ and $T_B^{2,B} < T_B^*(2,B)$. Moreover,

$$\frac{\partial T_B^{2,B}}{\partial \pi_B\left(3\right)} = -\frac{\left[e^{-rT_A^*(3)} - e^{-rT_B^*(3)}\right]/r + \left[\pi_B(3)\cdot e^{-rT_B^*(3)} + c'(T_B^*(3)\right]\cdot\frac{\partial T_B^*(3)}{\partial \pi_B(3)}}{-\pi_B\left(2,B\right)e^{-rT_B^{2,B}} - c'\left(T_B^{2,B}\right)} < 0$$

where the inequality holds because the first term in the numerator is positive, as $T_A^*(3) < T_B^*(3)$, and the second term is null by definition of $T_B^*(3)$.

Finally, by continuity, there is a region of the parameter space around the limit case $\pi_A\left(2\right) = \pi_B\left(2,B\right) = \pi_B\left(2,A\right)$ for which it holds that $t_2(ABB) < t_2(BAB)$.

Next, we show that given that $t_2(ABB) < t_2(BAB)$, condition (3) guarantees that $T_B^1 < T_A^1$. Consider the difference between the functions $D_B(t)$ and $D_A(t)$.

$$D_B(t) - D_A(t) =$$

$$\begin{aligned}
&[\pi_B(1) - \pi_A(1)]\int_t^{t_2(ABB)} e^{-rs}ds + \\
&+[\pi_B(1) - \pi_B(2,A)]\int_{t_2(ABB)}^{t_2(BAB)} e^{-rs}ds \\
&-\pi_A(2)\int_{t_2(ABB)}^{t_2(BAB)} e^{-rs}ds + \left[c\left(t_2(ABB)\right) - c\left(t_2(BAB)\right)\right]
\end{aligned}$$

The first term is negative, and vanishes for $\pi_A(1) - \pi_B(1)$ sufficiently small. The second term is positive. The expression in the third line is positive by definition of $T_A^*\left(2\right)$. To see this, notice that

$$\pi_A(2)\int_{t_2(ABB)}^{t_2(BAB)} e^{-rs}ds - \left[c\left(t_2(ABB)\right) - c\left(t_2(BAB)\right)\right]$$

$$= \pi_A(2)\int_{t_2(ABB)}^{+\infty} e^{-rs}ds - c\left(t_2(ABB)\right) - \left[\pi_A(2)\int_{t_2(ABB)}^{+\infty} e^{-rs}ds - c\left(t_2(BAB)\right)\right]$$

and the function $\pi_A(2)\int_t^{+\infty} e^{-rs}ds - c\left(t\right))$ is strictly increasing for $t < T_A^*\left(2\right)$, hence also in the interval from $t_2(ABB)$ to $t_2(BAB)$. Therefore, for $\pi_A(1) - \pi_B(1)$ sufficiently small $D_B(t) - D_A(t) > 0$ and so the earliest point in which $D_B(t)$ intersects zero is earlier than the earliest point in which $D_A(t)$ intersects zero.

Finally, we need to check that $D_B\left(t\right)$ has at least one intersection with zero. We evaluate it in $t_2(ABB)$.

$$D_B(t_2(ABB)) =$$

$$= \pi_B(1)\int_{t_2(ABB)}^{t_2(ABB)} e^{-rs}ds + [\pi_B(1) - \pi_B(2, A)]\int_{t_2(ABB)}^{t_2(BAB)} e^{-rs}ds - c(t_2(ABB)) + c\left(t_2(ABB)\right) > 0.$$

■

References

[1] Argenziano, R. and P. Schmidt-Dengler (2009), N-Player Preemption Games, mimeo

[2] Berry, S.M. (1992), Estimation of a Model of Entry in the Airline Industry, *Econometrica,* 60, 889–917.

[3] Corchón, L.C.(2008), *International Journal of Industrial Organization*, 26, 1120–1131.

[4] Dutta, P.J, Lach, S., and A. Rustichini (1995), Better Late than Early: Vertical Differentiation in the Adoption of a New Technology, *Journal of Economics and Management Strategy*, 4, 563-589.

[5] Einav, L. and D. Quint, (2005), Efficient Entry, *Economic Letters*, 88(2), 278-283.

[6] Fudenberg, D., and J. Tirole (1985), Preemption and Rent Equalization in the Adoption of a New Technology, *Review of Economic Studies*, 52, 383-401.

[7] Heracleous, L (2001), When Local Beat Global: the Chinese Beer Industry, *Business Strategy Review,* 12, 37-45

[8] Hoppe , H.C. and U. Lehmann-Grube (2001), Second-Mover Advantages in Dynamic Quality Competition, *Journal of Economics & Management Strategy*, 10(3), 419-433.

[9] Hoppe, H.C. and U. Lehmann-Grube (2005), Innovation Timing Games: A General Framework with Applications, *Journal of Economic Theory*, 121, 30-50.

[10] Katz, M. and C. Shapiro (1987), R & D Rivalry with Licensing or Imitation, *American Economic Review*, 77, 402-420.

[11] Marinov, M.A. and S.T. Marinova (1999), Foreign investor strategy development in the Central and Eastern European context, *Thunderbird International Business Review,* 41(1), 107-30.

[12] Riordan, M. H. (1992), Regulation and Preemptive Technology, *RAND Journal of Economics*, 23(3), 334-49.

[13] Simon, L., and M. Stinchcombe (1989), Extensive Form Games in Continuous Time: Pure Strategies, *Econometrica*, 57, 1171-1214.

[14] Tellis, G. J. and P.N. Golder. (1996), First to Market, First to Fail? The Real Causes of Enduring Market Leadership, *Sloan Management Review*, 37(2), 65-75.

[15] US Department of Justice and Federal Trade Commission (1992), Horizontal merger guidelines, Revised April 8 1997.