Conventional and Unconventional Monetary Policy Rules*

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Abstract

This essay examines the challenges in devising rules for unconventional monetary policy suitable for a post-crisis world. It is argued that unconventional monetary policy instruments are a poor substitute for conventional interest-rate policy in stabilizing the economy and in insulating monetary policy from political pressures. Some suggestions for the reform of inflation targeting are made to reduce the need for unconventional policy instruments in the future.

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1 Introduction

Since the financial crisis and the Great Recession, monetary policy has become constrained by a lower bound on the nominal interest rate. Central bankers, cautious by nature, have found themselves experimenting with a wide range of unconventional policy tools to substitute for their inability to cut interest rates further. This essay examines the challenges in devising rules for unconventional monetary policy suitable for a post-crisis world.

The first contribution of the essay is in setting up a simple economic model that can be used to analyse a variety of unconventional policy instruments. An unconventional policy instrument is defined as any instrument of the central bank that does not depend for its operation on changing the risk-free nominal interest rate now or in the future. This includes several types of balance-sheet policies where the central bank purchases assets (for example, quantitative easing or credit easing), emergency lending, and subsidized access to central-bank credit. Unconventional policy as defined also includes 'macroprudential' policy instruments that place restrictions on borrowing independently of interest rates.

The analysis of the economic model suggests that unconventional monetary policy instruments are a poor substitute for the conventional nominal interest rate. While both types of policy instrument have nominal and real effects on the economy, these effects are not equivalent. In particular, the use of unconventional policy to stabilize the economy today has a negative effect on the stability of the economy in the future. Consequently, it is not possible to use unconventional monetary policy to achieve an efficient allocation of resources, unlike conventional policy if it were unconstrained by the lower bound on interest rates. The optimal rule for conventional monetary policy can be expressed as a simple inflation target, but optimal unconventional policy is different and more complicated because of intertemporal trade-offs.

Following a rule for the unconventional instruments of monetary policy is also more difficult because unconventional policy has larger distributional effects than conventional policy. This means unconventional monetary policy is likely to attract greater political controversy than regular monetary policy, which makes it harder to insulate central banks from political pressure. The more monetary policy is swayed by distributional concerns, the less it is able to support an efficient allocation of credit in the economy.

While unconventional monetary policy has undesirable features, it has been argued that the low level of the natural rate of interest makes it hard to normalize monetary policy and return to using conventional instruments. However, the economic model applied in this essay has the property that the market-clearing real interest rate is not independent of the conduct of monetary policy. This suggests the concept of the natural interest rate should be applied with greater caution when thinking about monetary policy.

Finally, given the drawbacks of unconventional monetary policy tools, the essay closes with some thoughts on how inflation targeting might evolve to reduce the need to use unconventional policy instruments in the future. Rather than simply changing the level of inflation targets, this essay argues for a more radical reform that puts greater weight on asset prices when targeting inflation.

2 A simple model for monetary policy analysis

The analysis is based around a simple macroeconomic model taken from Sheedy (2017a), augmented below to study unconventional monetary policy. The model builds in several key features that are arguably essential to study monetary policy in a post-crisis world. First, there is private borrowing and lending in equilibrium because the model features heterogeneity among individuals. Second, asset prices play an important role because individuals will want to buy and sell houses over their lifetimes, which means that changes in house prices will affect the balance sheets of borrowers and lenders.[1] Third, there is a role for monetary policy to have real consequences because of nominal contracts, specifically, nominal debt contracts for borrowing to buy houses (mortgages) or to bring forward consumption. The use of nominal debt contracts by households and firms means that the economy has incomplete financial markets.[2]

The economy features overlapping generations of individuals who have deterministic lives spanning three discrete time periods. Individuals of different generations are referred to as the 'young', 'middle-aged', and 'old', indexed by $\tau \in \{\mathrm{y}, \mathrm{m}, \mathrm{o}\}$. There is no population growth, and the economy is at a stationary age distribution with a measure one of each cohort currently alive.

Individuals born at date t have the following lifetime expected utility function:

$$\mathscr{U}_t = \log C_{\mathrm{y},t} + \delta \mathbb{E}_t \left[\log C_{\mathrm{m},t+1} + \Theta(H_{\mathrm{m},t+1})\right] + \delta^2 \mathbb{E}_t \log C_{\mathrm{o},t+2}, \qquad [2.1]$$

where $C_{\tau,t}$ denotes per-person consumption of a composite good by individuals of age τ at time t. Utility is logarithmic in consumption of goods. Housing services $H_{\mathrm{m},t}$ is a continuous variable, and the housing utility function $\Theta(H)$ is strictly increasing, strictly concave, and satisfies an Inada condition ($\Theta'(H) > 0$, $\Theta''(H) < 0$, and $\lim_{H\to 0} \Theta'(H) = \infty$). The subjective discount factor is δ (satisfying $0 < \delta < \infty$), and $\mathbb{E}_t$ denotes expectations conditional on time-t information.

Two assumptions about individuals give rise to a simple pattern of gains from trade in financial markets. First, individuals receive labour income only while middle-aged. For simplicity, there is no choice of hours, so each middle-aged individual inelastically supplies one unit of labour $N_{\mathrm{m},t}$. Each unit of labour receives real wage w_t, so labour income $y_{\mathrm{m},t}$ is:

$$y_{\mathrm{m},t} = w_t N_{\mathrm{m},t}, \quad \text{where} \quad N_{\mathrm{m},t} = 1. \qquad [2.2]$$

Under these assumptions, youth can be interpreted as the very beginning of an individual's career, and old age as the beginning of retirement. The second assumption is that utility from housing services ($H_{\mathrm{m},t}$ in 2.1) is received only while middle-aged, and can only be obtained through homeownership. Houses must be purchased and held between $t-1$ and t to enjoy utility flows at time t. Consequently, individuals will want to buy and sell houses over their life-cycle, which can be interpreted as 'trading up' and 'trading down'. Furthermore, because of the first assumption, house purchases and consumption will need to be financed using debt when young, while later in the life-cycle, households will need to hold financial assets as a pension to enjoy consumption in

[1] The model adds housing to the earlier overlapping generations model of Sheedy (2013).

[2] Monetary policy with incomplete financial markets is also studied by Koenig (2013), and Sheedy (2014) analyses the quantitative importance of incomplete financial markets compared to other nominal frictions.

retirement.

Money is used as a unit of account, but the analysis abstracts from money's role as a medium of exchange. The nominal price of a unit of goods is denoted by P_t and the nominal price of a unit of housing by V_t.

Financial markets are incomplete. Individuals can hold only three assets: houses, nominal bonds, and corporate equity; and can issue only nominal bonds as liabilities. Let $B_{\tau,t}$ denote the quantity of nominal bonds purchased (or issued, if negative) by households of age τ at the end of time period t, and $U_{\tau,t}$ the number of shares purchased (short sales of equity are not allowed, $U_{\tau,t} \geq 0$). Each nominal bond is a riskless claim on one monetary unit of account at time $t+1$ (the face value is one unit of money). Each unit of corporate equity is a claim on a single dividend payment X_{t+1} (specified in nominal terms) at time $t+1$. At time t, the nominal prices of bonds and shares are Q_t and J_t respectively.

Individuals are born with no initial assets or liabilities and leave no bequests. The budget identities of the young, middle-aged, and old are respectively:

$$C_{\mathrm{y},t} + \frac{V_t H_{\mathrm{m},t+1}}{P_t} + \frac{Q_t B_{\mathrm{y},t}}{P_t} + \frac{J_t U_{\mathrm{y},t}}{P_t} = 0; \tag{2.3a}$$

$$C_{\mathrm{m},t} + \frac{V_t H_{\mathrm{o},t+1}}{P_t} + \frac{Q_t B_{\mathrm{m},t}}{P_t} + \frac{J_t U_{\mathrm{m},t}}{P_t} = y_{\mathrm{m},t} + \frac{V_t H_{\mathrm{m},t}}{P_t} + \frac{B_{\mathrm{y},t-1}}{P_t} + \frac{X_t U_{\mathrm{y},t-1}}{P_t}; \tag{2.3b}$$

$$C_{\mathrm{o},t} = \frac{V_t H_{\mathrm{o},t}}{P_t} + \frac{B_{\mathrm{m},t-1}}{P_t} + \frac{X_t U_{\mathrm{m},t-1}}{P_t}. \tag{2.3c}$$

Maximizing expected utility (2.1) with respect to house purchases $H_{\mathrm{m},t+1}$, net bond positions $B_{\mathrm{y},t}$ and $B_{\mathrm{m},t}$, and purchases of corporate equity $U_{\mathrm{m},t}$, subject to the budget identities in (2.3), implies the following first-order conditions:

$$\frac{V_t}{P_t C_{\mathrm{y},t}} = \delta \mathbb{E}_t \left[\Theta'(H_{\mathrm{m},t+1}) + \frac{\gamma V_{t+1}}{P_{t+1} C_{\mathrm{m},t+1}} \right]; \tag{2.4a}$$

$$\frac{Q_t}{P_t C_{\mathrm{y},t}} = \delta \mathbb{E}_t \left[\frac{1}{P_{t+1} C_{\mathrm{m},t+1}} \right]; \tag{2.4b}$$

$$\frac{Q_t}{P_t C_{\mathrm{m},t}} = \delta \mathbb{E}_t \left[\frac{1}{P_{t+1} C_{\mathrm{o},t+1}} \right]; \tag{2.4c}$$

$$\frac{J_t}{P_t C_{\mathrm{m},t}} = \delta \mathbb{E}_t \left[\frac{X_{t+1}}{P_{t+1} C_{\mathrm{o},t+1}} \right]. \tag{2.4d}$$

It is shown in Sheedy (2017a) that the equity short-sale constraint for the young is always binding ($U_{\mathrm{y},t} = 0$), so there is no first-order condition included for $U_{\mathrm{y},t}$. Similarly, house purchases are made only while young ($H_{\mathrm{o},t} = 0$), at least under conditions where monetary policy does not give rise to a rational bubble in the housing market, which is assumed in what follows.[3]

The supply of houses H_t is assumed to be inelastic and of measure one:

$$H_t = 1. \tag{2.5}$$

There are no maintenance costs.

[3]Bubbles are a possibility in an overlapping generations economy with incomplete financial markets, even though housing is a perpetual asset. The issue of rational bubbles and monetary policy is explored in Sheedy (2017b).

There is a unit continuum of representative firms in the economy. The representative firm has capital K_t and hires labour N_t to produce output Y_t according to the constant-returns Cobb-Douglas production function below:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{2.6}$$

where α is the elasticity of output with respect to capital (satisfying $0 < \alpha < 1$), and A_t is the level of total factor productivity. Capital is acquired through investment I_t, which transforms one unit of final goods into one unit of capital goods. There is a one-period time-to-build, and capital depreciates at rate 100% for tractability, thus the link between the capital stock and investment is:

$$K_t = I_{t-1}. \tag{2.7}$$

Firms hire units of labour in a competitive market at real wage w_t. Total factor productivity A_t is an exogenous stochastic process.

Firms live for one time period. They raise finance (debt and equity) in period t and invest in capital. At time $t+1$ they hire labour and use installed capital to produce output, and then pay out profits after debt repayments as dividends. Without loss of generality, assume each firm issues a unit measure of shares. Any bonds issued are one-period nominal bonds, with $B_{\mathrm{f},t}$ denoting the quantity of such bonds issued by the representative firm. The accounting identity for financing investment I_t is:

$$P_t I_t = \mathit{\Lambda}_t J_t, \quad \text{where } \mathit{\Lambda}_t \equiv 1 + \frac{Q_t B_{\mathrm{f},t}}{J_t}, \tag{2.8}$$

with the variable $\mathit{\Lambda}_t$ denoting the corporate leverage ratio. Given bond and share prices Q_t and J_t, the choice of the quantity $B_{\mathrm{f},t}$ of bonds to issue is the same as a choice of the leverage ratio $\mathit{\Lambda}_t$. At time $t+1$, the representative firm has nominal revenues $P_{t+1}Y_{t+1}$, a nominal wage bill $w_{t+1}P_{t+1}N_{t+1}$, and nominal debt repayments $B_{\mathrm{f},t}$. Nominal dividends X_{t+1} are revenues net of wages and debt repayments:

$$X_t = P_t Y_t - w_t P_t N_t - B_{\mathrm{f},t-1}. \tag{2.9}$$

Using the production function (2.6), the level of employment that maximizes dividends is:

$$(1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} = w_t, \tag{2.10}$$

and this equation must hold irrespective of the financing of investment and the ownership of the firm. It is shown in Sheedy (2017a) that any corporate leverage ratio $\mathit{\Lambda}_t \geq 1$ is consistent with equilibrium in this simple model.

In equilibrium, the goods, labour, housing, bond, and stock markets must clear:

$$C_t + I_t = Y_t, \quad \text{where} \;\; C_t = C_{\mathrm{y},t} + C_{\mathrm{m},t} + C_{\mathrm{o},t}; \qquad [2.11a]$$

$$N_{\mathrm{m},t} = N_t; \qquad [2.11b]$$

$$H_{\mathrm{m},t+1} + H_{\mathrm{o},t+1} = H_t; \qquad [2.11c]$$

$$B_{\mathrm{y},t} + B_{\mathrm{m},t} = B_{\mathrm{f},t}; \qquad [2.11d]$$

$$U_{\mathrm{y},t} + U_{\mathrm{m},t} = 1. \qquad [2.11e]$$

This completes the description of the model.

3 Conventional and unconventional policy instruments

The distinction between 'conventional' and 'unconventional' monetary policy instruments is now defined. The conventional monetary policy instrument is the central bank's control of the interest rate on risk-free nominal bonds. Unconventional monetary policy is then defined as a residual category: any other type of intervention by the central bank that does not depend for its operation on changing the risk-free nominal interest rate now or in the future.

Unconventional monetary policy instruments include balance-sheet policies, comprising various types of quantitative easing and credit easing policies (see, for example, Gagnon, Raskin, Remache and Sack, 2011, Woodford, 2012). Unconventional policy also includes emergency lending programmes and subsidized access to credit more generally. These policy instruments are seen as providing additional 'easing' even without any change in the risk-free nominal interest rate.

As defined, the unconventional instruments of monetary policy also include newer 'macroprudential' policies, which place additional restrictions on access to credit above and beyond the interest rate charged (for a survey, see Galati and Moessner, 2013). For example, more stringent loan-to-value ratios for mortgages might be imposed on banks. This type of policy is seen as tightening financial conditions even without any adjustment of the risk-free nominal interest rate.

Under the definition of 'conventional' policy instruments adopted here, policies such as 'forward guidance', where information is revealed about the future path of nominal interest rates (see Campbell, Evans, Fisher and Justiniano, 2012), are classified as conventional policies. This is because the policy is really a commitment to use a conventional policy instrument in a particular way at a future date.[4] Note also that the definition of 'conventional' applies only to the instruments of monetary policy, not to the goals (which could be 'unconventional' in a different sense).

3.1 The conventional monetary policy instrument

The central bank provides interest-bearing reserves that are a perfect substitute for risk-free nominal bonds. By setting the interest rate i_t paid on reserves, the central bank can determine the nominal

[4]This classification is justified to the extent that the transmission mechanism of successfully-communicated forward guidance is not so different from standard monetary policy, where expectations about the future also play a crucial role. See Woodford (2012) for further discussion.

bond price Q_t:

$$Q_t \equiv \frac{1}{1+i_t}, \quad \text{where } i_t \geq 0. \tag{3.1}$$

Reserves can be exchanged for physical cash, and it is assumed this cash can be stored costlessly.[5] These features of money place a zero lower bound $i_t \geq 0$ on the nominal interest rate.

3.2 Unconventional monetary policy instruments

While the economic model of section 2 naturally has a role for the nominal interest rate set by the central bank because this affects borrowing and saving decisions (with real effects due to the presence of nominal contracts), introducing unconventional policy instruments requires adding some additional features to the model to understand how these might affect individuals' behaviour. It is argued below that various types of unconventional monetary policy instrument all lead to borrowers' Euler equation (2.4b) being replaced by:

$$\frac{(1-\tau_t)Q_t}{P_t C_{\mathrm{y},t}} = \delta \mathbb{E}_t \left[\frac{1}{P_{t+1}C_{\mathrm{m},t+1}}\right], \tag{3.2}$$

where the variable τ_t is directly determined by the new policy instruments. The new policy instruments captured by τ_t thus appear as a wedge in borrowers' Euler equation that will affect the amount of borrowing in addition to the nominal interest rate i_t. It turns out that this will be the only change to the equilibrium conditions of the economy.

3.2.1 Balance-sheet policies

Assume the central bank makes purchases $\tilde{B}_t$ of nominal bonds. These purchases are targeted, with the central bank buying only household debt, not corporate bonds or equity. This can be interpreted as purchases of mortgage-backed securities, which formed a key component of the Federal Reserve's large-scale asset purchases. Suppose the central bank finances its asset purchases by issuing interest-bearing reserves B_t. From the perspective of those who will buy and hold these reserves, the reserves are essentially the same as nominal bonds. This means the market price of each unit of reserves sold by the central bank is Q_t.

The central bank is assumed to match the payoffs of the assets it purchases to its liabilities in the future so that there are no profits or losses to be passed on to future generations. This assumption is made to abstract from any direct intergenerational redistribution that would otherwise result from such asset purchases. In this simple analysis, the assets and liabilities on both sides of the central bank's balance sheet fundamentally have the same characteristics, so the central-bank balance sheet must be such that:

$$\tilde{B}_t = B_t. \tag{3.3}$$

A key assumption is that the central bank offers to buy bonds $\tilde{B}_t$ from borrowers at a price $\tilde{Q}_t$

[5] Cash is not explicitly modelled here. See Azariadis, Bullard, Singh and Suda (2015) for an analysis of the zero lower bound problem in an incomplete-markets economy with cash.

that may be different from the market price Q_t that would otherwise prevail. The central bank would not be able to find willing sellers of bonds if the price it offered were $\tilde{Q}_t < Q_t$, so that case is inconsistent with equilibrium in financial markets and attention is restricted to $\tilde{Q}_t \geq Q_t$. To sustain a differential between $\tilde{Q}_t$ and Q_t and ensure that financial markets can be in equilibrium without restricting the size $\tilde{B}_t$ of the asset purchases, the central bank must act in such a way that no arbitrage opportunities are available. One crude way to achieve this is by making its purchases and sales of bonds discriminatory: the central bank will only buy bonds from young borrowers,[6] and it will only offer interest-bearing reserves to individuals who are not young.[7]

With this asset purchase programme in place, the central bank is now a buyer and a seller in different segments of the bond market. The bond-market clearing condition (2.11d) is replaced by:

$$B_{\mathrm{y},t} + B_{\mathrm{m},t} + \tilde{B}_t = B_{\mathrm{f},t} + B_t, \quad \text{where it is required that } \tilde{B}_t \leq -B_{\mathrm{y},t}. \qquad [3.4]$$

The latter condition is needed because if bond purchases are made only from young borrowers, the amount $-B_{\mathrm{y},t}$ that the young borrow puts a limit on the maximum size of the asset purchases in equilibrium. Note that the young are always willing to sell to the central bank rather than the market because $\tilde{Q}_t \geq Q_t$.

Whenever $\tilde{Q}_t > Q_t$, equation (3.3) implies that the central bank's actions would result in an immediate loss. To abstract from any direct redistributional effects of the central bank's asset purchases, this loss is passed on as a lump-sum tax $-T_t$ (in real terms) paid by each young individual. The lump-sum tax is targeted to the young because they are the direct beneficiaries of being able to sell their bonds at a higher price to the central bank.[8] The required net transfer T_t is:

$$T_t = \frac{-(\tilde{Q}_t\tilde{B}_t - Q_tB_t)}{P_t}, \qquad [3.5]$$

and this is taken as given by each individual. The budget constraint of the young in (2.3a) is replaced by:

$$C_{\mathrm{y},t} + \frac{V_tH_{\mathrm{m},t+1}}{P_t} + \frac{Q_t(B_{\mathrm{y},t} + \tilde{B}_t)}{P_t} = \frac{\tilde{Q}_t\tilde{B}_t}{P_t} + T_t, \qquad [3.6]$$

where $B_{\mathrm{y},t} + \tilde{B}_t$ denotes the net bond trades of the young in the market, excluding sales of bonds to the central bank. If the central bank is the buyer of the marginal unit of bonds issued by young

[6] This limitation on the central bank's dealings is needed to ensure its asset purchases are targeted to supporting mortgage borrowing by the young. It is implicit that the young cannot trade with the central bank on behalf of others to exploit arbitrage.

[7] Consider an alternative assumption where the central bank would offer to buy bonds from the middle-aged rather than the young at price $\tilde{Q}_t > Q_t$. Since the middle-aged would need to be able to buy bonds on the market at price Q_t in order to have any financial wealth in retirement, there would be no equilibrium if the size of the central bank's purchases was unrestricted because the middle-aged could make unlimited profits from arbitrage. If the purchases were limited, arbitrage profits would be finite. However, under this alternative assumption, the marginal bond purchased by the middle-aged would have price Q_t, and the young would also sell at price Q_t, so the policy would have no effect apart from generating a lump-sum transfer to the middle-aged.

[8] It would also be possible to consider a lump-sum tax paid by all individuals. This would add some additional complications without changing the substance of the results. The key assumption is that the central bank and government cannot target lump-sum taxes and transfers in such a way as to offset the incompleteness of financial markets. Furthermore, such direct redistributions through lump-sum taxes ought to be classified as fiscal policy, not monetary policy.

borrowers then the Euler equation (2.4b) needs to be adjusted to reflect the higher price $\tilde{Q}_t$:

$$\frac{\tilde{Q}_t}{P_t C_{\mathrm{y},t}} = \delta \mathbb{E}_t \left[\frac{1}{P_{t+1} C_{\mathrm{m},t+1}} \right] \text{ if } \tilde{B}_t = -B_{\mathrm{y},t}, \quad \text{otherwise (2.4b) holds.} \qquad [3.7]$$

If the central bank is not the marginal buyer, the bond price in borrowers' Euler equation is simply the market price Q_t. The budget identity and Euler equation of middle-aged savers are unaffected by any of these changes to the central bank's operating procedures.

Observe that equations (3.3), (3.5) and (3.6) can be combined to deduce that the original budget identity of the young (2.3a) must hold in equilibrium. The original bond-market clearing condition (2.11d) also follows from (3.3) and (3.4) in equilibrium. Given the market price of bonds Q_t, the central bank's intervention price $\tilde{Q}_t$, and the size of the asset purchase $\tilde{B}_t$, define a variable τ_t as follows:

$$\tau_t = \begin{cases} -(\tilde{Q}_t - Q_t)/\tilde{Q}_t & \text{if } \tilde{B}_t = -B_{\mathrm{y},t} \\ 0 & \text{otherwise} \end{cases}. \qquad [3.8]$$

In the case where the central bank is the marginal buyer, the variable τ_t is equal to the negative of the premium paid by the central bank for borrowers' bonds above what would otherwise be the market price. It follows that $\tilde{Q}_t = (1 - \tau_t)Q_t$, and hence the new Euler equation for borrowers (3.7) is equivalent to (3.2) with τ_t as defined in (3.8). Note that this is the only change to the equilibrium conditions of the economy.

By setting the intervention price relative to the market price, and the size of the intervention, the central bank can determine the value of τ_t, which can in principle take any value satisfying $-1 < \tau_t \leq 0$. When the central bank is not the marginal buyer, $\tau_t = 0$, which means the asset purchases would have no effect on the economy's equilibrium conditions.

It is immediately apparent that asset purchases made at the market price ($\tilde{Q}_t = Q_t$) have no impact whatsoever on the economy. Since $\tau_t = 0$ in this case, the equilibrium conditions of the economy are identical to those in the absence of the asset purchases, so the policy has no effect on either nominal or real variables no matter what the size of $\tilde{B}_t$. This is just a version of the well-known neutrality results for balance-sheet policies (see, for example, Eggertsson and Woodford, 2003, Woodford, 2012). If the central bank lends more to the private sector, private agents lend less and simply hold more interest-bearing reserves instead, leaving the total amount of lending unaffected. Taking account of the central bank's budget constraint, this change in private behaviour delivers exactly the same allocation as before, so no market-clearing prices are affected.

This reasoning does not apply when the price paid for the assets purchased by the central bank is not the market price. In that case, the central bank's asset purchases effectively make credit available that would not have been forthcoming from private agents at the market price. This implicit subsidy (paid for by a reduction in central bank profits, all else equal) increases the total amount of borrowing, which can therefore have an effect on the economy's equilibrium.

Whether or not actual asset-purchase programmes have simply replaced private purchases of assets with an equivalent amount of public purchases is beyond the scope of this paper. Where central banks perform emergency lending to financial institutions, or make asset purchases in highly illiquid markets at times of panic, it is more likely the resulting expansion of the central bank's balance sheet occurs at non-market prices. In any case, such balance-sheet policies are feasible and provide a way of thinking about the effects of unconventional policy that is not circumscribed by the well-known neutrality theorems.

3.2.2 Subsidized credit

An equivalent way of thinking about the unconventional monetary policy described above is in terms of access to subsidized credit from either the central bank or government, ultimately financed by lump-sum taxes. This is modelled formally as a proportional subsidy ς_t to borrowing, with a lump-sum tax T_t levied on the group of beneficiaries of the subsidy. In this way, the subsidy rate ς_t affects the demand for loans without having any direct fiscal consequences.

The budget identity of the young (2.3a) is replaced by:

$$C_{\mathrm{y},t} + \frac{V_t H_{\mathrm{m},t+1}}{P_t} + \frac{(1+\varsigma_t) Q_t B_{\mathrm{y},t}}{P_t} = T_t, \tag{3.9}$$

with the required lump-sum tax $-T_t$ given by:

$$T_t = \frac{\varsigma_t Q_t B_{\mathrm{y},t}}{P_t}. \tag{3.10}$$

Maximizing utility (2.1) subject to (3.9) (with individuals taking T_t as given) implies that borrowers' Euler equation (2.4b) is replaced by:

$$\frac{(1+\varsigma_t) Q_t}{P_t C_{\mathrm{y},t}} = \delta \mathbb{E}_t \left[\frac{1}{P_{t+1} C_{\mathrm{m},t+1}} \right]. \tag{3.11}$$

Combining the budget identities (3.9) and (3.10) shows that (2.3a) continues to hold in equilibrium. The Euler equation (3.11) is equivalent to (3.2) with $\tau_t = -\varsigma_t$, that is, where the wedge τ_t in borrowers' Euler equation is equal to the negative of the subsidy rate ς_t. Apart from replacing (2.4b) with (3.2), all the equilibrium conditions of the economy are unchanged. As before, the variable τ_t can be treated as the unconventional monetary policy instrument.

While the approach taken here to modelling balance-sheet policies and credit subsidies is highly stylized, it provides a simple account of how the central bank can affect the interest rate paid by borrowers relative to the interest rate received by savers. Alternative approaches are based on the existence of a moral hazard problem between banks and depositors (Gertler and Karadi, 2011) or a costly state verification problem (Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez and Vardoulakis, 2015). The central bank is not subject to these financial frictions, so it is able to affect interest rate spreads through asset purchases. A common feature of these approaches and the one used here is that the central-bank balance sheet is unconstrained, unlike the balance sheets of private agents. One difference is that frictions such as moral hazard or costly state verification may allow the central bank to make profits on its asset purchases.

3.2.3 Macroprudential policies

While the previous two examples of unconventional monetary policies are designed to stimulate lending without reducing the risk-free nominal interest rate, it is also possible to envisage unconventional policies which do the opposite, namely restrict access to credit without raising the risk-free nominal interest rate. Recently, there has been an upsurge of interest in so-called 'macroprudential' policies that essentially have this aim (Galati and Moessner, 2013). While such policies are often considered as distinct from monetary policy, it will be seen that their effects work in a closely related, but opposite, manner to the unconventional monetary policies considered so far. For this reason, macroprudential policies are analysed as a type of monetary policy, albeit of an unconventional kind.

Consider a common type of macroprudential policy whereby the central bank imposes a regulation limiting access to credit. This restriction is specified as a maximum loan-to-value ratio for borrowers. Formally, the expected ratio of borrowers' debt burden to the value of the housing collateral they hold is limited to some policy-specified maximum χ_t:

$$-\mathbb{E}_t \frac{B_{\mathrm{y},t}}{V_{t+1}} \leq \chi_t. \qquad [3.12]$$

Maximizing utility (2.1) subject to the budget constraints in (2.3) and the collateral constraint (3.12) implies the first-order conditions:

$$\frac{Q_t}{P_t C_{\mathrm{y},t}} = \delta \mathbb{E}_t \left[\frac{1}{P_{t+1} C_{\mathrm{m},t+1}} \right] + \mathbb{E}_t \left[\frac{D_t}{V_{t+1}} \right], \quad D_t \geq 0, \quad \text{and} \quad \left(\chi_t + \mathbb{E}_t \left[\frac{B_{\mathrm{y},t}}{V_{t+1}} \right] \right) D_t = 0, \qquad [3.13]$$

where D_t is the Lagrangian multiplier attached to the collateral constraint. These first-order conditions along with (3.12) replace the Euler equation of borrowers in (2.4b), and D_t is an additional endogenous variable. It turns out that these equations are equivalent to (3.2) with τ_t being a function only of the macroprudential policy instrument χ_t:

$$\tau_t = \max\left\{0, 1 - \chi_t / d^*\right\}, \quad \text{and with} \;\; D_t = \tau_t \left(\mathbb{E}_t \left[\frac{P_t C_{\mathrm{y},t}}{Q_t V_{t+1}} \right] \right)^{-1}, \qquad [3.14]$$

where d^* is a positive constant. Since the solution for τ_t can be obtained without reference to any variable other than χ_t, this means that τ_t itself can be directly interpreted as the policy instrument. Hence, the equilibrium conditions change only in substituting (3.2) for (2.4b), noting that the new variable D_t does not appear in the other equations.

With balance-sheet policies and subsidized credit, the value of τ_t was negative. Here, with macroprudential policy, the borrowers' Euler equation (3.2) has the same general form, but now τ_t is non-negative. For high values of the loan-to-value ratio χ_t, the collateral constraint will not be binding, in which case $\tau_t = 0$ and the macroprudential policy has no effect. When the collateral constraint is binding, the implied value of τ_t is strictly positive.

It would also be possible to analyse macroprudential policies that operate as taxes on lending. These reduce the incentive to borrow, but do not impose any quantitative restrictions on borrowing. This case is essentially the opposite of the credit subsidy analysed in section 3.2.2. If τ_t is the tax rate on borrowing (with the proceeds rebated lump-sum to the group of borrowers as a whole), the only change to the equilibrium conditions is that (3.2) replaces (2.4b). In this case, the positive value of τ_t can be interpreted directly as the tax rate.

3.3 Monetary policy

The following result summarizes the analysis so far:

Result 1 *The various types of unconventional monetary policy considered in section 3.2 all imply that borrowers' Euler equation (2.4b) is replaced by (3.2), with no other alterations to the economy's equilibrium conditions. The new variable τ_t appearing in (3.2) is determined solely by the unconventional policy instruments.*

PROOF See appendix A.1. ■

This result allows several different types of unconventional monetary policy to be analysed using a common framework.

Turning first to what ought to be achieved through the use of monetary policy, note that in the economic model of section 2 the conditions for Pareto efficiency include:

$$\frac{C_{\mathrm{m},t}}{C_{\mathrm{y},t-1}} = \frac{C_{\mathrm{o},t}}{C_{\mathrm{m},t-1}}, \quad \text{and} \quad \frac{1}{C_{\mathrm{m},t}} = \delta \mathbb{E}_t\left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha}}{C_{\mathrm{o},t+1}}\right]. \qquad [3.15]$$

The first condition requires consumption smoothing and risk sharing across all cohorts of individuals. The second condition requires that the return on any resources used for investment is sufficient to compensate savers for delaying consumption. Since unconventional monetary policy only changes borrowers' Euler equation (2.4b), the results of Sheedy (2017a) show that the only way for monetary policy to ensure the efficiency conditions in (3.15) hold is by aiming to stabilize the following two ratios:

$$l_t \equiv -\frac{Q_t B_{\mathrm{y},t}}{P_t Y_t}, \quad \text{and} \quad s_t \equiv \frac{Y_t - C_t}{Y_t}. \qquad [3.16]$$

The variable l_t is new lending to households (for house purchases or consumption) relative to GDP, and s_t is the national saving rate. The national saving rate s_t is also equal to the ratio of investment to GDP given that the economy is closed. Given the equivalence between the efficiency conditions (3.15) and stabilizing l_t and s_t, if monetary policy is to support an efficient allocation of resources, it must try to prevent booms and busts in lending to households and firms.

Before considering the real effects of monetary policy, note that there are two relevant inflation rates in the economy: goods-price inflation ϖ_t, and asset-price (house-price) inflation π_t:

$$\varpi_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}}, \quad \text{and} \quad \pi_t \equiv \frac{V_t - V_{t-1}}{V_{t-1}}. \qquad [3.17]$$

Using (3.1) and (3.17), borrowers' Euler equation (3.2) can be written explicitly in terms of the inflation rate ϖ_{t+1} and the conventional and unconventional policy instruments i_t and τ_t:

$$\frac{1}{C_{\mathrm{y},t}} = \frac{\delta}{1-\tau_t}\mathbb{E}_t\left[\frac{(1+i_t)}{(1+\varpi_{t+1})C_{\mathrm{m},t+1}}\right]. \qquad [3.18]$$

Ignoring the zero lower bound constraint, Sheedy (2017a) shows that a feedback rule of the type $i_t = (1+\pi_t)^{\zeta}\mu_t - 1$ for the conventional policy instrument, where μ_t denotes an exogenous state-contingent shift in the stance of conventional monetary policy, results in a unique equilibrium with stable inflation if $\zeta > 1$. The monetary policy stance μ_t is an exogenous stochastic process that can be either interpreted as a reaction to the exogenous shocks hitting the economy (shocks to TFP A_t in this simple model), or as a pure monetary policy shock.

While the feedback rule $i_t = (1 + \pi_t)^\zeta \mu_t - 1$ has the interest rate react only to one endogenous variable, the inflation rate π_t, the rule is more general than it might first appear. If some equilibrium could be implemented using an alternative feedback rule, there is always an exogenous state-contingent policy stance $\{\mu_t\}$ for which the rule $i_t = (1 + \pi_t)^\zeta \mu_t - 1$ gives rise to the same equilibrium (and which is the unique equilibrium with stable inflation when $\zeta > 1$). This means that except for questions of the determinacy of the equilibrium, it is without loss of generality to focus on specifying the exogenous policy stance μ_t, rather than on the response of i_t to endogenous variables. Different targets for monetary policy can then be implemented by choosing different policy stances μ_t.

In this paper, the feedback rule for the nominal interest rate is modified so that it always respects the zero lower bound constraint:

$$i_t = \max\{(1 + \pi_t)^\zeta \mu_t - 1, 0\}. \qquad [3.19]$$

The new unconventional monetary policy instrument τ_t is set according to the feedback rule:

$$\tau_t = 1 - \frac{1}{\upsilon_t(1 + \min\{(1 + \pi_t)^\zeta \mu_t - 1, 0\})}, \qquad [3.20]$$

where υ_t is the exogenous stance of unconventional policy. When the zero lower bound constraint is not binding, the policy rule sets the unconventional instrument τ_t in accordance with the exogenous policy stance υ_t, but when the zero lower bound becomes binding, τ_t reacts endogenously to the inflation rate π_t to compensate for the inability to lower i_t below zero.

Monetary policy is specified as sequences of state-contingent policy stances $\{\mu_t\}$ and $\{\upsilon_t\}$ for the conventional and unconventional instruments. Both of these policy stances are exogenous variables, which could be either responses to the fundamental exogenous shocks faced by the economy (changes in TFP A_t) or pure monetary policy shocks.

4 Unconventional monetary policy rules

This section studies what can be achieved through the choice of a rule for unconventional monetary policy (a specification of the stance of unconventional policy $\{\upsilon_t\}$). The analysis below takes the stance of conventional monetary policy $\{\mu_t\}$ as given, as would be the case when the use of conventional monetary policy is constrained by the zero lower bound. The zero lower bound could become binding if the interest rate required to achieve an inflation target using only conventional monetary policy were to become negative following a shock (to TFP A_t in the model here).

As can be seen from equation (3.18), both conventional and unconventional monetary policy instruments appear in the economy's equilibrium conditions, but the unconventional policy instrument τ_t appears only in (3.18) (unlike the nominal interest rate i_t, which also appears in 2.4c through the bond price Q_t). This suggests that both types of monetary policy should affect the equilibrium of the economy, but that their effects might not be equivalent.

4.1 The control of inflation

The first finding is that unconventional monetary policy can be used to ensure there is a unique stable equilibrium for inflation.

Result 2 *Suppose the conventional and unconventional monetary policy instruments are set in accordance with the feedback rules* (3.19) *and* (3.20) *for given policy stances* μ_t *and* υ_t. *When* $\zeta > 1$, *there is a unique equilibrium with stable inflation.*

Proof See appendix A.2. ■

As explained in Woodford (2003), when the central bank conducts monetary policy using an interest rate rule, the 'Taylor principle' is required to ensure a unique stable equilibrium for inflation. This means that all else equal, the nominal interest rate must rise or fall more than any movement in inflation. Since the zero lower bound limits the maximum amount that the nominal interest rate can be cut, the Taylor principle cannot always be satisfied using conventional policy. This gives rise to multiple equilibria, one of which is a 'deflation trap' (see Benhabib, Schmitt-Grohé and Uribe, 2001). However, if monetary policy can be loosened through the unconventional instrument instead of lowering interest rates when inflation falls, the Taylor principle can be restored and a unique stable equilibrium is obtained.[9]

The ability of monetary policy to achieve a unique stable equilibrium for inflation is taken for granted in what follows, where attention turns to what outcomes can actually be achieved through the choice of the unconventional policy stance $\{\upsilon_t\}$.

Result 3 *Taking as given the stance of the conventional policy instrument* $\{\mu_t\}$, *any target path for the inflation rate* ϖ_t *or* π_t *can be achieved through an appropriate choice of the unconventional policy stance* $\{\upsilon_t\}$.

Proof See appendix A.3. ■

This result shows that for the control of inflation, the unconventional policy instrument is in principle a perfect substitute for conventional monetary policy. Intuitively, both types of policy instrument affect the demand for loans at a given rate of inflation. By varying the unconventional policy instrument appropriately, monetary policy can ensure the economy is in equilibrium at any target rate of inflation. The result also confirms that the particular form of the feedback rule (3.19) in

[9]It might be objected that the unconventional monetary policy instrument τ_t is itself subject to a lower bound. When τ_t is negative, this corresponds to the case of a credit subsidy or a central bank buying assets above the market price. This has a fiscal cost, which was assumed to be met by lump-sum taxes. In practice, only distortionary taxes are available to make up for central bank losses, which places a limit on the maximum size of unconventional monetary policy interventions. If the restriction $\tau_t \geq -\Bbbk$ is imposed for some $\Bbbk \geq 0$ then the problem of multiple equilibria returns.

reacting to house-price inflation π_t is without loss of generality because the same type of feedback rule can be used to achieve any alternative nominal target (which would correspond to some state-contingent sequence of house-price inflation $\{\pi_t\}$).

In what follows, the stance of conventional monetary policy $\{\mu_t\}$ is restricted so that the nominal interest rate is always bounded above zero in equilibrium ($i_t > 0$). It follows from (3.19) and (3.20) that the unconventional monetary policy instrument is equal to the following in equilibrium:

$$\tau_t = 1 - \frac{1}{\upsilon_t}. \tag{4.1}$$

This means the actual value of the unconventional policy instrument τ_t will be determined entirely by the exogenous unconventional policy stance $\{\upsilon_t\}$ on the equilibrium path.

4.2 The real effects of monetary policy

The results obtained so far provide no reason to prefer conventional to unconventional monetary policy instruments since both can be equally well used to control inflation. However, this is not the case when the real effects of the two policy instruments are considered.

The source of the real effects of monetary policy in the economic model is the presence of nominal contracts, specifically the nominal debt (mortgages) used to finance house purchases. Monetary policy matters because it affects nominal house prices, and in particular, how nominal house prices react to economic shocks (TFP shocks in the model). Since the nominal debt burden is predetermined, fluctuations in nominal house prices have real effects on the balance sheets of borrowers and lenders. Shocks to balance sheets affect relative consumption levels, and by changing the net worth of lenders, also affect the real quantity of new loans received by borrowers. This in turn has implications for the consumption levels of different cohorts of individuals. Finally, shocks to balance sheets also influence the real quantity of investment that is financed, which affects production in the economy.

The workings of the model are described in more detail in Sheedy (2017a). The old consume all of their financial wealth. The middle-aged are the lenders in the economy, and their net worth depends on the current value of the houses they bought when young and the amount of debt they must repay. Given logarithmic utility (2.1), income and substitution effects cancel out for savers, which implies the middle-aged will consume a constant fraction of their net worth. The remaining fraction is lent to the young to finance purchases of houses and consumption goods, and to firms to finance investment. This means the real supply of loans is a function only of the real net worth of lenders, not interest rates or any other factors such as uncertainty about the future. The young are borrowers, and for them income and substitution effects are reinforcing. This means that the demand for loans is negatively related to interest rates, and also to uncertainty about the future value of the houses that are bought using the loans. As explained earlier in section 3.2, the unconventional policy instruments also affect the demand for loans independently of the risk-free nominal interest rate.

Real house prices are determined in equilibrium by the real value of lending given the inelastic supply of houses and the dependence of homebuyers on mortgage financing. Equilibrium real interest rates are determined in the market for loans. The supply of loans is interest inelastic, but shifts with the net worth of lenders. The demand for loans is interest elastic and shifts with changes in the probability distribution of future house prices. Changing the stance of unconventional monetary policy also leads to a shift of the demand curve. Finally, inflation can be determined given its relationship with the economy's real variables and the stances of conventional and unconventional monetary policy.

The following result gives the real effects of unexpected changes in the stances of conventional and unconventional monetary policy μ_t and υ_t. These unexpected changes could be pure policy shocks or responses to unexpected economic shocks (TFP shocks in the model).

Result 4 *The real effects of an unexpected one-period loosening of the stances of conventional monetary policy (lower μ_t) and unconventional monetary policy (lower υ_t) are shown in the table below. The effects are reported for new lending l_t and the national saving rate s_t (both defined in 3.16), and the consumption-to-GDP ratios for the young, middle-aged, and old, as defined by $c_{y,t} \equiv C_{y,t}/Y_t$, $c_{m,t} \equiv C_{m,t}/Y_t$, and $c_{o,t} \equiv C_{o,t}/Y_t$.*

Variable	$l_{t+\ell}$		$s_{t+\ell}$		$Y_{t+\ell}$		$c_{y,t+\ell}$		$c_{m,t+\ell}$		$c_{o,t+\ell}$	
Policy	$\mu_t \downarrow$	$\upsilon_t \downarrow$	$\mu_t \downarrow$	$\upsilon_t \downarrow$	$\mu_t \downarrow$	$\upsilon_t \downarrow$	$\mu_t \downarrow$	$\upsilon_t \downarrow$	$\mu_t \downarrow$	$\upsilon_t \downarrow$	$\mu_t \downarrow$	$\upsilon_t \downarrow$
Time horizon (ℓ)												
0	+	+	+	+	0	0	+	+	+	+	−	−
1	0	−	0	−	+	+	0	−	0	−	0	+
≥ 2	0	0	0	0	+	?	0	0	0	0	0	0

Proof See appendix A.4. ■

Looser conventional and unconventional monetary policy can both stimulate additional lending to households and firms. Intuitively, both policy instruments affect asset-price inflation and thus have real effects on balance sheets in the presence of nominal debt. Looser policy of either kind boosts asset prices and improves the balance sheets of the middle-aged, whose lending to the next cohort of households and firms is proportional to their net worth. This leads to a positive shift in the supply of loans.

The consumption $c_{m,t}$ of the middle-aged relative to GDP is higher, reflecting the increase in net worth. The consumption $c_{y,t}$ of the young is also higher as a consequence of increased lending l_t. There is also an increase in investment given the higher national saving rate s_t. The consumption $c_{o,t}$ of the old declines relative to GDP because investment and the consumption of all other cohorts increase relative to GDP. This is a consequence of the declining real value of the pension wealth held in nominal bonds.

The real effects of monetary policy on GDP Y_t operate through investment in capital. Changes in investment have a persistent effect on the future capital stock and output. There is no effect on impact because of the one-period time-to-build. Owing to the three-period overlapping generations structure of the model, a shock to the net worth of the current cohort of lenders does not affect the net worth of future lenders (relative to GDP). This is why conventional monetary policy has a persistent effect only on GDP, not on other variables relative to GDP.

While conventional and unconventional policies have qualitatively identical real effects on impact, the two types of monetary policy are not equivalent because their subsequent effects differ. A past unconventional policy change pushes lending in the opposite direction in the subsequent time period, which does not happen with conventional policy. Therefore, while expansionary unconventional policy can stimulate lending in the short term, some of this increase in effectively pulled forward from future periods.

The explanation for this finding is that unconventional monetary policy also shifts the demand for loans at each risk-free nominal interest rate, in addition to the effect on the supply of loans that it shares with conventional monetary policy. This means the equilibrium real interest rate is higher than it would otherwise be when the same increase in borrowing is brought about directly through a conventional reduction in the nominal interest rate. Thus, expansionary unconventional policy leaves the balance sheets of those who will be providing loans in the future in a worse state than expansionary conventional policy. This leads to a subsequent reversal in lending.

One way that the negative effects of unconventional monetary policy might be smaller than argued here is if the central bank is able to make profits on its use of unconventional policy instruments. This could occur if the central bank's ability to allocate credit is not significantly lower than that of private financial institutions while not being subject to the financial frictions faced by those institutions. In that case, if the central bank distributes the profits it makes to borrowers then their balance sheets will improve.

4.3 Achieving an efficient allocation of resources

To achieve an efficient allocation of resources in the economy described in section 2, monetary policy needs to offset the credit cycle and stabilize the ratio of lending to GDP l_t and the national saving rate s_t. It is shown in Sheedy (2017a) that conventional monetary policy can do this, assuming it does not violate the interest rate lower bound. The policy works by leaning against the effects of TFP shocks on asset prices, ensuring that there is no amplification of those shocks through balance sheets. By stabilizing house-price inflation, monetary policy can tame the credit cycle.

Since unconventional monetary policy can also be used to control asset-price inflation, it might be thought that it too can be used to achieve an efficient allocation of resources, and thus come to the rescue when conventional policy is constrained by the zero lower bound. However, because of the differences in the real effects of unconventional policy described above, the unconventional policy action taken now to stabilize the economy would destabilize it in the future. A corollary is that there is no unconventional policy stance $\{\upsilon_t\}$ that can implement an efficient allocation of resources.

Result 5 *There is no stance of unconventional monetary policy $\{\upsilon_t\}$ that can stabilize the lending-to-GDP ratio l_t and the national saving rate s_t and thus achieve an efficient allocation of resources,* *unless an efficient allocation is already achieved through conventional policy.*

Proof See appendix A.5. ■

If unconventional monetary policy cannot achieve an efficient allocation of resources, what should be done? Assessing the trade-offs in optimal policy requires an explicit objective function. Consider a weighted sum of household utilities:

$$\mathscr{W}_{t_0} = \sum_{t=t_0-2}^{\infty} \Omega_t \mathbb{E}_{t_0-1} \mathscr{U}_t. \qquad [4.2]$$

From this point onwards, the analysis is performed using a simplified version of the model where there is no capital. This is the limiting case where the parameter α in the production function (2.6) tends to zero. Even in this special case, exactly characterizing the optimal policy rule is complicated owing to the trade-offs between stabilization now and stabilization in the future. This is analogous to difficulties faced by forward guidance where it entails a commitment to set the conventional policy instrument in the future at a different level from what would be warranted by the conditions then prevailing (Campbell, Evans, Fisher and Justiniano, 2012). Here, even without the issue of commitment to future unconventional policy, past unconventional policy actions have a destabilizing effect on current conditions.

Result 6 *Choosing the stance of unconventional policy* $\{\upsilon_t\}$ *to maximize the weighted sum of household utilities* (4.2)*, the resulting policy is not entirely passive* ($\tau_t \neq 0$)*, but does not aim for the same nominal target as the optimal conventional monetary policy unconstrained by the zero lower bound.*

Proof See appendix A.6. ■

It is not possible to say more in general about optimal unconventional policy even in the context of the simple model of this paper. The key message is that when switching from the conventional to the unconventional policy instrument, the optimal target for monetary policy will most likely need to change.

4.4 The distributional effects of unconventional policies

The distributional consequences of monetary policy have attracted much controversy in the years following the financial crisis (for some recent evidence, see Doepke, Schneider and Selezneva, 2015). If a policy instrument has large distributional effects, it becomes more difficult for policy to be set according to a rule aiming at stabilizing the economy. Political pressure for discretionary action seeking to exploit the distributional effects of policy will become harder for policymakers to resist.

Since the conventional instruments of monetary policy are known to have distributional effects, the challenge of insulating central banks from political pressure would not appear to be specific to times when unconventional policy is used. Nonetheless, debates about the distributional effects of monetary policy have risen up the political agenda in the era of unconventional policy after the financial crisis. Is there any sense in which the instruments of unconventional monetary policy have more potent distributional effects, and are thus more likely to result in political controversy?

When the instruments of unconventional monetary policy were introduced into the model in section 3, recall that by construction, all of those unconventional policies have no *direct* distributional consequences (lump-sum taxes or transfers were used to offset any direct distributional effects). With conventional policy, changes in the nominal interest rate do directly affect the nominal transfer made by borrowers to lenders when a loan is repaid. Hence, there should be no presumption that unconventional policy necessarily has stronger distributional effects than conventional policy in the economic model used here.

To address this issue satisfactorily, it is necessary to think about any distributional effects following from monetary policy's effect on the economy's equilibrium. Consider first a benchmark case where there is only a conventional monetary policy instrument and no zero lower bound constraint. It is shown in Sheedy (2017a) that depending on the weights assigned to different cohorts of individuals in (4.2), social welfare-maximizing monetary policy may or may not be swayed by distributional concerns. In some cases, monetary policy is used only to achieve an efficient allocation of resources, while in other cases, even though it is feasible to achieve an efficient allocation, monetary policy sacrifices this to pursue distributional goals.

To analyse the distributional consequences of unconventional monetary policy, suppose that a policymaker can set both the unconventional and conventional policy instruments (ignoring the zero lower bound constraint) to maximize the weighted sum of individuals' utilities in (4.2).

Result 7 *Suppose the stances $\{\mu_t\}$ and $\{\upsilon_t\}$ of conventional and unconventional policy are jointly determined to maximize the welfare function (4.2) for some weights Ω_t on different cohorts of individuals. The conventional policy stance is always set to stabilize asset-price inflation irrespective of the weights Ω_t. This action is what would be needed to achieve an efficient allocation of resources. The stance of unconventional policy is independent of the shocks hitting the economy and depends only on the weights Ω_t assigned to different cohorts.*

PROOF See appendix A.7. ■

The result suggests that it is actually the unconventional policy instrument which has the stronger distributional effects. If both instruments were available because the zero lower bound is not binding, a separation principle applies whereby the conventional policy instrument is used solely to achieve economic efficiency, and the unconventional instrument is used solely to pursue distributional goals. This finding suggests that it will be more difficult to insulate monetary policy rules from political pressure when unconventional policy instruments are used.

4.5 Pitfalls of the natural interest rate

Given that the use of unconventional monetary policy instruments has drawbacks, there has been much discussion of when economic conditions will allow for a 'normalization' of monetary policy, that is, a return to the use of conventional policy instruments. The concept of a 'natural' rate of interest has taken on an important role in these discussions (see, for example, Williams, 2016) because this determines, via the Fisher equation, whether a given target rate of inflation is consistent with a positive nominal interest rate. A natural interest rate that is too low means any positive nominal interest rate would be too high, and monetary policy must rely on unconventional instruments instead.

The natural interest rate is usually defined as the real interest rate consistent with desired saving equal to desired investment (or market clearing more generally). In the models typically used by central banks for monetary policy analysis (see, for example, Woodford, 2003), the natural rate of interest is determined by the economy's fundamentals and is independent of monetary policy. The presence of sticky prices or wages will mean that the actual real interest rate might be different from the natural interest rate in the short run.

If the natural interest rate is to provide a useful benchmark for when to normalize monetary policy, it is necessary that the natural interest rate is indeed independent of monetary policy. While this is typically the case in models with complete financial markets, even in the simple model of incomplete financial markets in this paper, the market-clearing real interest rate does depend on the conduct of monetary policy.

Result 8 *The market-clearing real interest rate ('natural' interest rate) ϱ_t depends on the stances of conventional and unconventional monetary policy. Unexpectedly expansionary conventional policy lowers ϱ_t, while expansionary unconventional policy has an ambiguous effect on ϱ_t.*

Proof See appendix A.8. ■

The mechanism through which the market-clearing real interest rate depends on monetary policy is the effect of changes in nominal asset prices on the balance sheets of lenders, and the effect of unconventional policy on incentives to borrow in addition to the risk-free nominal interest rate. These effects shift the supply of loans and the demand for loans and influence the market-clearing real interest rate. One consequence is that the concept of a natural rate of interest may be a poor guide to policy and should be used with greater caution.

5 Avoiding the need for unconventional monetary policies

The arguments put forward in section 4 suggest that unconventional monetary policy instruments may be a poor substitute for conventional ones. If it is difficult to design and put in place rules with desirable properties specified in terms of unconventional policy instruments, this naturally leads to the question of whether the need for unconventional instruments could be reduced by a change of monetary policy strategy. Specifically, are some monetary targets easier to implement using purely conventional instruments?

Consider two different versions of inflation targeting. First, the standard form of inflation targeting where the target is stated purely in terms of the prices of goods, not assets. Second, an alternative version of inflation targeting that aims to stabilize asset prices (house prices in the model) rather than goods prices.

For the economic model studied in this paper, Sheedy (2017a) shows that an efficient allocation of resources can be achieved if the central bank is able to use conventional monetary policy to stabilize asset-price inflation. This paper has established that the same policy implemented using unconventional policy does not attain economic efficiency, thus it is better to use the conventional instrument. But is it actually feasible to hit the target using conventional policy instruments if the nominal interest rate is subject to a zero lower bound?

Result 9 *A monetary policy that targets any non-negative rate of house-price inflation ($\pi_t = \bar{\pi} \geq 0$) can be implemented without unconventional instruments ($\tau_t = 0$) and without violating the zero lower bound on the nominal interest rate ($i_t \geq 0$), irrespective of the size of the TFP shocks hitting the economy. If the target is instead for goods-price inflation ($\varpi_t = \bar{\varpi}$), the zero lower bound will always bind for some realizations of shocks.*

Proof See appendix A.9. ■

This surprising result states that asset-price inflation targeting can be implemented without the risk of a binding zero lower bound, unlike goods-price inflation targeting. While this very strong result may be specific to the simple model studied here, intuition suggests that any successful attempt to stabilize nominal asset-price inflation cannot require large changes in nominal interest rates in equilibrium. Since asset prices are present discounted values, large swings in nominal interest rates would themselves destabilize asset prices. Consequently, the range of interest rates consistent with stable asset prices is likely to be smaller than the range required to keep goods prices stable.

In the case of goods-price inflation targeting, the range of nominal interest rates that might be required to achieve the target using only conventional instruments is basically determined by the range of the natural interest rate plus the inflation target. It has been noted above that the economic model with heterogeneous agents and incomplete financial markets implies the market-clearing real interest rate is not independent of monetary policy. It is therefore interesting to ask whether the standard version of inflation targeting itself contributes to an economic environment where the market-clearing real interest rate might fall to a point where the zero lower bound becomes binding.

Assume that the level of TFP A_t follows a random walk (allowing for drift). In this special case, the market-clearing real interest rate in a representative-agent version of the economic model would always be constant, so it would be easy to implement goods-price inflation targeting without facing a zero lower bound problem. However, with heterogeneous agents and incomplete financial markets, it turns out that the market-clearing real interest rate is lower on average and more volatile under goods-price inflation targeting than it would be under asset-price inflation targeting. A lower and more volatile market-clearing real interest rate both increase the risk of the interest rate lower bound becoming binding for a given level of the inflation target.

Result 10 *Suppose TFP A_t follows a random walk with drift and let ϱ_t denote the market-clearing real interest rate. Under a policy of goods-price inflation targeting ($\varpi_t = \bar{\varpi}$), the mean $\mathbb{E}[\varrho_t]$ is lower and the variance $\mathrm{Var}[\varrho_t]$ is higher than under a policy of asset-price inflation targeting ($\pi_t = \bar{\pi}$).*

Proof See appendix A.10 ■

In an economy with nominal debt contracts, if monetary policy ignores fluctuations in asset-price inflation and focuses only on goods-price inflation then this will lead to TFP shocks having an amplified effect on borrowers' and lenders' balance sheets. This gives rise to larger shifts in the supply of loans, which entails larger fluctuations in the market-clearing real interest rate. Furthermore, borrowing to buy houses is more risky if asset-price fluctuations are neglected by monetary policy. This leads to a precautionary reduction in the demand for loans, implying a lower market-clearing real interest rate on average.

6 Concluding remarks

This essay has discussed some of the difficulties and drawbacks of devising and following rules for the unconventional instruments of monetary policy. A simple economic model was used to show how various types of unconventional policy instruments are less effective at stabilizing the economy than conventional instruments, and also give rise to larger redistributional effects. To avoid these problems, central banks would need to follow monetary policy strategies that are less likely to require unconventional policy instruments. This essay suggests a reform to inflation targeting to give greater weight to asset prices, which it is argued would reduce the need to use unconventional policy instruments in the future. For monetary policy to go back to being conventional in its instruments, it needs to become unconventional in its targets.

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A Derivations of the results

A.1 Proof of Result 1

Introducing unconventional monetary policy instruments has been shown to change only one of the equilibrium conditions of the basic model from Sheedy (2017a) in section 2. The borrowers' Euler equation (2.4b) is replaced by (3.2), which contains the unconventional policy instrument τ_t. With all other equilibrium conditions unchanged, Steps 1–3 and Step 5 of Sheedy (2017a) continue to hold. Substituting the expressions for the consumption shares $c_{\mathrm{y},t} \equiv C_{\mathrm{y},t}/Y_t$ and $c_{\mathrm{m},t} \equiv C_{\mathrm{m},t}/Y_t$ and the ex-post real return r_t on bonds (defined by $1 + r_t = (P_{t-1}/P_t)/Q_{t-1}$) into (3.2):

$$\left(\frac{\delta\psi}{1+\delta\psi}\frac{\kappa}{\kappa+d_t}\right)^{-1} = \frac{\delta}{1-\tau_t}\mathbb{E}_t\left[\left(\frac{(\kappa+d_t)d_{t+1}}{\lambda(\kappa+d_{t+1})}\right)\left(\frac{1}{1+\delta\psi}\frac{\kappa}{\kappa+d_{t+1}}\right)^{-1}\right],$$

where $d_t \equiv -B_{\mathrm{y},t-1}/(V_t H_t)$ denotes the amount of outstanding debt relative to the value of houses. The terms ψ is shown in Sheedy (2017a) to be a constant that depends on the discount factor δ and $\theta \equiv \Theta'(1)$. The constants λ and κ are defined by $\lambda \equiv 1/(1-\psi)$ and $\kappa \equiv (1+\delta\psi)/(\delta(1-\psi))$. Simplifying the equation and using the definition of β (another constant from Sheedy (2017a) that depends on δ and θ) leads to:

$$\mathbb{E}_t d_{t+1} = \frac{\lambda}{\beta}(1-\tau_t). \qquad \text{[A.1.1]}$$

This result replaces Step 4 in Sheedy (2017a). Substituting $d_{t+1} = \lambda(1+i_t)/(1+\pi_{t+1})$ into the above implies:

$$\frac{1+i_t}{1-\tau_t} = \frac{1}{\beta\mathbb{E}_t[(1+\pi_{t+1})^{-1}]}. \qquad \text{[A.1.2]}$$

and hence:

$$d_t = \frac{\lambda}{\beta}(1-\tau_{t-1})\epsilon_t, \quad \text{where} \;\; \epsilon_t = \frac{(1+\pi_t)^{-1}}{\mathbb{E}_{t-1}[(1+\pi_t)^{-1}]}, \tag{A.1.3}$$

which satisfies $\mathbb{E}_{t-1}\epsilon_t = 1$.

Now consider the derivation of (3.2) and (3.14) in the case of the macroprudential policy instrument. Define a variable τ_t as follows:

$$\tau_t = D_t \mathbb{E}_t \left[\frac{P_t C_{\mathrm{y},t}}{Q_t V_{t+1}}\right]. \tag{A.1.4}$$

Observe that the first equation in (3.13) implies (3.2) holds in terms of the new variable τ_t, though τ_t is an endogenous variable. This replaces borrowers' bond Euler equation in (2.4b), noting that the equilibrium conditions must be augmented by additional equations because (3.2) refers to the new endogenous variable τ_t.

It is clear from the definition (A.1.4) that $\tau_t \geq 0$ if and only if $D_t \geq 0$, and $\tau_t = 0$ if and only if $D_t = 0$. Using $H_t = 1$ and the definition of d_{t+1} it follows that $d_{t+1} = -B_{\mathrm{y},t}/V_{t+1}$. Hence, together with the link between τ_t and D_t in (A.1.4), the macroprudential constraint (3.12) and the second and third optimality conditions in (3.13) are equivalent to:

$$\mathbb{E}_t d_{t+1} \leq \chi_t, \quad \tau_t \geq 0, \quad \text{and} \;\; \tau_t\left(\chi_t - \mathbb{E}_t d_{t+1}\right) = 0. \tag{A.1.5}$$

Therefore, the complete set of equilibrium conditions has (2.4b) replaced by (3.2), and also adds the conditions in (A.1.5). There is an additional endogenous variable τ_t, along with the new exogenous policy variable χ_t.

Note that Step 1 through to Step 3 in Sheedy (2017a) do not use the borrowers' bond Euler equation in (2.4b). Furthermore, since they do not make use of any equilibrium conditions referring to the new variable τ_t, they are unchanged in this version of the model. Using these steps and following exactly the same reasoning as above, (A.1.1) must hold in terms of the endogenous variable τ_t. This equation can be used to substitute for the terms involving $\mathbb{E}_t d_{t+1}$ in (A.1.5) to obtain:

$$\frac{\lambda}{\beta}(1-\tau_t) \leq \chi_t, \quad \tau_t \geq 0, \quad \text{and} \;\; \tau_t\left(\chi_t - \frac{\lambda}{\beta}(1-\tau_t)\right) = 0. \tag{A.1.6}$$

Observe that these conditions include only the new variable τ_t and the exogenous policy variable χ_t. The first condition is equivalent to $\tau_t \geq 1 - (\beta/\lambda)\chi_t$ and the third condition to $\tau_t(\tau_t - (1-(\beta/\lambda)\chi_t)) = 0$. Together with $\tau_t \geq 0$, the unique solution of this system of linear equations and inequalities is:

$$\tau_t = \max\left\{0, 1 - \frac{\beta}{\lambda}\chi_t\right\}, \tag{A.1.7}$$

and since $0 < \chi_t \leq \infty$, the solution satisfies $0 \leq \tau_t < 1$. It follows that the solution for the endogenous variable τ_t can be obtained with reference only to the single exogenous variable χ_t. This confirms the expression for τ_t in (3.14) with $d^* = \lambda/\beta$, and the equation for D_t follows immediately from the definition (A.1.4). This completes the proof.

A.2 Proof of Result 2

Using equations (3.19) and (3.20) for the conventional and unconventional policy instruments:

$$\frac{1+i_t}{1-\tau_t} = \frac{1+\max\{(1+\pi_t)^\zeta \mu_t - 1, 0\}}{1-\left(1-\frac{1}{\upsilon_t(1+\min\{(1+\pi_t)^\zeta\mu_t - 1, 0\})}\right)} = \max\{(1+\pi_t)^\zeta\mu_t, 1\}\upsilon_t \min\{(1+\pi_t)^\zeta\mu_t, 1\},$$

which uses $1+\max\{(1+\pi_t)^\zeta\mu_t - 1, 0\} = \max\{(1+\pi_t)^\zeta\mu_t, 1\}$ and $1+\min\{(1+\pi_t)^\zeta\mu_t - 1, 0\} = \min\{(1+\pi_t)^\zeta\mu_t, 1\}$. It follows that:

$$\frac{1+i_t}{1-\tau_t} = (1+\pi_t)^\zeta\mu_t\upsilon_t,$$

because $\max\{(1+\pi_t)^\zeta \mu_t, 1\} \min\{(1+\pi_t)^\zeta \mu_t, 1\} = (1+\pi_t)^\zeta \mu_t$. Substituting this into (A.1.2):

$$\beta \mu_t \upsilon_t (1+\pi_t)^\zeta \mathbb{E}_t[(1+\pi_{t+1})^{-1}] = 1. \qquad [A.2.1]$$

Since both μ_t and τ_t are exogenous stochastic processes, the equation above is isomorphic to the expectational difference equation considered in Step 9 of Sheedy (2017a) with the exogenous conventional monetary policy stance μ_t replaced by the combined exogenous policy stance $\tilde{\mu}_t \equiv \mu_t \upsilon_t$. When $\zeta > 1$, the unique stable solution for house-price inflation π_t is therefore given by:

$$1+\pi_t = \frac{\beta^{\frac{1}{1-\zeta}}}{\tilde{m}_t}, \quad \text{where } \tilde{m}_t \equiv \tilde{\mu}_t^{\frac{1}{\zeta}} \left(\mathbb{E}_t \tilde{\mu}_{t+1}^{\frac{1}{\zeta}} \left(\mathbb{E}_{t+1} \tilde{\mu}_{t+2}^{\frac{1}{\zeta}} (\cdots)^{\frac{1}{\zeta}} \right)^{\frac{1}{\zeta}} \right)^{\frac{1}{\zeta}} \text{ and } \tilde{\mu}_t \equiv \mu_t \upsilon_t, \qquad [A.2.2]$$

where the variable $\tilde{m}_t$ is determined by the current and expected future stances of conventional and unconventional monetary policy. This completes the proof.

A.3 Proof of Result 3

Consider a state-contingent path of house-price inflation $\{\tilde{\pi}_t\}$, and a state-contingent sequence of conventional monetary policy stances $\{\mu_t\}$. Setting the stance of unconventional monetary policy:

$$\upsilon_t = \frac{1}{\beta \mu_t (1+\tilde{\pi})^\zeta \mathbb{E}_t\left[(1+\tilde{\pi}_{t+1})^{-1}\right]},$$

comparison with (A.2.1) shows that $\pi_t = \tilde{\pi}_t$ is the unique stable equilibrium. Similarly, taking the house-price inflation path $\{\tilde{\pi}_t\}$ implied by a path of goods-price inflation $\{\tilde{\varpi}_t\}$, the unique equilibrium is $\varpi_t = \tilde{\varpi}_t$. This completes the proof.

A.4 Proof of Result 4

Given the solution for house-price inflation in (A.2.2), the implied debt ratio is:

$$d_t = \frac{\lambda}{\beta}(1-\tau_{t-1})\epsilon_t, \quad \text{where } \epsilon_t = \frac{\tilde{m}_t}{\mathbb{E}_{t-1}\tilde{m}_t}.$$

Using equation (4.1) to write $1-\tau_t$ in terms of υ_t:

$$d_t = \frac{\lambda}{\beta}\frac{\epsilon_t}{\upsilon_{t-1}}. \qquad [A.4.1]$$

Step 3 from Sheedy (2017a) remains valid in this version of the model. By substituting the expression for d_t from (A.4.1) into the formula for the new lending ratio l_t:

$$l_t = \frac{\beta\lambda}{\beta\kappa + \lambda\frac{\epsilon_t}{\upsilon_{t-1}}}, \qquad [A.4.2]$$

and the consumption ratios $c_{\mathrm{y},t}$, $c_{\mathrm{m},t}$, and $c_{\mathrm{o},t}$ can be obtained in the same way:

$$c_{\mathrm{y},t} = \frac{\delta\psi}{1+\delta\psi}\left(\frac{\beta\kappa}{\beta\kappa + \lambda\frac{\epsilon_t}{\upsilon_{t-1}}}\right), \quad c_{\mathrm{m},t} = \frac{1}{1+\delta\psi}\left(\frac{\beta\kappa}{\beta\kappa + \lambda\frac{\epsilon_t}{\upsilon_{t-1}}}\right), \quad \text{and } c_{\mathrm{o},t} = \frac{\lambda\frac{\epsilon_t}{\upsilon_{t-1}}}{\beta\kappa + \lambda\frac{\epsilon_t}{\upsilon_{t-1}}}. \qquad [A.4.3]$$

From (A.2.2), ϵ_t depends positively on current and expected future value of μ_t and υ_t. The result in (A.4.2) implies that an unexpected lowering of μ_t reduces ϵ_t and thus increases l_t, without having any effect on future $\epsilon_{t+\ell}$ and hence $l_{t+\ell}$. An unexpected lowering of υ_t also reduces ϵ_t with changing any future $\epsilon_{t+\ell}$. However, from (A.4.2), υ_t also has a direct positive effect on l_{t+1}. The surprise reduction in υ_t thus raises l_t but lowers l_{t+1}. Using (A.4.3), the effects on $c_{\mathrm{y},t}$ and $c_{\mathrm{m},t}$ go in the same direction as those on l_t, and the effects on $c_{\mathrm{o},t}$ go in the opposite direction.

The results can be extended to an economy with capital by following the same method as in Result 9 of Sheedy (2017a). The effects of surprise changes in μ_t and υ_t on the saving rate s_t are qualitatively the same as for l_t. Any effect on s_t changes the investment/GDP ratio and has a persistent effect on the subsequent capital stock and GDP. This completes the proof.

A.5 Proof of Result 5

Step 3 of Sheedy (2017a) yields an expression for l_t which remains valid here:

$$l_t = \frac{\lambda}{\kappa + d_t}. \tag{A.5.1}$$

Suppose that a combination of conventional monetary policy $\{\mu_t\}$ and unconventional monetary policy $\{\tau_t\}$ were to imply a constant ratio of lending to GDP $l_t = l$. According to (A.5.1), this is possible only if $d_t = d$, where $d = (\lambda/l) - \kappa$ is a constant. Clearly, this can happen only if $\mathbb{E}_t d_{t+1} = d$. Since (A.4.1) implies $\mathbb{E}_t d_{t+1} = (\lambda/\beta)/\upsilon_t$ because $\mathbb{E}_t \epsilon_{t+1} = 1$, a necessary condition is that unconventional policy is such that $\upsilon_t = \upsilon$, where $\upsilon = \lambda/\beta d$. With $d_t = d$ and $\upsilon_t = \upsilon$, equation (A.4.1) requires $d = (\lambda/\beta)(\epsilon_t/\upsilon)$, and given that $\upsilon = \lambda/\beta d$, this is possible only if $\epsilon_t = 1$.

Since $\upsilon_t = \upsilon$ at all times and in all states, $\tilde{\mu}_t = \mu_t \upsilon_t$ is proportional to μ_t. It then follows from equation (A.2.2) that $\tilde{m}_t = \upsilon m_t$, where m_t is the equivalent of $\tilde{m}_t$ in (A.2.2) when $\tilde{\mu}_t$ is replaced by μ_t. Given the expression for ϵ_t in (A.1.3), the conventional and unconventional policy combination $\{\mu_t, \upsilon_t\}$ achieves $\epsilon_t = 1$ if and only if this would be attained in the absence of unconventional policy ($\upsilon_t = 1$). It follows that there is no unconventional monetary policy which can achieve a constant ratio $l_t = l$ unless conventional monetary policy would already achieve this objective in the absence of any unconventional policy. An exactly equivalent argument applies to the national saving rate s_t.

The criteria for Pareto efficiency in Step 7 of Sheedy (2017a) remain valid here, so $c_{\mathrm{m},t}/c_{\mathrm{y},t-1} = c_{\mathrm{o},t}/c_{\mathrm{m},t-1}$ is necessary for an allocation to be efficient. The results in Step 3 are unaffected by the introduction of the unconventional policy instrument, hence the efficiency condition is equivalent to:

$$\frac{\frac{1}{1+\delta\psi}\left(\frac{\kappa}{\kappa+d_t}\right)}{\frac{\delta\psi}{1+\delta\psi}\left(\frac{\kappa}{\kappa+d_{t-1}}\right)} = \frac{\frac{d_t}{\kappa+d_t}}{\frac{1}{1+\delta\psi}\left(\frac{\kappa}{\kappa+d_{t-1}}\right)}.$$

By cancelling common terms, this requires $d_t = d$, where $d = \kappa/(\delta\psi(1+\delta\psi))$. Following the same steps as above shows that a conventional and unconventional monetary policy combination $\{\mu_t, \upsilon_t\}$ results in an equilibrium with this property only if the same conventional policy stance $\{\mu_t\}$ would already achieve $d_t = d$ in the absence of any unconventional policy ($\upsilon_t = 1$). There is thus no efficient allocation that can be attained through the use of unconventional monetary policy. This completes the proof.

A.6 Proof of Result 6

The welfare function $\mathscr{W}_{t_0}$ is defined in equation (4.2). Since $H_{\mathrm{m},t} = 1$ in equilibrium, the utility function $\mathscr{U}_t$ from (2.1) implies:

$$\mathscr{U}_t = \log C_{\mathrm{y},t} + \delta \mathbb{E}_t \log C_{\mathrm{m},t+1} + \delta^2 \mathbb{E}_t \log C_{\mathrm{o},t+2} + \text{t.i.p.},$$

where t.i.p. denotes terms independent of monetary policy (both conventional and unconventional). Since aggregate output is exogenous in the special case $\alpha = 0$, it follows that $\log C_{\tau,t} = \log c_{\tau,t} + \text{t.i.p.}$ for all $\tau \in \{\mathrm{y}, \mathrm{m}, \mathrm{o}\}$ where $c_{\tau,t} = C_{\tau,t}/Y_t$ is a ratio of consumption to output:

$$\mathscr{U}_t = \log c_{\mathrm{y},t} + \delta \mathbb{E}_t \log c_{\mathrm{m},t+1} + \delta^2 \mathbb{E}_t \log c_{\mathrm{o},t+2} + \text{t.i.p.}. \tag{A.6.1}$$

Substituting this into the welfare function (4.2):

$$\mathscr{W}_{t_0} = \sum_{t=t_0-2}^{\infty} \Omega_t \mathbb{E}_{t_0-1}\left[\log c_{\mathrm{y},t} + \delta \mathbb{E}_t \log c_{\mathrm{m},t+1} + \delta^2 \mathbb{E}_t \log c_{\mathrm{o},t+2}\right] + \text{t.i.p.}$$

$$= \sum_{t=t_0}^{\infty} \mathbb{E}_{t_0-1}\left[\Omega_t \log c_{\mathrm{y},t} + \delta \Omega_{t-1} \log c_{\mathrm{m},t} + \delta^2 \Omega_{t-2} \log c_{\mathrm{o},t}\right] + \text{t.i.p.}. \tag{A.6.2}$$

The results in Step 3 of Sheedy (2017a) remain valid here, so the equilibrium consumption ratios are:

$$c_{\mathrm{y},t} = \frac{\delta\psi}{1+\delta\psi}\left(\frac{\kappa}{\kappa+d_t}\right), \quad c_{\mathrm{m},t} = \frac{1}{1+\delta\psi}\left(\frac{\kappa}{\kappa+d_t}\right), \quad \text{and} \quad c_{\mathrm{o},t} = \left(\frac{d_t}{\kappa+d_t}\right). \tag{A.6.3}$$

Substituting these expressions into (A.6.2) leads to:

$$\mathscr{W}_{t_0} = \sum_{t=t_0}^{\infty} \Psi_t \mathbb{E}_{t_0-1}\left[(1-\omega_t)\log\frac{\kappa}{\kappa+d_t} + \omega_t \log\frac{d_t}{\kappa+d_t}\right] + \text{t.i.p.},$$

with the variables Ψ_t and ω_t being defined in terms of the welfare weights Ω_t assigned to each generation:

$$\omega_t = \frac{\delta^2\Omega_{t-2}}{\Omega_t + \delta\Omega_{t-1} + \delta^2\Omega_{t-2}}, \quad \text{and} \quad \Psi_t = \Omega_t + \delta\Omega_{t-1} + \delta^2\Omega_{t-2}. \qquad \text{[A.6.4]}$$

Since Ω_t must be measurable with respect to date-(t_0-1) information, this property is inherited by ω_t and Ψ_t. Adding and subtracting an arbitrary constant leads to the following expression for the welfare function:

$$\mathscr{W}_{t_0} = \sum_{t=t_0}^{\infty} \Psi_t \mathbb{E}_{t_0-1}\mathcal{W}_t + \text{t.i.p.}, \quad \text{where } \mathcal{W}_t = (1-\omega_t)\mathbb{E}_{t-1}\log\frac{\kappa+\lambda/\beta}{\kappa+d_t} + \omega_t\mathbb{E}_{t-1}\log\frac{(1+\beta\kappa/\lambda)d_t}{\kappa+d_t}. \qquad \text{[A.6.5]}$$

The period-t welfare function is seen to depend only on the variable d_t and the relative welfare weight ω_t.

Taking as given an arbitrary stance of conventional monetary policy $\{\mu_t\}$, the problem of choosing unconventional policy $\{\upsilon_t\}$ to maximize the welfare function $\mathscr{W}_{t_0}$ does not have an analytic solution. To provide an illustrative example, first consider a sequence of welfare weights $\{\Omega_t\}$ proportional to γ^t for some constant $0 < \gamma < 1$. Using (A.6.4), it follows that:

$$\omega_t = \omega = \frac{\delta^2}{\gamma^2 + \gamma\delta + \delta^2}, \quad \text{and} \quad \Psi_t = \left(1 + \frac{\delta}{\gamma} + \frac{\delta^2}{\gamma^2}\right)\gamma^t.$$

This implies the period-t welfare function is a time-invariant function of d_t, and that welfare at different dates is discounted by a term proportional to γ^t. Taking a second-order approximation of welfare around the non-stochastic steady state, the negative of the welfare function is approximately proportional to the following loss function:

$$\mathscr{L}_{t_0} = \frac{1}{2}\sum_{t=t_0}^{\infty} \gamma^{t-t_0}\mathbb{E}_{t_0-1}\mathsf{d}_t^2, \qquad \text{[A.6.6]}$$

where d_t denotes the log deviation of d_t from its non-stochastic steady state. In what follows, it is assumed that the first-order conditions are sufficient to characterize a global maximum of the welfare function (the requirements for this are discussed in Sheedy, 2017a), and thus that the optimal unconventional policy can be approximated by minimizing the loss function (A.6.6) subject to first-order approximations of the equilibrium conditions connecting monetary policy to the debt ratio d_t.

Using (4.1), (A.1.3), and (A.2.2) the first-order approximation of the debt ratio is given by:

$$\mathsf{d}_t = \epsilon_t - \upsilon_{t-1}, \quad \text{where } \epsilon_t = \tilde{\mathsf{m}}_t - \mathbb{E}_{t-1}\tilde{\mathsf{m}}_{t-1}, \quad \text{and hence } \mathsf{d}_t = \tilde{\mathsf{m}}_t - \mathbb{E}_{t-1}\tilde{\mathsf{m}}_t - \upsilon_{t-1}, \qquad \text{[A.6.7]}$$

where ϵ_t, υ_t, and $\tilde{\mathsf{m}}_t$ denote the log deviations of ϵ_t, υ_t, and $\tilde{m}_t$ from their non-stochastic steady-state values. The equation for $\tilde{m}_t$ in (A.2.2) can be written recursively as $\mathbb{E}_t\tilde{m}_{t+1} = \tilde{m}_t^{\zeta}/(\mu_t\upsilon_t)$, which can be log linearized as follows:

$$\tilde{\mathsf{m}}_t = \zeta^{-1}\mathbb{E}_t\tilde{\mathsf{m}}_{t+1} + \zeta^{-1}(\upsilon_t + \mu_t), \qquad \text{[A.6.8]}$$

with μ_t denoting the log deviation of μ_t from its steady-state value. The conventional monetary policy stance μ_t is taken as given here, with the unconventional policy stance υ_t being the choice variable. The loss function (A.6.6) can be minimized subject to the constraints in (A.6.7) and (A.6.8) by setting up the Lagrangian:

$$\mathcal{L}_{t_0} = \sum_{t=t_0}^{\infty} \gamma^{t-t_0}\mathbb{E}_{t_0-1}\left[\frac{\mathsf{d}_t^2}{2} - \xi_t\left(\mathsf{d}_t + \upsilon_{t-1} - \tilde{\mathsf{m}}_t + \mathbb{E}_{t-1}\tilde{\mathsf{m}}_t\right) - \varphi_t\left(\tilde{\mathsf{m}}_t - \zeta^{-1}\upsilon_t - \zeta^{-1}\mu_t - \zeta^{-1}\mathbb{E}_t\tilde{\mathsf{m}}_{t+1}\right)\right],$$

where ξ_t and φ_t are the Lagrangian multipliers attached to the two constraints (scaled by the discount factor γ^{t-t_0} without loss of generality). The first-order conditions of the constrained minimization problem are:

$$\mathsf{d}_t = \xi_t, \quad \varphi_t = \gamma^{-1}\zeta^{-1}\varphi_{t-1} + \xi_t - \mathbb{E}_{t-1}\xi_t, \quad \text{and} \quad \zeta^{-1}\varphi_t = \gamma\mathbb{E}_t\xi_{t+1}. \qquad \text{[A.6.9]}$$

The final equation implies $\gamma^{-1}\zeta^{-1}\varphi_{t-1} = \mathbb{E}_{t-1}\xi_t$, and substituting this into the second equation yields $\varphi_t = \xi_t$. Using the first equation to write $\mathsf{d}_{t+1} = \xi_{t+1}$, the third equation implies $\zeta^{-1}\varphi_t = \gamma\mathbb{E}_t\mathsf{d}_{t+1}$. Since the equilibrium condition for d_{t+1} in (A.6.7) implies $\mathbb{E}_t\mathsf{d}_{t+1} = -\upsilon_t$, it follows that $\varphi_t = \xi_t = \gamma\zeta\upsilon_t$. Finally, with $\mathsf{d}_t = \xi_t$, the optimal choice of υ_t must satisfy:

$$\mathsf{d}_t = -\gamma\zeta\upsilon_t, \quad \text{and} \quad \mathbb{E}_t\upsilon_{t+1} = \gamma^{-1}\zeta^{-1}\upsilon_t. \tag{A.6.10}$$

Iterating forwards the equation (A.6.8) for $\tilde{\mathsf{m}}_t$:

$$\tilde{\mathsf{m}}_t = \mathsf{m}_t + \sum_{\ell=0}^{\infty}\zeta^{-(\ell+1)}\mathbb{E}_t\upsilon_{t+\ell}, \quad \text{with} \;\; \mathsf{m}_t = \sum_{\ell=0}^{\infty}\zeta^{-(\ell+1)}\mathbb{E}_t\mu_{t+\ell}, \tag{A.6.11}$$

where m_t depends only on the exogenous stance $\{\mu_t\}$ of conventional monetary policy. The second equation in (A.6.10) implies $\mathbb{E}_t\upsilon_{t+\ell} = (\gamma\zeta)^{-\ell}\upsilon_t$ and hence:

$$\tilde{\mathsf{m}}_t = \mathsf{m}_t + \frac{\zeta^{-1}}{1-(\gamma\zeta^2)^{-1}}\upsilon_t = \mathsf{m}_t + \frac{\gamma\zeta}{\gamma\zeta^2-1}\upsilon_t,$$

assuming that $\gamma\zeta^2 > 1$. The unexpected components of the left- and right-hand sides must be equal:

$$\tilde{\mathsf{m}}_t - \mathbb{E}_{t-1}\tilde{\mathsf{m}}_t = (\mathsf{m}_t - \mathbb{E}_{t-1}\mathsf{m}_t) + \frac{\gamma\zeta}{\gamma\zeta^2-1}(\upsilon_t - \mathbb{E}_{t-1}\upsilon_t). \tag{A.6.12}$$

Note that (A.6.7) implies $\mathsf{d}_t - \mathbb{E}_{t-1}\mathsf{d}_t = \tilde{\mathsf{m}}_t - \mathbb{E}_{t-1}\tilde{\mathsf{m}}_t$, and the first equation in (A.6.10) implies $\mathsf{d}_t - \mathbb{E}_{t-1}\mathsf{d}_t = -\gamma\zeta(\upsilon_t - \mathbb{E}_{t-1}\upsilon_t)$. Together, it follows that:

$$-\gamma\zeta(\upsilon_t - \mathbb{E}_{t-1}\upsilon_t) = (\mathsf{m}_t - \mathbb{E}_{t-1}\mathsf{m}_t) + \frac{\gamma\zeta}{\gamma\zeta^2-1}(\upsilon_t - \mathbb{E}_{t-1}\upsilon_t),$$

and therefore:

$$\upsilon_t - \mathbb{E}_{t-1}\upsilon_t = -\gamma^{-1}\zeta^{-1}(1-(\gamma\zeta^2)^{-1})(\mathsf{m}_t - \mathbb{E}_{t-1}\mathsf{m}_t). \tag{A.6.13}$$

In combination with the second equation from (A.6.10), the solution for υ_t is:

$$\upsilon_t = \gamma^{-1}\zeta^{-1}\upsilon_{t-1} - \gamma^{-1}\zeta^{-1}(1-(\gamma\zeta^2)^{-1})(\mathsf{m}_t - \mathbb{E}_{t-1}\mathsf{m}_t). \tag{A.6.14}$$

Since $\upsilon_t \neq 0$, the optimal choice of the unconventional policy instrument is not entirely passive. Note that by combining (A.6.12) and (A.6.13):

$$\tilde{\mathsf{m}}_t - \mathbb{E}_{t-1}\tilde{\mathsf{m}}_t = (1-(\gamma\zeta^2)^{-1})(\mathsf{m}_t - \mathbb{E}_{t-1}\mathsf{m}_t),$$

and so $\tilde{\mathsf{m}}_t \neq \mathbb{E}_{t-1}\tilde{\mathsf{m}}_t$ whenever $\mathsf{m}_t \neq \mathbb{E}_{t-1}\mathsf{m}_t$. It follows that the optimal use of the unconventional policy instrument will not stabilize the same nominal target as when the conventional policy instrument is set optimally in the absence of a binding zero lower bound constraint. This completes the proof.

A.7 Proof of Result 7

Using equations (4.1), (A.1.3), and (A.2.2), the value of the debt ratio d_t in equilibrium is:

$$d_t = \frac{\lambda}{\beta}\frac{\epsilon_t}{\upsilon_{t-1}}, \quad \text{where} \;\; \epsilon_t = \frac{\tilde{m}_t}{\mathbb{E}_{t-1}\tilde{m}_t}, \tag{A.7.1}$$

where $\mathbb{E}_{t-1}\epsilon_t = 1$. The variable $\tilde{m}_t$ depends on the policy stances μ_t and υ_t as shown in equation (A.2.2). The period-t welfare function $\mathcal{W}_t$ from (A.6.5) is:

$$\mathcal{W}_t = \mathbb{E}_{t-1}\left[(1-\omega_t)\log\frac{\kappa+\frac{\lambda}{\beta}}{\kappa+\frac{\lambda}{\beta}\frac{\epsilon_t}{\upsilon_{t-1}}} + \omega_t\log\frac{\left(\kappa+\frac{\lambda}{\beta}\right)\frac{\epsilon_t}{\upsilon_{t-1}}}{\kappa+\frac{\lambda}{\beta}\frac{\epsilon_t}{\upsilon_{t-1}}}\right],$$

and by using $\beta\kappa/\lambda = \delta\psi(1+\delta\psi)$ this can be written as:

$$\mathcal{W}_t = \mathbb{E}_{t-1}\left[(1-\omega_t)\log\frac{1+\delta\psi+\delta^2\psi^2}{(\delta\psi+\delta^2\psi^2)+\frac{\epsilon_t}{\upsilon_{t-1}}} + \omega_t\log\frac{(1+\delta\psi+\delta^2\psi^2)\frac{\epsilon_t}{\upsilon_{t-1}}}{(\delta\psi+\delta^2\psi^2)+\frac{\epsilon_t}{\upsilon_{t-1}}}\right].$$

The period-t welfare function can be simplified as follows:

$$\mathcal{W}_t = \mathbb{E}_{t-1}\left[\omega_t \log \epsilon_t - \log((\delta\psi + \delta^2\psi^2)\upsilon_{t-1}) + (1-\omega_t)\log \upsilon_{t-1} + \log(1+\delta\psi+\delta^2\psi^2)\right]. \qquad \text{[A.7.2]}$$

Given this equation and (A.6.5), and the determination of the variable $\tilde{m}_t$ in (A.2.2), maximizing the welfare function $\mathscr{W}_{t_0}$ is equivalent to maximizing $\mathcal{W}_t$ in (A.7.2) for all $t \geq t_0$, subject to $\mathbb{E}_{t-1}\epsilon_t = 1$. Conditional on a particular relative welfare weight ω_t, this maximization problem has the following general form:

$$\max_{\epsilon,\upsilon} \mathbb{E}\mathcal{S}(\epsilon,\upsilon), \quad \text{subject to } \mathbb{E}\epsilon = 1, \quad \text{with } \mathcal{S}(\epsilon,\upsilon) = \omega\log\epsilon - \log((\delta\psi+\delta^2\psi^2)\upsilon + \epsilon) + (1-\omega)\log\upsilon,$$

where the formula for $\mathcal{S}(\epsilon,\upsilon)$ is obtained from (A.7.2) after dropping the final constant. Note that in this maximization problem, ϵ is a random variable (with mean one), whereas υ is not stochastic.

Using the proof of Result 4 in Sheedy (2017a), the optimal choice of ϵ is non-stochastic ($\epsilon = 1$) if:

$$\frac{\omega}{1-\omega}(\delta\psi + \delta^2\psi^2)\upsilon \geq 1. \qquad \text{[A.7.3]}$$

Conditional on $\epsilon = 1$, the partial derivative of $\mathcal{S}(\epsilon,\upsilon)$ with respect to υ is:

$$\frac{\partial\mathcal{S}(\epsilon,\upsilon)}{\partial\upsilon} = \frac{1-\omega}{\upsilon} - \frac{\delta\psi+\delta^2\psi^2}{(\delta\psi+\delta^2\psi^2)+1}.$$

This is positive for small υ and equals zero at a unique value of υ, which satisfies:

$$\frac{\omega}{1-\omega}(\delta\psi + \delta^2\psi^2)\upsilon = 1,$$

confirming that (A.7.3) holds. This means the optimal choice of ϵ_t is always equal to one, while the optimal υ_{t-1} can be obtained by rearranging the equation above:

$$\upsilon_{t-1} = \frac{(1-\omega_t)}{(\delta\psi+\delta^2\psi^2)\omega_t},$$

which depends only on parameters and the welfare weights $\{\Omega_t\}$ through ω_t. The choice of $\epsilon_t = 1$ can be implemented by setting $\mu_t = \tilde{\mu}/\upsilon_t$ for any constant $\tilde{\mu}$. With $\epsilon_t = 1$, house-price inflation is stabilized through conventional monetary policy, while unconventional monetary policy is used to pursue the distributional goals implicit in the welfare weights $\{\Omega_t\}$. This completes the proof.

A.8 Proof of Result 8

Using Step 3 of Sheedy (2017a), the equilibrium ex-post real return on bonds r_{t+1} between t and $t+1$ is:

$$1 + r_{t+1} = \frac{(1+g_{t+1})(\kappa + d_t)d_{t+1}}{\lambda(\kappa + d_{t+1})}, \qquad \text{[A.8.1]}$$

where g_t is the growth rate of real GDP. The equilibrium debt ratio d_t is:

$$d_t = \frac{\lambda}{\beta}(1-\tau_{t-1})\epsilon_t, \quad \text{where } \epsilon_t = \frac{\tilde{m}_t}{\mathbb{E}_{t-1}\tilde{m}_t}, \qquad \text{[A.8.2]}$$

and the variable $\tilde{m}_t$ is defined by:

$$\tilde{m}_t = \left(\frac{\mu_t}{1-\tau_t}\right)^{\frac{1}{\zeta}}\left(\mathbb{E}_t\left[\left(\frac{\mu_{t+1}}{1-\tau_{t+1}}\right)^{\frac{1}{\zeta}}\left(\mathbb{E}_{t+1}\left[\left(\frac{\mu_{t+2}}{1-\tau_{t+2}}\right)^{\frac{1}{\zeta}}(\cdots)^{\frac{1}{\zeta}}\right]\right)^{\frac{1}{\zeta}}\right]\right)^{\frac{1}{\zeta}}. \qquad \text{[A.8.3]}$$

The natural real interest rate is the market-clearing real interest rate ϱ_t, which is the ex-ante real return on bonds:

$$1 + \varrho_t = \mathbb{E}_t[1 + r_{t+1}]. \qquad \text{[A.8.4]}$$

Substituting (A.8.2) into (A.8.1) and that into (A.8.4):

$$1 + \varrho_t = \frac{\kappa + (\lambda/\beta)(1-\tau_{t-1})\epsilon_t}{\lambda}\mathbb{E}_t\left[\frac{(1+g_{t+1})(\lambda/\beta)(1-\tau_t)\epsilon_{t+1}}{\kappa + (\lambda/\beta)(1-\tau_t)\epsilon_{t+1}}\right]$$
$$= \frac{\beta\kappa + \lambda(1-\tau_{t-1})\epsilon_t}{\beta\lambda}\mathbb{E}_t\left[\frac{\lambda(1+g_{t+1})\epsilon_{t+1}}{\frac{\beta\kappa}{1-\tau_t} + \lambda\epsilon_{t+1}}\right]. \quad \text{[A.8.5]}$$

Unexpectedly expansionary conventional monetary policy (lower μ_t) implies $\tilde{m}_t$ and ϵ_t are lower according to (A.8.2) and (A.8.3), but there is no effect on future $\epsilon_{t+\ell}$. Holding constant unconventional monetary policy τ_t, equation (A.8.5) implies that the natural real interest rate is reduced.

An unexpected loosening of unconventional monetary policy (lower τ_t) also implies $\tilde{m}_t$ and ϵ_t are lower according to (A.8.2) and (A.8.3), with no effect on future $\epsilon_{t+\ell}$. Using equation (A.8.5), while lower ϵ_t reduces ϱ_t, the term inside the conditional expectation is decreasing in τ_t. The relative sizes of these effects depend on ζ and the realizations of τ_{t-1} and ϵ_{t+1}, so the overall effect on ϱ_t is ambiguous. This completes the proof.

A.9 Proof of Result 9

Using Step 3 of Sheedy (2017a), the ratio of house-price π_{t+1} to goods-price inflation ϖ_{t+1} between t and $t+1$ is:

$$\frac{1+\pi_t}{1+\varpi_t} = \frac{(1+g_{t+1})(\kappa + d_t)}{(\kappa + d_{t+1})}. \quad \text{[A.9.1]}$$

The real return on bonds is given by the Fisher equation in terms of the nominal interest rate i_t and the goods-price inflation rate ϖ_{t+1}:

$$1 + r_{t+1} = \frac{1+i_t}{1+\varpi_{t+1}}. \quad \text{[A.9.2]}$$

Using the expression for the ex-post real return r_{t+1} from (A.8.1) and combining this with (A.9.1) and (A.9.2):

$$\frac{1+i_t}{1+\pi_{t+1}} = \frac{d_{t+1}}{\lambda}. \quad \text{[A.9.3]}$$

If unconventional monetary policy instruments are not used, the equilibrium debt ratio d_{t+1} is given by:

$$d_t = \frac{\lambda}{\beta}\epsilon_t, \quad \text{where} \quad \epsilon_t = \frac{(1+\pi_t)^{-1}}{\mathbb{E}_{t-1}[(1+\pi_t)^{-1}]}, \quad \text{[A.9.4]}$$

where $\mathbb{E}_{t-1}\epsilon_t = 1$.

Suppose that monetary policy is able to achieve a target for house-price inflation $\pi_t = \bar{\pi}$ for all t for some $\bar{\pi} > 0$. It follows from (A.9.4) that $\epsilon_t = 1$, and hence $d_t = \lambda/\beta$. Substituting $\pi_t = \bar{\pi}$ and $d_t = \lambda/\beta$ into (A.9.3) implies:

$$1 + i_t = \frac{\lambda/\beta}{\lambda}(1+\bar{\pi}) = \frac{1+\bar{\pi}}{\beta},$$

which is strictly greater than 1 because $\bar{\pi} > 0$ and $0 < \beta < 1$. This demonstrates that any positive target for house-price inflation can be implemented without violating the zero lower bound ($i_t \geq 0$).

Now consider a target (positive or negative) for goods-price inflation: $\varpi_t = \bar{\varpi}$. If this target is achieved, the Fisher equation (A.9.2) implies that:

$$1 + r_{t+1} = \mathbb{E}_t[1 + r_{t+1}]. \quad \text{[A.9.5]}$$

Using this equation and the formula for r_{t+1} from (A.8.1), it follows that:

$$\frac{(1+g_{t+1})d_{t+1}}{(\kappa + d_{t+1})} = Z_t, \quad \text{[A.9.6]}$$

where Z_t is measurable with respect to date-t information. By rearranging this equation and taking expectations conditional on date-t information:

$$\frac{1+\mathbb{E}_t g_{t+1}}{Z_t} = 1 + \kappa \mathbb{E}_t d_{t+1}^{-1} > 1 + \frac{\kappa}{\mathbb{E}_t d_{t+1}} = 1 + \frac{\kappa}{\lambda/\beta} = \frac{\lambda+\beta\kappa}{\lambda},$$

which applies Jensen's inequality to the convex function d_{t+1}^{-1}. The inequality implies:

$$Z_t < \frac{\lambda(1+\mathbb{E}_t g_{t+1})}{\lambda+\beta\kappa}. \qquad \text{[A.9.7]}$$

Substituting (A.9.6) into (A.8.1), and then substituting that and $\varpi_t = \bar{\varpi}$ into (A.9.2):

$$1 + i_t = \frac{(1+\bar{\varpi})(\kappa + d_t)}{\lambda} Z_t. \qquad \text{[A.9.8]}$$

Using the result in (A.9.7), it follows that:

$$1 + i_t < \frac{(1+\bar{\varpi})(1+\mathbb{E}_t g_{t+1})(\kappa+d_t)}{\lambda+\beta\kappa} = \left(\frac{(1+\bar{\varpi})(1+\mathbb{E}_t g_{t+1})}{\beta}\right)\left(\frac{\kappa+d_t}{\kappa+\mathbb{E}_{t-1}d_t}\right).$$

The first term in brackets after the final equality on the right-hand side is the required gross nominal interest to achieve $\varpi_t = \varpi$ in a representative-agent model. The choice of a positive value of $\bar{\varpi}$ does not guarantee that this is not less than one. The second term in brackets has positive probability realizations less than one, so it follows that there are always realizations of shocks such that the required nominal interest rate would be negative if targeting goods-price inflation. This completes the proof.

A.10 Proof of Result 10

The natural interest rate is obtained by substituting (A.8.1) into (A.8.4):

$$1 + \varrho_t = \frac{\kappa + d_t}{\lambda} \mathbb{E}_t \left[\frac{(1+g_{t+1})d_{t+1}}{(\kappa + d_{t+1})}\right]. \qquad \text{[A.10.1]}$$

If TFP A_t follows a random walk with drift then the growth rate $1 + g_t = A_t/A_{t-1}$ is an i.i.d. stochastic process.

Consider first the policy of targeting house-price inflation, $\pi_t = \bar{\pi}$, which implies $d_t = \lambda/\beta$ for all t. Substituting this into (A.10.1) implies:

$$1 + \varrho_t = \left(\frac{\lambda/\beta}{\lambda}\right)\left(\frac{\kappa + (\lambda/\beta)}{\kappa + (\lambda/\beta)}\right) \mathbb{E}_t[1 + g_{t+1}] = \frac{1+\bar{g}}{\beta},$$

where $\mathbb{E}_t g_{t+1} = \mathbb{E} g_t = \bar{g}$ follows from the i.i.d. property of g_t. This immediately implies $\mathbb{E}[\varrho_t] = (1+\bar{g})/\beta - 1$ and $\text{Var}[\varrho_t] = 0$.

Now consider the policy of targeting goods-price inflation, $\varpi_t = \bar{\varpi}$. The Fisher equation (A.9.2) implies the ex-post real return on bonds is predictable, and thus (A.9.5) holds. Comparison with (A.8.1) confirms that (A.9.6) holds, where Z_t is measurable with respect to date-t information. Rearranging the equation for Z_t in (A.9.6) to solve for d_{t+1}:

$$d_{t+1} = \kappa \left(\frac{1+g_{t+1}}{Z_t} - 1\right)^{-1},$$

and by taking expectations of both sides conditional on date-t information:

$$\frac{\lambda}{\beta} = \kappa \mathbb{E}_t \left[\left(\frac{1+g_{t+1}}{Z_t} - 1\right)^{-1}\right].$$

The left-hand side of this equation follows from (A.9.4). Since g_{t+1} is independent of variables known at date t, and as Z_t is known at date t, the right-hand side of the equation above is a time-invariant function of Z_t. As the left-hand side is a constant, it follows that $Z_t = Z$ for some $Z > 0$.

By substituting (A.9.6) into (A.10.1) and using $Z_t = Z$:

$$1 + \varrho_t = \frac{(\kappa + d_t)Z}{\lambda}. \qquad \text{[A.10.2]}$$

Since $d_t = (\lambda/\beta)\epsilon_t$ according to (A.9.4), and $\text{Var}[\epsilon_t] > 0$ unless π_t is predictable, it follows that $\text{Var}[\varrho_t] > 0$.

With $Z_t = Z$ and $\mathbb{E}_t g_{t+1} = \bar{g}$, the inequality in (A.9.7) implies:

$$Z < \frac{\lambda(1+\bar{g})}{\lambda + \beta\kappa},$$

and together with (A.10.2) it follows that:

$$1 + \varrho_t < \frac{(1+\bar{g})(\kappa + d_t)}{\lambda + \beta\kappa} = \frac{(1+\bar{g})(\kappa + d_t)}{\beta(\kappa + (\lambda/\beta))}.$$

Since (A.9.4) implies $\mathbb{E}d_t = \lambda/\beta$, it must be the case that:

$$\mathbb{E}[\varrho_t] < \frac{(1+\bar{g})(\kappa + \mathbb{E}d_t)}{\beta(\kappa + (\lambda/\beta))} - 1 = \frac{1+\bar{g}}{\beta} - 1.$$

This demonstrates that $\mathbb{E}[\varrho_t]$ is smaller when goods-price inflation is the target, rather than house-price inflation. It has already been shown that ϱ_t is more volatile when goods-price inflation is the target, completing the proof.