The Ins and Outs of Selling Houses: Understanding Housing-Market Volatility*

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Abstract

This paper documents the role of inflows (new listings) and outflows (sales) in explaining the volatility and co-movement of housing-market variables. An 'ins versus outs' decomposition shows that both flows are quantitatively important for housing-market volatility. The correlations between sales, prices, new listings, and time-to-sell are stable over time, while the signs of their correlations with houses for sale are found to be time varying. A calibrated search-and-matching model with endogenous inflows and outflows and shocks to housing demand matches many of the stable correlations and predicts that the correlations with houses for sale depend on the source and persistence of shocks.

KEYWORDS: housing-market cyclicality; inflows and outflows; search frictions; match quality.

JEL CLASSIFICATIONS: E32; E22; R21; R31.

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1 Introduction

The importance of search frictions in buying and selling houses is widely acknowledged, with buyers and sellers spending considerable amounts of time searching. The essence of the search approach to markets is to understand how the stocks of buyers and sellers evolve through inflows and outflows. Applied to the labour market, this has been the subject of an extensive literature. However, for the housing market, there has been little work that aims to understand inflows and outflows jointly, especially with regard to cyclical fluctuations.

This paper assembles a collection of stylized facts about the cyclical properties of a broad set of U.S. housing-market variables over the last three decades, including house prices and the key stocks and flows, comprising houses for sale, sales transactions, new listings, and the average time taken for houses to sell. A calibrated search-and-matching model with both endogenous inflows (new listings) and outflows (sales) is used to explain the empirical findings.

One contribution of the paper is to document two novel facts. First, both inflows and outflows are quantitatively important in understanding housing-market volatility. This is shown using an 'ins versus outs' decomposition of the type that has been applied to the labour market. Here, the stock of houses for sale is the equivalent of unemployment, the evolution of which depends on the difference between new listings and sales. The second novel fact is that houses for sale does not have a stable correlation with house prices, sales, or new listings, while correlations among all other pairs of variables remain stable. The correlations among prices, sales, and new listings are all positive, while the correlations of these with time-to-sell are all robustly negative (with the possible exception of prices). In contrast, though the correlation of houses for sales with time-to-sell has been positive throughout the period studied, the correlations of houses for sale with prices, sales, and new listings have changed from positive to negative in recent times.

A second contribution of this paper is to demonstrate two new quantitative results using a stochastic search-and-matching model with endogenous inflows and outflows. Central to the model is the idea of idiosyncratic match quality between a house and its owner, and the dynamics of the distribution of ongoing match quality. Decisions to buy houses are described by a cut-off rule whereby a sale occurs when a draw of new match quality is above a certain threshold. Individual match quality is a persistent variable, but is subject to occasional idiosyncratic shocks that degrade it. After such shocks, homeowners decide whether to move house, and the moving decision is also described by a cut-off rule for match quality. These decision processes give rise to an endogenous distribution of match quality across all homeowners.

The first novel quantitative result is that housing-demand shocks coming from changes in interest rates and expenditures complementary to housing can explain most of the patterns of co-movement among housing-market variables. In the model, since moving house represents an investment in match quality, interest rates affect the incentive to invest in better match quality by changing the relative importance of future payoffs compared to current costs. A fall in the real interest rate increases the total surplus from a transaction and raises the price paid by buyers. Hence, a lower interest rate has a positive effect on house prices and new listings. A positive expenditure shock, associated with an increase in the flow utility received from occupying a house, raises the total surplus from a transaction and thus increases house prices. This shock increases the rate at which transactions occur, lowering time-to-sell. The positive expenditure shock also boosts homeowners' incentives to invest in better match quality by moving house, which leads to a rise in new listings, and these listings ultimately result in more sales.

Match quality plays a crucial role in the workings of the model and its ability to explain the stylized facts. The presence of a distribution of new match quality is central to generating a positive correlation between sales and prices. Given the equilibrium distribution of match quality among existing homeowners, a persistent housing-demand shock increases the incentive to invest in better match quality, leading to more listings. This explains the positive correlation between new listings and sales and prices.

The second quantitative result is that the model predicts different correlations between houses for sale and other variables when there is a change to the source or persistence of housing-market shocks. By simulating the model for two sub-sample periods, the lower measured persistence of the empirical proxy for the housing-demand shock can explain the switch from positive to negative in the correlations of houses for sale with sales, prices, and listings, as is seen empirically in recent times. Therefore, the model can offer an explanation of why the signs of the correlations between houses for sale and other variables have not been stable over time, while also being consistent with most of the empirically stable correlations among other housing-market variables.

The relative importance of interest-rate and expenditure shocks also matters because positive housing demand shocks from these two sources have opposite effects on time-to-sell. Lower interest rates raise the return to searching and thus increase time-to-sell, leaving more houses on the market. In contrast, positive expenditure shocks increase the desire to complete transactions and hence reduce time-to-sell, depleting the stock of houses for sale.

The main reason for the switch in the sign of the correlations between houses for sale and sales, prices, and new listings is a reduction in the measured persistence of the expenditure shock in the second sub-sample. The key point is that new listings rise by more than sales with a more persistent shock, which increases the stock of houses for sale. On the contrary, the less persistent shock fails to induce enough moving to replenish the stock of houses for sale. This explanation comes from understanding moving decisions as investments in match quality: a less persistent shock has a smaller effect on the present value of future housing utility flows, so homeowners are less willing to pay the upfront costs of moving.

The plan of the paper is as follows. Related literature is discussed below. Section 2 performs a decomposition of housing-market volatility into inflow and outflow components, presents stylized facts on housing-market cyclicality, and documents how the correlations among variables have changed over time. Section 3 presents the search-and-matching model with endogenous inflow and outflow decisions. Section 4 performs simulations of the calibrated model subject to aggregate shocks and assesses the model's performance in accounting for the joint time-series behaviour of sales, prices, new listings, time-to-sell, and houses for sale. Section 5 concludes.

Related literature There is a strand of literature starting from Wheaton (1990), and followed by many others, including the current paper, that studies frictions in the housing market with a search-and-matching model.[1] Han and Strange (2015) is a recent survey of this literature. The key contribution of this paper to the literature is in studying the role of new listings (inflows) alongside that of sales (outflows) in understanding the cyclical patterns of volatility and co-movement among housing-market variables.

Ngai and Sheedy (2020a) construct a time series for the inflow rate to the housing market using a stock-flow accounting identity and show that it accounts for most of the long-run changes in the level of sales. The current paper uncovers two new facts about housing-market cyclicality. First, inflows are volatile and as important as outflows in accounting for housing-market fluctuations. Second, inflows and outflows are positively correlated, and thus are associated with opposing effects on the number of houses for sale. This observation is closely related to the fact that correlations between houses for sale and other housing-market variables are not stable over time. In contrast, correlations among other pairs of variables are stable. This paper uses a stochastic version of the model of Ngai and Sheedy (2020a) to highlight how the source and persistence of shocks affect the predicted responses of housing-market variables, which allows the model to replicate the changing correlations between houses for sale and sales, prices, and new listings that are seen over time.[2]

Smith (2020) also documents and studies the patterns of volatility and co-movement among new listings, sales, and houses for sale using data from the South Central Wisconsin Multiple Listing Service (SCWMLS) for Dane County between January 1997 and December 2007. The data in the current paper cover the whole of the U.S. and span three decades, and one contribution is in showing that the correlations between houses for sale and other variables have been time varying. While Smith (2020) focuses on generating hot and cold spells in sales in a stock-flow matching model with endogenous entry of sellers, the model in the current paper explores how moving decisions respond to aggregate shocks, generating endogenous entry of buyers and sellers to understand the cyclical

[1]See, for example, Albrecht, Anderson, Smith and Vroman (2007), Anenberg and Bayer (2020), Aruoba, Davis and Wright (2016), Caplin and Leahy (2011), Coles and Smith (1998), Díaz and Jerez (2013), Gabrovski and Ortego-Marti (2019), Garriga and Hedlund (2020), Guren (2018), Han, Ngai and Sheedy (2022), Head, Lloyd-Ellis and Sun (2014), Hedlund (2016a), Ioannides and Zabel (2019), Krainer (2001), Moen, Nenov and Sniekers (2021), Ngai and Tenreyro (2014), Ngai and Sheedy (2020a), Novy-Marx (2009), Piazzesi and Schneider (2009), Piazzesi, Schneider and Stroebel (2020), and Smith (2020).

[2]Davis and Heathcote (2005) is one of the first studies to look at housing and the business cycle, exploring the role of residential investment. Another strand of the literature focuses on credit constraints, for example, see Fisher and Gervais (2011), Iacoviello (2005), Ortalo-Magné and Rady (2005), Stein (1995), and Ungerer (2015). Davis and Van Nieuwerburgh (2015) provide a survey of housing and business cycles.

behaviour of the housing market.

In exploring fluctuations in the housing market, Díaz and Jerez (2013) is the closest to the current paper in terms of its goals of examining a range of important housing-market statistics and explaining their cyclical patterns using a search-and-matching model. The main empirical contributions here relative to theirs are to document new business-cycle facts related to new listings, and to show that the correlations of houses for sale with sales, prices, and new listings have not been stable over time.[3]

Following Díaz and Jerez (2013), this paper uses real expenditures on 'furnishings and durable household equipment' to calibrate a housing-demand shock. In their model, this demand shock on its own cannot generate the observed positive correlations between sales and prices, or between houses for sale and prices.[4] Here, this persistent demand shock *alone* successfully generates these two positive correlations. In the model, the endogeneity of moving decisions means that a housing-demand shock induces more moving, acting like a moving-rate shock, as well as increasing the supply of houses on the market, acting like a housing-supply shock. Thus, one housing-demand shock replicates the three correlated, reduced-form shocks needed in Díaz and Jerez (2013).[5]

Motivated by the positive correlation between houses for sale and prices documented by Díaz and Jerez (2013) prior to 2010, Gabrovski and Ortego-Marti (2019) argue that the housing market features an upward-sloping Beveridge curve, that is, a positive correlation between houses for sale and the number of buyers. Using an exogenous-moving model, they show that endogenous entry of houses and buyers can generate such a positive correlation. The current paper shows that the endogenous moving decision of homeowners (related to 'own-to-own' moves) naturally implies a positive correlation between houses for sale and the number of buyers in response to aggregate shocks. The quantitative analysis demonstrates that persistent demand shocks can generate the observed positive correlation between houses for sale and prices seen prior to 2010 by inducing plenty of moving by homeowners. Furthermore, less persistent demand shocks in the period after 2010 induce smaller increases in moving — not enough to replenish the stock of houses for sale, and thus generate the observed post-2010 negative correlation between houses for sale and prices.

Anenberg and Bayer (2020) and Moen, Nenov and Sniekers (2021) also emphasize the role of own-to-own moves in amplifying fundamental shocks. They focus on the decision to buy first or sell first, while assuming an exogenous moving rate; here, the focus is on how the moving rate responds to fundamental shocks. The main objective of Anenberg and Bayer (2020) is to demonstrate own-to-own moves are very volatile and can amplify cyclical house-price volatility. The objective

[3] Hedlund (2016b) also documents cyclical facts about sales, time-to-sell, prices, and foreclosures. His focus is on foreclosures, and he does not study houses for sale or new listings.

[4] See Garriga and Hedlund (2020), Hedlund (2016b), and Gabrovski and Ortego-Marti (2019) for the roles of endogenous housing illiquidity and entry of buyers and sellers in generating a positive correlation between prices and sales.

[5] To be precise, their Table 4 reports negative correlations between prices and sales when there are only demand and/or supply shocks. They show a positive correlation is obtained only when they introduce a third correlated moving shock (Table 5), or in a model with a match-quality distribution (Table 9). They cannot obtain a positive correlation between houses for sale and prices in any of these cases.

here is similar, but the current paper looks at how own-to-own moves can *endogenously* respond to fundamental housing-demand shocks. Moen, Nenov and Sniekers (2021) present evidence on the importance of the order of buying and selling houses, and show that strategic complementarity in the order of transactions can give rise to multiple equilibria. Here, the current paper presents evidence on the cyclical behaviour of listings, and explains the observed patterns of correlation by studying homeowners' moving decisions.

2 The cyclical behaviour of housing-market variables

This section presents new empirical facts about housing-market cyclicality. The data used cover the U.S. from January 1991 to December 2019. The Federal Housing Finance Agency (FHFA) provides a monthly repeat-sales house-price index for single-family homes. Here, the purchase-only index is used, which excludes refinancing. Data on this variable begin in January 1991. The repeat-sales index is the best available price index that controls for the quality of the housing stock because it is designed to measure price changes of the same houses. Real house prices are obtained dividing by the Personal Consumption Expenditure (PCE) price index.

The National Association of Realtors (NAR) provides monthly estimates of the number of sales transactions and inventories of unsold houses at the end of each month, available for both single-family homes and condominiums. The coverage of the NAR data is existing homes only, so newly constructed houses are excluded. For consistency with the FHFA house-price index, NAR data for single-family homes are used, which constitute about 90% of total sales of existing homes.[6]

Following Ngai and Sheedy (2020a), a time series for houses newly listed for sale during a month is constructed using a stock-flow accounting identity. Sales during month t are denoted by S_t, and the inventory of all houses listed for sale but unsold as of the end of month t by I_t. Using NAR data on S_t and I_t, new listings N_t during month t are given by $N_t = I_t - I_{t-1} + S_t$ because the change in inventory (the stock of all properties listed for sale) is equal to the difference between inflows (new listings) and outflows (sales).

A measure of the average number of houses available for sale during a month can be obtained assuming inflows N_t and outflows S_t occur uniformly within a month. The term 'houses for sale' is used to distinguish carefully between the total stock of properties listed for sale and the flow (new listings). Houses for sale U_t during month t are $U_t = (I_t + I_{t-1})/2$, the average of the inventory levels at the ends of two adjacent months.[7] Using houses for sale U_t, 'time-to-sell' T_t is defined as the ratio of the houses on the market U_t to sales S_t during a month, that is, $T_t = U_t/S_t$.[8]

[6] Methodology and data for FHFA data are available at http://www.fhfa.gov. Methodology and recent data for NAR are available at https://www.nar.realtor/research-and-statistics.

[7] Since the time series for inventories has a high degree of serial correlation, the measure of houses for sale U_t turns out to be very closely related to inventories I_t (the correlation coefficient is 0.99).

[8] This measure is highly correlated with the 'months supply' number reported by NAR, which is defined as inventories of unsold houses at the end of the previous month divided by the number of houses sold in the current month. The mean

The non-seasonally adjusted data on prices and sales, and the constructed new listings, houses for sale, and time-to-sell series are deseasonalized by removing multiplicative month effects.[9] To smooth out excess volatility due to measurement errors in the data, quarterly time series are constructed from the monthly series.[10] The series used here cover the period from 1991Q1 to 2019Q4.

2.1 The ins and outs of houses for sale

In studying the housing market as a market subject to search frictions, the stock of houses for sale is analogous to unemployment in the labour market. As in the labour literature, it is possible to understand fluctuations in houses for sale in terms of changes in the rates of inflow and outflow to and from the housing market. A higher inflow rate (more new listings) increases houses for sale; a higher outflow rate (more sales) decreases houses for sale. Methodologically, this section follows the 'ins versus outs' decompositions of unemployment fluctuations (Petrongolo and Pissarides, 2008, Fujita and Ramey, 2009, Elsby, Hobijn and Şahin, 2013) to investigate the source of cyclical fluctuations in houses for sale using the same techniques that have been applied in research on labour markets.

The inflow and outflow rates in the housing market are respectively the rate at which houses are listed for sale and the rate at which they are subsequently sold. The sales rate $s_t = S_t/U_t$ is measured as the ratio of sales transactions S_t to houses for sale U_t. This is the inverse of the time-to-sell measure $T_t = U_t/S_t$ introduced earlier. The listing rate n_t is the ratio of the number of new listings N_t to the number of houses not currently listed for sale, that is, the difference between the total housing stock K and houses for sale U_t. The formula for the listing rate is $n_t = N_t/(K - U_t)$. In practice, since the total housing stock K far exceeds the number of houses for sale, the listing rate n_t is close to being proportional to the number of new listings N_t.

The inflow and outflow rates n_t and s_t are calculated from the NAR data on sales and inventories described earlier. These data are used to construct series for new listings N_t using the stock-flow accounting identity, and houses for sale U_t. A measure of the total housing stock K is also needed in calculating the inflow rate n_t, however, the main effect of K is on the average level of the inflow rate n_t, not the cyclical fluctuations that are the focus of this paper.[11] It turns out to make little difference to the following inflow-outflow decomposition exactly what value of K is used within a reasonable range. For the purposes of this study, the total housing stock should measure all houses that are either for sale or might be put up for sale, and the number should be consistent with the sales and inventories data from NAR on existing single-family homes. Using information from the U.S.

of T_t is 6.4 months, compared to 6.6 for 'months supply', and the correlation coefficient is 0.99.

[9] In logarithms, the differences between the average for each month of the year and the overall average are subtracted for each variable.

[10] Sales and new listings are summed for the months of a quarter; houses for sale are averaged over the months in a quarter.

[11] The total housing stock K is treated as a constant here because high-frequency data are not available. The role of a time trend in the housing stock in explaining long-run changes in sales volumes is explored in Ngai and Sheedy (2020a).

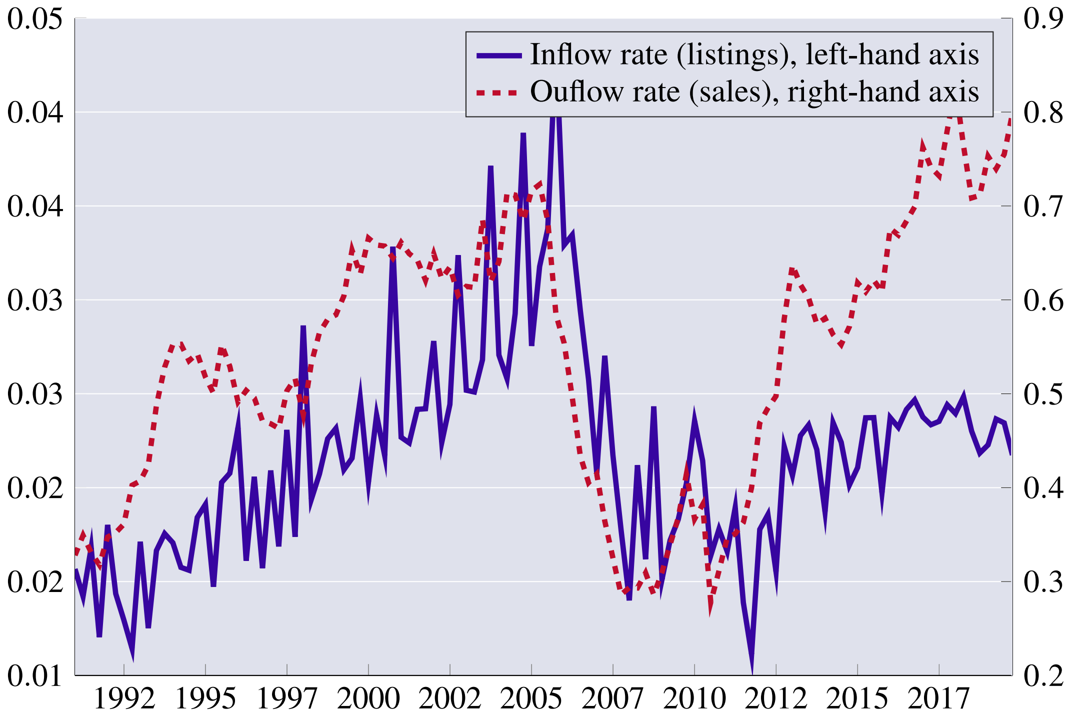
Census Bureau American Housing Survey and New Residential Construction data, the total housing stock K is set to be 50 million as an approximation.

Figure 1 plots the quarterly inflow and outflow rates. These are used to perform an inflow-outflow decomposition of fluctuations in houses for sale $u_t = U_t/K$ as a fraction of the total housing stock. Using the stock-flow accounting identity, the law of motion for u_t is approximately

$$\Delta u_t \approx n_t(1-u_t) - s_t u_t, \tag{1}$$

where $n_t(1-u_t)$ is the inflow and $s_t u_t$ is the outflow, both relative to the total stock of houses.[12]

Figure 1: *Inflow and outflow rates in the housing market*



Notes: Quarterly time series from 1991Q1 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency.
Source: NAR.

Several commonly used methods for performing the decomposition are based on the time-varying

[12] A refinement of this equation uses estimates of the continuous-time inflow and outflow rates to account explicitly for flows occurring within time periods, as is done, for example, in Petrongolo and Pissarides (2008). Note however that houses for sale u_t is calculated using an average of beginning-of-period and end-of-period inventory, which partially addresses this issue. In practice, there is no significant impact on the results presented below if continuous-time rates n_t and s_t are calculated using the method of Petrongolo and Pissarides (2008).

steady state u_t^* of the fraction of houses for sale, that is, the value of u_t such that $\Delta u_t = 0$ in (1):

$$u_t^* = \frac{n_t}{s_t + n_t}. \tag{2}$$

The argument for focusing on u_t^* instead of the actual u_t is that convergence to the steady state is expected to be rapid: the rate of convergence is the sum of the inflow and outflow rates. It is implicitly assumed that u_t is close enough to u_t^* to study the contributions of inflow and outflow rates to fluctuations in u_t through the effects of n_t and s_t on u_t^* in (2).

Fujita and Ramey (2009) note that changes in $\log u_t^*$ over time are approximately given by

$$\Delta \log u_t^* \approx (1 - u_t^*)(\Delta \log n_t - \Delta \log s_t), \tag{3}$$

where $\Delta \log n_t$ and $\Delta \log s_t$ are the changes in log inflow (listing) and outflow (sales) rates. From this equation, the inflow-outflow decomposition is obtained by calculating the coefficients γ_n and γ_s:

$$\gamma_n = \frac{\mathrm{Cov}[\Delta \log u_t^*, (1-u_t^*)\Delta \log n_t]}{\mathrm{Var}[\Delta \log u_t^*]}, \quad \text{and} \quad \gamma_s = \frac{\mathrm{Cov}[\Delta \log u_t^*, -(1-u_t^*)\Delta \log s_t]}{\mathrm{Var}[\Delta \log u_t^*]}. \tag{4}$$

The method in Petrongolo and Pissarides (2008) is similar, but uses an exact decomposition of Δu_t^* rather than the approximation of $\Delta \log u_t^*$ in (3), though this difference between the methods does not have a quantitatively significant effect on the results.[13] More importantly, Petrongolo and Pissarides (2008) calculate the decomposition coefficients γ_n and γ_s using only data points where the difference between Δu_t and Δu_t^* is no more than 10% of u_t, which excludes time periods where the steady-state equation (2) does not accurately describe houses for sale u_t. Another way to address this is to use the decomposition method proposed by Elsby, Hobijn and Şahin (2013), which explicitly takes into account the transitional dynamics of u_t when it is not close to u_t^*.[14]

The results of the three decomposition methods are shown in Table 1. From the size of the γ_n coefficients, all methods indicate that changes in the inflow (listing) rate are quantitatively important in explaining fluctuations in houses for sale. Those methods that account for deviations of u_t from u_t^*, and thus the presence of transitional dynamics, also find that changes in outflow (sales) rates are quantitatively important.

[13] The method is based on the exact decomposition of $\Delta u_t^* = u_t^* - u_{t-1}^*$ that follows from equation (2):

$$\Delta u_t^* = (1-u_t^*)u_{t-1}^* \frac{\Delta n_t}{n_{t-1}} - (1-u_{t-1}^*)u_t^* \frac{\Delta s_t}{s_{t-1}}.$$

[14] This is based on the following approximation:

$$\Delta \log u_t = \rho_t \left((1-u_t^*)(\Delta \log n_t - \Delta \log s_t) + \frac{(1-\rho_{t-1})}{\rho_{t-1}} \Delta \log u_{t-1} \right),$$

where $\rho_t = 1 - e^{-(n_t + s_t)}$ is the fraction of the gap between u_t and u_t^* closed in one time period.

Table 1: *Inflow-outflow decompositions of fluctuations in houses for sale*

Method	New listings (γ_n)	Sales (γ_s)
Fujita and Ramey (2009)	0.898	0.101
Petrongolo and Pissarides (2008)	0.576	0.424
Elsby, Hobijn and Şahin (2013)	0.467	0.525

Notes: With the Petrongolo and Pissarides (2008) method, $\gamma_n + \gamma_s = 1$, but with the Fujita and Ramey (2009) and Elsby, Hobijn and Şahin (2013) methods, the coefficients γ_n and γ_s need not sum to one exactly. There are residual terms coming from first-order approximations (see equation 3) of $\Delta \log u_t^*$ (0.001 using the Fujita and Ramey method, and 0.008 for the Elsby, Hobijn and Şahin method). For the Elsby, Hobijn and Şahin method, there is also an initial component of the decomposition coming from a deviation from steady state at the start of the sample (which is negligible here).

While the methods used in the labour literature can be carried over and applied to study fluctuations in the housing market, one fundamental difference in the behaviour of inflow and outflow rates should be noted. As Figure 1 clearly shows, listing and sales rates are positively correlated. This means that the effects on u_t of increases in both n_t and s_t go in opposite directions (see equation 1 for Δu_t, or 2 for u_t^*.). In contrast, while there is a debate in the labour literature about whether inflows or outflows are more important in explaining unemployment fluctuations, both effects are reinforcing because job-separation and job-finding rates are negatively correlated. Consequently, it is not obvious whether to expect a positive or negative correlation of houses for sale with other housing-market variables. Moreover, these correlations may not be stable over time. For example, Figure 1 shows the U.S. housing market experiences a boom with rising sales and listing rates up to 2006, followed by a collapse and then a recovery. During the boom period, the inflow rate rises proportionately more than the outflow rate. However, during the post-2010 recovery period, the outflow rate rises proportionately more than the inflow rate.

2.2 Volatility and co-movement of housing-market variables

This section documents patterns of volatility and co-movement across housing-market variables. Standard deviations and correlation coefficients of sales transactions, house prices, new listings, time-to-sell, and houses for sale are shown in Table 2. The data have been transformed into natural logarithms to make the magnitudes of the cyclical fluctuations comparable across variables, and the standard deviations are reported as percentages. A linear time trend is removed from all series to isolate the cyclical components of variables.

Díaz and Jerez (2013) present business-cycle facts for the housing market using data up to 2010.[15] The current paper builds on this earlier empirical work in two important ways. First of

[15] A results table directly comparable to Díaz and Jerez (2013), where data are detrended using the Hodrick-Prescott filter, and a table based on data without any detrending are provided in appendix A.1.

Table 2: *Cyclical properties of housing-market variables*

	Sales	Prices	New listings	Time-to-sell	Houses for sale
			Standard deviations, %		
	16.8	10.3	24.3	27.3	20.5
			Correlation coefficients		
Sales	1				
Prices	0.67	1			
New listings	0.83	0.60	1		
Time-to-sell	−0.66	−0.13	−0.55	1	
Houses for sale	−0.06	0.37	−0.06	0.79	1

Notes: Calculated from linearly-detrended natural logarithms of quarterly time series from 1991Q1 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency.
Sources: FHFA and NAR.

all, new listings is included as an additional variable, which the ins-and-outs decomposition has shown to be quantitatively important in accounting for housing-market fluctuations. Second, this paper assembles data on sales transactions, average time-to-sell, and the number of houses for sale from the same source rather than the three different sources used by Díaz and Jerez (2013). More specifically, in Díaz and Jerez (2013), sales data are taken from NAR as here, time-to-sell is measured only for newly constructed houses ('New Residential Sales' from the U.S. Census Bureau), and data on houses for sale come from the 'vacant for sale' measure provided by the U.S. Census Bureau Housing Vacancy Survey. Note that this 'vacant for sale' data include only a small fraction of the houses that are actually for sale because houses that are occupied but available for sale are excluded. Vacant houses are only around 11% of all single-family homes sold.[16]

Consistent with what is known in the literature, Table 2 shows house prices and sales positively co-move with a correlation coefficient of 0.67, there is a negative correlation between time-to-sell and sales with correlation coefficient −0.66, and the volume of sales transactions is highly volatile. In addition to these familiar facts, Table 2 reveals that new listings are also volatile, like sales.[17] New listings positively co-move with sales and prices with correlation coefficients of 0.83 and 0.60 respectively, and negatively co-move with time-to-sell with correlation coefficient −0.55. Finally, houses for sale are uncorrelated with sales volume and new listings, but positively correlated with prices and time-to-sell. These last two positive correlations are also documented by Díaz and Jerez (2013) using 'vacant for sale' as the measure of houses for sale.

[16]See Table 1 of NAR's methodological documentation.

[17]This is consistent with Bachmann and Cooper (2014), who show that housing turnover is volatile using data obtained from the Panel Study of Income Dynamics on flows within the owner-occupied segment of the housing market.

2.3 Is there time variation in correlations among housing-market variables?

To investigate whether the overall patterns of co-movement documented in Table 2 are stable or not over time, correlation coefficients in rolling ten-year windows are calculated for pairs of housing-market variables. The top panel of Figure 2 shows correlations of houses for sale with sales, prices, new listings, and time-to-sell plotted at the midpoints of ten-year windows over the sample period. It reveals all these correlations, with the exception of that with time-to-sell, change sign during the sample period, becoming negative in the last decade. The middle panel of Figure 2 displays correlations of time-to-sell with sales, prices, and new listings. It demonstrates there are stable negative correlations of time-to-sell with new listings and sales, but the correlation of time-to-sell with prices is unstable. Finally, the bottom panel of Figure 2 shows that correlations of sales with prices and new listings are both stable over time. These conclusions are robust to detrending the data with the Hodrick-Prescott filter or performing no detrending at all, as is shown in appendix A.1.

The findings provide evidence that there is no invariant structural relationship between houses for sale and prices, new listings, or sales, nor between time-to-sell and prices. As shown later in section 4 using a calibrated search-and-matching model, the changing sign of the correlation coefficients can be explained through variation in the persistence and nature of the shocks affecting the housing market.

2.4 Directly measured listings using data from Redfin

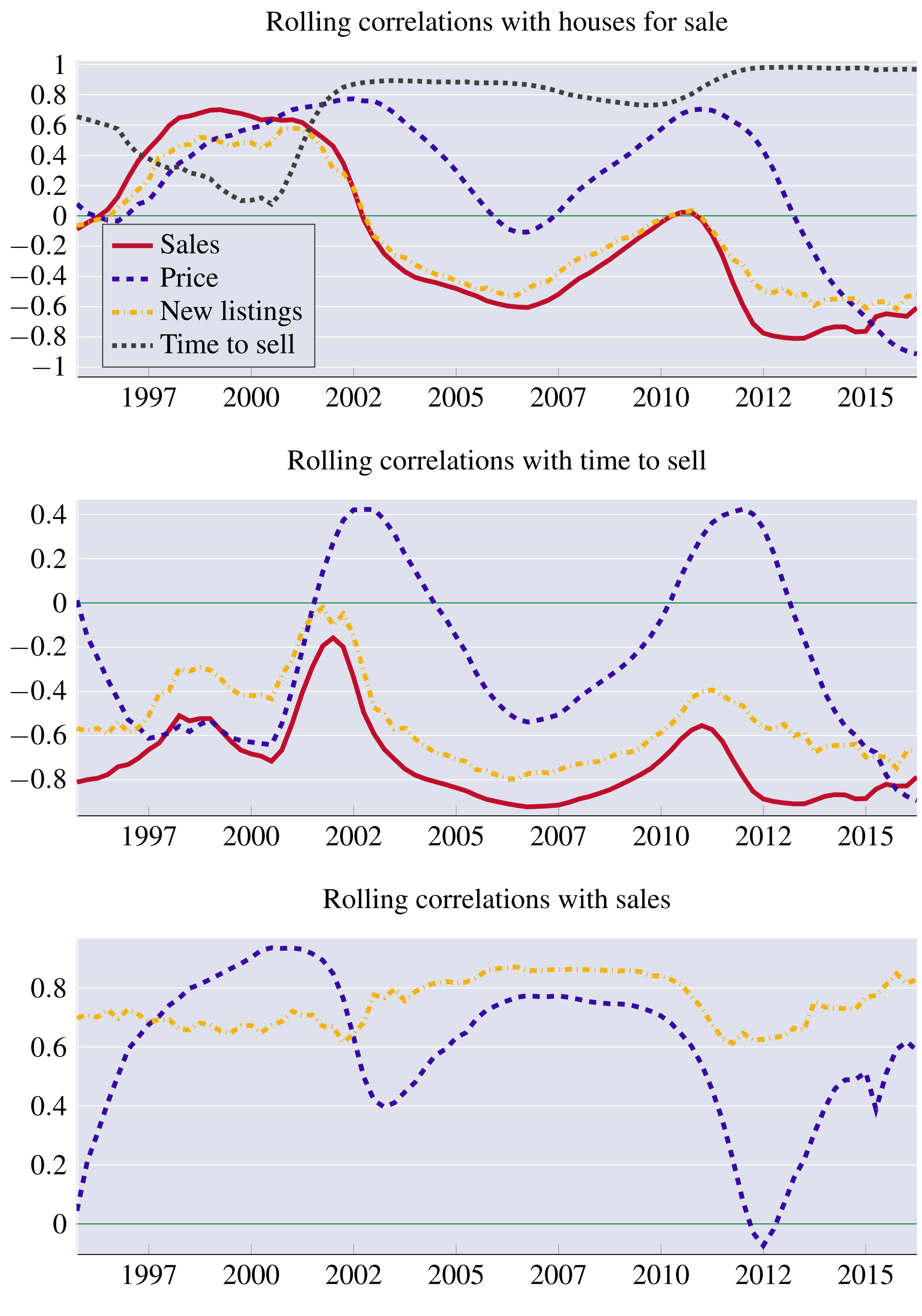
Since the earlier findings on the behaviour of new listings in Table 2 and Figure 2 were based on numbers imputed from NAR data using a stock-flow accounting identity, directly measured data on new listings from Redfin are used as a robustness check.[18] Redfin data on new listings, sales transactions, inventories, prices, and days on the market are available monthly from February 2012.

The Redfin house-price series is divided by the PCE price index to obtain the real price of housing, as was done earlier for the FHFA price data. The stock of houses for sale is calculated as the average of beginning- and end-of-month inventory, as was done with the NAR data. Days on the market is divided by 30 to obtain a direct monthly measure of time-to-sell, which is used instead of the $T_t = U_t/S_t$ variable derived from the sales and inventory data. The Redfin data are seasonally adjusted and converted to a quarterly frequency in the same way as was done for the NAR data earlier, but given that there are only seven years of data, the cyclical properties reported here are computed without any detrending. The results using linearly detrended data can be found in Table A.3.

Table 3 reports standard deviations and correlation coefficients of the variables from the Redfin dataset (numbers in bold) shown alongside the equivalent statistics calculated using the NAR and FHFA data only for the period 2012Q2–2019Q4 where Redfin data are available. As seen in the

[18] Redfin is a real-estate brokerage with direct access to data from local Multiple Listing Services (MLS). Methodology and data can be found at `http://www.redfin.com/news/data-center/`.

Figure 2: *Rolling correlations of housing-market variables*



Notes: Correlation coefficients in ten-year windows are calculated using linearly detrended and seasonally adjusted quarterly time series in logarithms. The date on the horizontal axis is the mid-point of each ten-year window.
Sources: FHFA and NAR.

Table 3: *Comparison with Redfin data, 2012Q2–2019Q4*

	Sales		Prices		New listings		Time-to-sell		Houses for sale	
	Standard deviations, %									
	9.5	5.8	**11.6**	9.8	**7.2**	10.2	**19.1**	14.0	**9.9**	9.4
	Correlation coefficients									
Sales	1									
Prices	**0.94**	0.77	1							
New listings	**0.92**	0.81	**0.89**	0.57	1					
Time-to-sell	−**0.95**	−0.87	−**0.97**	−0.90	−**0.90**	−0.68	1			
Houses for sale	−**0.83**	−0.67	−**0.88**	−0.87	−**0.67**	−0.50	**0.90**	0.95	1	

Notes: Calculated from natural logarithms of quarterly time series from 2012Q2 to 2019Q4 with no detrending. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency. Redfin statistics are in bold, adjacent to the equivalent NAR and FHFA statistics.
Sources: Redfin, NAR, and FHFA.

table, the direct measure of new listings is slightly less volatile than the measure imputed from NAR data, but its correlations with other housing-market variables are similar. This confirms the patterns discussed earlier where houses for sale becomes negatively correlated with sales and prices in the later part of the sample period. New listings are strongly positively correlated with sales and prices, and negatively correlated with time-to-sell.

3 A search model with endogenous inflows and outflows

This section presents a stochastic version in discrete time of the endogenous-moving model of Ngai and Sheedy (2020a). The model studies the decisions to buy and sell houses, and the decision to move house. The key message of the model is that moving house is like an investment decision. As with the analysis of the data in section 2, the model focuses on the market for existing homes. This abstracts from new entry of houses due to either new construction or houses that were previously rented, and abstracts from the entry of first-time buyers into the market.[19]

Households and houses There is an economy with a unit continuum of households and a unit continuum of houses. Each house is owned by one household. Each house is either occupied by its

[19] It is implicit in the model that households moving house might temporarily use the rental market in between selling and buying. The flow utility of renting net of rent payments is normalized to zero. The rental market is treated as a distinct segment of the housing market, a view supported by Glaeser and Gyourko (2007) and Bachmann and Cooper (2014), especially where the focus is on fluctuations in housing turnover within the owner-occupied segment of the market.

owning household in the sense of yielding a stream of utility flow values, or is listed for sale on the market while the household searches for a buyer. A household can occupy at most one house at any time, and searches for a house to buy and occupy if the household does not own a house that is not listed for sale.[20]

Time is indexed by t, and households make decisions at discrete time intervals of length $\tau > 0$. All units of time are measured in years throughout. During an interval of time $[t, t+\tau)$, households discount future payoffs beyond $t+\tau$ at an exogenous and stochastic rate r_t using the discount factor $\beta_t = e^{-\tau r_t}$. Expectations conditional on information available at time t are denoted by $\mathbb{E}_t[\cdot]$. Within each time interval, households first decide whether to move house following the realization of shocks, which gives rise to new listings that are added to the existing stock of houses for sale. Search and matching then occurs between buyers and sellers, which leads to transactions in the housing market when successful.

3.1 Behaviour of buyers and sellers

Search frictions The housing market is subject to search frictions. First, it is time-consuming and costly for buyers and sellers to arrange viewings of houses. Let u_t denote the measure of houses listed for sale and b_t the measure of buyers. Each buyer and each house can have at most one viewing in the time interval $[t, t+\tau)$.[21] For houses, this event has Poisson arrival rate $M(u_t, b_t)/u_t$, where $M(u, b)$ is a constant-returns-to-scale meeting function (noting that not all viewings will lead to matches). For buyers, the corresponding arrival rate is $M(u_t, b_t)/b_t$. During this process of search, buyers incur flow search costs τF per interval of time τ.

Given the unit measure of houses, there are $1-u_t$ houses that are already matched in the sense of being occupied by a household. As there is also a unit measure of households, there must be u_t households not matched with a house, and thus in the market to buy. This means the measures of buyers and sellers are the same ($b_t = u_t$). Given that the function $M(u, b)$ features constant returns to scale, the arrival rates of viewings for buyers and sellers are then both equal to $m = M(1, 1)$. This m summarizes all that needs to be known about the frictions in locating houses to view.

The second aspect of the search frictions in the housing market is heterogeneity in buyer tastes and the extent to which any given house will conform to these. The idiosyncratic utility flow value of an occupied house is match specific, that is, particular to both the house and the household occupying

[20]In principle, households can own multiple houses. The assumptions made here imply that utility flows from houses and listing decisions do not depend on the number of houses owned.

[21]Later, the model is calibrated so that a discrete time period $[t, t+\tau)$ is one week ($\tau = 1/52$).

it.[22] When a viewing takes place, match quality ε is drawn from the probability distribution

$$\varepsilon \sim \text{Pareto}(1, \lambda), \quad \text{where } \mathbb{P}[\varepsilon \leq w] = 1 - w^{-\lambda}. \tag{5}$$

The Pareto distribution is chosen for analytical tractability. The minimum value of ε is normalized to one, and the parameter $\lambda > 1$ determines the shape of the distribution. The variance of new match quality is inversely related to the shape parameter λ.

Transactions When a viewing occurs, ε is drawn and becomes common knowledge among the buyer and the seller. The value to a household of occupying a house with match quality ε is denoted $H_t(\varepsilon)$. By purchasing and occupying this house, the buyer loses the option of continuing to search, which has present value $\beta_t \mathbb{E}_t B_{t+\tau}$, where B_t is the value of being a buyer at time t. If the seller agrees to an offer to buy, the gain is the transaction price, and the loss is the option value of continuing to search, namely $\beta_t \mathbb{E}_t V_{t+\tau}$, where V_t is the value of owning a house for sale. The buyer and seller also face a combined transaction cost C. The total surplus $\Sigma_t(\varepsilon)$ resulting from a transaction with match quality ε at time t is therefore

$$\Sigma_t(\varepsilon) = H_t(\varepsilon) - \beta_t \mathbb{E}_t J_{t+\tau} - C, \quad \text{where } J_t = B_t + V_t, \tag{6}$$

with J_t denoting the combined value of being a buyer and having a house for sale. Since the value function $H_t(\varepsilon)$ is increasing in ε, transactions occur if match quality ε is no lower than a threshold y_t, defined by $\Sigma_t(y_t) = 0$. Intuitively, given that ε is observable to both buyer and seller and the surplus is transferable between the two, the transactions that occur are those with positive surplus. The transaction threshold y_t satisfies the following equation:

$$H_t(y_t) = \beta_t \mathbb{E}_t J_{t+\tau} + C. \tag{7}$$

The proportion π_t of viewings that lead to transactions implied by the Pareto distribution of ε in (5) with transaction threshold y_t is

$$\pi_t = y_t^{-\lambda}. \tag{8}$$

[22] A measure of the importance of the second friction is the average number of viewings needed before a house is sold (or equivalently, before a buyer makes a purchase). Ngai and Sheedy (2020a) report that viewings per transaction range from 9 to 15 using U.S. data from Genesove and Han (2012) and UK data from the Hometrack 'National Housing Survey'. The data reveal that the number of viewings per transaction is far greater than one, indicating there is substantial uncertainty about match quality prior to a viewing. Moreover, the data show that variation in time-to-sell is associated with movements in viewings-per-transaction in the same direction, and not simply due to variation in the time taken to meet buyers.

Using the density function $\lambda \varepsilon^{-(\lambda+1)}$ of the Pareto distribution (5), the expected surplus Σ_t from a viewing before the value of ε becomes known is

$$\Sigma_t = \int_{\varepsilon=y_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \Sigma_t(\varepsilon) \mathrm{d}\varepsilon. \tag{9}$$

Given the viewing rate m in the interval $[t, t+\tau)$ for both buyers and sellers, there is a probability $\mu = 1 - e^{-m\tau}$ that a buyer or a seller will make or receive a viewing in one discrete time period. Hence, the Bellman equation for the combined buyer and seller value J_t is

$$J_t = -\tau(F + D) + \mu \Sigma_t + \beta_t \mathbb{E}_t J_{t+\tau}, \tag{10}$$

where D is the flow maintenance cost of owning a home, which is incurred irrespective of whether the owner is trying to sell.[23] Intuitively, the first two terms capture the flow costs and benefits of being a buyer and a seller, while the final term is the continuation value.

Bargaining If a transaction occurs, the house price $p_t(\varepsilon)$ is agreed according to Nash bargaining. The surpluses of the buyer and the seller, conditional on the match quality between the buyer and the house being ε, are

$$\Sigma_{B,t}(\varepsilon) = H_t(\varepsilon) - \beta_t \mathbb{E}_t B_{t+\tau} - p_t(\varepsilon) - (1-\kappa)C, \quad \text{and } \Sigma_{V,t}(\varepsilon) = p_t(\varepsilon) - \beta_t \mathbb{E}_t V_{t+\tau} - \kappa C, \tag{11}$$

where κ is the fraction of the total transaction cost C borne directly by the seller. The value functions B_t of the buyer and V_t of the seller satisfy the Bellman equations

$$B_t = -\tau F + \beta_t \mathbb{E}_t B_{t+\tau} + \mu \int_{\varepsilon=y_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \Sigma_{B,t}(\varepsilon) \mathrm{d}\varepsilon, \quad \text{and} \tag{12a}$$

$$V_t = -\tau D + \beta_t \mathbb{E}_t V_{t+\tau} + \mu \int_{\varepsilon=y_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \Sigma_{V,t}(\varepsilon) \mathrm{d}\varepsilon. \tag{12b}$$

The Nash bargaining solution with bargaining power ω of the seller implies the surplus-splitting equation $(1-\omega)\Sigma_{V,t}(\varepsilon) = \omega \Sigma_{B,t}(\varepsilon)$, and hence $\Sigma_{V,t}(\varepsilon) = \omega \Sigma_t(\varepsilon)$, noting $\Sigma_t(\varepsilon) = \Sigma_{B,t}(\varepsilon) + \Sigma_{V,t}(\varepsilon)$ using (6) and (11). This equation determines the transaction price for a house with match quality ε to its buyer:

$$p_t(\varepsilon) = \kappa C + \beta_t \mathbb{E}_t V_{t+\tau} + \omega \Sigma_t(\varepsilon). \tag{13}$$

[23]The flow cost D also enters the value of being a homeowner $H_t(\varepsilon)$, which appears in the expected surplus Σ_t.

Transactions occur if $\varepsilon \geq y_t$, and the distribution of ε conditional on $\varepsilon \geq y_t$ is Pareto(y_t, λ), which has density function $\lambda y_t^{\lambda} \varepsilon^{-(\lambda+1)}$. The average price P_t of all houses sold at time t is therefore

$$P_t = \kappa C + \beta_t \mathbb{E}_t V_{t+\tau} + \omega \int_{\varepsilon=y_t}^{\infty} \lambda y_t^{\lambda} \varepsilon^{-(\lambda+1)} \Sigma_t(\varepsilon) \mathrm{d}\varepsilon. \tag{14}$$

3.2 Behaviour of owner-occupiers

Match quality A homeowner with match quality ε at time t receives a utility flow value of $\tau \varepsilon \theta_t$ during the time period $[t, t+\tau)$, where θ_t is an exogenous and stochastic component of housing utility common to all homeowners. A flow maintenance cost τD is also incurred during that period.

Individual match quality ε is a persistent variable. However, households are sometimes subject to idiosyncratic shocks that degrade match quality. These shocks can be thought of as life events that make a house less well suited to the household's current circumstances. At most one such shock occurs in the time interval $[t, t+\tau)$. The arrival of these shocks follows a Poisson process with arrival rate a. If a shock arrives, match quality ε is scaled down by a parameter δ with $\delta < 1$. If no shock occurs, match quality remains unchanged. Given match quality ε at time t, the stochastic process for match quality ε' at time $t+\tau$ is

$$\varepsilon' = \begin{cases} \varepsilon & \text{w.p. } \alpha \\ \delta\varepsilon & \text{w.p. } 1-\alpha \end{cases}, \tag{15}$$

where $\alpha = e^{-a\tau}$ is the probability that no idiosyncratic shock is received during $[t, t+\tau)$.

Listing decisions Following the arrival of idiosyncratic shocks, homeowners decide whether or not to list their home for sale on the market. The value function $H_t(\varepsilon)$ for an owner-occupier satisfies the Bellman equation

$$\begin{aligned} H_t(\varepsilon) = \tau\varepsilon\theta_t + \alpha\beta_t \mathbb{E}_t \max\{H_{t+\tau}(\varepsilon) - \tau D, J_{t+\tau} - \zeta\} \\ + (1-\alpha)\beta_t \mathbb{E}_t \max\{H_{t+\tau}(\delta\varepsilon) - \tau D, J_{t+\tau}\}, \end{aligned}$$

where ζ is an inconvenience cost of moving faced only by those who do not experience an idiosyncratic shock. This cost represents the inertia of families to remain in the same house. It is assumed the model parameters are such that ζ is large enough to deter moving if no idiosyncratic shock is received, that is, $\zeta > J_{t+\tau} - H_{t+\tau}(\varepsilon) + \tau D$, which holds when the cost ζ is large relative to the size of the aggregate shocks specified below. In this case, the Bellman equation simplifies to

$$H_t(\varepsilon) = \tau\varepsilon\theta_t + \alpha\beta_t \mathbb{E}_t[H_{t+\tau}(\varepsilon) - \tau D] + (1-\alpha)\beta_t \mathbb{E}_t \max\{H_{t+\tau}(\delta\varepsilon) - \tau D, J_{t+\tau}\}. \tag{16}$$

When a shock to match quality is received, a homeowner decides to move if the match quality ε is now below a moving threshold x_t defined by

$$H_t(x_t) = J_t + \tau D. \tag{17}$$

If no idiosyncratic shock is received, a homeowner chooses not to move given that the inconvenience cost ζ is sufficiently large. For those receiving idiosyncratic shocks, the decision to move depends on all relevant variables including their own idiosyncratic match quality, and current and expected future conditions in the housing market.

Aggregate shocks Analogous to an investment decision, homeowners compare the upfront costs of moving such as C to expected discounted flows of housing utility. The two exogenous random variables θ_t and r_t act as sources of aggregate housing-demand shocks by varying future utility flows and how they are discounted. Since the degree of persistence affects expected future flows, housing utility θ_t (in logarithms) and the discount rate r_t are modelled as exogenous AR(1) processes

$$\log\theta_t = \phi_\theta \log\theta_{t-\tau} + \eta_{\theta,t}, \quad \text{where } \eta_{\theta,t} \sim \text{i.i.d.}(0, \sigma_\theta^2), \text{ and} \tag{18a}$$

$$r_t = (1-\phi_r)r + \phi_r r_{t-\tau} + \eta_{r,t}, \quad \text{where } \eta_{r,t} \sim \text{i.i.d.}(0, \sigma_r^2), \tag{18b}$$

and ϕ_θ and ϕ_r are the persistence parameters, and σ_θ and σ_r are the standard deviations of the innovations $\eta_{\theta,t}$ and $\eta_{r,t}$ respectively. The unconditional expected values of $\log\theta_t$ and r_t are zero (a normalization) and $r > 0$ respectively, where r is the steady-state discount rate.

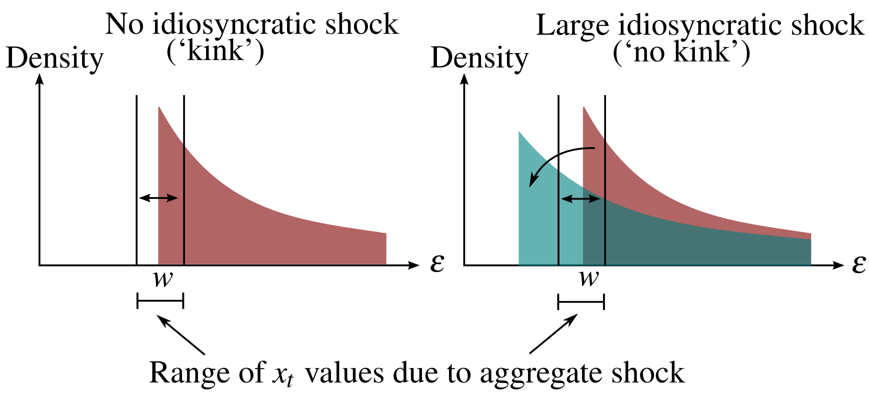
3.3 Solving the model

In the case of no aggregate shocks ($\eta_{\theta,t} = 0$ and $\eta_{r,t} = 0$ for all t, so $\theta_t = 1$ and $r_t = r$ in 18), the model becomes a discrete-time version of Ngai and Sheedy (2020a). With aggregate shocks, the solution of the model for aggregate variables is obtained approximately using a first-order perturbation (log linearization) around the deterministic steady state ($\sigma_\theta = 0$ and $\sigma_r = 0$). The well-known problem of non-differentiability in models with endogenous 'lumpy' adjustments — here, the decision to list a house for sale — is overcome given two parameter restrictions, while the Pareto distribution of new match quality significantly reduces the size of the model's state space.

Large idiosyncratic shocks First, idiosyncratic shocks are assumed to be large (in 15, δ is sufficiently far below 1) relative to aggregate shocks (the standard deviations σ_θ and σ_r in 18 are sufficiently small), and large relative to the difference between the transaction and moving thresholds y_t and x_t, which depends mainly on the transaction cost C. Second, the inconvenience cost ζ faced by those who do not receive an idiosyncratic shock is large relative to the size of the aggregate shocks.

Intuitively, the role of relatively large idiosyncratic shocks is illustrated in Figure 3, which shows the distribution of ε for existing matches, which has been previously truncated at some point w. The left panel shows the case where no idiosyncratic shock occurs. Without the cost ζ, the endogenous moving decision would imply a 'kinked' response of the overall number of homeowners who move: if the moving threshold falls due to an aggregate shock then there is no change in the number of homeowners who move, unlike the case where the moving threshold rises. The right panel shows the case where there are idiosyncratic shocks that are large relative to changes in the moving threshold due to aggregate shocks. In that case there is no problem of non-differentiability. Thus, when no idiosyncratic shock is received, the non-differentiability problem is avoided by a sufficiently large cost ζ.

Figure 3: *Differentiability and idiosyncratic shocks*



The magnitude of fluctuations in the transaction and moving thresholds y_t and x_t is small relative to the changes in ε brought about by idiosyncratic shocks when the standard deviations σ_θ and σ_r from (18) are relatively low. This avoids the non-differentiability problem for matches that have received multiple idiosyncratic shocks. For matches receiving their first shock, the problem is avoided if $\delta y_t < x_{t'}$ for all t and t'. This condition implies homeowners with match quality close to the transaction threshold always choose to move if an idiosyncratic shock is subsequently received, but not necessarily owners with higher match qualities.

Under these assumptions, the equations describing the equilibrium values of the aggregate variables are differentiable, and thus a perturbation method is admissible. The model allows an endogenous moving decision for those households most likely to consider moving, with a considerable gain in computational tractability by ruling out moving for those not hit by idiosyncratic shocks.

Pareto distribution In principle, solving the model requires finding the value function $H_t(\varepsilon)$ for all values of match quality ε, and keeping track of the whole distribution of surviving match quality. This means the model has an infinite-dimensional state space. However, with the assumption of a Pareto distribution for new draws of ε, the Bellman equations required to characterize the behaviour of aggregate variables can be reduced to a finite number of variables. Furthermore, the laws of motion for the stock of houses for sale and new listings can be written in terms of a finite and low number of state variables, ensuring the model remains tractable.

3.4 Laws of motion

The measure of houses listed and available for viewings and transactions in the interval of time $[t, t+\tau)$ is u_t. The fraction s_t of these houses sold is the product of the probability μ of a viewing and the probability π_t of a transaction conditional on a viewing, which gives the sales rate s_t/τ per unit of time. The reciprocal of the sales rate gives the average time T_t taken for houses to sell:

$$s_t = \mu \pi_t, \quad \text{and} \quad T_t = \frac{\tau}{s_t}. \tag{19}$$

The volume of transactions S_t during the interval $[t, t+\tau)$ is the product of s_t and u_t:

$$S_t = s_t u_t. \tag{20}$$

The stock of houses listed for sale evolves in line with the difference between inflows and outflows:

$$u_t - u_{t-\tau} = N_t - S_{t-\tau}, \tag{21}$$

where N_t denotes new listings occurring in the interval $[t-\tau, t)$. An equation for new listings N_t is found by noting that these listings must come from the existing matches $1 - u_{t-\tau} + S_{t-\tau}$ at date $t-\tau$ that receive an idiosyncratic shock (probability $1-\alpha$) during the interval $[t-\tau, t)$. It follows that N_t is equal to $(1-\alpha)(1 - u_{t-\tau} + S_{t-\tau})$ minus the measure of those homeowners who receive an idiosyncratic shock but who decide not to move.

Aggregating listing decisions All matches begin as draws from the distribution of match quality $\varepsilon \sim \text{Pareto}(1, \lambda)$. Surviving matches that receive an idiosyncratic shock during the interval $[t-\tau, t)$ can be characterized by their initial match quality ε, their vintage v, where $v \in \{1, 2, 3, \ldots\}$ denotes the number of discrete time intervals τ since the match formed, and the number $q \in \{0, 1, \ldots, v-1\}$ of previous idiosyncratic shocks that have occurred. At time t immediately after an idiosyncratic shock, current match quality is now $\varepsilon' = \delta^{q+1}\varepsilon$ given original match quality ε. A match survives the current shock only if $\varepsilon' \geq x_t$, or equivalently, $\varepsilon \geq x_t/\delta^{q+1}$ in terms of its original match quality.

Matches with vintage v at time t originate from the measure $\mu u_{t-\tau v}$ of past viewings. Depending on the timing of the realizations of past idiosyncratic shocks, matches with vintage v by time t and q previous shocks are those that remain after truncating the distribution of original match quality ε to the left at various points. These truncations occur with the first transaction decision ($\varepsilon \geq y_{t-\tau v}$) and subsequent moving decisions ($\varepsilon \geq x_{t-\tau i}/\delta^{j+1}$ for some $i = 1, \ldots, v-1$ and some $j = 0, \ldots, q$).

Let $G_{t,v,q}(w)$ denote the distribution function of the truncation points w of the distribution of original match quality for the cohort of vintage v by time t with q previous idiosyncratic shocks.

The properties of Pareto distributions imply that the distribution of ε conditional on $\varepsilon \geq w$ is Pareto(w, λ) with the same shape parameter λ. If $x_t/\delta^{q+1} \geq w$ for all w in the distribution $G_{t,v,q}(w)$, that is, $G_{t,v,q}(x_t/\delta^{q+1}) = 1$, then the probability of a match surviving the current shock conditional on any particular w and the original match having $\varepsilon \geq w$ is $\mathbb{P}[\varepsilon \geq x_t/\delta^{q+1} | \varepsilon \geq w] = (x_t/(\delta^{q+1}w))^{-\lambda}$. Since the possible truncation points are $w = y_{t-\tau v}$ or $w = x_{t-\tau i}/\delta^{j+1}$ for some $i \in \{1, \ldots, v-1\}$ and $j \in \{0, \ldots, q\}$, this formula is valid for a given range of fluctuations in the thresholds y_t and x_t if δ is sufficiently far below 1 because it implies $\delta x_t < x_{t'}$ and $\delta y_t < x_{t'}$ for all t and t'.

Conditional on vintage v, the independence of successive idiosyncratic shocks implies $q \sim$ Binomial$(v-1, 1-\alpha)$, where $v-1$ is the maximum number of previous shocks and $1-\alpha$ is the probability of each shock. With the original match quality of the mass $\mu u_{t-\tau v}$ of viewings previously truncated to the left of $\varepsilon = w$, a fraction $w^{-\lambda}$ of the initial draws of ε survived as matches up to the point where the current idiosyncratic shock occurs. Putting together these observations, the measure of matches receiving and surviving an idiosyncratic shock in the interval $[t-\tau, t)$ is

$$
\begin{aligned}
&(1-\alpha)\sum_{v=1}^{\infty} \mu u_{t-\tau v} \sum_{q=0}^{v-1} \frac{(v-1)!}{q!(v-1-q)!}(1-\alpha)^q \alpha^{v-1-q} \int_w \left(\frac{x_t}{\delta^{q+1}w}\right)^{-\lambda} w^{-\lambda} \mathrm{d}G_{t,v,q}(w) \\
&= \mu(1-\alpha)\delta^{\lambda} x_t^{-\lambda} \sum_{v=1}^{\infty} u_{t-\tau v} \left(\sum_{q=0}^{v-1} \frac{(v-1)!}{q!(v-1-q)!} \left((1-\alpha)\delta^{\lambda}\right)^q \alpha^{v-1-q} \int_w \mathrm{d}G_{t,v,q}(w)\right) \\
&\qquad\qquad = \mu(1-\alpha)\delta^{\lambda} x_t^{-\lambda} \sum_{v=1}^{\infty} \left(\alpha + (1-\alpha)\delta^{\lambda}\right)^{v-1} u_{t-\tau v}.
\end{aligned}
$$

The first line uses the probability ${}_{v-1}C_q(1-\alpha)^q\alpha^{v-1-q}$ of drawing q from the Binomial distribution, the second line notes that the terms in $w^{-\lambda}$ cancel out, which comes from the properties of the Pareto distribution of ε, and the third line makes use of $\int_w \mathrm{d}G_{t,v,q}(w) = 1$ and the binomial theorem to simplify the expression. It follows that aggregate listings N_t are given by

$$
N_t = (1-\alpha)(1-u_{t-\tau} + S_{t-\tau}) - \mu(1-\alpha)\delta^{\lambda} x_t^{-\lambda} \sum_{v=1}^{\infty} \left(\alpha + (1-\alpha)\delta^{\lambda}\right)^{v-1} u_{t-\tau v}. \tag{22}
$$

The key result is that the exact history of the number and timings of past idiosyncratic shocks (the distribution of q and the distribution $G_{t,v,q}(w)$ of past truncation thresholds w) can be eliminated from the formula for aggregate listings N_t. All that remains is the current moving threshold x_t and a weighted average of $u_{t-\tau v}$ over vintages $v = 1, 2, \ldots$.

4 Quantitative results

This section presents the results of simulating the model described in section 3 with aggregate shocks to housing utility and discount rates. Both of these shocks can be seen as shifts in housing demand. The aim is to study whether a model with endogenous inflows and outflows can jointly match the cyclical behaviour of sales, prices, new listings, time-to-sell, and houses for sale documented in section 2. The simulation results are obtained using a first-order perturbation around the model's equilibrium in the absence of aggregate shocks. The log-linearized equations of the model characterizing aggregate variables are derived in appendix A.3.

4.1 Calibration

Steady state In the absence of aggregate shocks, the steady state of the model is equivalent to that in Ngai and Sheedy (2020a), except for some small differences owing to the use of discrete time here. The length of a discrete time period τ is set to one week ($\tau = 1/52$) in the current paper. The other parameters are set to be the discrete-time equivalents of the continuous-time calibration of Ngai and Sheedy (2020a). This calibration strategy does not use any information derived from fluctuations in the time series of housing-market variables, only their average values. Table 4 reports the parameter values that are used.

Table 4: *Calibrated parameters*

Parameter description	Notation	Value	Continuous-time rate
Length of a discrete time period	τ	$1/52$	
Discount factor (steady state)	β	0.9989	$r = 0.057$
Probability of no idiosyncratic shock	α	0.9978	$a = 0.116$
Size of shocks	δ	0.903	
Distribution of new match quality	λ	17.6	
Probability of a viewing	μ	0.2994	$m = 18.5$
Total transaction costs	C	0.611	
Flow search costs	F	0.153	
Flow maintenance costs	D	0.275	
Share of total transaction costs directly borne by seller	κ	$1/3$	
Bargaining power of sellers	ω	$1/2$	

Notes: These parameters are taken from the calibrated continuous-time model in Ngai and Sheedy (2020a), with discrete-time equivalents $\beta = e^{-r\tau}$, $\alpha = e^{-a\tau}$, and $\mu = 1 - e^{-m\tau}$ calculated given the weekly length of a discrete time period ($\tau = 1/52$).

The sources of information used in the calibration are discussed in detail in Ngai and Sheedy (2020a). In brief, the annual discount rate is set to 5.7%, which determines $\beta = e^{-r\tau}$. Buyers and sellers are assumed to have equal bargaining power. The parameters F, D, C, and κ are calibrated to match the costs of owning a house and the costs involved in buying and selling houses relative to house prices, and how those costs are distributed across buyers and sellers. The hazard function for moving house provides information about the idiosyncratic shocks, and hence the parameters α and δ. Averages of time-to-sell and the number of viewings per sale provide information about the arrival rate of viewings and the distribution of new match quality, and hence parameters μ and λ.

Aggregate shocks The model features aggregate shocks to housing utility θ_t and the discount rate r_t. The empirical counterparts to these variables are taken to be real expenditures on furnishings and durable household equipment (as is also done by Díaz and Jerez, 2013) and the real mortgage interest rate. A formal justification is provided in Appendix A.13 of Ngai and Sheedy (2020a). Intuitively, θ_t enters households' utility multiplicatively with match quality ε, which reflects an underlying Cobb-Douglas utility function in the quantity of housing services and expenditures complementary with housing.[24] Such a Cobb-Douglas specification is commonly employed in the literature on life-cycle models of housing. For the discount rate r_t, note that in a general-equilibrium setting, market interest rates are linked to the rate at which future utility flows are discounted.

The stochastic properties of the shocks are calibrated using quarterly data on real expenditures on 'furnishings and durable household equipment' from the BEA (Table 2.4.6), and the 30-year conventional mortgage rate minus PCE inflation converted to a quarterly series. Data cover the same 1991Q1–2019Q4 period studied in section 2. The real expenditures series is converted into natural logarithms and the real interest rate series is divided by 100. A linear time trend is removed from both series to isolate the cyclical components. These cyclical components are modelled as independent AR(1) processes in equation (18). The persistence parameters ϕ_θ and ϕ_r are set to be the weekly equivalents of the first-order autocorrelation coefficients calculated from the quarterly data. This yields $\phi_\theta = 0.9873^{1/13}$ and $\phi_r = 0.8033^{1/13}$. The standard deviations σ_θ and σ_r of the innovations to the AR(1) processes in (18) are set so that θ_t and r_t have standard deviations matching those of the cyclical components of the data. This yields $\sigma_\theta = \sqrt{1-\phi_\theta^2} \times 0.0965$ and $\sigma_r = \sqrt{1-\phi_r^2} \times 0.0086$. Based on how they are measured, the two shocks are referred to respectively as 'expenditure' and 'interest rate' shocks.

4.2 Baseline results

This section compares the predictions of the calibrated model to the patterns of volatility and co-movement of housing-market variables documented in section 2, and also to the correlations between housing-market variables and the shocks themselves. Table 5 reports the model-implied standard

[24] Benmelech, Guren and Melzer (2023) provide evidence that expenditures on home-related durables and home improvement increase following home purchases. The model here does not need to take a stance on the exact cause of such a shift in complementary housing demand and housing-related expenditures: it could be due to an increase in real income or a change in preferences.

deviations and correlation coefficients among housing-market variables, as well as correlations of these with the shocks and the empirical counterparts of the latter. The predicted correlations between the shocks and housing variables match most of those observed in the data fairly closely.

Table 5: *Baseline results of the calibrated model*

Expenditure	Interest rate	Sales	Prices	New listings	Time-to-sell	Houses for sale
			Standard deviations, %			
9.7	0.86	6.9	8.1	7.0	6.5	1.8
				Correlations among housing-market variables		
	Sales	1				
	Prices	0.99	1			
	New listings	0.98	0.99	1		
	Time-to-sell	−0.97	−0.98	−0.95	1	
	Houses for sale	0.35	0.24	0.34	−0.10	1
				Correlations between housing variables and shocks		
	Expenditure (data)	0.78	0.93	0.68	−0.34	0.19
	Expenditure (model)	0.98	0.99	0.97	−0.99	0.14
	Interest rate (data)	−0.03	−0.10	0.04	−0.13	−0.21
	Interest rate (model)	−0.16	−0.12	−0.25	−0.05	−0.83

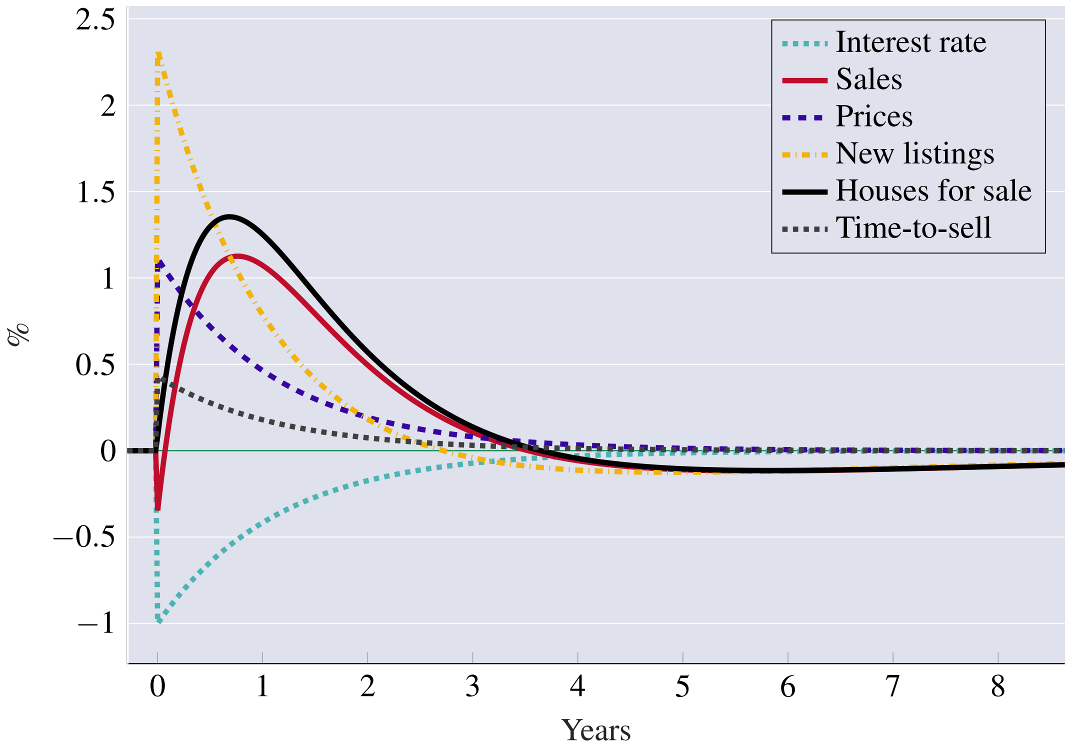
Notes: Simulated moments of the theoretical model with $\phi_\theta = 0.9873^{1/13}$, $\phi_r = 0.8033^{1/13}$, $\sigma_\theta = \sqrt{1-\phi_\theta^2} \times 0.0965$, and $\sigma_r = \sqrt{1-\phi_r^2} \times 0.0086$.

When compared to the empirical volatilities reported in Table 2, the two measured aggregate shocks produce a fair amount of model-implied volatility in all housing variables except houses for sale. Perhaps not surprisingly, a simple model with two shocks does not account for the full extent of volatility in the data.

Figure 2 shows that there are five stable correlations among housing-market variables: positive between houses for sale and time-to-sell, negative between time-to-sell and both sales and new listings, and positive between sales and both prices and new listings. The model matches four of these, the exception being the one between houses for sale and time-to-sell. The model also matches the correlations of prices with time-to-sell and houses for sale over the whole sample, but predicts mildly positive correlations of houses for sale with sales and new listings, while the empirical correlations are close to zero. But as seen in Figure 2, those correlations are not stable over time, and section 4.3 later examines whether the model can replicate this when the properties of the shocks themselves vary over time. The remainder of this section explores the economics behind how the two shocks successfully generate the stable correlations in the model.

Shocks to interest rates Figure 4 shows the impulse responses of sales, house prices, new listings, average time-to-sell, and the number of houses for sale to a negative unit shock (one percentage point) to the real interest rate r_t. The responses are given as percentage deviations from the steady-state values of variables. A fall in the real interest rate lowers the discount rate applied to future housing utility flows, increasing the total surplus from a transaction and raising the price paid. A lower interest rate increases homeowners' incentive to invest in better match quality by moving house because it raises the relative importance of future payoffs compared to current costs. Therefore, a lower interest rate has a positive effect on prices and new listings. These effects persist during the time taken for the interest rate to return to its steady-state level.

Figure 4: *Impulse responses of variables to a lower interest rate*



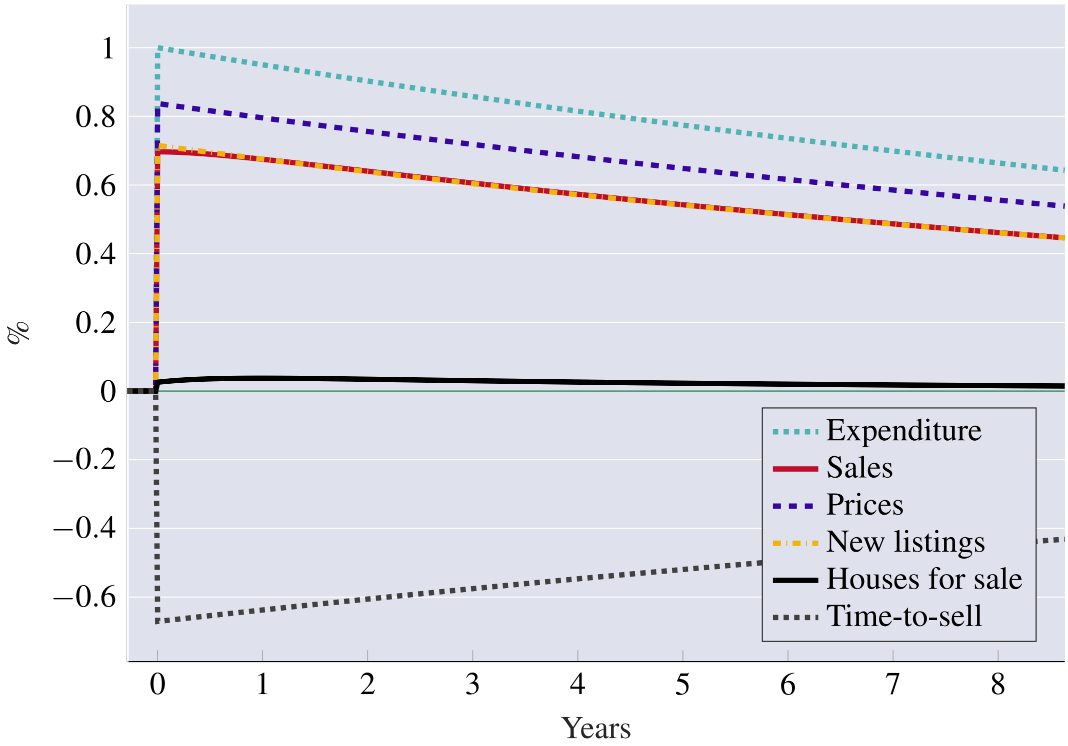
Notes: The interest-rate shock has persistence given by $\phi_r = 0.8033^{1/13}$.

While a lower interest rate stimulates listings, it also has the effect of lengthening time-to-sell. Since the lower interest rate increases the relative importance of future payoffs, it raises the returns to searching, leading to houses taking longer to sell. This initially reduces sales, and together with there being more new listings, the stock of houses for sale rises. Hence, an interest-rate shock can generate a positive co-movement between houses for sale and time-to-sell. Once the extra houses for sale are subsequently sold, sales are higher, so there is a positive co-movement between sales, prices, and new listings overall.[25]

[25] Note that the effects on sales, new listings, and houses for sale in Figure 4 become slightly negative after around four

Shocks to expenditure The impulse responses of housing-market variables to a positive unit (1%) expenditure shock are reported in Figure 5. A positive expenditure shock is associated with an increase in the flow utility from all occupied houses, raising the total surplus from a transaction, and thus increasing house prices as long as sellers have some positive bargaining power. The expenditure shock also raises the rate at which transactions occur, lowering time-to-sell. Furthermore, the shock increases homeowners' incentives to invest in better match quality, which leads to an increase in new listings.[26] These listings ultimately result in more sales.

Figure 5: *Impulse responses of variables to an expenditure shock*



Notes: The expenditure shock has persistence given by $\phi_\theta = 0.9873^{1/13}$.

As can be seen from the law of motion (21), the response of houses for sale depends on the difference between the changes in listings and sales. In the case shown here, the demand shock causes listings to rise by slightly more than sales initially, so houses for sale also increase slightly. More generally, the persistence of demand shocks affects the relative size of the listings and sales responses, and therefore the model does not make an unambiguous prediction about whether houses for sale will rise or fall. Later in section 4.3, the model is simulated using the stochastic properties

years. This overshooting arises because the interest-rate shock pulls forward some moving decisions. As homeowners' match quality is subsequently higher, new listings do not go back to zero even when interest rates return to steady state.

[26] The prediction of an increase in moving following a positive demand shock is consistent with the finding from Bachmann and Cooper (2014) that "changing residence appears to be something that happens in times of greater economic activity".

of housing-demand shocks in two sub-samples to illustrate this point.

The distribution of match quality Match quality plays a crucial role in the workings of the model. The presence of a distribution of new match quality is central to generating a positive co-movement between sales and prices. When a house is viewed by a potential buyer, new match quality is drawn from a probability distribution, and there is a transaction threshold at which the buyer is willing to trade. A shock associated with higher utility from housing raises the total surplus from a transaction and thus increases both the willingness to trade and the price paid, which gives rise to a positive correlation between sales and prices.

On the other hand, the equilibrium distribution of match quality among existing homeowners is key to explaining the positive correlation between sales and new listings. Homeowners' match quality is a persistent variable subject to occasional idiosyncratic shocks. At any point in time, there is an endogenous distribution of match quality across existing homeowners, and a moving threshold below which an owner will choose to move house, which can be seen as an investment in improving match quality. Persistent housing-demand shocks (either to expenditures or interest rates) increase the incentive to invest, leading to more listings. This explains the positive correlations between new listings and sales and prices, and why new listings have a similar volatility to sales.

In the presence of both expenditure and interest-rate shocks, the model's predictions for the correlations of time-to-sell and houses for sale with other variables are generally ambiguous, depending on the relative sizes of the two shocks. For example, time-to-sell co-moves positively with prices for an interest-rate shock, but negatively for an expenditure shock. In the data, the sign of this correlation does indeed vary over time (see Figure 2). For the whole sample, the overall correlation coefficient is negative, which the calibrated model matches. The correlations between houses for sale and other variables also vary over time. The following section shows how the model's predictions depend on the persistence of the shocks, and moreover, how changes in the observed serial correlation of shocks can explain the time-varying signs of the correlation coefficients among housing variables.

4.3 Can the model explain changes in housing-market cyclicality?

Section 2.3 showed that the correlations of houses for sale with other housing-market variables have changed sign over time, in contrast to the stability of correlations among other pairs of variables. It is of interest to explore whether the theoretical model can generate predictions consistent with both the stable and unstable correlations. This is studied by considering a shift from one regime of exogenous shocks to another with a different mixture expenditure and interest rate shocks, and differences in their degree of persistence.

Specifically, the full 1991Q1–2019Q4 sample is split at the beginning of 2007, dividing it into approximately two halves. This split captures the end of the U.S. housing boom and the beginning of the subsequent financial crisis.[27] The statistical properties of the measured expenditure and interest rate shocks are calculated separately for the 1991Q1–2006Q4 and 2007Q1–2019Q4 sub-samples.[28] The key difference found is in the persistence of the expenditure shock. The quarterly serial correlation coefficient of this shock changes from 0.9918 to 0.9765 across the two sub-samples. This is equivalent to an increase in the rate of reversion to the mean from 0.82% to 2.35% per quarter, reflecting the more transitory fluctuations of expenditure in the second sub-sample.

Table 6: *Model-predicted cyclicality in the two sub-sample periods*

	Sales		Prices		New listings		Time-to-sell		Houses for sale	
					Standard deviations, %					
	7.7	**4.7**	9.9	**5.0**	7.8	**4.8**	6.3	**6.2**	2.6	**2.3**
					Correlation coefficients					
Sales	1									
Prices	0.99	**0.97**	1							
New listings	0.98	**0.96**	0.98	**0.97**	1					
Time-to-sell	−0.95	**−0.94**	−0.98	**−0.97**	−0.93	**−0.91**	1			
Houses for sale	0.67	**−0.46**	0.56	**−0.60**	0.66	**−0.46**	−0.41	**0.73**	1	

Notes: The results for the 1991Q1–2006Q4 and 2007Q1–2019Q4 sub-samples are on the left and right of each column respectively, with the numbers for 2007–2019 given in bold. In the 1991–2006 sub-sample, the theoretical model is simulated with $\phi_\theta = 0.9918^{1/13}$, $\phi_r = 0.8466^{1/13}$, $\sigma_\theta = \sqrt{1-\phi_\theta^2} \times 0.0969$, and $\sigma_r = \sqrt{1-\phi_r^2} \times 0.0079$. In the 2007–2019 sub-sample, the theoretical model is simulated with $\phi_\theta = 0.9765^{1/13}$, $\phi_r = 0.7689^{1/13}$, $\sigma_\theta = \sqrt{1-\phi_\theta^2} \times 0.084$, and $\sigma_r = \sqrt{1-\phi_r^2} \times 0.0096$.

Using the measured properties of the shocks, the model-implied standard deviations and correlation coefficients of housing-market variables in the two sub-samples are reported in Table 6. Compared to Figure 2, the predicted correlations of sales with prices, new listings, and time-to-sell, and correlations of prices with new listings and time-to-sell remain of the same sign, as is found empirically. The model also predicts an increase in the correlation coefficient between houses for sale and time-to-sell, turning it significantly positive as in the data. More importantly, the model predicts the correlations of houses for sale with sales, prices, and new listings all turn from positive to negative, consistent with what is observed in the data.

The main reason for the switch in the signs of the correlations between houses for sale and sales, prices, and new listings is the reduction in the persistence of the expenditure shock in the

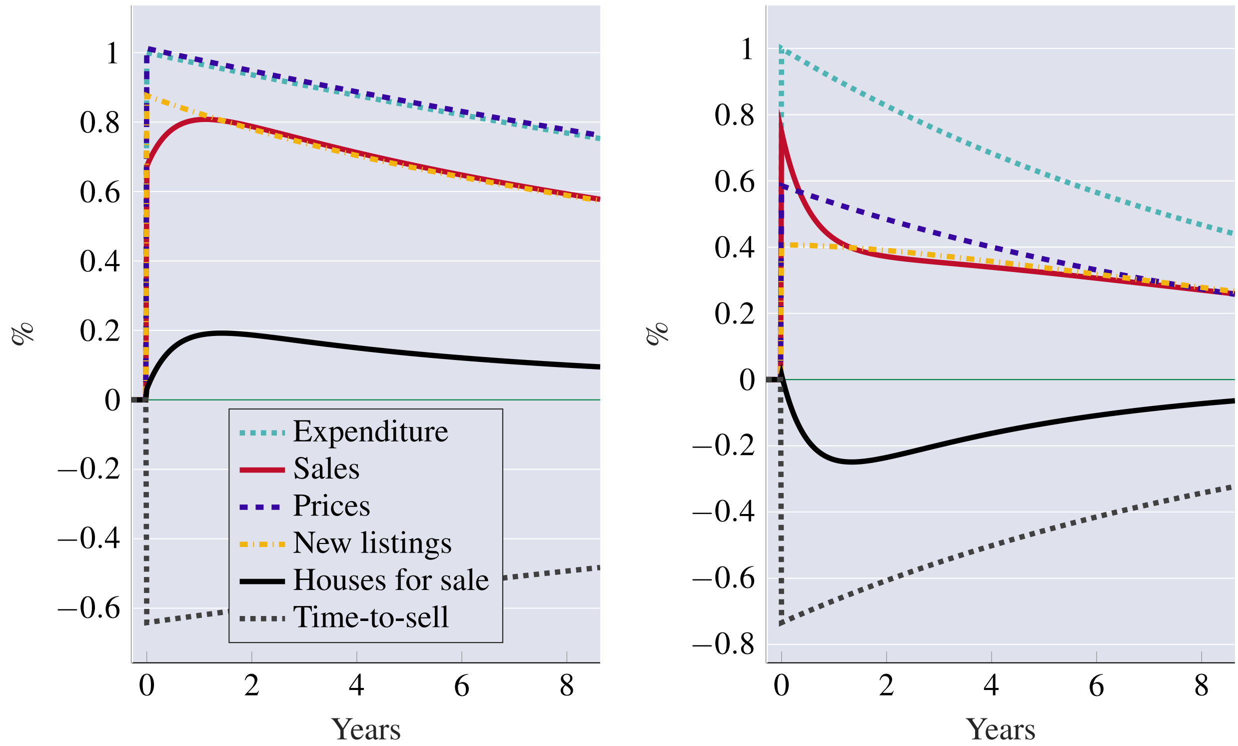
[27] Note that for the rolling correlations displayed in Figure 2, the date on the horizontal axis is the mid-point of the ten-year window, so data from 2007 begins to enter the correlation coefficients from the point labelled 2002 onwards.

[28] The data are first detrended using observations over the whole sample period.

second sub-sample.[29] The impulse responses to expenditure shocks in the two sub-sample periods are shown in Figure 6. The essential difference between the two cases is that new listings rise by more than sales with the more persistent shock, which increases the stock of houses for sale. On the contrary, the less persistent shock fails to induce enough moving to replenish the stock of houses for sale. The explanation comes from understanding moving decisions as investments in match quality: a less persistent shock has a smaller effect on the present value of future housing utility flows, so homeowners are less willing to pay the upfront costs of moving.[30]

For the correlation between houses for sale and time-to-sell, observe that these variables positively co-move with the less persistent expenditure shock, which now reinforces the positive co-movement coming from the interest-rate shock. Consequently, the predicted correlation coefficient becomes positive, in line with the data.

Figure 6: *Impulse responses to expenditure shocks in the two sub-sample periods*



Notes: The left panel is for the expenditure shock in the first sub-sample with $\phi_\theta = 0.9918^{1/13}$, and the right panel is for the less persistent expenditure shock in the second sub-sample with $\phi_\theta = 0.9765^{1/13}$.

[29] The impulse responses to interest rate shocks in the two sub-sample periods are similar to those in Figure 4 for the full sample.

[30] Recall from Figure 1 that the size of the movements in the inflow rate relative to the outflow rate changes in the second part of the sample. The model-implied impulse response of the inflow rate is essentially identical to the impulse response of new listings in Figure 6. The model-implied impulse response of the outflow rate is the negative of the impulse response of time-to-sell in that figure. As can be seen, the model generates a smaller rise in the inflow rate compared to the outflow rate in the second subsample owing to a less persistent change in expenditure.

4.4 Housing-market inflows: a discussion

The main lesson from the quantitative model is that with endogenous inflows, persistent housing-demand shocks naturally induce changes in the moving rate and housing supply (houses for sale) that are positively correlated with changes in housing demand. Correlated shocks to these variables are shown to be essential in understanding housing-market cyclicality by Díaz and Jerez (2013). As a point of comparison, appendix A.4 presents results from simulating the model with only the expenditure shock (a shock to housing utility θ_t, matching the stochastic properties of equipment expenditure), which is the same as the housing-demand shock used in their model. As seen by comparing Table A.4 to the empirical evidence presented in Table 2, the model with only an expenditure shock can generate the positive correlations of sales with prices and new listings, the positive correlations of prices with new listings and houses for sale, and the negative correlations of time-to-sell with sales, prices, and new listings.[31]

The role of endogenous inflows is illustrated more starkly by considering a special case of the model where moving is exogenous. As shown in appendix A.5, in the absence of an endogenous response of moving, the model predicts very low volatility in new listings, a perfect negative correlation between new listings and houses for sale, and more generally, the correlation coefficients between new listings and other variables always being the negative of those between houses for sale and other variables. This is because listings are proportional to the number of homeowners who receive shocks that lead them automatically to sell irrespective of market conditions. An earlier working-paper version of this paper (Ngai and Sheedy, 2020b) shows that adding an aggregate shock to the moving rate itself results in similarly negative conclusions. The main intuition stems from the fact that this version of the model features changes in the aggregate moving rate that are orthogonal to the factors that matter for transactions. This conclusion is consistent with the findings of Díaz and Jerez (2013) that in a search model with exogenous moving, three correlated shocks (housing demand, housing supply, and the moving rate) are needed to account for the cyclical behaviour of housing-market variables.

A further reason to use a housing model with endogenous inflows is the direct finding from the ins-and-outs decomposition of Table 1 that inflows account for a significant proportion of housing-market volatility. Applying the same Fujita and Ramey (2009) decomposition to simulated data generated by the calibrated model yields a contribution of 0.9 from inflows and 0.1 from outflows, closely matching the empirical decomposition.[32]

Finally, it should be noted that the model falls short of explaining a number of facts. For example, houses for sale is more volatile than the model can explain, and its correlation with time-to-sell is robustly positive, in contrast to the model. Introducing other shocks offers one possible way forward.

[31]It is also worth noting that the model with endogenous inflows can generate a positive correlation between houses for sale and prices, whereas this correlation remains negative in Díaz and Jerez (2013), even with three correlated shocks and an extended model where the timing of transactions is reversed and buyer valuations are heterogeneous.

[32]Sampling error in the estimator is minimized by simulating a 1000-year sample.

Another limitation of the model is in abstracting from fluctuations in market tightness, for example due to new construction, entry of first-time buyers, or the sequence of buying and selling.[33]

5 Conclusions

This paper has assembled a set of stylized facts about the cyclical behaviour of house prices, sales, new listings, average time-to-sell, and houses for sale in terms of volatilities and patterns of co-movement among these variables. It demonstrates that both inflows (new listings) and outflows (sales) are quantitatively important in understanding housing-market fluctuations. Many of the patterns of co-movement are found to be stable over a period of three decades, but importantly, the correlations of houses for sale with prices, sales, and new listings change sign from positive to negative during the sample period.

This paper has presented a stochastic search-and-matching model of the housing market with endogenous inflows and outflows. Simulations of the model were performed and compared to the empirical evidence on cyclical fluctuations and patterns of co-movement among housing-market variables. The model also demonstrates that the source and persistence of aggregate shocks matters for understanding the empirical evidence, particularly the time-varying correlations of houses for sale with other variables.

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A Appendices

A.1 Robustness of cyclical patterns in the data

Hodrick-Prescott filtered data (1991Q1–2019Q4) To compare the cyclical properties of the data with the findings of Díaz and Jerez (2013), the seasonally adjusted quarterly time series in natural logarithms are detrended using the Hodrick-Prescott filter (with smoothing parameter 1600). The standard deviations and correlation coefficients are shown in Table A.1.

Table A.1: *Cyclical properties of HP-filtered housing-market variables*

	Sales	Prices	New listings	Time-to-sell	Houses for sale
			Standard deviations, %		
	6.7	2.5	15.0	11.0	7.3
			Correlation coefficients		
Sales	1				
Prices	0.40	1			
New listings	0.46	0.29	1		
Time-to-sell	−0.76	−0.16	−0.36	1	
Houses for sale	−0.22	0.12	−0.12	0.80	1

Notes: Calculated from HP-filtered (smoothing parameter 1600) natural logarithms of quarterly time series from 1991Q1 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency.
Sources: FHFA and NAR.

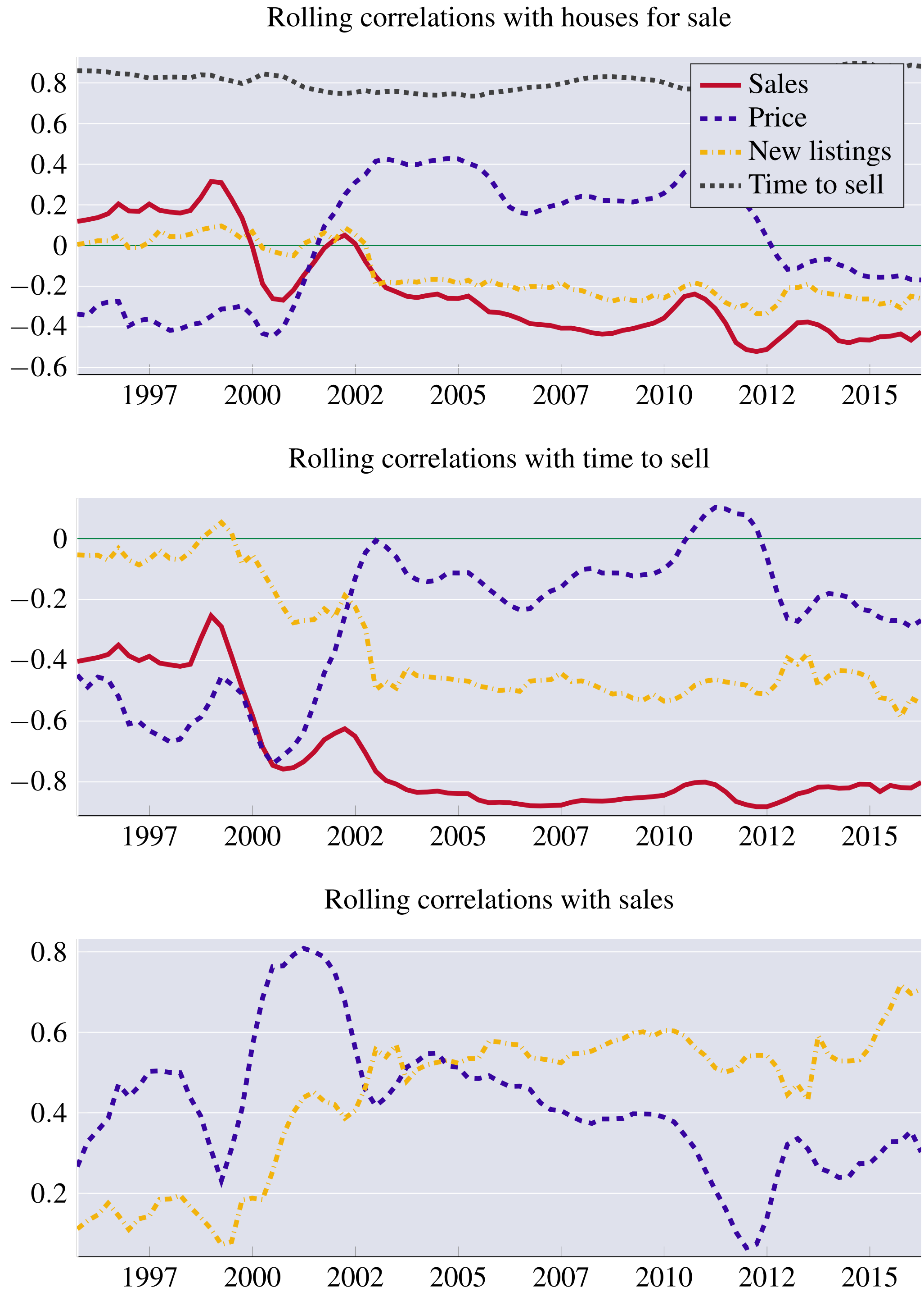
The statistics related to sales, prices, time-to-sell, and houses for sale are similar to those reported in Díaz and Jerez (2013). In addition to the differences in the measurement of time-to-sell and houses for sale discussed in section 2, note also that Table 1 of Díaz and Jerez (2013) uses different time periods for different variables, while the time series here all cover the period 1991Q1–2019Q4. For example, their measure of sales starts from 1968, but the price series starts from either 1975 or 1990.

The overall cyclical patterns are broadly consistent with those presented in Table 2, though the levels of the standard deviations are lower. To highlight a few differences in the correlation coefficients compared to Table 2, the positive correlations between house prices and sales, new listings and sales, and new listings and prices are all weaker. The negative correlation between time-to-sell and new listings is also weaker. Figure A.1 reports rolling correlations in ten-year windows for the HP-filtered data on housing-market variables. This exhibits the same patterns seen in Figure 2.

Data with no detrending (1991Q1–2019Q4) Table A.2 and Figure A.2 report the cyclical properties of the data without any detrending. The standard deviations and correlation coefficients are similar to Table 2 and the patterns of rolling correlations are the same as those in Figure 2.

Redfin data with linear detrending (2012Q2–2019Q4) Table A.3 reports cyclical properties of the Redfin data with linear detrending in comparison with the NAR and FHFA data. The levels of standard deviations and correlation coefficients are the same as those calculated using NAR and FHFA data for the same period. They are both similar to those patterns seen in the raw data from Table 3, except for the mild positive correlation between houses for sale and new listings in the linearly detrended Redfin data.

Figure A.1: *Rolling correlations of housing-market variables using HP-filtered data*



Notes: Correlation coefficients in ten-year windows are calculated using HP-filtered (smoothing parameter 1600) and seasonally adjusted quarterly time series in logarithms. The date on the horizontal axis is the mid-point of each ten-year window.
Sources: FHFA and NAR.

Table A.2: *Cyclical properties of housing-market variables without detrending*

	Sales	Prices	New listings	Time-to-sell	Houses for sale
			Standard deviations, %		
	18.7	16.3	25.4	28.6	20.5
			Correlation coefficients		
Sales	1				
Prices	0.72	1			
New listings	0.84	0.59	1		
Time-to-sell	−0.70	−0.31	−0.59	1	
Houses for sale	−0.06	0.22	−0.06	0.76	1

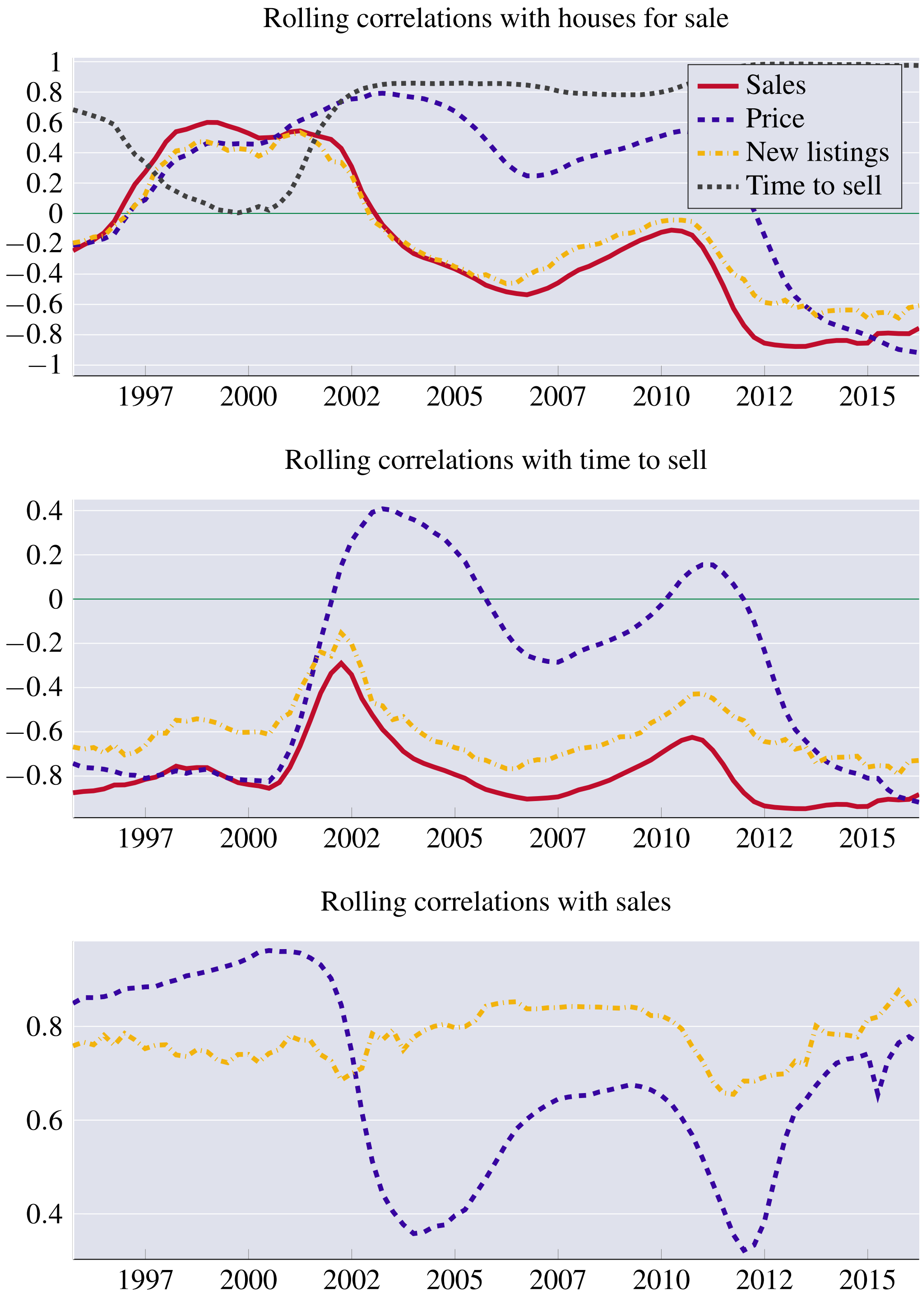
Notes: Calculated from natural logarithms of quarterly time series from 1991Q1 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency.
Sources: FHFA and NAR.

Table A.3: *Comparison with linearly detrended Redfin data, 2012Q2–2019Q4*

	Sales		Prices		New listings		Time-to-sell		Houses for sale	
					Standard deviations, %					
	4.1	3.9	**2.2**	8.0	**4.0**	8.6	**6.4**	6.4	**4.6**	4.7
					Correlation coefficients					
Sales	1									
Prices	**0.68**	0.49	1							
New listings	**0.73**	0.73	**0.68**	0.39	1					
Time-to-sell	**−0.67**	−0.67	**−0.65**	−0.42	**−0.65**	−0.51	1			
Houses for sale	**−0.14**	−0.08	**−0.10**	−0.16	**0.25**	−0.08	**0.44**	0.78	1	

Notes: Calculated from linearly detrended natural logarithms of quarterly time series from 2012Q2 to 2019Q4. The original monthly data are seasonally adjusted by removing multiplicative month effects and then converted to a quarterly frequency. Redfin statistics are in bold, adjacent to the equivalent NAR and FHFA statistics.
Sources: Redfin, NAR, and FHFA.

Figure A.2: *Rolling correlations of housing-market variables without detrending*



Notes: Correlation coefficients in ten-year windows are calculated using the seasonally adjusted quarterly time series in logarithms. The date on the horizontal axis is the mid-point of each ten-year window.
Sources: FHFA and NAR.

A.2 Characterizing aggregate dynamics with a finite number of variables

This section derives a set of equations in a finite number of variables that characterizes the aggregate dynamics of the housing market. Under the assumptions made in section 3.3, the idiosyncratic shock is sufficiently large (δ is sufficiently far below 1) so that $\delta x_t < x_{t'}$ and $\delta y_t < x_{t'}$ for all t and t'. Consequently, there exists a threshold ξ, which lies above y_t and x_t for all t, such that $\delta\varepsilon < x_{t+\tau}$ for any $\varepsilon \leq \xi$. Since $H_{t+\tau}(\varepsilon)$ is increasing in ε, it follows using (17) that $H_{t+\tau}(\delta\varepsilon) - \tau D < J_{t+\tau}$ for all $\varepsilon \leq \xi$ and thus $\max\{H_{t+\tau}(\delta\varepsilon) - \tau D, J_{t+\tau}\} = J_{t+\tau}$. The Bellman equation (16) for $\varepsilon \leq \xi$ becomes

$$H_t(\varepsilon) = \tau\varepsilon\theta_t + \alpha\beta_t\mathbb{E}_t[H_{t+\tau}(\varepsilon) - \tau D] + (1-\alpha)\beta_t\mathbb{E}_t J_{t+\tau}. \tag{A.1}$$

Differentiating with respect to ε gives $H_t'(\varepsilon) = \tau\theta_t + \alpha\beta_t\mathbb{E}_t H_{t+\tau}'(\varepsilon)$, which can be iterated forwards to deduce:

$$H_t'(\varepsilon) = \Theta_t, \quad \text{where } \Theta_t = \tau\mathbb{E}_t\left[\theta_t + \alpha\beta_t\theta_{t+1} + \alpha^2\beta_t\beta_{t+1}\theta_{t+2} + \cdots\right].$$

The new variable Θ_t depends only on the exogenous variables θ_t and β_t and satisfies the expectational difference equation

$$\Theta_t = \tau\theta_t + \alpha\beta_t\mathbb{E}_t\Theta_{t+\tau}. \tag{A.2}$$

Since $H_t'(\varepsilon)$ is independent of ε for $\varepsilon \leq \xi$, it follows that $H_t(\varepsilon)$ is linear for $\varepsilon \in [0,\xi]$, that is:

$$H_t(\varepsilon) = \Lambda_t + \Theta_t\varepsilon, \tag{A.3}$$

for some variable Λ_t independent of ε. Substituting back into (A.1) implies $\Lambda_t + \Theta_t\varepsilon = \tau\varepsilon\theta_t + \alpha\beta_t\mathbb{E}_t[\Lambda_{t+\tau} + \Theta_{t+\tau}\varepsilon - \tau D] + (1-\alpha)\beta_t\mathbb{E}_t J_{t+\tau}$, and then replacing Θ_t using (A.2) yields

$$\Lambda_t = \alpha\beta_t\mathbb{E}_t\Lambda_{t+\tau} - \alpha\beta_t\tau D + (1-\alpha)\beta_t\mathbb{E}_t J_{t+\tau}. \tag{A.4}$$

Since $x_t < \xi$, equation (A.3) can be evaluated at $\varepsilon = x_t$, hence $H_t(x_t) = \Lambda_t + \Theta_t x_t$. Using equation (17) that defines the moving threshold x_t, it follows that $\Lambda_t = J_t + \tau D - \Theta_t x_t$. Substituting into (A.4) implies

$$J_t + \tau D - \Theta_t x_t = \beta_t\mathbb{E}_t J_{t+\tau} - \alpha\beta_t\mathbb{E}_t[\Theta_{t+\tau}x_{t+\tau}].$$

Combining this with the Bellman equation (10) to eliminate the joint value function J_t:

$$x_t\Theta_t + \tau F = \alpha\beta_t\mathbb{E}_t[x_{t+\tau}\Theta_{t+\tau}] + \mu\Sigma_t. \tag{A.5}$$

This gives an expectational difference equation for the moving threshold x_t in terms of the surplus Σ_t and the exogenous variable Θ_t.

Using equations (7) and (17) defining the transaction and moving thresholds y_t and x_t, it follows that $H_t(y_t) - H_t(x_t) = \beta_t\mathbb{E}_t J_{t+\tau} + C - J_t - \tau D$. Substituting the Bellman equation (10) implies $H_t(y_t) - H_t(x_t) = \tau F + C - \mu\Sigma_t$. Furthermore, since $y_t < \xi$ and $x_t < \xi$, equation (A.3) yields $H_t(y_t) - H_t(x_t) = \Theta_t(y_t - x_t)$. Putting these equations together leads to the following relationship between the thresholds y_t and x_t:

$$\Theta_t(y_t - x_t) = C + \tau F - \mu\Sigma_t.$$

The term in the surplus Σ_t can be eliminated using (A.5) to leave a simpler relationship between y_t and $x_{t+\tau}$:

$$\Theta_t y_t = C + \alpha\beta_t\mathbb{E}_t[\Theta_{t+\tau}x_{t+\tau}], \tag{A.6}$$

and this equation contains only the thresholds and the exogenous variable Θ_t.

Now consider an arbitrary variable z_t that always satisfies $z_t \leq \xi$. Given z_t, define $\Psi_t(z_t)$ as follows:

$$\Psi_t(z_t) = \int_{\varepsilon=z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \left(H_t(\varepsilon) - H_t(z_t)\right) \mathrm{d}\varepsilon. \tag{A.7}$$

Since $z_t \leq \xi$, equation (A.1) applies and hence $H_t(z_t) = \tau z_t \theta_t + \alpha\beta_t \mathbb{E}_t[H_{t+\tau}(z_t) - \tau D] + (1-\alpha)\beta_t \mathbb{E}_t J_{t+\tau}$. Subtracting this from (16) and using (17) yields

$$\begin{aligned} &H_t(\varepsilon) - H_t(z_t) = \tau\theta_t(\varepsilon - z_t) + (1-\alpha)\beta_t \max\left\{H_{t+\tau}(\delta\varepsilon) - H_{t+\tau}(x_{t+\tau}), 0\right\} \\ &+ \alpha\beta_t \mathbb{E}_t[H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_t)] = \tau\theta_t(\varepsilon - z_t) + \alpha\beta_t \mathbb{E}_t[H_{t+\tau}(z_{t+\tau}) - H_{t+\tau}(z_t)] \\ &\qquad + \alpha\beta_t \mathbb{E}_t[H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau})] + (1-\alpha)\beta_t \max\left\{H_{t+\tau}(\delta\varepsilon) - H_{t+\tau}(x_{t+\tau}), 0\right\}, \end{aligned} \tag{A.8}$$

noting that $\max\{H_{t+\tau}(\delta\varepsilon) - \tau D, J_{t+\tau}\} = J_{t+\tau} + \max\{H_{t+\tau}(\delta\varepsilon) - \tau D - J_{t+\tau}, 0\} = J_{t+\tau} + \max\{H_{t+\tau}(\delta\varepsilon) - H_{t+\tau}(x_{t+\tau}), 0\}$ because $H_{t+\tau}(x_{t+\tau}) = \tau D + J_{t+\tau}$. Considering the following integral and making the change of variable $\varepsilon' = \delta\varepsilon$, and noting $\delta z_t < x_{t+\tau}$ because $z_t < \xi$:

$$\begin{aligned} &\int_{\varepsilon=z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \max\left\{H_{t+\tau}(\delta\varepsilon) - H_{t+\tau}(x_{t+\tau}), 0\right\} \mathrm{d}\varepsilon \\ &= \delta^{\lambda} \int_{\varepsilon'=\delta z_t}^{\infty} \lambda (\varepsilon')^{-(\lambda+1)} \max\left\{H_{t+\tau}(\varepsilon') - H_{t+\tau}(x_{t+\tau}), 0\right\} \mathrm{d}\varepsilon' = \delta^{\lambda} \int_{\varepsilon'=\delta z_t}^{x_{t+\tau}} \lambda (\varepsilon')^{-(\lambda+1)} 0 \mathrm{d}\varepsilon' \\ &\qquad + \delta^{\lambda} \int_{\varepsilon'=x_{t+\tau}}^{\infty} \lambda (\varepsilon')^{-(\lambda+1)} \left(H_{t+\tau}(\varepsilon') - H_{t+\tau}(x_{t+\tau})\right) \mathrm{d}\varepsilon' = \delta^{\lambda} \Psi_{t+\tau}(x_{t+\tau}), \end{aligned} \tag{A.9}$$

which uses $H_{t+\tau}(\varepsilon') < H_{t+\tau}(x_{t+\tau})$ for $\varepsilon' < x_{t+\tau}$, and the definition of $\Psi_t(z_t)$ from (A.7). Note also:

$$\int_{\varepsilon=z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \mathrm{d}\varepsilon = z_t^{-\lambda}, \quad \text{and} \quad \int_{\varepsilon=z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} (\varepsilon - z_t) \mathrm{d}\varepsilon = \frac{z_t^{1-\lambda}}{\lambda - 1}. \tag{A.10}$$

Since $z_t \leq \xi$ and $z_{t+\tau} \leq \xi$, it follows from (A.3) that $H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau}) = \Theta_{t+\tau}(\varepsilon - z_{t+\tau})$ for all ε between z_t and $z_{t+\tau}$. Breaking up the range of integration in the following equations and using the definition of $\Psi_t(z_t)$ from (A.7) leads to

$$\begin{aligned} &\int_{\varepsilon=z_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \left(H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau})\right) \mathrm{d}\varepsilon = \int_{\varepsilon=z_t}^{z_{t+\tau}} \lambda \varepsilon^{-(\lambda+1)} \left(H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau})\right) \mathrm{d}\varepsilon \\ &+ \int_{\varepsilon=z_{t+\tau}}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \left(H_{t+\tau}(\varepsilon) - H_{t+\tau}(z_{t+\tau})\right) \mathrm{d}\varepsilon = \Psi_{t+\tau}(z_{t+\tau}) + \Theta_{t+\tau} \int_{\varepsilon=z_t}^{z_{t+\tau}} \lambda \varepsilon^{-(\lambda+1)} (\varepsilon - z_{t+\tau}) \mathrm{d}\varepsilon \\ &\qquad = \Psi_{t+\tau}(z_{t+\tau}) + \Theta_{t+\tau} \left(\frac{\lambda}{\lambda-1}\left(z_t^{1-\lambda} - z_{t+\tau}^{1-\lambda}\right) + z_{t+\tau}\left(z_{t+\tau}^{-\lambda} - z_t^{-\lambda}\right)\right). \end{aligned} \tag{A.11}$$

Note also that $H_{t+\tau}(z_{t+\tau}) - H_{t+\tau}(z_t) = \Theta_{t+\tau}(z_{t+\tau} - z_t)$ using (A.3). By combining equations (A.7), (A.8), (A.9), (A.10), and (A.11), the following result holds for all $z_t \leq \xi$:

$$\begin{aligned} &\Psi_t(z_t) = \tau\theta_t \frac{z_t^{1-\lambda}}{\lambda-1} + \alpha\beta_t \mathbb{E}_t \Psi_{t+\tau}(z_{t+\tau}) + (1-\alpha)\delta^{\lambda}\beta_t \mathbb{E}_t \Psi_{t+\tau}(x_{t+\tau}) \\ &+ \alpha\beta_t \mathbb{E}_t\left[\left((z_{t+\tau} - z_t)z_t^{-\lambda} + \frac{\lambda}{\lambda-1}\left(z_t^{1-\lambda} - z_{t+\tau}^{1-\lambda}\right) + z_{t+\tau}\left(z_{t+\tau}^{-\lambda} - z_t^{-\lambda}\right)\right)\Theta_{t+\tau}\right] \\ &\quad = \tau\theta_t \frac{z_t^{1-\lambda}}{\lambda-1} + \alpha\beta_t \mathbb{E}_t \Psi_{t+\tau}(z_{t+\tau}) + (1-\alpha)\delta^{\lambda}\beta_t \mathbb{E}_t \Psi_{t+\tau}(x_{t+\tau}) + \alpha\beta_t \mathbb{E}_t\left[\left(\frac{z_t^{1-\lambda} - z_{t+\tau}^{1-\lambda}}{\lambda-1}\right)\Theta_{t+\tau}\right]. \end{aligned}$$

Grouping the terms in $z_t^{1-\lambda}$ and using equation (A.2) for Θ_t implies

$$\Psi_t(z_t) - \frac{\Theta_t z_t^{1-\lambda}}{\lambda - 1} = \alpha\beta_t \mathbb{E}_t \left[\Psi_{t+\tau}(z_{t+\tau}) - \frac{\Theta_{t+\tau} z_{t+\tau}^{1-\lambda}}{\lambda - 1}\right] + (1-\alpha)\delta^\lambda \beta_t \mathbb{E}_t \Psi_{t+\tau}(x_{t+\tau}), \tag{A.12}$$

which holds for any $z_t \leq \xi$ and for all t.

Making the following definition of a variable χ_t, and noting the relationship between the unconditional surplus Σ_t given in (9) and $\Psi_t(z_t)$ from (A.7):

$$\chi_t = \int_{\varepsilon = x_t}^{\infty} \lambda \varepsilon^{-(\lambda+1)} \left(H_t(\varepsilon) - H_t(x_t)\right) \mathrm{d}\varepsilon = \Psi_t(x_t), \quad \text{and } \Sigma_t = \Psi_t(y_t). \tag{A.13}$$

With $x_t < \xi$ and $y_t < \xi$, equation (A.12) can be evaluated at $z_t = x_t$ and $z_t = y_t$ and stated in terms of the variables from (A.13):

$$\chi_t - \frac{\Theta_t x_t^{1-\lambda}}{\lambda - 1} = \alpha\beta_t \mathbb{E}_t \left[\chi_{t+\tau} - \frac{\Theta_{t+\tau} x_{t+\tau}^{1-\lambda}}{\lambda - 1}\right] + (1-\alpha)\delta^\lambda \beta_t \mathbb{E}_t \chi_{t+\tau}, \text{ and} \tag{A.14}$$

$$\Sigma_t - \frac{\Theta_t y_t^{1-\lambda}}{\lambda - 1} = \alpha\beta_t \mathbb{E}_t \left[\Sigma_{t+\tau} - \frac{\Theta_{t+\tau} y_{t+\tau}^{1-\lambda}}{\lambda - 1}\right] + (1-\alpha)\delta^\lambda \beta_t \mathbb{E}_t \chi_{t+\tau}, \tag{A.15}$$

which yields a pair of equations for χ_t and Σ_t in terms of the thresholds x_t and y_t and the exogenous variable Θ_t. The solution for x_t, y_t, χ_t, and Σ_t is determined by (A.5), (A.6), (A.14), and (A.15), with the exogenous variable Θ_t obtained from (A.2).

Given y_t, the value of π_t comes from equation (8), and s_t and T_t from (19). The laws of motion involve equations (20) and (21) for S_t and u_t. Considering equation (22) for new listings N_t, make the following definitions of a new variable Υ_t and a constant ψ:

$$\Upsilon_t = (1-\psi)\sum_{\ell=0}^{\infty} \psi^\ell u_{t-\tau\ell}, \quad \text{where } \psi = \alpha + (1-\alpha)\delta^\lambda. \tag{A.16}$$

Using this new variable, equation (22) for listings becomes

$$N_t = (1-\alpha)(1 - u_{t-\tau} + S_{t-\tau}) - \frac{\mu(1-\alpha)\delta^\lambda}{(1-\psi)} x_t^{-\lambda} \Upsilon_{t-\tau}. \tag{A.17}$$

Equation (A.16) defining Υ_t can be stated equivalently as follows:

$$\Upsilon_t = \psi \Upsilon_{t-\tau} + (1-\psi) u_t. \tag{A.18}$$

Given x_t and y_t, the solution for π_t, s_t, T_t, S_t, N_t, u_t, and the auxiliary variable Υ_t is determined by (8), (19), (20), (21), (A.17), and (A.18).

Using the price equation (14), the equations for π_t and Σ_t in (8) and (9), and the Bellman equation (12b) for V_t, the average price paid is given by:

$$P_t = \kappa C + \beta_t \mathbb{E}_t V_{t+\tau} + \omega \frac{\Sigma_t}{\pi_t} = \kappa C + \tau D + V_t + \omega(1 - \mu\pi_t)\frac{\Sigma_t}{\pi_t}.$$

By subtracting $\beta_t \mathbb{E}_t P_{t+\tau}$ from P_t, it follows that:

$$P_t - \beta_t \mathbb{E}_t P_{t+\tau} = (1-\beta_t)(\kappa C + \tau D) + V_t - \beta_t \mathbb{E}_t V_{t+\tau} + \omega\left((1-\mu\pi_t)\frac{\Sigma_t}{\pi_t} - \beta_t \mathbb{E}_t\left[(1-\mu\pi_{t+\tau})\frac{\Sigma_{t+\tau}}{\pi_{t+\tau}}\right]\right),$$

and using the Bellman equation (12b) again to eliminate the value function V_t leads to:

$$P_t = \beta_t \mathbb{E}_t [P_{t+\tau} - \tau D] + (1-\beta_t)\kappa C + \omega \frac{\Sigma_t}{\pi_t} - \omega \beta_t \mathbb{E}_t \left[(1-\mu\pi_{t+\tau}) \frac{\Sigma_{t+\tau}}{\pi_{t+\tau}} \right]. \tag{A.19}$$

A.3 A log-linear approximation of the model

Deterministic steady state The deterministic steady state of the model is defined by the absence of aggregate shocks, though individual households still face uncertainty about draws of match quality and the occurrence of idiosyncratic shocks. With $\sigma_\theta = 0$ and $\sigma_r = 0$ in (18), the innovations $\eta_{\theta,t}$ and $\eta_{r,t}$ are always zero, and so $\theta_t = 1$ and $r_t = r$ for all t, the latter implying $\beta_t = \beta$. Using (A.2), this leads to

$$\Theta = \frac{\tau}{1-\alpha\beta}, \tag{A.20}$$

where a variable without a time subscript denotes the steady-state value of that variable. Equation (A.5) implies the steady-state moving threshold x and surplus Σ are related as follows:

$$x + F = \frac{\mu}{\tau}\Sigma. \tag{A.21}$$

The steady-state thresholds y and x are linked in accordance with equation (A.6):

$$y = \alpha\beta x + \left(\frac{1-\alpha\beta}{\tau}\right) C. \tag{A.22}$$

The steady-state value of χ can be deduced from equation (A.14):

$$\chi = \frac{x^{1-\lambda}}{(\lambda-1)}\left(\frac{\tau}{1-\psi\beta}\right), \tag{A.23}$$

where $\psi = \alpha + (1-\alpha)\delta^\lambda$ is as defined in (A.16). A relationship between Σ and χ can be derived using equations (A.14) and (A.15):

$$\Sigma - \frac{y^{1-\lambda}}{(\lambda-1)}\left(\frac{\tau}{1-\alpha\beta}\right) = \chi - \frac{x^{1-\lambda}}{(\lambda-1)}\left(\frac{\tau}{1-\alpha\beta}\right) = \frac{x^{1-\lambda}}{(\lambda-1)}\left(\frac{\beta(1-\alpha)\delta^\lambda}{1-\psi\beta}\right)\left(\frac{\tau}{1-\alpha\beta}\right),$$

where the second equality follows by substituting from (A.23) and noting $\psi - \alpha = (1-\alpha)\delta^\lambda$. The steady-state value Σ follows immediately from this:

$$\Sigma = \frac{1}{(\lambda-1)}\left(\frac{\tau}{1-\alpha\beta}\right)\left(y^{1-\lambda} + \beta\delta^\lambda\left(\frac{1-\alpha}{1-\psi\beta}\right)x^{1-\lambda}\right). \tag{A.24}$$

Eliminating Σ from equations (A.21) and (A.24) implies another equation linking the steady-state thresholds x and y:

$$x + F = \frac{1}{(\lambda-1)}\left(\frac{\mu}{\tau}\right)\left(\frac{\tau}{1-\alpha\beta}\right)\left(y^{1-\lambda} + \beta\delta^\lambda\left(\frac{1-\alpha}{1-\psi\beta}\right)x^{1-\lambda}\right). \tag{A.25}$$

The steady-state thresholds x and y are the solution of the simultaneous equations (A.22) and (A.25). Equation (A.22) implies a positive relationship between x and y, while equation (A.25) implies a negative relationship between x and y. If a solution exists, it must then be unique. Since (A.22) implies x is positive when $y = 0$, and because (A.25) implies $y \to 0$ as $x \to \infty$, while x tends to a positive number when $y \to \infty$, it follows that a unique solution $x > 0$ and $y > 0$ exists. However, the equations are only meaningful if $y > 1$ and

$\delta y < x$. The solution features $y > 1$ if and only if

$$\left(\frac{1-\left(\frac{1-\alpha\beta}{\tau}\right)C}{\alpha\beta}\right)+F<\frac{1}{(\lambda-1)}\left(\frac{\mu}{\tau}\right)\left(\frac{\tau}{1-\alpha\beta}\right)\left(1+\beta\delta^{\lambda}\left(\frac{1-\alpha}{1-\psi\beta}\right)\left(\frac{1-\left(\frac{1-\alpha\beta}{\tau}\right)C}{\alpha\beta}\right)^{1-\lambda}\right),$$

and it can also be verified whether δ is sufficiently far below 1 so that $\delta y < x$.

The steady-state acceptance probability is $\pi = y^{-\lambda}$ from (8), the steady-state selling probability $s = \mu\pi$ and time-to-sell $T = (1/\pi)(\tau/\mu)$ from (19). Equations (20) and (21) imply $S = su$ and $N = S$, hence $S = N = \mu y^{-\lambda}u$. Noting that $\Upsilon = u$ from (A.18), equation (A.17) in steady state implies

$$N=(1-\alpha)(1-u+\mu y^{-\lambda}u)-\frac{\mu(1-\alpha)\delta^{\lambda}}{(1-\psi)}x^{-\lambda}u.$$

Combined with $N = \mu y^{-\lambda}u$, this can solved for the steady-state u:

$$u=\frac{(1-\alpha)}{(1-\alpha)+\mu\left(\alpha y^{-\lambda}+\delta^{\lambda}x^{-\lambda}\frac{(1-\alpha)}{(1-\psi)}\right)}=\frac{1}{1+\mu\left(\frac{\alpha}{1-\alpha}y^{-\lambda}+\frac{\delta^{\lambda}}{1-\psi}x^{-\lambda}\right)}. \tag{A.26}$$

The steady state implied by the price equation (A.19) is:

$$P=\kappa C-\beta\left(\frac{\tau}{1-\beta}\right)D+\omega\left(\frac{1-\beta(1-\mu\pi)}{1-\beta}\right)\left(\frac{\tau}{\mu}\right)\left(\frac{x+F}{\pi}\right), \tag{A.27}$$

which uses (A.21) to substitute for Σ.

Log linearizations Log deviations of variables from their deterministic steady-state values are denoted using sans serif letters, for example, $\mathsf{x}_t = \log x_t - \log x$. The log linearization of equation (A.2) for Θ_t is

$$\mathsf{\Theta}_t=(1-\alpha\beta)\mathsf{\theta}_t+\alpha\beta\mathsf{\beta}_t+\alpha\beta\mathbb{E}_t\mathsf{\Theta}_{t+\tau},$$

which uses the steady-state values $\theta = 1$ and Θ from (A.20). The discount factor is $\beta_t = e^{-\tau r_t}$ in terms of the discount rate r_t, and $\beta = e^{-\tau r}$ is its steady-state value. It follows that $\mathsf{\beta}_t = \log\beta_t - \log\beta = -\tau(r_t - r) = -\tau\mathsf{r}_t$, where $\mathsf{r}_t = r_t - r$ is the deviation of the discount rate from its steady-state level. The log-linearized equation for Θ_t can then be written as

$$\mathsf{\Theta}_t=(1-\alpha\beta)\mathsf{\theta}_t-\alpha\beta\tau\mathsf{r}_t+\alpha\beta\mathbb{E}_t\mathsf{\Theta}_{t+\tau}. \tag{A.28}$$

Noting (A.20) and (A.21), the log linearization of the moving-threshold equation (A.5) is

$$\mathsf{x}_t=\alpha\beta\mathbb{E}_t\mathsf{x}_{t+\tau}+(1-\alpha\beta)\frac{(x+F)}{x}\mathsf{\Sigma}_t-(1-\alpha\beta)\mathsf{\theta}_t. \tag{A.29}$$

The transaction threshold equation (A.6) can be log linearized as follows:

$$\mathsf{y}_t=\frac{x}{y}\alpha\beta\left(\mathbb{E}_t\mathsf{\Theta}_{t+\tau}+\mathbb{E}_t\mathsf{x}_{t+\tau}-\tau\mathsf{r}_t\right)-\mathsf{\Theta}_t, \tag{A.30}$$

and this can be used to deduce that

$$
\begin{aligned}
&\mathsf{y}_t - \alpha\beta\mathbb{E}_t\mathsf{y}_{t+\tau} = \frac{x}{y}\alpha\beta\left(\mathbb{E}_t\left[\Theta_{t+\tau} - \alpha\beta\mathbb{E}_{t+\tau}\Theta_{t+2\tau}\right] + \mathbb{E}_t\left[\mathsf{x}_{t+\tau} - \alpha\beta\mathbb{E}_{t+\tau}\mathsf{x}_{t+2\tau}\right] - \tau(\mathsf{r}_t - \alpha\beta\mathbb{E}_t\mathsf{r}_{t+\tau})\right) \\
&-(\Theta_t - \alpha\beta\mathbb{E}_t\Theta_{t+\tau}) = \frac{x}{y}\alpha\beta\mathbb{E}_t\left[((1-\alpha\beta)\theta_{t+\tau} - \alpha\beta\tau\mathsf{r}_{t+\tau}) + (1-\alpha\beta)\left(\frac{(x+F)}{x}\Sigma_{t+\tau} - \theta_{t+\tau}\right)\right] \\
&+\frac{x}{y}\alpha\beta\tau(\mathsf{r}_t - \alpha\beta\mathbb{E}_t\mathsf{r}_{t+\tau}) - ((1-\alpha\beta)\theta_t - \alpha\beta\tau\mathsf{r}_t) \\
&\qquad = \frac{(x+F)}{y}(1-\alpha\beta)\alpha\beta\mathbb{E}_t\Sigma_{t+\tau} - (1-\alpha\beta)\theta_t + \frac{(y-x)}{y}\alpha\beta\tau\mathsf{r}_t, \quad \text{(A.31)}
\end{aligned}
$$

where the subsequent expressions follow from substituting (A.28) and (A.29).

For equation (A.14) for χ_t, by using (A.20) and (A.23), the log linearization is

$$
\begin{aligned}
\chi_t = \left(\alpha + (1-\alpha)\delta^\lambda\right)\beta\mathbb{E}_t\chi_{t+\tau} + \left(\frac{1-\psi\beta}{1-\alpha\beta}\right)((\Theta_t - \alpha\beta\mathbb{E}_t\Theta_{t+\tau}) + (1-\lambda)(\mathsf{x}_t - \alpha\beta\mathbb{E}_t\mathsf{x}_{t+\tau})) \\
-\left(\left(\alpha + (1-\alpha)\delta^\lambda\right) - \alpha\left(\frac{1-\psi\beta}{1-\alpha\beta}\right)\right)\beta\tau\mathsf{r}_t,
\end{aligned}
$$

and with the definition of $\psi = \alpha + (1-\alpha)\delta^\lambda$ from (A.16):

$$
\chi_t = \psi\beta\mathbb{E}_t\chi_{t+\tau} + \frac{(1-\psi\beta)}{(1-\alpha\beta)}((\Theta_t - \alpha\beta\mathbb{E}_t\Theta_{t+\tau}) + (1-\lambda)(\mathsf{x}_t - \alpha\beta\mathbb{E}_t\mathsf{x}_{t+\tau})) - \frac{(1-\alpha)\delta^\lambda}{(1-\alpha\beta)}\beta\tau\mathsf{r}_t.
$$

Substituting from (A.28) and (A.29):

$$
\chi_t = \psi\beta\mathbb{E}_t\chi_{t+\tau} + (1-\psi\beta)\left(\theta_t - \frac{\alpha}{1-\alpha\beta}\beta\tau\mathsf{r}_t + (1-\lambda)\left(\frac{(x+F)}{x}\Sigma_t - \theta_t\right)\right) - \frac{(1-\alpha)\delta^\lambda}{(1-\alpha\beta)}\beta\tau\mathsf{r}_t,
$$

and by collecting terms and simplifying:

$$
\chi_t = \psi\beta\mathbb{E}_t\chi_{t+\tau} + (1-\lambda)\frac{(x+F)}{x}(1-\psi\beta)\Sigma_t + \lambda(1-\psi\beta)\theta_t - \psi\beta\tau\mathsf{r}_t, \quad \text{(A.32)}
$$

which again uses the definition of $\psi = \alpha + (1-\alpha)\delta^\lambda$.

Taking equation (A.15) for Σ_t and log linearizing, making use of the steady-state equations (A.21), (A.23), and (A.24):

$$
\begin{aligned}
\Sigma_t = \alpha\beta\mathbb{E}_t\Sigma_{t+\tau} + \frac{\mu}{(\lambda-1)}\frac{x^{1-\lambda}}{(x+F)}\frac{(1-\alpha)\delta^\lambda\beta}{(1-\psi\beta)}\mathbb{E}_t\chi_{t+\tau} \\
+\frac{1}{(\lambda-1)}\frac{y^{1-\lambda}}{(x+F)}\frac{\mu}{(1-\alpha\beta)}((\Theta_t - \alpha\beta\mathbb{E}_t\Theta_{t+\tau}) + (1-\lambda)(\mathsf{y}_t - \alpha\beta\mathbb{E}_t\mathsf{y}_{t+\tau})) \\
-\left(\alpha - \frac{1}{(\lambda-1)}\left(\frac{y^{1-\lambda}}{(x+F)}\frac{\mu\alpha}{(1-\alpha\beta)} - \frac{x^{1-\lambda}}{(x+F)}\frac{\mu(1-\alpha)\delta^\lambda}{(1-\psi\beta)}\right)\right)\beta\tau\mathsf{r}_t.
\end{aligned}
$$

Substituting (A.28) and (A.31) into this equation yields

$$\Sigma_t = \alpha\beta\mathbb{E}_t\Sigma_{t+\tau} + \frac{\mu}{(\lambda-1)}\frac{x^{1-\lambda}}{(x+F)}\frac{(1-\alpha)\delta^\lambda\beta}{(1-\psi\beta)}\mathbb{E}_t\chi_{t+\tau} + \frac{\mu}{(\lambda-1)}\frac{y^{1-\lambda}}{(x+F)}\left(\theta_t - \frac{\alpha}{1-\alpha\beta}\beta\tau\mathsf{r}_t\right)$$
$$-\mu\frac{y^{1-\lambda}}{(x+F)}\left(\frac{(x+F)}{y}\alpha\beta\mathbb{E}_t\Sigma_{t+\tau} - \theta_t + \frac{(y-x)}{y}\frac{\alpha}{1-\alpha\beta}\beta\tau\mathsf{r}_t\right)$$
$$-\left(\alpha - \frac{1}{(\lambda-1)}\left(\frac{y^{1-\lambda}}{(x+F)}\frac{\mu\alpha}{(1-\alpha\beta)} - \frac{x^{1-\lambda}}{(x+F)}\frac{\mu(1-\alpha)\delta^\lambda}{(1-\psi\beta)}\right)\right)\beta\tau\mathsf{r}_t,$$

and cancelling terms, simplifying, and writing the equation in terms of $\pi = y^{-\lambda}$:

$$\Sigma_t = \alpha\beta(1-\mu\pi)\mathbb{E}_t\Sigma_{t+\tau} + \mu\pi\frac{(y/x)^\lambda}{(\lambda-1)}\frac{x}{(x+F)}\frac{(1-\alpha)\delta^\lambda\beta}{(1-\psi\beta)}\mathbb{E}_t\chi_{t+\tau} + \mu\pi\frac{\lambda}{(\lambda-1)}\frac{y}{(x+F)}\theta_t$$
$$-\left(\alpha\left(1+\frac{(y-x)}{(x+F)}\frac{\mu\pi}{1-\alpha\beta}\right) + \mu\pi\frac{(y/x)^\lambda}{(\lambda-1)}\frac{x}{(x+F)}\frac{(1-\alpha)\delta^\lambda}{(1-\psi\beta)}\right)\beta\tau\mathsf{r}_t. \quad \text{(A.33)}$$

Log linearizations of the transaction probability, sales rate, and time-to-sell equations from (8) and (19) are

$$\pi_t = -\lambda\mathsf{y}_t, \quad \mathsf{s}_t = \pi_t, \quad \text{and } \mathsf{T}_t = -\pi_t. \quad \text{(A.34)}$$

Using (A.21) and (A.27), the price equation (A.19) is log linearized as follows:

$$\left(\kappa C - \frac{\beta\tau D}{(1-\beta)} + \omega\frac{(1-\beta(1-\mu\pi))}{(1-\beta)}\frac{\tau}{\mu}\frac{(x+F)}{\pi}\right)(\mathsf{P}_t - \beta\mathbb{E}_t\mathsf{P}_{t+\tau}) = \omega\frac{\tau}{\mu}\frac{(x+F)}{\pi}(\Sigma_t - \pi_t)$$
$$-\left(\kappa C - \frac{\beta\tau D}{(1-\beta)} + \omega\frac{(1-\beta(1-\mu\pi))}{(1-\beta)}\frac{\tau}{\mu}\frac{(x+F)}{\pi} - \tau D - \kappa C - \omega(1-\mu\pi)\frac{\tau}{\mu}\frac{(x+F)}{\pi}\right)\beta\tau\mathsf{r}_t$$
$$-\beta\omega\frac{\tau}{\mu}\frac{(x+F)}{\pi}\mathbb{E}_t\left[(1-\mu\pi)(\Sigma_{t+\tau} - \pi_{t+\tau}) - \mu\pi\pi_{t+\tau}\right],$$

and simplifying the coefficients in this equation leads to:

$$\mathsf{P}_t = \beta\mathbb{E}_t\mathsf{P}_{t+\tau} + \frac{\frac{\omega\tau(x+F)}{\mu\pi}\left(\Sigma_t - \beta(1-\mu\pi)\mathbb{E}_t\Sigma_{t+\tau} - \pi_t + \beta\mathbb{E}_t\pi_{t+\tau}\right) - \frac{\tau(\omega(x+F)-D)\beta\tau}{(1-\beta)}\mathsf{r}_t}{\kappa C - \frac{\beta\tau D}{(1-\beta)} + \frac{\omega\tau(1-\beta(1-\mu\pi))(x+F)}{(1-\beta)\mu\pi}}. \quad \text{(A.35)}$$

Log-linearizations of the sales (20) and houses for sale (21) equations are

$$\mathsf{S}_t = \mathsf{s}_t + \mathsf{u}_t, \quad \text{and } \mathsf{u}_t - \mathsf{u}_{t-\tau} = \mu\pi(\mathsf{N}_t - \mathsf{S}_{t-\tau}), \quad \text{(A.36)}$$

where $\pi = y^{-\lambda}$ and the steady-state equation $N = S = su$ have been used. Equation (A.17) has the following log-linearization:

$$\mathsf{N}_t = \lambda\delta^\lambda\left(\frac{y}{x}\right)^\lambda\left(\frac{1-\alpha}{1-\psi}\right)\mathsf{x}_t + (1-\alpha)\mathsf{S}_{t-\tau} - \left(\frac{1-\alpha}{\mu\pi}\right)\mathsf{u}_{t-\tau} - \delta^\lambda\left(\frac{y}{x}\right)^\lambda\left(\frac{1-\alpha}{1-\psi}\right)\Upsilon_{t-\tau}, \quad \text{(A.37)}$$

which uses $N = S = su$, $s = \mu\pi$, and $\pi = y^{-\lambda}$. Finally, log-linearizing equation (A.18) for the auxiliary state variable Υ_t from (A.16):

$$\Upsilon_t = \psi\Upsilon_{t-\tau} + (1-\psi)\mathsf{u}_t, \quad \text{(A.38)}$$

which makes use of $\Upsilon = u$.

In summary, the system of equations (A.29), (A.30), (A.32), (A.33), (A.34), (A.35), (A.36), (A.37), and (A.38) can be solved for x_t, y_t, χ_t, Σ_t, π_t, s_t, T_t, P_t, S_t, N_t, u_t, and Υ_t. These equations are in recursive form with only contemporaneous (t), one-period lagged ($t-\tau$), and expected one-period ahead ($t+\tau$) values of the variables appearing.

The auxiliary variable χ_t can be eliminated as follows. Note that (A.33) implies

$$\Sigma_t - \psi\beta\mathbb{E}_t\Sigma_{t+\tau} = \alpha\beta(1-\mu\pi)\mathbb{E}_t\left[\Sigma_{t+\tau} - \psi\beta\Sigma_{t+2\tau}\right] + \mu\pi\frac{\lambda}{(\lambda-1)}\frac{y}{(x+F)}(\theta_t - \psi\beta\mathbb{E}_t\theta_{t+\tau})$$
$$-\left(\alpha\left(1+\frac{(y-x)}{(x+F)}\frac{\mu\pi}{1-\alpha\beta}\right)+\mu\pi\frac{(y/x)^\lambda}{(\lambda-1)}\frac{x}{(x+F)}\frac{(1-\alpha)\delta^\lambda}{(1-\psi\beta)}\right)\beta\tau(\mathsf{r}_t - \psi\beta\mathbb{E}_t\mathsf{r}_{t+\tau})$$
$$+\mu\pi\frac{(y/x)^\lambda}{(\lambda-1)}\frac{x}{(x+F)}\frac{(1-\alpha)\delta^\lambda\beta}{(1-\psi\beta)}\mathbb{E}_t\left[\chi_{t+\tau} - \psi\beta\chi_{t+2\tau}\right],$$

which makes use of the law of iterated expectations, and then by using equation (A.32):

$$\mu\pi\frac{(y/x)^\lambda}{(\lambda-1)}\frac{x}{(x+F)}\frac{(1-\alpha)\delta^\lambda\beta}{(1-\psi\beta)}\mathbb{E}_t\left[\chi_{t+\tau}-\psi\beta\chi_{t+2\tau}\right] = \mu\pi\frac{\lambda}{(\lambda-1)}\frac{x}{(x+F)}\left(\frac{y}{x}\right)^\lambda(1-\alpha)\delta^\lambda\beta\mathbb{E}_t\theta_{t+\tau}$$
$$-\mu\pi(1-\alpha)\delta^\lambda\left(\frac{y}{x}\right)^\lambda\beta\mathbb{E}_t\Sigma_{t+\tau} - \mu\pi\frac{(y/x)^\lambda}{(\lambda-1)}\frac{x}{(x+F)}\frac{(1-\alpha)\delta^\lambda\psi\beta}{(1-\psi\beta)}\beta\tau\mathbb{E}_t\mathsf{r}_{t+\tau}.$$

Combining these two equations yields

$$\Sigma_t = \left(\psi + (1-\mu\pi)\alpha - \mu\pi(1-\alpha)\delta^\lambda\left(\frac{y}{x}\right)^\lambda\right)\beta\mathbb{E}_t\Sigma_{t+\tau} - (1-\mu\pi)\alpha\psi\beta^2\mathbb{E}_t\Sigma_{t+2\tau}$$
$$+\mu\pi\frac{\lambda}{\lambda-1}\left(\frac{y}{(x+F)}(\theta_t - \psi\beta\mathbb{E}_t\theta_{t+\tau}) + \frac{x}{(x+F)}\left(\frac{y}{x}\right)^\lambda(1-\alpha)\delta^\lambda\beta\mathbb{E}_t\theta_{t+\tau}\right)$$
$$-\alpha\left(1+\frac{(y-x)}{(x+F)}\frac{\mu\pi}{1-\alpha\beta}\right)\beta\tau(\mathsf{r}_t-\psi\beta\mathbb{E}_t\mathsf{r}_{t+\tau}) - \mu\pi\frac{(y/x)^\lambda}{(\lambda-1)}\frac{x}{(x+F)}\frac{(1-\alpha)\delta^\lambda}{(1-\psi\beta)}\beta\tau\mathsf{r}_t.$$

The auxiliary variable Υ_t can also be eliminated by using (A.37) to obtain an equation for $\mathsf{N}_t - \psi\mathsf{N}_{t-\tau}$ and then substituting (A.38):

$$\mathsf{N}_t = \psi\mathsf{N}_{t-\tau} + \frac{\lambda\delta^\lambda(y/x)^\lambda(1-\alpha)}{(1-\psi)}(\mathsf{x}_t - \psi\mathsf{x}_{t-\tau}) + (1-\alpha)(\mathsf{S}_{t-\tau} - \psi\mathsf{S}_{t-2\tau})$$
$$-(1-\alpha)\left(\frac{1}{\mu\pi}+\delta^\lambda\left(\frac{y}{x}\right)^\lambda\right)\mathsf{u}_{t-\tau} + \frac{(1-\alpha)\psi}{\mu\pi}\mathsf{u}_{t-2\tau}.$$

A.4 A single housing-demand shock

Table A.4 reports the standard deviations and correlation coefficients predicted by the model with only a single housing-demand shock through changes in expenditure θ_t.

Table A.4: *Model-predicted correlations with only expenditure shocks*

	Sales	Prices	New listings	Time-to-sell	Houses for sale
	Correlation coefficients among housing-market variables				
Sales	1				
Prices	0.99	1			
New listings	1.00	0.99	1		
Time-to-sell	−0.99	−1.00	−0.99	1	
Houses for sale	0.95	0.95	0.95	−0.90	1
	Correlations between housing variables and shocks				
Expenditure (data)	0.78	0.93	0.68	−0.34	0.19
Expenditure (model)	1.0	1.0	1.0	−1.0	0.95

Notes: Simulated moments of the theoretical model with $\phi_\theta = 0.9873^{1/13}$, $\sigma_\theta = \sqrt{1-\phi_\theta^2} \times 0.0965$, and $\sigma_r = 0$ so that only the expenditure shock occurs.

A.5 The special case of exogenous moving decisions

A model with exogenous moving is a special case of the parameters of the model in section 3 for which the moving decision effectively becomes exogenous. If the size of the idiosyncratic shock to match quality becomes very large, that is, $\delta = 0$, then moving occurs if and only if an exogenous idiosyncratic shock is received. Adjusting the parameter α so that the average length of time between moving house remains the same provides an otherwise identical model with exogeneity of the moving decision as the only difference. The model-implied standard deviations and correlation coefficients subject to the same aggregate shocks are displayed in Table A.5.

The model with exogenous moving predicts that new listings are perfectly negatively correlated with houses for sale. This is because, irrespective of market conditions, listings are proportional to the previous number of homeowners not trying to sell. Furthermore, given that houses for sale are small on average as a fraction of all houses, the predicted volatility of new listings is tiny. Empirically, new listings are highly volatile and have a correlation with houses for sale that changes between positive and negative over time (see Table 2 and Figure 2). More generally, the exogenous-moving model predicts that correlations of houses for sale with other variables are always the negative of correlations of new listings with those variables. Hence, the model can only predict a change in the sign of the correlation between houses for sale and sales or prices if the sign of the new listings correlation with prices or sales changes. According to Figure 2, the correlations among sales, price, and new listings are stable.

Table A.5: *Predictions of the exogenous-moving special case of the model*

Expenditure	Interest rate	Sales	Prices	New listings	Time-to-sell	Houses for sale
		Standard deviations, %				
9.7	0.86	0.41	8.20	0.06	2.32	2.23
				Correlation coefficients		
	Sales	1				
	Prices	0.25	1			
	New listings	0.15	0.96	1		
	Time-to-sell	−0.32	−0.97	−0.98	1	
	Houses for sale	−0.15	−0.96	−1.00	0.98	1
				Correlations with shocks		
	Expenditure (data)	0.78	0.93	0.68	−0.34	0.19
	Expenditure (model)	0.29	0.99	0.98	−0.99	−0.98
	Interest rate (data)	−0.03	−0.10	0.04	−0.13	−0.21
	Interest rate (model)	0.25	−0.12	0.10	−0.14	−0.10

Notes: Simulated moments of the $\delta = 0$ special case of the theoretical model with $\phi_\theta = 0.9873^{1/13}$, $\phi_r = 0.8033^{1/13}$, $\sigma_\theta = \sqrt{1-\phi_\theta^2} \times 0.0965$, and $\sigma_r = \sqrt{1-\phi_r^2} \times 0.0086$.