Revising Claims and Resisting Ultimatums in Bargaining Problems*

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December 27, 2010

Abstract

We propose a simple mechanism which implements a unique solution to the bargaining problem with two players in subgame-perfect equilibrium. The mechanism incorporates two important features of negotiations; players can revise initial claims in an attempt to reach a compromise or pursue their claims in an ultimate take-it-or-leave-it offer. We show that when revisions are restricted, players avoid weak bargaining positions by showing restraint in the formulation of their initial claims. If no revisions are allowed, compatible claims implement the Nash solution. If all revisions are allowed, maximal claims implement the Kalai-Smorodinsky solution.

JEL Classification Numbers: C78, D74.

Keywords: Bargaining solutions, Nash program, Ultimatums

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*We are grateful to Arthur Campbell, Florian Ederer, Glenn Ellison, Luc Lauwers and Muhamet Yildiz for many helpful comments. We also wish to thank seminar participants at the Katholieke Universiteit Leuven and the Massachusetts Institute of Technology.

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1 Introduction

Negotiations in conflicts often share the following two features. First, players make claims that are subsequently revised in order to find a compromise. Their ability to make revisions depends on the context of the negotiations and may differ among players. Second, concessions may be induced by the threat of an ultimate take-it-or-leave-it offer. However, negotiators discourage such uncompromising behavior by adopting a firm posture - threatening to walk away from negotiations without agreement when facing such an ultimatum. These two features are extensively discussed in the negotiation literature (Sebenius 1992, Lewicki et al. 1994) and also appear in practical guides for negotiators, as in the defense procurement and acquisition guidelines by the US Department of Defense:[1] "Aim high" but "Give yourself room to compromise" and "Be willing to walk away from or back to negotiations".

This paper analyzes the outcome of a stylized two-player mechanism in subgame-perfect equilibrium. We aim to make two important contributions to the existing bargaining literature. First, the mechanism allows to analyze how the aforementioned negotiation features play out in equilibrium. In particular, players face a trade-off between claiming more so as to achieve more in an ultimatum and claiming less so as to leave room to compromise in the revision of claims. Also, the less room for compromise, the higher the resistance needed to block an ultimatum of an uncompromising opponent, which impedes a leading position in the negotiations. This reduces the gap between the game-theoretic bargaining literature and the negotiation literature. Second, the mechanism contributes to the Nash program. Different specifications of the mechanism implement a class of solutions to the bargaining problem. The single distinguishing feature is the extent to which claims can be subsequently revised. The mechanism nests the solution of Nash (1950) and the solution of Kalai and Smorodinsky (1975) in the extreme cases excluding or admitting all revisions. While solution concepts are comparable on the basis of their axiomatic properties, we extend the comparability to their non-cooperative foundations. The non-cooperative literature has previously focused on the implementation of a single solution (see Moulin (1984), Binmore et al. (1986) and Howard (1992)).

Restrictions on later revisions induce a negotiator to lower initial claims so as to avoid a weak bargaining position in which no feasible revision of his claim is acceptable to his opponent. In that case, one cannot lose in equilibrium by facing fewer restrictions regarding the revisions one can make. Still, we believe that such restrictions can be justified in many ways. Restrictions on revisions can be explicitly specified in the mandate given to the negotiator by his principal or arise from costs of revising initial plans. The restrictions may also arise from frustration from unfulfilled expectations

[1] The Contract Pricing Reference Guides (Vol5, Ch6) of the DPAP of the US Department of Defense, http://www.acq.osd.mil/dpap/cpf/docs/contract_pricing_finance_guide/vol5_ch6.pdf .

raised by the initial claims. In the context of negotiations, loss aversion appears as the aversion to making concessions (Kahneman and Tversky, 1995). Negotiators who do not fear to disappoint their principals or suppress their frustration obtain better agreements.

The mechanism has four stages. Players start by making *claims* in the first stage. In the second stage, they bid a *resistance probability*, which determines the probability of disagreement when an uncompromising opponent pursues his initial claim in a take-it-or-leave-it offer. The mediator gives leadership to the player who has bid the lowest resistance probability. In the third stage, the leader proposes a compromise within the set of feasible compromises which depends on the initial claims but remains beyond his control in all other respects. In the final stage, the opponent accepts or rejects the compromise. If he rejects, then he obtains his initial claim in an ultimatum unless he meets resistance to which the proposer is committed by the second stage; the negotiations end in disagreement with the proposer's resistance probability.

We characterize the unique allocation which is implemented in subgame-perfect equilibrium in two steps. In a first step, we analyze the trade-off players face in determining their resistance given the initial claims. This allows us to determine the equilibrium claims in a second step.

When determining their resistance probability, players trade off obtaining leadership - by bidding the lowest resistance probability - and discouraging uncompromising behavior - by bidding higher resistance probability. The follower will reject any feasible compromise and pursue his initial claim in an ultimatum when the leader's resistance falls short of some minimum level. Above this minimum, the leader's payoff in his best achievable compromise is increasing in his resistance. However, competition for leadership induces a player to bid the minimal resistance probability for which his opponent is willing to accept the maximal revision within his mandate. This determines the equilibrium outcome, depending on the feasible revisions for a pair of claims. First, if the *proportional solution* is feasible, then it is implemented. The proportional solution is the Pareto-efficient allocation for which the players' concessions are proportional to their claims. Both players need the same minimum resistance probability for imposing that solution as a leader. Since leadership is given to the lowest bidder, bidding this minimum guarantees the payoff of the proportional solution in equilibrium. Second, if the proportional solution is not feasible, then the *maximal revisions* of the initial claims are incompatible. The equilibrium implements the maximal revision of the player whose initial claim has the largest *extended* Nash product, which is the product of this player's claim and his opponent's payoff for the maximal revision of that claim. This player needs a lower resistance probability to make the own maximal revision acceptable, which puts him in a strong bargaining position. To obtain a strong bargaining position, a player should not only aim high when formulating claims, but also leave sufficient room for compromise.

The two previous observations determine the equilibrium claims. For both the proportional solution and the own maximal revision, a player's payoff is decreasing in his claim. Hence, one is willing to show restraint in the formulation of claims only if that increases one's room to compromise significantly. As long as the players' revisions of their initial claims are incompatible, competition for the strong bargaining position induces players to lower their claim so as to increase their extended Nash product. In restrictive environments, players lower their claims until the maximal revisions meet in a proportional solution with equal extended Nash products. This naturally holds in the Nash solution when the players cannot make concessions and the Nash products and the extended Nash products coincide. In less restrictive environments, revisions may be compatible for maximal claims so that the proportional solution is implemented for any pair of claims. Hence, no player has reason to show restraint in the formulation of his claim. This naturally holds in the extreme case in which all revisions are allowed. In equilibrium, players start by making maximal claims and the Kalai-Smorodinsky solution is implemented.

The mechanism underlines that room for compromise is essential for a strong bargaining position. When a negotiator is able - for a given claim - to increase his opponent's payoff in a compromise, his larger extended Nash product results in a stronger bargaining position with at least as good a payoff. The mechanism also clarifies the role of ultimatums and the resultant resistance needed for imposing a compromise. In the extreme case without revisions, both are irrelevant on the equilibrium path as players start by making compatible claims, corresponding to the Nash solution. When revisions are feasible in our mechanism, they both matter. We illustrate their importance by introducing ultimatums in the alternating-offer game first considered by Rubinstein (1982). In each round the responder can stop negotiations in an ultimatum and the proposer needs time to build resistance in order to deter such ultimatum for a better deal. With unrestricted revisions, as in the original alternating-offer game, the introduction of ultimatums moves the equilibrium outcome away from the Nash solution - the equilibrium solution of the alternating-offer game with equal waiting times - towards the Kalai-Smorodinsky solution - the equilibrium solution of the 4-stage mechanism with unrestricted revisions.

Related Literature According to Nash (1953), the relevance of a solution concept is enhanced if one arrives at it from very different points of view. The Nash program, as reviewed in Thomson (2010), attempts to complement the axiomatic properties of solution concepts with non-cooperative foundation. This paper takes the Nash program one step further by making solution concepts comparable based on their respective non-cooperative foundations. In a similar spirit, Miyagawa (2002) proposes a mechanism that can implement the solutions that maximize a set of quasi-concave welfare functions. The welfare functions are an explicit part of the rules of the mechanism that he considers. Our approach is to consider a stylized mechanism that captures features from negotiations as they are observed in practice. The mechanism we propose combines the approaches taken by Harsanyi (1977) and Moulin (1984).

Harsanyi (1977) proposed a two-stage demand game that implements the Nash solution. Players make demands in the first stage. Initial claims cannot be revised. The player with the highest *risk limit* imposes his demand in the second stage. A player's risk limit is the minimum resistance probability to which his opponent has to commit when the player pursues his demand in a take-it-or-leave-it offer and for which he accepts his opponent's demand. With revisable claims, a player's risk limit is higher if the extended Nash product of his claim is higher. By endogenizing resistance in the negotiation process for given claims in the 4-stage mechanism, we show how the strategic advantage for the strong player in a two stage-demand game is justified, whether or not claims are revisable.

Moulin (1984) proposed an auction game that implements the Kalai-Smorodinsky solution. Moulin's auction game is recasted in stages two to four of the 4-stage mechanism. However, players are assumed to make maximal claims and revisions are unrestricted. By endogenizing claims, we show that the assumption that players make maximal claims is justified as long as the revisions of claims are not too restricted. As Moulin, we give first-mover advantage to the player bidding the lowest resistance. However, if a better compromise can be negotiated for higher resistance and the building of resistance against an ultimatum takes time, then the recognition procedure in which the mediator gives leadership to the lowest bidder and the one in which players wait until they are strong enough to lead are equivalent. The equivalence is similar to the equivalence between the first-price sealed-bid auction and the Dutch auction.

Our mechanism with ultimatums thus provides another rationale for explaining delay when the order and timing of offers are determined endogenously in bargaining with complete information, as in Perry and Reny (1993) and Sakovics (1993). Ultimatums add another dimension to bargaining power. Bargaining power in most models is related to the probability of making subsequent offers by both parties after an offer is rejected. It is endogenized when players take costly measures to be recognized as leader (Yildirim 2007). Immediate agreement may be made impossible if players hold optimistic beliefs about the recognition process without a common prior in Yildiz (2003). Players wait to persuade in Yildiz (2004). In our mechanism with ultimatums, the player who proposes first is (recognized as) the leader. However, proposals get accepted only when a player has acquired and persuaded his opponent of his resistance to the threat of a take-it-or-leave-it offer. Since the building of resistance causes delay, the player who needs less resistance to make a feasible compromise acceptable to his opponent is in a stronger bargaining position.

The paper is organized as follows. The next section defines the bargaining problem, the 4-stage mechanism and the revision procedures. Section 3 analyzes the equilibrium strategies in the later stages, taking initial claims as given, and introduces risk limits to analyze the bidding strategies in particular. Section 4 characterizes the equilibrium claims and shows how the 4-stage mechanism justifies simplifying assumptions in Harsanyi's demand game and Moulin's auction game. The robustness of the mechanism is analyzed in section 5 and some examples of revision procedures are given in section 6. The final section concludes.

2 The model

In this section, we define the bargaining problem and a mechanism for selecting a solution in the bargaining set.

2.1 The bargaining problem

The bargaining problem is defined as follows. Let $N = \{1, 2\}$ be the set of players. The players are male and the mediator is female. Player $-i$ is player i's opponent for $i \in N$. The bargaining set S^* consists of the utility allocations $u = (u_1, u_2)$ associated with feasible bargaining outcomes. Player i's dictatorial outcome is the Pareto-efficient outcome for which his utility is maximized. The disagreement outcome obtains when no agreement can be reached. We assume that a player's utility is lowest in the disagreement outcome. The players and the mediator are completely informed about the players' private valuations of the outcomes. We abstract from the outcomes that give rise to the utility allocations in the bargaining set S^*.

The bargaining set S^* is compact and convex. As a normalization, a player's utility associated with his dictatorial outcome and the disagreement outcome are equal to 1 and 0 respectively. Hence, the bargaining set S^* is contained in $D \equiv [0, 1] \times [0, 1]$. Let $u^*_{-i}(u_i) = \sup \{ u_{-i} | \, u \in S^* \}$. Since S^* is convex, the function u^*_{-i} is concave on $[0, 1]$. Since S^* is closed, u^*_{-i} is continuous. The convex set $S^* \subseteq D$ is comprehensive if and only if $u^*_1(0) = u^*_2(0) = 1$. We assume that the bargaining set S^* is comprehensive, so that u^*_i is non-increasing on $[0, 1]$ for all $i \in N$. The set of Pareto-efficient utility pairs in S^* is $PO(S^*) \equiv \{ u \in [u^*_1(1), 1] \times [u^*_2(1), 1] | \, u_2 = u^*_2(u_1) \}$ and $\dot{S}^* \equiv \{ u \in S^* | \, u_i < u^*_i \, (u_{-i}) \, , i \in N \}$.

2.2 The mechanism

The mechanism Γ^R for selecting a solution in S^* has four stages.

The first stage is a demand stage, inspired by Harsanyi's demand game. Both players simultaneously formulate their *utility claims* $p \in D$. The second stage is a bidding stage, inspired by Moulin's auction game. Both players simultaneously bid *resistance probabilities* $q \in D$. The bid q_i determines the probability that player i stops negotiations in disagreement in the fourth stage, if his compromise were rejected by his opponent. The third stage is the compromising stage. The player with the lowest bid is selected by the mediator as the leader L, who makes a compromise proposal. The fourth and final stage is the approval stage. The follower F accepts the compromise or pursues his initial claim in a take-it-or-leave-it offer with a risk that negotiations end in disagreement.

The claims p of the first stage serve a double purpose. First, claims define the Pareto-efficient demands, $(p_1, u_2^*(p_1))$ for player 1 and $(u_1^*(p_2), p_2)$ for player 2, which a player pursues as follower in a take-it-or-leave-it offer in stage 4 after rejecting the leader's compromise. Second, the claim of player $i \in N$ determines the compromises he can propose in stage 3. The revised demand of player i after claiming p_i is $u^i(p_i) = \left(u_1^i(p_i), u_2^i(p_i)\right)$, where $u_i^i(p_i)$ denotes player i's (maximally) revised claim and $u_{-i}^i(p_i) = u_{-i}^*\left(u_i^i(p_i)\right)$ the utility of player $-i$ in the Pareto efficient allocation for this revised claim. Define $U^i(p_i) \equiv \left\{u \in S^* | \, u_i^i(p_i) \leq u_i \leq p_i\right\}$ accordingly.[2] The correspondences linking the set of feasible compromises to each player's claim are beyond the control of the players and the single distinguishing feature of each mechanism Γ^R. For now, we assume that a leader can also propose those compromises which are feasible for the follower.[3] Hence, the set of feasible compromises is

$$U(p) \equiv U^1(p_1) \cup U^2(p_2).$$

Claims are incompatible if $p \notin S^*$. Revised claims are incompatible if $\left(u_1^1(p_1), u_2^2(p_2)\right) \notin S^*$, implying $U^1(p_1) \cap U^2(p_2) = \emptyset$.

The probability bids q in the second stage serve a double purpose as well. First, the bid q_i determines the probability with which player i as a leader will resist his uncompromising opponent in the final stage so that negotiations end in disagreement. Second, the player who bids the lowest resistance probability, is rewarded with first-mover advantage. In case of equal bids, leadership is assigned by the mediator to player i with probability $\alpha_i(p)$. Hence,

$$\Pr(L(q,p) = i) = \begin{cases} 1 & \text{if } q_i < q_{-i}, \\ \alpha_i(p) & \text{if } q_i = q_{-i}. \end{cases}$$

Unless otherwise stated, $\alpha_i(p) = \frac{1}{2}$ and each player wins with equal probability in a tie.

The rules of the mechanism Γ^R can be summarized as follows:

- **Stage** 1. All $i \in N$ formulate claims $p = a^1 \in D$.

[2] For readability, we drop the reference to the revision procedures of the particular mechanism Γ^R in the notation of the revised demands. The notation is also simplified by ignoring inefficient demands, revised demands with upward revised claims ($u_i^i(p_i) > p_i$) or inefficient revised demands ($u_{-i}^i(p_i) < u_{-i}^*(p_i)$), since such demands would not occur in equilibrium or would not change the allocation in subgame-perfect equilibrium.

[3] This assumption is relaxed in section 5.

- **Stage** 2. All $i \in N$ bid resistance probabilities $q = a^2 \in D$.
- **Stage** 3. $L(q,p)$ proposes the compromise $u^c = a^3_L \in U(p)$.
- **Stage** 4. $F \in N \setminus \{L(q,p)\}$ chooses $a^4_F \in \{Y, N\}$.

The payoffs for player $i \in N$ are

$$\begin{array}{lll} u_i^c & \text{if} & a_F^4 = Y, \\ (1-q_L)p_F & \text{if} & a_F^4 = N \text{ and } i = F, \\ (1-q_L)u_L^*(p_F) & \text{if} & a_F^4 = N \text{ and } i = L. \end{array}$$

A mediator guarantees that players abide by the rules of the mechanism. She designates the leader, imposes disagreement with the probability chosen by L and limits F's payoff to his claim if F rejects L's compromise proposal.

2.3 The revision procedures and extended Nash products

We assume that player $-i$'s utility in the revised demand $u^i(p_i)$ of player $i \in N$ for his claim $p_i \in [0,1]$ is equal to

$$u_{-i}^i(p_i) = \sup \left\{ u_{-i} | \, u_i = p_i, u \in S^i \right\},$$

where S^i is a closed convex set, $S^* \subseteq S^i \subseteq D$. Since $S^* \subseteq S^i$, player i's own utility is revised downward ($u_i^i(p_i) \leq p_i$). Since S^i is a convex set, u_{-i}^i is a concave function on $[0,1]$. Moreover, comprehensiveness of S^* implies comprehensiveness of S^i so that u_{-i}^i is a non-increasing function. Since S^i is a closed set, u_{-i}^i is a continuous function.

We introduce a new concept, related to the Nash product, which determines the strength of a player's bargaining position for a given revision procedure.

Definition 1. *The product $p_i u_{-i}^i(p_i)$ is the extended Nash product of player i's claim p_i.*

The extended Nash product equals the product of player i's claim and player $-i$'s utility for the maximal revision of that claim. It is unimodal in p_i, since S^i is comprehensive, and reaches its maximum in $\hat{p}_i$ beyond which u_{-i}^i is strictly decreasing. We denote by $\check{p}_{-i}(p_i)$ the largest claim of player $-i$ with the same extended Nash product as the claim p_i of player i, if it exists. If claims cannot be revised ($u_{-i}^i = u_{-i}^*$), the extended Nash product is equal to the Nash product of claim p_i. Nash (1950) proposed $u^N \in \arg\max_{u \in S^*} u_1 \times u_2$ as a solution to the bargaining problem. Hence, $u^N = \left(\hat{p}_1, u_2^1(\hat{p}_1)\right)$ when $u_{-i}^i = u_{-i}^*$.

The reduced bargaining problem $S(p)$ for $p \notin \dot{S}^*$ is defined by

$$S(p) = \{ u \in S^* | \, u \leq p \}.$$

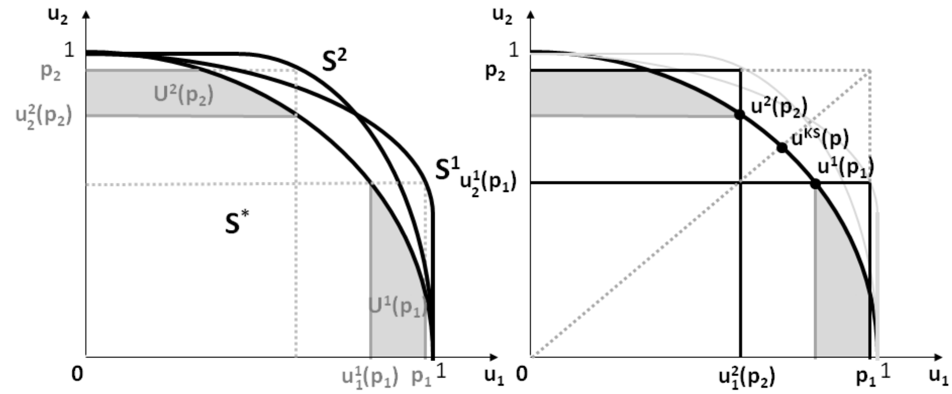

Figure 1: Feasible compromises and extended Nash products

Kalai and Smorodinsky (1975) proposed the Pareto efficient allocation $u^{KS}(p)$ for which $u_1^{KS}(p))/p_1 = u_2^{KS}(p))/p_2$ as a solution to the comprehensive bargaining problem $S(p)$ for $p \notin \dot{S}^*$. In this solution, the concessions are proportional to the claims and the payoffs are increasing in the own claim. Moreover, the extended Nash products are equal for p if the revised claims $\left(u_1^1(p_1), u_2^2(p_2)\right)$ meet exactly in $u^{KS}(p)$. More generally by Remark 1, the proportional solution for claims with equal extended Nash products is feasible if and only if the revised claims are compatible.

Remark 1. *For* $p = (p_1, \check{p}_2(p_1)) \notin \dot{S}^*$,

$$\text{if } U^1(p_1) \cap U^2(p_2) \neq \emptyset \text{ then } u^{KS}(p) \in U^1(p_1) \cap U^2(p_2).$$

Proof. *See appendix.* ■

Figure 1 illustrates the revision procedures associated with the convex sets S^1 and S^2. The left panel shows a pair of claims p and the associated set of feasible compromises $U(p) = U^1(p_1) \cup U^2(p_2)$. The right panel shows the associated extended Nash products as surfaces of two rectangles. Since the extended Nash products are equal and the revised claims are incompatible, the proportional solution does not belong to the compromise set, by Remark 1. Notice that the revised demands eventually meet in the proportional solution when claims are reduced while maintaining the equality of the extended Nash products, unless some player i first reaches his claim $\hat{p}_i$ with the largest extended Nash product.

3 Risk limits and bidding strategies

In this section, we take the initial claims as given and solve for the equilibrium strategies in Γ^R from the second stage on. When bidding resistance probabilities, players face a trade-off between obtaining leadership and discouraging uncompromising behavior by the follower. We find that, for any pair of incompatible claims, the player who formulated the claim with the larger extended Nash product is *strong* and has the privilege of dictating his revised demand or the proportional solution in equilibrium, as he prefers. If the proportional solution belongs to the compromise set, each player can guarantee himself the associated payoff by bidding the maxmin resistance probability. If the proportional solution is not a feasible compromise, the strong player will be the leader as he needs a lower resistance probability to make his own revised demand acceptable. This follows from the relation between the extended Nash products of claims and players' risk limits, as introduced by Zeuthen (1930) and used by Harsanyi (1977) to justify the Nash solution.

3.1 Risk limits

In the fourth stage, the follower F chooses between the compromise u^c and the lottery with his demand and disagreement as prizes. The probability of disagreement equals the leader's resistance q_L. Thus, when rejecting the compromise, F obtains $(1-q_L)p_F$ in expectation. If $p_F > 0$, the risk limit of player F is defined as

$$r_F(u_F^c, p_F) = \max\left\{1 - \frac{u_F^c}{p_F}, 0\right\}.$$

The risk limit measures the resolve of F in pursuing his own claim when u^c is proposed as a compromise. Given the resistance $q_L \in [0,1]$ of L to the uncompromising F, the risk limit of F defines his response in the last stage as

$$a_F^4(u^c, p, q) = \begin{cases} \text{Y if } q_L \geq r_F(u_F^c, p_F) \text{ and } u_F^c > 0, \\ \text{N otherwise.} \end{cases} \quad (1)$$

That is, F will accept L's proposal u^c if L's resistance q_L to F's initial claim is at least equal to F's risk limit. A proposal becomes acceptable by either reducing F's resolve in an improved compromise or by increasing the resistance to F's uncompromising behavior. In particular, F always accepts a compromise that repeats F's claim (because $r_F(p_F, p_F) = 0$). We assume that F accepts the compromise in a tie when $u_F^c = (1-q_L)p_F$.

In the third stage, the leader L proposes a compromise u^c, which is determined by the response of F as in

$$a_L^3(p,q) \in \arg\max_{u \in U(p)} \left\{ u_L \,|\, a_F^4(u,p,q) = \text{Y} \right\}. \quad (2)$$

When feasible, L proposes the Pareto efficient allocation u^c that leaves F indifferent between accepting or rejecting the compromise, i.e. $r_F(u_F^c, p_F) = q_L$. Clearly, L's payoff is increasing in his resistance probability q_L.

Lemma 1 summarizes the equilibrium strategies in the third and fourth stage for all possible histories. Let $\sigma = (\sigma_1, \sigma_2)$ be a strategy profile for Γ^R. The history $h^{\tau-1} \in \mathcal{H}^{\tau-1}$ at stage $\tau = 1, ..., 4$ is recursively defined by $h^\tau = \left(a^\tau, h^{\tau-1}\right)$ and $h^0 = \emptyset$. The strategy of player i at stage τ in the subgame for the history $h^{\tau-1}$ in σ is denoted by $a_i^\sigma(h^{\tau-1})$.

Lemma 1. *Assume that σ is a subgame-perfect equilibrium of Γ^R. Then for all $h^2 \in \mathcal{H}^2$, $u^c = a_L^\sigma(h^2) = a_L^3$ satisfies condition* (2) *and, for all $\hat{u} \in U(p)$, $a_F^\sigma(\hat{u}, h^2) = a_F^4$ satisfies condition* (1)*. Moreover,*

(i) *if $q_L \in \left[0, r_F(u_F^F(p_F), p_F)\right] \cup \left[r_F(u_F^L(p_L), p_F), r_F(u_F^*(p_L), p_F)\right]$, then $u^c \in PO(S^*)$ with $r_F(u_F^c, p_F) = q_L$. On both intervals, u_L^c is strictly increasing in q_L,*

(ii) *if $r_F(u_F^F(p_F), p_F) < q_L < r_F(u_F^L(p_L), p_F)$, then $u^c = u^F(p_F)$,*

(iii) *if $r_F(u_F^*(p_L), p_F) < q_L$, then $u_L^c = p_L$ and $u_F^c = u_F^*(p_L)$.*

Proof. *See appendix.* ■

3.2 Bidding Strategies

We first establish for each player the relationship between the extended Nash product of his claim, his risk limit and his opponent's resistance probability for a pair of claims. By the proportionality property of $u^{KS}(p)$,

$$q^{KS}(p) = r_{-i}(u_{-i}^{KS}(p), p_{-i}) = 1 - \frac{u_{-i}^{KS}(p)}{p_{-i}} \text{ for } i \in N \text{ and } p \notin \dot{S}^*.$$

Both players need the same resistance probability to make the proportional solution $u^{KS}(p)$ acceptable. If it is feasible, any player can make sure that it is implemented by bidding $q^{KS}(p)$. If it is not feasible, the minimal resistance probability needed by player i to impose his revised demand $u^i(p_i)$ is

$$q^i(p) = r_{-i}(u_{-i}^i(p_i), p_{-i}) \text{ for } i \in N. \tag{3}$$

For $q^i(p) = 1 - \frac{u_{-i}^i(p_i)}{p_{-i}} \geq 0$ (all $i \in N$), we find the equivalence

$$q^i(p) \geq q^{-i}(p) \text{ if and only if } p_i u_{-i}^i(p_i) \leq p_{-i} u_i^{-i}(p_{-i}).$$

The player who formulates the claim with the larger extended Nash product is in a strong bargaining position, as he needs less resistance to implement his revised demand.

Definition 2. *Assume that $\{s, w\} = N$ and $p_s u_w^s(p_s) \geq p_w u_s^w(p_w)$ for a pair of claims p. Player s is strong and player w is weak.*

The strong player's strategic advantage is driven by the first-mover advantage of the player bidding the lowest resistance probability. If the strong player bids $q_s = q^s(p)$, when $q^s(p) < q^w(p)$ and the proportional solution is not feasible, then the weak player accepts the strong player's revised demand $u^s(p_s)$. The weak player's resistance is too low for imposing his own revised demand $u^w(p^w)$ when he becomes leader by underbidding the strong player with $q_w \leq q^s(p) < q^w(p)$.

3.2.1 Incompatible claims

The equilibrium bidding strategies depend on whether the strong player prefers the proportional solution to his revised demand by the following remark.

Remark 2. *Assume $p \notin \dot{S}^*$. For all $i \in N$,*

$$u_i^{KS}(p) > u_i^i(p_i) \text{ if and only if } q^{KS}(p) > q^i(p).$$

Proof. *See appendix.* ■

Case 1: $u_s^{KS}(p) > u_s^s(p_s)$. If the strong player strictly prefers the proportional solution to his own revised demand for incompatible initial claims, then $u^{KS}(p) \in U^s(p_s)$. Competition for leadership puts downward pressure on the bids if the leader were to win with $q_L > q^{KS}(p)$. The follower would lead the compromise stage by undercutting his opponent with resistance exceeding $q^{KS}(p)$. He could propose an acceptable compromise that he prefers to the proportional solution rather than being forced to accept a compromise that his opponent prefers to the proportional solution. Conversely, no player would want to lead the compromise stage with $q_L < q^{KS}(p)$, since the leader's compromise is accepted only if the follower prefers it to the proportional solution. Profitable deviations are thus excluded only if both players bid resistance equal to $q^{KS}(p)$. Since both players propose the proportional solution, the identity of the leader is irrelevant. Hence, $u^{KS}(p)$ is the unique allocation implemented in subgame-perfect equilibrium.

Lemma 2. *Assume that σ is a subgame-perfect equilibrium of Γ^R. For all $h^1 \in \mathcal{H}^1$ with $p \notin S^*$ for which $u_s^{KS}(p) > u_s^s(p_s)$, $u^{KS}(p)$ is proposed by s and accepted by w and*

$$q_i = a_i^\sigma(h^1) = q^{KS}(p) \text{ for all } i \in N. \tag{4}$$

Proof. *See appendix.* ■

Bidding the resistance probability $q^{KS}(p)$ corresponds to a maxmin strategy, as shown before by Moulin (1984). If player i leads the negotiations with bid q_i $(\geq q^i(p))$, his payoff equals

$$\tilde{u}_i(q_i) \in \arg\max_{u \in U(p)} \{u_i | \, u_{-i} \geq (1 - q_i)p_{-i}\}.$$

If the opponent leads the negotiations with bid q_{-i}, player i's payoff is bounded from below by $(1 - q_i)\, p_i \leq (1 - q_{-i})\, p_i$. Hence, $\min\{\tilde{u}_i(q_i), (1 - q_i)\, p_i\}$ is a lower bound for his payoff. As higher resistance increases $\tilde{u}_i(q_i)$, but decreases $(1 - q_i)\, p_i$, the minimum of the two reaches a maximum when $u_i = \tilde{u}_i(q_i) = (1-q_i)\,p_i$. Given that $u_{-i} = (1-q_i)\,p_{-i}$, the opponent's payoff stands in the same proportion to his claim and the maxmin resistance probability equals $q^{KS}(p)$ for both players.

Case 2: $u_s^s(p_s) \geq u_s^{KS}(p)$**.** We distinguish between two subcases for incompatible initial claims. In the first subcase with $q^s(p) < q^w(p)$, the revised demand $u^s(p_s)$ of the strong player s is implemented for a bid $q_s \in [q^s(p), q^w(p))$. Indeed, s is strong enough to impose $u^s(p_s)$ as a leader and can reject any proposal in $U^w(p_w)$ as a follower. Hence, w chooses $q_w \geq r_s(u_s^s(p_s), p_s)$ so as to be strong enough to impose $u^s(p_s)$ as a leader and $q_w \leq q^s(p)$ so that he can reject any proposal in $U^s(p_s) \setminus \{u^s(p_s)\}$ as a follower. In the second subcase with $q^1(p) = q^2(p)$, the leader's revised demand is implemented for $q_1 = q_2 = q^1(p) = q^2(p)$. Indeed, when bidding q_i for $q_{-i} = q^{-i}(p)$, $i \in N$ is just strong enough to impose $u^i(p_i)$ as a leader and can reject $U^{-i}(p_{-i}) \setminus \{u^{-i}(p_{-i})\}$ as a follower.

Lemma 3. *Assume that σ is a subgame-perfect equilibrium of Γ^R. For all $h^1 \in H^1$ with $p \notin S^*$ for which $u_s^s(p_s) \geq u_s^{KS}(p)$,*

$$\text{if } q^s(p) < q^w(p), \quad q_s = a_s^\sigma(h^1) \in [q^s(p), q^w(p))$$

$$q_w = a_w^\sigma(h^1) \in \begin{cases} [r_s(u_s^s(p_s), p_s), q^s(p)] & \text{if } u_s^s(p_s) \neq p_s \\ [0,1] & \text{if } u_s^s(p_s) = p_s \end{cases}, \tag{5}$$

$$\text{if } q^s(p) = q^w(p), \quad q_i = a_i^\sigma(h^1) = q^i(p) \text{ for all } i \in N$$

If $q^s(p) < q^w(p)$, then $u^s(p_s)$ is proposed and accepted. If $q^s(p) = q^w(p)$, $u^i(p_i)$ is proposed and accepted if $i \in N$ is leader.

Proof. *See appendix.* ■

3.2.2 Compatible claims

For subgames with compatible claims, the opponent's demand is preferred by each player to the own demand. Moreover, the opponent always accepts his own demand as a follower, regardless of the resistance of the leader. Hence, competition for first-mover advantage excludes resistance of the leader to the opponent's demand.

Lemma 4. *Assume that σ is a subgame-perfect equilibrium of Γ^R. For all $h^1 \in \mathcal{H}^1$ the follower's demand is proposed and accepted with $q = a^\sigma(h^1) = \mathbf{0}$ for $p \in \dot{S}^*$ and $q = a^\sigma(h^1) \in D$ for $p_2 = u_2^*(p_1)$.*

Proof. *See appendix.* ■

4 Formulating Claims

In this section, we complete the subgame-perfect equilibrium by characterizing the claims in the first stage. By formulating lower claims, players have more room to compromise. We find that players show restraint in the formulation of their initial claims only to obtain the strong bargaining position associated with the larger extended Nash product. As shown in the previous section, the strong player $s(p)$ has strategic advantage allowing him to impose either his own revised demand $u^s(p_s)$ or the proportional solution $u^{KS}(p)$ as a compromise. For both compromises, the payoff is increasing in the own claim. Hence, a player faces the trade-off between increasing his claim - which increases his own payoff as the strong player - and reducing his claim - which may make him strong. This trade-off is strongest when no revisions are allowed and disappears when all revisions are allowed.

4.1 Simple Demand Game

We characterized the strategies in subgame-perfect equilibrium following any pair of initial claims for the mechanism Γ^R. It remains to determine the claims in the first stage for the complete characterization of a subgame-perfect equilibrium for Γ^R. From Lemmas 1-4, it follows immediately that the mechanism Γ^R implements the same solution as the following simple mechanism $\hat{\Gamma}^R$:

- **Stage** a**.** All $i \in N$ formulate claims $p \in D$.
- **Stage** b**.** Player $s(p)$ selects an allocation in $\hat{U}^{s(p)}(p)$,

$$\hat{U}^i(p) = \begin{cases} \left\{u^i(p_i), u^{KS}(p)\right\} & \text{if } p \notin \dot{S}^*, \\ \left\{\left(p_1, u_2^*(p_1)\right), \left(p_2, u_1^*(p_2)\right)\right\} & \text{if } p \in \dot{S}^*. \end{cases}$$

Stage a of $\hat{\Gamma}^R$ is equivalent to stage 1 of Γ^R and stages 2 to 4 of Γ^R are compressed in stage b of $\hat{\Gamma}^R$.

If $p \in \dot{S}^*$, then by increasing his claim to $u_i^*(p_{-i})$ player i obtains this payoff exceeding p_i with certainty and has a profitable deviation. Hence, claims that are strictly compatible are never formulated in equilibrium. For $p, p' \notin \dot{S}^*$ such that $p'_i \geq p_i$ and $p'_{-i} = p_{-i}$, the following two inequalities determine the equilibrium claims,

$$\min_{u \in \hat{U}^{-i}(p)} u_i \leq \max_{u \in \hat{U}^i(p)} u_i \leq \max_{u \in \hat{U}^i(p')} u_i. \tag{$\bigstar$}$$

By the first inequality of ($\bigstar$), it is advantageous for player i to be strong and to select his preferred allocation from his own set $\hat{U}^i(p)$ which yields $\max\left\{u_i^i(p_i), u_i^{KS}(p)\right\}$ as a payoff. If player $-i$ were to choose in $\hat{U}^{-i}(p)$, player i obtains $\min\left\{u_i^{-i}(p_{-i}), u_i^{KS}(p)\right\}$ and could not be better off. The advantage is strict, unless both players weakly prefer proportional concessions and the identity of the leader does not matter. By comprehensiveness of S^i, $\max\left\{u_i^i(p_i), u_i^{KS}(p)\right\}$ is strictly increasing in a player's claim. By the second inequality of ($\bigstar$), the strong player gains by increasing his claim if he remains strong for the increased claim, so that i claims $p_i \geq \hat{p}_i$ as a strong player. The two inequalities together imply that players prefer to be strong with a claim that is as high as possible. The equilibrium claims will depend on the revision procedure. We first consider two extreme cases, when no and all revisions are allowed.

4.2 Nash Solution

Consider the mechanism Γ^N for which claims and revised claims coincide when no revisions are feasible, $S^1 = S^2 = S^*$. In this case, the proportional solution $u^{KS}(p)$ is not feasible for any pair $p \notin S^*$ of incompatible claims. If incompatible claims define unequal extended Nash products, the strong player will impose his demand, which he prefers to the opponent's demand. However, the weak player can become leader and impose his demand by reducing his claim as long as claims remain incompatible. The competition to become strong results in players formulating claims that maximize the extended Nash product. When no revisions are feasible, the extended Nash product of a claim coincides with the Nash product. The Nash solution maximizes the Nash product. For $\hat{p} = u^N$ in equilibrium, the (revised) demands and the proportional solution coincide and $u^1(\hat{p}_1) = u^2(\hat{p}_2) = u^{KS}(\hat{p})$ for $\hat{p} = u^N \in PO(S^*)$.

Proposition 1. *Any subgame-perfect equilibrium σ of Γ^N for which $S^1 = S^2 = S^*$ implements the Nash solution u^N in S^*.*

Proof. If some player i receives a payoff below u_i^N, then he remains or becomes strong by claiming u_i^N for which the maximized (extended) Nash product of his claim exceeds his opponent's. He imposes u^N as the strong player by Lemma 3 and has a profitable deviation. Profitable deviations are excluded in subgame-perfect equilibrium if and only if $p = u^N$ is the pair of initial claims. ■

Harsanyi (1977) derived the Nash solution c^N as an equilibrium for a demand game in which, according to the conjecture of Zeuthen (1930), the player with the lowest risk limit submits. Notice that the simple demand game $\hat{\Gamma}^N$ for Γ^N with $u_i^i(p_i) = p_i$ coincides with Harsanyi's demand game. We augmented Harsanyi's demand game and find that, by keeping resistance within bounds, the strong player imposes his demand on the weak player and the weak player cannot impose his demand on the strong player. A player is therefore vulnerable if he has a lower risk limit, confirming Zeuthen's conjecture.

4.3 Kalai-Smorodinsky Solution

Consider the mechanism Γ^M for which all revisions are feasible, $S^1 = S^2 = D$. In this case $u_i^i(p_i) = 0$ for all $i \in N$, so that revised claims are compatible and the proportional solution $u^{KS}(p)$ is always feasible for $p \notin \dot{S}^*$. Since the own payoff in the proportional solution is strictly increasing in the own claim, nobody shows restraint in the formulation of claims.

Proposition 2. *Any subgame-perfect equilibrium σ of Γ^M for which $S^1 = S^2 = D$ implements the Kalai-Smorodinsky solution $u^{KS}(\mathbf{1})$ in S^*.*

Proof. For $S^i = D$, u_i^i is the constant function which is equal to zero and the proportional solution for $p \notin \dot{S}^*$ is preferred to the own revised demand. By Lemma 2, the proportional solution is implemented. Since the proportional solution is strictly increasing in the own claim, $p = \mathbf{1}$ and $u^{KS}(\mathbf{1})$ is implemented. ■

The stages 2 to 4 of Γ^R recast the mechanism proposed by Moulin (1984). The non-cooperative justification of the Kalai-Smorodinsky solution relies on two features of the negotiation procedure. First, the player showing lower resistance to an uncompromising opponent is given first-mover advantage. Although moderation is rewarded with first-mover advantage, higher resistance improves the own payoff in a compromise proposal, as in Moulin's model in which each player is assumed to make a maximal claim $p_i = 1$. The second feature of the negotiation procedure implementing the Kalai-Smorodinsky solution is the set of feasible compromises $U(p) = S(p)$. Since in the Kalai-Smorodinsky solution for $S(p)$, $u_i^{KS}(p)$ is monotone in p_i, nobody shows restraint in the formulation of claims. By augmenting Moulin's model with a demand stage, Γ^M justifies Moulin's assumption that players make maximal claims for $S^1 = S^2 = D$.

4.4 Implementing Bargaining Solutions

We now characterize the solution u^R of Γ^R for a partition of the set of comprehensive revision procedures. When the restrictions on revisions are insufficient to induce restraint, as for the mechanism Γ^M, players formulate maximal claims in equilibrium. When the restrictions on revisions are sufficient to induce restraint, as for the mechanism Γ^N, players formulate claims associated with equal extended Nash products in equilibrium.

In the first subset, some player $i \in N$ is strong for his maximal claim, whatever his opponent claims,

$$\exists i \text{ such that } p_i u_{-i}^i(p_i) \geq \hat{p}_{-i} u_i^{-i}(\hat{p}_{-i}) \text{ for } p_i = 1. \tag{6}$$

This subset contains the mechanism Γ^M in which (6) holds for $\hat{p}_{-i} = 1$. In the second subset, the revised claims are compatible when the extended Nash products are equal and player i's claim is maximal,

$$\exists i \text{ such that } u_{-i}^i(1) \geq u_{-i}^{-i}(\check{p}_{-i}(1)) \text{ and } \hat{p}_{-i} < 1. \tag{7}$$

In both subsets, none of the players gains by showing restraint in the formulation of their claims. This is true for player i by the second inequality of ($\bigstar$) provided that player $-i$ cannot gain by making a claim which makes him strong for player i's maximal claim. The latter is obvious if case (6) holds, since player i is always strong for his maximal claim. This is also the case if (7) holds, since $p_{-i} = \check{p}_{-i}(1)$ is the highest claim for which player $-i$ can become strong for $p_i = 1$. For this pair of claims, the revised claims are compatible. By Remark 1, the proportional solution is then feasible and implemented. Its payoff constitutes an upper bound on the payoffs obtained for even lower claims of player $-i$ by the second inequality of ($\bigstar$). In both subsets, the unique equilibrium allocation u^c is the strong player's preferred allocation in $\{u^{s(\mathbf{1})}(1), u^{KS}(1)\}$, as stated in the first part of Proposition 3. The strong player formulates the maximal claim $p_s = 1$. The weak player's equilibrium claim is not unique when the strong player strictly prefers $u^{s(\mathbf{1})}(1)$ to $u^{KS}(1)$.

In the third subset, (6) and (7) do not hold so that $\check{p}_i(1)$ is well-defined for all $i \in N$ and the revised claims are incompatible for the pair of maximal claims. *If* $p \neq \mathbf{1}$ *and* $q^s(p) < q^{KS}(p)$, then the proportional solution is feasible and implemented for p_s as well as for a small increase in p_s by Remark 2. By the monotonicity of the proportional solution, the strong player for p would have a profitable deviation. Hence, $p \neq 1$ implies that $q^s(p) \geq q^{KS}(p)$ and that $u^s(p)$ is implemented. Players thus compete for the privilege enjoyed by the strong player. This competition induces restraint in the formulation of claims by the first inequality of ($\bigstar$), as long as the revised claims are incompatible, and equalizes the extended Nash products as in the mechanism Γ^N, so that $p_1 = \check{p}_1(p_2)$ for $p \notin \dot{S}^*$ and $p \geq \hat{p}$. As long as $p > \hat{p}$, each player can outperform his opponent by a small reduction in his claim. A small additional concession implements his own revised demand with certainty and excludes the risk of being imposed his opponent's revised demand. In equilibrium, either the revised demands coincide for $p \geq \hat{p}$ or $p_w = \hat{p}_w$.[4]

In the first case, by Remark 1, $u^1(p_1) = u^2(p_2) = u^{KS}(p)$ for $p = (\check{p}_1(p_2), p_2)$ and the identity of the leader does not matter. This pair defines an equilibrium. A lower claim cannot increase a player's payoff, whichever of the three alternatives is chosen. As a player becomes weak for a higher claim, his opponent chooses his revised demand as the strong player and leaves the allocation unchanged.

In the second case, the revised claims are incompatible when $p_w = \hat{p}_w$. By Lemma 3, both players bid the same resistance probability. If leadership is randomly assigned when the default ($\alpha_s(p) = \frac{1}{2}$) is applied as a tie-breaking rule, then player s's expected payoff of claiming is discontinuous at $\check{p}_s(\hat{p}_w)$. At smaller (resp. higher) values he always (resp. never) wins and winning is random at $\check{p}_s(\hat{p}_w)$. His optimal response is well defined, if as a tie-breaking rule we replace the default by $\alpha_s(p) = 1$ when $p_s = \check{p}_s(\hat{p}_w)$ and $p_w = \hat{p}_w$. For $p_s = \check{p}_s(\hat{p}_w)$, player w's claim does not change the allocation.

[4] Notice that $\check{p}_w(\hat{p}_s)$ is not defined, unless the pair $\hat{p}$ defines equal extended Nash products.

Proposition 3. *Assume that* u^c *is the allocation implemented for* $p = a^{\sigma}(h^0)$ *in a subgame-perfect equilibrium of* Γ^R*.*

- *If* (6) *or* (7) *hold, then* u^c *is player* $s(\mathbf{1})$*'s preferred allocation in*

$$\{u^{s(\mathbf{1})}(1), u^{KS}(\mathbf{1})\}.$$

  If $u^{KS}_{s(\mathbf{1})}(\mathbf{1}) \geq u^{s(\mathbf{1})}_{s(\mathbf{1})}(1)$*, then* $p = \mathbf{1}$*. If* $u^{KS}_{s(\mathbf{1})}(\mathbf{1}) < u^{s(\mathbf{1})}_{s(\mathbf{1})}(1) = u^{KS}_{s(\mathbf{1})}(\tilde{p})$*, then* $w(\mathbf{1})$ *claims in* $[\tilde{p}_{w(\mathbf{1})}, 1]$ *for* $\tilde{p}_{s(\mathbf{1})} = 1$*.*

- *If* (6) *and* (7) *do not hold, then, for some* $s \in N$*,* $p_s = \check{p}_s(p_w)$*,* $u^c = u^s(\check{p}_s(p_w))$ *and*

$$u^c = \begin{cases} u^s(p_s) = u^w(p_w) = u^{KS}(p) & \text{if } u^s_s(\check{p}_s(\hat{p}_w)) \leq u^w_s(\hat{p}_w), \\ u^s(\check{p}_s(\hat{p}_w)) & \text{if } u^s_s(\check{p}_s(\hat{p}_w)) > u^w_s(\hat{p}_w). \end{cases}$$

Proof. *See appendix.* ■

5 Robustness

In this section, we justify some of the simplifying features of the mechanism Γ^R.

5.1 Unmediated leadership

We assumed that the mediator gives leadership to the player bidding the lowest resistance probability. If the mediator were to give leadership to the highest bid, players can lead the negotiations with maximal resistance against uncompromising followers. When the leader has bid one as a resistance probability, the follower will accept any compromise, including the leader's dictatorship, anticipating the disagreement outcome when he would reject the compromise. Hence both players' bids and claims are maximal in equilibrium, resulting in random dictatorship.

The incentives to limit resistance against a take-it-or-leave-it offer can be endogenized in an unmediated mechanism, if we assume that building up resistance to be armed against a take-it-or-leave-it offer needs time. Arms races and burning bridges are examples. In a similar spirit, Schelling (1956) argues that adherence to a commitment - preferring to leave the negotiation table empty-handed to the acceptance of an ultimatum - must be obliged and motivated and this obligation and motivation must be communicated to and recognized by the other party. The higher the commitment to resist, the longer the time needed to persuade.

Assume that in the second stage of the mechanism with unmediated leadership both players increase resistance, simultaneously and at the same pace, until one of the players stops and proposes a compromise. The resistance built by the proposer at that point is the disagreement probability to which he is committed in a mediated recognition process. A player takes the lead as soon as he is confident that his resistance to an ultimatum is sufficiently strong and a compromise within his mandate is acceptable. The equilibrium strategies when players bid for leadership or when resistance is built until one player takes the lead are the same. The strategy space and the allocation of leadership do not change. The equivalence between bidding and building resistance is the same as the equivalence between a sealed-bid first-price auction and a Dutch auction. Of course, mediated leadership is desirable as it implements a solution without costly delay.

5.2 Unmediated revisions

We also assumed that $U^1(p_1) \cup U^2(p_2)$ is the set of feasible compromises. That is, any opportunity for compromise available to one player is also made available to the other player by the mediator. We now consider *unmediated revisions* where each player i must make compromises within his own set $U^i(p_i)$ and the feasible revisions are player specific. This will be the case when frustration or loss aversion restrict the revision of claims, as in Example 2 in the next section.

Since the proportional solution is implemented when it is feasible for both players, the strong player gains under this alternative assumption only if $u^{KS}(p) \notin U^s(p_s) \cap U^w(p_w)$ and $q^w(p) > q^s(p)$. The strong player can raise his resistance probability $\tilde{q}_s$ above $q^s(p)$ and impose the compromise $\tilde{u}^c \in U^s(p_s)$ with $\tilde{u}^c_w = (1-\tilde{q}_s)p_w$ as a leader, which he strictly prefers to his revised demand $u^s(p_s)$. In the equilibrium for which the strategy of player w is weakly undominated, both players bid a resistance probability equal to $q^w(p)$ (assuming $\alpha_s(p) = 1$). The weak player cannot gain from taking the lead by bidding below $q^w(p)$. As he can no longer propose in $U^s(p_s)$ with unmediated revisions, the strong player pursues his ultimatum after rejecting any proposal in $U^w(p_w)$ when $\tilde{q}_w < q^w(p)$. However, the weak player can close the gap between $q^w(p)$ and $q^s(p)$ by reducing his claim in the competition for leadership as with mediated revisions when (6) and (7) do not hold for $p \neq \mathbf{1}$. Mediated and player-specific revisions thus yield the same equilibrium allocation. When (6) or (7) holds, the strong player formulates the maximal claim $p_s = 1$ and proposes $\tilde{u}^c_w = (1-q^w(p))\,p_w = u^w_s(p_w)\,p_w$. The second equality follows by (3) for $p_s = 1$. Hence, as long as the strong player prefers this compromise to the proportional solution $u^{KS}(p)$, the weak player gains from decreasing his claim p_w if this increases the associated extended Nash product. In equilibrium, $\tilde{u}^c_w = p_w u^w_s(p_w)$ and $p_s = 1$. Either $p_w = \hat{p}_w$, or $\tilde{u}^c = u^w(p_w) = u^{KS}(p)$ for $p_w > \hat{p}_w$. The equilibrium with player-specific revisions satisfies the following property.

Proposition 4. *Assume that $\tilde{\sigma}^X$ and $\tilde{\sigma}^Y$ are a subgame-perfect equilibrium for respectively the mechanism $\tilde{\Gamma}^X$ and $\tilde{\Gamma}^Y$ with player-specific revisions and that the weak player's bidding strategy is weakly undominated if (6) or (7) holds. If $\tilde{\Gamma}^X$ and $\tilde{\Gamma}^Y$ only differ in the comprehensive revision procedure of player i with player-specific revisions and if $S^{X,i} \subseteq S^{Y,i}$, then player i weakly prefers the equilibrium allocation $\tilde{u}^Y$ to $\tilde{u}^X$.*

Proof. See appendix. ■

The more a negotiator is susceptible to feelings of frustration from unfulfilled expectations, the less proficient he will be in negotiating. The higher his fear of disappointing his principal, the less ambitious the targets set by his principal and the less favorable the resulting agreement. In particular, the equilibrium payoff of a player can never decrease if larger concessions become feasible. If he can make larger concessions at his equilibrium claim, then his extended Nash product for that claim and his opponent's resistance probability for the same pair of claims increase. This allows him to increase his claim which, possibly after a reduction of his opponent's claim, increases his payoff when the equality of the extended Nash products is restored.[5]

5.3 Alternating offers

The 4-stage mechanism Γ^R allows for take-it-or-leave-it offers, possibly resulting in disagreement. This contrasts with the alternating offer game proposed by Rubinstein (1982) in which players respond by accepting a compromise or by proposing a new compromise. We propose a modification $\tilde{\Gamma}^{R_\varepsilon}$ of Γ^R in line with Rubinstein's alternating offer game which shows that this contrast is essential. After claims are made in the first stage, players take turns in making proposals in their compromise set until one player accepts his opponent's proposal or pursues his initial demand in a take-it-or-leave-it offer. The proposer i builds up resistance q_i before he proposes in $U^i(p_i)$. Building up resistance to block a take-it-or-leave-it response of his opponent to his compromise is requires time. It is assumed that the depreciation of the payoffs in this time interval is $1 - \exp(-\varepsilon q_i)$.

In line with Rubinstein's alternating offer game, we focus on the mechanism where all revisions are feasible and players start by making maximal claims $p = \mathbf{1}$ in equilibrium. For each element of a decreasing sequence of small positive ε, consider a compromise proposal $c^i(\varepsilon) \in PO(S^*)$ and resistance probability $q_i(\varepsilon)$ for each player $i \in N$. For the compromises to be proposed and accepted in equilibrium, one needs

$$c_i^{-i}(\varepsilon) = \exp(-\varepsilon q_i(\varepsilon)) c_i^i(\varepsilon) = (1 - q_{-i}(\varepsilon)) \text{ for all } i \in N. \tag{8}$$

By the equality of the first two terms in (8), accepting the opponent's offer $c^{-i}(\varepsilon)$ is as good as waiting for a time $\varepsilon q_i(\varepsilon)$ before proposing $c^i(\varepsilon)$ for all $i \in N$. For equal waiting times $\varepsilon q_i(\varepsilon) = \varepsilon q_{-i}(\varepsilon)$, the Nash products of the proposals are equal, as in Rubinstein's game. By the equality of the first and last term in (8), accepting the opponent's offer $c^{-i}(\varepsilon)$ is as good as stopping with a take-it-or-leave-it offer, in which case the initial demand is obtained with probability $(1 - q_{-i}(\varepsilon))$. Before proposing $c^{-i}(\varepsilon)$, player $-i$ has built up a resistance probability $q_{-i}(\varepsilon) = r_i\left(c_i^{-i}(\varepsilon), 1\right)$ which deters the making

[5] The same property holds for mediated revisions, unless the revised demand of the strong player is implemented for $p_s = 1$.

of a take-it-or-leave-it offer, as in Γ^R.

By combining the equalities in (8) for both $i \in N$, it follows that

$$\frac{\ln c_2^2(\varepsilon) - \ln c_2^1(\varepsilon)}{\ln c_1^1(\varepsilon) - \ln c_1^2(\varepsilon)} = \frac{r_1\left(c_1^2(\varepsilon), 1\right)}{r_2\left(c_2^1(\varepsilon), 1\right)}.$$

For $\varepsilon \to 0$, $c^1(\varepsilon)$ and $c^2(\varepsilon)$ converge to c^*, which by l'Hopital's rule satisfies

$$-\left.\frac{d \ln c_2}{d \ln c_1}\right|_{c_2^* = u_2^*(c_1^*)} = \frac{r_1(c_1^*, 1)}{r_2(c_2^*, 1)} = \frac{1 - c_1^*}{1 - c_2^*}.$$

The lefthand side is increasing in c_1^* and equal to 1 for $c^* = u^N$. The righthand side is decreasing in c_1^* and equal to 1 for $c^* = u^{KS}(\mathbf{1})$. Hence, the compromise c^* lies strictly in between the Nash solution u^N and the proportional solution $u^{KS}(\mathbf{1})$, unless both solutions coincide.

Assume that $u_i^N > u_i^{KS}(\mathbf{1})$. The introduction of take-it-or-leave-it offers in Rubinstein's alternating offer game moves the equilibrium solution away from the Nash solution towards the proportional solution. When the Nash solution is proposed as a compromise, player $-i$'s risk limit when pursuing his dictatorial outcome is greater than player i's risk limit since $u_i^N > u_{-i}^N$. Player i needs more time to build up the necessary resistance, so that his higher impatience inhibits him to obtain a compromise as good as u^N. In Rubinstein's game, the player's impatience is exogenously determined by the waiting time for making a counterproposal. In $\tilde{\Gamma}^{R_\varepsilon}$, the impatience of the players is endogenized by the choice of resistance. A player's impatience thus increases with his own payoff in his compromise proposal, as he requires a higher resistance to make this compromise acceptable.

Similarly, the introduction of alternating offers in an extension of the 4-stage mechanism with unrestricted revisions moves the equilibrium solution away from the proportional solution towards the Nash solution. Since the solution to Γ^R implies equal risk limits, its implementation in $\tilde{\Gamma}^{R_\varepsilon}$ would imply equal waiting times between alternating offers equal to $\varepsilon q^{KS}(\mathbf{1})$. However, for short equal waiting times, a compromise is proposed and accepted only if the payoffs are close to those in the Nash solution. Proposing an offer which deters ultimatums is necessary but not sufficient for its acceptance with an option to counter offers. The anticipation of counteroffers with ultimatums results in unequal waiting times. The player preferring the proportional solution to the Nash solution will make a proposal which his opponent prefers to the proportional solution. As this reduces the opponent's risk limit, he can reduce his resistance needed to block an ultimatum below $q^{KS}(\mathbf{1})$ and thus the time he must wait before making his proposal.[6]

[6] Player-specific revision procedures with incompatible revisions for $p(\varepsilon)$ and $\varepsilon > 0$ result in gaps which could allow a player to propose within his feasible set of compromises and to reject any feasible compromise of his opponent given the waiting times needed to resist ultimatums. Hence, if this gap can be closed, players make claims such that $p(\varepsilon) \to p^*$ and $c^i(\varepsilon) = u^i(p_i(\varepsilon)) \to c^* = u^i(p^*)$ for all

6 Examples

The maximal revision of a player's claim in a comprehensive revision procedure can be obtained by specifying the opponent's payoff $u^i_{-i}(p_i)$ as a concave function of the player's claim p_i or by specifying the convex sets S^1 and S^2 in the square D, containing the bargaining set S^*. We provide three examples.

Example 1: Flexibility. Consider the convex set

$$S^i = \nu_i S^* \cap D \text{ for } \nu_i \geq 1 \text{ and all } i \in N.$$

By varying ν_i, one can construct a family of nested comprehensive revision procedures, as in Proposition 4. Obtain u^N for $\nu_1 = \nu_2 = 1$. Obtain $u^{KS}(\mathbf{1})$ for $\nu_1 = \nu_2$ above some threshold. All Pareto efficient allocations in between these two solutions can be obtained for intermediate values.

The parameter ν_i captures the flexibility of the negotiators in the revision of their claims. This flexibility may depend on the cost of replanning. Plans must often be well documented before they are workable and the duties of the parties must be correctly specified before they can be written in a contract for complex relationships. Player i never loses when his flexibility ν_i is increased for given ν_{-i}, as in Proposition 4.□

Example 2: Accountability and Frustration. Let μ_i and λ_i be increasing functions defined on $[0,1]$ for $i = 1, 2$. The function μ_i is convex and $p_i - \mu_i(p_i) \geq 0$ with equality for $p_i = 0$. The function λ_i is concave and $\lambda_i(u_i) - u_i \geq 0$ with equality for $u_i = 1$. For $\kappa \leq 1$, $0 \leq \varphi \leq 1$ and $i = 1, 2$, the opponent's payoff for the maximal revision of player i's claim p_i is defined by

$$u^i_{-i}(p_i) = \left\{\varphi \left[u^*_{-i}(\mu_i(p_i))\right]^\kappa + (1-\varphi)\left[\lambda_{-i}\left(u^*_{-i}(p_i)\right)\right]^\kappa\right\}^{\frac{1}{\kappa}}.$$

If $\varphi = 1$, a player's maximal revision depends on the own maximal utility loss $p_i - \mu_i(p_i)$ he can bear. If $\varphi = 0$, a player's maximal revision depends on the opponent's maximal utility gain $\lambda_{-i}\left(u^*_{-i}(p_i)\right) - u^*_{-i}(p_i)$ he can tolerate. The restrictions on μ_i and λ_{-i} ensure that u^i_{-i} is decreasing and concave in p_i. By combining the two approaches, the opponent's payoff in the maximal revision is a weighted average ($\kappa = 1$) or converges to the minimum ($\kappa \to -\infty$) of the payoffs obtained in the two cases.

In many applications, negotiators are agents defending interests of their principal.

$i \in N$ and for $\varepsilon \to 0$. Moreover,

$$-\left.\frac{d \ln c_2}{d \ln c_1}\right|_{c_2^* = u_2^*(c_1^*)} = \frac{r_1(c_1^*, p_1^*)}{r_2(c_2^*, p_2^*)} = \frac{p_1^* p_2^* - p_1^* u_2^1(p_1^*)}{p_1^* p_2^* - -p_2^* u_1^2(p_2^*)}.$$

Similar remarks hold for the alternating offer game with restricted revisions when $u^1(p_1) = u^2(p_2) = u^{KS}(p)$ in the equilibrium of Γ^R.

Crawford (1982) refers to costs for negotiators who retreat on a position that they have agreed to defend. Negotiators have limited authority and are accountable to their principals. If revising targets must be justified, revisions of claims will be limited. These features can be modelled in the first approach. However, experienced negotiators will try to convince their principals that room is needed for compromise and complement the first approach by features belonging to the second approach. Similarly, revisions are limited by frustration or concession aversion (Kahneman and Tversky, 1995). Frustration may arise not only because claims raise unfulfilled expectations, but also because one's opponent gains in a disproportionate way. This again requires a two-sided approach.□

Example 3: Mediated revision procedures. As in Miyagawa (2002), consider a mediator who steers the negotiations and designs the revision procedures. The mediator can implement her preferred solution $u^c \in PO(S^*)$ by the revision procedures associated with

$$S^1 = S^2 = \{u \in D | \, u_1 u_2^c + u_2 u_1^c \leq W\} \equiv S^c,$$

and with $\sup\{u_1 u_2^c + u_2 u_1^c | \, u \in S^*\} \leq W \leq u_1^c u_2^c + \min\{u_1^c, u_2^c\}$. The lower bound ensures that $S^* \subseteq S^c$. The upper bound ensures that the claims are feasible in equilibrium. For any pair of claims $p \geq \hat{p}$ associated with equal extended Nash products (i.e., $p_i = \check{p}_i(p_{-i})$ for both $i \in N$), the payoffs for the follower in the revised demands are proportional to u^c (i.e., $\left(u_1^2(p_2), u_2^1(p_1)\right) = \tilde{\lambda} u^c$). Obtain the pair of equilibrium claims p when the revised demands meet in u^c (i.e., $u^1(p_1) = u^2(p_2) = u^{KS}(p) = u^c$).[7,8]

When player i increases his claim, his opponent's revised payoff decreases at the rate $\frac{u_{-i}^c}{u_i^c}$. The lower this rate, the more player i can increase his claim without losing his ability to find acceptable compromises. If the mediator sets equal rates for both players ($u_{-i}^c / u_i^c = 1$ for both $i \in N$), the proportional solution $c^{KS}(\mathbf{1})$ is implemented. If the mediator sets unequal rates, the player with the lower rate obtains a higher utility in the equilibrium allocation. When $u_{-i}^N / u_i^N < 1$ for an asymmetric bargaining set S^*, the mediator can implement the Nash solution by decreasing the rate for player i to $u_{-i}^N / u_i^N < 1$ (and thus increasing player $-i$'s rate to u_i^N / u_{-i}^N).□

7 Conclusion

We analyzed a simple, intuitive mechanism that implements a unique solution to the bargaining problem with two players. The mechanism introduces take-it-or-leave-it

[7] The pair $\hat{p}$, which defines for each player the claim that maximizes his extended Nash product, is the intersection of the half-line $L = \{\lambda u^c | \, \lambda \geq 0\}$ (with slope $\frac{u_2^c}{u_1^c}$) and the frontier of S^c (with slope $-\frac{u_2^c}{u_1^c}$). Since $S^1 = S^2$, it follows that $u^1(\hat{p}_1) = u^2(\hat{p}_2)$.

[8] If $W > u_1^c u_2^c + \min\{u_1^c, u_2^c\}$, then $p_i = \frac{W}{u_{-i}^c} - \frac{u_i^c}{u_{-i}^c} u_{-i}^c > 1$ for some $i \in N$ and the constraint $p_i \leq 1$ would be violated.

offers and the need to build resistance to discourage negotiators from making such offers. We generate a whole family of solutions by varying the extent to which initial claims can be revised during the negotiations. The Nash solution is the unique equilibrium solution, if negotiators cannot revise earlier claims. The ability to revise claims was assumed to be beyond the control of the negotiators in the course of negotiations. However, if a negotiator were to suppress his feelings of frustration or if he did not fear to disappoint his principal by making large concessions, he would achieve better deals. In the evaluation of the performance of a negotiator, results loom larger than circumstances under which his results were achieved. Hence, it seems plausible that professional negotiators will strive for more room to maneuver. Similarly, principals will learn by experience to give discretionary power to their negotiators as to decide which concessions have to be made. If restrictions on revisions are loosened, restraint in the formulation of claims will eventually disappear. The predicted allocation in conflicts between experienced negotiators would be the Kalai-Smorodinsky solution.

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8 Appendix

8.1 Proofs of Propositions

Proof of Proposition 3

Assume that $p \in \dot{S}^*$, that is, players make compatible claims defining unequal initial demands. By Lemma 4, the follower's initial demand is implemented. If some player $j \in N$ increases his claim to $p'_j = u^*_i(p_{-j})$, the initial demand of $-j$ will be implemented by Lemma 4, independently of the identity of the leader, and increase j's payoff. Since no player can have a profitable deviation in subgame-perfect equilibrium, it suffices to consider equilibrium claims $p \notin \dot{S}^*$. By Lemma 2 and 3, s imposes his preferred solution in $\{u^s(p_s), u^{KS}(p)\}$ for p as a leader. He leads with certainty if $p_s < \check{p}_s(p_w)$ and with probability $\alpha_s(p)$ if $p_s = \check{p}_s(p_w)$. If $u_s^s(p_s) \geq u_s^{KS}(p)$, then u_s^s is increasing by comprehensiveness of S^s. Since the proportional solution is increasing in one's claim, the strong player's payoff is increasing in his claim. It follows that $p_s \geq \hat{p}_s$. If, on the contrary, $p_s < \hat{p}_s$, then s would remain strong by claiming $\hat{p}_s$ and increase his payoff, a contradiction. If $p_s = \check{p}_s(p_w)$, then $p_w \geq \hat{p}_w$ for the same reason.

We distinguish between two cases. In the first case, $p \neq \mathbf{1}$ in subgame-perfect equilibrium. In that case, $q^s(p) \geq q^{KS}(p)$. Assume to the contrary that $q^s(p) < q^{KS}(p)$ so that $u^{KS}(p) \in U^s(p_s) \setminus \{u^s(p_s)\}$ by Remark 2 and implemented by Lemma 2. There exists p' such that $1 \geq p'_i > p_i$ and $p'_{-i} = p_{-i}$ for some $i \in N$ and $u^{KS}(p') \in U^s(p'_s)$ is implemented. Since the proportional solution is strictly monotone in player i's claim, player i has a profitable deviation, a contradiction. Hence $u^s(p_s)$ is weakly preferred to $u^{KS}(p)$ and implemented by Lemma 2. It then follows that, $q^w(p) = q^s(p)$. Assume to the contrary that $q^w(p) > q^s(p)$ so that s remains strong for a small increase in p_s which is feasible for $p \neq \mathbf{1}$ and the increase in max $\{u_s^s(p_s), u_s^{KS}(p)\}$ implies that s has a profitable deviation, a contradiction. We distinguish between two subcases. In the first subcase, $q^w(p) = q^s(p) = q^{KS}(p)$, so that $u^w(p_w) = u^s(p_s) = u^{KS}(p)$ and $p \geq \hat{p}$. Since the payoffs of both the proportional solution and the own revised demand are non-decreasing in the own claim, a player cannot gain by reducing his claim. Since a player is always weak after increasing his claim, his opponent chooses his revised demand and that the allocation remains unchanged. As profitable deviations are excluded, $u^{KS}(p)$ is implemented. In the second subcase, $q^w(p) = q^s(p) > q^{KS}(p)$, so that $u^s(\check{p}_s(\hat{p}_w))$ is preferred by s to $u^{KS}(p)$ by Remark 2 and implemented for $\alpha_s(p) = 1$ and for $p_w = \tilde{p}_w$ and $p_s = \check{p}_s(\hat{p}_w) \geq \hat{p}_s$ by Lemma 5. Player w cannot change the allocation since he remains weak for all his claims. Player s's payoff would strictly be reduced for any other claim than $\check{p}_s(\hat{p}_w)$: for lower claims, because s's payoff of the own revised demand is strictly decreasing for $u_s^s(p_s) \geq u_s^{KS}(p)$ and for higher claims because $u^w(\hat{p}_w)$ would be implemented when w becomes strong. Hence, none of the players has a profitable deviation for p and would have a profitable deviation for any other pair of claims equalizing the extended Nash products for $q^s(p) > q^{KS}(p)$.

In the second case, $p = \mathbf{1}$ in subgame-perfect equilibrium. In that case, there does not exist $p \neq \mathbf{1}$ such that $q^w(p) = q^s(p) \geq q^{KS}(p)$ and (6) or (7) hold. By Lemma 5, $s(\mathbf{1})$ implements his preferred solution in $\{u^s(1), u^{KS}(\mathbf{1})\}$. If he chooses $u^s(1)$ and $u^{s(\mathbf{1})}(1) = u^{KS}(\tilde{p})$, then $w(\mathbf{1})$ claims in $[\tilde{p}_{w(\mathbf{1})}, 1]$ for $\tilde{p}_{s(\mathbf{1})} = 1$ ■

Proof of Proposition 4

We show that for both players bidding $q^w(p)$ is an equilibrium bidding strategy when $u^{KS}(p) \notin U^w(p_w)$ for $p \notin S^*$ and $\alpha_s(p) = 1$. For this bidding strategy, the

Pareto-efficient allocation $\tilde{u}^c \in U^s(p_s)$ is implemented for which, by (3),

$$u_w^*(p_s) \leq \tilde{u}_w^c = (1 - q^w(p))\, p_w = \frac{p_w u_s^w(p_w)}{p_s} \leq u_w^s(p_s).$$

Leadership and the allocation does not change if w raises his bid. Since s rejects any proposal in $U^w(p_w)$ for a bid below $q^w(p)$, w's payoff as a leader for $q_w < q^w(p)$ is $(1 - q_w)\, u_w^*(p_s) < u_w^*(p_s) \leq \tilde{u}_w^c$. For $q_s > q^w(p) = q_w$, the payoff of s as a follower is $(1 - q^w(p))\, p_s = u_s^w(p_w) < u_s^s(p_s) \leq \tilde{u}_s^c$. Since $\tilde{u}_w^c$ is decreasing in s's bid and since any proposal in $U^s(p_s)$ is rejected for a bid below $q^s(p)$, s cannot gain as a leader by decreasing his bid. It follows that for bids equal to $q^w(p)$, none of the players has a profitable deviation. When $u^{KS}(p) \in U^w(p_w)$ for $p \notin S^*$ and $\alpha_s(p) = 1$, both players bid $q^{KS}(p)$ and $u^{KS}(p)$ is implemented, by Lemma 2.

Since for any $p \notin S^*$ and $\alpha_s(p) = 1$, player s achieves a payoff, which is bounded below by $\max\left\{u_s^s(p), u_s^{KS}(p)\right\}$, the payoff with mediated revisions. Hence, the equilibrium remains the same in the game with unmediated revisions, when competition to become strong results in equalized extended Nash products in the game with mediated revisions. Hence, if (6) and (7) do not hold, the equilibria coincide by Proposition 3. Otherwise, player s formulates the maximal claim $p_s = 1$ and proposes his preferred allocation in $\left\{\tilde{u}^c, u^{KS}(p)\right\}$, with $\tilde{u}_w^c = p_w u_s^w(p_w)$. Player w formulates the claim that minimizes the maximum payoff of player s. If $u_s^{KS}(\mathbf{1}) \geq \tilde{u}_s^c$, $p_w = 1$. Otherwise, either $p_w = \hat{p}_w$ (and $\tilde{u}_s^c \geq u_s^{KS}(p)$), or $\tilde{u}^c = u^w(p_w) = u^{KS}(p)$ for $p_w > \hat{p}_w$.

For player-specific revisions, we compare the equilibrium allocation $\tilde{u}^X$ for p^X in $\tilde{\Gamma}^X$ with $\tilde{u}^Y$ for p^Y in $\tilde{\Gamma}^Y$ by extending $S^{X,i}$ to $S^{Y,i} \supset S^{X,i}$ for some $i \in N$, so that $U^{X,i}(p_i) \subset U^{Y,i}(p_i)$ and $U^{X,-i}(p_{-i}) = U^{Y,-i}(p_{-i})$ for all p. We distinguish between three cases. In the first case, $p_i^X = 1$ and $s\left(p^X\right) = i$. It follows that $\tilde{u}^Y = \tilde{u}^X$. Either $\tilde{u}_{-i}^X = u_{-i}^{KS}(p)(\leq u_{-i}^{X,i}(1))$ or $\tilde{u}_{-i}^X = \hat{p}_{-i}^X u_i^{X,-i}\left(\hat{p}_{-i}^X\right)(\leq u_{-i}^{X,i}(1))$. The increased payoff for player $-i$ in the revised demand of player i, $u_{-i}^{X,i}(1) < u_{-i}^{Y,i}(i)$, does not affect the equilibrium allocation. In the second case, the extended Nash products of the claims are equal in $\tilde{\Gamma}^X$. Since the extended Nash product of player i's claims are at least as large in $\tilde{\Gamma}^Y$, there may exist a claim $\tilde{p}_i^Y > p_i^X$ for which $u_i^i\left(\tilde{p}_i^Y\right) > u_i^i\left(p_i^X\right)$ and $\tilde{p}_i^Y u_{-i}^i\left(\tilde{p}_i^Y\right) = p_i^X u_{-i}^i\left(p_i^X\right)$ or for which $u^i\left(p_i^X\right)$ is still feasible and $\tilde{p}_i^Y u_{-i}^i\left(p_i^X\right) > p_i^X u_{-i}^i\left(p_i^X\right)$. In equilibrium, if the extended Nash products are equalized in $\tilde{\Gamma}^Y$, then $u_i^{Y,i}\left(p_i^Y\right) \geq u_i^{X,i}\left(p_i^X\right)$. Otherwise, there exists $S^{Z,i}$, $S^{Y,i} \supseteq S^{Z,i} \supseteq S^{X,i}$ for which $p_i^Z = 1$ and the extended Nash products are equalized in $\tilde{\Gamma}^Z$ with $u_i^{Z,i}(1) \geq u_i^{X,i}\left(p_i^X\right)$. The extension of $S^{Z,i}$ to $S^{Y,i}$ does not change the allocation. In the third case, $p_{-i}^X = 1$ and $s\left(p^X\right) = -i$. If $\tilde{u}_i^X = \tilde{u}_i^c = \left(1 - q^i\left(p^X\right)\right) p_i^X = p_i^X u_{-i}^{X,i}\left(p_i^X\right)$, then either $\tilde{u}_i^Y = p_i^Y u_{-i}^{Y,i}\left(p_i^Y\right) \geq \tilde{u}_i^X$ or there exists $S^{Z,i}$, $S^{Y,i} \supseteq S^{Z,i} \supseteq S^{X,i}$ for which $\tilde{u}_i^Z \geq \tilde{u}_i^X$ and the extension of $S^{Z,i}$ to $S^{Y,i}$ increases i's payoff as in case 2. Otherwise, $\tilde{u}^X = u^{KS}(\mathbf{1}) = \tilde{u}^Y$.

We conclude the proof by showing that both player bidding $q^w(p)$ is the unique equilibrium which survives elimination of weakly dominated bidding strategies for the weak player for $p_s = 1$. There do not exist other equilibria in which the leader's bid exceeds $q^w(p)$. In that case, the follower can lead for the bid $q^w(p)$ and impose a proposal in $U^F(p_F)$ rather than being imposed a proposal in $U^L(p_L)$ and gain when revised claims are incompatible. There do not exist equilibria in which the leader's bid is smaller than $q^s(p)$, when both players prefer their ultimatum to any of their opponent's feasible offers and both players prefer to be the follower. If the Pareto-efficient allocation $\hat{u} \in U^s(p_s)$, with $\hat{u}_w = (1-\hat{q})\,p_w$ and $\hat{q} \in [q^s(p), q^w(p)]$, satisfies $(1-\hat{q})\,p_s \le \hat{u}_s$ when $U^s(p_s) \cap U^w(p_w) = \emptyset$, then bidding $\hat{q}$ is also an equilibrium strategy for both players. However, the weak player's bidding strategy q_w, $q^s(p) \le q_w < q^w(p)$, is weakly dominated by a bid equal to $q^w(p)$. For $q_s < q^s(p) \le q_w$, s is leader and w rejects any feasible compromise of s so that w obtains $(1-q_s)\,p_w$ as a payoff, which is independent of q_w. For $q^s(p) \le q_s < q_w \le q^w(p)$, w is leader and s rejects any feasible compromise of w so that w obtains $(1-q_s)\,u_w^*(p_s)$ as a payoff which is smaller than $(1-q^w(p))\,p_w$ obtained for $q_w = q^w(p)$. For $q_s \ge q^w(p)$, w is leader and obtains $u_s^w(p_w)$ as a payoff. Since $q^s(p) = q^w(p)$ in a subgame-perfect equilibrium for $p_s < 1$, the refinement which eliminates weakly dominated strategies of the weak player for $p_s = 1$ suffices for the property of Proposition 4.■

8.2 Proofs of Lemmas

Proof of Lemma 1.

The choices of L after the history h^2 and of F after the history $h^3 = (\hat{c}, h^2)$ in σ are determined by respectively (4.1) and (4.2) as in a Stackelberg two-stage game. Since $U(p)$ is non-empty, closed and bounded, the maxima in condition (4.2) are well defined.

For all $i \in N$, the unrestricted compromise

$$\tilde{u}^i(q_i) \in \arg\max_{u \in S^*} [u_i |\, r_{-i}(u_{-i}, p_{-i}) \le q_i]$$

is a uniquely defined Pareto efficient outcome in S^*. Hence, $\tilde{u}_i^i(.)$ and $-\tilde{u}_{-i}^i(.)$ are strictly increasing for $u_{-i} \ge p_{-i}$. It follows that F accepts u if and only if $u \in U(p)$ and $u_F \ge \tilde{u}_F^L(q_L)$. Therefore, L proposes $u \in PO(S^*)$ such that either $q_L' < q_L$ or $\tilde{u}_L^L(q_L') \notin U(p)$ whenever $\tilde{u}_L^L(q_L') \ge u_L$.

Since $r_F(., p_F)$ is non-increasing, $0 = r_F(p_F, p_F) \le r_F(u_F^F(p_F), p_F)$ and $r_F(u_F^L(p_L), p_F) \le r_F(u_F^*(p_L), p_F)$, obtain $\tilde{u}^L(q_L)$ in $U^F(p_F) \cup U^L(p_L)$ by varying q_L on $\left[0, r_F(u_F^F(p_F), p_F)\right] \cup \left[r_F(u_F^L(p_L), p_F), r_F(u_F^*(p_L), p_F)\right]$. For any q_L in these intervals, $u^c = \tilde{u}^L(q_L)$ will be proposed by L and accepted by F and u_L^c is increasing in q_L. If $U^F(p_F) \cap U^L(p_L) = \emptyset$, then $u_F^F(p_F) > u_F^L(p_L)$. For $r_F(u_F^F(p_F), p_F) < q_L < r_F(u_F^L(p_L), p_F)$ in that case, $u^L(p_L)$ will be rejected by F and $u^F(p_F)$ is proposed by L and accepted by F. Finally, for $q_L \in \left(r_F(u_F^L(p_L), p_F), 1\right]$, L's demand is proposed by L and accepted by F.■

Proof of Lemma 2

For $p \notin S^*$ and $u_s^{KS}(p) > u_s^s(p_s)$, $p_s > u_s^{KS}(p) > u_s^s(p_s)$ and $u_w^s > u_w^{KS}(p) > u_w^*(p)$ and $u^{KS}(p) \in U(p)$. If both players bid $q^{KS}(p) = r_F\left(u_F^{KS}, p_F\right)$, then L proposes $u^{KS}(p)$ and F accepts by Lemma 1. Since L is the lowest bidder, the allocation remains unchanged for a higher bid and its utility is reduced for the lower bidder by the monotonicity of $r_F(., p_F)$. Since no player has a profitable deviation, both bidding $q^{KS}(p)$ is an optimal response. The payoff of player $i \in N$ is bounded below by $u_i^{KS}(p)$ for $q_i = q^{KS}(p)$. If this bound were not tight for some bidding strategy, then one of the players would obtain a payoff below this bound and would have a profitable deviation. Conclude that $q^{KS}(p) = a_i^\sigma(h_1)$ for $p \notin S^*$ and $u_s^{KS}(p) > u_s^s(p_s)$ and for $i \in N$.■

Proof of Lemma 3

For $p \notin S^*$ and $u_s^s(p_s) \geq u_s^{KS}$ by Remark 1 and 2, $q^w(p) \geq q^s(p) \geq q^{KS}(p)$ and $u^{KS}(p) \notin U(p)$, unless $q^w(p) = q^s(p) = q^{KS}(p)$. Since $u_s^w(p_w) \leq u_s^s(p_s) \leq p_s$, it follows that $0 \leq r_s(u_s^s(p_s), p_s) \leq q^s(p)$. The bidding strategies in (5) are well defined. By Lemma 1, a proposal u^c of L is proposed and accepted for $q_F \geq q_L$ if and only if $q_L \geq r_F(u_F^c, p_F)$. We distinguish between two cases.

In the first case, $q^w(p) > q^s(p)$. We show that for bidding strategies satisfying (5), $u^s(p_s)$ is implemented. If $w = L$, then $q_w \leq q_s < q^s(p)$ and s rejects proposals in $U^w(p_w)$. As a result, w cannot do better than by proposing $u_s^s(p_s)$ in $U^s(p_s)$ which is accepted by s because $q_w \geq r_s(u_s^s(p_s), p_s)$. If $s = L$, then $q_s \geq q^s(p)$ implies that $u^s(p_s)$ is accepted by w. If $u_s^s = p_s$, then u^s is the best outcome in $U^s(p_s)$ for s. If $u_s^s < p_s$, then $q^s(p) \leq q_s \leq q_w \leq q^s(p)$ implies that $q_s = q^s(p)$ and that any better proposal for w in $U^s(p_s) \setminus \{u^s(p_s)\}$ is rejected by s. We show that for any other pair of bids q, some player has a profitable deviation. Player w never wants to lead with $q_w < r_s(u_s^s(p_s), p_s)$ because he is not strong enough to impose proposals which are as good as $u^s(p_s)$ or has to accept s's ultimatum. For $q_w > q^s(p)$ and $u_s^s < p_s$, player s can lead for q_s, $q_w > q_s > q^s(p)$, and impose a better compromise in $U^s(p_s) \setminus \{u^s(p_s)\}$. For $q_s < q^s(p)$, player s will lead for $q_s < q_w < q^s(p)$ and, since he is not strong enough to impose a compromise in $U^s(p_s)$, has to propose in $U^w(p_w)$ or has to accept w's ultimatum. Finally, for $q_s \geq q^w(p)$, player w will lead with positive probability and can propose in $U^w(p_w)$.

In the second case, $q^w(p) = q^s(p)$. Since $p \notin S^*$, the case $u_i^i = p_i$ and $q^i(p) = q^{KS}(p)$ for all $i \in N$ is excluded. For $q_w = q_s = q^w(p) = q^s(p)$, with probability $\alpha_L(p)$, L proposes $u^L(p_L)$ which is accepted by F. One loses with a higher bid, because one's opponent proposes his revised demand with probability 1. One loses with a lower bid, because one is to weak to impose one's own revised demand for a lower bid. Hence, equal bids do not allow for a profitable deviation and unequal bids or other equal bids would allow for a profitable deviation for some player.

It follows that the bidding strategies in subgame-perfect equilibrium are as in (5).■

Proof of Lemma 4

Since $r_F(p_F, p_F) = 0$, F accepts his demand for all q_L in the unit interval. If $p_2 = u_2^*(p_1)$, the demands of the players coincide and $U(p) = \{(p_1, u_2^*(p_1))\}$. Hence, for all $q \in D$, L proposes $(p_1, u_2^*(p_1))$ and F accepts. If $p \in \dot{S}^*$, the opponent's demand is a player's preferred allocation in $U(p)$ and $q = \mathbf{0}$. Each player is leader with equal probability, L proposes F's demand and F accepts by Lemma 1. If some player were to increase his bid, then his opponent would lead with certainty and reduce his expected payoff. For $q \neq \mathbf{0}$, one of the players with a positive bid has a profitable deviation. By a zero bid, he increases his probability of winning leadership. Hence, $a^\sigma(h^1) = \mathbf{0}$.■

8.3 Proofs of Remarks

Proof of Remark 1 and 2

Assume that $p \notin \dot{S}^*$, so that $p_i \geq u_i^*(p_{-i}) \geq u_{-i}^i(p_i) \geq 0$ for all $i \in N$. It follows that $1 - q^i(p) = \dfrac{u_{-i}^i(p_i)}{p_{-i}} = \dfrac{p_i u_{-i}^i(p_i)}{p_1 p_2} \geq 0$ for all $i \in N$, so that

$$q^i(p) \geq q^{-i}(p) \text{ if and only if } p_i u_{-i}^i(p_i) \leq p_{-i} u_i^{-i}(p_{-i}).$$

Hence, by monotonicity of $r_{-i}(., p_{-i})$, the definition of $q^{KS}(p)$ and condition (3), Remark 2 holds and $u^{KS}(p) = u^i(p_i)$ if and only if $q^{KS}(p) = q^i(p)$. Assume that $p = (p_1, \check{p}_2(p_1))$. It follows that $q^1(p) = q^2(p)$ and that $q^{KS}(p) \geq q^1(p) = q^2(p)$ if and only if each player weakly prefers $u^{KS}(p)$ to his revised demand, which implies Remark 1.■