Insurance and Perceptions: How to Screen Optimists and Pessimists

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Abstract

People have very different beliefs about the risks they face, even when these risks are identical. I analyze how heterogeneous risk perceptions affect the insurance contracts offered by profit-maximizing firms. An essential distinction is how risk perceptions affect the willingness to pay for insurance relative to the willingness to exert risk-reducing effort. This determines both the sign of the correlation between risk and insurance coverage in equilibrium, shedding new light on a recent empirical puzzle, and the type of individuals screened by either monopolistic or competing firms. Even with perfect competition, heterogeneous risk perceptions may well strengthen the case for government intervention in insurance markets.

Keywords: Insurance Markets, Risk perceptions, Adverse Selection, Moral Hazard

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1 Introduction

The perception of risk is inherently subjective.[1] Financial traders disagree about the risk of investments, mortgage bankers about the risk of defaulting homeowners, old and young drivers about the risk of a car accident, homeowners and renters about the risk of flooding. One of two neighbours may perceive the risk of a hurricane as very high, while the other perceives the risk as very small, even though the risk is exactly the same.[2] At the same time, the perception of the extent to which precautionary efforts mitigate the risk may differ as well. As a result, the one neighbour may take precautionary measures, while the other does not. Risk perceptions thus affect both the willingness to pay for insurance and the cost of being insured, which makes them central to the design of insurance contracts.

The heterogeneity driving the variation in demand has played a central role in the analysis of insurance markets. The canonical model by Akerlof (1970) and Rothschild and Stiglitz (1976) considers heterogeneity in risks and predicts that types with higher risk buy more insurance in equilibrium. Empirically, many papers find a correlation between risk and insurance coverage that is not significantly positive (Chiappori and Salanié 1997 and 2000, Cardon and Hendel 2001) or even negative (Cawley and Philipson 1999, Finkelstein and McGarry 2006, Fang, Keane and Silverman 2008). This puzzle has inspired a recent literature to explore the potential role of heterogeneous preferences, both theoretically (De Meza and Webb 2001, Jullien, Salanié and Salanié 2007) and empirically (Cohen and Einav 2007, Einav, Finkelstein and Cullen 2010a, Einav, Finkelstein and Schrimpf 2010b). The potential role of heterogeneous risk perceptions has been largely underexplored.

This paper aims to fill this gap by analyzing how heterogeneous risk perceptions among the insured affect the equilibrium contracts offered by profit-maximizing insurers. In a model with both adverse selection and moral hazard, I find that incentive compatibility imposes a simple structure on the equilibrium contracts, allowing to relate the correlation between risk and insurance coverage to the nature of the heterogeneity in risk perceptions. I also characterize the screening distortions as a function of the interaction between market competition and risk perceptions and I contrast the welfare consequences of heterogeneity in risk perceptions in this framework with earlier welfare results regarding adverse selection.

The paper presents a simple model with two states, a good state and a bad state. Insurees exert costly effort to increase the probability that the good state occurs, but have different beliefs about this probability as a function of effort. While the insurer cannot observe an insuree's belief or effort, he perceives her risk to depend only on the

[1] Slovic (2000) surveys the research documenting the heterogeneity in the perception of risk and its determinants.

[2] Peacock et al. (2005) find that people in Florida have very different perceptions about the risk of a hurricane damaging their property, even when controlling for geographic location of their property.

effort she exerts. The insuree does not change her belief in response to the menu of insurance contracts being offered. That is, the insurer and the insurees 'agree to disagree' about the true underlying risk. The insurees' preferences satisfy a single-crossing property if the one insuree perceives the likelihood of the risk as lower than the other insuree for any given insurance contract. This is conditional on the effort levels chosen by the respective insurees. Optimism can therefore arise for two reasons. First, an insuree may be more optimistic about the baseline likelihood of the risk for the same level of effort, referred to as *baseline optimism*. Second, an insuree may be more optimistic about the marginal return of effort, referred to as *control optimism*, and therefore exert higher effort for the same insurance contract. If the single-crossing property is satisfied, the insurer can only separate the (more) optimistic insuree by offering her less insurance coverage than the (more) pessimistic insuree. This monotonicity property is independent of the nature of competition between insurers.

Optimistic agents receive less insurance, but still may be more risky ex-post if they are pessimistic about their control and exert less precautionary effort. The correlation between risk and insurance coverage crucially depends on the correlation between the perceptions about the baseline risk and the own control. With two types of insurees who only differ in their beliefs, I show that it is sufficient that the one type is more *baseline-optimistic* and *control-optimistic* for the equilibrium to satisfy the positive correlation property. For the correlation to be negative, it is necessary that the control-pessimistic type is also more optimistic about the likelihood of the risk. The model thus provides an alternative explanation, based only on heterogeneity in risk perceptions, why the correlation between risk and insurance is found to be positive in some insurance markets and negative in other.

The previous monotonicity and correlation results depend on the equilibrium contracts being incentive compatibility. The binding incentive compatibility constraints determine which types' contracts are distorted compared to the case where the insurer could observe the insuree's beliefs. This depends on the interaction between the nature of competition and the dimension in which beliefs are biased. Competing insurers distort the contract offered to the insuree who can be insured at lower cost, which depends on the exerted precautionary effort and thus the insuree's control beliefs. A monopolistic insurer distorts the contract offered to the insuree whose willingness to pay is lower, which depends on the insuree's baseline beliefs. Compared to someone who is unbiased, an optimist's willingness to pay is lower for an *insurance contract* providing more insurance than her outside option, but higher for an *incentive contract* providing less insurance than her outside option.

The distortions due to the screening of types imply that agents with heterogeneous perceptions impose information externalities on each other. An agent with biased beliefs imposes a negative externality on an agent with unbiased beliefs, when private insurers distort the unbiased agent's contract to discourage the biased agent from taking this contract. The externality is only positive when a monopolistic insurer pays a rent to the unbiased agent not to take the contract offered to the biased type. For agents with biased beliefs, the screening distortions may aggravate the distortion due to the biases in their beliefs (Spinnewijn 2009). Hence, heterogeneity in optimistic beliefs may strengthen the case for (paternalistic) government intervention through mandating insurance. This contrasts with the result in Sandroni and Squintani (2007) that heterogeneity in beliefs reduces the scope for government intervention. The heterogeneity in optimistic beliefs they consider implies that some agents with different risks perceive their risk to be the same and are pooled in equilibrium. The heterogeneity I consider implies that agents with the same underlying risk are separated in equilibrium.

Related Literature Starting with the work by Chiappori and Salanié (1997, 2000), a large literature has re-examined the heterogeneity in insurance choices and risks in insurance markets. The surprising empirical evidence (see Cohen and Siegelman 2010) that in some insurance markets risks do not explain the variation in insurance demand or are even negatively related to the demand for insurance, has changed the focus of this literature from heterogeneity in risks to heterogeneity in preferences. De Meza and Webb (2001) and Jullien, Salanié and Salanié (2007) use heterogeneous risk preferences to explain the presence of advantageous selection in models with endogenous effort. Chiappori, Jullien, Salanié and Salanié (2006) argue that such heterogeneity is generally not sufficient to explain the negative correlation if the competition in the insurance market is perfect. A number of empirical papers (see Einav, Finkelstein and Levin 2010c) attributes the variation in insurance choices, unexplained by heterogeneous risks, to heterogeneous preferences and concludes that this strongly reduces the scope for government intervention in insurance markets. Spinnewijn (2012), however, finds that if this unexplained heterogeneity is driven by plausible heterogeneity in risk perceptions, welfare conclusions are substantially different.

A number of papers has analyzed equilibrium contracts in the presence of biased beliefs. Spinnewijn (2009) focuses on moral hazard and analyzes how the biases in baseline and control beliefs affect the optimal contract in the context of unemployment insurance when these biases are known to the insurer. Jeleva and Villeneuve (2004), Chassagnon and Villeneuve (2005) and Villeneuve (2005) focus on adverse selection only, analyzing equilibrium contracts in the standard Rothschild-Stiglitz model, but allowing risk types to misperceive their risks. As discussed before, Sandroni and Squintani (2007) also start from the Rothschild-Stiglitz model, but assume that some agents of the high-risk type are optimistic and think they are a low-risk type. Sandroni and Squintani (2010) generalize the analysis for a monopolistic insurer, focusing on the relation between observable variables in equilibrium. They find that in contrast with the competitive case the presence of optimistic high-risk types does not lead to major changes in testable implications in the case of monopoly. In contrast with these previous models, I consider a model with moral hazard and adverse selection and with differing beliefs as the only source of heterogeneity. That is, both types have the same ex ante risk, but differ in their beliefs about this risk. Since they can exert effort as well, the beliefs (about one's control) drive the heterogeneity in risks as well. The analysis distinguishes the results that are independent of the market structure, like the monotonicity and correlation results, but also shows that the impact of the market structure on the positive and normative results depends crucially on the dimension in which beliefs are different. Regarding the correlation between risk and insurance coverage, Koufopoulos (2008) and Huang, Liu and Tzeng (2007) also use heterogeneity in perceptions to illustrate the possibility of negative correlation. In particular, they consider a model a la Rothschild and Stiglitz, where one of the two types does exert precautionary effort, but is still more pessimistic about the probability of the risk and thus buys more insurance in the competitive equilibrium. I provide a more general framework to capture this intuition and relate the correlation between risk occurrence and insurance coverage explicitly to the correlation between baseline and control beliefs.

Finally, the paper also relates to the literature that explores what happens when boudedly rational consumers meet profit-maximizing firms (see Spiegler 2011, Ellison 2006). In particular, Grubb (2009) and Eliaz and Spiegler (2008) analyze how firms exploit differences in overconfidence and optimism about future demand respectively with a menu of screening contracts. In the spirit of this literature, I also consider the externalities that biased agents and unbiased agents impose on each other (e.g., DellaVigna and Malmendier 2004, Gabaix and Laibson 2006) and how these are affected by the market structure.

The remainder of the paper is organized as follows. Section 2 introduces the model and defines the agent's beliefs. Section 3 analyzes properties of incentive compatible contracts with heterogeneity in beliefs. Section 4 characterizes the optimal screening contracts, contrasting the competitive equilibrium and the monopolistic optimum. Section 5 discusses welfare and policy implications. Section 6 presents a simple application with continuous output and linear contracts, as in Holmström and Milgrom (1987). Section 7 concludes the paper. All proofs are in the appendix.

2 Model

I consider a principal-agent model with two states. In the good state, the total endowment equals W. In the bad state, a loss L is incurred and the total endowment equals $W - L$. The agent's unobservable choice of effort determines the probability that the good or bad state occurs. When she exerts effort at additive cost $e \in E$, the good state occurs with probability $\pi(e)$ with $\pi' \geq 0, \pi'' < 0$. The bad state occurs with probability $1 - \pi(e)$. A risk-neutral principal offers a contract to the risk-averse agent. For notational convenience, we describe the contract by the payoff-relevant terms for the agent. A contract is denoted by (w, Δ), where w is the agent's wealth net of the premium and Δ is the deductible. That is, the consumption levels of the agent, conditional on accepting the contract, are w and $w - \Delta$ in the good and bad state respectively. The deductible determines the consumption risk left to the agent. The higher the deductible, the less insured the agent is. The agent's outside opportunity is denoted by (w_0, Δ_0), such that the difference $w_0 - w$ equals the insurance premium that the agent pays to reduce her consumption risk from Δ_0 to Δ. Hence, the payoff for the principal depends on both (w, Δ) and (w_0, Δ_0), but this will be only introduced in Section 4.

I will allow the agent's outside option (w_0, Δ_0) to be different from the outcome without insurance (W, L). If the contract's deductible $\Delta < \Delta_0$, I call the contract an *insurance contract*. If the contract's deductible $\Delta > \Delta_0$, I call the contract an *incentive contract*. The principal's outside option equals $(W - w_0, L - \Delta_0)$ and the set of contracts that he can offer is restricted to

$$C \equiv \{(w, \Delta) \,|\Delta \in [0, L], w \in [\Delta, W]\}.$$

The agent cannot be overinsured, i.e. $\Delta \geq 0$, which follows immediately if the agent could make the bad state occur with certainty at zero cost.

2.1 The Agent's Beliefs

The agent's perception of the probability of success as a function of effort may differ from the true probability. I denote the agent's belief as $\hat{\pi}(e)$ with $\hat{\pi}' \geq 0, \hat{\pi}'' < 0$. I introduce these beliefs in the most general way, but the analysis shows that the differences in the levels and margins of the perceived probability functions are essential.

Definition 1 *Agent i is baseline-optimistic if $\hat{\pi}_i(e) \geq \pi(e)$ for all $e \in E$. Agent i is more baseline-optimistic than agent j if $\hat{\pi}_i(e) \geq \hat{\pi}_j(e)$ for all $e \in E$.*

Definition 2 *Agent i is control-optimistic if $\hat{\pi}'_i(e) \geq \pi'(e)$ for all $e \in E$. Agent i is more control-optimistic than agent j if $\hat{\pi}'_i(e) \geq \hat{\pi}'_j(e)$ for all $e \in E$.*

For expositional purposes, I consider the sign of the differences to be the same for all effort levels. Baseline and control beliefs are related, but optimism in the one dimension does not exclude pessimism in the other dimension. Whether agents who are more optimistic about the baseline probability are also more optimistic about their control depends on the context, as in the following two examples.

Example I $\pi(e) = \theta e$ *and* $\hat{\pi}(e) = \hat{\theta} e$ *with* $e \in [0, \min\{1/\theta, 1/\hat{\theta}\}]$*:*

> When for a project the probability of success is complementary in the entrepreneur's ability θ and effort e, an entrepreneur who overestimates his ability (i.e. $\hat{\theta} > \theta$) is at the same time baseline-optimistic and control-optimistic.

Example II $1-\pi(e)=\phi(1-e)$ *and* $1-\hat{\pi}(e)=\hat{\phi}(1-e)$ *with* $e\in[0,1]$:

A driver who underestimates the probability to have an accident when exerting no effort (i.e. $\hat{\phi}<\phi$) is baseline-optimistic, but control-pessimistic.

The first two definitions are about the primitives of the probability functions. I introduce a third definition which involves the endogenous choice of effort by the respective agents. This third definition allows to describe a single-crossing property for the preferences of agents with different beliefs and will determine jointly with the control beliefs the characterization of the equilibrium contracts.

Definition 3 *Agent* i *is more optimistic than agent* j *if* $\hat{\pi}_i(\hat{e}_i(c))\geq\hat{\pi}_j(\hat{e}_j(c))$ *for all* $c\in C$*.*

An agent can be more optimistic either because she perceives the likelihood of the good state to be higher for the same level of effort or because she perceives the marginal return to effort to be higher and thus exerts more effort.

Lemma 1 *An agent who is more baseline- and control-optimistic is more optimistic as well.*

2.2 The Agent's Preferences

The agent chooses the effort level that maximizes her perceived expected utility. Given the contract (w,Δ), the agent solves

$$U(w,\Delta)=\max_{e}\hat{\pi}(e)\,u(w)+(1-\hat{\pi}(e))\,u(w-\Delta)-e.$$

The agent's choice of effort $\hat{e}(w,\Delta)$ solves

$$\hat{\pi}'(\hat{e}(w,\Delta))\,[u(w)-u(w-\Delta)]=1.$$

The second order condition is satisfied since $\hat{\pi}''<0$. The effort choice is increasing in the agent's perceived return to search $\hat{\pi}'(\cdot)$ and the deductible Δ. When the outside option is chosen, the agent's perceived expected utility equals

$$U(w_0,\Delta_0)\equiv\hat{\pi}(\hat{e}(w_0,\Delta_0))\,u(w_0)+(1-\hat{\pi}(\hat{e}(w_0,\Delta_0)))\,u(w_0-\Delta_0)-\hat{e}(w_0,\Delta_0)\,.$$

The utility in the outside option is increasing in the baseline belief about the probability $\hat{\pi}(\cdot)$ that the good state occurs. This increase is higher, the less insurance the outside option provides.

3 Incentive Compatibility with Heterogeneity in Beliefs

Pessimistic agents are willing to pay more for insurance coverage than optimistic agents because they perceive the risk as more likely, regardless of whether optimism is driven by baseline and/or control optimism. This single-crossing property of the preferences implies that only contracts providing more insurance to pessimistic agents than to optimistic agents can be incentive compatible. Whether pessimistic agents are also more risky ex-post depends on the agents' efforts and thus the agents' control beliefs. The monotonicity in insurance coverage, therefore implies simple conditions for the correlation between risk occurrence and insurance coverage to be positive or negative, which depend only on the relative optimism and control-optimism of the agents.

I consider two types of agents who only differ in their beliefs. Type 1 and type 2 hold the beliefs $\hat{\pi}_1(\cdot)$ and $\hat{\pi}_2(\cdot)$ respectively, with $\hat{\pi}_1(\cdot) \neq \hat{\pi}_2(\cdot)$. These beliefs are unobservable to the insurers, but the insurers know the true probability of success $\pi(\cdot)$, which is the same function of effort for both types. For the characterization itself of the equilibrium contracts. It is sufficient that the agents' probability functions are perceived to be the same by the insurers. The outside option is the same for both types, but the perceived expected utility of the outside option may be different.

3.1 Single-Crossing Property

If the one type always perceives the probability of the good state to be greater than the other type for any possible contract, the two types' preferences satisfy a single-crossing property.

Assumption 1 *Type* 1 *is more optimistic than type* 2*.*

The higher the perceived probability that the bad state occurs, the higher the willingness to give up wealth w to decrease the deductible Δ. The perceived marginal rate of substitution between w and Δ for type i equals

$$\left.\frac{d\Delta}{dw}\right|_{\hat{\pi}_i} = \frac{\hat{\pi}_i(\hat{e}_i(w,\Delta))}{1-\hat{\pi}_i(\hat{e}_i(w,\Delta))}\frac{u'(w)}{u'(w-\Delta)} + 1.$$

The effect through changes in effort on the perceived expected utility in response to dw and $d\Delta$ is of second order because of the envelope condition and does not impact the marginal rate of substitution. For different types, the marginal rates of substitution for a given contract (w, Δ) is ranked based on the respective perceived probability of success $\hat{\pi}_i(\hat{e}_i(w,\Delta))$. If type 1 is more optimistic than type 2, the marginal rates of substitution are ranked the same for any contract.

Lemma 2 *For any $c \in C$,*

$$\left.\frac{d\Delta}{dw}\right|_{\hat{\pi}_1} \geq \left.\frac{d\Delta}{dw}\right|_{\hat{\pi}_2}.$$

The profit-maximizing insurer cannot observe the type of insuree he is facing. By the revelation principle, we can restrict the analysis to contracts that are incentive compatible such that the different types will self-select into the contracts designed for them. A pair of contracts $\{(w_1, \Delta_1), (w_2, \Delta_2)\}$ is incentive compatible if and only if

$$U^i(w_i, \Delta_i) \geq U^i(w_j, \Delta_j) \text{ for } i, j = 1, 2,$$

with

$$U^i(w, \Delta) \equiv \max_e \hat{\pi}_i(e) u(w) + (1 - \hat{\pi}_i(e)) u(w - \Delta) - e.$$

Clearly, for any pair of incentive compatible contracts, one contract cannot offer more consumption in both states than the other contract. That is, if $w_1 > w_2$, then $w_1 - \Delta_1 < w_2 - \Delta_2$ and vice versa. I introduce the relation $x \rhd y$ to describe that the contract x provides less insurance than contract y in the sense that x provides lower coverage at a lower insurance premium than contract y.

Notation 1 $(w_i, \Delta_i) \rhd (w_j, \Delta_j) \Leftrightarrow w_i > w_j$ *and* $w_i - \Delta_i < w_j - \Delta_j$

Notation 2 $(w_i, \Delta_i) \unrhd (w_j, \Delta_j) \Leftrightarrow w_i \geq w_j$ *and* $w_i - \Delta_i \leq w_j - \Delta_j$

I use the particular notation because $(w_i, \Delta_i) \rhd (w_j, \Delta_j)$ implies $(w_i, \Delta_i) > (w_j, \Delta_j)$. Notice that the opposite does not hold.

3.2 Monotonicity

In standard adverse selection problems the incentive compatibility constraints imply a monotonicity constraint on the separating contracts offered to different types, if the preferences satisfy a single-crossing property. The same is true here despite the presence of moral hazard.

The utility from one insurance contract can be expressed as the utility from any other insurance contract, plus the sum of the utility gains, positive or negative, from the incremental changes that lead from the latter to the former insurance contract. That is,

$$U^i(w_i, \Delta_i) = U^i(w_j, \Delta_j) + \int_{\Delta_j}^{\Delta_i} \left\{ U^i_w(\widetilde{w}(\Delta), \Delta) \widetilde{w}'(\Delta) + U^i_\Delta(\widetilde{w}(\Delta), \Delta) \right\} d\Delta,$$

for any continuous, differentiable function $\widetilde{w}(\Delta)$ with $\widetilde{w}(\Delta_j) = w_j$ and $\widetilde{w}(\Delta_i) = w_i$. I denote the gain in perceived expected utility for type i from switching from contract (w_2, Δ_2) to (w_1, Δ_1) by

$$G^i[(w_1, \Delta_1), (w_2, \Delta_2)] \equiv U^i(w_1, \Delta_1) - U^i(w_2, \Delta_2).$$

For contracts to be incentive compatible, the gain from switching to the other type's contract has to be negative for both types,

$$G^1 \left[(w_2, \Delta_2), (w_1, \Delta_1) \right] \geq 0 \qquad (IC_1)$$

$$G^2 \left[(w_1, \Delta_1), (w_2, \Delta_2) \right] \geq 0. \qquad (IC_2)$$

When choosing between two contracts, the more optimistic type puts relatively more weight on the change in consumption when successful and relatively less weight on the change in consumption when unsuccessful. This difference in weights is not sufficient to sign the difference for two types in utility gains from switching contracts, because the exerted effort levels differ as well. However, the single-crossing property can be used the evaluate the utility gains from all marginal changes in Δ and $\widetilde{w}(\Delta)$ for which changes in effort are of second order. When changing the contract from (w_j, Δ_j) to (w_i, Δ_i), the sign of the difference in utility gains for type i and type j from the marginal changes along the linear function,

$$\widetilde{w}(\Delta) = w_j + (\Delta - \Delta_j) \frac{w_i - w_j}{\Delta_i - \Delta_j},$$

exactly equals the sign of the difference in perceived likelihoods, evaluated along the linear path,

$$\hat{\pi}_i \left(\hat{e}_i \left(\widetilde{w}(\Delta), \Delta \right) \right) - \hat{\pi}_j \left(\hat{e}_j \left(\widetilde{w}(\Delta), \Delta \right) \right).$$

If an agent is more optimistic, she suffers less from each marginal increase in Δ and gains more from the associated marginal increase in $\widetilde{w}(\Delta)$, leading from the contract providing more to the contract providing less insurance. This observation implies the following lemma.

Lemma 3 *If $(w_1, \Delta_1) \rhd (w_2, \Delta_2)$, then*

$$G^1 \left[(w_1, \Delta_1), (w_2, \Delta_2) \right] > G^2 \left[(w_1, \Delta_1), (w_2, \Delta_2) \right].$$

The utility gain from switching to an insurance contract for which the insurance coverage and the insurance premium is lower, is greater for someone who is more optimistic about the probability of the good state. This implies that for two contracts to be incentive compatible, the insurance contract designed for the more optimistic type must provide less insurance, but at a lower insurance premium.

Proposition 1 *Type 1 receives less insurance than type 2 in any incentive compatible equilibrium, i.e.*

$$(w_1, \Delta_1) \unrhd (w_2, \Delta_2).$$

This monotonicity property follows immediately from the incentive compatibility constraints and Lemma 3. Assume, by contradiction, that (w_2, Δ_2) provides less insurance than (w_1, Δ_1). Since type 1 is more optimistic than type 2, the utility gain from switching to the contract providing less insurance is higher for type 1 than for type 2. However, for (w_2, Δ_2) to be incentive compatible for type 2, her gain from switching from (w_1, Δ_1) to (w_2, Δ_2) must be positive, which implies that the gain from switching from (w_1, Δ_1) to (w_2, Δ_2) is positive for type 1 as well. By consequence, (w_1, Δ_1) is not incentive compatible for type 1.

3.3 Positive vs. Negative Correlation

With heterogeneity in perceptions, either positive or negative correlation can arise between the endogenous probability that the risk occurs for a type and the insurance coverage provided to that type. An optimistic type necessarily receives more insurance than a pessimist, but whether the optimistic type is more risky depends on both her control beliefs and the insurance coverage.

Corollary 1 *If type 1 is more optimistic and control-optimistic than type 2, the equilibrium satisfies the 'positive correlation'-property, i.e.*

$$(w_1, \Delta_1) \trianglerighteq (w_2, \Delta_2) \;\; \textit{and} \; \pi\left(\hat{e}_1\left(w_1, \Delta_1\right)\right) \geq \pi\left(\hat{e}_2\left(w_2, \Delta_2\right)\right).$$

If type 1 is more control-optimistic, she exerts more effort than type 2 for the same level of insurance. Since in addition type 1 receives less insurance, she exerts more effort in equilibrium and is less likely to suffer a loss. The observed correlation between risk occurrence and insurance coverage is positive.[3]

Corollary 2 *Only if the optimistic type* 1 *is more control-pessimistic than type* 2*, the equilibrium may satisfy the 'negative correlation'-property, i.e.*

$$(w_1, \Delta_1) \trianglerighteq (w_2, \Delta_2) \;\; \textit{and} \; \pi\left(\hat{e}_1\left(w_1, \Delta_1\right)\right) \leq \pi\left(\hat{e}_2\left(w_2, \Delta_2\right)\right).$$

If type 1 is more control-pessimistic, she exerts less effort than type 2 for the same level of insurance. If she is sufficiently more control-pessimistic, she will still exert less effort despite receiving less insurance as well. The negative correlation between optimism and control-optimism across types is necessary for the negative correlation between risk occurrence and insurance coverage to occur.

3.4 Discussion

The empirical analysis of insurance markets, often finding negative and insignificant correlations between insurance coverage and risk, suggests that the standard model of adverse selection with only heterogeneity in risks, robustly predicting a positive correlation (Chiappori et al. 2006), falls short. In the model with heterogeneous perceptions, negative correlation arises naturally when one type believes her effort has no impact at

[3]Notice that the property is trivially satisfied for a pooling equilibrium.

all, but still perceives the probability that the good state occurs to be more likely than the other type. This extreme case is also considered by Koufopoulos (2008) and Huang et al. (2007). More generally, the previous results relate the sign of the correlation property to the correlation between risk and control perceptions. Interestingly, the heterogeneity in perceptions is the only source of heterogeneity in this model required to predict different signs of the correlation. Previous work has introduced heterogeneity in risk preferences, next to the heterogeneity in risks. Clearly, any other source of heterogeneity influencing the insurance choice will reduce the positive correlation between insurance and risk. When this alternative source of heterogeneity is negatively correlated with risks, this could also explain the negative correlation between insurance and risk (Einav et al. 2010a, Spinnewijn 2012). One way in which this negative correlation endogenously arises is when individuals with heterogeneous risk preferences also have some control over their risk (De Meza and Webb 2001, Jullien et al. 2006).

The question which heterogeneity is driving the demand for insurance and the cost of providing insurance is an empirical one. Despite the focus of the literature, very little evidence relates insurance choices to direct preference measures. For example, Fang et al. (2008) find that the negative correlation is driven by differences in cognitive ability rather than by differences in measures of risk aversion. While the heterogeneity in risk perceptions is well documented - even for people facing similar risks - (see for example Slovic 2000 or Peacock et al. 2005), these perceptions are rarely linked to insurance choices and precautionary efforts. Complementing insurance data with information on perceptions of risk and control would allow testing directly to which extent the correlation between the perceived risk and control drives the correlation between insurance and risk. This will depend on the particular risk being considered, as illustrated in the examples before. For instance, young drivers tend to overestimate the probability to avoid an accident, but underestimate the returns to driving safely (Finn and Bragg 1986, Tränkle et al. 1990). Similarly, women who overestimate the probability not to have breast cancer are less likely to take mammograms (Katapodi et al. 2004), plausibly because they underestimate the returns to preventive efforts, as argued by Polednak et al. (1991).

4 Optimal Insurance Contracts

In this section, I analyze the actual distortions in the insurance contracts relative to the full-information benchmark due to the unobservable heterogeneity in risk perceptions. Insurers may distort the insurance contract designed for one type to discourage the other type from choosing that contract. Once we know which contract is distorted, we know in which direction the distortion goes by the single-crossing property; the contract for the more optimistic type would be distorted towards less insurance coverage, while for the more pessimistic type this would be towards more insurance coverage.

To determine which contract will be distorted in equilibrium, I find that the interaction between the nature of the heterogeneity in beliefs and the nature of competition plays a central role. The first reason is that the heterogeneity in beliefs may affect the insurer's cost of providing insurance and the insuree's willingness to pay for being insured in different directions. While the insurer's cost of providing insurance depends on the insuree's effort choice and thus her control beliefs, her willingness to pay for insurance also depends on her baseline beliefs. This contrast with a model with only heterogeneity in risks, where a riskier type values insurance more, but is also more costly to insure. The second reason is that competing insurers screen types based on the difference in cost, while a monopolistic insurer screens types based on their willingness to pay. That is, competing insurers distort the contract offered to the 'low-cost' type to discourage the 'high-cost' type from pretending she has low cost. Control beliefs are thus central under competition. A monopolistic insurer distorts the contract to the 'low-valuation' type to discourage the 'high-valuation' type from pretending she has low valuation. Baseline beliefs become thus central under monopoly.

For the competitive equilibrium, I assume that insurers are competing as in Rothschild and Stiglitz (1976) with any contract offered in equilibrium making zero profit in expectation. For the monopolistic optimum, the insurees' participation constraints are central to the analysis. For a contract to be accepted, the insuree needs to expect higher utility from that contract than from her outside option. For the competitive case, I assume that the outside option provides no insurance and that the participation constraints are never binding in the competitive equilibrium. I relax both assumptions for the monopolistic case.

4.1 Full-Information Benchmark

I briefly characterize the profit-maximizing contract when the insurer knows the agent's perceived probability function $\hat{\pi}(e)$. This full-information contract will be the benchmark for the equilibrium contracts when risk perceptions are unobservable, but is discussed at length in Spinnewijn (2009).

When an insuree of type i accepts the contract (w, Δ), the insurer's expected profit equals

$$\Pi^i(w, \Delta) = w_0 - w - (1 - \pi(\hat{e}_i(w, \Delta)))(\Delta_0 - \Delta),$$

where (w_0, Δ_0) denotes the outside option. Notice that while the insurer expects to pay insurance coverage $\Delta_0 - \Delta$ with probability $1 - \pi(\hat{e}_i(w, \Delta))$, which is decreasing in the agent's effort level $\hat{e}_i(w, \Delta)$, the agent herself expects to receive this coverage with probability $1 - \hat{\pi}(\hat{e}(w, \Delta))$. A monopolistic insurer offers (w_m^*, Δ_m^*), solving

$$\max_{(w,\Delta)} w_0 - w - (1 - \pi(\hat{e}(w, \Delta)))(\Delta_0 - \Delta)$$

such that

$$u(w) - (1 - \hat{\pi}(\hat{e}(w,\Delta)))[u(w) - u(w-\Delta)] - \hat{e}(w,\Delta) \geq U(w_0,\Delta_0).$$

Competing insurers offer (w_c^*, Δ_c^*), solving the dual problem with the equilibrium profits equal to zero,

$$\max_{(w,\Delta)} u(w) - (1 - \hat{\pi}(\hat{e}(w,\Delta)))[u(w) - u(w-\Delta)] - \hat{e}(w,\Delta)$$

such that

$$w_0 - w - (1 - \pi(\hat{e}(w,\Delta)))(\Delta_0 - \Delta) \geq 0.$$

This implies the following proposition, similar to Proposition 2 in Spinnewijn (2009).

Proposition 2 *The profit-maximizing contract* (w^*, Δ^*) *is characterized by*

$$\frac{\frac{1-\hat{\pi}(\hat{e})}{1-\pi(\hat{e})}\frac{\pi(\hat{e})}{\hat{\pi}(\hat{e})}u'(w^*-\Delta^*) - u'(w^*)}{u'(w^*)} = \varepsilon_{1-\pi(\hat{e}),w-\Delta}\frac{L-\Delta^*}{w^*-\Delta^*}, \tag{1}$$

with $\hat{e} = \hat{e}(w^*, \Delta^*)$. *In monopoly, the perceived expected utility* $U(w^*, \Delta^*) = U(w_0, \Delta_0)$. *In competition, the expected profit* $\Pi(w^*, \Delta^*) = 0$.

The contracts optimally trade off the moral hazard cost of insurance, given by the elasticity $\varepsilon_{1-\pi(\hat{e}),w-\Delta}$ that captures the insuree's responsiveness with respect to coverage , and the perceived consumption smoothing benefits of insurance. The optimal contract response to baseline optimism is to lower the insurance coverage, since a baseline optimist perceives insurance as less valuable than an unbiased agent (i.e. $\frac{1-\hat{\pi}(\hat{e})}{1-\pi(e)}\frac{\pi(\hat{e})}{\hat{\pi}(\hat{e})} < 1$). The optimal response to control optimism is ambiguous though. If an insuree becomes more control-optimistic, less risk is required to induce her to exert the same level of effort. Hence, the insurers substitute towards inducing more effort, but given the control optimism, could do so by giving at the same time more insurance. The insurance coverage can therefore be either higher or lower for the more control-optimistic type in the full-information equilibrium. This is in stark contrast with Proposition 1. If beliefs are private and one insuree is more optimistic than the other (e.g. because she is more control-optimistic), she receives less insurance coverage in any incentive compatible equilibrium.

4.2 Binding Incentive Compatibility

I now continue by analyzing which incentive compatibility (IC) constraints are binding in equilibrium. While the difference in control optimism determines which IC constraint is binding in the competitive equilibrium, the difference in optimism determines which IC constraint is binding at the monopolistic optimum.

Control-Optimism and the Zero-Profit Constraint The insurer's expected profit is increasing in the effort choice and thus the control-optimistic beliefs of the agent accepting the contract. Since the expected profit from any contract equals zero in a competitive equilibrium, the more control-optimistic type can be offered better terms than the more control-pessimistic type. By revealed preference, the more control-optimistic type always prefers her full information contract to the full information contract offered to the other type. The latter contract would make non-negative profits when accepted by the more control-optimistic type, but since it is not offered in the full-information equilibrium, it must be that the more control-optimistic type prefers the former contract. However, the other type may prefer the full information contract offered to the more control-optimistic type. This implies the following lemma.

Lemma 4 *If type i is more control-optimistic than type j, the IC constraint for type i is never binding in a separating competitive equilibrium.*

Since the true risk is the same function of effort for both types, the control beliefs need to differ and effort needs to have a non-negligible impact on the outcome for the zero-profit conditions not to coincide. If the zero-profit conditions coincide, types with different beliefs prefer different contracts that satisfy this zero-profit condition. Hence, if only beliefs differ and there is no moral hazard, the full-information contracts are incentive compatible. The presence of the one type does not distort the contract offered to another type.

Optimism and the Participation Constraint An insuree's willingness to accept a contract depends on her risk perception and whether she bears more or less risk than in her outside option. The perceived utility increase from taking the contract (w_i, Δ_i) rather than the outside option (w_0, Δ_0) has to be non-negative,

$$G^i \left[(w_i, \Delta_i), (w_0, \Delta_0) \right] \geq 0 \text{ for } i = 1, 2.$$

Interestingly, the perceived expected utility in the outside option $U^i(w_0, \Delta_0)$ depends on the agent's beliefs unless the outside option provides full insurance.[4] Optimistic types require less compensation for a decrease in coverage, but value an increase in coverage less. If contracts provide more insurance than the outside option, the pessimistic type is tempted to take the insurance contract offered to the optimistic type at a more favorable premium. That is, the insurance contract that leaves the optimistic type indifferent with the outside option is strictly preferred by the pessimistic type to the outside option. If contracts provides less insurance than the outside option, the optimistic type is tempted to take the favorable incentive contract offered to the pessimistic type. Hence, it is the combination of the insurance provided in the outside

[4] Jullien (2000) analyzes screening contracts when the utility of outside options is type-dependent.

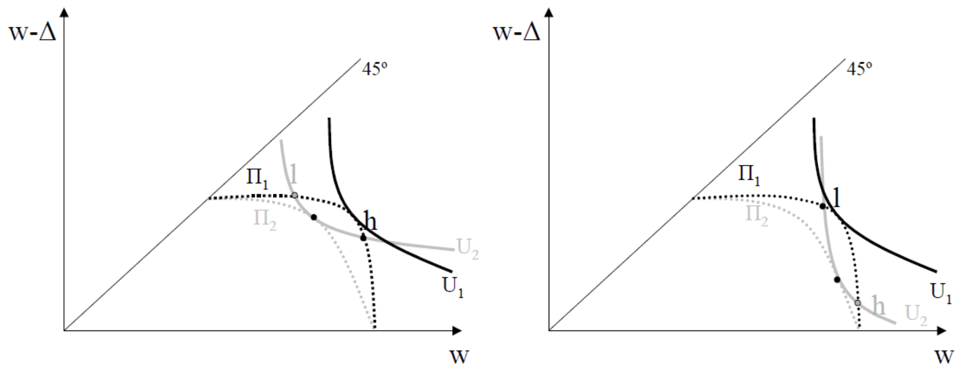

Figure 1: Competitive Equilibrium: Positive vs. Negative Correlation

option together with the difference in optimistic beliefs that determines which incentive compatibility constraint will be binding for the monopolist.

Lemma 5 *In a separating monopolistic optimum with type i more optimistic than type j, the IC constraint is binding for type i and the IR constraint is binding for type j when the outside option provides full insurance ($\Delta_0 = 0$). The reverse is true when the outside option provides no insurance ($\Delta_0 = L$).*

4.3 Competitive Equilibrium

The previous insights drive the characterization of the competitive equilibrium. I restrict the analysis to insurance contracts for clarity of the exposition.[5] The control beliefs are central. In this subsection, I therefore assume that type 1 is more control-optimistic than type 2 and characterize the competitive equilibrium depending on whether type 1 is more optimistic or more pessimistic.

Assumption 1' *Type* 1 *is more control-optimistic than type* 2.

The contract offered under full information in the competitive equilibrium to type 1 would make negative profit if chosen by type 2. There are two exceptions. Two contracts always make zero profits, regardless of the beliefs of the agent: the full-insurance contract with $(w, \Delta) = (W - (1 - \pi(0)) L, 0)$ and the no-insurance contract with $(w, \Delta) = (W, L)$. I show this graphically in Figure 1. The respective zero-profit curves are denoted by Π_1 and Π_2. Both curves connect the full-insurance contract (on the 45°-line) and the no-insurance contract (on the x-axis). However, the zero-profit curve for type 1 connects contracts that provide more consumption in the good and

[5] As argued before, changing the outside option from no to full insurance has an important impact on the participation constraints, but these play no significant role in the competitive equilibrium. While changing the outside option also changes the zero-profit constraints, contracts making zero-profit on less control-optimistic types will still make positive profits on more control-optimistic types. Hence, the insights from this analysis will naturally generalize for other outside options.

bad state than the zero-profit curve for type 2. The indifference curves are represented by U_1 and U_2. U_1 crosses U_2 once by the single-crossing property: from above if type 1 is more optimistic, from below if type 1 is more pessimistic. The full-information equilibrium contract for a type is determined by the tangency point between the zero-profit curve and the indifference curve for that type. The single-crossing property allows to fully characterize the separating equilibrium, if it exists. I first introduce the two contracts $\left(w^h, \Delta^h\right)$ and $\left(w^l, \Delta^l\right)$.

Definition 4 *Contracts* $\left(w^h, \Delta^h\right)$ *and* $\left(w^l, \Delta^l\right)$ *satisfy*

$$\begin{cases} \left(w^i, \Delta^i\right) \sim_2 \left(w^*_{c,2}, \Delta^*_{c,2}\right) \\ W - w^i = \left(1 - \pi\left(\hat{e}_i\left(w^i, \Delta^i\right)\right)\right)\left(L - \Delta^i\right) \end{cases} \quad \text{for } i = h, l,$$

moreover $\left(w^h, \Delta^h\right) \rhd \left(w^l, \Delta^l\right)$.

Both contracts satisfy the zero-profit condition of type 1 and leave type 2 indifferent between that contract and her full-information contract $\left(w^*_{c,2}, \Delta^*_{c,2}\right)$. The contract $\left(w^h, \Delta^h\right)$ provides less insurance coverage at a lower insurance premium than $\left(w^l, \Delta^l\right)$. The contracts are indicated by h and l in Figure 1.

Proposition 3 characterizes the separating equilibrium when type 1 is both more optimistic and more control-optimistic than type 2. I assume that the perceived expected utility is concave in consumption in the good the and the bad state.

Proposition 3 *If a separating equilibrium exists and type* 1 *is both more optimistic and more control-optimistic than type* 2*, the equilibrium contracts equal*

$$\begin{aligned} \left(w^{**}_{c,1}, \Delta^{**}_{c,1}\right) &= \left(w^h, \Delta^h\right) \\ \left(w^{**}_{c,2}, \Delta^{**}_{c,2}\right) &= \left(w^*_{c,2}, \Delta^*_{c,2}\right), \end{aligned}$$

unless $\left(w^*_{c,2}, \Delta^*_{c,2}\right) \succeq_2 \left(w^*_{c,1}, \Delta^*_{c,1}\right)$*, in which case the full-information contracts are separating.*

The presence of type 1 who is more control-optimistic has no impact on the contract offered to type 2, by Lemma 4. The presence of type 2 has no impact on the equilibrium contract offered to type 1 either if the full information equilibrium is separating. For instance, if type 1 is very optimistic, she will not be offered any insurance, regardless of the presence of a pessimistic type. Therefore, if the full information equilibrium is not separating, it is because the more control-pessimistic type 2 prefers type 1's full information contract. In that case, contracts h and l are natural alternatives, since type 2 is exactly indifferent between her full information contract $\left(w^*_{c,2}, \Delta^*_{c,2}\right)$ and these contracts. Contract h will be offered in equilibrium though, since the optimistic type 1 prefers the high deductible contract $\left(w^h, \Delta^h\right) \rhd \left(w^*_{c,1}, \Delta^*_{c,1}\right)$ to the low deductible contract $\left(w^l, \Delta^l\right) \lhd \left(w^*_{c,1}, \Delta^*_{c,1}\right)$ by the single-crossing property. The two types are thus separated by decreasing the insurance coverage for the optimistic type 1. I show this graphically in the left panel of Figure 1. The correlation between the ex-post risk and insurance coverage will be positive by Corollary 1.

Type 1's contract is distorted in the opposite direction if she is more control-optimistic, but at the same time more pessimistic than type 2.

Proposition 4 *If a separating equilibrium exists, utility is concave in consumption and type* 1 *is more control-optimistic, but more pessimistic than type* 2*, the equilibrium contracts are*

$$\left(w^{**}_{c,1}, \Delta^{**}_{c,1}\right) = \left(w^{l}, \Delta^{l}\right)$$
$$\left(w^{**}_{c,2}, \Delta^{**}_{c,2}\right) = \left(w^{*}_{c,2}, \Delta^{*}_{c,2}\right),$$

unless $\left(w^{*}_{c,2}, \Delta^{*}_{c,2}\right) \succeq_2 \left(w^{*}_{c,1}, \Delta^{*}_{c,1}\right)$*, in which case the full-information equilibrium is separating.*

The separating equilibrium contract for the control-pessimistic type 2 is still $\left(w^{*}_{c,2}, \Delta^{*}_{c,2}\right)$ as a consequence of Lemma 4. However, the pessimistic type 1 now prefers $\left(w^{l}, \Delta^{l}\right)$ to $\left(w^{h}, \Delta^{h}\right)$, because of the reversed single-crossing property. Since $\left(w^{l}, \Delta^{l}\right) \lhd \left(w^{*}_{c,2}, \Delta^{*}_{c,2}\right)$, the two types are separated by increasing the insurance coverage for type 1. I show this graphically in the right panel of Figure 1. Type 1 now receives less insurance than type 2. If type 2 is sufficiently control-pessimistic compared to type 1, the correlation between ex post risk and insurance coverage is negative, in line with Corollary 2.

Propositions 3 and 4 together imply that the insurance coverage of the control-optimistic type 1 may be non-monotonic in the baseline beliefs of type 2. Consider an increase in the baseline-optimism of type 2, keeping the control beliefs fixed, starting from more baseline-pessimistic to very baseline-optimistic relative to type 1. In Figure 1, this change steepens the indifference curves of type 2, decreasing the insurance coverage in the contract where her indifference curve is tangent to her own zero-profit curve, but also changing the intersections with type 1's zero-profit curve. When type 2 is very baseline-pessimistic, type 1's equilibrium contract $\left(w^{*}_{c,1}, \Delta^{*}_{c,1}\right)$ is separating. When the baseline-pessimism of type 2 is reduced, the incentive compatibility constraint for type 2 will become eventually binding and type 1's separating contract becomes $\left(w^{h}, \Delta^{h}\right)$, providing less insurance than $\left(w^{*}_{c,1}, \Delta^{*}_{c,1}\right)$ by Proposition 3. When reducing type 2's baseline pessimism further, type 1's contract needs to be distorted towards less insurance to keep the contracts separated, i.e. Δ^{h} increases. Eventually type 2 becomes so baseline-optimistic that despite her relative control-pessimism she becomes more optimistic than type 1, the equilibrium contract for type 1 jumps to $\left(w^{l}, \Delta^{l}\right)$ with $\Delta^{l} < \Delta^{h}$ by Proposition 4. Hence, the insurance coverage for type 1 will have jumped up again. This contract provides more insurance than $\left(w^{*}_{c,1}, \Delta^{*}_{c,1}\right)$.[6] When now increasing type 2's baseline optimism further, type 1's insurance coverage decreases, i.e. Δ^{l} increases, until eventually the full information contract $\left(w^{*}_{c,1}, \Delta^{*}_{c,1}\right)$ becomes

[6] This holds with certainty if type 1's zero-profit curve is decreasing in $(w, w\text{-}\Delta)$ - space.

separating again. Notice that this ignores pooling equilibria, which may survive if the single-crossing property does not hold any longer when changing type 2's beliefs. If the single-crossing property does hold, a pooling contract can never be offered in an equilibrium because of cream skimming (Rothschild and Stiglitz 1976). However, they may inhibit the existence of a separating equilibrium if the ratio of type 2 agents is sufficiently low.

4.4 Monopolistic Optimum

For the monopolistic optimum, I allow the outside option to provide some or even full insurance. Hence, depending on the outside option, the equilibrium contracts will provide more or less insurance coverage. I characterize the monopolistic optimum depending on the extent to which the agent is insured the outside option. I generalize the methodology in Jullien et al. (2007) who analyze monopolistic screening in the presence of moral hazard and adverse selection due to unobserved heterogeneity in CARA preferences. The beliefs about the likelihood of the risk are central to the analysis. I therefore assume again that type 1 is more optimistic than type 2 and

Assumption 1 *Type* 1 *is more optimistic than type* 2.

If the outside option provides no insurance, the monopolist needs to pay a rent to the pessimist to induce her not to choose the contract offered to the optimist. Since the optimist needs to be compensated less than the pessimist for an increase in risk, the monopolist reduces the rent paid to the pessimist by imposing more risk on the optimist. The separating contract offered to the pessimist, however, is constrained efficient. If the outside option provides full insurance, it is the optimist who needs to be paid a rent in order to be separated from the pessimist. The monopolist now imposes less risk on the pessimist to reduce the rent paid to the optimist. The separating contract offered to the optimist is (constrained) efficient. That is, given the rent given to the agent, the contract satisfies condition (1) in Proposition 2.

Proposition 5 *If the monopolist separates types, the optimal contract satisfies that*

$$\begin{aligned} \left(w_{m,1}^{**}, \Delta_{m,1}^{**}\right) \rhd \left(w_{m,1}^{*}, \Delta_{m,1}^{*}\right) \text{ when } \Delta_0 &= L, \\ \left(w_{m,2}^{**}, \Delta_{m,2}^{**}\right) \lhd \left(w_{m,2}^{*}, \Delta_{m,2}^{*}\right) \text{ when } \Delta_0 &= 0. \end{aligned}$$

In both cases, the contract offered to the other type is constrained efficient.

The monopolist excludes a type if the profit from this type does not compensate for the rents paid to the other type required to induce the other type to select the contract proposed to her. In that case, the other type is proposed the full-information contract. If $\Delta_0 = L$ and the agent is optimistic with sufficiently low probability κ, the optimistic type is excluded. If $\Delta_0 = 0$ and the agent is optimistic with sufficiently high probability κ, the pessimistic type is excluded. Finally, a pooling contract may dominate any separating contract. This happens when the solution to the profit-maximizing problem constrained to a binding participation constraint for one type and a binding incentive compatibility constraint for the other type gives more insurance to the optimistic type than to the pessimistic type.[7]

A sufficient condition under which these results generalize for outside opportunities that provide some but not full insurance is that the full-information problem is convex. One special case may arise when the outside opportunity provides partial insurance; if the full information contracts specify deductibles $\Delta_2^* < \Delta_0 < \Delta_1^*$, then these contracts are incentive compatible.[8]

Proposition 6 *If type* 1 *is more optimistic than type* 2 *and the full-information problem is convex, the optimal contract with types separated satisfies*

$$\begin{aligned} \left(w_{m,1}^{**}, \Delta_{m,1}^{**}\right) &\rhd \left(w_{m,1}^{*}, \Delta_{m,1}^{*}\right) \;\; \textit{if } \Delta_0 > \max\left\{\Delta_1^*, \Delta_2^*\right\}, \\ \left(w_{m,2}^{**}, \Delta_{m,2}^{**}\right) &\lhd \left(w_{m,2}^{*}, \Delta_{m,2}^{*}\right) \;\; \textit{if } \Delta_0 < \min\left\{\Delta_1^*, \Delta_2^*\right\}. \end{aligned}$$

The contract offered to the other type is constrained efficient in both cases. Also, if $\Delta_2^* < \Delta_0 < \Delta_1^*$, $\left(w_{m,i}^{**}, \Delta_{m,i}^{**}\right) = \left(w_{m,i}^{*}, \Delta_{m,i}^{*}\right)$ *for* $i = 1, 2$.

If both contracts are incentive contracts (i.e. $\Delta_i > \Delta_0$), the optimistic type receives a rent and the contract for the pessimistic type is distorted towards less incentives. If both contracts are insurance contracts (i.e. $\Delta_i < \Delta_0$), the pessimistic type receives a rent and the contract of the optimistic type is distorted towards less insurance.

5 Welfare Analysis

This section analyzes the normative implications of heterogeneous risk perceptions in this model. The analysis has shown that insurers change the terms of the contracts to screen indistinguishable optimistic and pessimistic types. The presence of one type thus imposes an externality on the other type. This externality can be very different when insurees face a monopolistic or competing insurers. Moreover, the screening distortion may aggravate, but also reduce the distortion due to an agent's bias in beliefs.

I first focus on the informational externality biased agents ($\hat{\pi} \neq \pi$) impose on unbiased agents ($\hat{\pi} = \pi$), since the evaluation of an unbiased agent's welfare is unambiguous.

[7] Notice that with CARA preferences and monetary costs of efforts, as considered in Jullien et al. (2007) and in Section 6, the monopolist pays a rent to the potentially imitating agent i by increasing w_i, but keeping Δ_i unchanged at Δ_i^*. Hence, if for the full-information contracts $\Delta_1^* \geq \Delta_2^*$, then a pooling contract can never be optimal.

[8] If for a given outside opportunity (w_0, Δ_0) the full information contracts are such that $\Delta_1^* < \Delta_0 < \Delta_2^*$, then the two types are likely to be pooled. This 'irregular case' is also treated by Jullien et al. (2007).

In a competitive equilibrium, the insurance contract for the more control-optimistic type may be distorted compared to the full-information contract by Lemma 4, while the contract offered to the more control-pessimistic type is unchanged. The welfare of the unbiased agent may thus be lower, but only in the presence of a more control-pessimistic type. Notice that if the set of contracts that make zero profits is the same for two types of insurees, the presence of the one does not affect the contract offered to the other. Moral hazard and disagreement about the returns to effort are therefore necessary ingredients for informational externalities to occur in the competitive equilibrium.

Corollary 3 *In a separating competitive equilibrium, an agent with unbiased beliefs never gains from the presence of an agent with biased beliefs and may strictly lose only if that agent is control-pessimistic.*

Heterogeneity in risk perceptions induces screening by the insurers and the corollary shows that screening decreases welfare. Hence, the heterogeneous risk perceptions increase the gain from a government intervention that regulates insurance coverage towards the full-information contract. This is very different from the conclusions in the model considered by Sandroni and Squintani (2007). Central to their analysis is that some types of agents perceive their risk to be the same, although their true risk is different. In particular, some optimistic high risk types perceive their risk to be low and thus are necessarily pooled with the low risk types. This increases the insurance premium for the low risk types in the competitive equilibrium, such that an insurance mandate may actually decrease their welfare. Hence, an insurance mandate would not be Pareto improving as in the standard Rothschild-Stiglitz model. In the model considered here, the heterogeneity in perceptions increases rather than decreases the dispersion in the perceived risk relative to the dispersion in the actual risk, which creates screening distortions and thus increases the scope for government intervention.[9] Related to this, Spinnewijn (2012) analyzes the welfare consequences of inefficient pricing due to adverse selection in a model with continuous heterogeneity and without moral hazard. In that model, the welfare gain of an insurance mandate is increased either by an increase in the dispersion of the perceived risks or by a reduction in the correlation between the perceived and actual risks.

When facing a monopolistic insurer, the externality that a biased agent exerts on an unbiased agent is different. An important difference is that the contracts offered to both types may change when the insurees' perceptions are not observable. If for one type the IC constraint is binding, that type receives a rent not to switch to the other type's contract and thus ends up strictly better off than with the full-information

[9] As discussed in Section 4.3, the non-monotonicity of the distortion of the more control-optimistic type's contract in the beliefs of the more control-pessimistic type, suggests that the direction of the optimal policy is ambiguous. If the more control-optimistic type is more optimistic as well, the distortion is towards less insurance and vice versa.

contract. The second type will still be indifferent about switching to the outside option, but her contract is distorted compared to the full-information contract to reduce the rent paid to the first type. The cases in which an agent with unbiased beliefs is paid a rent follow immediately from Proposition 6.

Corollary 4 *In a separating monopolistic optimum, an agent with unbiased beliefs gains from the presence of a pessimistic agent when offered incentive contracts and from the presence of an optimistic agent when offered insurance contracts.*

The previous welfare results generalize for an agent with biased beliefs when welfare is evaluated in terms of her perceived expected utility, but not necessarily when it is evaluated in terms of her true expected utility. Insurers who can observe the insuree's beliefs offer too little insurance coverage to an optimistic agent, who underestimates the value of insurance, and to a control-optimistic agent, who already exerts too much effort and thus should be incentivized less.[10] Hence, the distortions due to the unobservability of risk perceptions can mitigate, but also aggravate the distortion due to biased beliefs depending on the type of bias. Consider an economy with an unbiased agent and biased agent, where the latter is at the same time too optimistic and too control-optimistic. This case corresponds to Example I and is analyzed further in the next section. The biased agent would already receive too little insurance when the insurance company could observe her optimistic bias, evaluating welfare based on their true expected utility. In the presence of an unbiased agent, the insurance company would screen the optimistic agent by providing even less insurance, either in a competitive equilibrium by Proposition 3 or in the monopolistic optimum by Proposition 6. In this example, the heterogeneity in perceptions unambiguously increases the welfare gain from an insurance mandate. In addition, the example illustrates two important misconceptions regarding the importance of competition and the presence of rational agents for the welfare of boundedly rational agents, as discussed in Spiegler (2011). First, the presence of the unbiased agent does not necessarily exert a positive externality on the biased agent, as the optimistic agent in the example is even worse off in the presence of the unbiased agent. Second, competition is not effective in eliminating the exploitation of the biased agent and may make the distortions even worse.

6 An Example with Continuous Risk

The analysis has sofar considered insurance against binary risks. In this section, I consider a standard setting with continuous risk, following Holmström and Milgrom

[10] Spinnewijn (2009) finds that increasing the insurance coverage in the full-information equilibrium would increase the agent's true expected utility, when

$$\{\hat{\pi}(e) - \pi(e)\} \geq \{\pi'(e) - \hat{\pi}'(e)\} \frac{\hat{\pi}'(e)}{-\hat{\pi}''(e)}. \tag{2}$$

(1987), and show how the previous insights naturally generalize. The characterization of the equilibrium contracts is particularly simple and shows clearly the relation between the distortions and the heterogeneity in beliefs.

Output $q = \theta e + \varepsilon$ is additive in effort and a normal noise term ε with variance σ^2. The return to effort depends on the agent's ability θ. However, the agent may perceive her ability differently. There are two types, where an agent is of type 1 with probability κ. Type 1 perceives the return to effort to be $\hat{\theta}_1$. Type 2 perceives this return to be $\hat{\theta}_2$, with $\hat{\theta}_1 > \hat{\theta}_2$. Hence, type 1 is both more baseline-optimistic and control-optimistic, and thus more optimistic than type 2 by Lemma 1. Both types have the same constant absolute risk aversion η and face the same monetary costs of effort $\psi \frac{e^2}{2}$. The outside option is the same, but the perceived expected utility of the outside option depends on the ability perception. Denote the respective expected utility levels by $\hat{u}_1$ and $\hat{u}_2$.

I restrict the analysis to linear contracts $t + sq$. Based on her perceived expected utility, the certainty equivalent of this contract for type i equals

$$CE^i (s,t) = t + s\hat{\theta}_i \hat{e}_i (s) - \psi \frac{\hat{e}_i (s)^2}{2} - \frac{\eta}{2} s^2 \sigma^2,$$

where $\hat{e}_i (s) = \frac{\hat{\theta}_i}{\psi} s$. The expected profit for the insurer equals

$$\Pi^i (s,t) = (1-s)\, \theta \hat{e}_i (s) - t.$$

I first characterize the optimal linear sharing rule under full information as a benchmark.

Result 1 *With θ and $\hat{\theta}_i$ the true and perceived ability, the linear sharing contract offered by a monopolist or competing insurers under full information is characterized by*

$$s_i^* = \left[1 + \frac{1}{\hat{\theta}_i \theta} \psi \eta \sigma^2 - \frac{\hat{\theta}_i - \theta}{\theta}\right]^{-1} \equiv \left[\Xi_i - \frac{\hat{\theta}_i - \theta}{\theta}\right]^{-1}.$$

The contract is more performance-dependent the more optimistic the agent is about her ability. An increase in $\hat{\theta}_i$ increases the agent's perceived ability to increase output and thus her responsiveness to incentives. An increase in $\hat{\theta}_i$ also increases the probabilistic weight the agent puts on states with high output.[11]

When types cannot be distinguished, the full information contracts may not be incentive compatible. The single-crossing property implies that in any equilibrium the optimist's contract is at least as performance-dependent as the pessimist's contract, in line with Proposition 1.

[11] While private insurers respond to the agent's baseline optimism by decreasing $[s_i^*]^{-1}$ by $\frac{\hat{\theta}_i - \theta}{\theta}$, the linear sharing rule that maximizes the true expected utility is not affected by baseline optimism, but corrects for control optimism. The socially optimal sharing rule equals $\left[\Xi_i + \frac{\hat{\theta}_i - \theta}{\theta}\right]^{-1}$, providing less incentives (and more insurance) than the competitive sharing rule when $\hat{\theta}_i > \theta$.

Result 2 *In any incentive compatible contract, type 1's contract is more performance-dependent than type 2's contract,*

$$s_1^{**} \geq s_2^{**}.$$

A competing insurer can offer better terms to the optimistic type. Hence, the pessimist prefers the full information contract offered to the optimist in a competitive equilibrium. The full-information contract of the optimist is thus distorted towards more incentives to discourage the pessimist from switching to the optimist's contract. This result is in line with Proposition 3.

Result 3 *In any competitive equilibrium, the performance-dependence is distorted upward for type* 1*,*

$$s_{c,1}^{**} \geq s_1^* \text{ and } s_{c,2}^{**} = s_2^*.$$

With a monopolist, the outside utilities are relevant, denoted by $\hat{u}_i$. When agents receive full insurance in the outside option (i.e. $\hat{u}_1 = \hat{u}_2$), the monopolist can pay a relatively low transfer t to the optimist to convince her to bear more risk. The optimist may be better off by taking the full information contract offered to the pessimist. In the separating optimum, the optimist thus receives a rent and the linear sharing rule for the pessimist is distorted downward, providing less incentives. The distortion depends on the difference in beliefs, $(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2$. When agents receive no insurance in the outside option (i.e. $\hat{u}_1 > \hat{u}_2$), the monopolist can only ask a relatively small payment $-t$ from the optimist for providing insurance. The pessimist prefers the full information contract offered to the optimist. In the separating optimum, the linear sharing rule of the optimist's contract is distorted upward, providing more incentives, unless the optimists are excluded. For some intermediate coverage in the outside option (i.e. $\hat{u}_1 \in [l, h]$), the full-information contracts are incentive compatible.[12] This is summarized in the following result.

Result 4 *If the monopolist separates type* 1 *and type* 2 *with* $\hat{\theta}_1 > \hat{\theta}_2$*, the linear sharing rules of the separating contracts equal*

$$\begin{bmatrix} s_{m,1}^{**} \\ s_{m,2}^{**} \end{bmatrix} = \begin{cases} \begin{bmatrix} [\Xi_1 - \frac{\hat{\theta}_1 - \theta}{\theta}]^{-1} \\ [\Xi_2 - \frac{\hat{\theta}_2 - \theta}{\theta} + \frac{\kappa}{1-\kappa} \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{\theta \hat{\theta}_2}]^{-1} \end{bmatrix} & \text{for } \hat{u}_1 < l \\ \\ \begin{bmatrix} [\Xi_1 - \frac{\hat{\theta}_1 - \theta}{\theta} - \frac{1-\kappa}{\kappa} \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{\theta \hat{\theta}_1}]^{-1} \\ [\Xi_2 - \frac{\hat{\theta}_2 - \theta}{\theta}]^{-1} \end{bmatrix} & \text{for } \hat{u}_1 > h. \end{cases}$$

The sharing rules are the same as in the full-information contracts $\left(s_{m,1}^*, s_{m,2}^*\right)$ *for* $\hat{u}_1 \in [l, h]$.

[12] The bounds are $l \equiv \hat{u}_2 + \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{2\psi} (s_2^*)^2$ and $h \equiv \hat{u}_2 + \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{2\psi} (s_1^*)^2$, as derived the appendix.

If $\hat{\theta}_1 > \hat{\theta}_2 > \theta$, private insurers provide too little insurance to the optimistic workers. The screening distortion aggravates this distortion in the competitive equilibrium. In the monopolistic case, this is only true if the optimistic type perceives the expected utility of the outside option to be sufficiently higher than the pessimistic type.

7 Conclusion

People have very different perceptions about the likelihood and the controllability of the risks they face. I analyze how insurance companies separate people based on the heterogeneity in their perceptions. While people with heterogeneous abilities or risks, but identical perceptions cannot be separated, people with different perceptions can be separated with a menu of screening contracts, even when the true abilities or risks are the same.

The differences in the perceptions of the likelihood and control drive the design of the equilibrium contracts. While an individual's equilibrium coverage depends on her optimism about the risk, the cost of providing insurance depends on her optimism about her control. Heterogeneous beliefs, in contrast with heterogeneity in risks, may affect the willingness to pay and the cost to the insurer in different directions. This drives why a monopolistic insurer and competing insurers respond differently to different beliefs. A monopolistic insurer wants to separate the types with high willingness to pay. Competition induces insurers to separate the types with low cost.

Heterogeneity in beliefs can explain why contracts offer too little insurance in some markets (e.g., health insurance, car insurance) and in other markets provide no incentives at all, although a small risk suffices to induce effort (e.g., no-limit contracts on rented cars mileage, cell phone usage). However, unobserved differences in risk or ability could have the same positive implications. Heterogeneity in beliefs can also explain why in some insurance markets the correlation between risk occurrence and insurance coverage is positive, but negative in others. However, unobserved differences in preferences, positively or negative correlated with risks, could explain this as well. Direct evidence is mostly lacking, but the literature has mostly attributed unexplained heterogeneity in choices to heterogeneity in preferences. The underlying heterogeneity is, however, crucial for policy and welfare conclusions, as argued here and in Sandroni and Squintani (2007) and developed further in Spinnewijn (2012). Identifying to what extent results are driven by heterogeneity in perceptions or heterogeneity in preferences is clearly a challenge (Manski 2004). Potential approaches to quantify the importance of heterogeneity in beliefs rather than preferences or risks are, either by eliciting expectations directly through surveys (Spinnewijn 2009) or by relating certain behavior to biases in perceptions (Köszegi and Rabin 2007, Landier and Thesmar 2009) or experiences that are likely to have changed perceptions (Malmendier, Tate and Yan 2011). Extending these empirical approaches to the analysis of insurance markets seems a promising avenue for future research.

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Appendix: Proofs

Proof of Lemma 1

If $\hat{\pi}_i(e) \geq \hat{\pi}_j(e)$ for any e and $\hat{e}_i(c) = \hat{e}_j(c)$, than $\hat{\pi}_i(\hat{e}_i(c)) \geq \hat{\pi}_j(\hat{e}_j(c))$ for any c. If also $\hat{\pi}_i'(e) \geq \hat{\pi}_j'(e)$ for any e, than $\hat{e}_i(c) \geq \hat{e}_j(c)$ for any c. The Lemma follows, since $\hat{\pi}'(e) > 0$.□

Proof of Lemma 2

The marginal rate of substitution (MRS) between Δ and w equals

$$\left.\frac{d\Delta}{dw}\right|_{\hat{\pi}_i} = \frac{\hat{\pi}_i(\hat{e}_i(c))\,u'(w) + (1-\hat{\pi}_i(\hat{e}_i(c)))\,u'(w-\Delta)}{(1-\hat{\pi}_i(\hat{e}_i(c)))\,u'(w-\Delta)} = \frac{\hat{\pi}_i(\hat{e}_i(c))}{(1-\hat{\pi}_i(\hat{e}_i(c)))}\frac{u'(w)}{u'(w-\Delta)} + 1.$$

Since $\frac{u'(w)}{u'(w-\Delta)} > 0$, the MRS is increasing in $\hat{\pi}_i(\hat{e}_i(c))$. The lemma follows, since $\hat{\pi}_1(\hat{e}_1(c)) \geq \hat{\pi}_2(\hat{e}_2(c))$ for any c.□

Proof of Lemma 3

Given $\widetilde{w}(\Delta) = w_j + (\Delta - \Delta_j)\frac{w_i - w_j}{\Delta_i - \Delta_j}$,

$$U_w^i(\widetilde{w}(\Delta),\Delta)\,\widetilde{w}'(\Delta) + U_\Delta^i(\widetilde{w}(\Delta),\Delta) =$$
$$\hat{\pi}_i(\hat{e}_i(\widetilde{w}(\Delta),\Delta))\,u'(\widetilde{w}(\Delta))\frac{w_1-w_2}{\Delta_1-\Delta_2} - (1-\hat{\pi}_i(\hat{e}_i(\widetilde{w}(\Delta),\Delta)))\,u'(\widetilde{w}(\Delta)-\Delta)\frac{[w_2-\Delta_2]-[w_1-\Delta_1]}{\Delta_1-\Delta_2}.$$

Hence,

$$G^1[(w_1,\Delta_1),(w_2,\Delta_2)] - G^2[(w_1,\Delta_1),(w_2,\Delta_2)] =$$
$$\int_{\Delta_2}^{\Delta_1}\{[\hat{\pi}_1(\hat{e}_1(\widetilde{w}(\Delta),\Delta)) - \hat{\pi}_2(\hat{e}_2(\widetilde{w}(\Delta),\Delta))]\times$$
$$[u'(\widetilde{w}(\Delta))\frac{w_1-w_2}{\Delta_1-\Delta_2} + u'(\widetilde{w}(\Delta)-\Delta)\frac{[w_2-\Delta_2]-[w_1-\Delta_1]}{\Delta_1-\Delta_2}]\}d\Delta.$$

Since $\frac{w_1-w_2}{\Delta_1-\Delta_2} > 0$ and $\frac{[w_2-\Delta_2]-[w_1-\Delta_1]}{\Delta_1-\Delta_2} > 0$, given $(w_1,\Delta_1) \rhd (w_2,\Delta_2)$, the term in the integral between squared brackets is greater than zero. Since the integration is from Δ_2 to Δ_1 with $\Delta_1 > \Delta_2$,

$$G^1[(w_1,\Delta_1),(w_2,\Delta_2)] - G^2[(w_1,\Delta_1),(w_2,\Delta_2)] > 0,$$

if $\hat{\pi}_1(\hat{e}_1(\widetilde{w}(\Delta),\Delta)) \geq \hat{\pi}_2(\hat{e}_2(\widetilde{w}(\Delta),\Delta))$ for all $\Delta \in [\Delta_2,\Delta_1]$ and $\hat{\pi}_1(\hat{e}_1(\widetilde{w}(\Delta),\Delta)) > \hat{\pi}_2(\hat{e}_2(\widetilde{w}(\Delta),\Delta))$ for some $\Delta \in [\Delta_2,\Delta_1]$.□

Proof of Lemma 4

If type 1 is (strictly) more control-optimistic than type 2, $\Pi_1(w,\Delta) > \Pi_2(w,\Delta)$. In any separating equilibrium,

$$\Pi_1 (w_1, \Delta_1) = \Pi_2 (w_2, \Delta_2) = 0.$$

Assume by contradiction that $(w_2, \Delta_2) \sim_1 (w_1, \Delta_1)$ in a separating equilibrium, but $\Pi_2 (w_2, \Delta_2) = 0$. Hence, if preferences are continuous and the single-crossing property is satisfied, an insurer can change the contract (w_2, Δ_2) to (w_2', Δ_2') such that $\Pi_2 (w_2', \Delta_2') = 0$, but $(w_2', \Delta_2') \succ_1 (w_1, \Delta_1)$ and $\Pi_1 (w_2', \Delta_2') > 0$. This is a profitable deviation.□

Proof of Lemma 5

If $\Delta_0 = L$, then for any interior solution $\Delta_1 < \Delta_0$ by the risk aversion of the agent and the risk neutrality of the principal. This implies $G^2 [(w_1, \Delta_1), (w_0, \Delta_0)] > G^1 [(w_1, \Delta_1), (w_0, \Delta_0)]$. Given the binding IC/IR constraints, this implies

$$G^2 [(w_2, \Delta_2), (w_0, \Delta_0)] = G^2 [(w_1, \Delta_1), (w_0, \Delta_0)] > 0.$$

Moreover, since $(w_1, \Delta_1) > (w_2, \Delta_2)$ if the contracts are separating, $G^1 [(w_1, \Delta_1), (w_0, \Delta_0)] > G^1 [(w_2, \Delta_2), (w_0, \Delta_0)]$ by Lemma 3. This in turn implies that $G^1 [(w_1, \Delta_1), (w_0, \Delta_0)] = 0$.

If $\Delta_0 = 0$, then for any interior solution $\Delta_2 > \Delta_0$, since the agent cannot overinsure. This implies $G^1 [(w_2, \Delta_2), (w_0, \Delta_0)] > G^2 [(w_2, \Delta_2), (w_0, \Delta_0)]$. Given the binding IC/IR constraints, this implies

$$G^1 [(w_1, \Delta_1), (w_0, \Delta_0)] = G^1 [(w_2, \Delta_2), (w_0, \Delta_0)] > 0.$$

Moreover, since $(w_1, \Delta_1) > (w_2, \Delta_2)$ if the contracts are separating, $G^2 [(w_2, \Delta_2), (w_0, \Delta_0)] > G^2 [(w_1, \Delta_1), (w_0, \Delta_0)]$ by Lemma 3. This in turn implies that $G^2 [(w_2, \Delta_2), (w_0, \Delta_0)] = 0$.□

Proof of Proposition 1

Assume, by contradiction, that $(w_2, \Delta_2) \rhd (w_1, \Delta_1)$, although type 1 is more optimistic than type 2. In that case,

$$G^1 [(w_2, \Delta_2), (w_1, \Delta_1)] \geq G^2 [(w_2, \Delta_2), (w_1, \Delta_1)] \geq 0,$$

by Lemma 3. But this contradicts $G^1 [(w_1, \Delta_1), (w_2, \Delta_2)] \geq 0$, since $G^1 [x, y] = -G^1 [y, x]$.□

Proof of Proposition 2

The competing insurers and the monopolist solve

$$\max \hat{\pi} (e) [u (w) - u (w - \Delta)] + u (w - \Delta) - e$$

such that

$$\begin{aligned} \hat{\pi}'(e)\left[u(w)-u(w-\Delta)\right] &= 1 \\ W-w-(1-\pi(e))\left[L-\Delta\right] &\geq \bar{\Pi}, \end{aligned}$$

where $\bar{\Pi}$ equals 0 in the competitive equilibrium and $\bar{\Pi}$ equals the expected profits such that $U(w,\Delta)=U(w_0,\Delta_0)$ in the monopolistic optimum.

I define the insurance coverage $b=w-\Delta$ and the tax on the good state $\tau=W-w$, as in Spinnewijn (2009). Denote by $\hat{e}_i(b)$ the level of effort and $\hat{\tau}_i(b)$ the tax that solve the IC constraint and the respective profit constraints for a given level of insurance coverage b with $i=c,m$. The change in the required tax τ when the insurance coverage increases equals

$$\tau_i'(b)=\frac{1-\pi(\hat{e}_i(b))}{\pi(\hat{e}_i(b))}\left[1+\frac{\tau_i(b)+b-W+L}{b}\varepsilon_{1-\pi(\hat{e}_i(b)),b}\right].$$

Notice that $\bar{\Pi}$ does not enter the expression for this derivative directly.

The first order condition with respect to b gives

$$(1-\hat{\pi}(\hat{e}_i(b)))\,u'(b)-\hat{\pi}(\hat{e}_i(b))\,u'(w-\tau_i(b))\,\tau_i'(b)=0.$$

The effect through effort is of second order by the envelope condition. Plugging in for $\tau_i'(b)$ and rewriting the expression, one finds

$$\frac{\frac{1-\hat{\pi}(\hat{e}_i(b))}{1-\pi(\hat{e}_i(b))}u'(b)-\frac{\hat{\pi}(\hat{e}(b))}{\pi(\hat{e}(b))}u'(w-\tau_i(b))}{\frac{\hat{\pi}(\hat{e}_i(b))}{\pi(\hat{e}_i(b))}u'(w-\tau_i(b))}=\frac{\tau_i(b)+b-W+L}{b}\varepsilon_{1-\pi(\hat{e}_i(b)),b}.$$

With $b=w-\Delta$ and $\tau=W-w$, the proposition immediately follows.□

Proof of Proposition 3

If the full-information contracts are incentive compatible, the competitive equilibrium coincides with the full-information equilibrium. If the full-information contracts are not incentive compatible, then the IC constraint of type 1 is still not binding in a separating equilibrium, by Lemma 4. Hence, if a separating equilibrium exists, type 2's equilibrium contract equals the full-information contract. The actuarial contract that maximizes the perceived expected utility of type 1, but is not strictly preferred by type 2 to its full-information contract is $\left(w^h,\Delta^h\right)$. Since $\left(w^h,\Delta^h\right)\sim_2\left(w^l,\Delta^l\right)$ and $\left(w^h,\Delta^h\right)\rhd\left(w^l,\Delta^l\right)$, $\left(w^h,\Delta^h\right)\succ_1\left(w^l,\Delta^l\right)$ by Lemma 3. Then, since utility is concave in consumption, $\left(w^h,\Delta^h\right)\succ_1(w,\Delta)$ for any actuarial contract for which $(w,\Delta)\rhd\left(w^h,\Delta^h\right)$ or $(w,\Delta)\lhd\left(w^l,\Delta^l\right)$. Hence, type 1's equilibrium contract equals $\left(w^h,\Delta^h\right)$.□

Proof of Proposition 4

The proof is analogue to the proof of Proposition 3. Since the single-crossing property is reversed now, $\left(w^h, \Delta^h\right) \sim_2 \left(w^l, \Delta^l\right)$ and $\left(w^h, \Delta^h\right) \rhd \left(w^l, \Delta^l\right)$ imply that $\left(w^l, \Delta^l\right) \succ_1 \left(w^h, \Delta^h\right)$. Type 1's equilibrium contract equals $\left(w^l, \Delta^l\right)$.□

Proof of Proposition 5

The monopolist solves

$$\max \kappa \left\{W - w_1 - (1 - \pi(\hat{e}_1(1)))[L - \Delta_1]\right\} + (1-\kappa)\left\{W - w_2 - (1 - \pi(\hat{e}_2(2)))[L - \Delta_2]\right\}$$

such that

$$\begin{aligned} G_1((w_1, \Delta_1), (w_0, \Delta_0)) &= \max\{0, G_1((w_2, \Delta_2), (w_0, \Delta_0))\} \\ G_2((w_2, \Delta_2), (w_0, \Delta_0)) &= \max\{0, G_2((w_1, \Delta_1), (w_0, \Delta_0))\}. \end{aligned}$$

If $\Delta_0 = L$, then by Lemma 5, the IC/IR constraints for a separating optimum simplify to[13]

$$\begin{aligned} G_1((w_1, \Delta_1), (w_0, \Delta_0)) &= 0 \\ G_2((w_2, \Delta_2), (w_1, \Delta_1)) &= 0. \end{aligned}$$

Assume $(w_1, \Delta_1) < (w_1^*, \Delta_1^*)$, then $G_2((w_1, \Delta_1), (w_1^*, \Delta_1^*)) > 0$ by Lemma 3, so the utility rent paid to type 2 implied by the binding IC constraint is higher if type 1 receives (w_1, Δ_1) rather than (w_1^*, Δ_1^*). Since the profit made on type 1 is higher in (w_1^*, Δ_1^*) as well, (w_1, Δ_1) can never be optimal. Hence, $(w_1, \Delta_1) \geq (w_1^*, \Delta_1^*)$.

Assume $(w_1, \Delta_1) = (w_1^*, \Delta_1^*)$, one can find a contract (w_1', Δ_1') such that $(w_1', \Delta_1') \sim_1 (w_1^*, \Delta_1^*)$ and $(w_1', \Delta_1') > (w_1^*, \Delta_1^*)$ (and thus, $(w_1', \Delta_1') \prec_2 (w_1^*, \Delta_1^*)$), but sufficiently close to (w_1^*, Δ_1^*) such that the loss in profit on type 1 is of second order, but the reduction in the rent paid to type 2 is of first order. Hence, $(w_1, \Delta_1) > (w_1^*, \Delta_1^*)$.

Finally, if the optimum is separating, the incentive compatibility constraint for type 1 is slack. Hence, the problem becomes separable for type 2. The contract (w_2, Δ_2) solves the full information problem with type 2's outside opportunity equal to (w_1, Δ_1). Clearly, this contract is efficient.

If $\Delta_0 = 0$, then by Lemma 5, the IC/IR constraints for a separating optimum simplify to[14]

$$\begin{aligned} G_1((w_1, \Delta_1), (w_2, \Delta_2)) &= 0 \\ G_2((w_2, \Delta_2), (w_0, \Delta_0)) &= 0. \end{aligned}$$

[13] Notice that the optimum is only separating if the solution of this simplified constrained problem satisfies monotonicity, i.e. $(w_1, \Delta_1) \geq (w_2, \Delta_2)$. If not, the two types are pooled.

[14] Again, the optimum is only separating if the solution of this problem satisfies monotonicity, i.e. $(w_1, \Delta_1) \geq (w_2, \Delta_2)$.

Assume $(w_2, \Delta_2) > (w_2^*, \Delta_2^*)$, then $G_1((w_2, \Delta_2), (w_2^*, \Delta_2^*)) > 0$ by Lemma 3, so the utility rent paid to type 1 implied by the binding IC constraint is higher if type 2 receives (w_2, Δ_2) rather than (w_2^*, Δ_2^*). Since the profit made on type 2 is higher in (w_2^*, Δ_2^*) as well, (w_2, Δ_2) can never be optimal. Hence, $(w_2, \Delta_2) \leq (w_2^*, \Delta_2^*)$.

Assume $(w_2, \Delta_2) = (w_2^*, \Delta_2^*)$, one can find a contract (w_2', Δ_2') such that $(w_2', \Delta_2') \sim_2 (w_2^*, \Delta_2^*)$ and $(w_2', \Delta_2') < (w_2^*, \Delta_2^*)$ (and thus, $(w_2', \Delta_2') \prec_1 (w_2^*, \Delta_2^*)$), but sufficiently close to (w_2^*, Δ_2^*) such that the loss in profit on type 2 is of second order, but the reduction in the rent paid to type 1 is of first order. Hence, $(w_2, \Delta_2) < (w_2^*, \Delta_2^*)$.

Finally, if the optimum is separating, the incentive compatibility constraint for type 2 is slack. Hence, the contract (w_1, Δ_1) solves the full information problem with type 1's outside opportunity equal to (w_2, Δ_2). Clearly, this contract is efficient.□

Proof of Proposition 6

If $\Delta_0 < \min\{\Delta_1^*, \Delta_2^*\}$, then any contract $(w_2, \Delta_2) < (w_0, \Delta_0)$ such that $(w_2, \Delta_2) \succeq_2 (w_0, \Delta_0)$, is dominated by offering (w_0, Δ_0) to type 2 and (w_1^*, Δ_1^*) to type 1. These contracts are incentive compatible by Lemma 3. Given that the full-information problem is convex and $(w_2^*, \Delta_2^*) > (w_0, \Delta_0)$, the insurer makes lower profit when offering $(w_2, \Delta_2) < (w_0, \Delta_0)$. Moreover, the insurer cannot make higher profits than the full-information profits on type 1. Any contract $(w_1, \Delta_1) < (w_0, \Delta_0)$ such that $(w_1, \Delta_1) \succeq_1 (w_1^*, \Delta_1^*)$ is also dominated by offering (w_0, Δ_0) to type 2 and (w_1^*, Δ_1^*) to type 1. Again, the profit on type 1 cannot be higher than in (w_1^*, Δ_1^*). Moreover, by Lemma 3, $(w_1, \Delta_1) \succeq_2 (w_0, \Delta_0)$ and thus incentive compatibility requires $(w_2, \Delta_2) \succeq_2 (w_0, \Delta_0)$. Again, given that the full-information problem is convex and $(w_2^*, \Delta_2^*) > (w_0, \Delta_0)$, the insurer makes lower profit when offering a contract $(w_2, \Delta_2) < (w_0, \Delta_0)$ rather than (w_0, Δ_0). Hence, both (w_1, Δ_1) and (w_2, Δ_2) need to be greater than the outside option (w_0, Δ_0) in order to be optimal. If $\Delta_0 > \max\{\Delta_1^*, \Delta_2^*\}$, the argument is exactly the same, mutatis mutandum. In this case, both (w_1, Δ_1) and (w_2, Δ_2) need to be smaller than the outside option (w_0, Δ_0) in order to be optimal.

Now, if either

$$(w_1, \Delta_1) > (w_0, \Delta_0) \text{ and } (w_2, \Delta_2) > (w_0, \Delta_0)$$

or

$$(w_1, \Delta_1) < (w_0, \Delta_0) \text{ and } (w_2, \Delta_2) < (w_0, \Delta_0),$$

Lemma 5 and exactly the same argument as in Proposition 5 applies. This proves the first part of the proposition.

If $\Delta_2^* < \Delta_0 < \Delta_1^*$, $G_1[(w_1^*, \Delta_1^*), (w_0, \Delta_0)] = 0$ implies that $G_2[(w_1^*, \Delta_1^*), (w_0, \Delta_0)] < 0$ and $G_2[(w_2^*, \Delta_2^*), (w_0^*, \Delta_0^*)] = 0$ implies that $G_1[(w_2^*, \Delta_2^*), (w_0^*, \Delta_0^*)] < 0$, by Lemma 3. Hence, both $G_1[(w_1^*, \Delta_1^*), (w_2^*, \Delta_2^*)] \geq 0$ and $G_2[(w_2^*, \Delta_2^*), (w_1^*, \Delta_1^*)] \geq 0$. The incentive compatibility constraints are satisfied for the full-information contracts.□

Proof of Result 1

The optimal linear sharing rule maximizes the total surplus, that is the sum of the perceived certainty equivalent for the agent and the expected profit for the firm. With $\hat{e}_i(s) = \frac{\hat{\theta}_i}{\psi}s$, this simplifies to

$$\max_s (1-s)\,\theta\hat{\theta}_i \frac{s}{\psi} + s\hat{\theta}_i^2 \frac{s}{\psi} - \frac{\hat{\theta}_i^2}{\psi}\frac{s^2}{2} - \frac{\eta}{2}s^2\sigma^2.$$

The derivation of this problem yields

$$s_i^* = \frac{1}{1 + \frac{1}{\hat{\theta}_i\theta}\psi\eta\sigma^2 - \frac{\hat{\theta}_i - \theta}{\theta}},$$

for both the competitive and the monopolistic contract. The performance-independent transfer t is such that the profit of the firms is zero in competition and such that the perceived expected utility of the agent equals $\hat{u}_i$ with a monopolist.□

Proof of Result 2

For an agent with belief $\hat{\theta}$, the marginal rate of substitution between s and t equals

$$\left.\frac{ds}{dt}\right|_{\hat{\theta}} = \hat{\theta}\hat{e}(s) - \eta\sigma^2.$$

Hence, $\left.\frac{ds}{dt}\right|_{\hat{\theta}_1} \geq \left.\frac{ds}{dt}\right|_{\hat{\theta}_2}$, since $\hat{\theta}_1 \geq \hat{\theta}_2$. Given this single-crossing property, any competitive equilibrium must satisfy $s_1 > s_2$ and $t_1 < t_2$.□

Proof of Result 3

Since type 1 exerts more effort for any given contract, any contract making zero profit from type 2 makes non-negative profits from type 1. Hence, by revealed preference, type 1 prefers his full information contract. If type 2 prefers type 1's full information contract as well, type 1 will be given the contract that makes zero profit from type 1 and makes type 2 indifferent between the two contracts. Hence,

$$t^h = \left(1 - s^h\right)\theta\hat{\theta}_1 \frac{s^h}{\psi}$$

and

$$(1 - s_2^*)\, s_2^* \frac{\theta\hat{\theta}_2}{\psi} + (s_2^*)^2 \left[\frac{\left(\hat{\theta}_2\right)^2}{2\psi} - \eta\sigma^2\right] = \left(1 - s^h\right) s^h \frac{\theta\hat{\theta}_1}{\psi} + \left(s^h\right)^2 \left[\frac{\left(\hat{\theta}_2\right)^2}{2\psi} - \eta\sigma^2\right].$$

The result follows.□

Proof of Result 4

The monopolist solves

$$\max \kappa \left[(1-s_1)\theta e_1 - t_1\right] + (1-\kappa)\left[(1-s_2)\theta e_2 - t_2\right]$$

such that

$$\begin{aligned} t_i + s_i\hat{\theta}_i e_i - \psi\frac{e_i^2}{2} - \frac{\eta}{2}s_i^2\sigma^2 &\geq t_j + s_j\hat{\theta}_i e_i^j - \psi\frac{\left(e_i^j\right)^2}{2} - \frac{\eta}{2}s_j^2\sigma^2 && (IC_i) \\ t_i + s_i\hat{\theta} e_i - \psi\frac{e_i^2}{2} - \frac{\eta}{2}s_i^2\sigma^2 &\geq \hat{u}_i && (IR_i) \end{aligned}$$

and

$$e_i = \frac{\hat{\theta}_i}{\psi}s_i \text{ and } e_i^j = \frac{\hat{\theta}_i}{\psi}s_j \text{ for } i,j = 1,2.$$

First, consider the case that IC_2 and IR_1 are binding, then type 2 is given a rent

$$R = s_1\left(\hat{\theta}_2 e_2^1 - \hat{\theta}_1 e_1\right) - \psi\frac{\left(\left(e_2^1\right)^2 - \left(e_1\right)^2\right)}{2}$$

such that

$$t_2 + s_2\hat{\theta}_2 e_2 - \psi\frac{e_2^2}{2} - \frac{\eta}{2}s_2^2\sigma^2 = \hat{u}_1 + R.$$

The monopolist's problem simplifies to

$$\begin{aligned} \max \kappa &\left[\theta e_1 + s_1\left(\hat{\theta}_1 - \theta\right)e_1 - \psi\frac{\left(e_1\right)^2}{2} - \frac{\eta}{2}s_1^2\sigma^2\right] \\ &\qquad + (1-\kappa)\left[\theta e_2 + s_2\left(\hat{\theta}_2 - \theta\right)e_2 - \psi\frac{\left(e_2\right)^2}{2} - \frac{\eta}{2}s_2^2\sigma^2 - R\right] \end{aligned}$$

with

$$e_i = \frac{\hat{\theta}_i}{\psi}s_i \text{ for } i = 1,2.$$

Except for the rent R paid to type 2, this problem is the same as in the full-information problem. Since R only depends on s_1, the optimal sharing rule for type 2 is the same as in the full-information problem. The monopolist reduces the rent by the distorting the sharing rule for type 1. The result in the proposition follows immediately from differentiating with respect to s_1.

Second, consider the case that IC_1 and IR_2 are binding. The problem is the same, mutatis mutandum. A rent is paid to type 1 now, so only the sharing rule for type 2 will be different from the full-information problem. The sharing rule for type 2 is distorted compared to the full-information problem.

Finally, from the IC constraints, we find that the full-information sharing rules for type 1 and type 2 are optimal if respectively

$$\hat{u}_1 \geq \hat{u}_2 + \frac{\left(\hat{\theta}_1\right)^2 - \left(\hat{\theta}_2\right)^2}{2\psi} (s_2^*)^2$$

and

$$\hat{u}_2 \geq \hat{u}_1 + \frac{\left(\hat{\theta}_2\right)^2 - \left(\hat{\theta}_1\right)^2}{2\psi} (s_1^*)^2 .$$

Since $\hat{\theta}_1 > \hat{\theta}_2$ and $s_1^* > s_2^*$, it immediately follows that for $\hat{u}_1 < l \equiv \hat{u}_2 + \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{2\psi} (s_2^*)^2$, IC_1 and IR_2 are binding. For $\hat{u}_1 \in [l, h]$ with $h \equiv \hat{u}_2 + \frac{(\hat{\theta}_1)^2 - (\hat{\theta}_2)^2}{2\psi} (s_1^*)^2$, the full-information contracts are optimal. For $\hat{u}_1 > h$, IC_2 and IR_1 are binding.□