Insurance and Perceptions:
How to Screen Optimists and Pessimists*

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Abstract

People have very different beliefs about the risks they face. I analyze how heterogeneous risk perceptions affect the insurance contracts offered by profit-maximizing firms. An essential distinction is how risk perceptions affect the willingness to pay for insurance relative to the willingness to exert risk-reducing effort. This determines both the sign of the correlation between risk and insurance coverage in equilibrium, shedding new light on a recent empirical puzzle, and the type of individuals screened by either monopolistic or competing firms. Even with perfect competition, heterogeneous risk perceptions may well strengthen the case for government intervention in insurance markets.

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1 Introduction

The perception of risk is inherently subjective.[1] Financial traders disagree about the risk of investments, mortgage bankers about the risk of defaulting homeowners, old and young drivers about the risk of a car accident, homeowners and renters about the risk of flooding. One of two neighbours may perceive the risk of a hurricane as very high, while the other perceives the risk as very small, even though the risk is exactly the same.[2] At the same time, the perception of the extent to which precautionary efforts mitigate the risk may differ as well. As a result, the one neighbour may take precautionary measures, while the other does not. Risk perceptions thus affect both the insured's

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[1] Slovic (2000) surveys the research documenting the heterogeneity in the perception of risk and its determinants.

[2] Peacock *et al.* (2005) find that people in Florida have very different perceptions about the risk of a hurricane damaging their property. While the actual risk is fully determined by one's geographic location, it only explains a small share of the variation in perceived risk.

willingness to pay for insurance and her risk exposure, which makes them central to the design of insurance contracts.

The canonical model for insurance analysis (Akerlof, 1970; Rothschild and Stiglitz, 1976) considers only heterogeneity in risks. Since higher risk types value insurance more, heterogeneity in risks gives rise to adverse selection with risk and insurance coverage being positively correlated in equilibrium. Empirically, however, this correlation is found to be insignificant or even negative in several insurance markets.[3] This puzzle has inspired a growing literature to introduce heterogeneous risk preferences in the analysis.[4] In particular, types who are more risk-tolerant take fewer precautions and are less inclined to buy insurance, breaking the positive link between risk and insurance coverage (de Meza and Webb, 2001; Jullien *et al.*, 2007). The introduction of heterogeneous risk preferences in the canonical model can help explaining the absence of positive correlation in some insurance markets. Direct evidence on the role of heterogeneity in risk preferences is, however, lacking. While other sources of heterogeneity could be underlying the empirical moments, the positive and normative implications of alternative sources have been largely unexplored.

This paper presents a tractable model in which individuals differ only in their risk perceptions and analyzes how the heterogeneity in risk perceptions affects the equilibrium contracts offered by profit-maximizing insurers. From a positive perspective, the analysis provides an alternative explanation for the recent empirical evidence, relating the sign of the correlation between risk and insurance coverage to the differences in risk perceptions, in particular regarding the risk itself and one's control over this risk. Depending on the correlation between these two dimensions, I find that risk perceptions can break or strengthen the positive link between risk and insurance coverage. From a normative perspective, I characterize the screening distortions as a function of the differences in risk perceptions and find that this relation crucially depends on the nature of competition. The welfare consequences of the screening distortions in this framework are different from earlier welfare results regarding adverse selection.

I consider a simple model with two states, a good state and a bad state. Insurees exert costly effort to increase the probability that the good state occurs, but have different beliefs about this probability as a function of effort. While the insurer cannot observe an insuree's belief or effort, he perceives her risk to depend only on the effort she exerts. The insuree does not change her belief in response to the menu of insurance contracts being offered. That is, the insurer and the insurees 'agree to disagree' about the true underlying risk. The insurees' preferences satisfy a single-crossing property if the one insuree perceives the likelihood of the risk as lower than the other insuree for any given insurance contract. This is conditional on the effort levels chosen by the respective insurees. Optimism can therefore arise for two reasons. First, an insuree may be more optimistic about the baseline likelihood of the risk for the same level of effort, referred to as *baseline optimism*. Second, an insuree may be more optimistic about the marginal return of effort, referred to as *control*

[3] See Cohen and Siegelman (2010) for an overview. For example, Chiappori and Salanié (1997; 2000), Cardon and Hendel (2001) find insignificant correlations, while Cawley and Philipson (1999), Finkelstein and McGarry (2006), and Fang *et al.* (2008) find negative correlations.

[4] See Einav *et al.* (2010a) and Chiappori and Salanié (2012) for recent reviews.

optimism, and therefore exert higher effort for the same insurance contract. If the single-crossing property is satisfied, the insurer can only separate the (more) optimistic insuree by offering her less insurance coverage than the (more) pessimistic insuree. This monotonicity property is independent of the nature of competition between insurers.

Optimistic agents receive less insurance, but still may be more risky ex-post if they are pessimistic about their control and take less precautions. The correlation between risk and insurance coverage crucially depends on the correlation between the perceptions about the baseline risk and the own control. With two types of insurees who only differ in their beliefs, I show that it is sufficient that the one type is more baseline-optimistic and control-optimistic for the equilibrium to satisfy the positive correlation property. For the correlation to be negative, it is necessary that the control-pessimistic type is also more optimistic about the likelihood of the risk. The model thus provides an alternative explanation, based only on heterogeneity in risk perceptions, why the correlation between risk and insurance is found to be positive in some and negative in other insurance markets.

The previous results depend only on the equilibrium contracts being incentive compatible. I also characterize and evaluate the contract distortions relative to the case where the insurer could observe the insuree's beliefs and find that these depend crucially on the interaction between the nature of competition and the dimension in which beliefs differ. The reason is that competing insurers screen types based on the difference in cost, while a monopolistic insurer screens types based on the difference in valuation. The difference in cost is determined by the control beliefs, while the difference in valuation is determined by the baseline beliefs. The mere presence of a different type thus entails distortions due to the screening of unobservable types, just like with heterogeneity in risk types or risk preferences. However, not only the direction of the distortions may be different, the evaluation of these distortions is different as well. For example, insurance companies may screen both risk-tolerant and optimistic types by providing less coverage at a lower premium. While the reduction in coverage comes at a low cost for the risk-tolerant type, it comes at a high cost for the optimistic type who underestimates the actual risk she is facing. The example clearly illustrates that the screening distortions due to heterogeneity in beliefs may well strengthen the case for (paternalistic) government intervention through insurance mandates. Interestingly, this contrasts with the finding in Sandroni and Squintani (2007) that heterogeneity in beliefs reduces the scope for government intervention. The heterogeneity in optimistic beliefs they consider implies that some agents with different risks perceive their risk to be the same and are pooled in equilibrium. The heterogeneity I consider implies that agents facing the same risk are separated in equilibrium.

Related literature Starting with the work by Chiappori and Salanié (1997; 2000), a large literature has re-examined the reasons for the heterogeneous insurance choices and risks in insurance markets. In particular, de Meza and Webb (2001) consider a parsimonious model with a risk-averse and risk-tolerant type; the risk-tolerant type values insurance less and takes no precautions regardless of the insurance received. The paper shows the existence of a competitive equilibrium with advantageous selection; only the risk-averse type buys insurance, but is less risky due to the precautions she takes. In contrast with the standard model by Rothschild and Stiglitz (RS), a small tax on insurance can yield a Pareto gain in their model. Jullien *et al.* (2007) consider monopolistic screening when agents differ in risk aversion and choose a continuous effort level. The paper derives conditions on risk preferences to obtain a single-crossing property. They focus on CARA preferences to show how single-crossing implies monotonicity in insurance coverage, to illustrate the possibility of negative correlation between insurance and risk and to characterize the screening distortions. In terms of modelling, my paper is closely related to both papers combining moral hazard and adverse selection in respectively a competitive and monopolistic market.[5] The approach in my paper is more general by allowing for continuous effort and general risk preferences. The paper also clearly distinguishes between the results that do and do not depend on the market structure, and shows the important interactions between market forces and type heterogeneity. The mechanism underlying the negative correlation is very similar in the case of heterogeneous risk preferences and perceptions. Also Huang *et al.* (2007) and Koufopoulos (2008) use heterogeneity in perceptions to illustrate the possibility of advantageous selection. Similar to de Meza and Webb (2001), they consider a RS-type model where one of the two types does exert precautionary effort, but is still more pessimistic about the probability of the risk and thus buys more insurance in the competitive equilibrium. In contrast with all previous papers, I provide a more general framework to shed light on this mechanism and explicitly relate the sign of the correlation property to conditions on the type heterogeneity.

Next to characterizing the positive implications, I also show how the welfare and policy implications differ depending on the source of heterogeneity. Einav *et al.* (2010b; 2010c) attribute the variation in insurance choices, unexplained by heterogeneous risks, to heterogeneous preferences and estimate a very small welfare cost due to inefficient pricing, implying a limited scope for government intervention in insurance markets. Spinnewijn (2012) finds that if a plausible share of this unexplained heterogeneity is driven by heterogeneity in risk perceptions, welfare and policy conclusions are substantially different. The previous papers consider the welfare cost of inefficient pricing, assuming that the set of contract is fixed, which simplifies the analysis. In contrast with these papers, the current paper analyzes the welfare implications of inefficient contract distortions and allows for moral hazard. A number of papers has analyzed equilibrium contracts in the presence of biased risk perceptions. Spinnewijn (2009) focuses on moral hazard and analyzes how the biases in baseline and control beliefs affect the optimal contract in the context of unemployment insurance when these biases are known to the insurer. Jeleva and Villeneuve (2004), Chassagnon and Villeneuve (2005) and Villeneuve (2005) focus on adverse selection only, analyzing equilibrium contracts in the RS model, but allowing risk types to misperceive their risks. As discussed before, Sandroni and Squintani (2007) also start from the RS model, but assume that some agents of the high-risk type are optimistic and think they are a low-risk type. Sandroni and Squintani

[5] Chassagnon and Chiappori (2005) also considers competitive screening with moral hazard and adverse selection in case of heterogeneous effort cost. This paper also shows how some standard results change substantially without single-crossing property, which I assume holds in my analysis.

(2010) generalize the analysis for a monopolistic insurer, focusing on the relation between observable variables in equilibrium. They find that in contrast with the competitive case the presence of optimistic high-risk types does not lead to major changes in testable implications in the case of monopoly. In contrast with these previous models, I consider a model with moral hazard and adverse selection and with differing beliefs as the only source of heterogeneity. Finally, the paper also relates to the literature that explores what happens when boundedly rational consumers meet profit-maximizing firms (see Ellison, 2006; Spiegler, 2011).[6] In the spirit of this literature, I also consider the externalities that biased agents and unbiased agents impose on each other (e.g., DellaVigna and Malmendier, 2004; Gabaix and Laibson 2006) and how these are affected by the market structure.

The remainder of the paper is organized as follows. Section 2 introduces the model and defines the agent's beliefs. Section 3 analyzes properties of incentive compatible contracts with heterogeneity in beliefs. Section 4 characterizes the equilibrium screening contracts, contrasting the competitive equilibrium and the monopolistic optimum. Section 5 discusses welfare and policy implications. Section 6 concludes the paper. All proofs are in the appendix.

2 Model

I consider a principal-agent model with two states. In the good state, the total endowment equals W. In the bad state, the total endowment equals $W-L$. The agent's unobservable choice of effort determines the probability that the good or bad state occurs. When she exerts effort at additive cost $e \in E$, the good state occurs with probability $\pi(e)$ with $\pi' \geq 0, \pi'' < 0$. The bad state occurs with probability $1-\pi(e)$. A risk-neutral principal offers a contract to the risk-averse agent. For notational convenience, we describe the contract by the payoff-relevant terms for the agent. A contract is denoted by $c = (w, \Delta)$, where w is the agent's wealth net of the premium and Δ is the deductible. That is, the consumption levels of the agent, conditional on accepting the contract, are w and $w - \Delta$ in the good and bad state respectively. The deductible determines the consumption risk left to the agent. The higher the deductible, the less insured the agent is. The set of contracts that the principal can offer is restricted to

$$C \equiv \{(w, \Delta) \,|\Delta \in [0, L], w \in [\Delta, W]\}.$$

Hence, the agent cannot be overinsured, i.e. $\Delta \geq 0$, and cannot take on more risk than the difference in total endowments, i.e., $\Delta \leq L$.[7]

I denote the agent's and principal's outside option by (w_0, Δ_0) and $(W - w_0, L - \Delta_0)$ respectively. In the one extreme case, the agent owns the total endowment and thus bears the entire risk,

[6] In particular, Grubb (2009) and Eliaz and Spiegler (2008) analyze how firms exploit differences in overconfidence and optimism about future demand respectively with a menu of screening contracts.

[7] Notice that in a standard setting with common priors both restrictions would not be binding in equilibrium. Moreover, even with different priors, a principal will never overinsure an agent if the agent can make the bad state happen with certainty at zero cost.

$(w_0, \Delta_0) = (W, L)$. In the other extreme case, the principal owns the total endowment such that $(w_0, \Delta_0) = (0, 0)$. If the contract's deductible $\Delta < \Delta_0$, I call the contract an *insurance contract*. In this case, the difference $w_0 - w$ equals the insurance premium that the agent pays to reduce her risk from Δ_0 to Δ. If the contract's deductible $\Delta > \Delta_0$, I call the contract an *incentive contract*. Hence, the difference $w_0 - w$ equals the wage premium that the principals pays for increasing the agent's incentives from Δ_0 to Δ.

2.1 The Agent's Beliefs

The agent's perception of the probability of success as a function of effort may differ from the true probability. I denote the agent's belief as $\hat{\pi}(e)$ with $\hat{\pi}' \geq 0, \hat{\pi}'' < 0$. I introduce these beliefs in the most general way, but the analysis shows that the differences in the levels and margins of the perceived probability functions are essential like in Spinnewijn (2009).

Definition 1 *Agent i is baseline-optimistic if $\hat{\pi}_i(e) \geq \pi(e)$ for all $e \in E$. Agent i is more baseline-optimistic than agent j if $\hat{\pi}_i(e) \geq \hat{\pi}_j(e)$ for all $e \in E$.*

Definition 2 *Agent i is control-optimistic if $\hat{\pi}'_i(e) \geq \pi'(e)$ for all $e \in E$. Agent i is more control-optimistic than agent j if $\hat{\pi}'_i(e) \geq \hat{\pi}'_j(e)$ for all $e \in E$.*

For expositional purposes, I consider the sign of the differences to be the same for all effort levels. Baseline and control beliefs are related, but optimism in the one dimension does not exclude pessimism in the other dimension. Whether agents who are more optimistic about the baseline probability are also more optimistic about their control depends on the context, as in the following two examples with $\rho'(e) > 0$, $\rho''(e) < 0$ and $\hat{\pi}(e)$ bounded between 0 and 1.

Example I $\pi(e) = \theta\rho(e)$ *and* $\hat{\pi}(e) = \hat{\theta}\rho(e)$*:*

When for a project the probability of success is complementary in the entrepreneur's ability θ and effort e, an entrepreneur who overestimates her ability (i.e., $\hat{\theta} > \theta$) is at the same time baseline-optimistic and control-optimistic.

Example II $1 - \pi(e) = \alpha + \phi(1 - \rho(e))$ *and* $1 - \hat{\pi}(e) = \alpha + \hat{\phi}(1 - \rho(e))$:

A driver who underestimates the probability to have a car accident when exerting no effort (i.e., $\hat{\phi} < \phi$) is baseline-optimistic, but also underestimates the return to exerting effort and is thus control-pessimistic.

2.2 The Agent's Preferences

The agent chooses the effort level that maximizes her perceived expected utility. Given the contract (w, Δ), the agent solves

$$U(w, \Delta) \equiv \max_e \hat{\pi}(e) u(w) + [1 - \hat{\pi}(e)] u(w - \Delta) - e. \tag{1}$$

The agent's choice of effort $\hat{e}(w,\Delta)$ equalizes the perceived marginal return and marginal cost,

$$\hat{\pi}'(\hat{e}(w,\Delta))[u(w)-u(w-\Delta)]=1. \tag{2}$$

The agent exerts more effort for a higher deductible Δ, but she also exerts more effort the higher she believes the increase in the success probability to be when increasing her effort level, $\hat{\pi}'(\cdot)$. Comparing two agents, the more control-optimistic agent of the two always exerts more effort.

I now introduce a third definition regarding any two agents' beliefs which involves the endogenous choice of effort by the respective agents.

Definition 3 *Agent i is more optimistic than agent j if $\hat{\pi}_i(\hat{e}_i(c))\geq\hat{\pi}_j(\hat{e}_j(c))$ for all $c\in C$.*

An agent can be more optimistic either because she perceives the likelihood of the good state to be higher for the same level of effort or because she exerts more effort. Hence, if the more baseline-optimistic agent is more control-optimistic as well, then she will be more optimistic than the other agent. If the more baseline-optimistic agent is more control-pessimistic, then the higher effort exerted by the other agent may induce that agent to perceive the good state to be more likely. However, when the incentives provided by the feasible contracts are not too large, the difference in baseline beliefs dominates the difference in control beliefs and the baseline-optimistic agent will be more optimistic.[8]

Lemma 1 *Agent i is more optimistic than agent j for any set C when she is more baseline-optimistic and control-optimistic than agent j. For some non-empty set C, agent i is more optimistic than agent j when she is more baseline-optimistic and control-pessimistic than agent j.*

Notice that when agent i is sufficiently more baseline-optimistic such that $\hat{\pi}_i(e)\geq\max_{e\in E}\hat{\pi}_j(e)$ for any effort level, this agent will be more optimistic for any C, regardless of her control beliefs. Hence, even when the more baseline-optimistic agent is more control-pessimistic she may be more optimistic without restrictions on the set C. A natural example of this is Example II with parameters $\hat{\phi}_i=0$ and $\hat{\phi}_j>0$.

The agent's expected utility in the outside option $c_0=(w_0,\Delta_0)$ also depends on her beliefs,

$$U(c_0)\equiv\hat{\pi}(\hat{e}(c_0))\,u(w_0)+[1-\hat{\pi}(\hat{e}(c_0))]\,u(w_0-\Delta_0)-\hat{e}(c_0). \tag{3}$$

The perceived expected utility is increasing in the baseline belief about the probability $\hat{\pi}(\cdot)$ that the good state occurs. This increase is higher, the less insurance the outside option provides.

[8] With full insurance, the difference in perceived probabilities equals $\hat{\pi}_i(0)-\hat{\pi}_j(0)$. The change in response to an increase in incentives equals

$$\frac{d[\hat{\pi}_i(\hat{e}_i(c))-\hat{\pi}_j(\hat{e}_j(c))]}{d[u(w)-u(w-\Delta)]}=\left[\frac{\hat{\pi}_i'(\hat{e}_i(c))^2}{-\hat{\pi}_i''(\hat{e}_i(c))}-\frac{\hat{\pi}_j'(\hat{e}_j(c))^2}{-\hat{\pi}_j''(\hat{e}_j(c))}\right]/\,[u(w)-u(w-\Delta)].$$

As the first-order derivatives are equal by the FOC, this change depends only on how the second-order derivative changes with the endogenous effort choice.

3 Incentive Compatibility with Heterogeneous Beliefs

Pessimistic agents are willing to pay more for insurance coverage than optimistic agents because they perceive the risk as more likely, regardless of whether optimism is driven by baseline and/or control optimism. This single-crossing property of the preferences implies that only contracts providing more insurance to the more pessimistic agent can be incentive compatible. Whether the more pessimistic agent is also more risky ex-post depends on the agents' efforts and thus the agents' control beliefs. The monotonicity in insurance coverage, therefore implies simple conditions for the correlation between risk occurrence and insurance coverage to be positive or negative, which depend only on the relative optimism and control-optimism of the agents.

3.1 Single-Crossing Property

I consider two types of agents who only differ in their beliefs. Type 1 and type 2 hold the beliefs $\hat{\pi}_1(\cdot)$ and $\hat{\pi}_2(\cdot)$ respectively, with $\hat{\pi}_1(\cdot) \neq \hat{\pi}_2(\cdot)$. The share of type i-agents is denoted by κ_i. Types are unobservable to the insurers, but the insurers know the true probability of success $\pi(\cdot)$, which is the same function of effort for both types.[9]

Assumption 1 *Type 1 is more optimistic than type 2.*

The higher the perceived probability that the bad state occurs, the higher the willingness to give up wealth w to decrease the deductible Δ. The perceived marginal rate of substitution between w and Δ for type i equals

$$\left.\frac{d\Delta}{dw}\right|_{\hat{\pi}_i} = \frac{\hat{\pi}_i(\hat{e}_i(w,\Delta))}{1-\hat{\pi}_i(\hat{e}_i(w,\Delta))} \frac{u'(w)}{u'(w-\Delta)} + 1. \tag{4}$$

The effect through changes in effort on the perceived expected utility in response to dw and $d\Delta$ is of second order because of the envelope condition and does not affect the marginal rate of substitution. For different types, the marginal rates of substitution for a given contract (w, Δ) is ranked based on the respective perceived probability of success $\hat{\pi}_i(\hat{e}_i(w,\Delta))$. If type 1 is more optimistic than type 2, the marginal rates of substitution are ranked the same for any contract.

Lemma 2 *For any contract $c \in C$, type 1 values additional insurance less than type 2,*

$$\left.\frac{d\Delta}{dw}\right|_{\hat{\pi}_1} \geq \left.\frac{d\Delta}{dw}\right|_{\hat{\pi}_2}.$$

The profit-maximizing insurer cannot observe the type of insuree he is facing. By the revelation principle, we can restrict the analysis to contracts that are incentive compatible such that the different types will self-select into the contracts designed for them. A pair of contracts

[9] For the characterization of the equilibrium contracts, it is sufficient that the agents' true probability functions are perceived to be the same by the insurers.

$\{(w_1, \Delta_1), (w_2, \Delta_2)\}$ is incentive compatible if and only if

$$U^i(w_i, \Delta_i) \geq U^i(w_j, \Delta_j) \text{ for } i, j = 1, 2, \tag{5}$$

with

$$U^i(w, \Delta) \equiv \max_e \hat{\pi}_i(e) u(w) + (1 - \hat{\pi}_i(e)) u(w - \Delta) - e. \tag{6}$$

Clearly, for any pair of incentive compatible contracts, one contract cannot offer more consumption in both states. That is, if $w_1 > w_2$, then $w_1 - \Delta_1 < w_2 - \Delta_2$ and vice versa. I introduce the relation $x \rhd y$ to describe that the contract x provides less insurance than contract y in the sense that x provides lower coverage at a lower insurance premium than contract y.

Notation 1 $(w_i, \Delta_i) \rhd (w_j, \Delta_j) \Leftrightarrow w_i > w_j$ *and* $w_i - \Delta_i < w_j - \Delta_j$

Notation 2 $(w_i, \Delta_i) \unrhd (w_j, \Delta_j) \Leftrightarrow w_i \geq w_j$ *and* $w_i - \Delta_i \leq w_j - \Delta_j$

I use the particular notation because $(w_i, \Delta_i) \rhd (w_j, \Delta_j)$ implies $(w_i, \Delta_i) > (w_j, \Delta_j)$. Notice that the opposite does not hold.

3.2 Monotonicity

In standard adverse selection problems the incentive compatibility constraints imply a monotonicity constraint on the separating contracts offered to different types, if the preferences satisfy a single-crossing property. I show that the same is true here for general preferences, despite the presence of moral hazard.

The utility from one insurance contract can be expressed as the utility from any other insurance contract, plus the sum of the utility gains, positive or negative, from the incremental changes that lead from the latter to the former insurance contract. That is,

$$U^i(w_i, \Delta_i) = U^i(w_j, \Delta_j) + \int_{\Delta_j}^{\Delta_i} \left[U_w^i(\widetilde{w}(\Delta), \Delta) \widetilde{w}'(\Delta) + U_\Delta^i(\widetilde{w}(\Delta), \Delta)\right] d\Delta, \tag{7}$$

for any continuous, differentiable function $\widetilde{w}(\Delta)$ with $\widetilde{w}(\Delta_j) = w_j$ and $\widetilde{w}(\Delta_i) = w_i$. I denote the gain in perceived expected utility for type i from switching from contract (w_y, Δ_y) to (w_x, Δ_x) by

$$G^i((w_x, \Delta_x), (w_y, \Delta_y)) \equiv U^i(w_x, \Delta_x) - U^i(w_y, \Delta_y). \tag{8}$$

For contracts to be incentive compatible, the gain from switching to the other type's contract has to be negative for both types,

$$G^1((w_2, \Delta_2), (w_1, \Delta_1)) \leq 0 \tag{IC_1}$$

$$G^2((w_1, \Delta_1), (w_2, \Delta_2)) \leq 0. \tag{IC_2}$$

When choosing between two contracts, the more optimistic type puts relatively more weight on the change in consumption when successful and relatively less weight on the change in consumption when unsuccessful. This difference in weights is not sufficient to sign the difference for two types in utility gains from switching contracts, because the exerted effort levels differ as well. However, the single-crossing property can be used the evaluate the utility gains from all marginal changes in Δ and $\widetilde{w}(\Delta)$ for which changes in effort are of second order. When changing the contract from (w_j, Δ_j) to (w_i, Δ_i), the sign of the difference in utility gains for type i and type j from the marginal changes along the linear function, $\widetilde{w}(\Delta) = w_j + (\Delta - \Delta_j)[(w_i - w_j)/(\Delta_i - \Delta_j)]$, exactly equals the sign of the difference in perceived likelihoods, evaluated along the linear path,

$$\hat{\pi}_i(\hat{e}_i(\widetilde{w}(\Delta), \Delta)) - \hat{\pi}_j(\hat{e}_j(\widetilde{w}(\Delta), \Delta)). \tag{9}$$

If an agent is more optimistic, she suffers less from each marginal increase in Δ and gains more from the associated marginal increase in $\widetilde{w}(\Delta)$, leading from the contract providing more to the contract providing less insurance. This observation implies the following lemma.

Lemma 3 *If contract c_x provides less insurance than contract c_y, $c_x \rhd c_y$, then type 1 gains more than type 2 when switching from c_y to c_x, $G^1(c_x, c_y) > G^2(c_x, c_y)$.*

The utility gain from switching to an insurance contract for which the insurance coverage and the insurance premium is lower, is greater for someone who is more optimistic about the probability of the good state. This implies that for two contracts to be incentive compatible, the insurance contract designed for the more optimistic type must provide less insurance, but at a lower insurance premium.

Proposition 1 *Type 1 receives less insurance than type 2 in any incentive compatible equilibrium, i.e.*

$$(w_1, \Delta_1) \unrhd (w_2, \Delta_2).$$

This monotonicity property follows immediately from the incentive compatibility constraints and Lemma 3. Assume, by contradiction, that (w_2, Δ_2) provides less insurance than (w_1, Δ_1). Since type 1 is more optimistic than type 2, the utility gain from switching to the contract providing less insurance is higher for type 1 than for type 2. However, for (w_2, Δ_2) to be incentive compatible for type 2, her gain from switching from (w_1, Δ_1) to (w_2, Δ_2) must be positive, which implies that the gain from switching from (w_1, Δ_1) to (w_2, Δ_2) is positive for type 1 as well. By consequence, (w_1, Δ_1) is not incentive compatible for type 1.

3.3 Positive vs. Negative Correlation

With heterogeneity in perceptions, either positive or negative correlation can arise between the endogenous probability that the risk occurs for a type and the insurance coverage provided to that type. An optimistic type necessarily receives less insurance than a pessimistic type, but whether the optimistic type is more risky depends on both her control beliefs and the insurance coverage.

Corollary 1 *If type 1 is more optimistic and control-optimistic than type 2, the equilibrium satisfies the 'positive correlation'-property, i.e.*

$$(w_1, \Delta_1) \trianglerighteq (w_2, \Delta_2) \ \ \text{and}\ \pi\left(\hat{e}_1\left(w_1, \Delta_1\right)\right) \geq \pi\left(\hat{e}_2\left(w_2, \Delta_2\right)\right).$$

If type 1 is more control-optimistic, she exerts more effort than type 2 for the same level of insurance. Since in addition type 1 receives less insurance, she exerts more effort in equilibrium and is less likely to suffer a loss. The observed correlation between risk occurrence and insurance coverage is positive.[10]

Corollary 2 *Only if the optimistic type* 1 *is more control-pessimistic than type* 2*, the equilibrium may satisfy the 'negative correlation'-property, i.e.*

$$(w_1, \Delta_1) \trianglerighteq (w_2, \Delta_2) \ \ \text{and}\ \pi\left(\hat{e}_1\left(w_1, \Delta_1\right)\right) \leq \pi\left(\hat{e}_2\left(w_2, \Delta_2\right)\right).$$

If type 1 is more control-pessimistic, she exerts less effort than type 2 for the same level of insurance. However, she needs to be sufficiently more control-pessimistic such that she will exert less effort even when less insured. The negative correlation between optimism and control-optimism across types is thus necessary, but not sufficient for the negative correlation between risk and insurance coverage to occur. The corollary does not prove the existence of equilibria with the negative correlation property, but it is straightforward to construct an example. Consider the extreme case in which one agent perceives the marginal return to effort to be zero. Clearly, this agent will be more risky, but still be given less insurance in any separating equilibrium if she is also more optimistic than the other type. This will be the case for $\hat{\phi}_1 = 0$ and $\hat{\phi}_2 > 0$ in Example II.[11]

3.4 Discussion

Recent estimates of the correlation between risk and insurance coverage in different insurance markets suggest that the standard model with only heterogeneity in risks falls short empirically (Cohen and Siegelman, 2010). The analysis above shows how heterogeneity in perceptions as the only source of heterogeneity can cause some markets to satisfy the positive and other markets to satisfy the negative correlation property. Corollaries 1 and 2 relate the sign of this correlation to the correlation between the insurees' perception of control and their perception of the risk itself. The model's predictions suggest that a positive correlation between risk and insurance coverage is more likely when those who believe they have more control are also more optimistic about the final outcome, as illustrated in Example I. This type of heterogeneity could for example explain part of the positive correlation in the market for crop insurance (see Makki and Somwaru, 2001). The model's predictions suggest also that a negative correlation is more likely when those who perceive a risk as smaller also believe that the returns to reducing this risk are smaller. This type

[10] Notice that the property is trivially satisfied for a pooling equilibrium.
[11] This extreme case is also considered by Koufopoulos (2008) and Huang et al. (2007).

of heterogeneity relates to Example II and may explain why the correlation is not significantly positive in the market for automobile insurance (see Chiappori and Salanié, 1997, 2000) and even negative in the market for health insurance (see Fang *et al.*, 2008).[12]

Other sources of heterogeneity affecting the insurance choice or the risk of the insurees will affect the equilibrium correlation between risk and insurance as well. A natural source of heterogeneity explored in the literature is heterogeneity in risk preferences. In particular, risk-averse individuals are more likely to buy insurance and to exert precautionary effort. This reduces the positive correlation between risk and insurance and even reverses the correlation when risk-averse individuals exert more effort despite being more insured, as argued by de Meza and Webb (2001) and Jullien *et al.* (2007). The result in Corollary 2 is thus similar to this alternative explanation for the negative correlation property. An important difference between the two sources of heterogeneity is that risk aversion increases both the willingness to buy insurance and the willingness to take precautionary measures. Pessimistic perceptions increase the willingness to buy insurance, but may as well decrease the willingness to take precautionary measures and thus strengthen the positive link between insurance and risk. This distinction depends in a natural way on the type of heterogeneity in risk perceptions and clearly disentangles the two relevant forces determining the correlation in insurance markets more generally. Notice also that preference heterogeneity, in contrast with perception heterogeneity, is generally not sufficient to explain negative correlation in a competitive market, as argued by Chiappori *et al.* (2006). The reason is that a competitive equilibrium with negative correlation requires individuals to forego on additional insurance coverage offered at an actuarially fair price. Risk-averse individuals are not willing to do this, unless they do not perceive the offered price as actuarially fair.

The remaining part of the paper analyzes the screening distortions and welfare implications due to heterogeneity in risk perceptions and contrasts the welfare implications for alternative sources of heterogeneity. The empirical question which source of heterogeneity is driving the variation in insurance demand and risk remains open. The similar positive implications of preference and perception heterogeneity suggest the need for additional information to test for the importance of the different models. The recent empirical insurance literature has focused on preference heterogeneity and simply assumes that the heterogeneity in insurance choices is driven by heterogeneity in risk preferences (e.g., Cohen and Einav, 2007; Einav *et al.*, 2010c; Barseghyan *et al.*, 2011), but very little evidence relates insurance choices directly to preference measures.[13] The heterogeneity in risk perceptions is, however, well documented in the psychological literature (see Slovic, 2000) and also a growing literature in economics studies risk perceptions and its measurement (see Manski, 2004). Still, risk perceptions are rarely linked to insurance choices and risks. Complementing insurance data with individual information on the perceptions of risk and control would allow for a direct

[12] Notice that young drivers tend to overestimate the probability to avoid an accident, but underestimate the returns to driving safely (Finn and Bragg 1986, Tränkle et al. 1990). Similarly, women who overestimate the probability not to have breast cancer are less likely to take mammograms (Katapodi et al. 2004), plausibly because they underestimate the returns to preventive efforts, as argued by Polednak et al. (1991).

[13] For example, Fang et al. (2008) find that the negative correlation is driven by differences in cognitive ability rather than by differences in measures of risk aversion.

test of the model by relating individuals' insurance choices and risk to their specific perceptions.[14]

4 Equilibrium Insurance Contracts

In this section, I analyze the screening distortions in the insurance contracts relative to the full-information benchmark. I find that the interaction between the nature of the heterogeneity in beliefs and the nature of competition plays a central role in this analysis. The reason is that competing insurers screen types based on the difference in cost, while a monopolistic insurer screens types based on the difference in valuation. As shown before, the cost and valuation of a type are directly related to the type's risk perceptions. That is, competing insurers distort the contract offered to the 'low-cost' type to discourage the 'high-cost' type from pretending she has low cost. Control beliefs are thus central under competition. A monopolistic insurer distorts the contract to the 'low-valuation' type to discourage the 'high-valuation' type from pretending she has low valuation. Baseline beliefs thus become central under monopoly. From the single-crossing property, we know in which direction a contract would be distorted; the contract for the more optimistic type would be distorted towards less insurance coverage, while for the more pessimistic type this would be towards more insurance coverage.

The characterization of the screening distortions is not specific to heterogeneous risk perceptions, but applies to other sources of heterogeneity affecting the cost and valuation of types. This also implies that the screening distortions are similar for heterogeneous risk preferences and risk perceptions when the more optimistic type is also more control-pessimistic, as analyzed in de Meza and Webb (2001) and Jullien *et al.* (2007) in the competitive and monopolistic case respectively. The analysis highlights the importance of the difference in effort choice in the competitive case and the difference in insurance value in the monopolistic case.

4.1 Principal's Profits and Market Equilibrium

I consider profit-maximizing insurers in a competitive and monopolistic market. An insurer's expected profit depends on the effort level and thus the control beliefs of the agent accepting his contract. When an insuree of type i accepts the contract (w, Δ), the expected profit equals

$$\Pi(w, \Delta; i) = w_0 - w - [1 - \pi(\hat{e}_i(w, \Delta))](\Delta_0 - \Delta). \tag{10}$$

Notice that the insurer expects to pay $\Delta_0 - \Delta$ with probability $1 - \pi(\hat{e}_i(w, \Delta))$, which is decreasing in the agent's effort level $\hat{e}_i(w, \Delta)$, while the agent expects to receive this coverage with probability $1 - \hat{\pi}(\hat{e}_i(w, \Delta))$.

For the competitive equilibrium, I assume that insurers are competing like in Rothschild and

[14] One notable exception in the recent empirical literature is Barseghyan *et al.* (2012) who jointly estimate the heterogeneity in risk preferences and risk perceptions using only choice data, but taking a parametric approach. In ongoing work, Kircher and Spinnewijn analyze the non-parametric identification of the heterogeneity in risk preferences and risk perceptions, also using choice data only, but exploiting exogenous price variation.

Stiglitz (1976); the pair of contracts $\{(w_{c,i}^{**}, \Delta_{c,i}^{**})\}_{i=1,2}$ constitutes a competitive equilibrium if and only if no contract offered in equilibrium makes negative profits and no new contract can be offered and make positive profits. For the competitive case, I assume that the agent's outside option provides no insurance and that the participation constraints are not binding in equilibrium. The characterization of the screening distortions follows directly from the standard RS analysis, accounting for the endogenous effort level like in de Meza and Webb (2001) and Chassagnon and Chiappori (2005).

For the monopolistic optimum, I assume that the monopolistic insurer offers the pair of contracts $\{(w_{m,i}^{**}, \Delta_{m,i}^{**})\}_{i=1,2}$ that maximizes his expected profits, given the insurees' incentive compatibility and participation constraints. Since the participations constraints play a central role in the monopolistic case, I characterize the monopolistic optimum for different outside options for the agent, ranging from no insurance to full insurance. The characterization of the screening distortions follows the analysis in Jullien *et al.* (2007), characterizing monopolistic screening contracts with heterogeneous risk preferences.[15]

4.2 Full-Information Benchmark

In order to screen agents with unobservable types, the insurer changes the contracts relative to those that would be offered when he knows the agent's beliefs. This full-information benchmark is discussed at length in Spinnewijn (2009); the contract trades off the moral hazard cost of insurance, captured by the elasticity of the true loss probability with respect to a revenue-neutral change in insurance coverage, and the consumption smoothing benefits of insurance as perceived by the agent.

Proposition 2 *The profit-maximizing contract* (w^*, Δ^*) *is characterized by*

$$\frac{\frac{1-\hat{\pi}(\hat{e})}{1-\pi(\hat{e})}\frac{\pi(\hat{e})}{\hat{\pi}(\hat{e})}u'\left(w^* - \Delta^*\right) - u'\left(w^*\right)}{u'\left(w^*\right)} = \varepsilon_{1-\pi(\hat{e}),w-\Delta}\frac{\Delta_0 - \Delta^*}{w^* - \Delta^*}, \tag{11}$$

with $\hat{e} \equiv \hat{e}\left(w^*, \Delta^*\right)$ *and* $\varepsilon_{1-\pi(\hat{e}),w-\Delta} \equiv \frac{d[1-\pi(\hat{e})]}{d(w-\Delta)}|_{\Pi}\frac{w-\Delta}{1-\pi(\hat{e})}$*. In monopoly, the perceived expected utility* $U\left(w_m^*, \Delta_m^*\right) = U\left(w_0, \Delta_0\right)$*. In competition, the expected profit* $\Pi\left(w_c^*, \Delta_c^*\right) = 0$*.*

As discussed in Spinnewijn (2009), the more optimistic type does not necessarily receive less insurance than the more pessimistic type in the respective full-information contracts. This contrasts with Proposition 1. The reason is that the optimal response to control-optimism is ambiguous; if an insuree becomes more control-optimistic, less incentives are required to induce her to exert the same level of effort. Hence, the insurers substitute towards inducing more effort, but given the control optimism, could still do so by giving more insurance as well. Importantly, profit-maximizing contracts are inefficient when evaluating welfare based on the agent's true expected utility; profit-maximizing insurers provide too much insurance in response to baseline-pessimistic

[15] Notice that in the case of heterogeneous preferences, the utility of the outside option is also type-dependent, as analyzed in Jullien (2000) and Jullien et al. (2007)

beliefs, exploiting the overestimation of the value of insurance, and in response to control-pessimistic beliefs, not correcting for the lack of effort provision. These welfare inefficiencies are analyzed in Spinnewijn (2009).[16]

4.3 Binding Incentive Compatibility

I continue by analyzing which incentive compatibility (IC) constraints are binding in equilibrium. While the difference in control optimism determines which IC constraint is binding in the competitive equilibrium, the difference in optimism determines which IC constraint is binding at the monopolistic optimum.

4.3.1 Control-optimism and the zero-profit constraint

The insurer's expected profit is increasing in the effort choice and thus the control-optimistic beliefs of the agent accepting the contract. Since the expected profit from any contract equals zero in a competitive equilibrium, the more control-optimistic type can be offered better terms than the more control-pessimistic type. Hence, by revealed preference, the more control-optimistic type always prefers her full-information contract to the full-information contract offered to the other type. However, the other type may prefer the full-information contract offered to the more control-optimistic type. This implies the following Lemma.

Lemma 4 *The IC constraint is never binding for the more control-optimistic type in a separating competitive equilibrium.*

If the sets of contracts satisfying the zero-profit condition coincide, types with different beliefs may prefer different contracts in this set. This implies that without moral hazard or significant differences in the control beliefs, the full-information contracts would be incentive compatible and the presence of the one type would not distort the contract offered to another type.

4.3.2 Optimism and the participation constraint

An insuree's willingness to accept a contract depends on her risk perception and whether she bears more or less risk than in her outside option. The perceived utility increase from taking the contract (w_i, Δ_i) rather than the outside option (w_0, Δ_0) has to be non-negative,

$$G^i\left((w_i, \Delta_i), (w_0, \Delta_0)\right) \geq 0 \text{ for } i = 1, 2. \tag{12}$$

More optimistic types value an increase in coverage less and require less compensation for a decrease in coverage. Hence, if contracts provide more insurance than the outside option, the insurer needs

[16] For example, increasing the insurance coverage in the full-information equilibrium would increase the agent's true expected utility when

$$\left[\hat{\pi}\left(\hat{e}\left(c\right)\right) - \pi\left(\hat{e}\left(c\right)\right)\right] \geq \left[\pi'\left(\hat{e}\left(c\right)\right) - \hat{\pi}'\left(\hat{e}\left(c\right)\right)\right] \frac{\hat{\pi}'\left(\hat{e}\left(c\right)\right)}{-\hat{\pi}''\left(\hat{e}\left(c\right)\right)}.$$

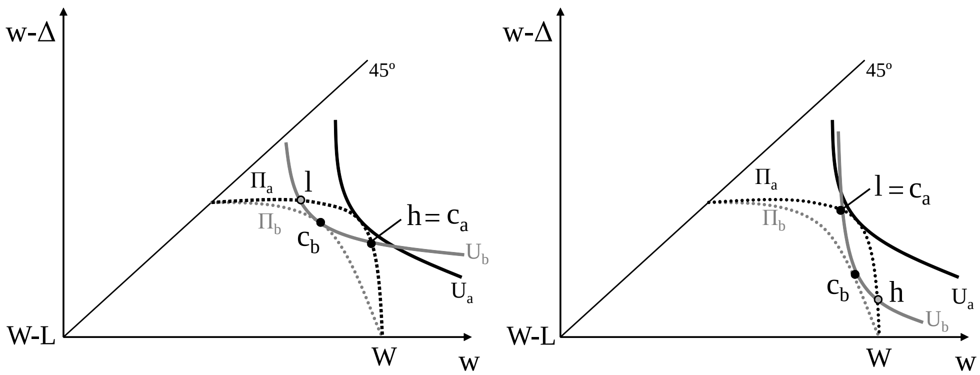

Figure 1: Competitive Equilibrium: Positive vs. Negative Correlation

to provide a more favorable premium to the more optimistic type to buy the insurance contract. However, the more pessimistic type is tempted to take this insurance contract offered at a more favorable premium. In the opposite case, when contracts provides less insurance than the outside option, the more optimistic type is tempted to take the favorable incentive contract offered to the more pessimistic type. Hence, it is the combination of the insurance provided in the outside option together with the difference in optimistic beliefs that determines which incentive compatibility constraint will be binding for the monopolist.

Lemma 5 *In a separating monopolistic optimum with the outside option providing no insurance ($\Delta_0 = L$), the IR constraint is binding for (the more optimistic) type* 1 *and the IC constraint is binding for type* 2*. The reverse is true when the outside option provides full insurance ($\Delta_0 = 0$).*

4.4 Competitive Equilibrium

Since control beliefs are central, I assume that type a is more control-optimistic than type b and characterize the competitive equilibrium depending on whether type a is more optimistic or more pessimistic than type b.

Assumption 2 *Type a is more control-optimistic than type b.*

For the competitive case, I restrict the analysis to insurance contracts with the agent receiving no insurance in her outside option $(w_0, \Delta_0) = (W, W - L)$.[17] The contract offered under full information in the competitive equilibrium to type a would make negative profit if chosen by type b. There are two exceptions. Two contracts always make zero profits, regardless of the beliefs of

[17] As argued before, changing the outside option from no to full insurance has an important impact on the participation constraints, but these play no significant role in the competitive equilibrium. While changing the outside option also changes the zero-profit constraints, contracts making zero-profit on less control-optimistic types will still make positive profits on more control-optimistic types. Hence, the insights from this analysis will naturally generalize for other outside options.

the agent: the full-insurance contract $(W - [1 - \pi(0)] L, 0)$ and the outside option $(W, W - L)$. I show this graphically in Figure 1. The respective zero-profit curves are denoted by Π_a and Π_b. Both curves connect the full-insurance contract (on the 45°-line) and the no-insurance contract (on the x-axis). However, the zero-profit curve for type a connects contracts that provide more consumption in the good and bad state than the zero-profit curve for type b. The indifference curves are represented by U_a and U_b. U_a crosses U_b once by the single-crossing property: from above if type a is more optimistic, from below if type a is more pessimistic. I assume that the indifference curves are convex and the zero-profit curve is concave.[18] The full-information contract is then determined by the tangency point between the zero-profit curve and the indifference curve for the respective type.

The single-crossing property allows me to fully characterize the separating equilibrium, if it exists. For this, I need to define the contracts $\left(w^h, \Delta^h\right)$ and $\left(w^l, \Delta^l\right)$. Both contracts satisfy the zero-profit condition of type a and leave type b indifferent between that contract and her full-information contract $\left(w^*_{c,b}, \Delta^*_{c,b}\right)$. The contract $\left(w^h, \Delta^h\right)$ provides less insurance coverage at a lower insurance premium than $\left(w^l, \Delta^l\right)$. The contracts are indicated by h and l in Figure 1.

Definition 4 *Contracts $\left(w^h, \Delta^h\right)$ and $\left(w^l, \Delta^l\right)$ satisfy*

$$\begin{cases} \left(w^i, \Delta^i\right) \sim_b \left(w^*_{c,b}, \Delta^*_{c,b}\right) \\ W - w^i = \left[1 - \pi\left(\hat{e}_i\left(w^i, \Delta^i\right)\right)\right]\left(L - \Delta^i\right) \end{cases} \quad \text{for } i = h, l,$$

moreover $\left(w^h, \Delta^h\right) \rhd \left(w^l, \Delta^l\right)$.

Proposition 3 characterizes the separating equilibrium when type a is both more optimistic and more control-optimistic than type b.

Proposition 3 *If type a is both more optimistic and more control-optimistic than type b, the contracts in the separating equilibrium equal*

$$\begin{aligned} (w^{**}_{c,a}, \Delta^{**}_{c,a}) &= (w^h, \Delta^h) \\ (w^{**}_{c,b}, \Delta^{**}_{c,b}) &= (w^*_{c,b}, \Delta^*_{c,b}), \end{aligned}$$

unless $(w^*_{c,b}, \Delta^*_{c,b}) \succeq_b (w^*_{c,a}, \Delta^*_{c,a})$, in which case the full-information contracts are separating. In the former case, the separating equilibrium exists only if $\kappa_a < \bar{\kappa}$ for some $\bar{\kappa} \in (0, 1)$.[19]

If the full-information equilibrium is separating, the presence of type b has no impact on the equilibrium contract offered to type a. For instance, if type a is very optimistic, she will not

[18] This assumes a sufficiently smooth response in effort when changing the terms of the insurance contract. Notice that with effort fixed, the zero-profit function is linear with slope $\pi(e) / [1 - \pi(e)]$ - increasing in e - and the concavity of the indifference curves is simply implied by the concavity of the Bernouilli function $u(\cdot)$. However, with binary effort choices, like in De Meza and Webb (2001), the zero-profit function jumps, while the indifference curves are kinked at the consumption pairs for which the agent switches her effort level.

[19] The cut-off value $\bar{\kappa}$ is defined in the proof of the Proposition. Notice that the existence of this upper bound depends crucially on the single-crossing property, as discussed in Chassagnon and Chiappori (2005).

be offered any insurance, regardless of the presence of a pessimistic type. If the full-information equilibrium is not separating, it is because the more control-pessimistic type b prefers type a's full-information contract, by Lemma 4. In that case, contracts h and l are natural alternatives, since type b is exactly indifferent between her full-information contract $(w^*_{c,b}, \Delta^*_{c,b})$ and these contracts. Since the optimistic type a prefers the high deductible contract $\left(w^h, \Delta^h\right)$ to the low deductible contract $\left(w^l, \Delta^l\right)$ by the single-crossing property, contract h will be offered in equilibrium. The two types are thus separated by decreasing the insurance coverage for the optimistic type a, i.e., $\left(w^h, \Delta^h\right) \rhd (w^*_{c,a}, \Delta^*_{c,a})$. I show this graphically in the left panel of Figure 1. Since the optimistic type is also more control-optimistic, the correlation between the ex-post risk and insurance coverage is positive in equilibrium, in line with Corollary 1. Like in the original analysis by Rothschild and Stiglitz (1976), no pooling equilibrium can exist due to the single-crossing property. Moreover, a separating equilibrium exists only if no pooling contract making non-negative profits would be preferred by both types. This is avoided when the share of the (low-risk) type a is sufficiently small.

Type a's contract is distorted in the opposite direction if she is more control-optimistic, but at the same time more pessimistic than type b.

Proposition 4 *If type a is more control-optimistic, but more pessimistic than type b, the contracts in the separating equilibrium equal*

$$\begin{aligned}(w^{**}_{c,a}, \Delta^{**}_{c,a}) &= (w^l, \Delta^l)\\ (w^{**}_{c,b}, \Delta^{**}_{c,b}) &= (w^*_{c,b}, \Delta^*_{c,b}),\end{aligned}$$

unless $(w^*_{c,b}, \Delta^*_{c,b}) \succeq_b (w^*_{c,a}, \Delta^*_{c,a})$, in which case the full-information equilibrium is separating. In the former case, the separating equilibrium exists only if $\kappa_a < \bar{\kappa}$ for some $\bar{\kappa} \in (0, 1)$.[20]

The separating equilibrium contract for the control-pessimistic type b is still $(w^*_{c,b}, \Delta^*_{c,b})$. However, the pessimistic type a now prefers $\left(w^l, \Delta^l\right)$ to $\left(w^h, \Delta^h\right)$, as the single-crossing property is reversed. The two types are separated by increasing the insurance coverage for type a, i.e., $\left(w^l, \Delta^l\right) \lhd (w^*_{c,b}, \Delta^*_{c,b})$. Not surprisingly, the direction of the distortion is opposite to the previous case where type b is more pessimistic.[21] I show this graphically in the right panel of Figure 1. Type a now receives less insurance than type b. Since type b is more control-pessimistic than type a, the correlation between ex post risk and insurance coverage may be negative, in line with Corollary 2.[22]

[20] The cut-off value $\bar{\kappa}$ is again defined in the proof of the Proposition.

[21] Notice that the insurance coverage of the control-optimistic type is non-monotonic in the baseline beliefs of type b. While the contract $\left(w^h, \Delta^h\right)$ or $\left(w^l, \Delta^l\right)$ is offered depending on whether type b is more pessimistic or optimistic, in the more extreme cases that type b is either much more optimistic or much more pessimistic than type a, type a is offered the full-information contract $\left(w^*_{c,a}, \Delta^*_{c,a}\right)$ in equilibrium.

[22] For the negative correlation property to hold, the more optimistic type b needs to be optimistic as well, in line with Chiappori *et al.* (2006).

4.5 Monopolistic Optimum

I characterize the monopolistic optimum depending on the insurance provided to the agent in the outside option and again use the assumption that type 1 is more optimistic than type 2.

If the outside option provides no insurance, the monopolist needs to pay a rent to type 2 to induce her not to choose the contract offered to type 1. Since type 1 needs to be compensated less for an increase in risk, the monopolist reduces the rent paid to type 2 by imposing more risk on type 1. Still, the separating contract offered to type 2 trades off the provision of insurance and incentives efficiently from the insurer's perspective. That is, the contract satisfies the full-information condition (11) in Proposition 2. If the outside option provides full insurance, type 1 needs to be paid a rent in order to be separated from type 2. The monopolist now imposes less risk on type 2 to reduce the rent paid to type 1. The separating contract offered to type 1 now satisfies the full-information condition (11).

Proposition 5 *If both types are served, but separated in the monopolistic optimum, the optimal contracts satisfy*

$$\begin{aligned}\left(w_{m,1}^{**}, \Delta_{m,1}^{**}\right) \rhd \left(w_{m,1}^{*}, \Delta_{m,1}^{*}\right) \ \textit{when}\ \Delta_0 &= L,\\ \left(w_{m,2}^{**}, \Delta_{m,2}^{**}\right) \lhd \left(w_{m,2}^{*}, \Delta_{m,2}^{*}\right) \ \textit{when}\ \Delta_0 &= 0.\end{aligned}$$

In both cases, the contract offered to the other type satisfies condition (11).

Notice that the control beliefs do not affect the direction of the distortions in the separating optimum, but will affect which type is exerting more effort in line with Corollaries 1 and 2. The Proposition applies when the monopolist serves both types and is willing to pay a rent to separate them. First, the monopolist will exclude a type if the expected profit from this type does not compensate for the rents paid to the other type to discourage contract switching. In that case, the other type is proposed the full-information contract. The expected profit depends on the share of this type, while the rent paid to the other type does not. Hence, if $\Delta_0 = L$ and the share κ_1 is sufficiently small, type 1 is excluded. If $\Delta_0 = 0$ and the share κ_2 is sufficiently small, type 2 is excluded. Second, the monopolist prefers to pool the two types when it is too costly to separate them. Incentive compatibility requires that type 1 is given less insurance than type 2, but the full-information contracts may provide more insurance to type 1 than type 2. The distortions required to separate types may be substantial and thus not profitable.[23]

The results can be generalized for outside opportunities providing partial insurance.

Proposition 6 *If type* 1 *is more optimistic than type* 2 *and the full-information problem is convex,*

[23] Analyzing heterogeneity in risk preferences with monetary cost of effort, Jullien *et al.* (2007) show how pooling can be optimal in the 'non-responsive case'. The 'non-responsive case' applies when the solution to the profit-maximizing problem constrained to a binding participation constraint for one type and a binding incentive compatibility constraint for the other type gives more insurance to type 1 than to type 2.

the optimal contracts with types separated satisfy

$$\begin{aligned}\left(w_{m,1}^{**},\Delta_{m,1}^{**}\right) &\rhd \left(w_{m,1}^{*},\Delta_{m,1}^{*}\right) \text{ if } \Delta_0 > \max\left\{\Delta_1^*,\Delta_2^*\right\},\\ \left(w_{m,2}^{**},\Delta_{m,2}^{**}\right) &\lhd \left(w_{m,2}^{*},\Delta_{m,2}^{*}\right) \text{ if } \Delta_0 < \min\left\{\Delta_1^*,\Delta_2^*\right\}.\end{aligned}$$

The contract offered to the other type satisfies condition (11) in both cases. Also, if $\Delta_2^* < \Delta_0 < \Delta_1^*$, $\left(w_{m,i}^{**},\Delta_{m,i}^{**}\right) = \left(w_{m,i}^{*},\Delta_{m,i}^{*}\right)$ *for* $i = 1, 2$.

The possibility arises that the full-information contract provides less insurance than the outside option to type 1, but more insurance than the outside option to type 2, $\Delta_2^* < \Delta_0 < \Delta_1^*$. In this case, the full-information contracts are incentive compatible. Otherwise, the conclusions are similar to the extreme cases with no and full insurance in the outside option. If both contracts are incentive contracts (i.e. $\Delta_i > \Delta_0$), the more optimistic type receives a rent and the contract for the more pessimistic type is distorted towards less incentives. If both contracts are insurance contracts (i.e. $\Delta_i < \Delta_0$), the more pessimistic type receives a rent and the contract of the more optimistic type is distorted towards less insurance.

5 Welfare Analysis

Insurers distort the terms of the contracts to screen otherwise indistinguishable optimistic and pessimistic types. The presence of a type with particular beliefs thus imposes an externality on a type with different beliefs. This inefficiency affects welfare and may justify government interventions, like in the analysis by Rothschild and Stiglitz (1976). The screening distortions depend on the differences in the willingness to pay and the expected cost, regardless of the heterogeneity underlying these differences. However, the evaluation of the screening distortions is different in the presence of biased beliefs. The screening distortion aggravates or reduces the contract distortion due to the agent's bias in beliefs.

Unbiased Beliefs I first focus on the informational externality that biased agents ($\hat{\pi} \neq \pi$) impose on unbiased agents ($\hat{\pi} = \pi$). In a competitive equilibrium, the insurance contract for the more control-optimistic type may be distorted compared to the full-information contract by Lemma 4, while the contract offered to the more control-pessimistic type is unchanged. The welfare of the unbiased agent may thus be lower, but only in the presence of a more control-pessimistic type.

Corollary 3 *In a separating competitive equilibrium, an agent with unbiased beliefs never gains from the presence of an agent with biased beliefs and may strictly lose only if that agent is control-pessimistic.*

Heterogeneity in risk perceptions induces screening by the insurers and this decreases welfare. The heterogeneity thus increases the gain from a government intervention that regulates insurance coverage towards the full-information contract. This is different from the policy recommendation in the model considered by Sandroni and Squintani (2007). Central to their analysis is that some types of agents perceive their risk to be the same, although their true risk is different. In particular, some optimistic high risk types perceive their risk to be low and thus are necessarily pooled with the low risk types.[24] This increases the insurance premium for the low risk types in the competitive equilibrium, such that an insurance mandate may actually decrease their welfare. Hence, an insurance mandate would not be Pareto improving as in the RS model. In the model considered here, the heterogeneity in perceptions increases rather than decreases the dispersion in the perceived risk relative to the dispersion in the actual risk. This creates screening distortions and thus increases the scope for government intervention.

When facing a monopolistic insurer, the externality that a biased agent exerts on an unbiased agent is different. The contracts offered to both types may change when the insurees' perceptions are not observable. If for one type the IC constraint is binding, that type receives a rent not to switch to the other type's contract and thus ends up strictly better off than with the full-information contract. The contract for the second type is distorted compared to the full-information contract to reduce the rent paid to the first type, but the second type will still be indifferent about switching to the outside option and thus ends up as well off as with the full-information contract. The cases in which an agent with unbiased beliefs is paid a rent follow immediately from Proposition 6.

Corollary 4 *In a separating monopolistic optimum, an agent with unbiased beliefs gains from the presence of a pessimistic agent when offered incentive contracts and from the presence of an optimistic agent when offered insurance contracts.*

Biased Beliefs The previous welfare implications from the screening of types are not specific to heterogeneity in risk perceptions. Similar conclusions would be drawn when the screening distortions are driven by a different source of heterogeneity. However, the distinction becomes crucial when considering the contract distortions for an agent with biased beliefs. As shown in Spinnewijn (2009) and briefly discussed in Section 4.2, the full-information contract does not maximize the true expected utility of an agent with biased beliefs and is therefore no longer the natural benchmark for welfare analysis. In contrast with the positive characterization of screening contracts, the absolute bias in beliefs becomes relevant to evaluate the welfare implications. This has three important implications.

First, screening distortions are more or less costly depending on whether they increase or decrease the initial distortion due to the biases in beliefs. For example, if the full-information contract already provides too little insurance to the more optimistic type, the screening distortion towards even less insurance comes at a higher cost.[25] In particular, the same screening distortion towards less insurance may successfully separate the more optimistic type in the case of heterogeneous

[24] Notice that the pooled low-risk and optimistic high-risk types in Sandroni and Squintani (2007) correspond to a medium-risk type who believes she is low-risk. Hence, from a positive perspective, the case considered in Sandroni and Squintani (2007) is captured in the more general setup here.

[25] Similarly, the screening distortion towards more insurance for the more pessimistic type is more costly if the full-information contract already provides too much insurance.

perceptions and the more risk-tolerant type in the case of heterogeneous preferences. However, while the anticipated utility loss from a reduction in insurance coverage may be perceived to be the same by both types, the true expected utility loss will be higher for the optimistic than for the risk-tolerant type. The consequence is that a welfare gain from an insurance mandate is higher in the case of heterogeneous perceptions than in the case of heterogeneous preferences.

Second, the sign of the correlation between risk and insurance is no longer sufficient to evaluate whether insurance coverage is inefficiently low or high in a competitive market. As argued by de Meza and Webb (2001) and Einav *et al.* (2010b), markets provide too much insurance when the low risk types buy less or are less willing to buy insurance due to heterogeneous preferences. However, the same 'advantageous selection' may arise in a market with heterogeneous risk perceptions, as described in Corollary 2. In this case, the market provides indeed more insurance than the full-information contract, but for an agent with biased beliefs, this is no longer sufficient to conclude that the market provides too much insurance.[26]

Finally, the analysis illustrates two important misconceptions regarding the importance of competition and the presence of rational agents for the welfare of boundedly rational agents, as discussed in Spiegler (2011). First, the presence of the unbiased agent does not necessarily exert a positive externality on the biased agent, as the screening distortion may aggravate the contract distortion due to the bias in beliefs. Second, competition is not effective in eliminating the exploitation of the biased agent and may make the distortions even worse.

6 Conclusion

People have very different perceptions about the likelihood and the controllability of the risks they face.This article analyzes how insurance companies separate agents who differ in their perceptions. Interestingly, while individuals with heterogeneous abilities or risks, but identical perceptions cannot be separated, individuals with different perceptions can be separated with a menu of screening contracts, even when their true abilities or risks are the same. The analysis shows that the differences in risk and control perceptions drive the design of the equilibrium contracts. Heterogeneous perceptions may affect the insurance value for the insuree and the insurance cost for the insurer in different directions. This determines the equilibrium correlation between risk and insurance and causes differential responses by a monopolistic insurer and competing insurers to differences in beliefs.

Heterogeneity in beliefs can explain why contracts offer very little insurance in some markets (e.g., health insurance, car insurance) and provide no incentives at all in other markets, although a small financial incentive suffices to induce effort (e.g., no-limit contracts on rented cars mileage, cell phone usage). However, unobserved differences in risk could have similar positive implications. Heterogeneity in beliefs can also explain why the correlation between risk and insurance coverage

[26] Spinnewijn (2012) explicitly relates the cost of adverse selection to the different sources of heterogeneity, but considers inefficient pricing rather than insurance distortions and ignores moral hazard.

is positive in some, but negative in other insurance markets. Again, unobserved differences in preferences, positively or negative correlated with risks, could explain this as well. Direct evidence is lacking, but the literature has mostly attributed unexplained heterogeneity in choices to heterogeneity in preferences. Understanding the underlying heterogeneity is, however, crucial for evaluating welfare and designing policies. Identifying to what extent results are driven by heterogeneity in perceptions or heterogeneity in preferences is clearly a challenge (Manski 2004). Potential approaches to quantify the importance of heterogeneity in beliefs rather than preferences or risks are, either by eliciting expectations directly through surveys (Spinnewijn 2009) or by relating certain behavior to biases in perceptions (Köszegi and Rabin 2007; Landier and Thesmar, 2009) or to experiences that are likely to have changed perceptions (Malmendier *et al.*, 2011). Extending these empirical approaches to the analysis of insurance markets seems a promising avenue for future research.

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Appendix: Proofs

Proof of Lemma 1

Consider the first case with $\hat{\pi}_i(e) \geq \hat{\pi}_j(e)$ and $\hat{\pi}'_i(e) \geq \hat{\pi}'_j(e)$ for any e. The second condition implies $\hat{e}_i(c) \geq \hat{e}_j(c)$ for any c, since $\hat{\pi}''(e) < 0$, and thus $\hat{\pi}_i(\hat{e}_i(c)) \geq \hat{\pi}_i(\hat{e}_j(c))$ for any c, since $\hat{\pi}'(e) > 0$. The first condition then implies $\hat{\pi}_i(\hat{e}_i(c)) \geq \hat{\pi}_j(\hat{e}_j(c))$ for any c.

Consider the second case $\hat{\pi}_i(e) \geq \hat{\pi}_j(e)$ and $\hat{\pi}'_i(e) \leq \hat{\pi}'_j(e)$ for any e. For $c = (w, 0)$, $\hat{e}_i(c) = \hat{e}_j(c)$ and thus $\hat{\pi}_i(\hat{e}_i(c)) \geq \hat{\pi}_j(\hat{e}_j(c))$. It immediately follows that a set $C = \{(w, \Delta) \,|\Delta \in [0, L], w \in [\Delta, W]\} \supseteq \{(w, 0) \,|w \in W\}$ exists for which agent i is more optimistic than agent j. More generally, this is the case for any C containing only contracts c such that

$$\hat{\pi}_i(0) + \int_0^{\hat{e}_i(c)} \hat{\pi}'_i(e)\, de \geq \hat{\pi}_j(0) + \int_0^{\hat{e}_j(c)} \hat{\pi}'_j(e)\, de. \quad \square$$

Proof of Lemma 2

The marginal rate of substitution (MRS) between Δ and w equals

$$\begin{aligned} \left.\frac{d\Delta}{dw}\right|_{\hat{\pi}_i} &= \frac{\hat{\pi}_i(\hat{e}_i(c))\, u'(w) + [1 - \hat{\pi}_i(\hat{e}_i(c))]\, u'(w - \Delta)}{[1 - \hat{\pi}_i(\hat{e}_i(c))]\, u'(w - \Delta)} \\ &= \frac{\hat{\pi}_i(\hat{e}_i(c))}{1 - \hat{\pi}_i(\hat{e}_i(c))} \frac{u'(w)}{u'(w - \Delta)} + 1. \end{aligned}$$

Since $\frac{u'(w)}{u'(w-\Delta)} > 0$, the MRS is increasing in $\hat{\pi}_i(\hat{e}_i(c))$. The lemma follows, since $\hat{\pi}_1(\hat{e}_1(c)) \geq \hat{\pi}_2(\hat{e}_2(c))$ for any c.$\square$

Proof of Lemma 3

Given $\widetilde{w}(\Delta) = w_j + (\Delta - \Delta_j)\frac{w_i - w_j}{\Delta_i - \Delta_j}$,

$$\begin{aligned} &U^i_w(\widetilde{w}(\Delta), \Delta)\, \widetilde{w}'(\Delta) + U^i_\Delta(\widetilde{w}(\Delta), \Delta) = \\ &\quad \hat{\pi}_i(\hat{e}_i(\widetilde{w}(\Delta), \Delta))\, u'(\widetilde{w}(\Delta))\, \tfrac{w_1 - w_2}{\Delta_1 - \Delta_2} - [1 - \hat{\pi}_i(\hat{e}_i(\widetilde{w}(\Delta), \Delta))]\, u'(\widetilde{w}(\Delta) - \Delta)\, \tfrac{(w_2 - \Delta_2) - (w_1 - \Delta_1)}{\Delta_1 - \Delta_2}. \end{aligned}$$

Hence,

$$\begin{aligned} &G^1((w_1, \Delta_1), (w_2, \Delta_2)) - G^2((w_1, \Delta_1), (w_2, \Delta_2)) = \\ &\quad \int_{\Delta_2}^{\Delta_1} \{[\hat{\pi}_1(\hat{e}_1(\widetilde{w}(\Delta), \Delta)) - \hat{\pi}_2(\hat{e}_2(\widetilde{w}(\Delta), \Delta))] \times \\ &\quad\quad [u'(\widetilde{w}(\Delta))\, \tfrac{w_1 - w_2}{\Delta_1 - \Delta_2} + u'(\widetilde{w}(\Delta) - \Delta)\, \tfrac{(w_2 - \Delta_2) - (w_1 - \Delta_1)}{\Delta_1 - \Delta_2}]\} d\Delta. \end{aligned}$$

Since $\frac{w_1 - w_2}{\Delta_1 - \Delta_2} > 0$ and $\frac{(w_2 - \Delta_2) - (w_1 - \Delta_1)}{\Delta_1 - \Delta_2} > 0$, given $(w_1, \Delta_1) \rhd (w_2, \Delta_2)$, the second factor in the integral between squared brackets is greater than zero. Since the integration is from Δ_2 to Δ_1 with

$\Delta_1 > \Delta_2$,

$$G^1((w_1,\Delta_1),(w_2,\Delta_2)) - G^2((w_1,\Delta_1),(w_2,\Delta_2)) > 0,$$

if $\hat{\pi}_1(\hat{e}_1(\widetilde{w}(\Delta),\Delta)) \geq \hat{\pi}_2(\hat{e}_2(\widetilde{w}(\Delta),\Delta))$ for all $\Delta \in [\Delta_2,\Delta_1]$ and $\hat{\pi}_1(\hat{e}_1(\widetilde{w}(\Delta),\Delta)) > \hat{\pi}_2(\hat{e}_2(\widetilde{w}(\Delta),\Delta))$ for some $\Delta \in [\Delta_2,\Delta_1]$.□

Proof of Proposition 1

Assume, by contradiction, that $(w_2,\Delta_2) \rhd (w_1,\Delta_1)$, although type 1 is more optimistic than type 2. In that case,

$$G^1((w_2,\Delta_2),(w_1,\Delta_1)) \geq G^2((w_2,\Delta_2),(w_1,\Delta_1)) \geq 0,$$

by Lemma 3. But this contradicts $G^1((w_1,\Delta_1),(w_2,\Delta_2)) \geq 0$, since $G^1(x,y) = -G^1(y,x)$.□

Proof of Proposition 2

This proof follows Spinnewijn (2009) using consumption in the good and bad state as controls; $c_g = w$ and $c_b = w - \Delta$. The competing insurers and the monopolist solve

$$\max_{c_g,c_b} \hat{\pi}(e)[u(c_g) - u(c_b)] + u(c_b) - e$$

such that

$$\begin{aligned} \hat{\pi}'(e)[u(c_g) - u(c_b)] &= 1 \\ \pi(e)(w_0 - c_g) + (1-\pi(e))(w_0 - \Delta_0 - c_b) &= \bar{\Pi}, \end{aligned}$$

where $\bar{\Pi}$ equals 0 in the competitive equilibrium and $\bar{\Pi}$ equals the expected profits such that $U(w,\Delta) = U(w_0,\Delta_0)$ in the monopolistic optimum.

Using the IC constraint and the profit constraint, we can implicitly define $\tilde{e}(c_b)$ and $\tilde{c}_g(c_b)$ as a function of c_b. From implicit differentiation of the profit constraint, we find

$$\pi'(\tilde{e}(c_b))\tilde{e}'(c_b)[(w_0 - c_g) - (w_0 - \Delta_0 - c_b)]dc_b - [1 - \pi(\tilde{e}(c_b))]dc_b = -\pi(\tilde{e}(c_b))dc_g.$$

Hence,

$$\begin{aligned} \tilde{c}_g'(c_b) &= \frac{1-\pi(\tilde{e}(c_b))}{\pi(\tilde{e}(c_b))} - \frac{\pi'(\tilde{e}(c_b))}{\pi(\tilde{e}(c_b))}\tilde{e}'(c_b)[\Delta_0 - (c_g - c_b)] \\ &= \frac{1-\pi(\tilde{e}(c_b))}{\pi(\tilde{e}(c_b))}\left[1 + \varepsilon_{1-\pi(\tilde{e}(c_b)),c_b}\frac{\Delta_0 - (c_g - c_b)}{c_b}\right], \end{aligned}$$

where $\varepsilon_{1-\pi(\tilde{e}(c_b)),c_b} = \frac{d[1-\pi(\tilde{e}(c_b))]}{dc_b}\frac{c_b}{1-\pi(\tilde{e}(c_b))}$. Substituting the implicit functions in the objective function, the first order condition of the unconstrained problem equals

$$\hat{\pi}(\tilde{e}(c_b))u'(\tilde{c}_g(c_b))\tilde{c}_g'(c_b) + [1 - \hat{\pi}(\tilde{e}(c_b))]u'(c_b) = 0.$$

The effect through effort is of second order by the envelope condition. Substituting for $\tilde{c}_g'(c_b)$, we find

$$\frac{1-\hat{\pi}\left(\tilde{e}\left(c_b\right)\right)}{\hat{\pi}\left(\tilde{e}\left(c_b\right)\right)}\frac{u'\left(c_b\right)}{u'\left(\tilde{c}_g\left(c_b\right)\right)}=\frac{1-\pi\left(\tilde{e}\left(c_b\right)\right)}{\pi\left(\tilde{e}\left(c_b\right)\right)}\left[1+\varepsilon_{1-\pi(\tilde{e}(c_b)),c_b}\frac{\Delta_0-\left(c_g-c_b\right)}{c_b}\right]$$

and thus

$$\frac{\frac{1-\hat{\pi}(\tilde{e}(c_b))}{1-\pi(\tilde{e}(c_b))}\frac{\pi(\tilde{e}(c_b))}{\hat{\pi}(\tilde{e}(c_b))}u'\left(c_b\right)-u'\left(\tilde{c}_g\left(c_b\right)\right)}{u'\left(\tilde{c}_g\left(c_b\right)\right)}=\varepsilon_{1-\pi(\tilde{e}(c_b)),c_b}\frac{\Delta_0-\left(c_g-c_b\right)}{c_b}.$$

With $c_b = w - \Delta$ and $c_g = w$, the expression in the proposition immediately follows.□

Proof of Lemma 4

If type 1 is (strictly) more control-optimistic than type 2, $\Pi\left(w,\Delta;1\right) > \Pi\left(w,\Delta;2\right)$. In any separating equilibrium,

$$\Pi\left(w_1,\Delta_1;1\right)=\Pi\left(w_2,\Delta_2;2\right)=0.$$

Assume by contradiction that $\left(w_2,\Delta_2\right)\sim_1\left(w_1,\Delta_1\right)$ in a separating equilibrium, but $\Pi\left(w_2,\Delta_2;2\right)=0$. Hence, if preferences are continuous and the single-crossing property is satisfied, an insurer can change the contract $\left(w_2,\Delta_2\right)$ to $\left(w_2',\Delta_2'\right)$ such that $\Pi\left(w_2',\Delta_2';2\right)$, but $\left(w_2',\Delta_2'\right)\succ_1\left(w_1,\Delta_1\right)$ and $\Pi\left(w_2',\Delta_2';1\right)>0$. This is a profitable deviation.□

Proof of Lemma 5

If $\Delta_0 = L$, then $\Delta_1 < \Delta_0$ for any interior solution and thus $G^2\left(\left(w_1,\Delta_1\right),\left(w_0,\Delta_0\right)\right) > G^1\left(\left(w_1,\Delta_1\right),\left(w_0,\Delta_0\right)\right)$. Since not both the IR and IC constraint can be slack for a type, this implies

$$G^2\left(\left(w_2,\Delta_2\right),\left(w_0,\Delta_0\right)\right)=G^2\left(\left(w_1,\Delta_1\right),\left(w_0,\Delta_0\right)\right)>0.$$

Moreover, since $\left(w_1,\Delta_1\right)\rhd\left(w_2,\Delta_2\right)$ if the contracts are separating, $G^1\left(\left(w_1,\Delta_1\right),\left(w_0,\Delta_0\right)\right) > G^1\left(\left(w_2,\Delta_2\right),\left(w_0,\Delta_0\right)\right)$ by Lemma 3. This in turn implies that $G^1\left(\left(w_1,\Delta_1\right),\left(w_0,\Delta_0\right)\right)=0$.

If $\Delta_0 = 0$, the implications are exactly opposite. Since $\Delta_2 > \Delta_0$ for any interior solution, $G^1\left(\left(w_2,\Delta_2\right),\left(w_0,\Delta_0\right)\right) > G^2\left(\left(w_2,\Delta_2\right),\left(w_0,\Delta_0\right)\right)$. This now implies

$$G^1\left(\left(w_1,\Delta_1\right),\left(w_0,\Delta_0\right)\right)=G^1\left(\left(w_2,\Delta_2\right),\left(w_0,\Delta_0\right)\right)>0.$$

Moreover, since $\left(w_1,\Delta_1\right)\rhd\left(w_2,\Delta_2\right)$ if the contracts are separating, $G^2\left(\left(w_2,\Delta_2\right),\left(w_0,\Delta_0\right)\right) > G^2\left(\left(w_1,\Delta_1\right),\left(w_0,\Delta_0\right)\right)$ by Lemma 3. This now implies that $G^2\left(\left(w_2,\Delta_2\right),\left(w_0,\Delta_0\right)\right)=0$.□

Proof of Proposition 3

If the full-information contracts are incentive compatible, the competitive equilibrium coincides with the full-information equilibrium. If the full-information contracts are not incentive compatible, then the IC constraint of type a is still not binding in a separating equilibrium, by Lemma 4. Hence, if a separating equilibrium exists, type b's equilibrium contract equals the full-information contract. The actuarial contract that maximizes the perceived expected utility of type a, but is not strictly preferred by type b to its full-information contract is $\left(w^h,\Delta^h\right)$. Since $\left(w^h,\Delta^h\right)\sim_b\left(w^l,\Delta^l\right)$ and $\left(w^h,\Delta^h\right)\rhd\left(w^l,\Delta^l\right)$, $\left(w^h,\Delta^h\right)\succ_a\left(w^l,\Delta^l\right)$ by Lemma 3. Then, when the indifference curves are convex and the zero-profit curve is concave, $\left(w^h, \Delta^h\right) \succ_a (w, \Delta)$ for any contract on type a's zero-profit curve for which $(w, \Delta) \rhd \left(w^h, \Delta^h\right)$ or $(w, \Delta) \lhd \left(w^l, \Delta^l\right)$. Hence, if a separating equilibrium exists, type a's equilibrium contract equals $\left(w^h, \Delta^h\right)$. The zero-profit curve when both types accept a contract is in between Π_a and Π_b. Define by $\bar{\kappa}$ the share of type a such that the 'pooling' zero-profit curve is tangent to the indifference curve for type a through $\left(w^h, \Delta^h\right)$. When Π_a and Π_b only coincide at full insurance and no insurance, $\bar{\kappa} \in (0,1)$ by the single-crossing property. For any $\kappa < \bar{\kappa}$, no contract on the pooling zero-profit curve is preferred by type a to $\left(w^h, \Delta^h\right)$. Hence, a separating equilibrium exists and equals $\left\{\left(w^*_{c,b}, \Delta^*_{c,b}\right), \left(w^h, \Delta^h\right)\right\}$.□

Proof of Proposition 4

The proof is analogue to the proof of Proposition 3. Since the single-crossing property is reversed now, $\left(w^h, \Delta^h\right) \sim_b \left(w^l, \Delta^l\right)$ and $\left(w^h, \Delta^h\right) \rhd \left(w^l, \Delta^l\right)$ imply that $\left(w^l, \Delta^l\right) \succ_a \left(w^h, \Delta^h\right)$. Type a's equilibrium contract equals $\left(w^l, \Delta^l\right)$. The upper bound $\bar{\kappa}$ now equals the share of type a for which the 'pooling' zero-profit curve is tangent to the indifference curve for type a through $\left(w^l, \Delta^l\right)$.□

Proof of Proposition 5

The monopolist solves

$$\max \kappa \left\{w_0 - w_1 - \left[1 - \pi\left(\hat{e}_1\left(c_1\right)\right)\right]\left(\Delta_0 - \Delta_1\right)\right\} + (1-\kappa)\left\{w_0 - w_2 - \left[1 - \pi\left(\hat{e}_2\left(c_2\right)\right)\right]\left(\Delta_0 - \Delta_2\right)\right\}$$

such that

$$\begin{aligned} G_1\left(\left(w_1, \Delta_1\right), \left(w_0, \Delta_0\right)\right) &= \max\left\{0, G_1\left(\left(w_2, \Delta_2\right), \left(w_0, \Delta_0\right)\right)\right\} \\ G_2\left(\left(w_2, \Delta_2\right), \left(w_0, \Delta_0\right)\right) &= \max\left\{0, G_2\left(\left(w_1, \Delta_1\right), \left(w_0, \Delta_0\right)\right)\right\}. \end{aligned}$$

If $\Delta_0 = L$, then by Lemma 5, the binding IC/IR constraints for a separating optimum imply

$$\begin{aligned} G_1\left(\left(w_1, \Delta_1\right), \left(w_0, \Delta_0\right)\right) &= 0 \\ G_2\left(\left(w_2, \Delta_2\right), \left(w_1, \Delta_1\right)\right) &= 0. \end{aligned}$$

Notice that the optimum is only separating if the solution of this problem satisfies monotonicity, i.e. $(w_1, \Delta_1) \rhd (w_2, \Delta_2)$.

Assume $(w_1, \Delta_1) \lhd (w_1^*, \Delta_1^*)$, then $G_2\left(\left(w_1, \Delta_1\right), \left(w_1^*, \Delta_1^*\right)\right) > 0$ by Lemma 3, so the utility rent paid to type 2 implied by the binding IC constraint is higher if type 1 receives (w_1, Δ_1) rather than (w_1^*, Δ_1^*). Since the profit made on type 1 is higher in (w_1^*, Δ_1^*) as well, (w_1, Δ_1) can never be optimal. Hence, $(w_1, \Delta_1) \trianglerighteq (w_1^*, \Delta_1^*)$.

Assume $(w_1, \Delta_1) = (w_1^*, \Delta_1^*)$, one can find a contract (w_1', Δ_1') such that $(w_1', \Delta_1') \sim_1 (w_1^*, \Delta_1^*)$ and $(w_1', \Delta_1') \rhd (w_1^*, \Delta_1^*)$ (and thus, $(w_1', \Delta_1') \prec_2 (w_1^*, \Delta_1^*)$), but sufficiently close to (w_1^*, Δ_1^*) such that the loss in profit on type 1 is of second order, but the reduction in the rent paid to type 2 is of first order. Hence, $(w_1, \Delta_1) > (w_1^*, \Delta_1^*)$.

Finally, if the optimum is separating, the incentive compatibility constraint for type 1 is slack.

Hence, the problem becomes separable for type 2. The contract (w_2, Δ_2) solves the full-information problem with type 2's outside opportunity equal to (w_1, Δ_1). Clearly, this contract satisfies condition (11).

If $\Delta_0 = 0$, then by Lemma 5, the IC/IR constraints for a separating optimum simplify to

$$\begin{aligned} G_1\left(\left(w_1, \Delta_1\right), \left(w_2, \Delta_2\right)\right) &= 0 \\ G_2\left(\left(w_2, \Delta_2\right), \left(w_0, \Delta_0\right)\right) &= 0. \end{aligned}$$

Again, the optimum is only separating if the solution of this problem satisfies monotonicity, i.e. $(w_1, \Delta_1) \rhd (w_2, \Delta_2)$. The steps are now exactly opposite to the previous case.

Assume $(w_2, \Delta_2) \rhd (w_2^*, \Delta_2^*)$, then $G_1\left(\left(w_2, \Delta_2\right), \left(w_2^*, \Delta_2^*\right)\right) > 0$ by Lemma 3, so the utility rent paid to type 1 implied by the binding IC constraint is higher if type 2 receives (w_2, Δ_2) rather than (w_2^*, Δ_2^*). Since the profit made on type 2 is higher in (w_2^*, Δ_2^*) as well, (w_2, Δ_2) can never be optimal. Hence, $(w_2, \Delta_2) \unlhd (w_2^*, \Delta_2^*)$.

Assume $(w_2, \Delta_2) = (w_2^*, \Delta_2^*)$, one can find a contract (w_2', Δ_2') such that $(w_2', \Delta_2') \sim_2 (w_2^*, \Delta_2^*)$ and $(w_2', \Delta_2') \lhd (w_2^*, \Delta_2^*)$ (and thus, $(w_2', \Delta_2') \prec_1 (w_2^*, \Delta_2^*)$), but sufficiently close to (w_2^*, Δ_2^*) such that the loss in profit on type 2 is of second order, but the reduction in the rent paid to type 1 is of first order. Hence, $(w_2, \Delta_2) \lhd (w_2^*, \Delta_2^*)$.

Finally, if the optimum is separating, the incentive compatibility constraint for type 2 is slack. Hence, the contract (w_1, Δ_1) solves the full-information problem with type 1's outside opportunity equal to (w_2, Δ_2) and thus satisfies condition (11).$\square$

Proof of Proposition 6

If $\Delta_0 < \min\{\Delta_1^*, \Delta_2^*\}$, then any separating menu with contract $(w_2, \Delta_2) \lhd (w_0, \Delta_0)$ such that $(w_2, \Delta_2) \succeq_2 (w_0, \Delta_0)$, is dominated by offering the menu $\{(w_1^*, \Delta_1^*), (w_0, \Delta_0)\}$, which is incentive compatible by Lemma 3. The profits on type 1 are maximized. Moreover, if the full-information problem is convex and $(w_2^*, \Delta_2^*) \rhd (w_0, \Delta_0)$, the insurer makes higher profits when offering (w_0, Δ_0) than when offering $(w_2, \Delta_2) \lhd (w_0, \Delta_0)$. Any separating menu with contract $(w_1, \Delta_1) \lhd (w_0, \Delta_0)$ is also dominated by $\{(w_1^*, \Delta_1^*), (w_0, \Delta_0)\}$, since incentive compatibility requires $(w_2, \Delta_2) \lhd (w_1, \Delta_1)$ and offering a menu with contract $(w_2, \Delta_2) \lhd (w_0, \Delta_0)$ is dominated. As a consequence, both (w_1, Δ_1) and (w_2, Δ_2) need to provide less insurance than the outside option (w_0, Δ_0) in order to be optimal. If $\Delta_0 > \max\{\Delta_1^*, \Delta_2^*\}$, the argument is exactly the same, mutatis mutandum. In this case, both (w_1, Δ_1) and (w_2, Δ_2) need to provide more insurance than the outside option (w_0, Δ_0) in order to be optimal.

Now, if either

$$(w_1, \Delta_1) \rhd (w_0, \Delta_0) \text{ and } (w_2, \Delta_2) \rhd (w_0, \Delta_0)$$

or

$$(w_1, \Delta_1) \lhd (w_0, \Delta_0) \text{ and } (w_2, \Delta_2) \lhd (w_0, \Delta_0),$$

Lemma 5 and exactly the same argument as in Proposition 5 apply. This proves the first part of the proposition.

If $\Delta_2^* < \Delta_0 < \Delta_1^*$, $G_1\left(\left(w_1^*, \Delta_1^*\right), \left(w_0, \Delta_0\right)\right) = 0$ implies that $G_2\left(\left(w_1^*, \Delta_1^*\right), \left(w_0, \Delta_0\right)\right) < 0$ and $G_2\left(\left(w_2^*, \Delta_2^*\right), \left(w_0^*, \Delta_0^*\right)\right) = 0$ implies that $G_1\left(\left(w_2^*, \Delta_2^*\right), \left(w_0^*, \Delta_0^*\right)\right) < 0$, by Lemma 3. Hence, both $G_1\left(\left(w_1^*, \Delta_1^*\right), \left(w_2^*, \Delta_2^*\right)\right) \geq 0$ and $G_2\left(\left(w_2^*, \Delta_2^*\right), \left(w_1^*, \Delta_1^*\right)\right) \geq 0$. The incentive compatibility constraints are satisfied for the full-information contracts.$\square$